

# Elementary Effects Sensitivity Analysis of Lunar PNT Performance Metrics

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## BIOGRAPHY

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## ABSTRACT

Multiple space agencies have announced plans to deploy position, navigation, and timing (PNT) systems to support lunar operations. Indeed, it has consistently been evaluated as one of the most important aspects to support sustained operations. Most existing work has focused on optimizing combinations of frozen orbits to have acceptable Dilution of Precision (DoP) characteristics, then analyzing the effect of technology on specific designs. However, the most common dilution of precision measures assume that the error of each measurement is identical. By loosening the identical error assumption, geometric dilution of precision (GDOP) becomes user equivalent range error (UERE) weighted dilution of precision, or KDOP. KDOP introduces a recoupling of the geometry of a constellation and the technologies on board. The objective of this work is to conduct a full space sensitivity analysis across a number of performance measures: KDOP, UERE, GDOP, and coverage. To conduct sensitivity analysis, a full lunar PNT model with orbit determination and time synchronization support from GPS, high-fidelity orbital dynamics, and dynamic clock models will be utilized. Due to the size of the design space and computational requirements of the model, the elementary effects, or method of Morris, was selected for the sensitivity analysis. This sensitivity analysis method provides a qualitative ranking of the importance of parameters, operating as a good proxy of the total sensitivity index. It was found that the elementary effects method was capable of identifying known relationships in the design space. The comparison of the analysis with two and six orbital planes demonstrated interactions between the design parameters and the number of planes. KDOP and UERE were shown to have strong sensitivities to both orbital and technology parameters, showing that design space explorations should cover both classes of design variable.

## I. INTRODUCTION

LunaNet is a collaborative interoperable system-of-systems planned to provide Communication, Positioning, Navigation, and Timing support to spacecraft operating in lunar space (Israel and Gramling, 2023). The first phases of deployment will begin within the decade, if current plans hold (Melman et al., 2025). These systems will be interoperable and available to all users, but they are not planned to have the same exact performance (Melman et al., 2025), similar to the Global Navigation Satellite System. With the LunaNet systems' imminent arrival, spacecraft designers must be able to estimate their performance.

Positioning, Navigation, and Timing (PNT) systems are most often characterized by an error metric, such as User Equivalent Range Error (UERE) or Signal-in-Space Error (SISE), and a dilution of precision metric; in fact, SISE and Geometric Dilution of Precision (GDOP) are used as requirements for the proposed NASA component of LunaNet (Speciale, 2022). Dilution of

Precision measures the increase in the error seen by the user due to the non-ideal alignment of the measurements (Hegarty et al., 2017). Most commonly the error is assumed to be identical and identically distributed amongst the measurements, a necessary condition to yield GDOP. However, since the system performance is not identical and there are variations in the orbits, this assumption does not hold for all cases (Gabhart et al., 2025). This is not a new problem with it established that alternative performance measures are more apt when SISE varies (Hegarty et al., 2017). Some of the first work on lunar PNT derived a weighted dilution of precision measure (Sands et al., 2006) and a recent study utilized a weighted formulation to optimize phased deployments (Iiyama and Gao, 2025). In this work, the focus will be on UERE weighted dilution of precision (KDOP), which uses each of the measurements received equally while including the individual UERE values (Sairo et al., 2003).

With a weighted formulation, the separation between the orbit and the technologies is eliminated. The UERE or SISE errors are a function of the orbit for systems receiving updates from Earth, and the dilution of precision is a function of those errors in a weighted formulation. This couples the technology and orbit selection problems even tighter. An example of the effect of this is the result from Mina et al., 2025, where an unexpected GNSS outage and clock performance led to unacceptable SISE, in simulation. This work seeks to provide insight into this coupling through a combined sensitivity analysis across both orbital and technological parameters, utilizing a weighted dilution of precision metric as one of the performance measures. A combined analysis is novel among the existing work which tends to focus on either technological or orbital trades. Overall, the goal of the sensitivity analysis is to determine whether any input parameters can be assigned a lower priority in future work on the utility of KDOP. In Sec. II, sensitivity analysis techniques will be reviewed before one is selected and the existing literature on lunar PNT performance sensitivities is presented. These previous sensitivity analyses will be used to garner additional insight from the results. Section III will detail the modeling environment used, the performance measures, and the specific of the sensitivity analysis conducted. The results, including a discussion of convergence are presented in Sec. IV before conclusions and plans for future work in Sec. V.

## II. BACKGROUND

### 1. Sensitivity Analysis Techniques

Sensitivity analysis is a process through which the uncertainty in the estimate of a model can be attributed to the uncertainty in a specific input (Saltelli et al., 2008b). In specific problems, sensitivity analyses can be used to prioritize the investigation of certain factors for which the output has a high sensitivity (Reed et al., 2022; Saltelli et al., 2008b). If a parameter has a low enough sensitivity, an analyst might even be able to fix the value of that input and reduce the dimensionality of the problem (Saltelli et al., 2008b). This is the goal here since the design space of lunar PNT systems is of very high dimensionality without restrictions on the orbital designs.

To select a sensitivity analyses technique, the limitations of the techniques must be assessed. Sensitivity analyses can be conducted locally about one point or globally. The focus here will be on global methods, but local methods are useful when investigating a single design point (Reed et al., 2022). A common global technique is the evaluation of scatterplots (Reed et al., 2022). Scatterplot methods are less applicable when there are interactions between inputs or many inputs (Saltelli et al., 2008b). The most applicable technique is the global Sobol sensitivity analysis or other all at a time techniques (Reed et al., 2022), like that performed in Pereira et al., 2022. However, that technique requires  $O(1000d)$  model evaluations to converge, where  $d$  is the dimensionality of the problem (Reed et al., 2022). This is prohibitive for the models being evaluated here. A compromise between computational cost and information gained is the elementary effects method, or Method of Morris (Saltelli et al., 2008a), which required only  $O(10d)$  model evaluations (Reed et al., 2022). This is a one at a time method which can produce a good qualitative proxy of the total sensitivity index (Saltelli et al., 2008a). In the elementary effects method, the change in an output is quantified along trajectories which step through levels of the input variables one at a time (Saltelli et al., 2008a). These changes are then averaged to generate a composite index,  $\mu$  (Saltelli et al., 2008a). An adjustment to the calculation to take absolute values produces the measure,  $\mu^*$ , which can be a better measurement of non-monotonic relationships (Saltelli et al., 2008a). Since the elementary effects method is applicable to non-monotonic discontinuous problem (Reed et al., 2022), it can be applied to a wide range of problems without *a priori* knowledge. Therefore, the elementary effects method is selected for this effort.

### 2. Lunar PNT Sensitivity Analysis

Existing investigations into the sensitivities of lunar PNT system performance are divided by orbital and technology parameters. Orbital studies focus heavily on GDOP performance and technology studies on SISE or user performance. Starting with orbital parameter studies, Bender et al., 2023, Gabhart et al., 2024, and Iiyama and Gao, 2025 conducted analysis through analyzing plots, and Pereira et al., 2022 conducted a Sobol sensitivity analysis. All of these focused on GDOP derived measures of performance for PNT evaluation, and Iiyama and Gao, 2025 and Pereira et al., 2022 focused on frozen orbits. It is well understood that increasing the number of satellites improves the performance strongly (Bender et al., 2023; Gabhart et al., 2024; Iiyama and Gao, 2025; Pereira et al., 2022), whereas increasing only the number of planes does not have as direct of an effect (Iiyama and Gao, 2025; Pereira et al., 2022). Semimajor axis has a strong effect on coverage (Bender et al., 2023; Iiyama

and Gao, 2025), but could come at the cost of signal quality (Murata et al., 2022). Gabhart et al., 2024 found that elliptical constellations performed better over GDOP than circular constellations, achieving similar levels of performance with fewer assets. Inclination visually had a more complicated relationship with GDOP performance (Bender et al., 2023; Iiyama and Gao, 2025), which is quantified in its total Sobol index (Pereira et al., 2022). Pereira et al., 2022 also found strong secondary effects from the phasing of the satellites.

Technology parameter investigations have been primarily focused on trade studies around specific technology options. In this vein, Small et al., 2022 and Bhamidipati et al., 2023 both traded specific clock options and found significant performance changes. Hartigan and Lightsey, 2025 also found significant clock effects and that in the presence of a high precision clock ephemeris error dominates. Gabhart et al., 2025 studied monotonic relationships across several technologies assuming a continuous parameter space. Clock performance was again found to be important, but the availability of position and time updates from GPS were stronger effects. From these results, it is expected that the number of satellites and clock parameters should appear prominently in the combined sensitivity analysis.

### III. TECHNICAL APPROACH

#### 1. Modeling Environment

The studies will be conducted using an environment built with System ToolKit (STK) and its python API. This allows for custom calculations while leveraging industry standard high-fidelity physics models. A high level block diagram of the environment is shown in Fig. 1.

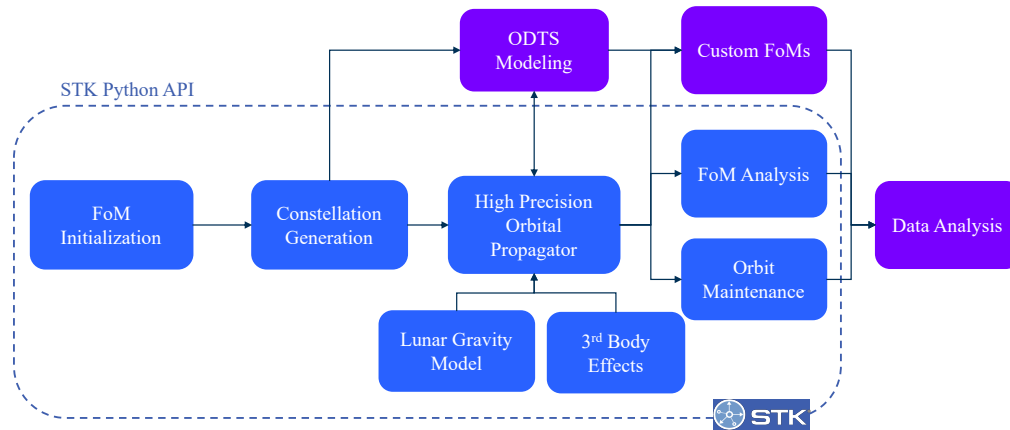


FIGURE 1 Evaluation Environment with STK Modules in Blue and Custom Modules in Purple

For the purposes of this work, the GRAIL 440B lunar gravity model was used with third body perturbations from the Earth and Sun in the High Precision Orbit Propagator. Data was collected at more than 300 points south of 85°S latitude. The evaluation time was set to two days, since previous work had found performance to converge at that time. Data was collected every 175 seconds. Position and time updates were assumed to be provided by GPS satellites that were propagated with the SPG4 propagator. Updates were received at regular intervals set by an input parameter when line-of-sight conditions were met with line of sight limited by a different input parameter. The on-board clocks were modeled as put forward by Bhamidipati et al., 2022. Both the clock and the position estimates were assumed to receive and track updates with a Kalman filter.

#### 2. Figures-of-Merit

When evaluating the performance of a lunar PNT system, there are Figures-of-Merit (FoMs) that can be used. They can be broken down into the broad categories of performance, orbital stability, and cost. In this work, the focus is on the performance of the service provided to a surface user. Four FoMs are defined to evaluate this performance—the percentage of time a point has at least four-fold coverage, the average UERE among the pseudorange measurements ( $\bar{\sigma}_{UERE}$ ), GDOP, and KDOP. These FoMs are averaged over time and locations for the sensitivity analysis. Equation (1) provides the mathematical definition of the four-fold coverage metric, where  $N$  is the number of locations and  $M\Delta t$  is the total evaluation time.

$$4 - Cov = \frac{1}{N} \sum_{i=1}^N \left( \frac{1}{M} \sum_{j=1}^M \begin{cases} 1 & \text{if coverage}_j \geq 4 \\ 0 & \text{otherwise} \end{cases} \right) \quad (1)$$

For UERE, the formulation for an individual measurement is adapted from Bhamidipati et al., 2023 and Sirbu et al., 2023, Jul and presented in Eq. (2). A similar formulation is also presented in Hartigan and Lightsey, 2025. For the purposes of this work, the multipath error is assumed to be zero, which could be achieved through waveform design (Bhamidipati et al., 2023).

$$\sigma_{UERE} = \sqrt{\sigma_{clk}^2 + \sigma_{rel}^2 + \sigma_{pos}^2 + \sigma_{gd}^2 + \sigma_{mp}^2 + \sigma_{rec}^2} \quad (2)$$

$\sigma_{clk}$  : Clock Error

$\sigma_{rel}$  : Relativistic Error

$\sigma_{pos}$  : Satellite Position Error

$\sigma_{gd}$  : Group Delay Error

$\sigma_{mp}$  : Multi-path Error

$\sigma_{rec}$  : Receiver Noise Error

Geometric Dilution-of-Precision is a common metric, given in Eq. (3) (Hegarty et al., 2017). The  $H$  matrix contains the normalized relative position vectors associated with the pseudoranges with a column of ones appended.

$$GDOP = \sqrt{\text{tr}[(H^T H)^{-1}]} \quad (3)$$

The formulation for KDOP is given in Eq. (4). It is similar to GDOP, but adds the weighting matrix  $D$  and retains the left pseudo-inverse of  $H$ ,  $H^-$ , defined as  $H^- \triangleq (H^T H)^{-1} H^T$ , which assumes that  $H$  has full column rank. Derivations of this form can be found in Sairo et al., 2003 and Gabhart et al., 2025. Other weighting matrices can be used as the one in Iiyama and Gao, 2025 to get related metrics. Importantly, KDOP is not monotonically decreasing with the number of pseudorange measurements received by the user.

$$KDOP = \sqrt{\text{tr}[H^- D (H^-)^T]} \quad \text{where} \quad D_{i,j} = \begin{cases} \frac{\sigma_{UERE,i}^2}{\sigma_{UERE}^2} & \text{when } i = j \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

To obtain overall performance for the system, these metrics were averaged using Eq. (5). The metrics are only defined when there is four-fold coverage, so the time average is limited to those instances. This could add noise to the evaluation, since the number of available values to average could be low.

$$FoM_{avg} = \frac{1}{N} \sum_{i=1}^N \left( \frac{1}{\text{len}(\tau)} \sum_{t \in \tau} FoM_{i,t} \right) \quad \text{where} \quad \tau = \{t : \text{coverage}(t) \geq 4 \forall t \in [0 : \Delta t : M\Delta t]\} \quad (5)$$

### 3. Sensitivity Analysis Definition

To conduct the sensitivity analysis the python package SALib was used (Herman and Usher, 2017; Iwanaga et al., 2022, May). This package contained built-in functions to generate the samples and conduct the calculation of the sensitivity index,  $\mu^*$ , and its confidence intervals. An initial investigation of convergence was conducted, finding that 80d cases were needed. To achieve at least this number, six levels of the parameters were used. The sampling was conducted as put forward by Campolongo et al., 2007 where a large number of random initialized trajectories are down selected to optimally cover the space. One-thousand trajectories were generated and 90 optimal trajectories were selected. The random seed was set to 42. The parameter levels used for the technology parameters is given in Table 1. The range of levels was derived from relevant literature. A number of the parameters were sampled in log space to evenly sample over the orders of magnitude.

**TABLE 1**  
Levels Used for Technology Parameters

Parameter	Level						Reference
	1	2	3	4	5	6	
$\sigma_{gd}$ [m]	0.25	2.2	4.15	6.1	8.05	10	Sirbu et al., 2023, Jul
$\sigma_{rec}$ [m]	0	2	4	6	8	10	Sirbu et al., 2023, Jul; Small et al., 2022
$h_0$	$10^{-27}$	$10^{-25.4}$	$10^{-23.8}$	$10^{-22.2}$	$10^{-20.6}$	$10^{-19}$	Bhamidipati et al., 2022, 2023; Pereira and Selva, 2022
$h_{-1}$	$10^{-50}$	$10^{-44.4}$	$10^{-38.8}$	$10^{-33.2}$	$10^{-27.6}$	$10^{-22}$	Bhamidipati et al., 2022, 2023; Pereira and Selva, 2022
$h_{-2}$	$10^{-50}$	$10^{-44.8}$	$10^{-39.6}$	$10^{-34.4}$	$10^{-29.2}$	$10^{-24}$	Bhamidipati et al., 2022, 2023; Pereira and Selva, 2022
time update noise [s]	$10^{-9}$	$10^{-8.2}$	$10^{-7.4}$	$10^{-6.6}$	$10^{-5.8}$	$10^{-5}$	Iiyama et al., 2023; Murata et al., 2022; Small et al., 2022
clock bias [s]	$10^{-12}$	$10^{-10.8}$	$10^{-9.6}$	$10^{-8.4}$	$10^{-7.2}$	$10^{-6}$	Iiyama et al., 2023; Murata et al., 2022; Small et al., 2022
clock drift [‰]	$10^{-18}$	$10^{-16.8}$	$10^{-15.6}$	$10^{-14.4}$	$10^{-13.2}$	$10^{-12}$	Iiyama et al., 2023; Murata et al., 2022; Small et al., 2022
position update error [m <sup>2</sup> ]	$10^0$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	Capuano et al., 2015, Dec; Delépaut et al., 2020; Murata et al., 2022
initial error [m <sup>2</sup> ]	$10^1$	$10^{1.86}$	$10^{2.72}$	$10^{3.58}$	$10^{4.44}$	$10^{5.3}$	Pereira and Selva, 2022; Small et al., 2022
update interval [s]	30	744	1458	2172	2886	3600	Delépaut et al., 2020
view angle [°]	-65	-52	-39	-26	-13	0	Donaldson et al., 2020; Fischer, 2022

Table 2 contains the orbital parameter levels. Radius of Periapsis was used rather than semi-major axis to make it easier to avoid surface collisions in the sampling. Similarly the ranges of some of the parameters were limited to increase the likelihood of four-fold coverage around the lunar South Pole. The range for the number of satellites in a plane varied between the two number of planes to increase four-fold coverage. While not directly derived from a particular architecture, these ranges encompass the LunaNet service provider proposals (Melman et al., 2025).

**TABLE 2**  
Levels Used for Orbital Parameters

Parameter	Level					
	1	2	3	4	5	6
Radius of Periapsis (RoP) [km]	2500	4000	5500	7000	8500	10000
Eccentricity (Ecc)	0	0.19998	0.39996	0.59994	0.79992	0.9999
Inclination (Inc) [°]	30	54	78	102	126	150
Right Ascension of the Ascending Node (RAAN) [°]	0	60	120	180	240	300
Argument of Periapsis (AoP) [°]	0	60	120	180	240	300
Number of Satellites ( $N_{sat}$ )	2 planes	2	4	5	7	8
	6 planes	1	2	3	4	5
Initial True Anomaly ( $TA_0$ ) [°]	0	72	144	216	288	360
True Anomaly Offset ( $\Delta TA$ ) [°]	-120	-72	-24	24	72	120

For the calculation of the indices, only the orbital parameters were used for the four-fold coverage and GDOP. The bootstrap confidence intervals were calculated at the 95% level with 25000 resamples. The values were calculated for differing amounts of data proportional the dimensionality to confirm convergence. Figure 2 shows the convergence behavior for the GDOP. It is seen that there are amounts of data that significantly shift the values. This is expected to be due to the inclusion of a piece of data with a large difference from the previous point. Since some of the cases do get dropped from the final calculation, this could be due to discontinuity in the sampling trajectory. The  $\mu^*$  and ranks are calculated relative to the final value, and the confidence interval is shown directly. The  $\mu^*$  values convergence within the confidence intervals very rapidly, as seen by comparing Figs. 2a and 2b.

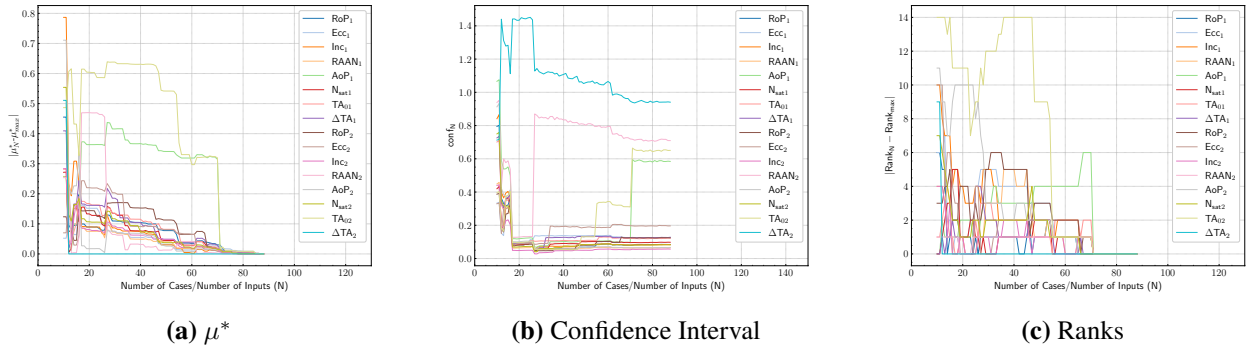
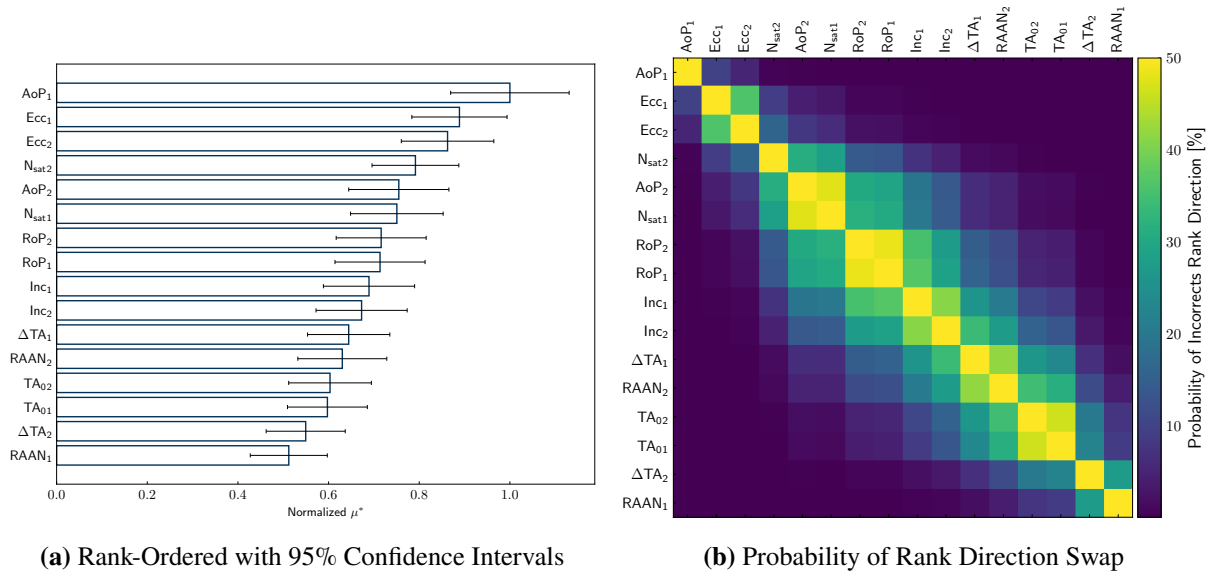


FIGURE 2 Convergence Behavior for GDOP

## IV. RESULTS

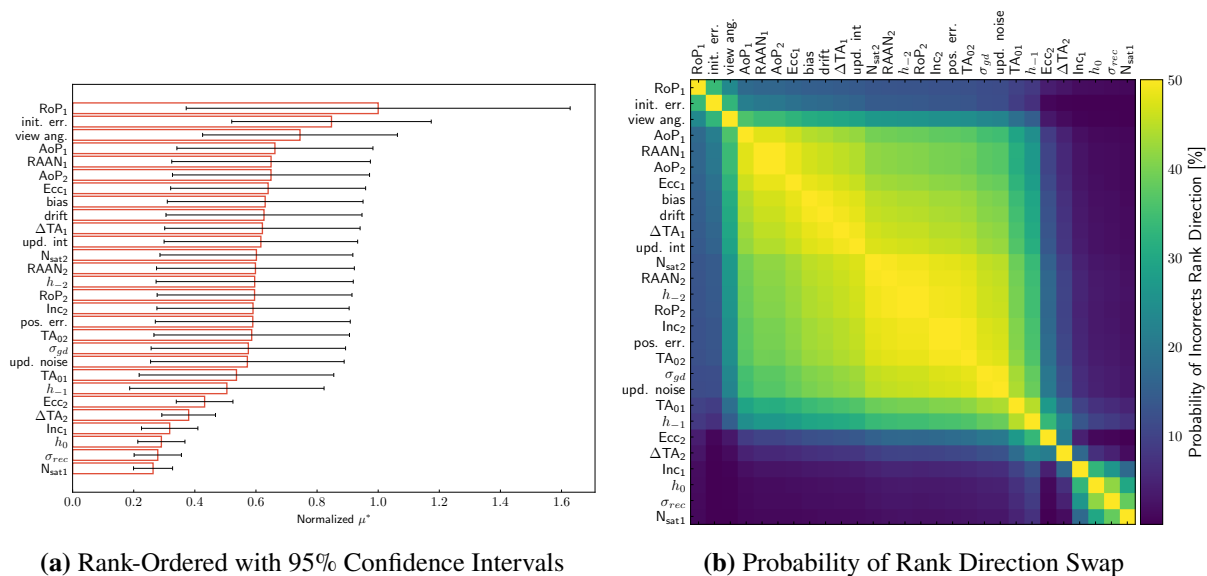
### 1. Two Planes

The first results are those collected for the two plane analysis. Roughly 9% of the cases contained no times with four-fold coverage and 8 cases failed due to satellite demise, out of 2700. There is expected to be some noise in the calculations for UERE, GDOP, and KDOP since around 10% of the data had average percent four-fold coverage of less than 5%. This could lead to averages over small samples, per Eq. (5). Figure 3 contains the findings from the four-fold coverage percentage. Here the orbital parameters have subscripts to denote to which plane they belong. The bar plot ordered by the rankings, Fig. 3a, shows some of the expected results. The same orbital elements are generally closely ranked. The highly ranked parameters also are logical. Argument of periapsis and eccentricity both contribute to the dwell time over a specific region of the central body. The eccentricity result could be related to the semimajor axis results in literature for coverage (Bender et al., 2023; Iiyama and Gao, 2025), since in this analysis, the shape of the orbit conic is set by eccentricity and radius of periapsis. This parametrization makes semimajor axis a function of eccentricity. Further, the shear number of satellites contributes greatly to whether four would be in view at any given time. None of the parameters have a sensitivity value that is less than half of the greatest value, showing high levels of sensitivity to all of the parameters. However, the confidence intervals are quite wide hurting the visual interpretability of the rankings. To create a more quantitative understanding of the certainty of any specific ranking, the likelihood of the parameter being less than (or greater than) another parameter was calculated. This calculation assumed that the confidence interval was a standard Gaussian distribution. The results are displayed in Fig. 3b, where a 50% indicates that two parameters are statistically equivalent in their  $\mu^*$  value. A completely certain set would look like bright yellow boxes along the diagonal and dark purple everywhere else. In Fig. 3b, it is seen that most of the parameters are relatively certain in their directionality compared to the other parameters with some noise in the middle rankings. It is of note that the initial true anomaly values and the radius of periapsis  $\mu^*$  values are nearly equivalent between planes.



**FIGURE 3** Normalized Sensitivities for Four-Fold Coverage Percentage (2-planes)

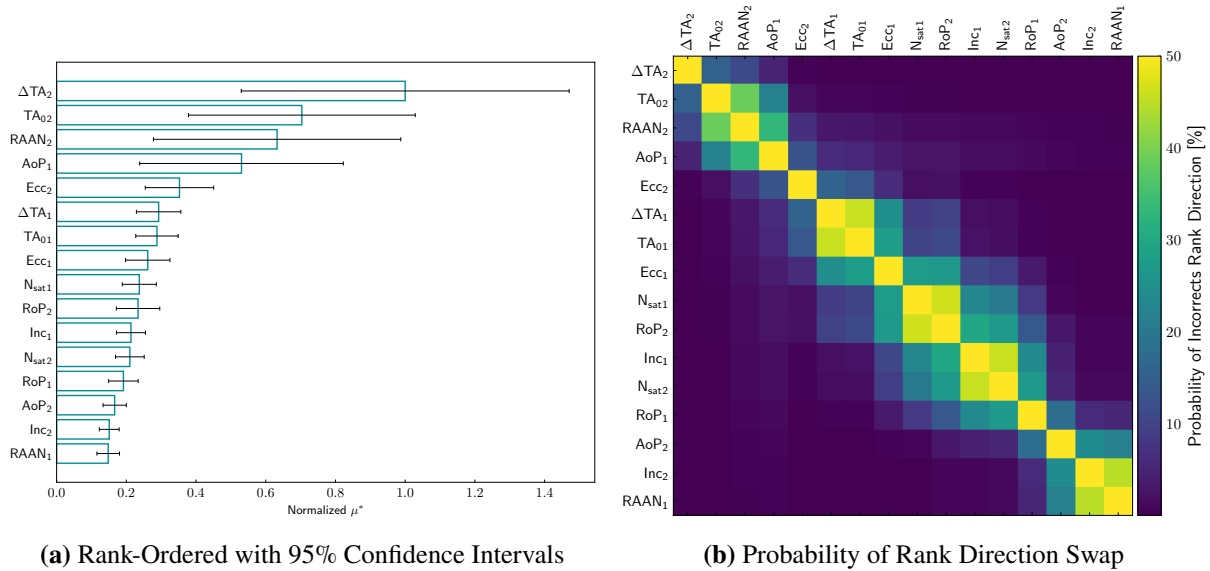
Figure 4 presents the results for the average UERE. It is immediately obvious that these results are less certain than the results for four-fold coverage. The top ranked parameters, seen in Fig. 4a, are again mostly intuitive. The initial error and the view angle that determines access to GPS-based orbit determination support should have strong effects and were found to have statistically significant monotonic relationships in previous work (Gabhart et al., 2025). The radius of periapsis is surprising. Upon further inspection of the data, there are four cases which have UERE values roughly six times greater than the next highest value. They all have a  $RoP_1$  value of 2500 km. These cases also have a very small, less than 0.1%, four-fold coverage percentage. No outliers were removed in the collection of this data, so neither were these. The radius of periapsis relationship should not be completely discounted, but the significance of its magnitude must be analyzed in the context of these outliers. These results show less of the grouping of the orbital parameters due to the interspacing of the technology parameters. In Fig. 4b, it is further enforced that the middle rankings are uncertain in their specific order with 19 of them having a greater than 30% chance of swapping.



**FIGURE 4** Normalized Sensitivities for Average UERE (2-planes)

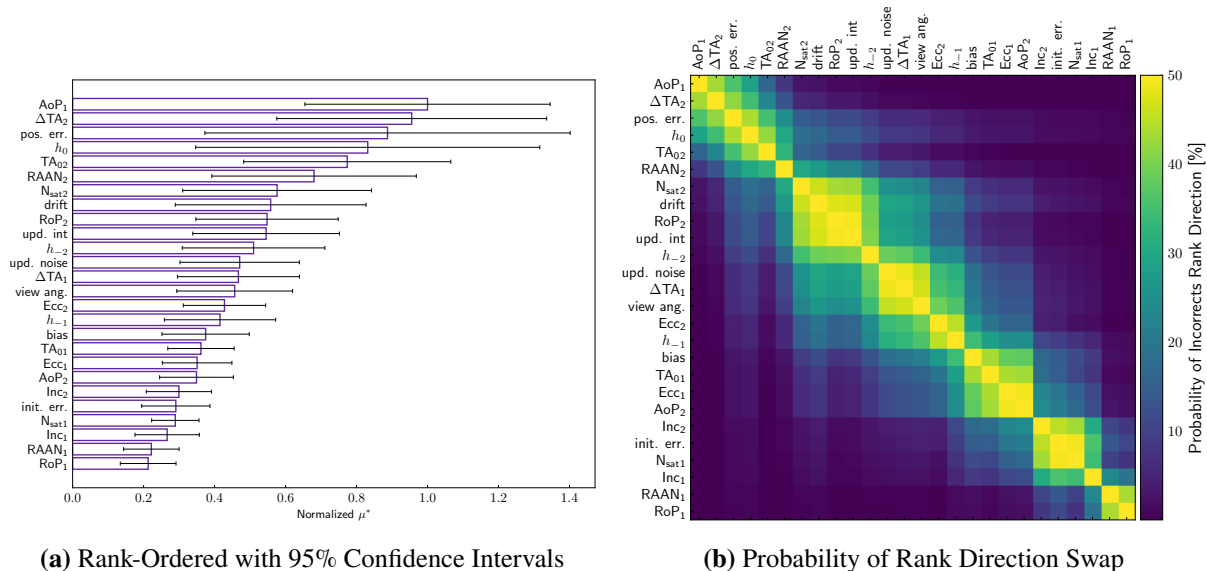
Figure 5 displays the results for the average GDOP. Only the orbital parameters were used similar to four-fold coverage since

they are the only inputs that will have an effect. Immediately, in Fig. 5a, it is obvious that the top ranked parameters are orientation parameters for the second plane. This is possibly due to the nature of the parameterization with one plane before the other. These high ranking do correlate with GDOP being a measure of geometric diversity. This also explains why  $\Delta TA_1$  is ranked highly. In comparison to other work, eccentricity is found to have decently high rankings like the result in Gabhart et al., 2024, but the number of satellites is ranked lower in contrast to the results in literature (Iiyama and Gao, 2025; Pereira et al., 2022). This could be due to the fact that those studies were looking at more structured constellations while this one treats each plane completely separately. The importance of the true anomaly parameters is aligned with the phasing parameter result from Pereira et al., 2022. Figure 5b shows that the ranking are much more certain than the UERE rankings. These results are more similar to the four-fold coverage results with relatively high certainty within a few ranks and some blocks of nearly equivalent parameters. However, unlike the four-fold coverage result, most of the  $\mu^*$  values are less than half the largest value, indicating that only a few parameters primarily drive the response.



**FIGURE 5** Normalized Sensitivities for Average GDOP (2-planes)

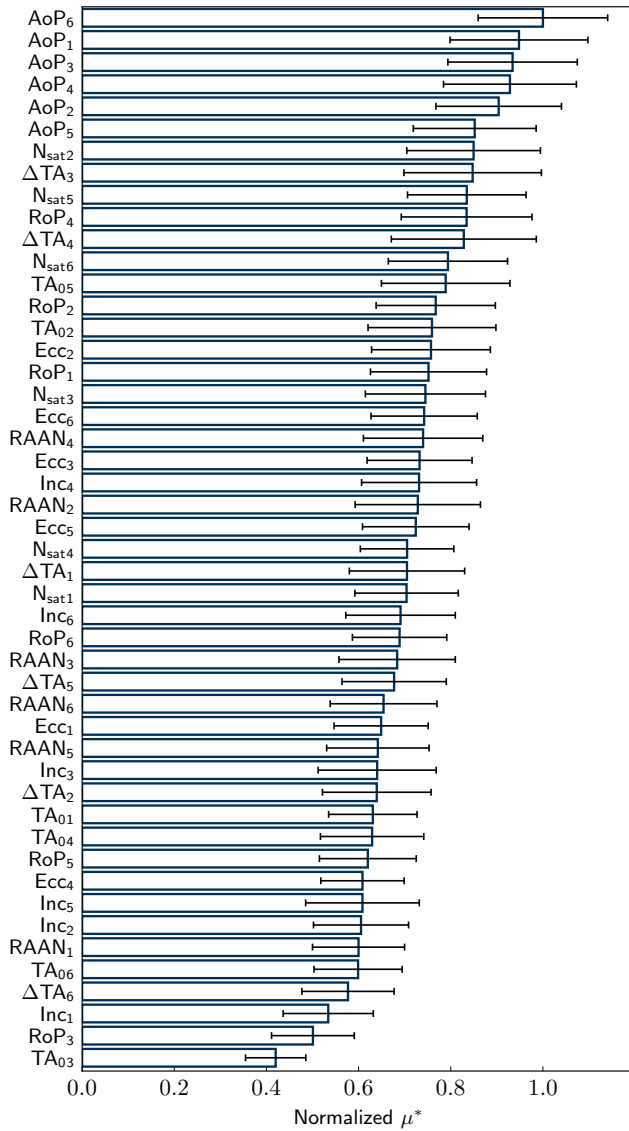
Figure 6 presents the sensitivity analysis for KDOP. In this analysis, the receiver and group delay noise were removed since they are known to have no effect. Importantly, KDOP is a function of error distribution and geometric diversity. In Fig. 6a, it is seen that in the top ranking there is a mix of technology and orbital parameters. The same top three from GDOP are in the top six showing the interconnected nature of these parameters. The angle of periapsis is an interesting result, but it could be due to increasing dwell time which could reduce error if the satellite is receiving updates. It is interesting that the position update error, pos. err. in the plot, is ranked so high when for UERE it was in the middle. This could indicate that it has a stronger effect on the distribution of error than the overall magnitude. In Fig. 6b, it is seen that the KDOP ranking are more certain than UERE but noisier than GDOP. Again, there are groups of two or three parameters that are nearly equivalent. It is worth noting that similar orbital parameters are not ranked similarly in contrast to the results from four-fold coverage. The notable exception is the inclinations which are only two ranks apart.



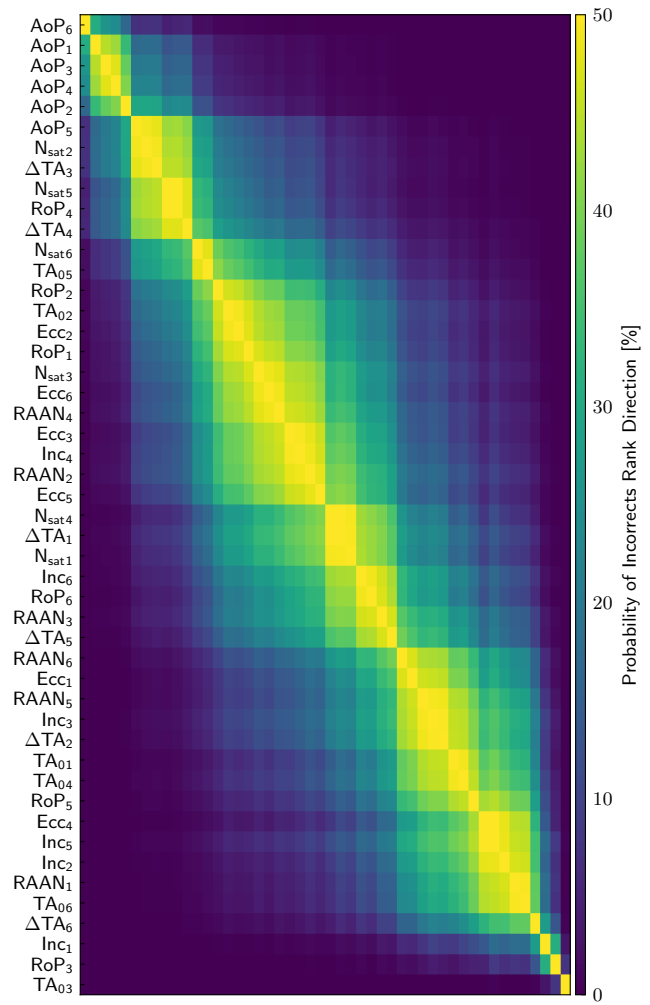
**FIGURE 6** Normalized Sensitivities for Average KDOP (2-planes)

## 2. Six Planes

The six plane data contained 5490 cases, of which 17 errored out due to satellite demise. All of the cases had some four-fold coverage with most having four-fold coverage 100% of the time. The sensitivity results for four-fold coverage are shown in Fig. 7. As seen in Fig. 7a, the highest ranked parameters are again the argument of perisapsis. Compared the the two plane results eccentricity is less significant. Figure 7b shows that the order of the rankings is less certain than the two plane case. Similar to the two plane cases all of the parameters have a similar  $\mu^*$  value with no significantly higher or lower values.



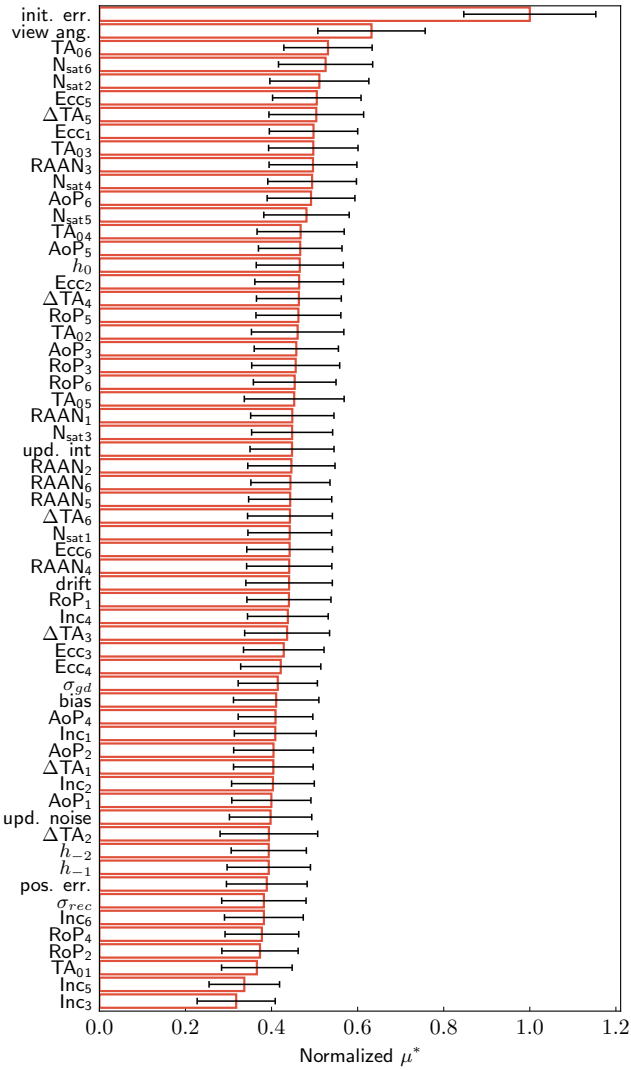
(a) Rank-Ordered with 95% Confidence Intervals



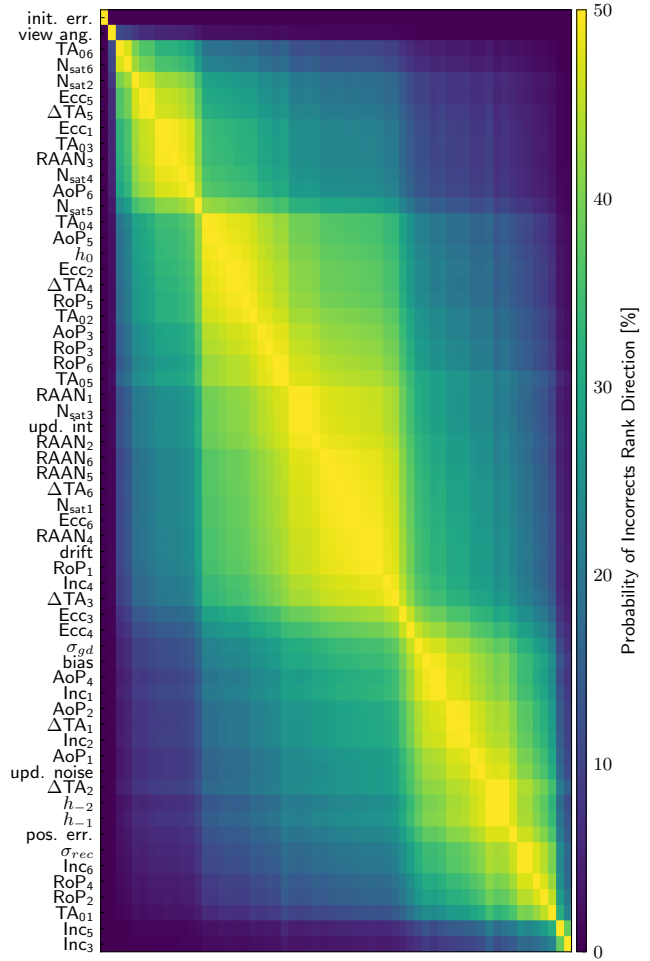
(b) Probability of Rank Direction Swap

FIGURE 7 Normalized Sensitivities for Four-Fold Coverage Percentage (6-planes)

Figure 8 shows the results for the average UERE with six planes. Immediately, in Fig. 8a, it is seen that the radius of periapsis result from the two plane case is not repeated. Radius of periapsis parameters are ranked in the top half, but they are not dominating. This further indicates that the two-plane result was due to a small sample of points to average over. Two technology parameters related to ODTS support dominate the responses. As with the two plane results, the rest of the parameters have similar responses without extreme certainty in the rankings, as seen in Fig. 8b. This indicates that the effects are similar and not a purely a function of the small samples in teh two plane case. The number of satellite parameters are ranked relatively high, which was not expected. This could be due to the averaging over the number of measurements.



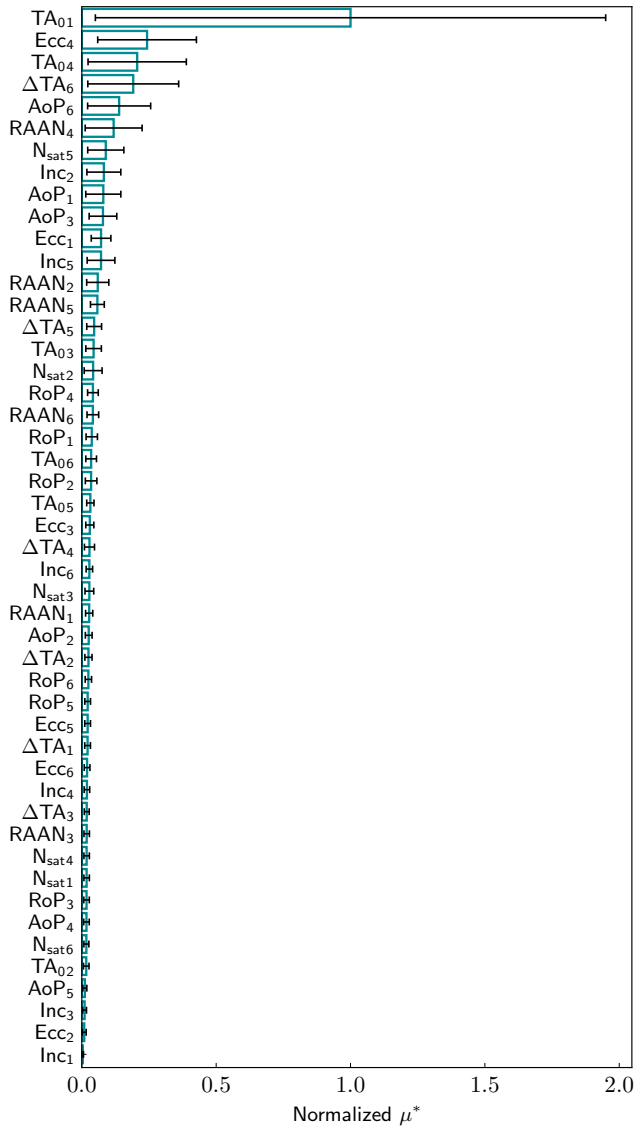
(a) Rank-Ordered with 95% Confidence Intervals



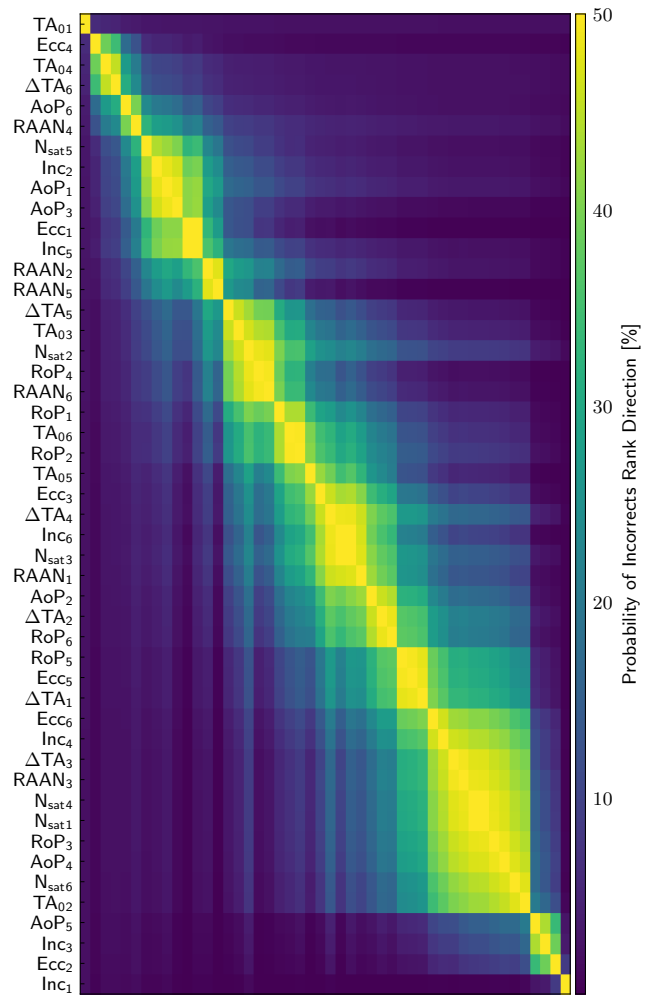
(b) Probability of Rank Direction Swap

FIGURE 8 Normalized Sensitivities for Average UERE (6-planes)

The GDOP results are shown in Fig. 9. The rankings in Fig. 9a are less intuitive than the two plane results. It is seen that the sorting of the orientation parameters by plane has not held from the two plane case, indicating that was a result of the sampling technique. Again, the number of satellite effect from literature (Iiyama and Gao, 2025; Pereira et al., 2022) is not seen. Three of the top four ranked parameters are related to the phasing of the satellites within the planes. These results have much tighter confidence intervals that translate to more certain rankings as seen in 9b. It is also worth noting that the lower rankings here are much smaller than the highest ranked value.



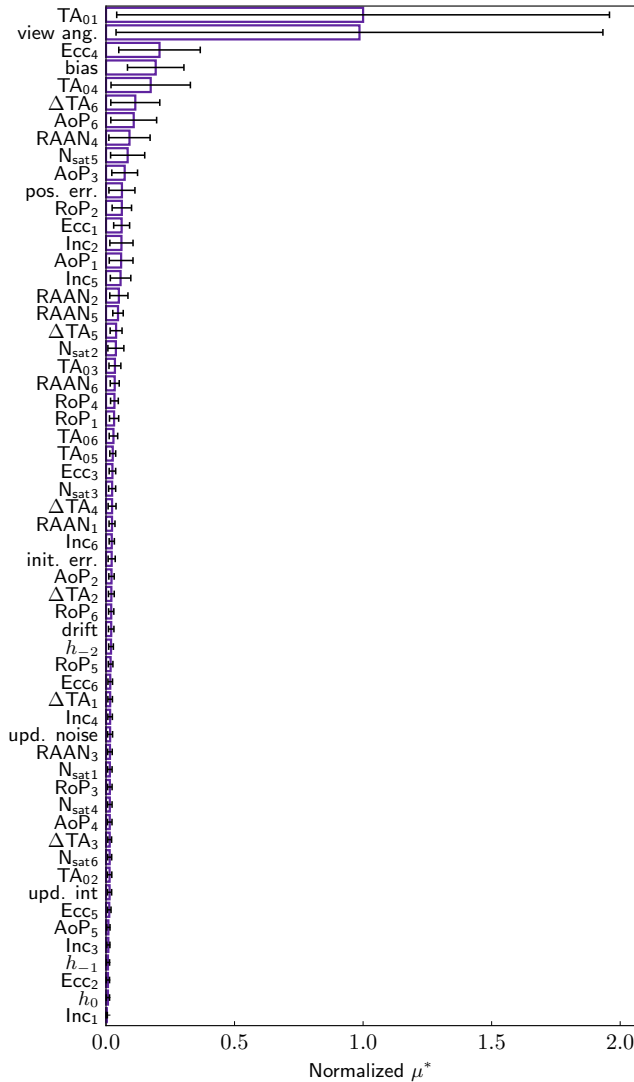
(a) Rank-Ordered with 95% Confidence Intervals



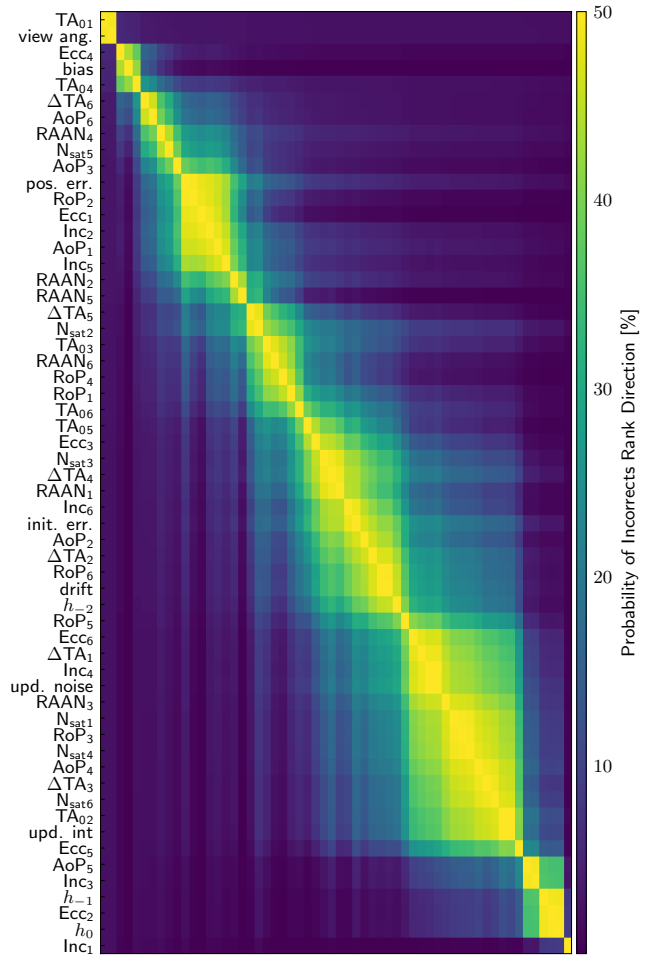
(b) Probability of Rank Direction Swap

**FIGURE 9** Normalized Sensitivities for Average GDOP (6-planes)

The results for the performance metric KDOP are given in Fig. 10. Similar to the two plane results, the rankings, shown in Fig. 10a, are the same orbital parameters as GDOP with technology parameters interspaced. As compared to the two plane results, the view angle and initial clock bias are more significant for the six plane case. The clock stability parameters drop significantly. The ranking certainties, shown in Fig. 10b, are very similar to those for GDOP.



(a) Rank-Ordered with 95% Confidence Intervals



(b) Probability of Rank Direction Swap

FIGURE 10 Normalized Sensitivities for Average KDOP (6-planes)

## V. CONCLUSIONS AND FUTURE WORK

This paper presented sensitivity analysis of a combined technology and orbital design space for lunar PNT systems. The Elementary Effects method was utilized and shown to make this high dimensional problem trackable under computational constraints. A general convergence criteria was derived relative to the dimensionality. Similar relationships to those found in literature were identified displaying to efficacy of the elementary effects method for this problem. These included a strong relationship between the number of satellites, the phasing parameters, and GDOP as seen in Pereira et al., 2022. Due to the large confidence intervals, specific rankings and relationships are not able to be assessed, specifically for UERE. Generally, KDOP is sensitive to both orbital and technological parameters showing that both must be considered when using it as a measure of performance for lunar PNT. There were significant shifts in the importance of parameters between the number of planes, so future studies cannot represent the full design space by studying a single number of planes. The importance of the phasing parameters to all of the performance metrics indicates that system designers should pay special attention to true anomaly values and explore beyond standard schemes. It was determined through this analysis that there are likely no parameters amongst the ones evaluated that meet the criteria of removal from performance investigation. The low sensitivity of KDOP and GDOP to some of the inputs for the six plane case indicate some dimensionality reduction might be possible when comparing those two parameters.

Going forward, these results could be augmented through additional sampling and study to reduce the confidence intervals, specifically for UERE. Further, the evaluation time could be increased to attempt to reduce the effect of small samples in the averaging. In a continued effort to understand the differences in utility between KDOP and GDOP, the sensitivities of that difference could be analyzed to determine which parameters should be prioritized in evaluating the portion of the design space where their differences are significant. The sensitivity analysis could also be extended to Sobol analysis to determine more quantitative results between the performance metrics and the design parameters.

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