

EFFECT OF A GROUND PLANE ON THE
INDUCED POWER REQUIRED BY A HOVERING HELICOPTER

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TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS	ii
NOTATIONS	iv
LIST OF TABLES	vi
LIST OF FIGURES	vii
SUMMARY	viii
Chapter	
I. INTRODUCTION	1
II. PROCEDURE	4
III. DISCUSSION OF RESULTS	8
IV. CONCLUSIONS	10
APPENDIX I. Calculation of Correction Factor for Wake Vortex Sheet Strength and Its Application	11
APPENDIX II. Thrust Coefficient Ratio Calculations	13
APPENDIX III. Tables of Calculated Results	15
APPENDIX IV. Figures	18
BIBLIOGRAPHY	22

NOTATION .

- c_{d_0} = Blade element profile drag coefficient.
 C_Q = Torque coefficient.
 C_T = Thrust coefficient.
 C_{T_∞} = Thrust coefficient out of ground effect.
 d_1 = Nondimensional shortest distance from a point P to the circular axis of a vortex ring.
 d_2 = Nondimensional longest distance from a point P to the vortex ring.
 $\frac{d\Gamma}{dz}$ = The strength of the wake vortex sheet per unit length in Z-direction.
 $E(\tau)$ = Complete elliptic integral of the first kind.
 $K(\tau)$ = Complete elliptic integral of the second kind.
 Q = Nondimensional volume flow.
 r = Radial distance from wake axis.
 R = Rotor radius.
 $\frac{z}{R}$ = Nondimensional ground height.
 s = Normal distance from a point P to the vortex ring axis of symmetry.
 T = Rotor thrust.
 V_i = Normal component of the nondimensional induced velocity.
 \bar{v} = Mean induced velocity over the rotor disk.
 y = Vertical coordinate of the wake element.
 Z = Distance of the rotor from ground plane.

Γ = Bound vortex ring strength.

δ_0 = Value of profile drag at $c_l = 0$

ϵ = Constant in power equation for c_{d_0} ($c_{d_0} = \delta_0 + \epsilon c_l^2$)

λ_i = Nondimensional mean induced velocity $\left(\frac{v}{\sqrt{\frac{T}{2 \rho \pi R^2}}} \right)$

η = Correction factor for wake vortex sheet strength.

ρ = Air density.

σ = Rotor solidity.

$$\tau = \frac{d_2 - d_1}{d_2 + d_1}$$

LIST OF TABLES

Table	Page
1. Approximate Nondimensional Wake Radii for $\frac{Z}{R} = \frac{1}{4}, \frac{1}{2},$ and 1.	15
2. Approximate Nondimensional Wake Radii for $\frac{Z}{R} = \infty$	16
3. Total Nondimensional Values of Induced Velocities Over the Rotor Radius	17

LIST OF FIGURES

Figure	Page
1. Spread of Hovering Wake Due to Presence of Ground Plane	18
2. Cylindrical Vortex System for Ground Effect Analysis	18
3. Induced Velocity Distribution Over the Rotor Radius	19
4. Variation of the Mean Nondimensional Induced Velocity with Ground Height	20
5. Thrust Ratio Variation with Ground Height at Constant Power	21

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SUMMARY

An analytic study of the influence of ground effect on the induced power required for a hovering helicopter is presented. The theory is extended by considering the expansion of the wake as it approaches the ground and also the effects of the bound vortex ring enclosing the periphery of the rotor.

Comparison of this extended theory with available experimental data over the range of ground heights of interest shows better agreement than previous analytical studies.

CHAPTER I

INTRODUCTION

The presence of the ground plane influences the performance of a hovering helicopter. The character and relative magnitude of this influence is dictated by the change in the wake vortex system which in turn is a function of the ground height. The problem of determining the exact values of the velocities induced by such a vortex system is extremely difficult because of the complexity of the flow patterns.

Glauert (1) investigated one phase of the problem in his early attempt to determine the influence of the slip stream contraction on the induced velocity. He found that the magnitude of the induced velocity was decreased by taking the effect of the slipstream contraction into account. However, he was unable to obtain a solution for hovering and low advance ratios.

Previous mathematical analyses of the ground effect problem were made by Betz (2) and Knight and Hefner (3).

Betz, in his approximate treatment of the ground effect problem, assumed constant thrust and simplified the analysis by taking very small and very large ratios of ground height to rotor radius. For the case of small ground heights the direction of the flow was assumed to be parallel to the ground plane. An approximate expression for the quantity of flow then was obtained and compared with the case where the rotor was at an infinite distance from the ground plane. For the case of the rotor operating at a large distance

from the ground, the reflected image of the rotor was replaced by a point sink that gave rise to an additional axial induced velocity at the plane of the rotor which is not in accord with the experimental data. The ground effect was given as the ratio of the induced velocity at the rotor to the velocity induced by a wake extending to infinity. The results obtained by the above method predicts a linear variation of induced power up to a ground height of half rotor radius. At larger ground heights Betz assumed the ground plane to have no effect on the induced power required by a hovering helicopter.

A more comprehensive mathematical analysis on the ground effect was made by Knight and Hefner. In this analysis, the complexity of the problem was reduced by the following stated assumptions:

1. The number of blades could be taken as infinite.
2. The usual small angle approximations could be made.
3. Rotational and radial components and tip effects could be neglected.
4. The slipstream contraction could be neglected.
5. The circulation of the blades could be taken as constant.

The wake was represented by a cylindrical vortex cylinder of uniform sheet strength. On the basis of an additional assumption that the thrust was independent of ground height, the method of reflection was employed in obtaining the velocities induced at the rotor by the cylindrical wakes. Knight's approach to the problem of determining the induced velocities at the rotor is precise from the mathematical standpoint. However, as a result of errors introduced by the simplifying assumptions, the effects of the ground plane are overestimated.

The present analysis is an attempt to obtain a more accurate theoretical treatment by taking into account the principal effects of the "bound vortex ring" presented by Castles (4) and the initial contraction and subsequent spread of the wake.

CHAPTER II

PROCEDURE

For rotors having a rotational tip speed which is large compared to the normal velocity and constant circulation along the blade radii, the wake boundary can be considered to be composed of an infinite number of very closely spaced vortex rings of infinitesimal strength. This vortex configuration is equivalent to a continuous vortex sheet. In the case of axially symmetric flows the planes of the wake vortex rings can be considered to remain parallel to the plane of the rotor.

The interactions of the successive rings generated by a hovering rotor in combination with the pressure gradients along the ground plane will cause the wake to spread as it approaches the ground, as shown schematically in Fig. 1.

It follows that the first step in determining the induced velocity distribution over the rotor is to find the approximate shape of the wake. The ground plane can be made a plane of symmetry by placing an image of the wake vortex system below the ground plane. Then an iteration procedure starting with a cylindrical wake and a bound vortex ring at the periphery of the rotor may be employed to determine the approximate shape of the wake. The radius of the wake at any axial station can be determined from continuity by equating the flux across the rotor to the flux crossing a horizontal plane through a given axial station. The flux at an axial station is computed from the vertical component of the mean velocity induced by the vortex system and its image. For

an approximate solution the induced velocities at the symmetrical axis can be substituted for the mean induced velocities.

The second step is the determination of the vertical components of velocities over the rotor disk that are induced by the new vortex system obtained from step one. Graphical integration is required for this evaluation.

In the course of investigating the ground effect problem, it was found necessary to reduce the complexity of the mathematical analysis by making the following assumptions:

1. The circulation is assumed constant along the radius.
2. The vertical component of the vortex sheet strength remains constant.
3. The tangential velocity of the blade tips can be considered very large compared to the induced velocities.
4. Thrust is independent of the ground height.
5. The bound vortex ring strength remains constant.

The expressions necessary to find the vertical component of the velocity induced by a ring and a cylindrical wake, respectively, are given by Castles and DeLeeuw (5). However, the use of the image method necessitates changes in the form of these equations. Thus, for the coordinate system shown in Fig. 2, the vertical component of the velocity induced by the bound vortex ring and its image at any point on the symmetrical axis is

$$v_{i(\text{ring})} = \frac{1}{2} \left\{ \frac{R^2}{[(z-y)^2 + R^2]^{3/2}} - \frac{R^2}{[(z+y)^2 + R^2]^{3/2}} \right\} \quad (1)$$

and similarly for the wake vortex sheet.

$$V_{i(\text{wake})} = \frac{1}{2} \frac{d}{dZ} \left[\frac{Z-y}{\sqrt{(Z-y)^2 + R^2}} + \frac{2y}{\sqrt{y^2 + R^2}} - \frac{Z+y}{\sqrt{(Z+y)^2 + R^2}} \right] \quad (2)$$

where,

$$= 0.257 \quad (\text{The strength of the vortex ring given by Castles (4)})$$

and for step one,

$$\frac{d\Gamma}{dZ} = 1 \quad (\text{The strength of the sheet per unit length in } Z \text{ - direction})$$

The radius of the wake at any given cross section is then given to a first approximation by

$$\frac{r}{R} = \sqrt{\frac{Q}{\pi (V_{i\text{ring}} + V_{i\text{wake}})}} \quad (3)$$

where,

Q = non-dimensional volume flow through rotor.

It is also shown by Castles and DeLeeuw (5) that the vertical component of velocity at any point induced by a vortex ring is given by the equation

$$V_{i\text{ring}} = \frac{\Gamma}{2\pi s} (A + CDE) \quad (4)$$

where,

s = normal distance from the point P to the ring axis of symmetry

$$A = K(\tau) - E(\tau)$$

$$\tau = \frac{d_2 - d_1}{d_1 + d_2}$$

d_1 = least distance from P to circular axis of ring

d_2 = greatest distance from P to circular axis of ring

$K(\tau)$ = complete elliptic integral of the second kind

$E(\gamma)$ = complete elliptic integral of the first kind

$$B = \frac{s-R}{d_1} + \frac{s+R}{d_2}$$

R = radius of the ring

$$C = d_1 + d_2$$

$$D = \frac{\gamma E(\gamma)}{1-\gamma^2}$$

$$E = \frac{1}{R} - \frac{y^2 + s^2 + R^2 - d_1 d_2}{2s^2 R} - \frac{(s+R)d_1^2 + (s-R)d_2^2}{2sRd_1 d_2}$$

Equation (4) is employed in the second step to find the velocities at the rotor induced by the wake vortex elements the positions of which have been determined by the previous equations. Because of the mathematical difficulties involved in evaluating the sheet strength at different axial stations, (i.e., determining the induced velocities at the sheet on each side of the sheet), an approximate correction factor to account for the variable wake sheet strength had to be employed. The derivation and the use of this correction factor is explained in Appendix I.

CHAPTER III

DISCUSSION OF THE RESULTS

The nondimensional induced velocity distribution over the rotor radius is plotted for different ground heights in Fig. 3. As can be seen from this figure the value of nondimensional induced velocities are equal to unity at $\frac{r}{R} = 0.954$ for all ground heights. This result is obtained on the basis of the assumptions of a constant thrust and bound vortex ring strength and on the basis of the results obtained by Castles (4) as follows: In order to eliminate physically impossible, infinite velocities and pressure gradients at the edge of the rotor, isolation of this point from the flow is necessary. The assumption is therefore made that the vortex sheet enclosing the wake unrolls from the periphery of the rotor in the form of a spiral. For mathematical purposes this phenomenon may be represented as a bound vortex ring with circular axis located at the periphery of the rotor. The nondimensional value of the velocity of the sheet where it leaves the plane of the rotor is shown by Castles (4) to be a function solely of the bound vortex ring strength and equal to unity. Since it is assumed in the present report that the bound vortex ring strength is constant it follows that the nondimensional wake vortex sheet velocity can be taken as constant for all ground heights at the radius at which the sheet leaves the rotor.

It is important to note at this point that the peaks of the curves at $\frac{r}{R} = 0.954$ as shown in Fig. 3 would be rounded in the actual case by the effects of viscosity.

In order to show the influence of the ground plane on the induced power required by a hovering helicopter, the nondimensional mean induced velocity λ_i is plotted on Fig. 4, versus nondimensional ground height where

$$\lambda_i = \frac{v}{\sqrt{\frac{T}{2\rho\pi R^2}}}$$

and v is defined as the average value of the normal component of the induced velocity over the rotor. For comparison purposes the λ_i values obtained from Knight's (3) results are also given in Fig. (4).

As can be seen from Fig. 4, the values of the mean induced velocities obtained by the present analysis are larger than those obtained by Knight (3). This difference is principally attributable to the neglect in Knight's analysis (3) of the effects of the bound vortex ring.

A summary of the results of helicopter model tests are shown by Gessow and Myers (6) in the form of a plot of the thrust ratio at constant power versus ground height. Several points obtained from full-scale flight tests are also given on this plot.

In order to compare the calculated results of the present thesis and the values obtained from Knight's (3) report, with the above experimental data thrust coefficient ratios $\frac{C_T}{C_{T_{\infty}}}$ at constant power were calculated by the method given in Appendix II for a typical single-rotor helicopter operating at different ground heights. The calculated and experimental thrust ratios are compared in Fig. 5.

The results of the present thesis indicate good agreement with the experimental data.

CHAPTER IV

CONCLUSIONS

1. The reduction in the induced power required by a hovering helicopter that arises from the presence of a ground plane amounts to about 36% at a rotor ground height of half a rotor radius and decreases with increasing ground distance to a value of about 14% at a height of a rotor radius.
2. The variation of thrust at constant power with rotor ground height as calculated by the present method is in good agreement with the experimental results over the range for which experimental data is available.
3. The results of the present investigation indicate that it is necessary to take into account the presence of the bound vortex ring in order to obtain the effects of the ground plane on the induced power with sufficient accuracy for performance calculations.

APPENDIX I

It is shown by Castles (4) that the strength of the wake vortex sheet does not remain constant along the wake. Although the approximate value of the sheet strength at the plane of the rotor is half the ultimate wake sheet strength, it is very difficult to determine its relative strength at any point between these limits.

However, an approximate correction factor for the effect of the variation of sheet strength can be found by considering the known value of the flux at the rotor due to a wake vortex sheet extending to infinity and comparing this value with the calculated flow for a similar wake having a constant sheet strength.

The actual value of the flux given by Castles (4) for a wake vortex sheet extending to infinity and having a unit strength at infinity is

$$Q_{\text{actual}} = \frac{\pi}{4}$$

The calculated value of the flux on the basis of the assumption that the vortex sheet strength has a constant value of unity is

$$Q_{\text{const. strength}} = 1.229$$

Therefore, the correction factor η for the uniform wake sheet strength can be expressed as the ratio of the calculated flux to the actual flux. Thus

$$\eta = \frac{1.229}{\frac{\pi}{4}} = 1.565$$

It follows that the approximate value of the correction to the velocity components induced by the assumed constant strength wake vortex sheet amounts to $1/\eta$.

The application of the factor η to obtain the nondimensional induced velocity distribution over the rotor radius for a ground distance of $\frac{Z}{R} = \frac{1}{4}$ is shown below.

Rotor sta.	V_{i_wake}	$\frac{V_i}{\eta}$	V_{i_ring}	V_{i_total}
0	0.0128	0.0082	0.0362	0.044
0.200	0.0132	0.0084	0.0399	0.048
0.400	0.0187	0.0120	0.0567	0.069
0.600	0.0315	0.0201	0.0965	0.117
0.800	0.0691	0.0442	0.2234	0.268
0.954	-	-	-	1.000
1.000	0	0	0	0

APPENDIX II

The torque coefficient C_Q for a hovering helicopter may be expressed for typical blade circulation distributions as

$$C_Q = \sqrt{\frac{1}{2}} \lambda_i C_T^{3/2} + \frac{1}{8} \delta_o \sigma \left(1 + \frac{4}{9} C_T\right) + \frac{9}{2} \epsilon \frac{C_T^2}{\sigma} \quad \text{where}$$

λ_i = nondimensional mean induced velocity

C_T = thrust coefficient

δ_o = value of profile drag coefficient at $c_l = 0$

ϵ = constant in power equation for c_{d_o} ($c_{d_o} = \delta_o + \epsilon c_l^2$)

σ = solidity of the blades

For a typical single-rotor helicopter,

$C_{T_{\infty}}$ = 0.0055 (thrust coefficient out of ground effect)

σ = 0.06

$\delta_o \approx \epsilon$ = 0.008

Using the above values and taking $\lambda_i = 1.09$, the value derived by Castles (4) for a hovering helicopter out of ground effect, the value of the torque coefficient is $C_Q = 0.0003927$.

This constant value of C_Q may then be substituted into the torque coefficient equation. The resulting values of the thrust coefficient C_T may then be calculated for the different ground heights using the calculated values of λ_i .

Because of the nature of the torque equation, graphical methods were employed to find the values of C_T for the different ground heights. The

results are expressed in the form of thrust coefficient ratios, $\frac{C_T}{C_{T\infty}}$, and are given below:

Ground Height Relative to Rotor Radius $\frac{Z}{R}$	Nondimensional Mean Induced Velocity λ_i	Thrust Coefficient C_T	Thrust Coefficient Ratio $\frac{C_T}{C_{T\infty}}$
0.25	0.512	0.00860	1.564
0.50	0.698	0.00715	1.300
1.00	0.944	0.00601	1.093
∞	1.090	0.0055	1

APPENDIX III

Table 1. Approximate Nondimensional Wake Radii

for $\frac{Z}{R} = \frac{1}{4}, \frac{1}{2}$ and 1

Nondimensional Axial Distance from Rotor	Nondimensional Wake Radii, $\frac{r}{R}$ at		
	$\frac{Z}{R} = 1$	$\frac{Z}{R} = \frac{1}{2}$	$\frac{Z}{R} = \frac{1}{4}$
0	0.954	0.954	0.954
1/6	0.936	1.017	1.038
1/3	0.956	1.112	1.152
1/2	1.013	1.230	1.352
2/3	1.173	1.521	1.616
5/6	1.640	2.133	2.282
1	∞	∞	∞

Table 2. Approximate Nondimensional Wake Radii For $\frac{Z}{R} = \infty$

Nondimensional Axial Distance from Rotor	Nondimensional Wake Radii, $\frac{r}{R}$ $\frac{Z}{R} = \infty$
0	0.954
.5	0.837
1.0	0.798
1.5	0.781
2.0	0.772
2.5	0.768
3.0	0.765
3.5	0.763
∞	0.707

Table 3. Total Nondimensional Values of Induced
Velocities Over the Rotor Radius

Rotor Station	Nondimensional Values of Induced Velocities			
	$\frac{Z}{R} = \infty$	$\frac{Z}{R} = 1$	$\frac{Z}{R} = \frac{1}{2}$	$\frac{Z}{R} = \frac{1}{4}$
0	0.431	0.283	0.130	0.044
0.200	0.432	0.292	0.137	0.048
0.400	0.438	0.318	0.163	0.069
0.600	0.456	0.375	0.224	0.117
0.800	0.556	0.523	0.385	0.268
0.954	1.000	1.000	1.000	1.000
1.000	0	0	0	0

APPENDIX IV

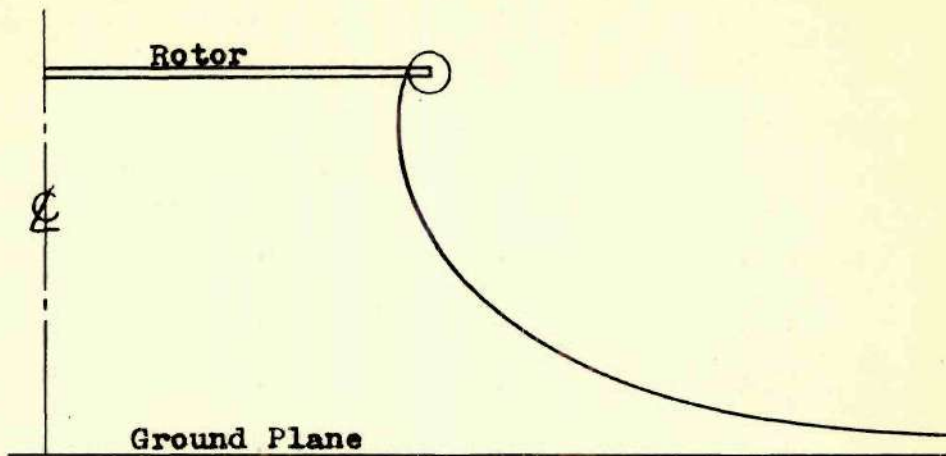


Figure 1
Spread of Hovering Wake Due to Presence of
Ground Plane

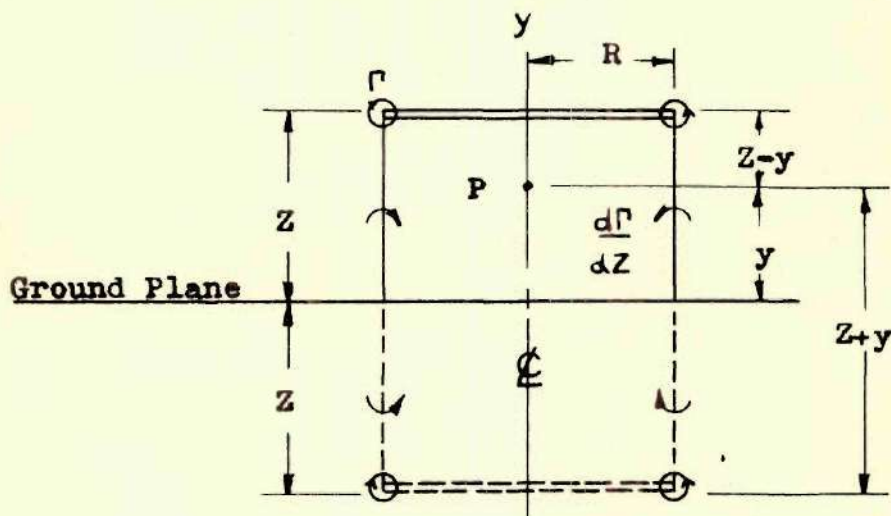


Figure 2
Cylindrical Vortex System for Ground
Effect Analysis

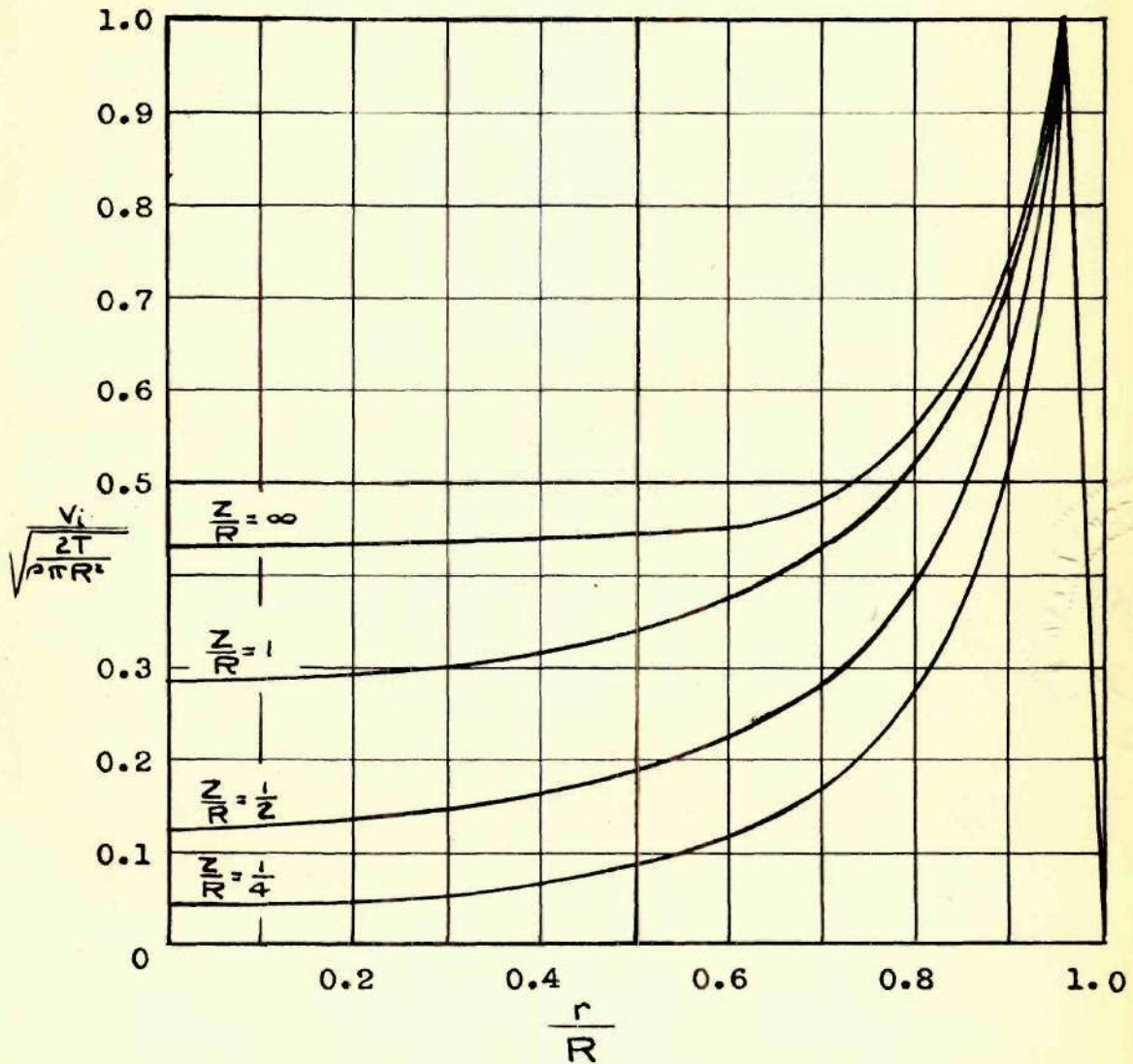


Figure 3
Induced Velocity Distribution over the
Rotor Radius

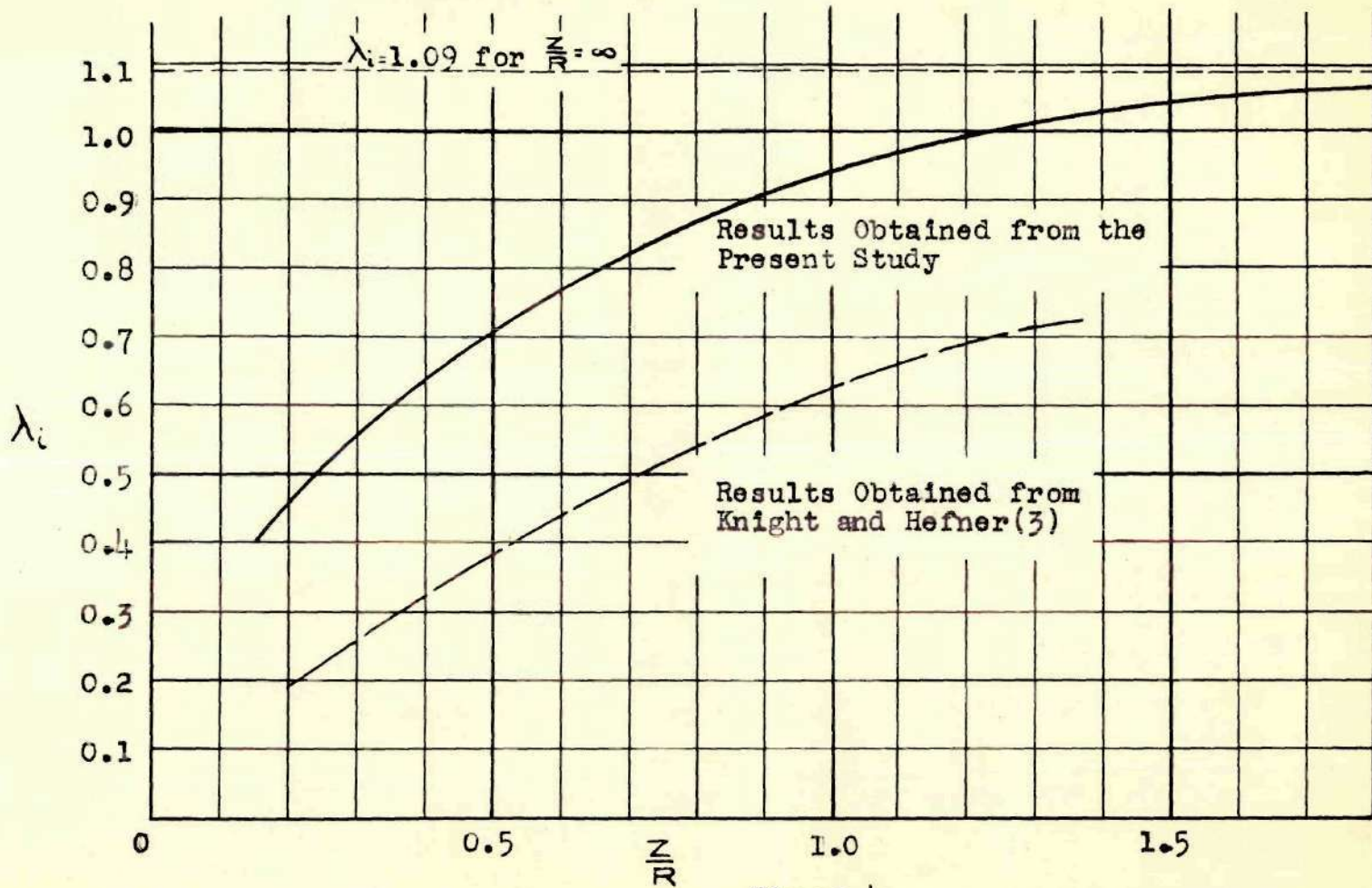


Figure 4
 Variation of the Mean Nondimensional Induced
 Velocity with Ground Height

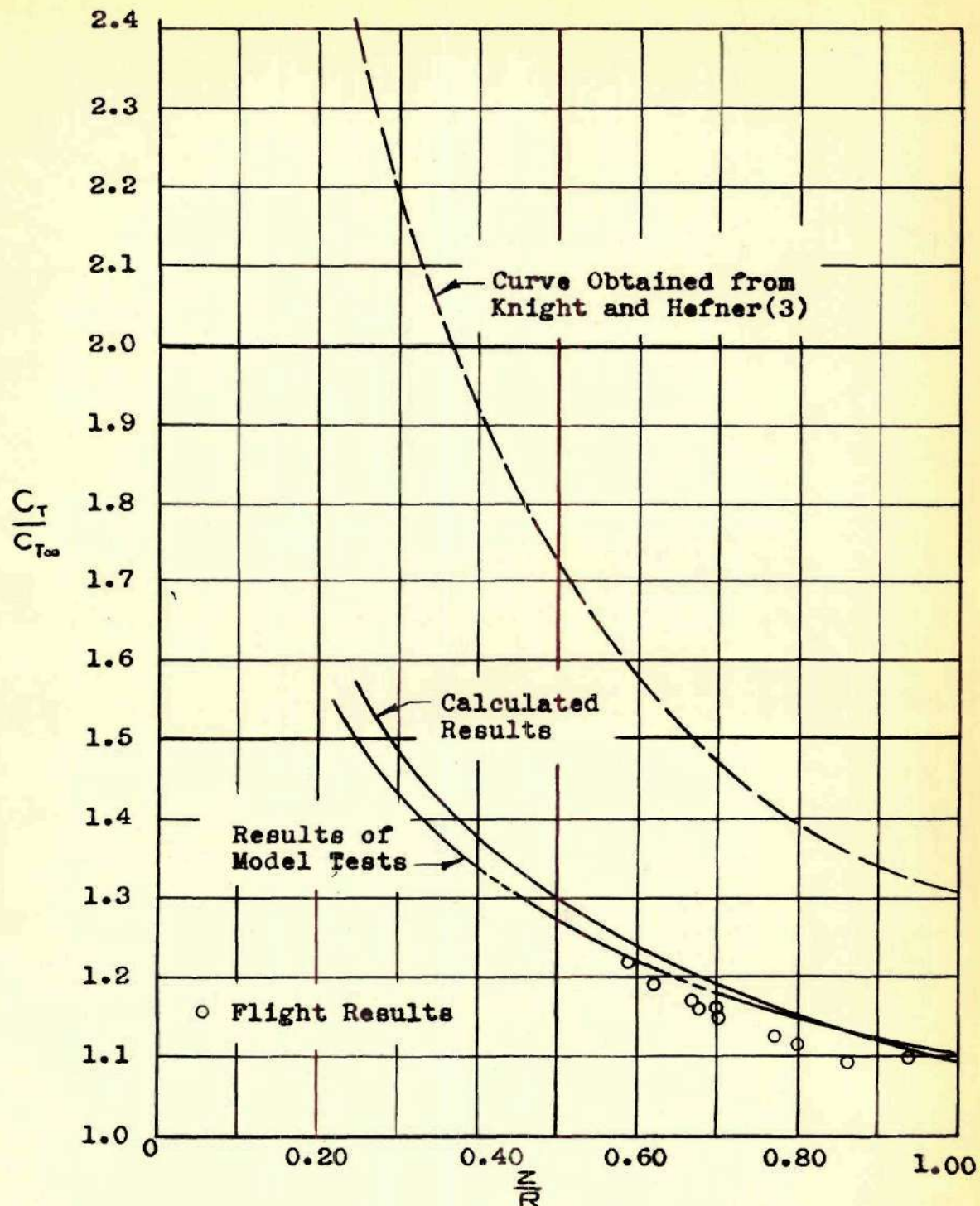


Figure 5
Thrust Ratio Variation with Ground Height
at Constant Power

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