A NOVEL MODEL FOR ACOUSTIC THROUGH ICE DATA TRANSFER

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A NOVEL MODEL FOR ACOUSTIC THROUGH ICE DATA TRANSFER

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[To the students of the Georgia Institute of Technology]
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I would like to thank mi cafécito con leche in the mornings ❤️ and all of the wonderful friends and mentors I have met during my long, wonderful, difficult, and rewarding time at Georgia Tech. Thank you, for some of the fondest memories of my life.
# TABLE OF CONTENTS

ACKNOWLEDGEMENTS iv  
LIST OF TABLES vii  
LIST OF FIGURES viii  
LIST OF SYMBOLS AND ABBREVIATIONS x  
SUMMARY xi  

CHAPTER 1. Introduction 1  

CHAPTER 2. BACKGROUND 4  
2.1 Spreading 5  
2.2 Absorption Loss 6  
2.3 Scattering Loss 6  
2.4 Ice Types 7  

CHAPTER 3. Review of published literature 9  
3.1 Simple Acoustic Signaling 9  
3.1.1 Price 1993, 2006 9  
3.1.2 SPATS: The South Pole Acoustic Test Set-up 10  
3.2 Data transfer through ice 11  
3.2.1 Lishman et al., 2013 11  
3.2.2 Han et al., 2019 12  

CHAPTER 4. methods 14  
4.1 Strategy for Addressing the Problem 14  
4.2 Acoustic Attenuation Theory 14  
4.2.1 Spreading Loss 14  
4.2.2 Absorption Loss 15  
4.2.3 Scattering Loss 18  
4.2.4 Fractures 24  
4.2.5 Seawater loss 25  
4.2.6 Combining absorption and attenuation 25  
4.3 Computer Modeling 27  
4.3.1 GNU Radio: A Software Architecture for use of the Acoustic Channel Model 27  
4.3.2 Design and Incorporation of the Acoustic Model with GNU Radio 27  

CHAPTER 5. comparative benchmarks 31  
5.1 Sea Ice 31  
5.1.1 Analysis 33  
5.2 Glacier Ice 36
# LIST OF TABLES

| Table 1 | Elastic Constants, i.e. stiffness coefficients, for hexagonal ice measured by Gannon, et al., (1987) for glacier ice. | 20 |
| Table 2 | Summary of mechanisms of attenuation, with corresponding attenuation coefficients. | 26 |
| Table 3 | Difference between acoustic attenuation data and SPATS data for attenuation in nepers/km. | 44 |
| Table 4 | Estimated minimum and maximum bounding values for $\tau_m$ and effects on overall attenuation at 325 m depth | 46 |
| Table 5 | Bounded minimum and maximum values for $\tau_m$ for absorption by impurities (325 m). | 47 |
| Table 6 | Bounded minimum and maximum values for grain boundary size and corresponding effect on attenuation. | 48 |
| Table 7 | Link budget comparison for three different media: ice, water, and air. | 53 |
| Table 8 | Standard acoustic equations for calculating Link budget values, including Power Level at transmitter and receiver, equivalent to $P_{tx}$ and $P_{rx}$, respectively. | 55 |
| Table 9 | Suggested/estimated link budget for an acoustic through-ice communication link, see supplementary materials for intermediate calculations and device specifications. | 56 |
LIST OF FIGURES

Figure 1 Visualization of inverse square law. Applied to acoustic point sources, one can visualize why intensity decreases by approximately 6dB for each doubling of distance for a point source (NASA, 2009).

Figure 2 Kuroiwa (1964) Measurements of relaxation time in varying species of ice, experimentally determined.

Figure 3 Structure of a plane of ice (left) depicting the four types of crystal lattice defects causing proton movement between bond sites (Price, 2006). Additional image of hexagonal structure of ice, in plane view (right) (Weeks & Ackley, 1982).

Figure 4 Images of grain boundaries within (A,B) Greenland glacier ice, (C) Antarctic iceberg ice, and (D) Le Comte glacier ice (left) (Kuroiwa, 1964b); initial disks formed during freezing of seawater at Thule, Greenland (d~1mm) (Weeks & Ackley, 1982).

Figure 5 Profiles of physical properties of the Antarctic ice sheet at Byrd station, from studies of deep-drill cores (2164 m) by Gow (1971).

Figure 6 GNU Radio blocks for each modelled mechanism of attenuation. Absorption attenuation equations are all contained within the absorption block (left). Scattering attenuation equations were broken into individual blocks (middle) for simplicity. “sa0” stands for “saltwater attenuation” (right).

Figure 7 Simple GNU Radio flowgraph modelling a 10 kHz signal frequency (leftmost block) through 1 km of glacier ice. The “QT GUI Time Sink” block plots calculated attenuation.

Figure 8 GNU Radio flowgraph for an FSK modulated signal with acoustic channel model applied. See code for details.

Figure 9 Reported acoustic attenuation coefficient calculated for transmission through sea ice at Tanquary, Fiord, Ellesmere Island (Langleben, 1969).

Figure 10 Comparison of acoustic model attenuation data for prescribed sea-ice environment with in-situ sea-ice data from Langleben.

Figure 11 Data plotted alongside acoustic model and Langleben data, for sea ice at T= - 6 °C (Wen, et al., 1991).
Figure 12  Acoustic model attenuation calculated for vertical transmission through sea ice, compared to Wen, et al., (1991) data for matching set-up.

Figure 13  Westphal (1965) acoustic attenuation data, with attenuation coefficient, α, in [nepers/m] and the x-axis in kHz.

Figure 14  Pierson acoustic model data vs Westphal data, recreation of in-ice environment of Blue Glacier, noted for 3 distinctive layers of ice within the ice column.

Figure 15  Plane view of ice at Blue Glacier, depicting all three types of ice in the ice column and both filled and unfilled crevasses. From the ice surface to the rock margin was measured to be approximately 300 m at the location of the acoustic test.

Figure 16  Acoustic model with layering implementation, all three ice types accounted for: coarse bubbly, coarse clear, and fine.

Figure 17  Comparison of two data sets for through-ice acoustic attenuation, both recorded at glaciers with potential for significant cracking or fractures present.

Figure 18  SPATS (South Pole Acoustic Test Set up) attenuation values [nepers/km] in deep glacial ice in Antarctica.

Figure 19  Acoustic model and SPATS data for deep Antarctic glacier ice at the South Pole.

Figure 20  Measurements of mean crystal size in deep drill cores (2146 m) at Byrd Station, Little America V, Maudheim, Antarctica, and ‘Site 2’ in Greenland.

Figure 21  GNU Radio flowgraph for a BPSK modulation scheme with acoustic channel model applied.

Figure 22  Recovered, demodulated text input from BPSK flowgraph including acoustic channel model for a modelled sea-ice environment.

Figure 23  Estimates of channel capacity calculated using the Shannon Hartley theorem.
LIST OF SYMBOLS AND ABBREVIATIONS

\( A_0 \) Initial Amplitude [m]
\( A \) Amplitude [m]
\( c \) Speed of Sound [m/s]
\( f \) Frequency [Hz]
\( f_{center} \) Center frequency [Hz]
\( \text{BW} \) Bandwidth [Hz]
\( P \) Pressure [Pa]
\( I \) Intensity [W/m\(^2\)]
\( \text{SPL} \) Sound pressure level [dB]
\( \lambda \) Wavelength [m]
\( \alpha \) Attenuation coefficient
\( r \) Distance [m]
\( \text{TL} \) Transmission loss [dB]
\( \text{TVR} \) Transmitting Voltage Response
\( \omega \) Radial frequency [rads]
\( t \) Time [s]
\( i \) Imaginary number \((\sqrt{-1})\)
SUMMARY

Acoustic communication links have been suggested as potentially optimal through-ice data transfer solutions for a variety of harsh environment investigations. Acoustic communication systems ("comms") have extensive heritage in underwater applications, proving capability of acoustic data transfer links through complex dispersive environments. Furthermore, acoustic comms have attestation as reliable, low-power science sensing and environmental characterization devices through ice at Earth’s polar regions, resulting in a wealth of data on the effects of the in-ice environment on acoustic signals. To understand and optimize a through-ice communication system for through-ice channels, it is necessary to understand and model the effects of the acoustic environment on such a system.

Due to the budding nature of acoustics through-ice research, no link budgets or channel models currently exist for estimating and optimizing hardware and software needs. To fill this knowledge gap, I created a novel, adaptable model for acoustic attenuation and acoustic data transfer through ice leveraging analytical and empirical data from acoustic characterization experiments in-situ over a large range of frequencies (f~1-100 kHz). Using GNU Radio, the acoustic model is incorporated into a signal chain for which input parameters include the target ice environment and signal type. Paired with a link budget, optimal system parameters may be estimated. The work serves as an important first step in designing potential communications architectures for a number of applications, including polar environments, ocean environments, and planetary environments (Lawrence et al., 2023; McCarthy et al., 2019; Schmidt et al., 2023; Snyder et al., 2023).
Acoustic communication through ice is a growing field of study with potential as both a data transfer and environmental characterization solution. Platforms for under-ice scientific investigation at Earth’s polar regions, like AUVs, ROVs, and glider surveys, are valuable tools for collecting ground-truth data; but in the GPS-denied under-ice environment, localization (positioning), and ultimately through-ice data return are hurdles to long-term, wide-range, untethered vehicle exploration (Lawrence et al., 2023; Schmidt et al., 2023; Snyder et al., 2023). Moreover, deployed sensors within and below the ice similarly suffer from limited data return and the logistics of this data transfer presently limit how and where sensors can be deployed. Meanwhile, missions envisioned by NASA to ocean worlds like Jupiter’s moon Europa require auxiliary through-ice communication links robust to an in-ice environment expected to contain potential hazards to communication links: brine pockets, salt flats, “slushy” ice (ice/water mixtures, or “mixed media”), etc. For both of these cases, wireless acoustic links have been suggested as a potential solution for through-ice data transfer (Bar-Cohen et al., 2021; McCarthy et al., 2019; Pierson, 2022).

Acoustic communications have extensive heritage in long-range (>10 km) underwater communications, with acoustic methods being leveraged for decades as environmental characterization devices both in and out of ice (Bentley & Kohnen, 1976; Stojanovic, 2007; Westphal, 1965). As such, acoustic transducer technology is mature in low-temperature, high-pressure, and low-power applications (IceCube Collaboration et al., 2017; Sánchez et al., 2012; Semburg, 2009). Moreover, transmission losses (TL)
experienced by propagating acoustic signals—passive or active—provide information about the propagation medium (Kinsler & Frey, 2000). The inherent ability for environmental characterization and proven robustness to ice/water mixtures are what make acoustic links an ideal system to leverage for through-ice communications.

Due to the budding nature of acoustics through-ice research, no link budgets or channel models exist for estimating and optimizing the hardware and software (data transfer/signaling techniques) needs of an acoustic through-ice system with respect to parameters of the target in-ice environment. This knowledge gap hinders technological advancement and investigative use-cases, necessitating optimization and range/data capability to occur in-situ—which, in harsh polar or remote environments, may become a costly, resource-intensive, and potentially iterative endeavor. To fill this knowledge gap, I have created a novel model for acoustic attenuation and signaling through ice. The acoustic model incorporates analytical and empirical equations, and data over a wide range of frequencies (~1-100kHz) for transmission loss parameters from in-situ acoustic characterization investigations. Using GNU Radio, an open-source flow-graph style software, the acoustic model for transmission loss is incorporated into a realistic signal chain. The user may input parameters of the target ice environment (temperature, density, ice crystal size, air bubble size and concentration, salt crystal size) and transducer parameters (range, propagation direction, frequency/frequency range, amplitude), as well as signal type (pulse, sweep, FSK/BPSK modulation scheme). As the signal passes through the acoustic channel model, the frequency of each sample is detected and attenuated accordingly; the model outputs the attenuated (and demodulated, if selected) signal at the desired receiver distance, and can be used to find total attenuation of the intended channel.
When paired with a link budget, the user may estimate optimal hardware parameters for the target acoustic environment. Here, I describe the creation of the acoustic channel model, benchmark its outputs with in-situ data and discuss novel use-cases, and finally discuss use of the model and link budget for future communications links.
CHAPTER 2. BACKGROUND

Acoustic waves are mechanical disturbances propagating through an elastic medium. As they propagate, these waves interact with the transmission media. Losses governed by the environmental composition and structure, such as absorption or scattering, will affect the received signal waveform and strength in predictable ways.

Acoustic losses have been studied extensively both in-situ and in isolation. This understanding of transmission loss allows investigators to model the structure of the environment with respect to the type of acoustic source. Models of the expected acoustic environment are then used to both increase the fidelity of environmental recreation, and allow for optimization of hardware and software (signal processing) techniques in the future. This application can be seen in seismic investigations (AlAli & Anifowose, 2022). Furthermore, advances in model fidelity create a positive feedback loop: good models improve understanding gained from in-situ data, better understanding of in-situ data improves future data collection and increases our understanding of the environment, increased understanding of the environment improves modeling efforts, and so on and so forth.

In order to understand how to design a through-ice communication system it is therefore necessary to understand and model environmental losses. To begin this discussion, it is first necessary to establish a few fundamental equations for acoustic waves which will be used throughout the text, and give a brief overview of acoustic losses (i.e.: attenuation, dispersion) – a major topic within the thesis. The main modes of acoustic losses pertinent to this work are Spreading, Absorption, and Scattering.
\[ p(r, t) = A_0 e^{i\omega(t-\frac{r}{c})} \]  

\[ I = \frac{\text{Power [Watts]}}{4\pi r^2} \]  

\[ c = \frac{f}{\lambda} \]

2.1 Spreading

As an acoustic source creates sound waves, the waves propagate out from the source in all directions (sources can also produce directional sound depending on design of the source and source housing). As the wavefront expands in all directions, intensity will decrease with the inverse of distance as demonstrated in Figure 1, i.e., the inverse square law, Equation (2). This type of energy loss of a wave is a purely geometric property and can be entirely constrained knowing the travel distance of the acoustic wave (and in the case of directional transducers, parameters of the housing which are given specifications).

![Figure 1. Visualization of inverse square law. Applied to acoustic point sources, one can visualize why intensity decreases by approximately 6dB for each doubling of distance for a point source. (NASA, 2009)](image-url)
2.2 Absorption Loss

Due to the mechanical nature of acoustic waves, as a wave passes through a medium heat conduction and material viscosity will contribute to “the conversion of acoustic energy into thermal energy,” referred to as absorptive losses (Kinsler & Frey, 2000). Absorptive losses affect acoustic signal amplitude (i.e., pressure) as:

\[ p(r, t) = A_0 e^{\alpha r} \]  \hspace{1cm} (4)

Where \( p \) is the pressure of the received acoustic signal, \( A_0 \) is the initial signal amplitude, \( \alpha \) is the attenuation coefficient, and \( r \) is range, i.e.: distance, between the source and receiver. The attenuation coefficient is often an analytically or empirically derived value. For pure, or homogenous materials, it is possible attenuation will only be caused by absorption, but in ice defects cause additional losses due to scattering.

2.3 Scattering Loss

Inhomogeneities within ice cause scattering losses, defined as “the reradiation of incident acoustic energy out of the incident beam” (Kinsler & Frey, 2000). For the purposes of my work, scattering loss refers to attenuation due to interface and/or boundary interactions; in ice this may be caused by salt crystals, refraction/velocity changes at boundaries of the ice crystal lattice, air bubbles, or cracks/fractures within the ice. Scattering losses and absorptive losses may be summed, contributing to a total attenuation coefficient:
\[ a_{total} = \sum a_{scattering} + a_{absorption} \] (5)

The in-ice environment can vary significantly with respect to location, type of ice (fresh/saltwater), and method of formation.

2.4 Ice Types

It is important to note that the structure of ice, whether formed from freshwater or saltwater, will greatly affect the attenuative effects experienced by a propagating acoustic wave.

Freshwater ices, found in glaciers, are formed through compaction of snow, and may vary in structures and fabric with depth due to the effects of pressure and stress over time. Other freshwater ice can be formed by accretion of melt water at the base of glaciers, or the freezing of freshwater lakes and rivers. For the purposes of the model and thesis, I focus on glacier ice.

Saltwater ice, on the other hand, forms through accumulation and bonding of frazil ice (small “needle-like” crystals a few millimeters in size that form on the surface of seawater) and/or by accretion onto the base of existing ice, called marine or congelation ice. The thickness of sea-ice sheets may vary with the seasons, depending on location. Marine ice layers on the bottom of ice shelves on Earth may vary from a few centimeters to hundreds of meters thickness (Wolfenbarger et al., 2021). Furthermore, while slowly freezing saltwater rejects salts, sea-ice sheets may contain brine pockets or regions of accumulation of rejected salt (Buffo et al., 2020).
On Earth, most polar environments will be comprised of one or the other of these ices, but in some ice shelves, glacier ice overlays marine ice. However, on ocean worlds like Europa, we expect complex layering of fresher and saltier ice to be found according to the formation of the original ice shell and thermal and geological evolution (Buffo et al., 2020; Chivers et al., 2021). Thus, designing a model that can incorporate these factors is important for future exploration of both the Earth and other ocean worlds.
CHAPTER 3. REVIEW OF PUBLISHED LITERATURE

While acoustic signaling through ice has a decades-long history as a tool for environmental characterization, acoustic signaling through ice with the purpose of data transfer, specifically, is a field in its infancy. At the time of writing, only one case of data transfer through ice was found in the literature, and one case of simple signaling with the purpose of assessing capability of data transfer. Also detailed here are two investigations of simple acoustic signaling through ice which are widely cited, and foundational for their work in attenuation; these investigations were instrumental in the creation of the acoustic channel model.

3.1 Simple Acoustic Signaling

3.1.1 Price 1993, 2006

Investigation of acoustic attenuation through ice by Price (1993, 2006) focused on quantifying the utility of acoustic transducers within Antarctic glacier ice for passive environmental characterization by neutrinos passing through the ice. Extensive analysis of the in-ice environment culminated in a comprehensive overview of “mechanisms of absorption and scattering” in polycrystalline ice. Analytical attenuation coefficient equations were determined for each mechanism of attenuation outlined; resulting data from these equations was benchmarked against empirical data. Price noted: “Acoustic attenuation in ice increases with temperature, impurity content, crystal size and degree of randomness of crystal orientation. …elastic constants are anisotropic, and longitudinal and shear waves propagate with different speeds in different crystallographic directions…”
(1993). Through further laboratory and in-situ experimentation, Price later predicted the propagating range of longitudinal acoustic signals in South Pole ice to be 9±3 km for frequencies above $f \sim 100$ Hz, corresponding to an attenuation coefficient of $\alpha \sim 1.2412$ dB/km (2006). The analytical derivations for TL-specific attenuation coefficients were incorporated in my own modeling work.

3.1.2 SPATS: The South Pole Acoustic Test Set-up

The work of Price helped inform the SPATS project (in association with the Ice Cube Collaboration), the first experiment of its kind and an enormous undertaking, to provide in-situ data on acoustic attenuation and sound speed in deep glacial ice in Antarctica. It is important to note that this project transmitted acoustic signals using two different sensors: SPATS sensors (main instrument) and two HADES sensors (Abbasi et al., 2010; Semburg, 2009). The former instrumentation contains three piezoelectric transducers in a steel housing, coupled to the housing wall with a load screw, while the latter sensor consists of a ring-shaped piezoelectric element (and associated necessary electrical components) encased in an ice-impedance-matched polyurethane resin.

Transducers were embedded into the ice at depths of 190–500 m and emitted signals between 1-100 kHz. Transmission distances ranged from ~0.5-1 km (for set-up details see Abbasi et al., 2011). Experimental results yielded an overall attenuation coefficient of $\alpha = 27.7952 \pm 4.9510$ km$^{-1}$, significantly higher than predicted by Price, but notably, this value was obtained only for transverse signal propagation, and included coupling losses and possible absorption due to cracks in the propagation path, expected at this depth within
the ice column. The discrepancy was also theorized to be due to disproportionate Rayleigh scattering and linear dislocation damping.

3.2 Data transfer through ice

3.2.1 Lishman et al., 2013

Following the above reported data, Lishman sought to further investigate the utility of an acoustic link (versus an RF link) for vertical transmission through both water and ice—from a subglacial lake to the glacier surface.

Lishman first conducted his own tests through glacier ice at Leverett Glacier, West Greenland—acoustic transducers were placed in boreholes <1 m in depth which were then filled with water, with (horizontal) transmission ranging from 5-25 m. Based on the SPATS study, an estimated optimal transmission frequency range of 10-30 kHz was used; the transducer and input power were also selected for their comparability. It was noted that this near-surface acoustic channel was vastly different than that of SPATS project, with higher temperatures, significantly larger ice crystal grain sizes, and excessive fractures oriented perpendicular to the direction of signal propagation. These differences contributed to large attenuation coefficient values: \( \alpha = 0.35 \text{ dB/m at } 14 \text{ kHz to } \alpha = 1.06 \text{ dB/m at } 26 \text{ kHz.} \)

Owing to these differences, Lishman went on to derive a link budget using the attenuation coefficient found by SPATS—expected to better match the attenuation environment of the theoretical link’s transmission path (vertical transmission from subglacial lake to glacier surface). Lishman concludes an acoustic link “may be a useful technique for through-ice communication in situations where there is too much water
present to permit effective RF communications,” estimating that a 1 W acoustic transmitter could feasibly transmit through 1 km of ice, and 100 m of water. The novelty of this study stems from its intent to incorporate a link budget into calculations of signal range, and varying link budget parameters (except TL) to inform hardware needs (power, frequency).

3.2.2 Han et al., 2019

The only true investigation of acoustic data transfer through both ice and ice/water mixtures in the literature comes from Han et al [2019]. In this experiment, a flextensional transducer with $f_{\text{center}} = 750$ Hz, and $BW = 500$ Hz, transmitted a BPSK modulated signal (500 bits of training sequence, followed by 3000 bits of data at 250 symbols/sec) to an array of 5 ice-mounted geophones, with 0.5 m spacing, at a 100 m distance; signals were sampled at 48 kHz. With the transducer placed at 2 m below the ice, the resulting transmission was through approximately 80 m of water and 20 m of ice.

The authors conducted a comprehensive analysis of the in-ice environment. Using a least squares algorithm, the authors estimated channel impulse response, concluding that individual transmission channels were very temporally coherent (coherency coefficient > 0.98). Estimating signal path amplitudes with a Rice distribution and calculating mean-to-standard deviation ratios (see Yang, 2012 for details), the authors also concluded that “the amplitudes of individual paths in cross-ice acoustic communication are very deterministic and the random components are very small.” Despite this, clear differences were found across channels with respect to amplitude and channel structure; for this reason, the authors used PTR (passive time reversal) and M-DFE (multi-channel decision feedback equalizer).
techniques to improve communication performance—these techniques are commonly used in underwater communication applications.

Output signal-to-noise ratio (OSNR) for each geophone (G1-G5) after equalization was: 14.2 dB, 15 dB, 18.5 dB, 17.8 dB, and 18.8 dB. Equalization of all five geophones results in an OSNR of 27 dB. Overall, the study achieved robust data transmission, concluding cross-ice data transfer is feasible over 100 m, with future work of communication over “relatively long distance” cited. Though it is important to note that while very promising, the limitations of this study with respect to my work lie in the use of geophones for signal reception (measuring 3-dimensional particle motion, a vector quantity) as opposed to hydrophones or more ideally, piezoelectric transducers (measure received signal pressure, a scalar quantity).
CHAPTER 4. METHODS

4.1 Strategy for Addressing the Problem

Extensive literature review covering attenuation through ice has demonstrated both the capability of acoustic signaling in ice and the highly varied nature of transmission loss within the in-ice environment. While transmission loss has been characterized extensively in various ice environments, when I began this work, no models existed for calculating transmission loss based on expected or targeted environmental parameters, and thus, it was not possible to use a link budget (see Chapter 4.4) to determine feasibility or capability of an acoustic link through ice without first experimentally determining TL in-situ. By collecting and integrating acoustic coefficient equations found in the literature I was able to create a novel, adaptable model for acoustic attenuation through ice, i.e., an acoustic channel model. Details on the theory contributing to creation of the model for acoustic attenuation are below.

4.2 Acoustic Attenuation Theory

4.2.1 Spreading Loss

Recall that acoustic attenuation through ice (for our purposes) consists of three main modes of loss: spreading, absorption, and scattering. Geometric losses, or spreading losses, can be fully constrained knowing the directionality of the transmitter and/or receiver (this is a decibel value describing the fraction of energy traveling in the desired direction) and travel distance of the signal. Geometric losses for isotropic sources are modeled as:
\[ \alpha_{\text{spreading}} = 20 \log_{10}(r) \text{ [dB]} \]  

Transmitter directivity index (DI) is provided, if applicable, for a given transducer, and may be calculated for a receiver as:

\[ DI_{tx} = \text{given} \]  

\[ DI_{rx} = \frac{Q}{\rho \cdot v} \]  

Where \( \rho \) is density of ice, \( Q \) is the quality factor, and velocity, \( v \), is the speed of propagation of the sound wave in the medium, which is dependent on direction of travel of the pressure wave, transverse or longitudinal (Abbasi, et al., 2010):

\[ v_{\text{transverse}} = v_t = -2.3 \cdot (T \degree C) + 3795 \left[ \frac{m}{s} \right] \]  

\[ v_{\text{longitudinal}} = v_l = -1.2 \cdot (T \degree C) + 1915 \left[ \frac{m}{s} \right] \]  

Note the dependence of velocity on temperature (in \( \degree C \)), and the significant variance in propagation speed as a function of direction of travel. This is due to the nature of ice crystal growth, where both sea ice and freshwater ice expresses a higher velocity in the growth direction (Abbasi et al., 2011; McCammon & McDaniel, 1985; Vogt et al., 2008; Wen et al., 1991).

4.2.2 Absorption Loss

In the literature, absorption losses were identified as a consequence of relaxation processes, defined as the “return of a perturbed system to equilibrium” (“Relaxation
When ice is disturbed by an acoustic wave, relaxation processes are mechanical in nature (i.e.: mechanical damping) and have an associated relaxation time, $\tau_m$, and logarithmic decrement, $\delta$ (Kuroiwa, 1964a, 1964b). From these values, an attenuation coefficient, $\alpha$, can be found, Price (1993):

$$\tau_m = \tau_0 \cdot e^{(U/kT)}$$

$$\delta = \frac{2\delta_{\text{max}}\omega \tau_m}{1 + \omega^2 \tau_m^2}$$

$$\alpha_{\text{absorption}} = \frac{\delta f}{v}$$

Note that $\delta_{\text{max}}$ and $v$ may change with respect to propagation direction of the acoustic wave, and that both $\delta$ and $\alpha$ are functions of frequency. Kuroiwa (1964a, 1964b) studied relaxation processes in ice grown from both liquid (“melt”) and glacier ice extensively, demonstrating that several processes may contribute to relaxation time as a function of temperature and frequency of excitation of ice (related to activation energy, $U$): Grain boundary sliding, proton reorientation, impurities, dislocation damping, thermoelastic loss, and phonon relaxation. Experiments by Clee et al. (1969) supported Kuroiwa’s research, and found that in glacier ice, grain boundary sliding dominated between 0-10 ºC. A brief description of each follows:

**Grain boundary sliding:** a consequence of internal friction, affects transverse waves more than longitudinal waves.
Proton reorientation: a rotation of the dipole moment of H$_2$O or other molecules to a preferred orientation through movement of defects in ice; defects are present at concentrations determined by temperature.

Impurities: refers to inclusions such as HF, HCl, NaCl, NaOH, etc., these effects were studied in-lab; it was determined these impurities concentrate at grain boundaries and reduce the activation energy for relaxation by grain boundary sliding. These impurities are typically noted by their percent weight within the medium; due to their effect on activation energy, $U$, changes in impurity type or percentage will ultimately effect $\tau_m$, $\delta_{max}$, $\delta$, and therefore, attenuation.

Dislocation damping: occurs due to a slight increase in temperature in the direction of compression of the travelling acoustic wave.

Thermoelastic loss and Phonon relaxation: will not be discussed further, as they were determined to occur at frequencies higher than applicable here, $f > 100$ kHz.

Due to a lack of data ($\tau_0$, $\delta_m$, $U$), the acoustic model currently takes in estimates of $\tau_m$ and therefore $\delta_m$, using the empirical data from Kuroiwa whose work contains a reasonable spread of values for different species of freshwater ice, Figure 2.
Figure 2. Kuroiwa (1964) Measurements of relaxation time in varying species of ice, experimentally determined.

Total attenuation for absorptive losses are modeled as:

\[ \alpha_{\text{absorption}} = \alpha_{\text{gbs}} + \alpha_{\text{proton reorientation}} + \alpha_{\text{impurities}} + \alpha_{\text{dislocations}} \]  

(14)

4.2.3 Scattering Loss

While absorption attenuation can be thought of as an atomic-scale form of energy loss, attenuation due to scattering can be represented by more macroscopic elements. In the literature, scattering was attributed to: Grain Boundary Scattering, Air bubbles, Salts, and fractures.

4.2.3.1 Grain boundary Scattering (GBS)
Grain boundaries scatter acoustic waves in ice due to abrupt changes in wave speed at crystal faces. Note that a “grain” of ice refers to one crystal of ice, and grain boundary size refers to the radius, $a$, or diameter, $d$, of the ice crystal. Virtually all ices occurring on Earth are hexagonal in structure and form in a crystalline lattice, Figure 3.

Figure 3. Structure of a plane of ice (left) depicting the four types of crystal lattice defects causing proton movement between bond sites$^1$ (Price, 2006). Additional image of hexagonal structure of ice, in plane view (right) (Weeks & Ackley, 1982)$^2$.

Crystal size and hexagonal shape affect acoustic attenuation as a function of five elastic constants; the elastic constant values used in this acoustic attenuation model are kept constant for simplicity, Table 1, and were obtained empirically by Gagnon et al. (1987) for glacier ice with a random distribution of c-axes. While this may prove inaccurate for deep glacier ice with a strong c-axis alignment, for most other environments it provides a conservative attenuation estimate.

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$^1$ Associated with absorption mechanisms, Price (2006) see study for more details
$^2$ Numbers (i.e.: [0011]) are related to crystallographic and/or c-axis orientation
Table 1. Elastic Constants, i.e. stiffness coefficients, for hexagonal ice measured by Gannon, et al., (1987) for glacier ice.

<table>
<thead>
<tr>
<th>Elastic constants, glacier ice at -55°C and ~0.09 kbar</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>--</td>
<td>kbar</td>
</tr>
<tr>
<td>C_{11}</td>
<td>147.44</td>
</tr>
<tr>
<td>C_{12}</td>
<td>75.11</td>
</tr>
<tr>
<td>C_{13}</td>
<td>60.64</td>
</tr>
<tr>
<td>C_{33}</td>
<td>158.94</td>
</tr>
<tr>
<td>C_{44}</td>
<td>31.85</td>
</tr>
</tbody>
</table>

Price (1993) determined that scattering in ice due to acoustic waves must be considered with respect to the ratio of grain radius, $a$, to acoustic wavelength, $\lambda$. This results in three distinct scattering regimes: Rayleigh, Stochastic, and Geometric.

4.2.3.1.1 Rayleigh: $\frac{\lambda}{4\pi a} > 1$

Using the attenuation coefficient values obtained by Merkulov, Price (1993) noted that in this regime attenuation follows Rayleigh law, where scattering is occurring by grains much smaller than the acoustic wavelengths, scaling roughly as $a \propto a_1^3 f^4$, with attenuation coefficients and associated coefficients $a_1, b_1, a_2, b_2, Y, \eta, \chi$:

\[
\alpha_{Rayleigh,L} = \frac{16\pi^4 a^3 f^4}{1350\rho^2 v_L^3} \left( a_1 \frac{b_1}{v_L^5} + \frac{b_1}{v_T^5} \right) 
\]

(15)

\[
\alpha_{Rayleigh,T} = \frac{16\pi^4 a^3 f^4}{1350\rho^2 v_T^3} \left( a_2 \frac{b_2}{v_L^5} + \frac{b_2}{v_T^5} \right) 
\]

(16)

\[
a_1 = \frac{88}{15} Y^2 + 40\chi^2 + 96\eta^2 + \frac{80}{3} \chi Y + \frac{128}{3} Y \eta + \frac{320}{3} \chi \eta 
\]

(17)

\[
b_1 = \frac{82}{15} Y^2 + 30\chi^2 + \frac{272}{3} \eta^2 + 20\chi Y + \frac{112}{3} Y \eta + 80\chi 2 
\]

(18)
\[ a_2 = \frac{41}{15} Y^2 + 15 \chi^2 + \frac{136}{3} \eta^2 + 10 \chi Y + \frac{56}{3} Y \eta + 40 \chi \eta \]  

(19)

\[ b_2 = \frac{8}{5} Y^2 + 28 \eta^2 + 8 Y \eta \]  

(20)

\[ \gamma = c_{11} + c_{33} - 2(c_{12} + c_{44}); \quad \chi = c_{13} - c_{12}; \quad \eta = c_{44} + \frac{c_{12} - c_{11}}{2} \]  

(21)

4.2.3.1.2 Stochastic: \( 0.5 < \frac{\lambda}{4 \pi a} < 1 \)

In the stochastic (a.k.a. “Mie”) regime we have a deviation from Rayleigh scattering; here, the wavelength of the acoustic wave is on the order of the size of the ice grains. Attenuation is proportional to \( \alpha \propto f^2 \) and for the purposes of this model is the same in both longitudinal and transverse directions. For the frequencies of interest to this acoustic model (\( f \sim 1-100 \) kHz) Stochastic scattering becomes important for large grain sizes coupled with high frequencies (ex. \( f \geq 100 \) kHz \& \( a \geq 0.3 \) cm, or \( f > 10 \) kHz \& \( a \geq 3 \) cm; the latter case can be seen in experiments by Langleben (1969) at Blue Glacier).

\[ \alpha_{\text{Stochastic, L}} = \left( \frac{32 \pi^2 a f^2}{1575 \nu_L^6 b^2} \right) (7Y^2 + 35\chi^2 + 140\eta^2 + 30\chi Y + 140\chi \eta) \]  

(22)

4.2.3.1.3 Geometric: \( \frac{\lambda}{4 \pi a} < 0.5 \)

In this regime the acoustic waves are much smaller compared to the size of the ice grains, and attenuation becomes similar to a diffusive process. For frequencies of interest to this
modeling effort, typically geometric scattering is not applicable, but in included for completeness:

\[
\alpha_{\text{geometric}} = \langle R \rangle = \sqrt{\frac{\langle (c'_{11} - \langle c'_{11} \rangle)^2 \rangle}{4\langle c'_{11} \rangle^2}}
\]  

(23)

Figure 4. Images of grain boundaries within (A,B) Greenland glacier ice, (C) Antarctic iceberg ice, and (D) Le Comte glacier ice (left) (Kuroiwa, 1964b); initial disks formed during freezing of seawater at Thule, Greenland (d~1mm) (Weeks & Ackley, 1982).

4.2.3.2 Air Bubbles
Attenuation due to air bubbles is dependent on air bubble size and concentration within the crystalline lattice. At Byrd Station Antarctica, Gow and Williamson (1976) found an upper limit of air bubble concentration, $n_0$, of approximately $n_0 \sim 220$ bubbles/cm$^3$, with a diameter, $d \sim 0.001$ cm, Figure 5. These dimensions allow the scattering effects of air bubbles on acoustic attenuation to be modeled following Rayleigh law (see Price, 1993, for details). Note that air bubble concentration in sea ice was not found to exceed this limit in the literature.

$$
\alpha_{\text{airbubbles}} = 2.68 \times 10^{-10} \left( \frac{n_0}{200 \text{ cm}^{-3}} \right) \left( \frac{a}{0.01 \text{ cm}} \right)^6 \left( \frac{f}{10 \text{ kHz}} \right)^4
$$

Figure 5. Profiles of physical properties of the Antarctic ice sheet at Byrd station, from studies of deep-drill cores (2164 m) by Gow (1971).

4.2.3.3 Salts
Note that in ice, there are two main considerations when considering salt’s contribution to attenuation. Salts (typically NaCl) may be modeled as impurities in the ice crystal structure as a percent weight, as noted in the equations for Absorption; or may be modeled as a distinct crystalline scatterer. When sea ice freezes, salt may be rejected and create salt crystals, salt flats (layers of salts entrained in ice), or pockets of hypersaline brine. To estimate the effect of regions of rejected salt within frozen sea ice, the acoustic model interpolates this attenuation coefficient from empirical data of pure salts with diameters of 0.5-2 cm (Price, 2006), see code for details.

\[
\alpha_{\text{salts}} = \alpha_{\text{interpolated}}
\]  

(235)

Total attenuation for scattering losses are modeled as:

\[
\alpha_{\text{scattering, total}} = \alpha_{gbs} + \alpha_{\text{airbubble}} + \alpha_{\text{salts}}
\]  

(246)

4.2.4 Fractures

Note that the precise contribution of cracks, or fractures, is difficult to constrain. Fine hairline cracks or large voids like crevasses, will have either small or large contributions to loss of energy in an acoustic wave, respectively; since cracks may be considered a boundary interaction, this loss will fluctuate with respect to the ratio of wavelength (e.g., frequency) to fracture size. Furthermore, incidence angle (normal vs oblique) and roughness of the ice surface will also contribute to the amount of loss experienced by the acoustic wave. At the time of writing, the acoustic model only considers the first three scattering types.
4.2.5 Seawater loss

While the acoustic attenuation model is intended for use to calculate transmission loss through ice, use-cases for the model may include UUV (unmanned underwater vehicles) signalling through both ice and ice/saltwater mixtures. Acoustic signal attenuation in sea water is empirically determined, and well established. A full discussion of the mechanisms of attenuation in seawater are beyond the scope of this paper, but are discussed in detail in the following references (P. C. H. Hansen, 1951; Kinsler & Frey, 2000). This mechanism of attenuation is included for completeness:

\[ a_{\text{seawater}} = \left( \frac{A}{f_1^2 + f^2} + \frac{B}{f_2^2 + f^2} + C \right) f^2 \]  

(27)

\[ f_1 = 780 \exp(T/29) \]

\[ f_2 = 42000 \exp(T/18) \]

\[ A = 0.083(S/35) \exp[T/31 - Z/91 + 1.8(pH - 8)] \]

(2825)

\[ B = 22(S/35) \exp(T/14 - Z/6) \]

\[ C = 4.9 \times 10^{-10} \exp(-T/26 - Z/25) \]

4.2.6 Combining absorption and attenuation

Acoustic theory dictates that coefficients of both absorption and attenuation can be combined as a sum of their constituent components, as if each loss mechanism were acting alone (Kinsler & Frey, 2000). Thus, attenuation can be combined simply:
\[ \alpha_{total} = \alpha_{spreading} + \alpha_{absorption} + \alpha_{scattering} \] (29)

Table 2: Summary of mechanisms of attenuation, with corresponding attenuation coefficients.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spreading</td>
<td>[ \alpha_{spreading} = 20 \log_{10}(r) ]</td>
</tr>
<tr>
<td>Absorption</td>
<td>[ \alpha_{absorption} = \frac{\delta f}{v} ]</td>
</tr>
<tr>
<td>Rayleigh, Longitudinal</td>
<td>[ \alpha_{Rayleigh,L} = \frac{16\pi^4 a^3 f^4}{1350 \rho^2 v_L^3} \left( \frac{a_1}{v_L^5} + \frac{b_1}{v_T^5} \right) ]</td>
</tr>
<tr>
<td>Rayleigh, Transverse</td>
<td>[ \alpha_{Rayleigh,T} = \frac{16\pi^4 a^3 f^4}{1350 \rho^2 v_T^3} \left( \frac{a_2}{v_L^5} + \frac{b_2}{v_T^5} \right) ]</td>
</tr>
<tr>
<td>Stochastic, Longitudinal</td>
<td>[ \alpha_{Stochastic,L} = \frac{32\pi^2 a f^2}{1575 v_L^6 \rho^2} \left( 7 \gamma^2 + 35 \chi^2 + 140 \eta^2 \right) \left( 30 \chi \gamma + 140 \chi \eta \right) ]</td>
</tr>
<tr>
<td>Geometric</td>
<td>[ \alpha_{geometric} = \langle R \rangle = \sqrt{\langle (c_{11} - \langle c_{11}' \rangle)^2 \rangle \over 4 \langle c_{11}' \rangle^2} ]</td>
</tr>
<tr>
<td>Scattering – Air bubbles</td>
<td>[ \alpha_{airbubbles} = 2.68 \times 10^{-10} \left( \frac{n_0}{200 \text{ cm}^{-3}} \right) \left( \frac{a}{0.01 \text{ cm}} \right)^6 \left( \frac{f}{10 \text{ kHz}} \right)^4 ]</td>
</tr>
<tr>
<td>Scattering – Salts</td>
<td>[ \alpha_{salts} = \alpha_{interpolated} ]</td>
</tr>
<tr>
<td>Seawater Absorption</td>
<td>[ \alpha_{seawater} = \left( \frac{A}{f_1^2 + f^2} + \frac{B}{f_2^2 + f^2} + C \right) f^2 ]</td>
</tr>
</tbody>
</table>
4.3 Computer Modeling

4.3.1 GNU Radio: A Software Architecture for use of the Acoustic Channel Model

GNU Radio is a well-established open-source, flow-graph style signal processing software currently used in academic and professional settings (GNU Radio - The Free & Open Source Radio Ecosystem · GNU Radio, n.d.). While originally made for use with radio frequency communication systems, the similarities in signal processing between acoustics and RF signals make the software applicable to acoustics needs. To that end, GNU Radio has been leveraged as a tool for modelling underwater acoustic links with underwater-specific signal processing blocks (ex: doppler shift correction) (Kassem et al., 2018), and as a rapid-prototyping alternative to traditional acoustic modems (Nesl/Uant, 2013/2022; Torres et al., 2015; Zhu, 2020). In my work, GNU Radio is leveraged as a user-friendly tool for integration of signal transmission schemes; by creating a “flowgraph” the “blocks” for acoustic signal attenuation may be easily incorporated into a signal chain, see below.

4.3.2 Design and Incorporation of the Acoustic Model with GNU Radio

Equations for acoustic coefficients summarized in Table 2 above are all functions of frequency and various parameters describing the ice environment. For each transmission loss mechanism, a code was written (Python) for which a user could input frequency and desired environmental parameters. Each code was then made into a signal processing “block” in GNU Radio, Figure 6. Note spreading loss is not a block. Since spreading
attenuation has no frequency dependence, it is more simply applied as a multiplier in GNU Radio at run-time.

Figure 6. GNU Radio blocks for each modelled mechanism of attenuation. Absorption attenuation equations are all contained within the absorption block (left). Scattering attenuation equations are broken into individual blocks (middle) for simplicity. “sa0” stands for “saltwater attenuation” (right).

Signal processing blocks may be used two ways. The simplest implementation is to input only one frequency to the acoustic blocks, and sum the calculated output attenuation (in dBs/m). In this use-case, the modelled estimate represents the attenuation with respect to desired transmission range and input frequency within a prescribed acoustic channel, see example below (Figure 7):
Figure 7. Simple GNU Radio flowgraph modelling a 10 kHz signal frequency (leftmost block) through 1 km of glacier ice. The “QT GUI Time Sink” block plots calculated attenuation.

The acoustic blocks can also be used as a channel model with modulation, applying attenuation with respect to frequency to each signal sample, and then outputting the attenuated signal. In the second use-case, the output signal is an initial estimate for data transfer capability within ice. Currently, the model has been incorporated into FSK and BPSK modulation schemes, see Chapter 6.1 for more details.

Figure 8. GNU Radio flowgraph for an FSK modulated signal with acoustic channel model applied. See code for details.
Thus, using GNU Radio, estimates of acoustic attenuation in a prescribed in-ice environment are possible for a range of input signal frequencies and/or signal transmission schemes. For the following analyses, GNU Radio flowgraphs were used to calculate the attenuation/data reported.
CHAPTER 5.  COMPARATIVE BENCHMARKS

My model calculations are only able to be validated for frequencies between 1-100 kHz, as this is the frequency range over which empirical data (used for validation of individual attenuation components) were taken. This range exceeds the frequencies of interest suggested for optimal acoustic communication through ice, which can be roughly bound between $f \sim 1$-50 kHz. To test the efficacy of the acoustic model, model outputs for attenuation were compared to that of in-situ data. Due to the varied nature of the in-ice environment with respect to ice type, as mentioned previously, environments representative of these differences were selected for comparison; these are highlighted/discussed below.

To compare model data to reported in-situ data, simple GNU Radio flowgraphs were created to calculate attenuation for the in-ice environment as prescribed in the study, with signals of matching frequency range. Output attenuation values were recorded in excel for plotting; note that where tabulated data in the literature was not available and only graphical representations of data were provided (ex. the glacier analysis in Chapter 5.2), a software package called “plotdigitizer” was used to extract data points for creating graphs of superimposed data (PlotDigitizer, n.d.).

5.1 Sea Ice

Langleben (1969) reported in-situ attenuation values for an acoustic sounding experiment in sea ice near Tanquary Fiord, Ellesmere Island. The sea ice was approximately 2.6 m in depth; piezoelectric transducers used for transmission and
reception of acoustic signals were placed in boreholes filled with kerosene (improves coupling) at a depth of 1.3 m. A horizontal transmission path was used, with ranges varying from 1.3-7.6 m. Salinity of the ice along the transmission path was approximately 0.1% by weight, and ice temperature was ~6°C. The structure of the ice was also determined through horizontal and vertical thin-section analysis, finding that crystal size in the growth direction ranged from 1 cm to greater than 5 cm, and approximately 2 cm in the horizontal plane (the direction of propagation of the acoustic wave). A continuous wave frequency sweep was used to generate the acoustic signal for \( f = 10-500 \) kHz; results for attenuation are shown in Figure 9 below.

![Graph](image)

**Figure 9.** Reported acoustic attenuation coefficient calculated for transmission through sea ice at Tanquary, Fiord, Ellesmere Island (Langleben, 1969).

Recreating this data with my own model, I plotted both sets of data for ease of comparison, Figure 10; the attenuation coefficients for my acoustic model are shown with pink triangles,
Langleben’s data is shown in yellow circles. Note that at approximately 50 kHz we see a discrepancy of 3 dBs in the data which grows with frequency.

![Sea ice attenuation,](Image)

**Figure 10. Comparison of acoustic model attenuation data for prescribed sea-ice environment with in-situ sea-ice data from Langleben.**

5.1.1 **Analysis**

For frequencies below 50 kHz, the model data provides a reasonable fit with the sea ice data, but the discrepancy at higher frequencies was a surprising finding. Comparison of Langleben’s data to yet another sea-ice study (Wen et al., 1991) of one-way vertical transmission—expected to experience less attenuation due to alignment of ice grains in the growth direction—demonstrates that the attenuation documented by Langleben is notably small (furthermore, this attenuation is also small compared to the glacier attenuation data seen in Chapter 5.2). A comparison of all three data sets is seen in Figure 11 below.
Figure 11. Data plotted alongside acoustic model and Langleben data, for sea ice at \( T = -6 \, ^\circ\text{C} \) (Wen, et al., 1991).

This pointed discrepancy between Langleben’s data and the two comparison cases (this work and Wen’s) may be due to two causes: a waveguide effect in the sea ice sheet where testing was conducted, and/or due to the use of a continuous wave frequency sweep.

An acoustic wave travelling horizontally through a 2.6 m thick ice sheet that is exposed to air above and water below is nearly guaranteed to encounter reflections in the propagation path. This, combined with the use of a continuous wave for transmission (and therefore, continuous sampling for reception) may have caused significant constructive interference in the received acoustic signal, contributing to the low attenuation recorded.

To further investigate this possibility the acoustic model was benchmarked against only the Wen, et al., (1991) data. This study used single-frequency pulses from 20-90 kHz to investigate acoustic attenuation for vertical, one-way transmission through first-year arctic sea ice. Changing the model velocity input to “vertical” as opposed to “horizontal”
and recreating the environment of the Wen, et al., experiment, we see the data matches well, with the largest discrepancy being ~4 dBs, Figure 12. It should also be noted that tabulated values for the Wen data were not available, instead the authors provided an equation of best fit, Equation 30, used for plotting data in Figure 12:

$$\alpha_{wen} = 0.19 \cdot (f[kHz]) \left(\frac{-6}{T[^{\circ}\text{C}]})\right)^{2/3} \quad (26)$$

It is possible then, depending on the original spread of the Wen data, that the acoustic model outputs could fall within the best-case scenario of ≤ 3 dB difference between data sets.

![Sea ice attenuation graph](image)

**Figure 12.** Acoustic model attenuation calculated for vertical transmission through sea ice, compared to Wen, et al., (1991) data for matching set-up.

5.1.1.1 Discussion: Limitations, Improvements, Conclusions
While there is a lack of analytical data on the complex nature of frazil ice and the potentially reflective environment of 1-way transmission through thin (<50 m) sheets of sea ice, it is possible this area of the code could be improved in the future by incorporating more empirical data for this type of ice.

Currently within a margin of 3 dBs from the in-situ data, it can be concluded that the acoustic model is capable of recreating the acoustic environment with reasonably high fidelity. Thus, the acoustic channel model could be confidently used to calculate the transmission loss of a vertical communications link through sea ice for the frequency ranges of interest for acoustic communication \( f \sim 1-50 \text{ kHz} \).

### 5.2 Glacier Ice

Westphal’s 1965 investigation of acoustic attenuation through ice was conducted at Blue Glacier in Washington state. Two boreholes were drilled in ice, 2.5 m apart; the first 65 m deep hole housed the piezoelectric transducer for reception of the transmitted acoustic signal; the second hole, 0.5 m deep, was used for detonating standard electric blasting caps (a.k.a. “seismic” blasting caps), an impulsive acoustic source. The in-ice environment in the locality of the experiment was documented by Kamb (1959) and described as containing two distinctive layers: a ‘coarse clear’ layer with grain sizes up to 20 cm in the growth direction (the direction of travel of the acoustic wave), and the second as a ‘coarse bubbly’ layer, with bubbles of 1-2 mm diameter comprising of up to 10% of the ice volume.

Recreation of the in-ice environment for acoustic modeling required some simple calculation to arrive at the necessary input values: bubble radius, \( r = 0.05 - 0.1 \text{ cm} \); bubble volume, \( v_0 = 0.00052 - 0.00419 \text{ cm}^3 \); concentration, \( n_0 = 24 - 191 \text{ cm}^{-3} \). Ice density used
was \( \rho = 900 \text{ kg/m}^3 \), and due to a predominantly vertical propagation path, the largest ice grain size, \( \sim 20 \text{ cm} \), was considered. Westphal’s original data can be seen in Figure 13. Acoustic model results plotted against Westphal’s data (for ease of comparison) can be seen in Figure 14.

Figure 13. Westphal (1965) acoustic attenuation data, with attenuation coefficient, \( \alpha \), in [nepers/m] and the x-axis in kHz\(^3\).

\(^3\) “kc” or “kilo-cycles” is used by the author, this is equivalent to kHz.
Figure 14. Pierson acoustic model data vs Westphal data, recreation of in-ice environment of Blue Glacier, noted for 3 distinctive layers of ice within the ice column.

5.2.1 Analysis

It is interesting to note that when considering the spread of data, the two curves match each other in shape quite well; following a similar trend across frequency. However, the model underestimates attenuation by over one order of magnitude. Kamb and Westphal describe layering within the ice environment at the study site, with coarse bubbly ice being layered with fine ice near the surface of the ice column and coarse clear ice being found near the bottom. Figure 15 depicts the ice structure of the region in plane view, with the expected ice types occurring in the regions indicated. It is possible the additional energy loss energy loss is due to an interface change between two ice types, or due to unaccounted-for reflectivity or scattering at an abrupt interface, such as a filled or unfilled crevasse.
Figure 15. Plane view of ice at Blue Glacier, depicting all three types of ice in the ice column and both filled and unfilled crevasses. From the ice surface to the rock margin was measured to be approximately 300 m at the location of the acoustic test.

To investigate this possible discrepancy the model is used to test a “layering” effect within the ice by cascading attenuation blocks to account for the different ice types, as opposed to just the most prominent ice type (coarse bubbly). The results can be seen in Figure 16.
Figure 16. Acoustic model with layering implementation, all three ice types accounted for: coarse bubbly, coarse clear, and fine.

We can see clear improvement in the model calculations, with output values nearly one order of magnitude higher, but still seeing an approximately 17 neper/m difference in the calculated data vs the in-situ data for $f \sim 15$ kHz. To put this in perspective, that is approximately a 147 dB difference! As was noted by Lishman (2013) in his assessment of through-ice acoustic communication through highly fractured glacier ice, several orders of magnitude of power may be lost due to significant cracking. Lishman provided the following empirical equation for acoustic attenuation at his test site:

$$\alpha_{\text{Lishman}} = 0.4182 + 0.0191 f [\text{Hz}]$$  \hspace{1cm} (31)
When using this equation and comparing to Westphal’s data we see the following results, Figure 17:

![Model vs Westphal 1965 Blue Glacier data](image)

**Figure 17.** Comparison of two data sets for through-ice acoustic attenuation, both recorded at glaciers with potential for significant cracking or fractures present.

The calculated attenuation for Lishman’s study also follows a similar trend and demonstrates even higher attenuation values for reportedly fracture-riddled ice.

5.2.1.1 **Discussion: Limitations, Improvements, Conclusions**

Due to these large jumps in the scale of attenuation presented between all three sets of work, I conclude that at the Westphal site the acoustic signal encountered a significant loss of energy from one or more filled or unfilled crevasses. It can also be inferred that
while both Westphal and Lishman encountered fractured ice, due to the nature of the geometry and structure of ice, Westphal’s experiment experienced less. Due to the geometric structure and growth of ice, cracks tend to propagate in the growth direction (vertically); therefore, the horizontally propagating acoustic waves in Lishman’s investigation inherently encountered more attenuation.

For the complicated structure of glacier ice, it can be concluded that the acoustic model can provide insight into the acoustic environment, but for high-fidelity recreation of the environment for enabling communication or data transfer, some a-priori knowledge of the glacier in-ice environment would be critical to accuracy.

In the future it is possible that incorporating analytical equations on losses due to reflectivity in air-ice interfaces could improve model calculations and shed more light on the very complex nature of attenuation due to cracks and fractures.

Single-layer and/or more uniformly structured glacier ices can be modeled with more confidence, as can be seen in the next section.

5.3 Deep Glacier Ice/Antarctic Ice

Finally, the last data to be compared comes from the SPATS project (Abdou et al., 2012). Attenuation values for a case of horizontal acoustic transmission across various depths is shown in Figure 18. This proves an interesting test case since not all environmental parameters were provided by the study specifically, and estimates of expected environmental parameters were bound using similar deep-ice studies of Antarctic
ice; such as temperature, air bubble concentration and radius, and grain size with respect to depth (Gow & Williamson, 1976; Price et al., 2002), though the temperature profile used was found through a complimentary study by the IceCube Collaboration (South Pole Ice Characteristics, n.d.).

![Graph showing attenuation coefficient vs. depth](image)

**Figure 18.** SPATS (South Pole Acoustic Test Set up) attenuation values [nepers/km] in deep glacial ice in Antarctica.

5.3.1 Analysis

In this case, the model outputs appear to transect the data reported the SPATS study; see the acoustic model data overlaid on the SPATS plot below, Figure 19. Note that for all depths, the attenuation coefficient calculated is an average of the maximum and minimum frequency values (10, 30 kHz), this data can be found labelled as “initial test” in Table 3.
Figure 19. Acoustic model and SPATS data for deep Antarctic glacier ice at the South Pole.

Table 3. Difference between acoustic attenuation data and SPATS data for attenuation in nepers/km.

<table>
<thead>
<tr>
<th>Depth (meters)</th>
<th>190</th>
<th>250</th>
<th>320</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPATS (nepers/km)</td>
<td>2.9</td>
<td>3.85</td>
<td>3.8</td>
<td>3</td>
<td>2.3</td>
</tr>
<tr>
<td>Initial Acoustic model data (nepers/km)</td>
<td>2.89</td>
<td>2.93</td>
<td>2.96</td>
<td>2.98</td>
<td>3.01</td>
</tr>
<tr>
<td>Difference (nepers/km)</td>
<td>0.01</td>
<td>0.92</td>
<td>0.84</td>
<td>0.02</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

At most, the model experiences a 0.92 dB difference in attenuation calculated by the initial model run, with a difference in the shape of the overall trend between the idealized, single-layer model and the real data. The acoustic model data follow a very predictable upward slope in this regime, with each value for attenuation increasing by approximately 0.03 nepers/m, and the spread of data no larger than 2.95±0.6 nepers/m. These results shed light on the weight of attenuative factors used in the acoustic model. For
the environment in this case, the modelled parameters changed very little with depth; the largest contributor to loss is absorption, followed by grain boundary scattering, and then air bubbles, where air bubble attenuation is nearly negligible in magnitude compared to absorption and grain boundary scattering. Two immediate possibilities come to mind to explain the attenuation variances with depth: absorption error, and/or error in grain boundary size estimate.

5.3.1.1 Absorption considerations

Since absorption is the main contributor to attenuation in this calculation, it is possible that inputs for $\tau_m$, which affect $\delta_{max}$ and therefore $\alpha$, may impact variance. Any discrepancy between in-situ values (which were not calculated and/or measured) and estimates used (approximated using Kuroiwa’s data) has potential to impact attenuation output. For now, input values for $\tau_m$ in the model may be changed manually within the python block code for “absorption”, and automatically default to values found in Price (1993).

In situations such as this, it is feasible to have no prior knowledge about certain parameters of the environment, like $\tau_m$. To investigate how $\tau_m$ affects attenuation, I begin by bounding the problem and select to vary values of $\tau_m$ by an order of magnitude, shown in Table 4, to determine the weight of these factors in the calculation of absorption. Note that in the acoustic code, these values are applied to absorption from impurities, dislocation damping, and proton reorientation only, as these were the largest sources of absorption from all possible absorption mechanisms, in that order. I then test the effects of changing
the $\tau_m$ value, noting if it shifts attenuation up or down; this change is only tested on the 320m depth due to the extremely linear nature of attenuation in this frequency range.

**Table 4. Estimated minimum and maximum bounding values for $\tau_m$ and effects on overall attenuation at 325 m depth.**

<table>
<thead>
<tr>
<th>$\tau_m$</th>
<th>$\tau_m$ Original value</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Adjusted Attenuation at 325 m depth [nepers/m] Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton reorientation (PR)</td>
<td>.0084</td>
<td>.00084</td>
<td>.084</td>
<td>3.2556</td>
<td>2.9272</td>
</tr>
<tr>
<td>Impurities (IM)</td>
<td>.0001</td>
<td>.00001</td>
<td>.001</td>
<td>15.0145</td>
<td>0.4851</td>
</tr>
<tr>
<td>Dislocation Damping (DD)</td>
<td>1e-9</td>
<td>1e-10</td>
<td>1e-8</td>
<td>2.9529</td>
<td>3.007875</td>
</tr>
</tbody>
</table>

Note that changes of one order of magnitude only prove to provide a significant change in attenuation for absorption by impurities. Changes in $\tau_m$ for proton reorientation and dislocation damping fail to adjust the final attenuation value enough to be capable of matching the in-situ SPATS data. Now, only adjusting $\tau_m$ for absorption by impurities, several iterations were run to determine bounding values of $\tau_m$ that would make the initial acoustic model data match the in-situ SPATS data. Meaning, the $\tau_m$ lower and upper bounding values should be capable of producing attenuation values matching the highest and lowest values for attenuation reported by the SPATS study. Results are seen in Table 5.
Table 5. Bounded minimum and maximum values for $\tau_m$ for absorption by impurities (325 m).

<table>
<thead>
<tr>
<th>Impurities</th>
<th>$\tau_m$ Min</th>
<th>$\tau_m$ Max</th>
<th>Adjusted attenuation at 325 m depth [nepers/m]</th>
<th>Maximum and minimum Attenuation reported by SPATS [nepers/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.000075</td>
<td>.000125</td>
<td>2.3034</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.8370</td>
<td>3.85</td>
</tr>
</tbody>
</table>

Note that it stands to reason that the changes in $\tau_m$ are only relevant for absorption by impurities, as the bounding values can now be seen to fall within reasonable limits as marked in the estimates of relaxation time plotted by Kuroiwa (1964), Figure 2.

The next source of possible discrepancy is the estimated grain boundary size input, which affects attenuation due to scattering. The same process is carried out here; for 325 m depth, grain boundary size is changed to analyse its effect on overall attenuation. This time, we use bounding values within reasonable limits, by consulting prior data/literature on the Antarctic environment, Figure 20 (Gow, 1970). Note that the minimum grain boundary size selected corresponds to the trendline for ‘Site 2’ at 325 m depth, and the maximum grain boundary size selected corresponds to the trendline for Byrd station at 325 m depth (the bottom of the plot). The attenuation results for these minimum and maximum grain boundary limits can be seen in Table 6.
Figure 20. Measurements of mean crystal size in deep drill cores (2146 m) at Byrd Station, Little America V, Maudheim, Antarctica, and ‘Site 2’ in Greenland.

Table 6. Bounded minimum and maximum values for grain boundary size and corresponding effect on attenuation.

<table>
<thead>
<tr>
<th>Original grain boundary radius [cm]</th>
<th>Grain boundary radius [cm]</th>
<th>Adjusted attenuation at 325 m depth [nepers/m]</th>
<th>Maximum and minimum Attenuation reported by SPATS [nepers/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>0.2764</td>
<td>0.126</td>
<td>0.293</td>
<td>2.8074</td>
</tr>
</tbody>
</table>
The resulting spread of adjusted attenuation values in Table 6 tell us that grain boundary size alone is not sufficient to explain the variances seen in the SPATS data, implying that absorption, specifically impurities, may be a prominent cause of the trends in the in-situ data.

5.3.1.2 Discussion: Limitations, Improvements, Conclusions

The acoustic model calculations of attenuation for deep glacier ice proved very well fit to the data overall, with less than 1 dB difference overall, but due to the estimated nature of certain inputs over the very large spatial range (190 – 500 m depth within the ice shell) discrepancies could be expected.

From this analysis it can be concluded that analytical equations for calculating attenuation in deep glacier ice produce high fidelity results, and with some knowledge of the in-ice environment, whether from ice-penetrating radar or in-situ data, the model can be useful for investigating effects of complex environmental parameters.
Currently, the acoustic model is expected to produce reliable results within a frequency range of \( f \sim 1-50 \) kHz for sea ice and glacier ice without cracks or fractures. With this in mind, it is possible to incorporate the acoustic channel model into a full GNU Radio flowgraph to demonstrate data transfer functionality.

6.1 A Novel Model for Acoustic Data Transfer with GNU Radio

Considering the case of a sensor or UUV deployed under sea-ice, environmental parameters matching that of Wen, et al. (1991) data for first-year arctic sea ice were used as inputs to the acoustic channel model. The transmission path is assumed to be vertical, with the acoustic wave propagating through a 5-m thick sea-ice sheet; see Figure 12 for calculation of attenuation (dB/m). The blocks representing the acoustic model (i.e.: ‘acoustic channel model,’ or ‘attenuation model’) were then incorporated into a full BPSK (Binary Phase Shift Keying) flowgraph, Figure 21. This flowgraph takes in a simple text file as an input, uses a BPSK scheme to modulate the data, passes the data through the acoustic channel model (boxed in yellow), and then demodulates the signal. The recovered signal can be seen in Figure 22, where both the original signal (blue) and received, demodulated signal (red) have been normalized to an amplitude of 1.

The acoustic attenuation code has also been incorporated into an FSK (Frequency Shift Keying) modulation scheme, Figure 8.
Figure 21. GNU Radio flowgraph for a BPSK modulation scheme with acoustic channel model applied.

Figure 22. Recovered, demodulated text input from BPSK flowgraph including acoustic channel model for a modelled sea-ice environment.
6.2 A Novel Link Budget

Using the flowgraphs above as a stepping stone, a user of the acoustic data transfer model could additionally use a link budget to begin to constrain the hardware necessary for a full acoustic communications link through ice. Below, I discuss the state of link budgets as related to acoustic through-ice communication and, continuing to use the sea-ice environment example, estimate/bound the needs and capabilities of an acoustic through-ice system.

6.2.1 Link Budget Theory

Historically, acoustic signaling through ice borrows both theory and technology from the established field of underwater acoustic communication. Underwater link budgets, like RF link budgets, are a design tool; these simple equations encompass all gains and losses experienced by a communication system “from transmitter output power to the power seen by the receiver” (CircuitDesign Inc., 2022). While one link budget was found in the literature (Lishman et al., 2013), this link was not cited or referenced to any other established forms. To ensure that all necessary components are accounted for, I compared Lishman’s link with those of Hansen’s underwater acoustic link (2002), Proakis and Salehi’s free-space RF link (2008), and my through-ice link (Pierson, 2022), in Table 7.

It is important to note that Lishman (2003) includes two extra terms: Transducer and Coupling losses, which, in his analysis are inherently linked. These terms would be beneficial to use in an end-to-end modelled system for two reasons:
1. $P_{tx}$, transmitted power, may not always be possible to measure empirically in an through-ice communication system. In air and water, a secondary measurement is typically made at a set distance of 1 meter to record a standardized “output power” in the target environment. Whether measured by the manufacturer (TVR, Transmitting Voltage Response, a plot provided with some transducers) or by the individual using the device, this measurement would inherently account for transducer and coupling losses. When conducting measurements in ice, however, this is not possible without introducing additional attenuative effects due to coupling.

2. Coupling losses are yet unconstrained in the literature. While coupling effects have been recognized (Abbasi et al., 2011; Semburg, 2009), they are difficult to constrain without bias in the ice environment.

It should also be noted that Lishman’s link budget lacks an ambient noise term, and directivity indices. As demonstrated by SPATS, directivity of acoustic sources is possible within the ice environment, and furthermore, gaussian noise is present within the ice column. Therefore, for the analysis presented here, I use a new link budget encompassing all terms, to fully constrain the communications of an acoustic through-ice link, in the final row of Table 3.

Table 7. Link budget comparison for three different media: ice, water, and air.

<table>
<thead>
<tr>
<th>Link</th>
<th>Author</th>
<th>Link Budget Equation</th>
<th>Variables</th>
</tr>
</thead>
</table>
| Acoustic | Lishman et al., 2013 | $P_{rx} = P_{tx} - P - A - R - T - C$ | $P_{tx}$, $P_{rx}$: power$^4$
$P$: path loss, |

$^4$ Subscripts “rx” and “tx” stand for “received” and “transmitted”, respectively. It should be noted that SNR and $P_{rx}$ are equivalent if both quantities are expressed in dBs. By convention SNR is expressed in dBs, while $P_{rx}$ is often expressed as [dB re 1 Watt]. See Table 9 below for conversion equations.
A: attenuation loss,  
R: reflection loss,  
T: transducer loss,  
C: coupling loss

| Underwater | J. T. Hansen, 2002 | \[ \text{SNR} = \text{PSL} - \text{TL} - \text{AN} + \text{DI}_{tx} + \text{DI}_{rx} \] | \( \text{SNR} \): signal to noise ratio  
\( \text{PSL} \): Pressure spectrum level  
\( \text{TL} \): transmission loss  
\( \text{AN} \): ambient noise  
\( \text{DI}_{tx}, \text{DI}_{rx} \): directivity

| RF         | Proakis & Salehi, 2008 | \( P_{rx} = P_{tx} + L_s + L_n + G_{tx} + G_{rx} \) | \( G_{tx}, G_{rx} \): antenna gain  
\( L_s \): free-space path loss  
\( L_n \): noise factor

| Acoustic   | Pierson, 2022           | \( P_{rx} = P_{tx} - \text{TL} - \text{AN} - \text{CT} + \text{DI}_{tx} + \text{DI}_{rx} \) | \( \text{CT} \): combined transducer (T) and coupling loss, (C)

### 6.3 Estimates for a Through-Ice Acoustic Link

Using the acoustic link and calculated transmission loss values from the GNU Radio flowgraph, one can now estimate optimal properties of a communications system.

\[
P_{rx} = P_{tx} - \text{TL} - \text{AN} - \text{CT} + \text{DI}_{tx} + \text{DI}_{rx}
\]  
(3227)

For our model system communicating through 5-m thick sea-ice, several link budget variables are known: Transmission loss, calculated above, Figure 12; Ambient noise, which may be modelled as gaussian but is also well documented in the literature; and Directivity, which are properties of the hardware used in the communications link and can be bound or calculated with their specifications. In this case, I model the link using a Neptune Sonar T235 transducer (\textit{T235.Pdf}, n.d.). This device transmits in the range of \( f \sim 10-25 \) kHz, but I select only a portion of this, from 10 to 14 kHz, to ensure transmission loss no larger than 5 dB/m; for transmission through 5 m of sea ice, this totals to 5 dB of transmission loss. Next, assuming an input power of 1 W, the transmission power of the
transducer may be deduced using the following equations, Table 8 (standard acoustic equations, see Kinsler & Frey, 2000 for details). Note pressures are referenced to 1 µPa (as is convention for underwater transducer calculations, since no reference pressures for through-ice communication currently exist), a flat TVR is assumed, directivity is assumed to be 1 (no directivity) for a conservative estimate, $\rho = 0.900$ kg/cm$^3$, and $c = \sim 3800$ m/s for a conservative estimate of vertical propagation of an acoustic wave through ice. The largest value for ambient sound found in the literature is no larger than $\sim 1.5$ dBs (Abdou et al., 2012; Stojanovic, 2007), but applying a safety factor of 2 the value used here is $AN = 3$ dBs. Lastly, I assume a conservative combined transducer and coupling loss of $CT = 6$ dBs (corresponding to 25% efficiency of power transfer from the transducer into the ice). This is far more than reported in the HADES studies ($\sim 89$% efficiency) but deemed appropriate for a through-sea-ice link; due to the difficulty of placing sensors in the environment it is possible coupling is not optimized. We see the results of this analysis in Table 9.

Table 8. Standard acoustic equations for calculating Link budget values, including Power Level at transmitter and receiver, equivalent to $P_{tx}$, and $P_{rx}$, respectively.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Units</th>
<th>Model Environment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sound Pressure Level at 1 m</strong></td>
<td>$SPL@1\text{m} = TVR + 10 \log_{10} V_{in}$</td>
<td>[dB re 1 µPa]</td>
</tr>
<tr>
<td><strong>Sound Pressure at 1 m</strong></td>
<td>$p@1\text{m} = 10^{-6} \times 10^{\frac{SPL}{20}}$</td>
<td>[Pa]</td>
</tr>
<tr>
<td><strong>Power at 1 m</strong></td>
<td>$P_{tx}@1\text{m} = \frac{4\pi p^2}{Q\rho c}$</td>
<td>[W]</td>
</tr>
<tr>
<td><strong>Power Level at 1 m</strong></td>
<td>$SWL@1\text{m} = 10 \log_{10} P_{ac}$</td>
<td>[dB re 1 W]</td>
</tr>
<tr>
<td><strong>Power Level at 5 m</strong></td>
<td>$SWL@5\text{m} = SWL@1\text{m} - TL$</td>
<td>[dB re 1 W]</td>
</tr>
</tbody>
</table>
Table 9. Suggested/estimated link budget for an acoustic through-ice communication link, see supplementary materials for intermediate calculations and device specifications.

<table>
<thead>
<tr>
<th>Prx</th>
<th>PtX</th>
<th>TL</th>
<th>AN</th>
<th>CT</th>
<th>DItx</th>
<th>DIrx</th>
</tr>
</thead>
<tbody>
<tr>
<td>-111.96 [dB re 1 W]</td>
<td>-106.60 [dB re 1 W]</td>
<td>5 dB</td>
<td>3 dB</td>
<td>6 dB</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The analysis above equates to a SPL at the receiver of ~20 dBs, well within the range of the sensitivity reported for the transducer in question; Furthermore, using the Shannon Hartley theorem for channel capacity, Equation 33, estimates of bit rate for the communications link can be made, Figure 23. For a BPSK acoustic link, like the one currently being analysed for $f \sim 4.5$-14 kHz bandwidth would be equivalent to the center frequency used. Assuming a transmission frequency of approximately 14 kHz, we can finally estimate a channel capacity for a theoretical acoustic link as approximately 20,000 to 30,000 bits/sec.

\[
\text{Channel Capacity} \left[\frac{\text{[bits]} }{\text{[sec]}}\right] = \text{BW} \cdot \log_2(1 + \text{SNR})
\]

(3228)
Figure 23. Estimates of channel capacity calculated using the Shannon Hartley theorem.
CHAPTER 7. CONCLUSIONS & FUTURE WORK

7.1 Summary of Results

The acoustic model presented here is a novel, adaptable tool for calculating attenuation in various in-ice environments. This acoustic model has been successfully incorporated into GNU Radio for use as a channel model, allowing a user to both calculate attenuation for a prescribed environment and test the data transfer capabilities in said, modelled environment.

Within the frequency range of interest for acoustic through-ice communication, \( f \sim 1-50 \text{ kHz} \), the acoustic model is currently capable of high-fidelity recreation of both sea-ice (\( \leq 4 \) dB difference between model estimates and in-situ data, Chapter 5.1) and deep-glacier environments (\( \leq \pm 0.6 \) nepers/km difference, Chapter 5.3).

Model results can be used to calculate attenuation, and furthermore, can be used to deduce complex environmental parameters, like relaxation time and grain boundary size, by varying inputs and analyzing model results as compared to in-situ data (Chapter 5.3). When paired with a link budget, the capabilities of an acoustic through-ice communications system may be estimated, leading to capability for estimating theoretical channel capacity for a modelled through-ice acoustic channel (Chapter 6). Available for use by the public immediately following publication of this thesis, the acoustic channel model blocks, made for use with GNU Radio, may be used by any investigator wishing to investigate acoustic through-ice communications links.
7.2 Future Work

Future work includes improvement of the acoustic model for complex and yet unconstrained in-ice acoustic effects, such as constructive interference from wave guides and loss due to fractures (Chapter 5.2). This could be done by incorporating analytical acoustic equations and/or by incorporating a third analysis tool, BELLHOP (acoustic ray tracing software); which would provide additional information on the reflecting nature of the medium. The model may also be improved by incorporating capability to “cascade” blocks in GNU to produce a layering effect automatically, instead of calculating this layering by hand by summing the contributions of attenuation calculated using two or more flowgraphs representing the different environments; greatly improving the current methods for analysis of acoustic signaling through ice.
APPENDIX A. CODE

Code for the acoustic attenuation blocks may be found on GitHub immediately following the acceptance of this thesis:

https://github.com/sarapierson234/gr-aa0.git


65