Modeling and optimization of the omnichannel retailing problem
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Abstract: In this paper, we focus on a problem faced by an omnichannel retailer that operates multiple stores and a fulfillment center operating in different zones. The retailer sells products to customers over a selling horizon of the period through online and physical channels. The aim of this study is to determine the joint tactical and operational decisions on fulfillment optimization and inventory management services in an integrated model. The multi-period horizon in our model allows more realistic planning, where various decisions can be taken at different periods. Besides, our model considers the lost-demand sale to reflect the actual sale. The proposed model helps omnichannel retailers to have an integrated plan for inventory management and fulfillment services by monitoring the real-time inventory across seller locations.

Keywords: Omnichannel retailer, Optimization, Inventory Management, Replenishment, fulfillment.

Conference Topic(s): logistics and supply networks; omnichannel & e-commerce logistics.

Physical Internet Roadmap (Link): Select the most relevant area for your paper: ☐ PI Nodes, ☐ PI Networks, ☒ System of Logistics Networks, ☐ Access and Adoption, ☐ Governance.

1 Introduction

The world of retailing has expanded dramatically in the past few decades. The digital channel finds its way through all stages of the customer shopping journey because it provides a more convenient way of shopping, a wider choice of items, and the ability to access more information about items through customer reviews (Verhoef, 2021). Consequently, a growing number of retailers have started to integrate their traditional physical sales with online channels, moving towards omnichannel retailing to leverage their physical store channels (Bayram & Cesaret, 2021).

There are different definitions of omnichannel retailing but the understanding of the concept still varies. In general, different definitions exhibit distinct features in the channel organization of the omnichannel, as discussed below:

1. Needs the consistent and fully integrated information, services, and process at any moment of its operation (Bayram & Cesaret, 2021; Bieberstein, 2015; Cummins et al., 2016; Fairchild, 2014; Fernie & Sparks, 2004a; Galipoglu et al., 2018; Kozlenkova et al., 2015a; Rigby, 2011; Saghir et al., 2018; Wollenburg et al., 2018; Yrjölä et al., 2018a).
2. Includes customer touchpoints (Chauhan & Sarabhai, 2019; Cortiñas et al., 2019; Cummins et al., 2016; Heuchert et al., 2018; Pawar & Sarmah, 2015; Picot-Coupey et
al., 2016; Verhoef et al., 2015; Yrjölä et al., 2018b, 2018a); any direct or indirect communication or contact with the prospect or customer which is not necessarily an interaction(Mirsch et al., 2016; Pawar & Sarmah, 2015b).

3. Eliminates borders between channels and manages them as an integrated channel (Heuchert et al., 2018; Hübner et al., 2016; Melacini & Tappia, 2018; Menrad, 2020; Pawar & Sarmah, 2015b; Picot-Coupey et al., 2016; Trenz et al., 2020; Verhoef et al., 2015).

4. Provide a seamless shopping experience for Customers(Abrudan et al., 2020; Hole et al., 2019; Jiu, 2022; Kozlenkova et al., 2015b; Menrad, 2020; Mosquera et al., 2017).

In this study, we define omnichannel retailing as a unique world of shopping (Abrudan et al., 2020) that provides a seamless shopping experience through fully integrated distribution and communication channels (Fairchild, 2014), allowing customers to purchase and return products from anywhere and allows retailers to fulfill orders from anywhere by eliminating borders between different channels (inspired by the definitions of (Bayram & Cesaret, 2021)). Performing the logistic operations in an omnichannel retailing network is not a simple task. This activity significantly increases the complexity in terms of fulfillment planning and inventory management.

An omnichannel retailer has the flexibility to fulfill online orders from a physical stores (Ship-From-Store (SFS) strategy), fulfillment center, or their combinations (Difrancesco et al., 2021). Using stores to fulfill online orders is the most adopted strategy among omnichannel retailers(Bayram & Cesaret, 2021). Appropriately, a ship-from-store strategy need the lowest initial investment(Gallagher & Vella-Brodrick, 2008), increase online sales by avoiding frustrating online stock-outs(Jiu, 2022b), and improve customer value and experience (Difrancesco et al., 2021). On the other hand, using store inventory increases the risk of stock-outs(Bendoly, 2004) and the following risk of dissatisfaction and disloyalty of customers who can't find items in-store(Goedhart et al., 2022). To overcome this challenge and benefit from the advantages of different fulfillment strategies, this study focuses on the combined ship-from-store as well as fulfillment facilities.

A suitable inventory system would be the one where the order size is just right and takes into account factors such as the ordering cost, the holding cost, the shortage cost, etc.(Shenoy & Rosas, 2018). Retailers need to decide how to retain their stocks across the channel to prevent risks of running out of stock, and stockouts. Excessive stock levels may result in increasing storage, labor, and insurance costs and causing revenue loss and quality reduction depending on the type of product. On the other hand, the lack of inventory can lead to an inability to meet customer demand, lost sales, and customer dissatisfaction(Kilimci et al., 2019).

The aim of this study is to provide the tactical and operational decisions on optimal amount of start inventory, replenishment, and fulfillment location for an omnichannel retailer with the goal of minimising the total cost over a finite horizon. We formulate the problem as an integer linear programming problem and examine the viability of the proposed model using a numerical example, inspired by an Australian omnichannel retailer.

The rest of the paper is organized as follows. In Section 2, the problem description and mathematical formulation in an omnichannel problem are described. Section 3 presents numerical examples. The solution approach and computational results are described in Sections 4. Finally, the conclusion and future outlines provide in Section 5.
2 Problem description

2.1 Problem statement

Figure 2 shows the structure of the omnichannel retailing network where solid lines represent customers visiting stores and dashed lines represent products delivered to customers. The notations of the figure will be used in the mathematical modeling.

The network is divided into several local zones based on customer locations; each includes exactly one store. Customers are classified by their shopping channels and physical/delivery addresses. With respect to the shopping channel, the retailer deals with two types of customers in each zone: (i) walk-in customers, who physically come to the store and collect products from a store shelf, and (ii) online customers, who place orders through the retailer’s website or mobile app (Govindarajan et al., 2021). Customers buying product(s) online would have the option of selecting the delivery services, with products directly shipped to their location, called home delivery, or collecting orders from a local store called click-and-collect (C&C).

C&C customers are defined as a part of online customers where the payment and purchase accrue at the online channel, but the pick-up happens at a dedicated store, meaning that customers cross-buy (buy online and pick up in-store) (Jara et al., 2018). We emphasize that with the introduction of C&C, the retailer requires an integrated information system to provide customers with immediate access to the real-time inventory information at each store (Gallino & Moreno, 2019). In another word, the actual inventory availability is promptly updated once a C&C order is processed by an employee, or a walk-in customer makes a purchase in the store.

The entire fulfillment process of walk-in customers only is taken care of at the corresponding store in the local zone. We assume that customers who face a stock-out don't switch to other stores, and it is because of the long distance between stores. Therefore, the unfulfilled in-store demand of one store cannot be fulfilled by other stores. This assumption is in line with research in Bayram & Cesaret, (2021), and Jiu, (2022b). The retailer needs to decide from which location (a store or an FC) fulfills online orders. We stress that the fulfillment process in a fulfillment center is more efficient than in a store (Gallino & Moreno, 2019). The flowchart of fulfilling online orders is shown in Figure 1.

In this research, we study omnichannel retail operations to determine the joint tactical and operational decisions on fulfillment optimization and inventory management in an integrated model. In particular, the retailer determines the amount of inventory replenishment at the beginning of each period, the inventory policy at each location, and the fulfillment of the online orders for different selling channels.
Assumptions used in the mathematical model have listed below:

- The retailer fulfills the demand of walk-in and click-and-collect customers from the inventory of a corresponding local store of the same zone as long as the inventory level is positive.
- Unfulfilled demand of walk-in and click-and-collect customers for products that are out of stock at the same zone as customers are immediately lost.
- Customers don't switch between selling channels or stores in case of stock out, i.e., a) A customer placing an online order will not intend to buy products physically in stores b) Walk-in customers do not choose delivery but collect products from a store.
- Online orders fulfill from a location that optimize the retailer's profit.
- No replenishment occurs during planning periods.
- Stores support walk-in, click-and-collect, and online demand.
- Sellers have a limited storage inventory.
- The fixed costs of replenishing and ordering are assumed to be zero.
- The retailer incurs all cost in fulfilling an online demand.
- The planning horizon splits into different periods with the length of a week.
- The model is built for the whole planning horizon.
- The actual inventory availability is changed at the time the picking begins, meaning that the retailer has an integrated information system.

2.2. Mathematical model

Notations:
Consider a retailer selling products \( p \in \mathcal{P} = \{1, ..., P\} \) to customers in different demand zones \( z \in \mathcal{Z} = \{1, ..., Z\} \) over a finite planning horizon of periods \( t \in \mathcal{T} = \{1, ..., T\} \). Let \( s \in \mathcal{S} = \{1, ..., S\} \) denotes the set of stores and \( f \in \mathcal{F} = \{1, ..., F\} \) denote the set of fulfillment centers respectively, and \( \mathcal{L} = \mathcal{S} \cup \mathcal{F} \) represent the set of all seller locations. We let exactly one store per zone, \( S = Z \). For the complete list of notations, please see Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( \mathcal{S} )</td>
<td>Set of stores (( s \in \mathcal{S} ))</td>
</tr>
<tr>
<td>( \mathcal{F} )</td>
<td>Set of fulfillment centers (( f \in \mathcal{F} ))</td>
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</table>
\( l \)  Set of sellers \((L = S \cup F)\)

\( P \)  Set of products \((p \in P)\)

\( T \)  Set of time periods \((t \in T)\)

\( Z \)  Set of zones \((z \in Z)\)

Parameters:

\( do_{pz}^{t} \)  The online demand of zone \( z \) for product \( p \) at period \( t \),

\( dw_{ps}^{t} \)  The online demand of store \( s \) for product \( p \) at period \( t \),

\( dc_{ps}^{t} \)  The click-and-collect demand of store \( s \) for product \( p \) at period \( t \),

\( i_p \)  The price of product \( p \),

\( ch_{plz}^{t} \)  The handling cost of delivering product \( p \) from the seller \( l \) to zone \( z \) at period \( t \),

\( cf_{ps}^{t} \)  The walk-in fulfilment cost of \( p \) from store \( s \) at period \( t \),

\( cr_{pl}^{t} \)  The replenishment cost or product \( p \) in seller \( l \) at period \( t \),

\( co_{pl}^{t} \)  Holding cost or product \( p \) in seller \( l \) at period \( t \),

\( u_l \)  The capacity of seller \( l \),

\( k_{lz} \)  The capacity of transportation vehicle from seller \( l \) to customer in zone \( z \),

\( lo_{pz}^{t} \)  Lost sale penalty cost for online customers in zone \( z \) for product \( p \) at period \( t \),

\( lw_{ps}^{t} \)  Lost sale penalty cost for walk-in customers in store \( s \) for product \( p \in P \) at period \( t \in T \)

Variables:

\( X_{plz}^{t} \)  The amount of inventory for product \( p \) ship to online customers of zone \( z \) from seller \( l \) at period \( t \),

\( Y_{ps}^{t} \)  The amount of inventory for product \( p \) to fulfill walk-in customers in store \( s \) at period \( t \),

\( C_{ps}^{t} \)  The amount of inventory for product \( p \) to fulfill click-and-collect orders in store \( s \) at period \( t \),

\( I_{pl}^{t} \)  Initial inventory of product \( p \) at location \( l \) at the beginning of period \( t \),

\( R_{pl}^{t} \)  Replenishment amount of product \( p \) for seller location \( l \) in period \( t \),

\( Q_{cpl}^{t} \)  The total sale of seller \( l \) for product \( p \) at period \( t \).
Before proceeding with definition, we describe the order of events during a period. The start inventory for each period is $I_{pt}^t$ product $p$ that is available at seller $l$. Online and walk-in demands occur during periods. Walk-in demands are immediately satisfied if the inventory level of the corresponding store is strictly positive, otherwise, they are lost. Online demand and the click-and-collect orders could be satisfied from either a store or by the fulfillment center (the pick-up location for click-and-collect orders is stored). At the beginning of each period, $R_{pt}^t$ product $p$ at seller $l$ is replenished due to the demand at that period.

**The amount of sale for sellers:** Let $X_{piz}^t$ denotes a decision variable representing the amount of inventory $p$ shipped from seller $l$ to satisfy online demand of zone $z$ at time period $t$ while $Y_{ps}^t$ shows the amount of inventory used to fulfill the walk-in customers from the local store $s$. Besides, the retailer reserve $C_{ps}^t$ amount of inventory and keep it in the store for C&C orders. Therefore, stores satisfy the walk-in and C&C demand of the same zone as well as the online demand may appear from the same and other zones. For ease of exposition, we substitute the total sale for the seller $l$ for product $p$ at time $t$ by $Q_{pt}^t$ as

$$Q_{pt}^t = \begin{cases} \sum_{z \in Z} X_{piz}^t + Y_{ps}^t + C_{ps}^t, & \forall t \in T, p \in P, s \in S, \\ \sum_{z \in Z} X_{piz}^t, & \forall t \in T, p \in P, l \in F, \end{cases}$$

where the phrases show the total sale of product $p$ at time $t$ for store $s$ and fulfillment center $f$, respectively.

**The start inventory level for sellers:** We consider a periodic inventory system, where the retailer has to make a regular decision on the amount of replenishment at the beginning of period $t$, shown by $R_{pt}^t$ for product $p$ at the seller $l$. The replenished products arrive immediately and will only be available to serve the demand that occurs during the same period. During periods, there is no replenishment opportunity. The start inventory ($I_{pt}^t > 0$) is determined on the basis of the amount of total sale, replenishment amount, and the leftover inventory from the previous period $t - 1$. We apply an order-up-to-level system, in that the start inventory raises to $I_{pt}^t$ at the beginning of each period with zero lead time. This is well-known as $(T, I_{pt}^t)$ in the inventory control system literature (Poormoaied, 2022). The start inventory of seller $l$ at period $t$ is the amount of product $p$ that is left over from the previous period in $t - 1$ given by

$$I_{pt}^t = I_{pt}^{t-1} + R_{pj}^{t-1} - Q_{pt}^{t-1} \quad \forall t \in T, p \in P, l \in L.$$  

The planning period is one day and we assume that the replenishment plan for product $p$ at seller $l$ is predetermined for the entire horizon $T$ according to a weekly calendar such that $R_{pt}^t \neq 0$. 


Supply constraint: To guarantee that the amount of inventory used by each seller to satisfy customers doesn't exceed the available inventory, we present constraints

\[ Q^t_{pl} \leq R^t_{pl} + I^t_{pl} \quad \forall t \in T, p \in P, l \in L. \]  

(3)

Capacity constraints: In our setting, each seller and transportation vehicles have a limited storage capacity. To respect the capacity of seller \( l \) after receiving replenishment products at period \( t \), the below constraints is given to restricting the retrieval quantity to each seller capacity:

\[ R^t_{pl} + I^t_{pl} \leq \bar{u}^t_{pl} \quad \forall t \in T, p \in P, l \in L, \]  

(4)

\[ \sum_{p \in P} x^t_{plz} \leq \bar{k}^t_{lz} \quad \forall t \in T, p \in P, l \in L, z \in Z. \]  

(5)

Fulfillment constraint: Let \( d_{0pz}^t \) denote the online demand of zone \( z \) for product \( p \) at period \( t \). The demand for the walk-in and click-and-collect customers from local store \( s \) is presented by \( d_{ws}^t \) and \( d_{cs}^t \), respectively. Constraints

\[ \sum_{l \in L} x^t_{plz} \leq d^t_{0pz} \quad \forall t \in T, p \in P, z \in Z, \]  

(6)

\[ y^t_{ps} \leq d_{ws}^t \quad \forall t \in T, p \in P, s \in S, \]  

(7)

\[ c^t_{ps} \leq d_{cs}^t \quad \forall t \in T, p \in P, s \in S, \]  

(8)

ensure that the sale is subject to the demand.

Income: The expected cumulative revenues from physical and online sales are

\[ A_{0p} = \sum_{t \in T} \sum_{p \in P} \sum_{l \in L} i^t_{p} Q^t_{pl}. \]  

(9)

The holding costs: Leftover products remaining at the end of the period incur a per unit holding cost equal to \( c_{0pl}^t > 0 \). This cost represents the costs of capital tied up, fulfillment center, space, insurance, taxes, and so on. In our setting, the holding cost is assessed only on inventory left at the end of a period and carries over to the next period. The other cost of holding products temporarily is considered a fixed cost and irrelevant to our setting. The total holding cost of products for each period is presented by
\[ A_{1c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}} c_{o_{pl}}^t I_{p_{l}}^t. \] (10)

**Unsatisfied demand cost:** Retailers policy prohibits deliberately planning for shortages of any of its products. However, a shortage of products occasionally crops up. Consequently, the amount of the product required (demand) exceeds the available stock. Unfulfilled demand for products that are out of stock at a store \((d_{w_{ps}}^t + d_{c_{ps}}^t - Y_{ps}^t + C_{ps}^t > 0)\) is immediately lost and incur per unit penalty cost \(l_{w_{ps}}^t\) and \(l_{o_{ps}}^t\) for online orders exceeding the sellers’ inventory and includes the loss of current revenue from not meeting the demand plus the cost of losing future business because of lost goodwill as shown by

\[ A_{2.1c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{z \in \mathcal{Z}} l_{o_{ps}}^t (d_{o_{ps}}^t - \sum_{l \in \mathcal{L}} X_{p_{lz}}^t). \] (11)

\[ A_{2.2c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}} l_{w_{ps}}^t (d_{w_{ps}}^t + d_{c_{ps}}^t - Y_{ps}^t - C_{ps}^t). \] (12)

**The replenishment cost:** The fixed cost of replenishing products is assumed to be zero. The retailer pays \(c_{r_{pl}}^t\) unit variable cost for replenishing product \(p\) for seller \(l\) in period \(t\). The replenishment cost is given by

\[ A_{3c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}} c_{r_{pl}}^t R_{p_{l}}^t. \] (13)

**The total handling cost:** The fixed cost of ordering assumes to be zero. The handling cost for online demand of zone \(z\) associated with preparing and shipping product \(p\) at time period \(t\) for seller \(l\) is given by \(c_{h_{p_{lz}}}^t\). The handling cost for online orders is calculated by

\[ A_{4c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}} \sum_{z \in \mathcal{Z}} c_{h_{p_{lz}}}^t X_{p_{lz}}^t \] (14)

**The store fulfillment cost:** The unit cost \(c_{f_{ps}}^t\) for fulfilling the demand of walk-in and click-and-collect orders including rent, overhead, labor, etc. are associated with products type, stores, and periods, that is the cost of store \(s\) for product \(p\) at time period \(t\). So, the total store fulfillment cost is given by

\[ A_{5c} = \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} c_{f_{ps}}^t (Y_{ps}^t + Z_{ps}^t). \] (15)

Constraints

\[ I_{p_{l}}^t, R_{p_{l}}^t, Q_{p_{l}}^t \geq 0 \quad \forall t \in \mathcal{T}, p \in \mathcal{P}, l \in \mathcal{L} \] (16)

restrict variables to a non-negative amount. The model formulation is given as
\[ \min (A_{0p} + A_{1c} + A_{2,1c} + A_{2,2c} + A_{3c} + A_{4c} + A_{5c}). \]  

(17)

4 Numerical experiments

This section presents the set of parameters, explains how the data was generated, and presents the numerical results.

Data Generation:

The time horizon \( T \) is fixed to 14 time periods (days) to represent two weeks of the retailer. For each instance, the real replenishment schedule of each seller was applied such that sellers visited once at the beginning of each week, during weeks there is no replenishment opportunity. The start inventory is set as 120 and 250 per product for each store and fulfillment center, respectively. The walk-in, online, and click-and-collect demand for each customer zone was generated randomly as \( U_d[1,100], U_d[1,60], U_d[1,35] \), respectively.

The available transportation and storage capacity for each store were estimated 1.2 higher than the total mean (walk-in, click-and-collect, and online) demand of a zone (7,74) while for the fulfillment center was 1.2 higher than the online mean demand of all zones (56,56).

The other values for the settings and parameters are present in Table 2.

<table>
<thead>
<tr>
<th>Table 1: Instance generation parameters</th>
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<tbody>
<tr>
<td>Parameters</td>
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<tr>
<td>Price per product: 40$</td>
</tr>
<tr>
<td>Replenishment charge per price</td>
</tr>
<tr>
<td>Holding charge per price</td>
</tr>
<tr>
<td>Handling charge per price</td>
</tr>
<tr>
<td>Lost sale cost per price</td>
</tr>
<tr>
<td>Walk-in fulfilment cost per price</td>
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The demand distribution for the walk-in, click-and-collect, and delivery orders for product 1 at time 2 is presented in Figure 2. The figure shows the demand and commutative demand for zone 1 over time.
Solution approach and computation result:

All experiments were carried out on a PC with Intel Corei7, CPU 2.6 GHz, 16 GB of RAM, using a Gurobi solver version 11.5. The maximization problem for the deterministic model was solved in 0.002323 seconds with 3418 Variables and 1204 constraints and the objective value are 636221.5 $. The most important results are presented as follows.

Start Inventory: The Start inventory of the fulfillment store and stores 1 are shown in Figure 3. The beginning inventory for sellers is raised to the highest point at the replenishment date (at the beginning of each period) and then continue to fall between two replenishment period.

Figure 2: Demand distribution for product 1 at time 2

Figure 3: Start inventory for the FC and store 1 over time.
The total online sale: The online sale for stores in comparison to the online sale of the fulfillment center is presented in Figure 4. The figure shows that the total online sale for stores is higher than the fulfillment center. Although the handling cost for the FC is less than a store, this cost increase on the distance and it is more profitable if the online orders are served by the stores at the same zones, however establishing more fulfillment centers and a close distance from customer zones, may improve the objective value.

![Figure 4: Start inventory for the FC and stores for product 1 over time](image)

The results are also drawn based on changes in demand as depicted in Figure 5. The results shows that the objective function is positively affected by increasing the customer demand. By changing the customer demand by 50%, the demand change to almost to 50%.

![Figure 5: The relationship between demand and objective function](image)

5 Conclusion

This study investigated a capacitated omnichannel retailing problem to integrate tactical and strategic decisions on inventory management and fulfilment optimization. By examining the results, some insights which abide by the market rules and the unique structure of the problem were drawn. Finally, a fruitful area of future research would be to expand the conclusions beyond the research by considering the role of rival retailer as
well as stochastic demand in the model. Combining different solution method such as the mix integer programming and machine learning is another interesting area for the future research.

Reference


