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Eugene Wesley Green ✓

AN ANALYSIS OF THE CHARACTERISTICS OF
ELEMENT-TIME DISTRIBUTION

A THESIS

Presented to
the Faculty of the Graduate Division
Georgia Institute of Technology

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Industrial Engineering

By
Eugene Wesley Green

June 1955

AN ANALYSIS OF THE CHARACTERISTICS OF
ELEMENT-TIME DISTRIBUTION

Approved:



Date Approved by Chairman: 3 JUNE 1955

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SUMMARY

The primary purpose of this research was to test the hypothesis that the element-time distribution characteristics do not vary significantly from the work-time distribution for the cycles of which these elements are a part. Secondary objectives were to investigate the relationship among elements in terms of their independency or dependency and to study the possible effects of the assignable causes of variation in work method on the distribution of element times. The data used in this study were obtained from previous studies of the research project which has been conducted since 1951 under the direction of Doctors Lehrer and Moder.

The histograms of the frequency distributions of the element times of a stable operator were plotted to determine right and left hand and effective element-time distribution characteristics and to determine the effect of six assignable cause variables on the distributions. Calculation of the mean, variance, skewness and peakedness was also completed to more clearly define these effects. The Normal, Log Normal and Pearson Type III Curves were fitted to the experimental distributions to determine whether or not any of these curves typified the underlying theoretical distribution. Finally, to compare element and cycle distribution characteristics, cumulative distributions of the element and cycle times were tested for significant differences.

The results of this study indicated that, generally, the characteristics of the element and cycle distributions are similar. Specifically,

the conclusions reached were as follows:

1. The characteristics of the element-time distributions and the work-time distributions of the cycle of which the elements are a part were similar.
2. Of the curves tested, the Log Normal and Pearson Type III curves were the curves of best fit to the experimental distributions, but the theoretical element-time distribution curve may not be one of those tested.
3. There was evidence of independency among the elements of this operation.
4. The effect of leveling the element times to compensate for the difference in the level of operator performance during the shift was to reduce the amount of peakedness and skewness of the distributions.

The results of this study must be viewed in the light of the following limitations: The sample size was limited, and only one operator working on one manually controlled, short cycle assembly operation was studied.

It was recommended that another study be made of more than one operator and more than one operation to test the above conclusions. Further, the exploratory study concerning the relationship among elements which was started in this thesis should be continued in a subsequent, more rigorous study.

CHAPTER I

INTRODUCTION

General Problem

The question of whether or not time study could be considered scientific has been the source of much controversy from the early days of Scientific Management. However, there seemed little doubt that time study was basic in the accomplishment of many of the objectives of the Scientific Management movement. The acceptance of the latter concept occurred largely as a result of a general realization that time study, despite its limitations, could be used effectively as a tool for work measurement.

The controversy concerning time study appeared to center about two general areas. These were, first, the independence of, or the interaction among elements, and second, the present rating techniques which were based on an assumed normal distribution of work-times and accomplished by subjective personal judgment.

Literature Survey

Element Relationships.—F. W. Taylor, who is known as the father of the Scientific Management movement, as quoted by Gombert (1), proposed the development of elemental standard data by combining the times for elements of several operations, as needed, into a standard time for doing almost any type of work.

However, it was suggested by Abruzzi (2) that the elements within one cycle tended to compensate one another. That is, the operator would speed up to compensate for lost time in an earlier element to avoid unduly long cycle times. The operator would also tend to slow down to compensate for an earlier exceptionally fast element to avoid an unduly short cycle time. If this were true, elements could not be considered as separate entities, and the time for completing one element would depend upon other elements within the same cycle. Consequently, valid results would not be obtained if an element from one cycle were combined with an element, or elements from other cycles to form a new cycle of work.

Abruzzi's conclusions verified those of Barnes and Mundel (3) who performed a number of studies of the time required to position pins in bushings with beveled holes. They concluded that the standard times for certain therbligs could not be given as independent values because of interdependency among the elements, or the effect of other therbligs in the cycle. A number of other studies seemed to bear out these findings.

This question had far-reaching repercussions in the development of standard data and the several "synthetic time systems," for the very basis for validity of these systems was the independency of the motions which, combined, made up the time standard. Although Gombert (4) reluctantly accepts standard data on the basis of consistency, he (5) lists four criteria for scientific validity, among which were the following:

2. That the arbitrary divisions into which time study elements are divided are independent of one another.
3. That these elements constitute an additive set.

He cited Abruzzi's work as an indication that neither of these criteria were met.

However, Abruzzi (6) found from experiments in which the stop watch and wink counter were used to measure times that the existence of independence among elements sometimes depended on the measurement method, especially for short elements. Davidson (7) even indicated that there was some question as to whether or not cycle times were independent, and suggested additional research along this line. It may be, then that the type of work, the length of the elements, and their definition as well as other factors would determine whether or not elements are truly independent.

Work-time Distribution.—There had long been wide acceptance of the "normal distribution of work-time" theory. Presgrave, as quoted by Davidson (8), in writing of the concept of the "normal" worker, stated, "Here . . . is the structure upon which incentives and time study must be built." Barnes (9) explained this concept when he wrote:

There is considerable evidence to show that if the working speed of each member of a large group of people, such as would be found in a factory, were arranged along the base line (of a frequency distribution) according to magnitude in per cent of normal, and if the vertical scale indicated frequency, the shape of the curve would fit fairly closely the normal bell curve. . . .

The next step is to establish the point on the curve that will represent normal speed. . . .

It is of course not expected that any group of workers would exactly fit the normal curve, although an examination . . . will show that the output of this group of 121 operators tends to fit the normal curve.

However, on the basis of a number of statistical tests, Davidson (10) indicated that the normal curve was not the curve of best fit. He also cited the work of C. M. Worthley as further supporting evidence of

this phenomena, and suggested (11) that "the search for appropriate concepts to serve as the theoretical bases for practical methodologies may be regarded as an activity of prime importance to the advancement of time study."

Davidson (12) tended to reject the concept of "constant chance cause system." However, Lehrer (13) wrote:

Some stable 'system of chance causes' is inherent in any particular scheme of production and evaluation. Variation within this pattern is inevitable. The reason for variation outside this stable pattern may be discovered and corrected.

Wiberg (14) proposed that the use of Work-Time Distribution theory would alleviate much of the controversy now concerned with the concept of the normal worker. He indicated that it would aid in quantitative definition of the average employee, that it could be used in time study rating, that it would reduce the length of time required for time studies and would be useful in personnel management as well as in the study of the psychological aspects of the individual. However, his contentions were not supported by adequate data to validate all these claims.

According to Wiberg (15),

A work-time distribution is defined as a frequency distribution of a specified number of time values obtained through time study as actual and unadjusted watch readings, on an element of repetitive manual work for the purpose of relating the distribution characteristics to the various influences which the worker, the work-method and the work-environment exert upon the efficiency of a particular work situation.

The characteristics which were studied in research in this field include the first four moments, the mean, variance, skewness and peakedness,

of the distribution. These were related, by Wiberg (16) to habituation and motivation. He wrote that positive skew indicated a high degree of motivation, negative skew indicated "soldiering," and small variance indicated strong habit in performing work. Lack of substantiating information, however, left these ideas open to question.

Previous Research.---Abruzzi conducted extensive research in the general field of the distribution of work-times. He used the four characteristic defined above as the basis for statistical analysis of the results of his experimentation. In addition to the findings listed earlier¹ as to the relationship of elements, he (17) concluded that at the cycle level there were wide differences among mean times of workers on the same operation but with about the same degree of variability. At the element level, workers differed in mean times and variability. He (18) also found that the absolute degree of variability increased as the mean time increased but that the relative degree of variability (coefficient of variability) decreased as the mean time increased.

Davidson (19) reported on a number of exploratory studies conducted by graduate students at Ohio State University. These studies had to do with operator stability as a means of determining whether or not Gomburg's criteria for time study were met. As has been the case in most research in this area, cycle times were used in the analysis. Reasons for this included the previously mentioned question as to the independence of elements which would introduce some question as to the validity of results if element times were used.

¹Supra, p. 2.

Results in all three studies cited were inconclusive. The first, conducted by Brague, indicated that while some of the data was found to be statistically stable, other data with larger sample size showed some cycles in control, or stable, and some unstable. In other words, the analysis results proved inconsistent. Damon and Losely, both of whom worked under Davidson, found essentially the same thing. One general conclusion was drawn. This was that there was considerable question as to whether or not the control chart technique, by which the degree of stability was tested, was a satisfactory test for the stated purpose.

Along with the analysis of cycle times, Brague also analyzed the element time variations for a part of his data. He found that these data indicated a greater degree of instability than the cycle times, and concluded that instability of elements would not produce the same degree of instability in the cycle. He also found, by the use of the statistical test of comparison of the sum of the element variances with the cycle variance, that the elements of the operations studied exhibited interdependence.

Davidson stressed the exploratory nature of these studies and indicated that their main purpose was in uncovering leads for other research. He reached the general conclusion that these three studies indicated a need for a "fairly large scale" investigation of cycle time distributions.

In the same study as that cited concerning Davidson's finding of non-normality of work time distributions², the following conclusion was also reached: ". . . . the sample displayed a definite positive skewness

²Supra, p. 3.

which was real, rather than the result of sampling errors alone."

Presgrave (20) concluded that a selection of operators through a good testing program would crowd a far greater proportion of the workers into the upper sections of the range and thus impart a marked skewness to the curve. He apparently tried to explain away the positive skewness indicated in his experimental distributions, by writing: "The question of skewness is not important to the time-study man unless it becomes unusually pronounced."

Recent Studies.—Research on the subject of the work-time distribution has been conducted in the School of Industrial Engineering at the Georgia Institute of Technology since 1951 under the direction of Doctors Lehrer and Moder.

The operation studied was the assembly of a ball point pen. This was a two-handed operation for which the work-place was established in such a way as to enable the hands to utilize simultaneous motions while performing the assembly operation. This enabled the operator to assemble two pens per cycle.

The first study, conducted by Lind (21), was an analysis of data obtained by a stop watch time study. The operation was broken down into twelve elements as defined below.

1. Get barrel
2. Place barrel in fixture
3. Get writing unit
4. Place writing unit in barrel
5. Get drive nut

6. Place drive nut on unit
7. Get ferrule
8. Place ferrule over drive nut
9. Get complete unit from fixture
10. Place complete unit in staking device
11. Stake ferrule to secure assembly and remove from staking device
12. Aside assembled unit to container.

These same elements were maintained by Taft (22) in a micromotion study recommended by Lind in his thesis. In this second study, hereinafter referred to as Study B, the film analysis revealed significant variation in the method of different operators in performing the first and last elements of the cycle. For this reason, these elements were excluded and a modified cycle time was obtained for the modified cycle composed of ten elements. Subsequent studies have retained this modified cycle time as the basis for analysis.

These first two studies revealed that the work-time distribution for all the operators analyzed were positively skewed. It was recommended that further studies be made to determine whether or not there is a distribution pattern that would typify the work-time distributions for operators performing manual operations.

Friedman (23) tested the hypothesis that ". . . there is a work-time distribution pattern which will typify the statistically stable distributions of operators performing short cycled manually controlled operations."

In the thesis based on his study, Friedman concluded that the

characteristics of the theoretical work-time distribution were as follows:

1. It differs significantly from the Normal Curve.
2. It is positively skewed.
3. Its peakedness is greater than that of the Normal Curve.
4. It can be reasonably approximated by a Pearson Type III Curve.

These results were amended somewhat by Summers (24) when he stated that the elimination of all assignable causes of variation from cycle time would result in relatively constant peakedness and skewness. He concluded that

This result suggests that the typical curve is not necessarily one of those tested, (Normal, Log Normal or Pearson Type III) but that it may be another curve with constant skewness and peakedness, and that the variance of this curve will be the only independent parameter which will influence its shape.

Summers based this conclusion on an analysis of the data of Study B and Study C. This latter study involved the elimination of six variables established as assignable cause variables in a study conducted by McLeod (25). He tested stability of the various operators by using \bar{X} and S control charts using three-sigma limits. The control chart technique was applied first to the raw data of Study B, and then the six variables were eliminated to determine whether or not the elimination of assignable causes of variation had any affect on the stability of the operators. These variables were as follows:

1. Fumble during the cycle
2. Drop part
3. Get two units and return one to box

4. Inspection delay
5. Bad part
6. Part stuck in staking device.

The data used in the present study was that obtained as a part of Studies B and C.

CHAPTER II

PURPOSE

With the exception of a portion of Bragus's exploratory study as discussed by Davidson¹, previous work-time distribution research was based upon the time required to complete a group of elements, or a cycle of work. Part of the reasoning behind the selection of cycle times was given above². Further, the amount of work required to make a thorough analysis of the element times for a large number of workers would make such a study prohibitive at the graduate student level. However, the relationship between the elements and the cycle of which these elements are a part should be determined in order to obtain a complete picture of the theory of work-time distribution.

Therefore, the primary purpose of this study was to analyze the characteristics of the element distributions in relation to those of the complete cycle in order to test the following null hypothesis:

The element-time distribution characteristics do not vary significantly from the work-time distribution for the cycle of which these elements are a part.

A secondary purpose was to make exploratory studies concerning the relationship among elements of a cycle in terms of their independency or dependency. Further, it was desired to investigate the possible effects, on the distribution, of the assignable causes of variation in method, and

¹Supra, p. 5.

²Supra, p. 5.

the possible differences in distribution characteristics of the element times as performed by the right and left hands as compared to the effective element time.

CHAPTER III

PROCEDURE

General Procedure.—Generally, the procedure followed in conducting this research involved two main steps. First, was the preparatory work. This included the selection of data and the calculations required to translate these data into a useable form. The second step was the application of analytical tools. This second step necessitated the preparation of histograms from the grouped data and the calculation of the first four moments of these distributions in order to determine what the characteristics of the element distributions were. Further, to determine the nature of the theoretical curve which typified the distributions, the element time distributions were fitted to the Normal, Log Normal and Pearson Type III curves. Finally, to test the hypothesis stated above¹, cumulative-percentage histograms were plotted for normalized element and cycle times.

Data Selection.—For this study, data for one operator was selected on the basis of two criteria. First, only stable operators were considered, and second, in order to obtain valid results, the data on the operator which would furnish the largest sample size for the element times were selected.

To fulfill the first criterion, the results of the analysis of McLeod (25) were analyzed. He found that, in addition to statistical

¹Supra, p. 11.

stability as indicated for three-sigma limits on the control charts, operator Q exhibited a production rate which was not exceeded by any other operator. The data for operator Q also furnished a sample size of 74 observations with the variables excluded, and 94 observations with the variables included. Since this sample size was greater than that of any other operator, these data were selected for the study. Selection on the basis of the above two criteria tended to randomize the effects of other factors which might have affected the selection of the sample to be studied. These included the degree of symmetry of motions, human characteristics of the operator, and thoroughness of film analysis.

Treatment of Data.—The data for operator Q was obtained in the form of film analysis sheets which listed the elements and the frame counter reading for the end of each element. A sample analysis sheet is included in the Appendix as Figure 21. To obtain the frame count, hereinafter referred to as element time, for each element as performed by the right and left hands, the counter reading for one element was subtracted from that of the preceding element.

The modified cycle time used in previous studies was defined as the time from the beginning of the second element in the cycle to the end of the eleventh element. This was, in effect, a composite time for the cycle as performed by the right and left hands. In order to make a valid comparison between cycle and element times it was necessary to compute an element time which would be a composite of the time for both hands. Therefore, the effective element time which was defined here as the time from the end point of one element to the end point of the next element for either hand was determined.

To compensate for the difference in level of performance of the operator during the periods of the shift when the eight "shots"² were taken, it was necessary to level the individual element times to a grand average for each element. To do this a mean element time for each "shot" was calculated and the mean for all "shots" was then determined. This is equivalent to dividing the sum of all element times by the total number of elements, and was termed the calculation of the element grand average. The individual elements within each "shot" were then leveled to this grand average by applying an algebraic addition factor of the magnitude of the difference between the "shot" average and the grand average to the element times within each "shot". The calculations required to obtain these leveled times, to be known hereafter as Study D, are indicated in Figure 22 in the Appendix.

Element-Time Frequency Distributions.---In order to determine the relationship between the element times of the right and left hands and the effective element time, histograms were plotted from the frequency distributions for each element. The interval used in the frequency distribution was obtained by rounding off to the nearest whole number the solution of the formula,

$$I = \frac{R}{1 + 3.322 \log N},$$

where I is the cell width or interval, R is the range of observations for

²"Shot" was the term used by Taft in referring to the group of ten to twelve consecutive cycles which were included in each of eight film shots of each operator taken at one hour intervals during the shift.

the element times, and N is the sample size. The frequency distributions appear as Tables 5 and 6 in the Appendix.

The histograms for Study C, variables excluded, and Study D, leveled element times, were plotted. The effect of the six variables listed previously was studied by superimposing the element-time frequency distributions for Study B, variables included, on the histograms of Study C.

Moments.—From the tabulations obtained from the grouped data, the first, second, and third moments, (the mean, variance, and skewness) for each distribution were calculated as shown in Figure 23 in the Appendix. The statistic "a", mean deviation divided by standard deviation, was calculated as a measure of peakedness for each distribution. This statistic was used in place of the fourth moment, due to the sample size which was less than the 200 minimum recommended for the fourth moment.

The Coefficient of Variation was calculated for each of the elements and cycles for Study C, unadjusted data, and Study D, leveled data, using the formula,

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100.$$

Curve-Fitting.—The element-time distributions for each element of Study C, unadjusted data, and Study D, leveled data, were fitted to a Normal curve having the formula,

$$Y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$$

and tested for goodness of fit by the use of the Chi-Square test. Sample calculations are shown in Figure 24 in the Appendix. The probability level

of the fit was obtained by the use of a nomograph (26) which showed probability level as a function of the value of the Chi-Square distribution and the degrees of freedom.

The element-time distributions for each element of the two studies were fitted to a Log Normal curve by translating the time intervals in terms of logarithms, and "t" values were used to enter the table of areas under the normal curve (27). They were tested for goodness of fit by the use of the Chi-Square test. Sample calculations are shown in Figure 25 in the Appendix. The probability level of the fit was determined by the use of the nomograph referred to above.

The element-time distributions for Studies C, unadjusted data, and D, leveled data, were fitted to a Pearson Type III curve which has the formula,

$$Y = Y_0 \left[\left(1 + \frac{\alpha_3}{2} t \right) \frac{\sqrt{4}}{\alpha_3} - 1 \right] \left[e^{-\frac{2}{\alpha_3} t} \right],$$

where α_3 is the skewness. "T" values were used to enter the tables (28) which show the area under this curve. The Chi-Square test was then used to test for goodness of fit. Sample calculations are shown in Figure 26 in the Appendix. The probability level of the fit was determined by the use of the nomograph (26).

Comparison of Element and Cycle Characteristics.—The cycle distribution characteristics for operator Q as determined for Study C, unadjusted data, were obtained from the results of Summers' (24) investigation. The characteristics of the cycle times for operator Q as determined for Study D, leveled data, were obtained from the results of an investigation being

conducted concurrently with this study by Wilson (29) in the preparation of his Master of Science thesis.³

In order to obtain a valid comparison between the characteristics of the cycle and the elements which make up the cycle, the cycle and element times were normalized by dividing the difference between the actual element time and the average time for that element by the standard deviation of the element distribution. Sample calculations are shown in Figure 27 in the Appendix.

Dixon (30) indicated that it was possible to predict, by means of confidence intervals, how close the cumulative distribution of a sample could be expected to be to the cumulative distribution of the population. This was a non-parametric test which involved only the reasonable assumption that the cumulative distribution function common to the two populations be continuous. The Z scores, or normalized values of the cycle times as calculated above and included as Tables 7, 8, 9 and 10 in the Appendix, were plotted along the abscissa and the cumulative-percentage frequency was plotted along the ordinate. This plot represented a cumulative-percentage histogram of normalized time values of the cycle. This histogram was used as the centerline for the band giving a confidence interval of 95 percent which was determined by the use of the formula,

$$C. I. = \frac{1.36}{\sqrt{N}},$$

³Wilson's study is being conducted to determine the effect on work-time distribution of removing the "level of performance" variable from the data.

where C. I. was the confidence interval, and N was the sample size. The validation of this formula was found by referring to the article by Dixon (31) which appeared in Volume 11 of the Annals of Mathematical Statistics.

On this same set of coordinates, the normalized element times were plotted in the form of a cumulative-percentage histogram. If the element cumulative histograms fell within the established confidence interval, the moments of the distribution, with the exception of the mean and variance which were neutralized by normalizing the data, could be considered similar.

Test for Element Independence.—Davidson (34) suggested the following as a means of determining whether or not elements were independent.

. . . if the elements are independent we know that the total cycle variance will be equal to the sum of the element variances. . . . In other words, a comparison of total cycle variance with the sum of element variances provides an indication of whether or not there are appreciable interaction effects among the element times.

Consequently, this test was performed for the data of both Study C, unadjusted data, and Study D, leveled data. The sum of the element variances and the cycle variance were tested at the 95 per cent confidence level for significant difference. The statistic F determined by the formula,

$$F = \frac{s_1^2}{s_2^2},$$

where s_1^2 is the sum of the variances of the elements and s_2^2 is the variance of the cycle, and the Fisher distribution were used for this test. The calculations required are shown in Figure 28 in the Appendix.

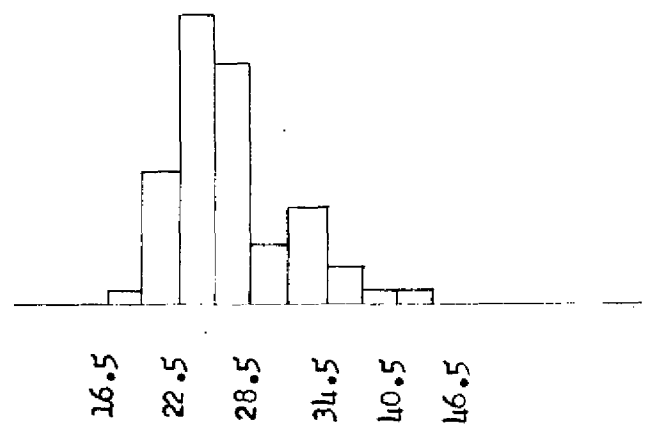
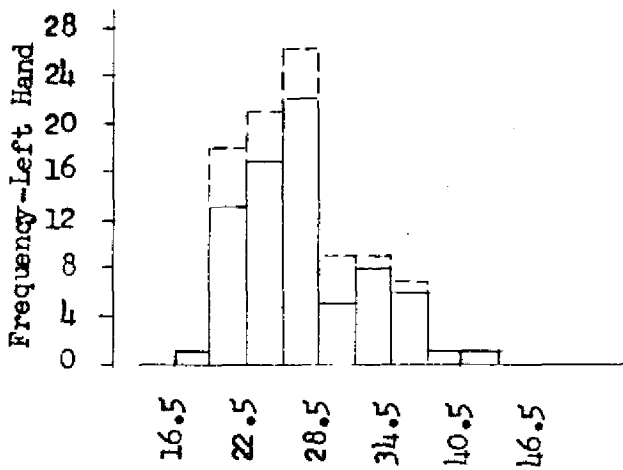
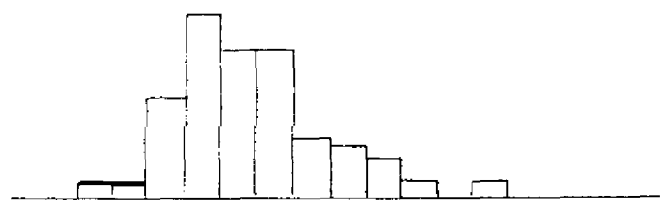
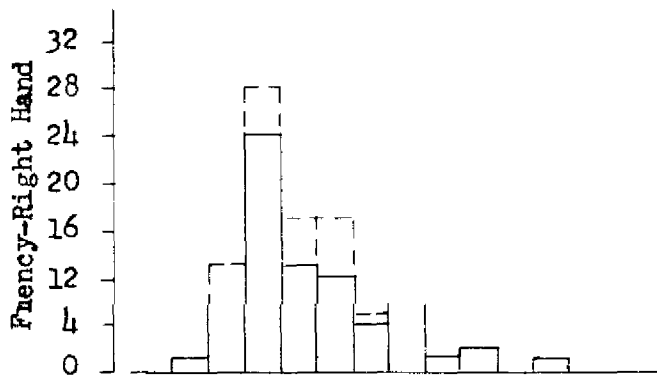
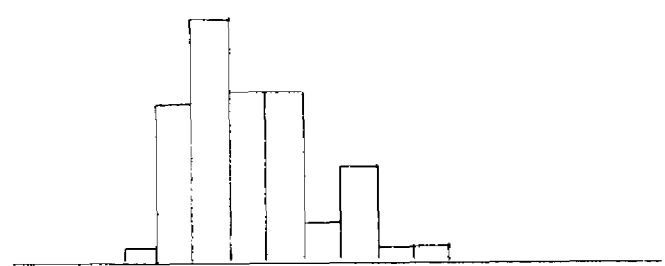
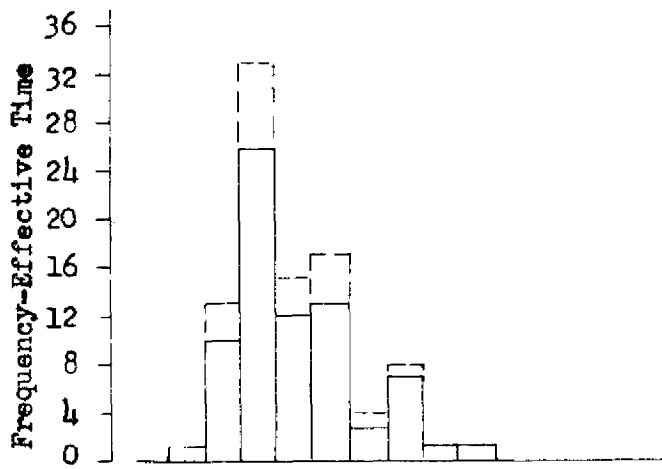
CHAPTER IV

DISCUSSION OF RESULTS

Element-Time Frequency Distributions.—No conclusive results could be obtained from visual analysis of the histograms for the element times, which appear in Figures 1 through 10. This was due to the uneven appearance of the histograms and to the lack of apparent trends in the histograms. The uneven appearance may have resulted from the limitations which were placed on the data on which they are based. In order to obtain a meaningful histogram the sample should consist of at least 100, and preferably 200 observations. There were only 7½ observations in Studies C (unadjusted data without variables) and D (leveled data). There were 9½ observations in Study B (unadjusted data with variables included).

Comparison of the histograms of the right and left hand, and effective element time revealed no exceptional differences. If there was a trend toward similarity, it was between the histograms of the effective times and the times for the left hand. However, this was not conclusive. When the histograms were grouped into elements according to the therbligs required to perform the element, little difference in the results was noted. The one characteristic which was common to all the histograms was the evidence of positive skew. This similarity will be discussed in greater detail in the section headed "Moments."

The element times from cycles containing methods variations were plotted on top of the histograms of unadjusted times. Reference to these



Study B (Variables included)
 Study C (Variables excluded)

Study D (Leveled Times)

Figure 1. Histograms of Element Time Distributions

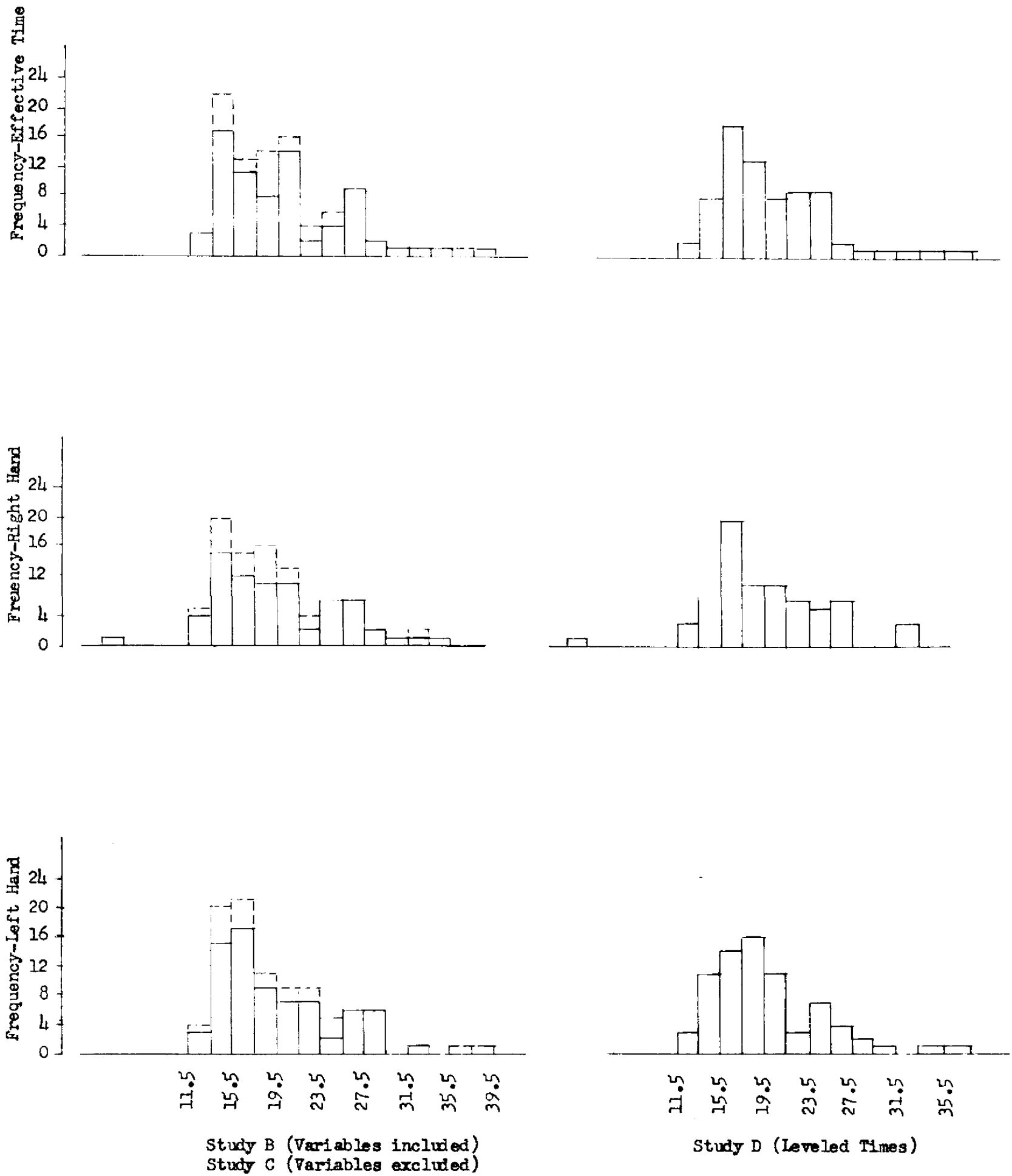
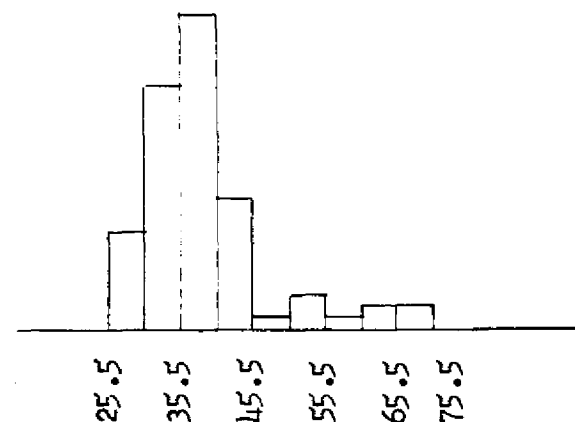
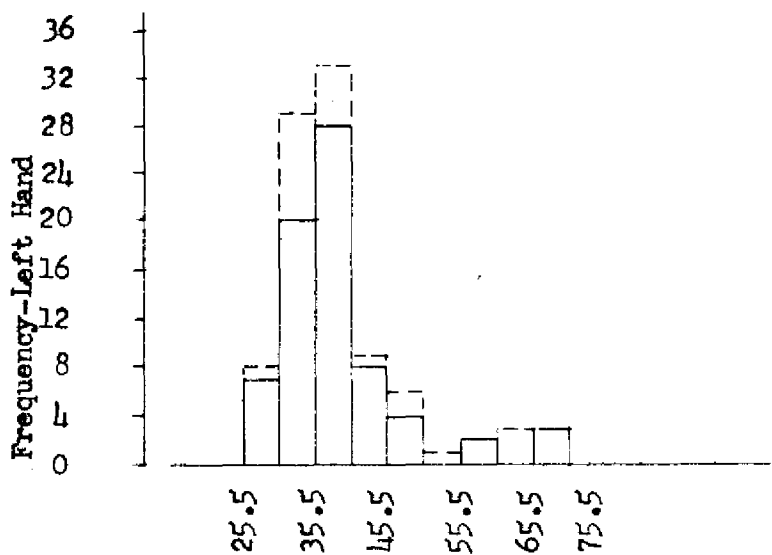
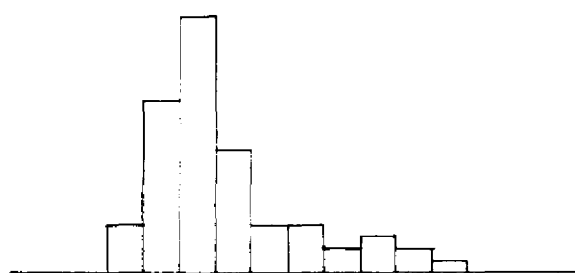
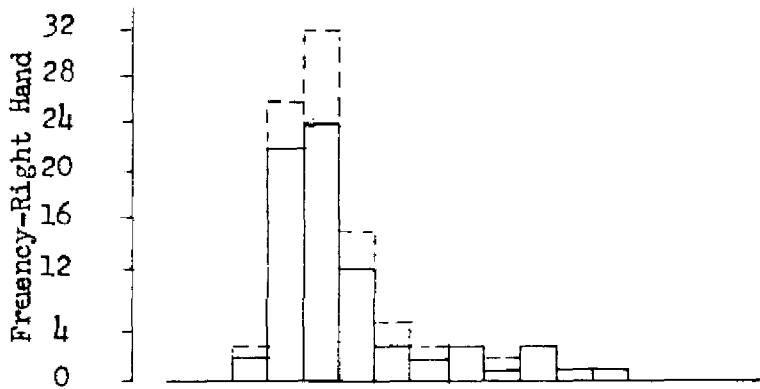
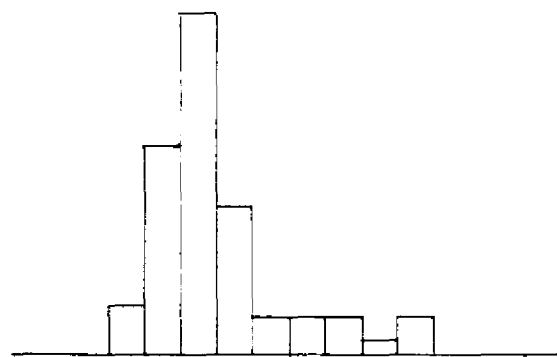
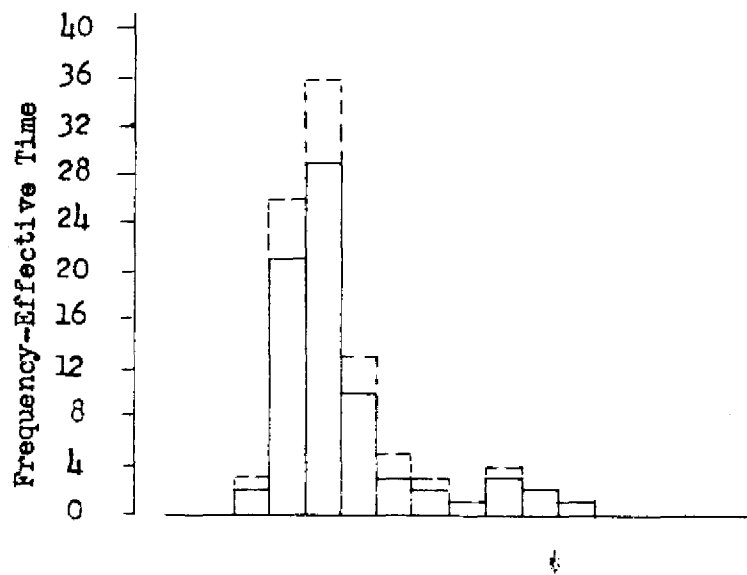


Figure 2. Histograms of Element Time Distributions

Element 3

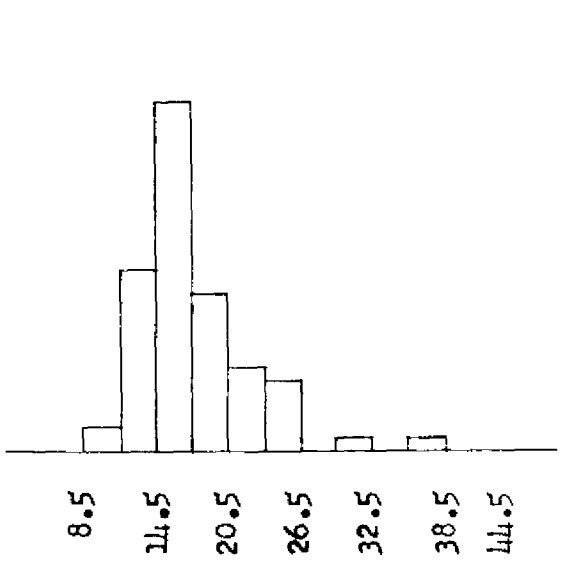
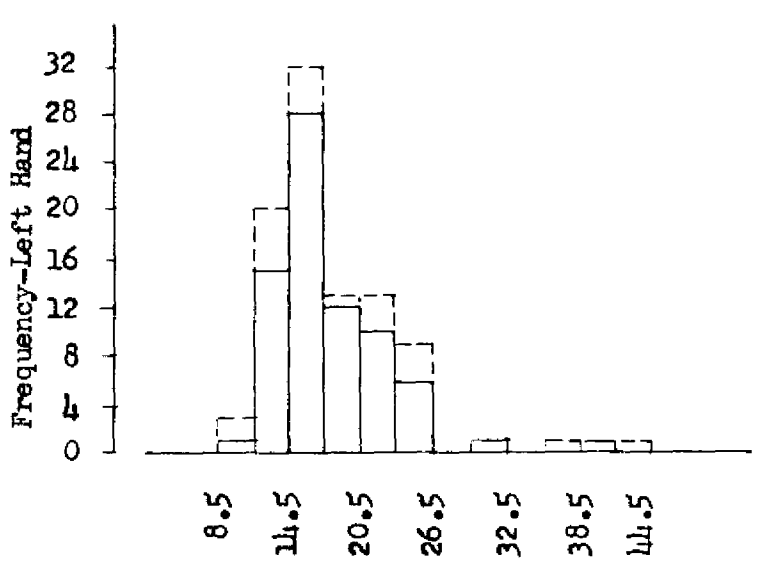
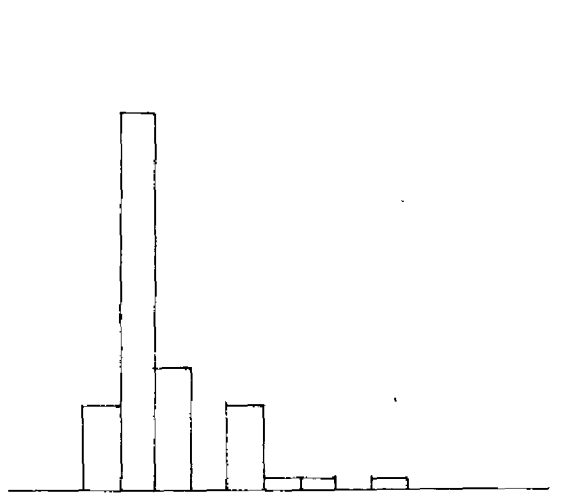
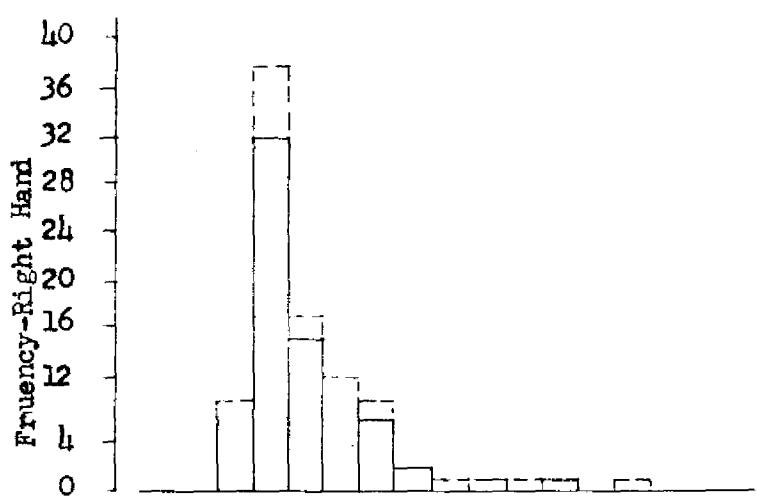
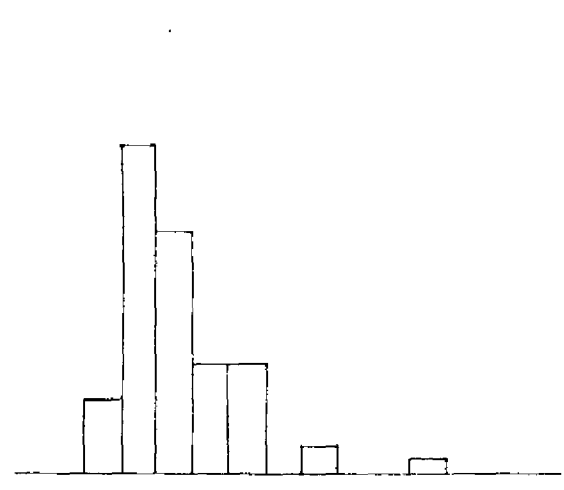
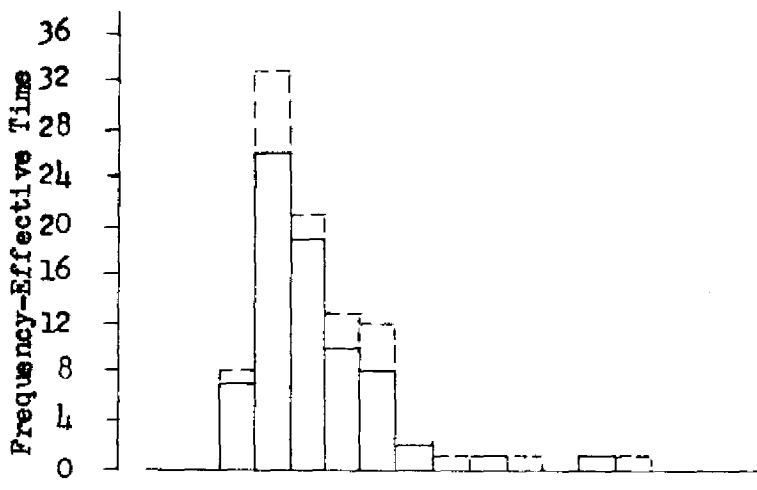


Study B (Variables included)
Study C (Variables excluded)

Study D (Levelled Times)

Figure 3. Histograms of Element Time Distributions

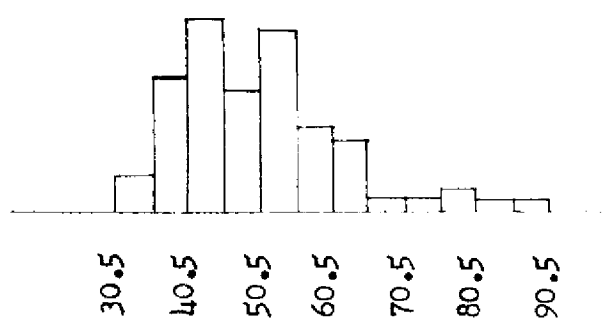
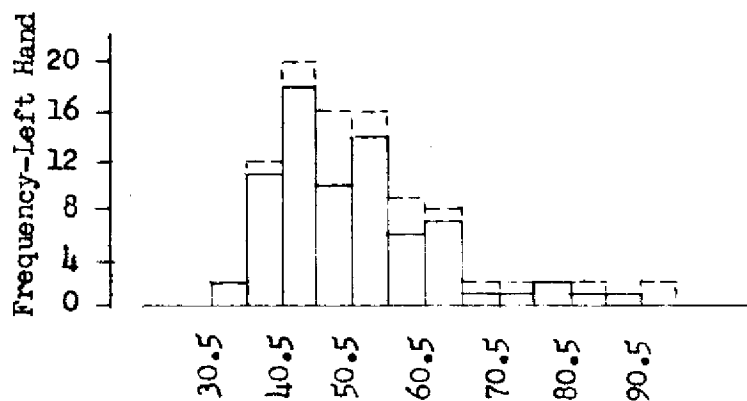
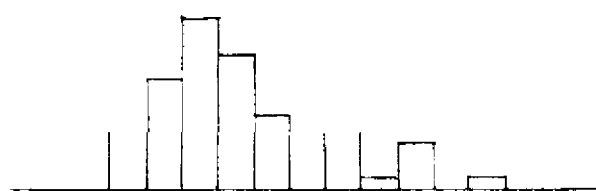
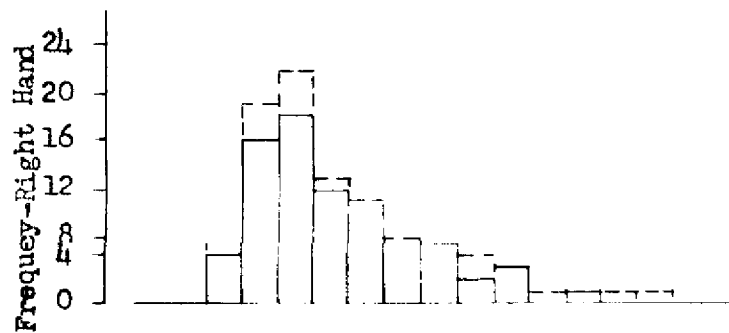
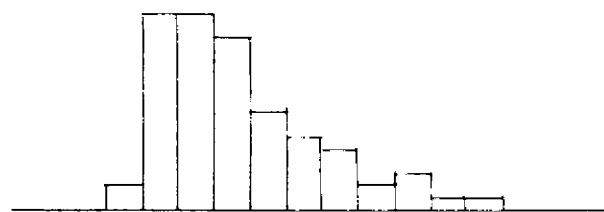
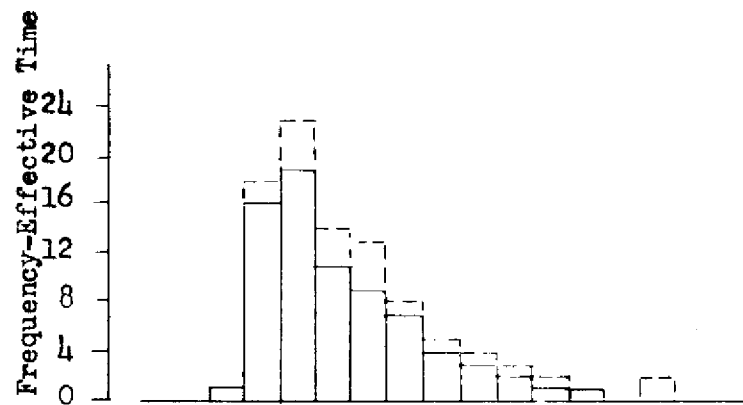
Element 4



Study B (Variables included)
 Study C (Variables excluded)

Study D (Leveled Times)

Figure 4. Histograms of Element Time Distributions



Study B (Variables included)
Study C (Variables excluded)

Study D (Leveled Times)

Figure 5. Histograms of Element Time Distributions

Element 6

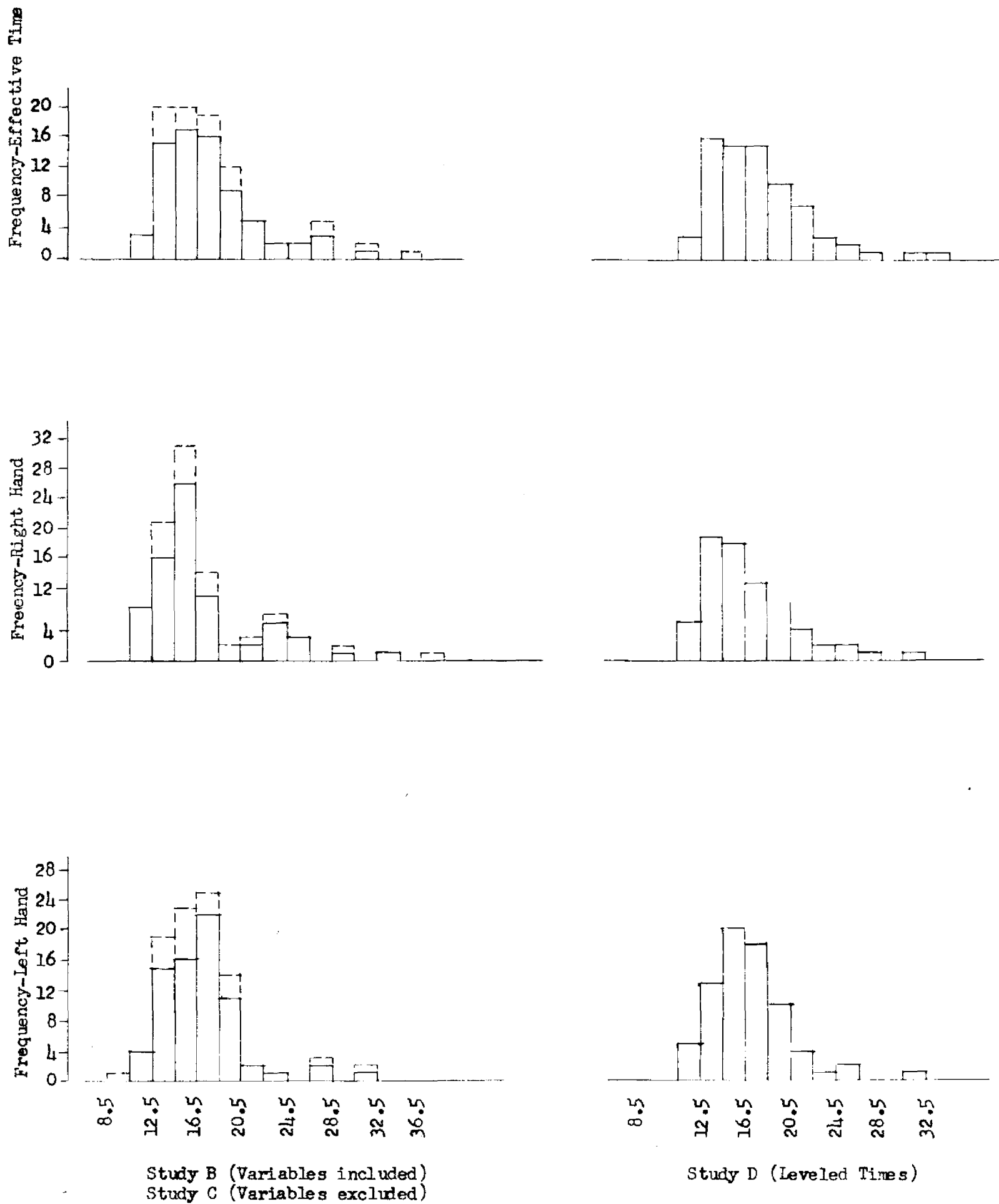
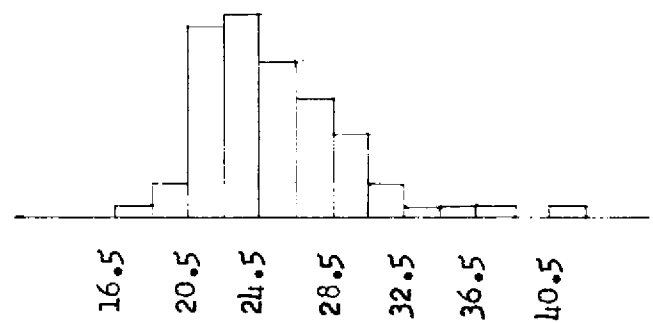
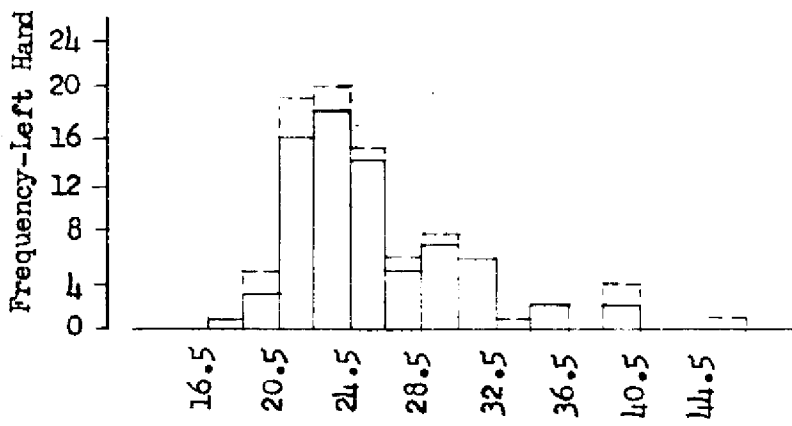
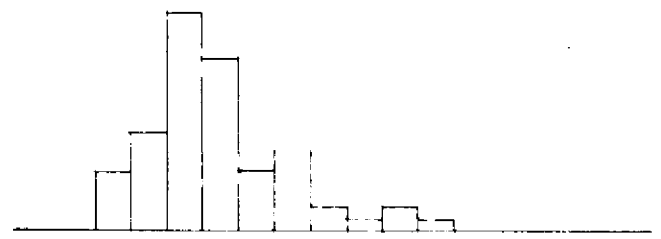
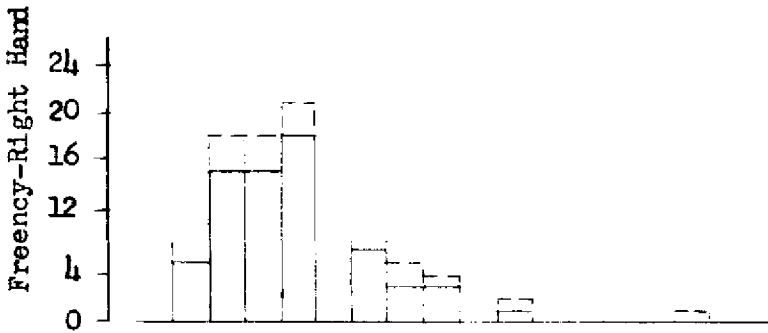
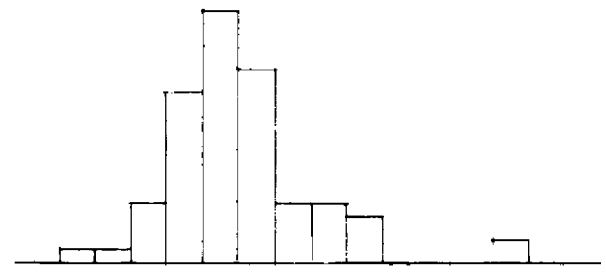
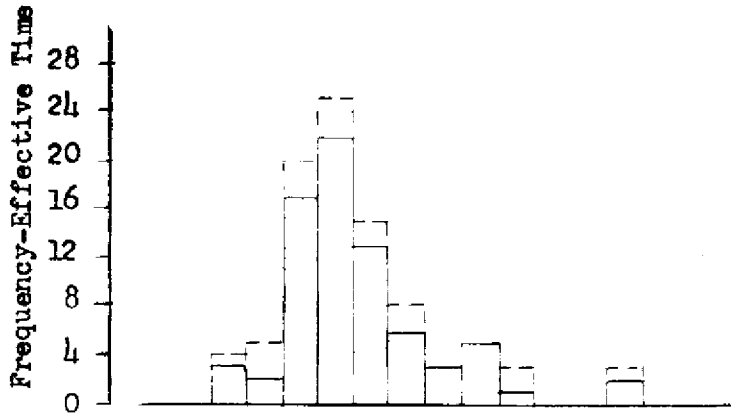


Figure 6. Histograms of Element Time Distributions

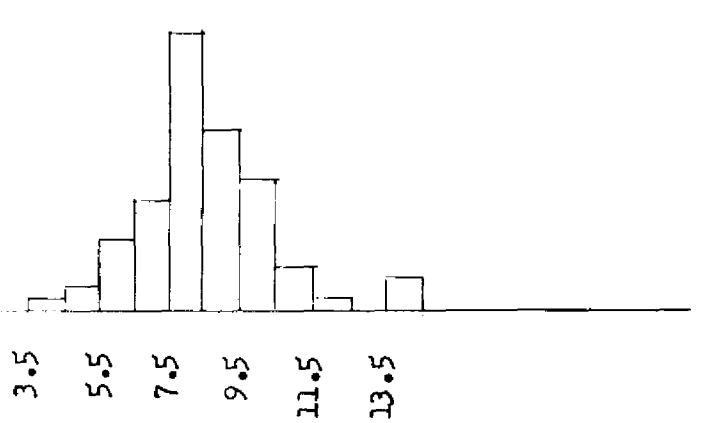
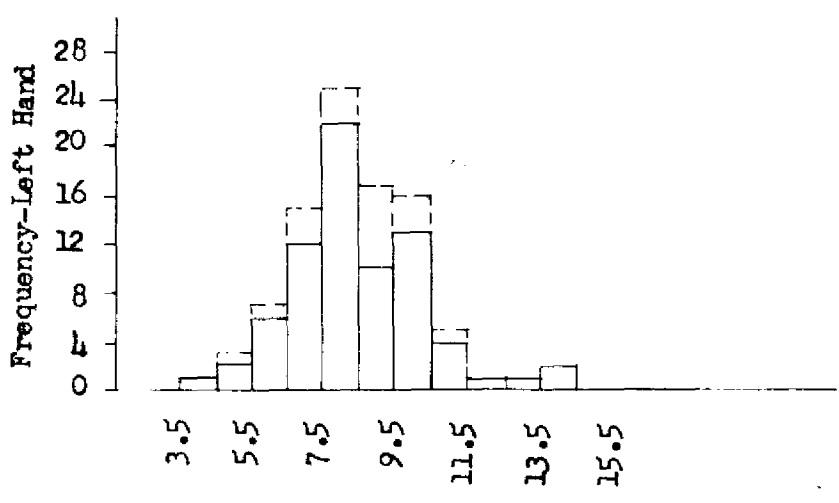
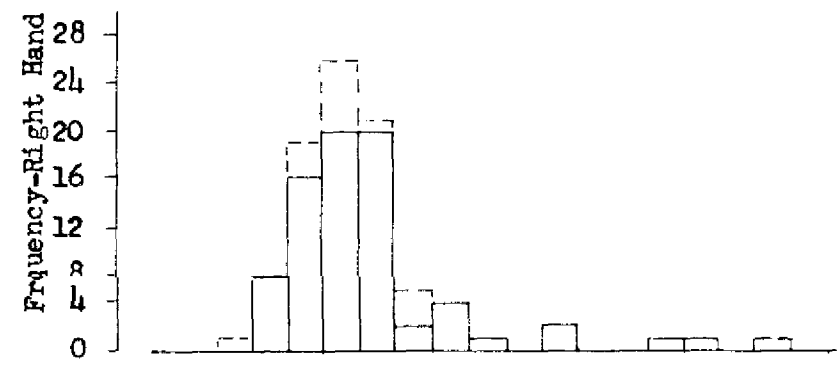
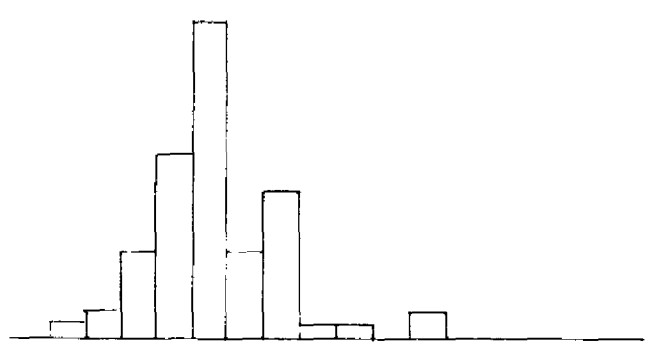
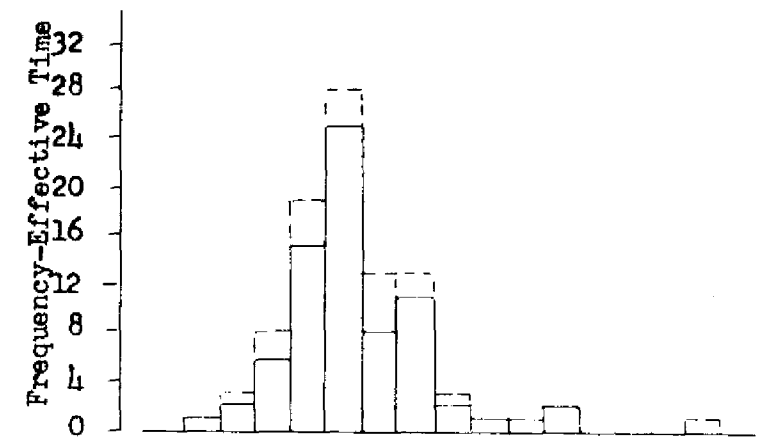
Element 7



Study B (Variables included)
Study C (Variables excluded)

Study D (Leveled Times)

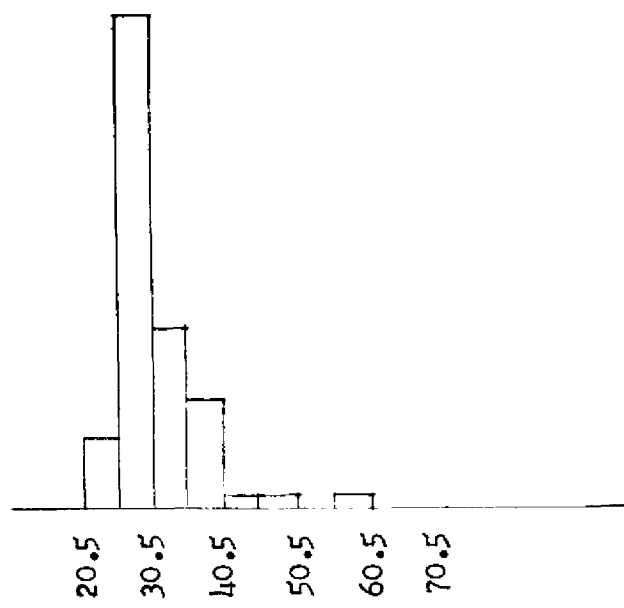
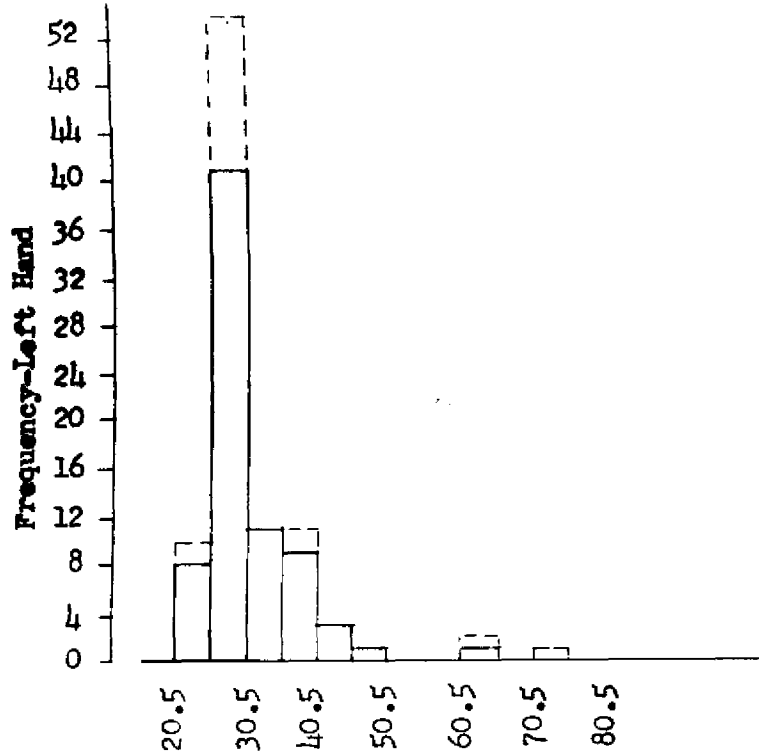
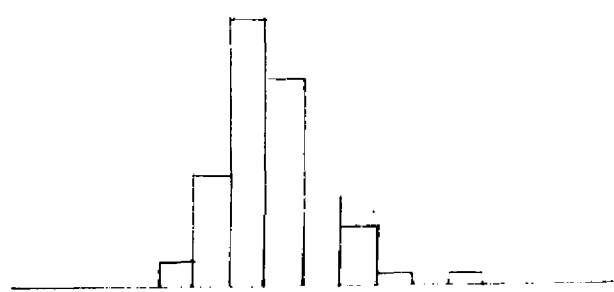
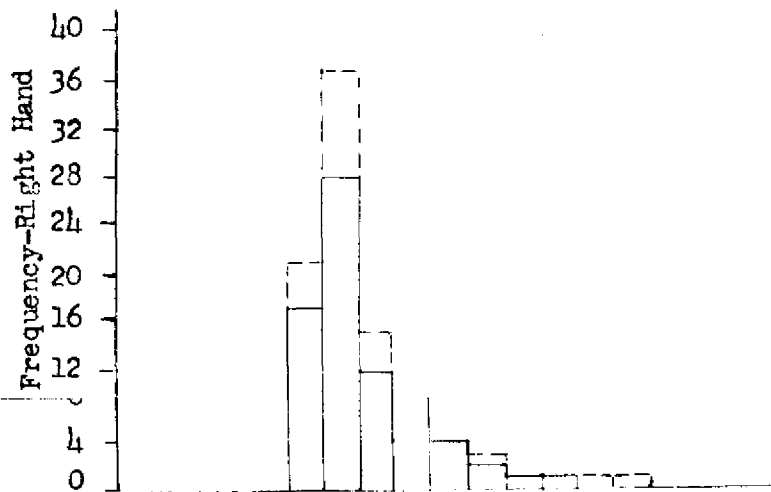
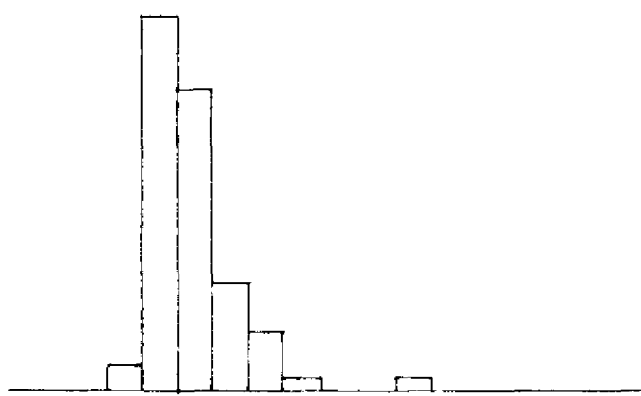
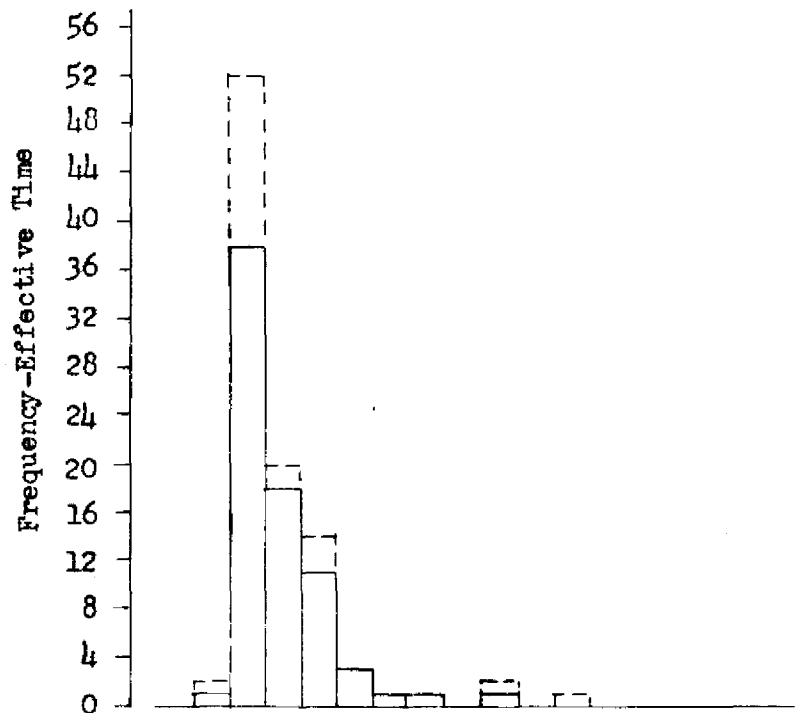
Figure 7. Histograms of Element Time Distributions



Study B (Variables included)
 Study C (Variables excluded)

Study D (Leveled Times)

Figure 8. Histograms of Element Time Distributions



Study B (Variables included)
 Study C (Variables excluded)

Study D (Levelled Times)

Figure 10. Histograms of Element Time Distributions

Element 11

histograms will show a general smoothing out of the shape of the histogram when these times were included. This smoothing effect could come from two possible sources. First, inclusion of these twenty times in the distributions created a sample size of 94, which was closer to the recommended minimum for preparation of histograms. Consequently, this would tend to form a more characteristically shaped histogram. Second, and the implication here was considerably more important, it was possible that this smoothing effect was caused by the absence of interaction among the elements. Usually a fumble, a dropped part or similar variable occurred during only one element of the cycle. If the theory of interaction among elements of a cycle were correct, this would mean that at least the preceding and succeeding element times, and possibly the times for other elements would be affected. The range of the histogram, or the difference between the minimum and maximum times, should be increased if the time for an element, which had been increased by the presence of a variable, was included in the data. By the same reasoning, if the succeeding element time were shortened by the operator (to compensate for the extra time taken in the previous element), it would be expected that this time would be less than the average value for that element. If this occurred frequently enough, a reverse, or negative skew should tend to appear in the histogram. As evidenced by the smoothing effect on histogram shape by including the variables, and by the absence of any additional skewing, this was not the case in the present study. This alone is not enough evidence to reject the theory of element independence. The grouping of element time into cells reduced the efficiency of the histogram as a test for this type of

hypothesis. A more rigorous test would be required to investigate this possibility. However, there was enough evidence to make further study of this possibility worthwhile.

For purposes of further analysis the elements were grouped according to the therbligs required to perform the element. Those which involved transport empty, select and grasp were classified as "get" elements and those which required transport loaded, position, assembly and release were classified as "place" elements. This analysis was also inconclusive as evidenced by the fact that elements two and four were quite peaked, while this peakedness was less pronounced for elements six and eight which were also classified as "place" elements. The absence of similarity was also noted in the case of the "get" elements. No appreciable difference was observed in the extent of skewness.

Because of the inconclusive results obtained in the above analyses, it was decided to exclude the variables in subsequent statistical analysis. Furthermore, the exclusion of these variables would make the testing of the primary hypothesis more objective.

One additional comparison was made of the histograms. This was for the purpose of studying the effect of compensating for level of performance by the leveling technique explained above as Study D. Due to the uneven appearance of the histograms, the results of this comparison were also inconclusive. However, there was a slight shift along the time axis of most of the histograms. This resulted in a pronounced overall negative shift in the maximum value, a less obvious negative shift in the minimum value and a slight positive shift in the modal value. More specifically, there was a negative shift of the maximum and minimum values

and a positive shift of the modal value in element number three. A negative shift also occurred in the maximum value of element four, while this negative shift of the maximum value with a positive shift of the modal value recurred in the fifth element. A negative shift appeared in the mode of the next two elements with no change in maximum or minimum. In element six the only change was a negative shift in the minimum value. The only other change appeared in the tenth element where a positive shift of both the mode and minimum occurred.

Moments.—A more precise evaluation of the differences between right and left hand, and effective element times was obtained by the analysis of the calculated moments for Study C (unadjusted data) and Study D (leveled data). There was no established pattern as to the relationship among the calculated mean times for the three values in either study, nor was there any apparent relationship between the values for Studies C and D. The element mean times for Study C (unadjusted data with variables excluded) and Study D (leveled data) were paired. This was done for the effective, the left hand and right hand element times. A "t" test was then applied to the pair whose mean times showed the greatest spread between the two studies. This was the effective time for element six. At the 5% level, there was no significant difference between the mean time for the two studies. This lead to the conclusion that there was no significant difference between the paired mean times of the two studies. This applies to the comparison between the right hand times, the left hand times and the effective element times.

A comparison of the means within the "get" and "place" groups

referred to above was equally unproductive since there was a difference of approximately equal magnitude among elements within these groups as there was between the groups.

Mention should be made of the relationships noted in elements ten and eleven. In both of these elements the mean effective time and that for the left hand were essentially the same. However, in element number ten, the value for the right hand was much less than either of those for the left hand or the effective time. The value for the right hand mean time was greater than either of the other two values for the eleventh element. In all probability this strange relationship may be explained by the fact that the operator gave visual direction to the reach and grasp of the right hand in removing the assembled unit from the fixture just before the staking operation. This resulted in a lag by the left hand. On the staking element, the cumulative time for both hands must be approximately equal since both pens were staked at the same time. Any variation in the cumulative time would have been introduced by a difference in the time required to disassemble the pens from the staking device. This equalization of cumulative times for the operation up to the end of element 11 would indicate that the right hand was required to wait until the left hand "caught up" in the performance of the cycle. The difference between the mean for the right hand and that of the left hand and effective time was of no special significance in the pursuit of the thesis objective, but did provide a source of possible further research into the relationship between symmetry of motions of right and left hands and visual guidance of either hand.

There was no apparent difference among the paired standard deviations

of the left hand, right hand and the effective element time for each study. However, with the exception of element six, the standard deviation was less for Study D (leveled element times) and Study C (unadjusted element times). It was concluded, therefore, that the variance in element times was reduced by the application of the leveling factor to account for the level of performance of the operator during the shift.

With the exceptions as indicated below, the application of the leveling factor served to reduce the amount of positive skew which was apparent in all distributions. The exceptions referred to were elements three and eight in which the degree of skew was increased from Study C to Study D. There was little difference between the skew of the distributions of the right and left hand or effective element times, and even the general decrease caused by the application of the leveling factor as indicated above was so small in most cases that it is doubtful if a rigorous test would reveal significant difference in the skewness.

Generally, the "a" values indicated that the distributions were more peaked than the normal curve. There was no particular difference in the degree of peakedness for the elements either between studies, unadjusted or leveled element times, or in the comparison of the three values within each study.

Table 1 summarizes the four moments of all distributions studied. For subsequent analyses only the effective element time was considered.

Curve-Fitting.--For Study C (unadjusted element times) the element distributions fitted the Normal curve at a probability level of 1.3 per cent or less. This compared with a fit to the Normal curve for the modified

Table 1. Summary of Statistical Moments for Study C
(Unadjusted Element Times with Variables Excluded)
and Study D (Leveled Element Times)

Element and Study	Mean (\bar{X})			Standard Deviation (σ_x)		
	LH	RH	EFF	LH	RH	EFF
2-C	27.081	27.771	27.000	5.052	5.898	4.980
2-D	26.645	27.659	27.050	4.686	5.772	4.980
3-C	19.636	19.148	19.932	5.342	5.334	5.546
3-D	19.686	19.360	20.064	4.888	4.952	4.862
4-C	39.215	41.245	40.365	9.325	10.635	9.450
4-D	38.860	41.430	40.550	8.815	9.935	9.125
5-C	17.866	15.634	16.204	4.749	4.779	4.962
5-D	17.592	15.603	16.173	4.581	4.401	4.638
6-C	50.500	48.205	52.080	11.455	10.915	10.785
6-D	50.480	48.455	49.200	11.515	10.990	11.075
7-C	16.878	16.636	17.690	3.538	4.300	4.474
7-D	16.874	16.794	17.686	3.482	3.872	4.290
8-C	25.446	21.312	24.716	4.410	4.096	5.888
8-D	25.388	21.306	24.686	4.220	3.818	4.080
9-C	8.473	8.807	8.320	1.890	5.196	1.840
9-D	8.509	8.577	8.199	1.779	2.537	1.752
10-C	23.496	15.522	23.442	3.804	2.380	3.776
10-D	23.632	15.036	23.578	3.726	2.278	3.698
11-C	30.820	46.090	32.375	6.475	7.610	6.490
11-D	30.755	46.090	32.580	5.705	6.915	6.160

LH - Left hand element time.
RH - Right hand element time.
EFF - Effective element time.

Table 1. Summary of Statistical Moments for Study C
(Unadjusted Element Times with Variables Excluded)
and Study D (Leveled Element Times)

(Continued)

Element and Study	Skewness ($\sqrt{\gamma}$)*			Peakedness (α)**		
	LH	RH	EFF	LH	RH	EFF
2-C	1.140	1.144	0.833	0.761	0.784	0.798
2-D	0.969	0.906	0.673	0.764	0.786	0.798
3-C	1.037	0.549	0.987	0.819	0.793	0.798
3-D	1.145	0.333	1.142	0.774	0.769	0.790
4-C	1.669	1.685	1.765	0.686	0.726	0.709
4-D	1.603	1.321	1.454	0.686	0.754	0.733
5-C	1.718	1.844	1.916	0.757	0.732	0.711
5-D	1.488	1.517	1.666	0.754	0.772	0.719
6-C	1.096	0.969	1.045	0.772	0.793	0.879
6-D	1.043	0.928	0.999	0.780	0.774	0.793
7-C	1.517	1.604	1.317	0.720	0.727	0.735
7-D	1.310	1.313	1.316	0.748	0.769	0.747
8-C	1.131	1.108	0.473	0.736	0.748	0.521
8-D	1.229	1.001	1.253	0.756	0.757	0.726
9-C	0.847	2.570	0.836	0.777	0.311	0.738
9-D	0.485	2.523	0.842	0.757	0.632	0.719
10-C	2.513	1.904	2.596	0.625	0.672	0.615
10-D	2.247	1.610	2.326	0.642	0.741	0.625
11-C	2.139	1.278	2.142	0.734	0.788	0.723
11-D	2.000	1.002	2.000	0.744	0.776	0.699

LH - Left hand element time.

RH - Right hand element time.

EFF - Effective element time.

*Normal curve value for $\sqrt{\gamma} = 0.0$

**Normal curve value for $\alpha = 0.797$

cycle distribution for the same operator of 1.0 per cent as reported by Summers (24). This indicated an equally poor fit to the Normal curve for both the element and cycle time distributions.

For Study D (leveled element times) a slightly better fit was obtained for the element distributions. This is probably accounted for by the reduction in skewness reported for the application of the leveling factor to the element times. The probability level of this fit to the Normal curve ranged from 0.1 per cent to 2.5 per cent as compared to the probability level for the cycle, as reported by Wilson (29), of 5.7 per cent. The highest probability level for the element distributions was well below that for the cycle and was considered to be a very poor fit.

The fit of the element distributions to the Log Normal curve was found to be much better than the fit to the Normal curve for most of the elements. The probability levels in Study C ranged from 0.0 per cent for two elements to 48.0 per cent. The probability level for the cycle was 6.0 per cent. In Study D (leveled data) it was found that the element distributions fitted the Log Normal curve with probability levels ranging from 0.22 per cent to 48.0 per cent. The probability level for six of the element distributions was at or above 9.0 per cent which indicated a fairly good fit. The probability level for the cycle was noted to be 15.0 per cent.

The fit of the element distributions to the Pearson Type III curve was not satisfactory since the skewness of the fourth, fifth, seventh, tenth and eleventh elements was greater than the maximum skewness value in the Pearson Type III area tables. However, for those elements which could be fitted to this curve, the probability level was above that for

the Log Normal in five of the nine fits of the Pearson Type III curve. Study D results were better than those for Study C. Again, the reduction in the amount of skew as a result of compensating for the level of performance probably accounted for this since a lower value of skewness allowed the use of the Pearson Type III area tables.

The results of the comparison of element and cycle distribution fits to the three curves indicated some similarity in the characteristics of these distributions since there was a tendency for the element distributions to fit the three curves in a similar way. There was a poor fit to the Normal curve for both element and cycle time distributions, and a fairly good fit to the Log Normal curve. A good fit to the Pearson Type III curve was obtained for these element and cycle distributions which could be fitted. The skewness of eleven elements was too great to allow this curve fit. There is some doubt, however, as to whether any of these curves typify the theoretical element-time distribution. The above results are summarized in Table 2.

Comparison of Element and Cycle Characteristics.--Analysis of the cumulative-percentage histograms of the comparison of Z scores for element and cycle times indicated a similarity between element and cycle time distribution characteristics. For the unadjusted element times it was found that there were three points outside of the confidence intervals. There was one point out of 20 for element three outside the interval, one point out of eleven in element nine and one point out of 20 in element 11 outside the confidence interval. It was felt that considering the number of elements and points within elements which were plotted, there was not

Table 2. Summary of Curve-Fitting Results
for Study C (Unadjusted Element Times)
and Study D (Leveled Element Times)
(Probability of Fit in Per Cent)

Element and Study	Normal Curve	Log Normal Curve	Pearson Type III Curve
2-C	0.0	6.6	2.2
2-D	0.4	10.0	15.0
3-C	0.0	0.06	0.15
3-D	1.4	13.0	37.0
4-C	0.0	0.19	—*
4-D	0.0	0.3	—*
5-C	0.3	28.0	—*
5-D	0.15	9.0	—*
6-C	0.07	48.0	1.0
6-D	0.4	12.0	57.0
7-C	0.55	44.0	—*
7-D	1.6	40.0	—*
8-C	0.045	2.2	0.5
8-D	2.5	16.0	—*
9-C	1.3	0.0	3.5
9-D	1.5	0.22	1.2
10-C	0.0	0.03	—*
10-D	0.05	2.8	—*
11-C	0.0	0.0	—*
11-D	0.014	1.3	—*
C-C**	1.0	6.0	3.0
C-D***	5.7	15.0	20.0

*Indicates that value of skewness is above the maximum value shown in area tables.

**C-C — Cycle curve fit, Study C.

***C-D — Cycle curve fit, Study D.

enough evidence to say that the element-time distribution characteristics varied significantly from those of the work-time distribution for the cycles of which these elements were a part. This conclusion was also supported by location of the points which were outside the confidence interval. Reference to these histograms revealed that these three points would fall within the confidence interval if a higher level had been chosen.

The analysis of the plot of the leveled data of Study D revealed no points outside the confidence interval. Therefore, the null hypothesis stated in Chapter II was accepted for this study. The results of these tests are shown in Figures 11 through 20. In view of the similarity within the groups of the "get", and "place" elements, a cumulative-percentage histogram was plotted for all of the "get" elements, one for all of the "place" elements, and one each for the other three elements.

Test for Element Independence.—Table 3 shows the result of the addition of element variances for the test of element independence. At the 95 per cent confidence level it was found that, for Studies C, unadjusted data, and D, leveled data, there was no significant difference between the sum of the element variances and the cycle variance. This result was an indication of independency of elements, but more rigorous tests would have to be made to obtain assurance that this was true.

Table 4 shows a comparison of element variance and the Coefficient of Variation with the element mean time arranged in order of magnitude from the maximum value to the minimum value. Analysis of this table revealed that the conclusions of Abruzzi¹ were not fully verified by the

¹Supra, p. 5.

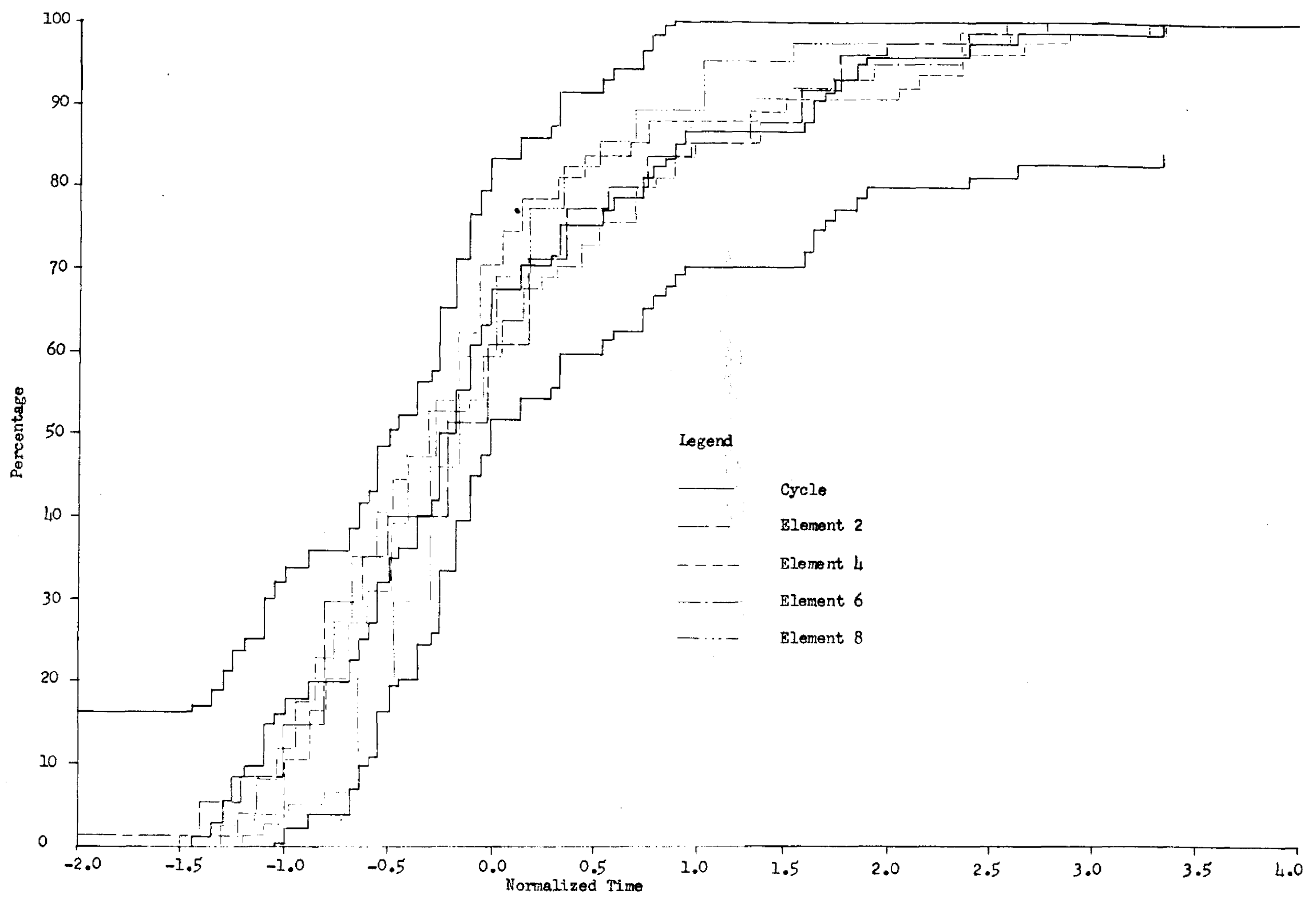


Figure 11. Cumulative-Percentage Histograms From Study C (Unadjusted Times) Elements 2, 4, 6, 8

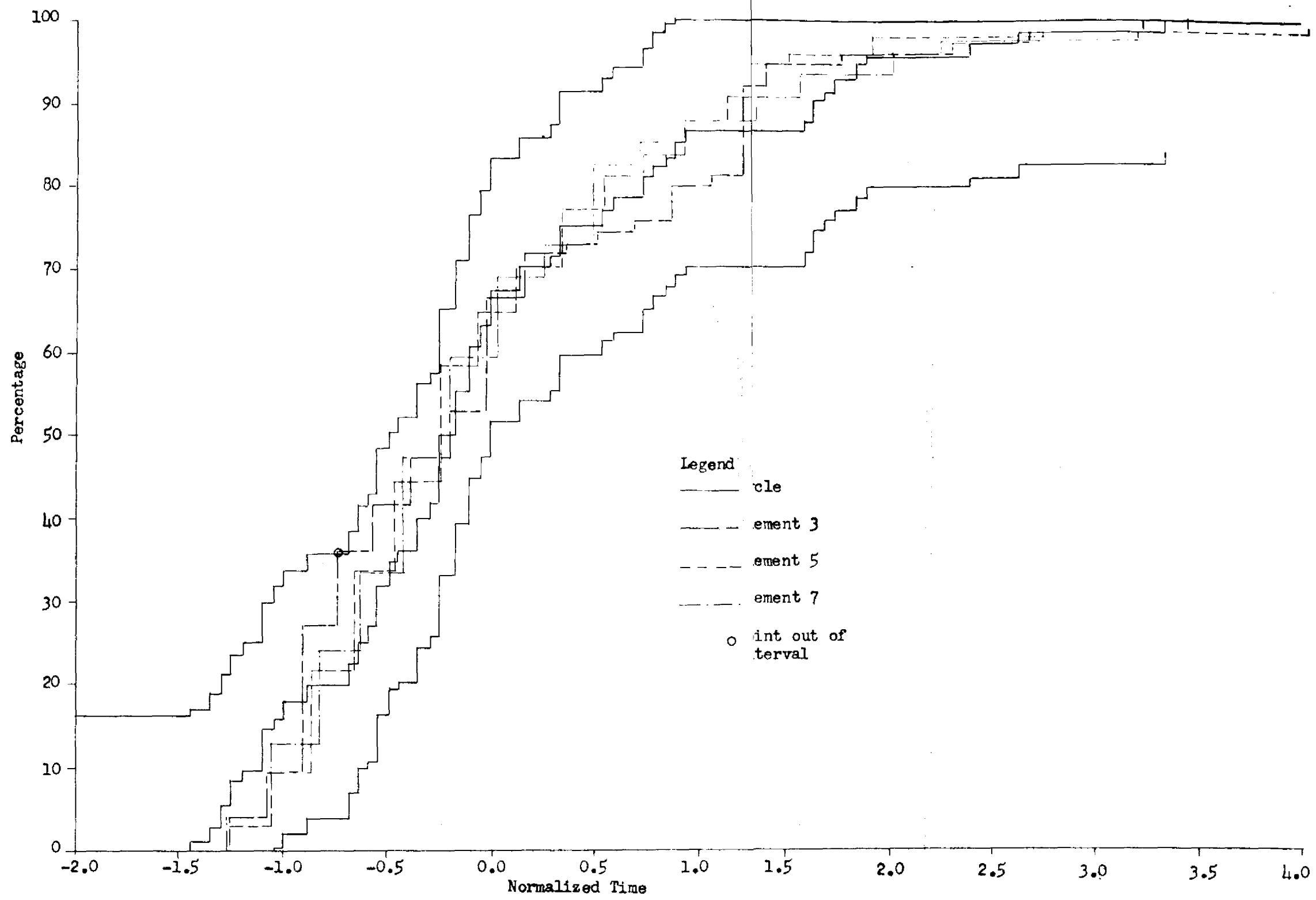


Figure 12. Cumulative-Percentage Histograms From Study C (Unadjusted Times) Elements 3, 5, 7

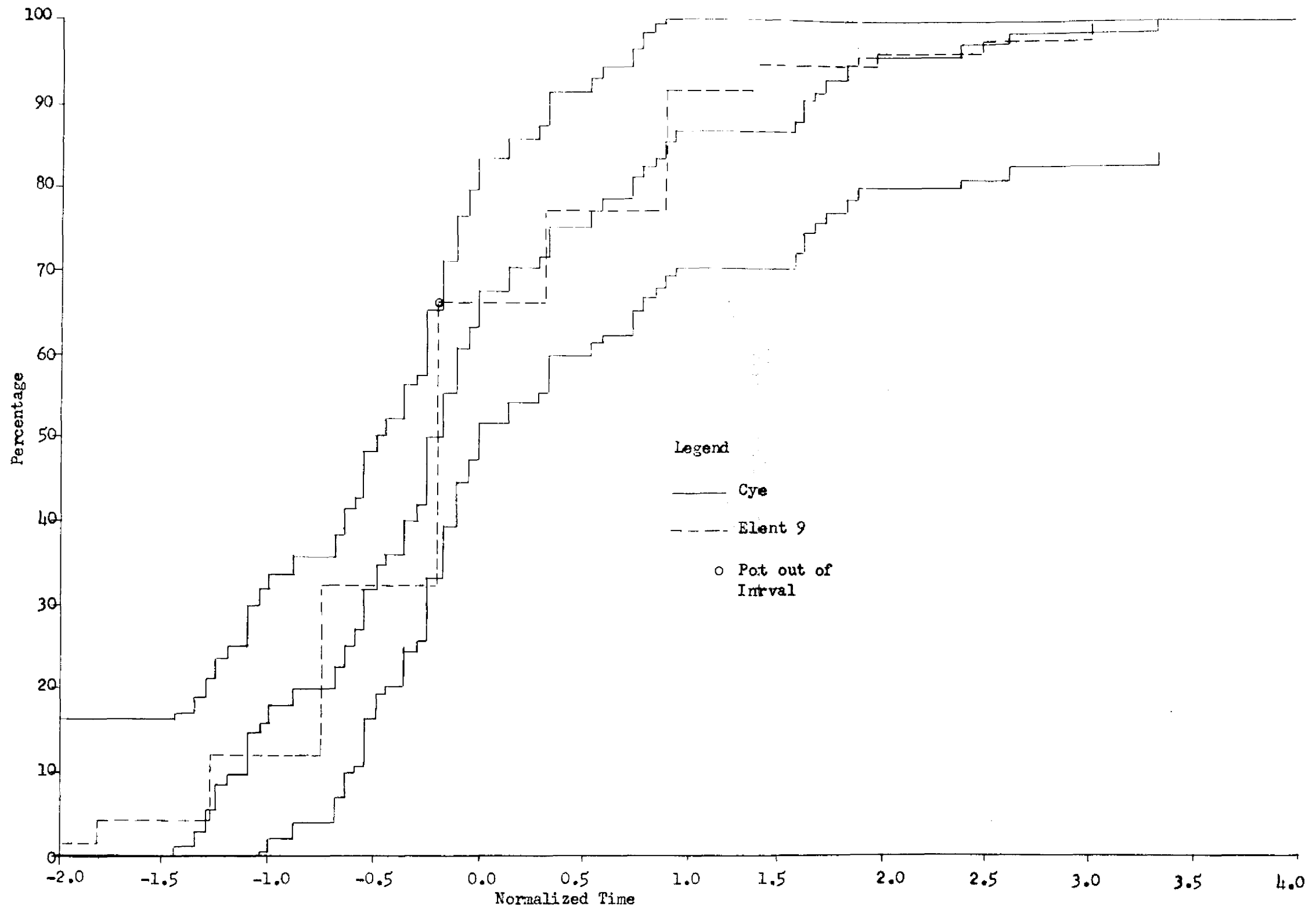


Figure 13. Cumulative-Percentage Histograms From Study C
(Unadjusted Times) Element 9

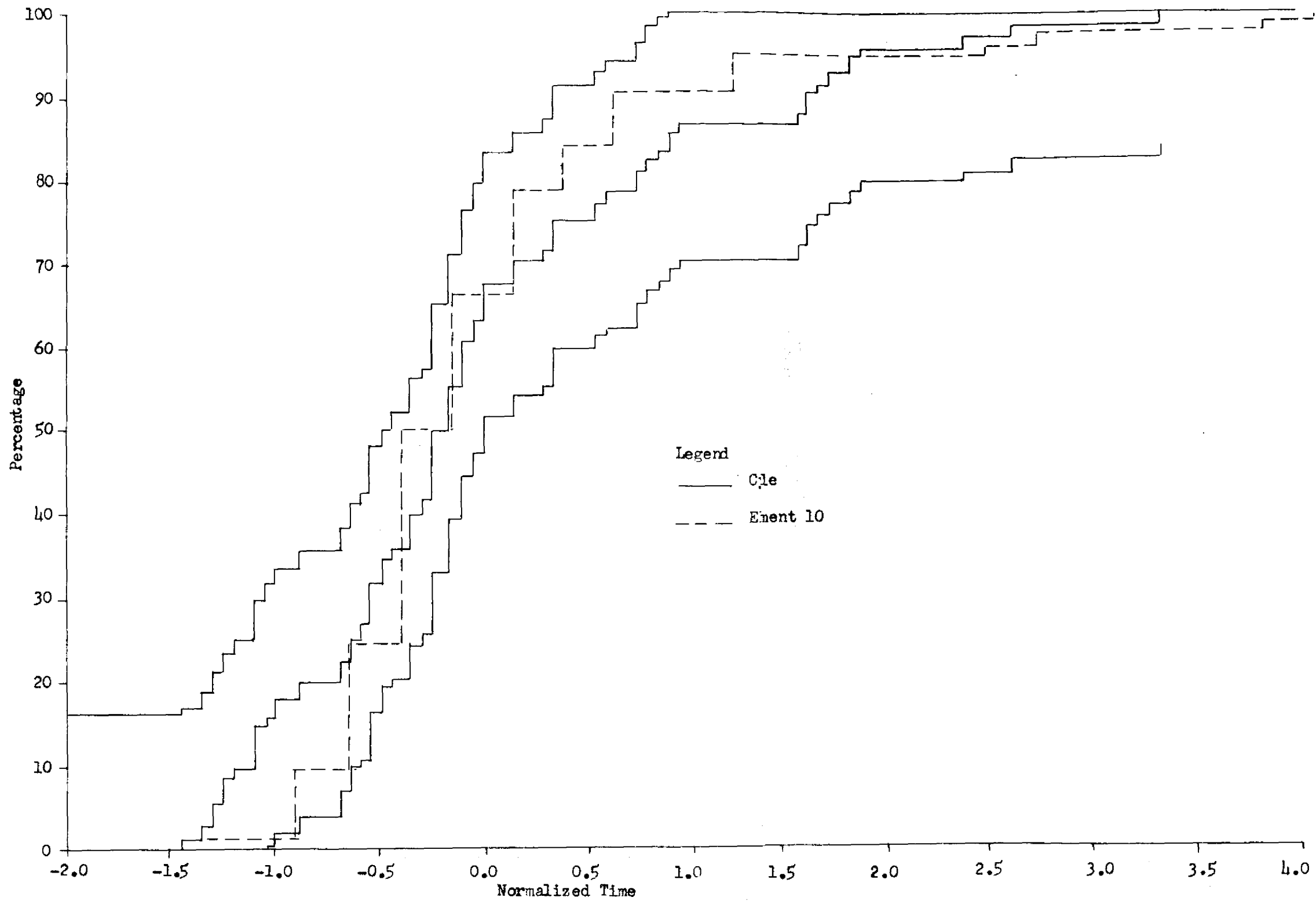


Figure 14. Cumulative-Percentage Histograms From Study C
(Unadjusted Times) Element 10

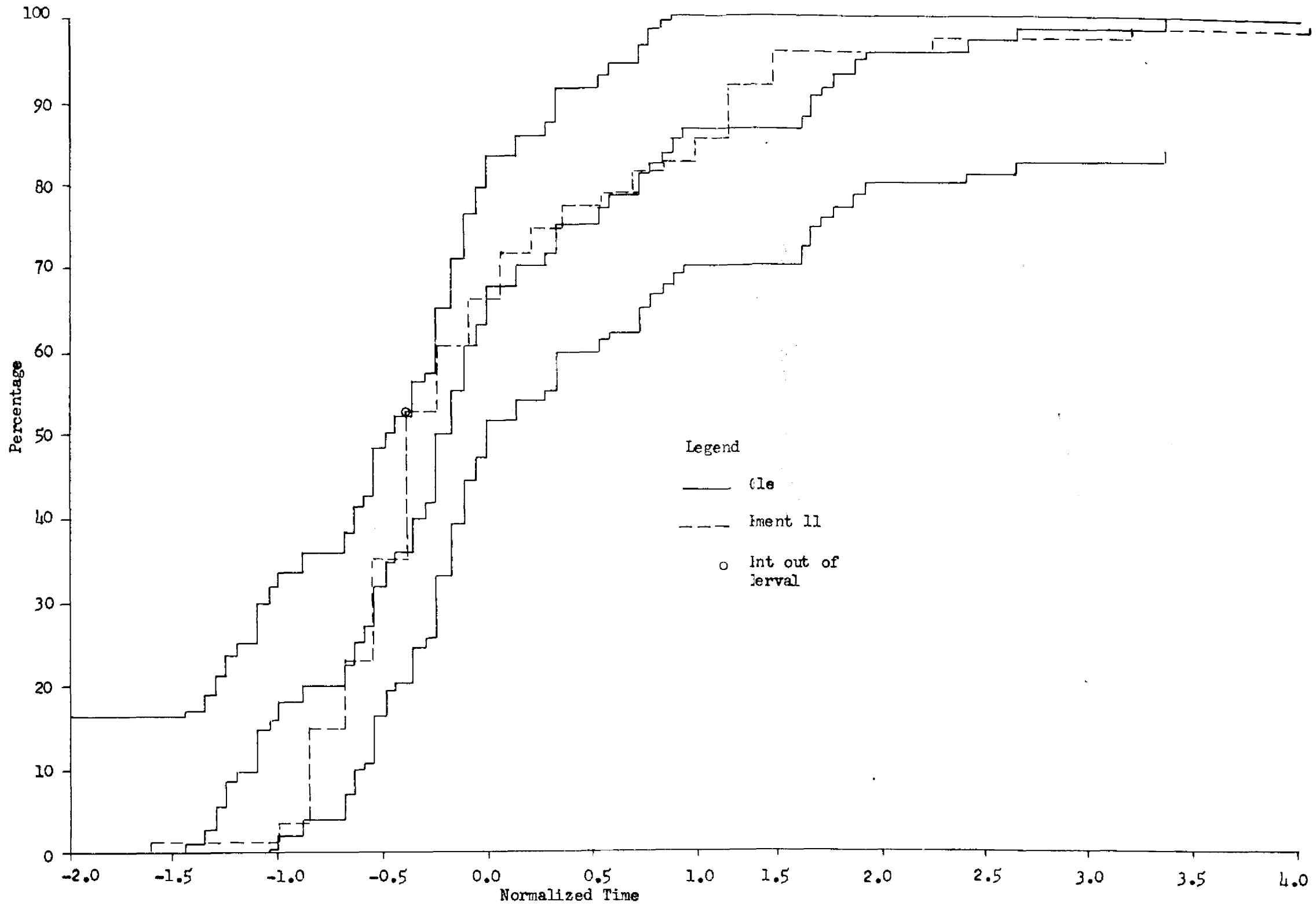


Figure 15. Cumulative-Percentage Histograms From Study C
(Unadjusted Times) Element 11

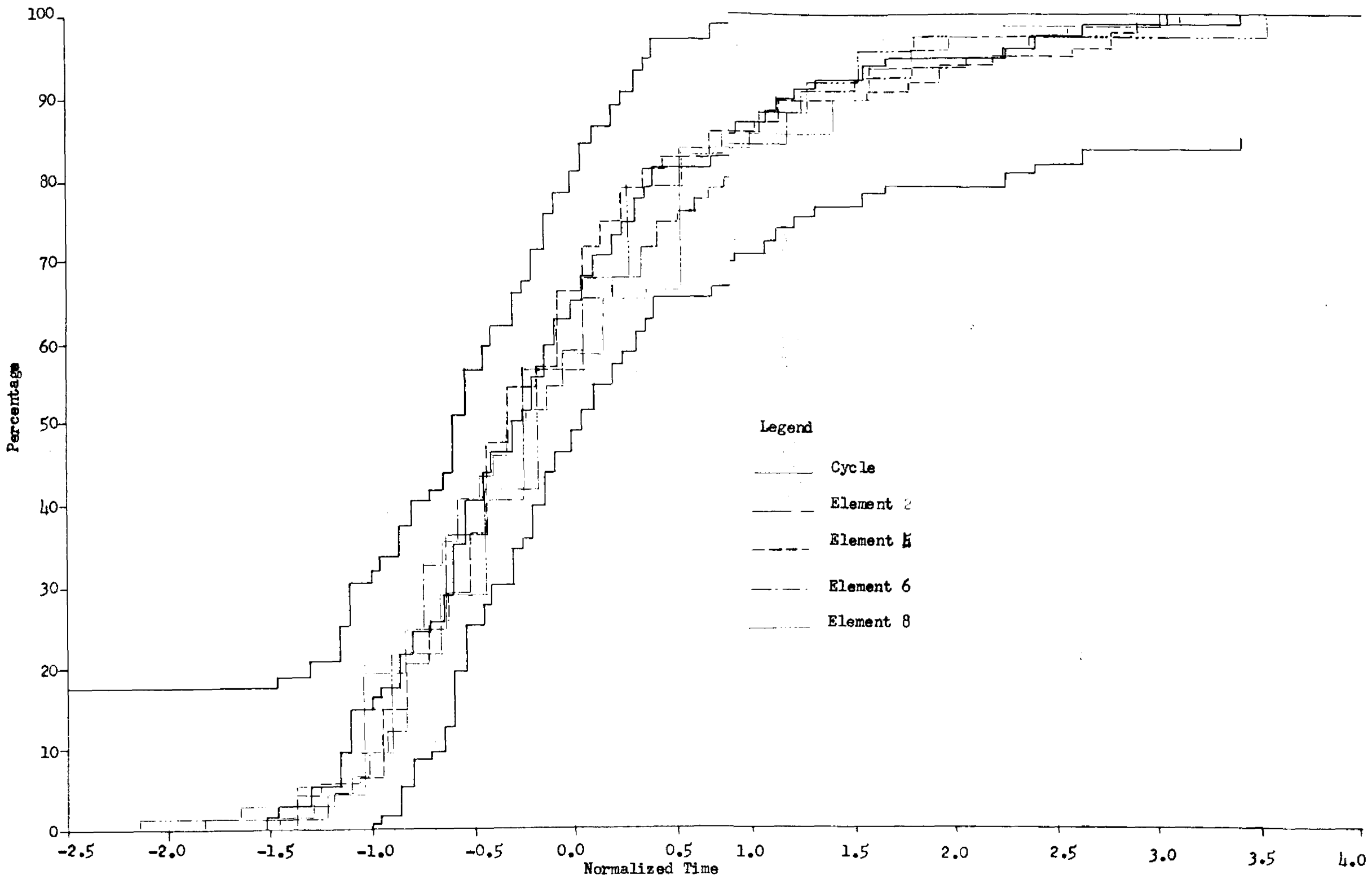


Figure 16. Cumulative-Percentage Histograms From Study D (Level Times) Elements 2, 4, 6, 8

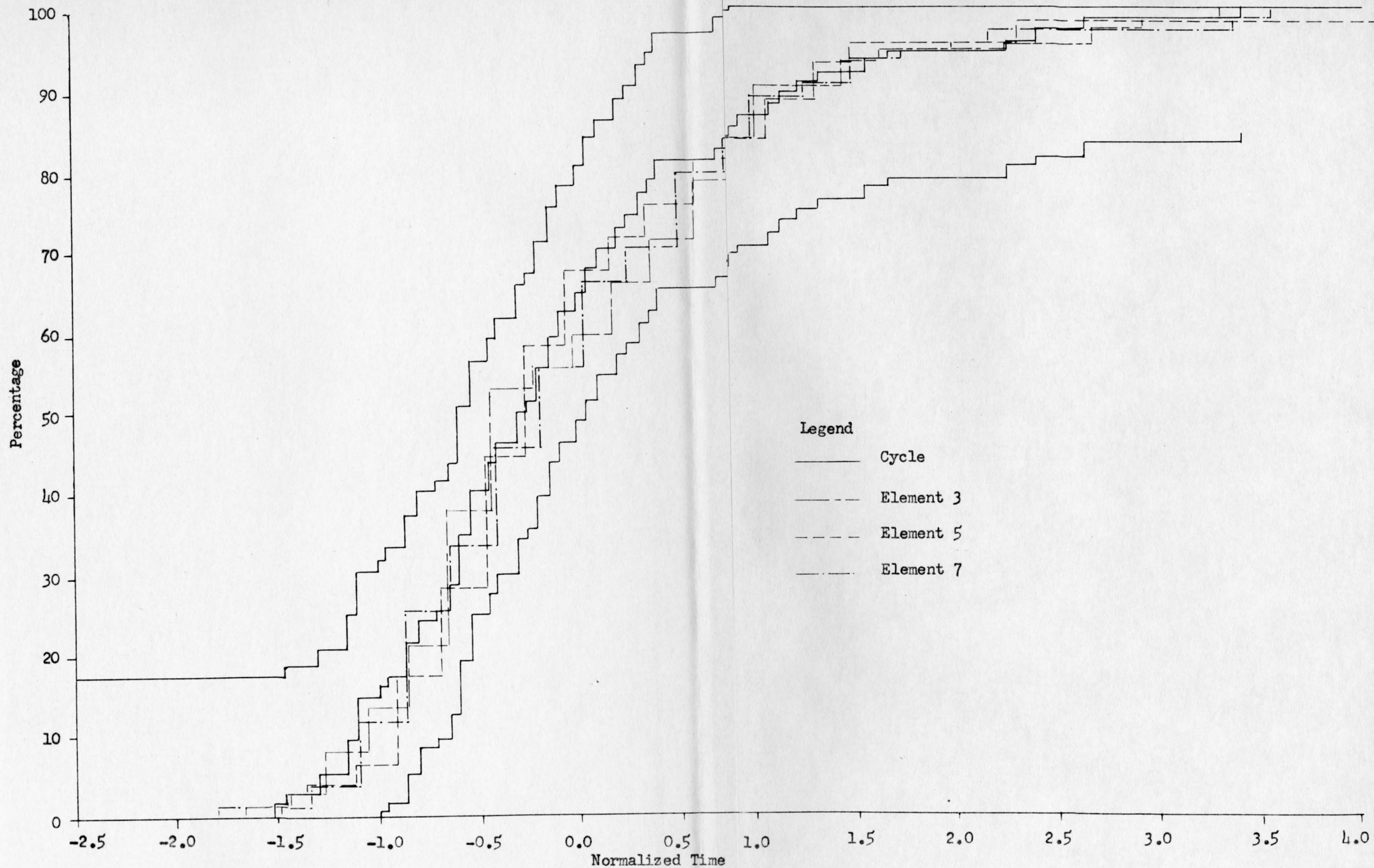


Figure 17. Cumulative-Percentage Histograms From Study D
(Level Times) Elements 3, 5, 7

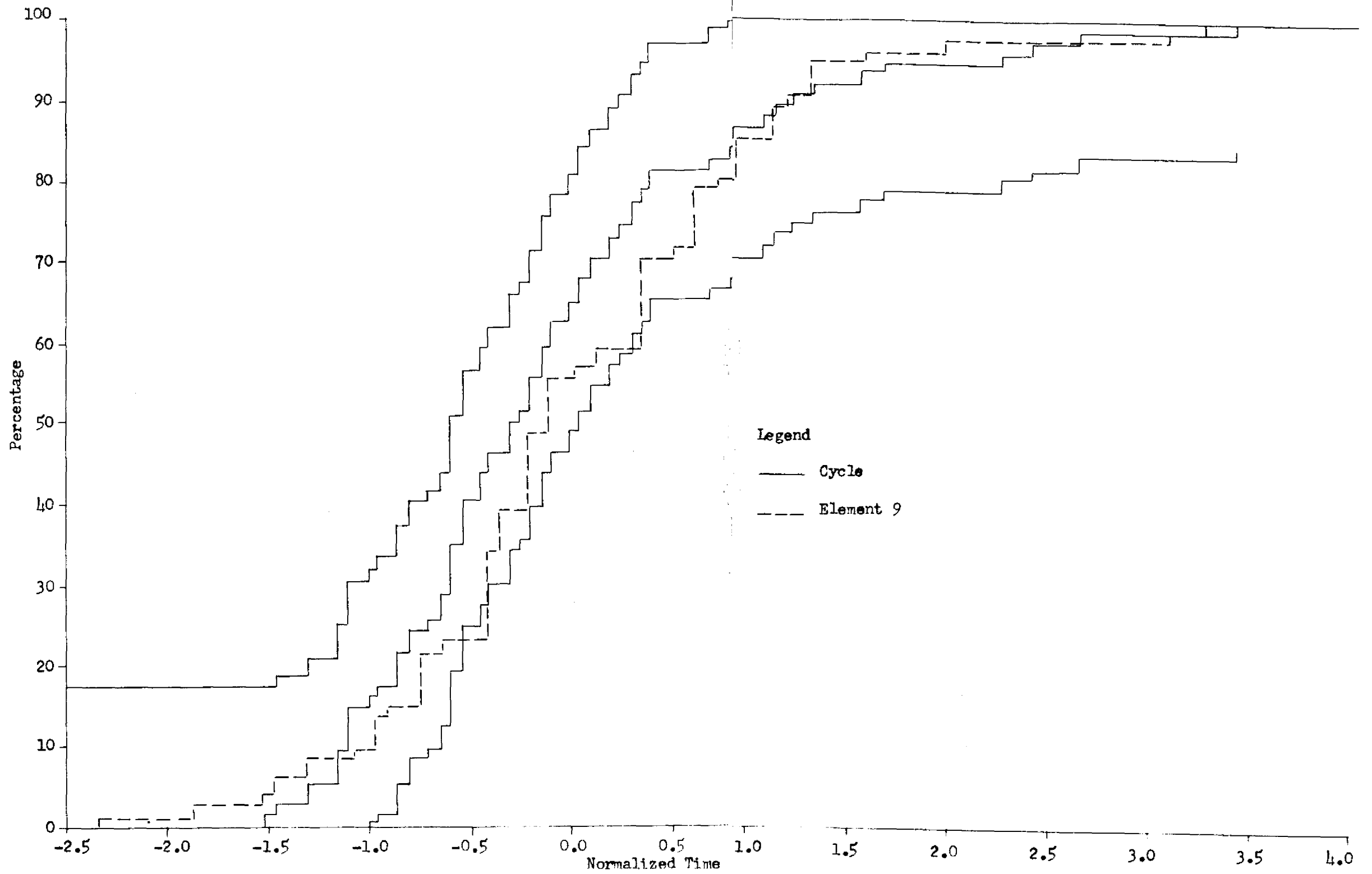


Figure 18. Cumulative-Percentage Histograms From Study D
(Level Times) Element 9

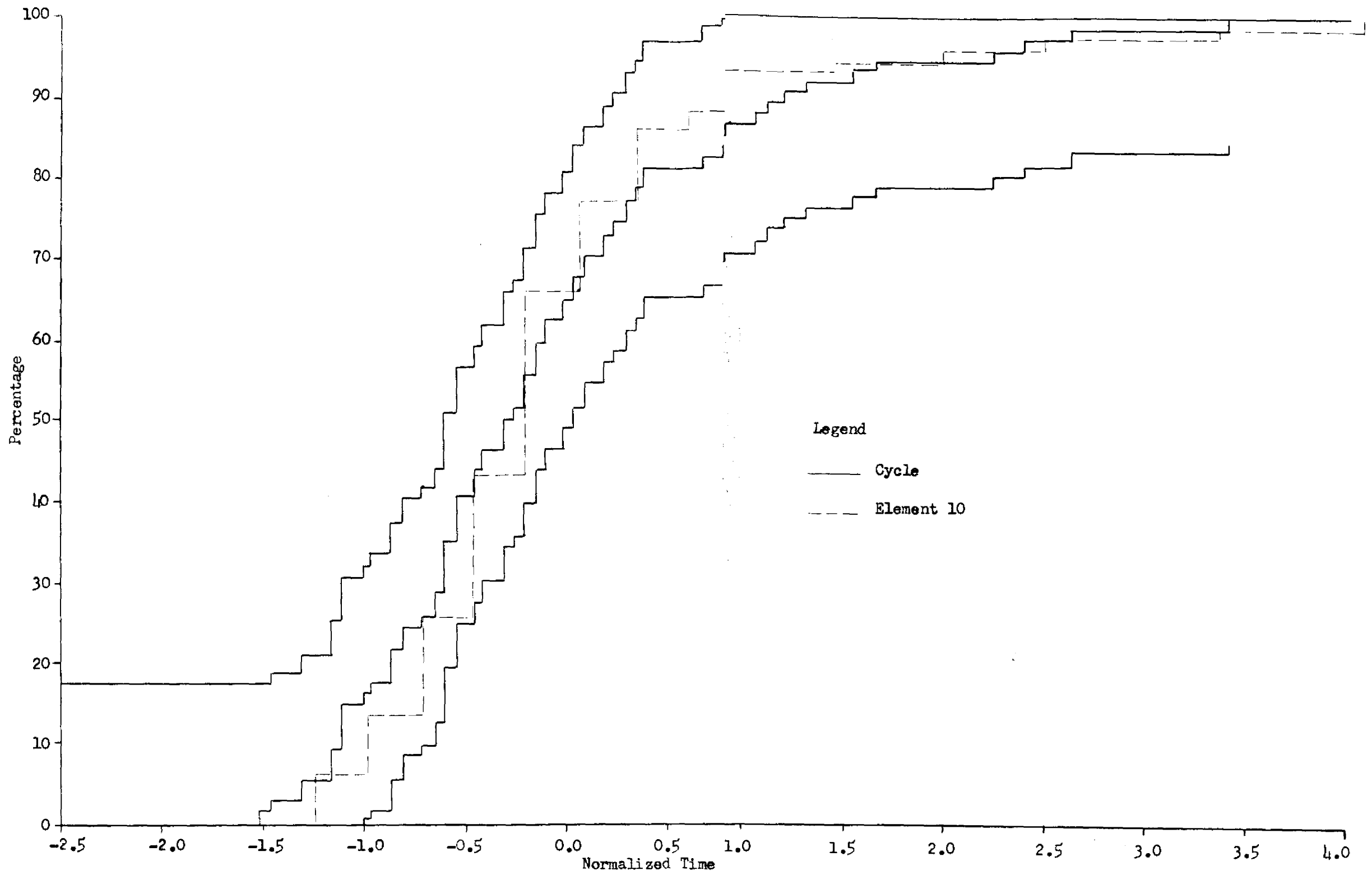


Figure 19. Cumulative-Percentage Histograms From Study D (Level Times) Element 10

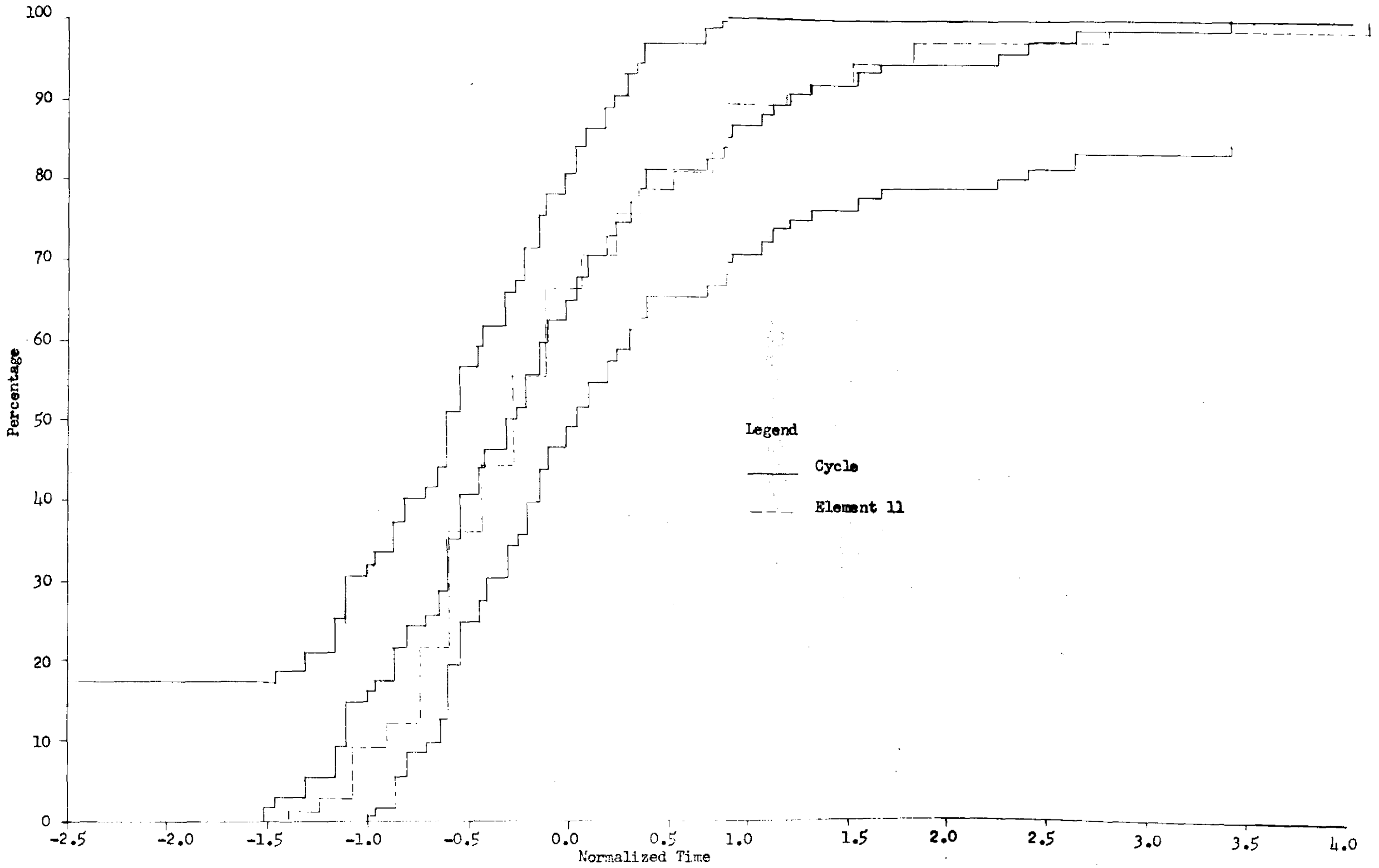


Figure 20. Cumulative-Percentage Histograms From Study D
(Level Times) Element 11

Table 3. Comparison of Element and Cycle Variance

Element	Study C Variances (Unadjusted Times)	Study D Variances (Leveled Times)
2	24.80	24.80
3	30.76	23.64
4	89.30	83.26
5	24.62	21.51
6	116.32	122.66
7	20.02	18.40
8	34.67	16.65
9	3.38	3.07
10	14.26	13.68
11	42.12	37.94
Cycle	402.80	388.09
Element Total	400.25	365.61
Per Cent Difference	0.63	5.79

Table 4. Comparison of Element Variance and Coefficient of Variability with the Element Mean Time—Study C (Unadjusted Times)

Element No.	Effective Mean Time	Element Variance (σ^2)	Coefficient of Variation
6	52.080	116.32	21.9
4	40.365	89.30	23.4
11	32.375	42.12	20.0
2	27.000	24.80	18.4
8	24.716	34.67	23.8
10	23.442	14.26	16.1
3	19.932	30.76	27.8
7	17.690	20.02	25.2
5	16.204	24.62	30.6
9	8.320	3.38	22.1

Table 4. Comparison of Element Variance and Coefficient of
 Variability with the Element Mean Time-Study D
 (Leveled Times)

(continued)

Element No.	Effective Mean Time	Element Variance	Coefficient of Variation
6	49.200	122.66	22.5
4	40.550	83.26	22.5
11	32.580	37.94	18.9
2	27.050	24.80	18.4
8	24.686	16.65	16.5
10	23.578	13.68	15.6
3	20.064	23.64	24.2
7	17.686	18.40	24.2
5	16.173	21.51	28.6
9	8.199	3.07	21.3

present research. He found a direct relationship between mean time and variance and an inverse relationship between mean time and Coefficient of Variation. The evidence here indicated that there was a direct relationship between mean and variance. However, there does not appear to be any relationship between the mean and Coefficient of Variation.

In the present study it was found that the Coefficient of Variation for the elements was greater than that of the cycle. The Coefficient of Variation ranges from 15.6 to 30.6 while that of the cycle for this operator was 8.0.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Previous investigations of work-time distributions have been confined to the consideration of cycle time. The present study was an investigation of element-time distribution characteristics. The primary objective was to determine the relationship between the elements and the cycles of which these elements are a part. The results of this study indicate that the characteristics are similar for element and cycle time distributions. A secondary objective was to make a preliminary analysis of the interaction of elements. No evidence was found of element interdependence.

The hypothesis tested was that the characteristics of the element-time distributions do not vary significantly from the characteristics of the work-time distribution of the cycle of which these elements are a part. On the basis of the results of this study, the hypothesis can be accepted. The characteristics considered were the mean, variance, skewness and peakedness of the distributions. There was no evidence of statistically significant difference between the skewness and peakedness of the element and cycle time distributions. This conclusion was based on the results of a nonparametric test which compared the characteristics of a sample and the population. At the 95 per cent confidence level the skewness and peakedness of the elements, or sample, were similar to those of the population as represented by the cycle.

There was no reason to test the mean and variance of the distributions since these characteristics would be inherently different due to the difference in the time for completing the elements and the cycles.

The results of fitting the element distributions to the Normal, Log Normal and Pearson Type III curves indicate that the Log Normal curve is generally the curve of best fit. However, there was a high probability level for the fit of part of the distributions to the Pearson Type III curve. The value of skewness of the remainder of the distributions was too great to allow a fit to this curve. The probability level of the fit to the Normal curve for all elements indicates a very poor fit. Consequently, there is not enough evidence to say that any one of these curves typifies the theoretical element-time distributions.

If the performance of the operator studied in this research can be considered typical, it can be concluded that the Normal curve does not typify the theoretical curve of element-time distributions. Further, it would appear that the theoretical curve would be positively skewed and would have a peakedness somewhat greater than that of the normal curve. In view of the similarity between the characteristics of the element and cycle time distributions, the same conclusions would be true for the cycle. This agrees with the previously stated conclusions of Summers (24) and Friedman (23).

The present study was conducted for only one operator performing a short-cycle, assembly operation. Further, the apparent high motivation level of this operator may not be typical of the average industrial worker. In view of these two limitations, further research should be conducted in

order to generalize from these conclusions. Consequently, it is recommended that motion pictures be taken of a number of different manually controlled operations. A sample of the performance of a number of operators on each operation should be obtained. Enough film should be taken to insure an adequate sample size for valid interpretation of the results of statistical analysis. The element and cycle time distributions of the sample thus obtained should be analyzed in order to compare the characteristics of the elements and the cycles. If the characteristics prove to be similar, then the cycle distributions should be treated mathematically to obtain the curve of best fit.

The general characteristics of the theoretical curve of the work-time distribution have been well defined by the results of the research conducted under the supervision of Doctors Lehrer and Moder. It would appear that these characteristics should be converted to a mathematical formula which would define this theoretical curve. If one typical curve can be established by the research suggested above, this need for a more clearly defined theoretical curve would be partially fulfilled.

The results of a number of studies, notably those of Barnes and Mundel (3), and Abruzzi (2), indicate that there is interaction among elements. The possibility of interaction among the elements analyzed in the present research was studied. The results indicated that there is evidence of element independence. This conclusion is based upon the results of analysis of histograms and of a "t" test of the difference between the cycle variance and the sum of the element variances. Histograms were plotted for the distributions of element times with deviations in

method of performance excluded. The element distributions with these deviations included were then superimposed upon the original histograms. Analysis revealed evidence of element independence. This conclusion was substantiated by the results of the "t" test. At the 95 per cent confidence level no significant difference was found.

Therefore, it can be concluded that elements similar to those defined in the present study and measured in time intervals of $1/2000$ of a minute would be independent. These results tend to support the conclusions of Abruzzi (6) who stated that element independence may be contingent upon the time measurement method and the motions which comprise the element.

In the present preliminary research, one operator performing a single activity was studied. Further, the maximum sample size of $9\frac{1}{4}$ observations was below the recommended minimum sample size for histogram analysis. In view of these limitations, further research would have to be conducted to determine whether or not these conclusions are true for all manually controlled activities. Consequently, it is suggested that the research recommended above be expanded. The data obtained in this new study should be tested by one of the multiple correlation techniques to determine the extent of element interaction.

The relationships between the mean time and the variance, and the mean time and Coefficient of Variation for the elements defined in this research were studied. The results of a visual comparison of these values revealed that the element variance tends to vary directly with the mean time. However, there appeared to be neither a direct nor an inverse relationship between the mean and the Coefficient of Variation. From

these results it may be concluded that for these elements, the direct relationship between mean and variance found by Abruzzi appears to be true. However, the inverse relationship which he found between mean time and Coefficient of Variation is not typical of the relationship between these values for all activities.

APPENDIX

Table 5. Frequency Distributions From Study B (Variables Included) and Study C (Variables Excluded)

Element 2

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
16.5-19.5	1	1	1	1	1	1
19.5-22.5	18	13	13	9	13	10
22.5-25.5	21	17	28	24	33	26
25.5-28.5	26	22	17	13	15	12
28.5-31.5	9	5	17	12	17	13
31.5-34.5	9	8	5	4	4	3
34.5-37.5	7	6	7	6	8	7
37.5-40.5	1	1	1	1	1	1
40.5-43.5	1	1	3	3	1	1
43.5-46.5	0	0	0	0	0	0
46.5-49.5	0	0	1	1	0	0
49.5-52.5	0	0	0	0	0	0
52.5-55.5	1	0	0	0	0	0
55.5-58.5	0	0	0	0	1	0
58.5-61.5	0	0	1	0	0	0
Totals	94	74	94	74	94	74

LH-B - Left hand, Study B	LH-C - Left hand, Study C
RH-B - Right hand, Study B	RH-C - Right hand, Study C
EFF-B - Effective, Study B	EFF-C - Effective, Study C

Table 5. Frequency Distributions From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 3

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
3.5-5.5	0	0	1	1	0	0
5.5-7.5	0	0	0	0	0	0
7.5-9.5	0	0	0	0	0	0
9.5-11.5	0	0	0	0	0	0
11.5-13.5	4	3	6	5	3	3
13.5-15.5	20	15	20	15	22	17
15.5-17.5	21	17	15	12	13	11
17.5-19.5	11	9	16	11	14	8
19.5-21.5	9	7	13	11	16	14
21.5-23.5	9	7	4	2	4	2
23.5-25.5	5	2	7	6	6	4
25.5-27.5	6	6	6	6	9	9
27.5-29.5	6	6	2	2	2	2
29.5-31.5	0	0	1	1	1	1
31.5-33.5	1	1	2	1	1	1
33.5-35.5	0	0	1	1	1	1
35.5-37.5	1	0	0	0	1	0
37.5-39.5	1	1	0	0	1	1
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 4

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
25.5-30.5	8	7	3	2	3	2
30.5-35.5	29	20	26	22	26	21
35.5-40.5	33	28	32	24	36	29
40.5-45.5	9	8	15	12	13	10
45.5-50.5	6	4	5	3	5	3
50.5-55.5	1	0	3	2	3	2
55.5-60.5	2	2	3	3	1	1
60.5-65.5	3	2	2	1	4	3
65.5-70.5	3	3	3	3	2	2
70.5-75.5	0	0	1	1	1	1
75.5-80.5	0	0	1	1	0	0
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 5

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
8.5-11.5	3	1	10	8	8	7
11.5-14.5	20	15	38	32	33	26
14.5-17.5	32	28	17	15	21	19
17.5-20.5	13	12	12	9	13	10
20.5-23.5	13	10	10	6	12	8
23.5-26.5	9	6	2	2	2	2
26.5-29.5	0	0	1	0	1	0
29.5-32.5	1	1	1	1	1	1
32.5-35.5	0	0	1	0	1	0
35.5-38.5	1	0	1	1	0	0
38.5-41.5	1	1	0	0	1	1
41.5-44.5	1	0	1	0	1	0
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 6

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
30.5-35.5	2	2	5	4	1	1
35.5-40.5	12	11	19	16	18	16
40.5-45.5	20	18	22	18	23	19
45.5-50.5	16	10	13	12	14	11
50.5-55.5	16	14	11	6	13	9
55.5-60.5	9	6	8	7	8	7
60.5-65.5	8	7	5	5	5	4
65.5-70.5	2	1	4	2	4	3
70.5-75.5	2	1	3	3	3	2
75.5-80.5	2	2	1	0	2	1
80.5-85.5	2	1	1	1	1	1
85.5-90.5	1	1	1	0	0	0
90.5-95.5	2	0	1	0	2	0
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 7

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
8.5-10.5	1	0	0	0	0	0
10.5-12.5	4	4	8	7	3	3
12.5-14.5	19	15	21	16	20	15
14.5-16.5	23	16	31	26	20	17
16.5-18.5	25	22	14	11	19	16
18.5-20.5	14	11	2	2	12	9
20.5-22.5	2	2	3	2	5	5
22.5-24.5	1	1	6	5	2	2
24.5-26.5	0	0	3	3	2	2
26.5-28.5	3	2	0	0	5	3
28.5-30.5	0	0	2	1	0	0
30.5-32.5	2	1	0	0	2	1
32.5-34.5	0	0	1	1	1	1
34.5-36.5	0	0	0	0	1	0
36.5-38.5	0	0	1	0	0	0
46.5-48.5	0	0	1	0	1	0
60.5-62.5	0	0	0	0	1	0
64.5-66.5	0	0	1	0	0	0
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 8

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
12.5-14.5	0	0	1	0	0	0
14.5-16.5	0	0	7	5	0	0
16.5-18.5	1	1	18	15	4	3
18.5-20.5	5	3	18	15	5	2
20.5-22.5	19	16	21	18	20	17
22.5-24.5	20	18	9	7	25	22
24.5-26.5	15	14	7	6	15	13
26.5-28.5	6	5	5	3	8	6
28.5-30.5	8	7	4	3	3	3
30.5-32.5	6	6	0	0	5	5
32.5-34.5	1	0	2	1	3	1
34.5-36.5	2	2	1	1	0	0
36.5-38.5	0	0	0	0	0	0
38.5-40.5	4	2	0	0	3	2
40.5-42.5	0	0	0	0	0	0
42.5-44.5	0	0	1	0	0	0
44.5-46.5	1	0	0	0	0	0
50.5-52.5	1	0	0	0	1	0
52.5-54.5	1	0	0	0	0	0
60.5-62.5	1	0	0	0	1	0
64.5-66.5	1	0	0	0	0	0
72.5-74.5	1	0	0	0	1	0
Totals	94	74	94	74	94	74

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 9

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
3.5-4.5	1	1	0	0	1	1
4.5-5.5	3	2	1	0	3	2
5.5-6.5	7	6	8	6	8	6
6.5-7.5	15	12	19	16	19	15
7.5-8.5	25	22	26	20	28	25
8.5-9.5	17	10	21	20	13	8
9.5-10.5	16	13	5	2	13	11
10.5-11.5	5	4	4	4	3	2
11.5-12.5	1	1	1	1	1	1
12.5-13.5	1	1	0	0	1	1
13.5-14.5	2	2	2	2	2	2
14.5-15.5	0	0	0	0	0	0
15.5-16.5	0	0	0	0	0	0
16.5-17.5	0	0	1	1	0	0
17.5-18.5	0	0	1	1	1	0
18.5-19.5	0	0	0	0	0	0
19.5-20.5	0	0	1	0	0	0
20.5-21.5	0	0	1	1	0	0
33.5-34.5	0	0	1	0	0	0
45.5-46.5	0	0	1	0	0	0
46.5-47.5	0	0	1	0	1	0
Totals	93*	74	94	74	94	74

*Frame count not indicated on film analysis sheet.

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 10

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFT-B	EFT-C
10.5-12.5	0	0	1	0	0	0
12.5-14.5	0	0	34	30	0	0
14.5-16.5	0	0	35	27	0	0
16.5-18.5	2	1	14	12	2	1
18.5-20.5	6	6	3	1	7	6
20.5-22.5	34	30	3	2	34	30
22.5-24.5	26	20	1	1	28	21
24.5-26.5	10	9	2	1	10	9
26.5-28.5	7	4	0	0	6	3
28.5-30.5	1	0	0	0	1	0
30.5-32.5	1	0	0	0	0	0
32.5-34.5	3	2	0	0	3	2
34.5-36.5	0	0	0	0	0	0
36.5-38.5	1	1	0	0	1	1
38.5-40.5	0	0	0	0	0	0
40.5-42.5	2	1	0	0	2	1
Totals	93*	74	93*	74	94	74

*One value outside the possible range for inclusion on the histogram.

Table 5. Frequency Distribution From Study B (Variables Included) and Study C (Variables Excluded)

(continued)

Element 11

Element Times (in Frames)	LH-B	LH-C	RH-B	RH-C	EFF-B	EFF-C
20.5-25.5	10	8	0	0	2	1
25.5-30.5	54	41	0	0	50	38
30.5-35.5	11	11	0	0	20	18
35.5-40.5	11	9	21	17	14	11
40.5-45.5	3	3	37	28	3	3
45.5-50.5	1	1	15	12	1	1
50.5-55.5	0	0	9	8	1	1
55.5-60.5	0	0	5	5	0	0
60.5-65.5	2	1	3	2	2	1
65.5-70.5	0	0	1	1	0	0
70.5-75.5	1	0	1	1	1	0
75.5-80.5	0	0	1	0	0	0
80.5-85.5	0	0	1	0	0	0
Totals	93	74	94	74	94	74

Table 6. Frequency Distributions From
Study D (Leveled Time)

Element Times (in Frames)	Element 2			Element Times (in Frames)	Element 3		
	LH	RH	EFF		LH	RH	EFF
13.5-16.5	0	1	0	1.5- 3.5	0	1	0
16.5-19.5	1	0	1				
19.5-22.5	11	12	13	11.5-13.5	3	3	2
22.5-25.5	24	19	20	13.5-15.5	11	8	8
25.5-28.5	20	12	14	15.5-17.5	14	20	18
28.5-31.5	5	16	14	17.5-19.5	16	11	13
31.5-34.5	8	5	3	19.5-21.5	11	11	8
34.5-37.5	3	4	8	21.5-23.5	3	6	9
37.5-40.5	1	3	0	23.5-25.5	7	5	9
40.5-43.5	1	1	1	25.5-27.5	4	6	2
43.5-46.5	0	0	0	27.5-29.5	2	0	1
46.5-49.5	0	1	0	29.5-31.5	1	0	1
				31.5-33.5	0	3	1
				33.5-35.5	1	0	1
				35.5-37.5	1	0	1
Totals	74	74	74	Totals	74	74	74

LH - Left hand
RH - Right hand
EFF - Effective

Table 6. Frequency Distributions From
Study D (Leveled Time)

(continued)

Element Times (in Frames)	Element 4			Element Times (in Frames)	Element 5		
	LH	RH	EFF		LH	RH	EFF
25.5-30.5	8	4	4	8.5-11.5	2	7	6
30.5-35.5	20	17	17	11.5-14.5	15	34	27
35.5-40.5	26	24	28	14.5-17.5	29	13	20
40.5-45.5	11	13	12	17.5-20.5	13	10	9
45.5-50.5	1	4	3	20.5-23.5	7	7	9
50.5-55.5	3	4	3	23.5-26.5	6	1	0
55.5-60.5	1	2	3	26.5-29.5	0	1	2
60.5-65.5	2	3	1	29.5-32.5	1	0	0
65.5-70.5	2	2	3	32.5-35.5	0	1	0
70.5-75.5	0	1	0	35.5-38.5	1	0	1
Totals	74	74	74	Totals	74	74	74

Table 6. Frequency Distributions From
Study D (Leveled Time)

(continued)

Element Times (in Frames)	Element 6			Element Times (in Frames)	Element 7		
	LH	RH	EFF		LH	RH	EFF
30.5-35.5	3	6	2	10.5-12.5	5	5	3
35.5-40.5	11	12	16	12.5-14.5	13	19	16
40.5-45.5	16	17	16	14.5-16.5	20	18	15
45.5-50.5	10	14	14	16.5-18.5	18	13	15
50.5-55.5	15	9	8	18.5-20.5	10	9	10
55.5-60.5	7	5	6	20.5-22.5	4	4	7
60.5-65.5	6	5	5	22.5-24.5	1	2	3
65.5-70.5	1	1	2	24.5-26.5	2	2	2
70.5-75.5	1	4	3	26.5-28.5	0	1	1
75.5-80.5	2	0	1	28.5-30.5	0	0	0
80.5-85.5	1	1	1	30.5-32.5	1	1	1
85.5-90.5	1	0	0	32.5-34.5	0	0	1
Totals	74	74	74	Totals	74	74	74

Table 6. Frequency Distributions From
Study D (Leveled Time)

(continued)

Element Times (in Frames)	Element 8			Element Times (in Frames)	Element 9		
	LH	RH	EFF		LH	RH	EFF
14.5-16.5	0	5	1	3.5-4.5	1	0	1
16.5-18.5	1	11	1	4.5-5.5	2	1	2
18.5-20.5	3	21	5	5.5-6.5	6	7	7
20.5-22.5	16	17	14	6.5-7.5	9	17	15
22.5-24.5	17	5	21	7.5-8.5	23	23	26
24.5-26.5	13	9	16	8.5-9.5	15	15	7
26.5-28.5	10	2	5	9.5-10.5	11	2	12
28.5-30.5	7	1	5	10.5-11.5	4	3	1
30.5-32.5	3	2	4	11.5-12.5	1	2	1
32.5-34.5	1	1	0	12.5-13.5	0	0	0
34.5-36.5	1	0	0	13.5-14.5	2	1	2
36.5-38.5	1	0	0	14.5-15.5	0	0	0
38.5-40.5	0	0	2	15.5-16.5	0	0	0
40.5-42.5	1	0	0	16.5-17.5	0	1	0
				17.5-18.5	0	1	0
				18.5-19.5	0	0	0
				19.5-20.5	0	1	0
Totals	74	74	74	Totals	74	74	74

Table 6. Frequency Distributions From
Study D (Leveled Time)

(continued)

Element Times (in Frames)	Element 10			Element Times (in Frames)	Element 11		
	LH	RH	EFF		LH	RH	EFF
10.5-12.5	0	3	0	20.5-25.5	6	0	2
12.5-14.5	0	35	0	25.5-30.5	41	0	31
14.5-16.5	0	24	0	30.5-35.5	15	2	25
16.5-18.5	0	7	0	35.5-40.5	9	12	9
18.5-20.5	10	1	10	40.5-45.5	1	25	5
20.5-22.5	22	3	22	45.5-50.5	1	20	1
22.5-24.5	24	1	25	50.5-55.5	0	8	0
24.5-26.5	8	0	8	55.5-60.5	1	5	0
26.5-28.5	5	0	4	60.5-65.5	0	1	1
28.5-30.5	1	0	1	65.5-70.5	0	0	0
30.5-32.5	1	0	1	70.5-75.5	0	1	0
32.5-34.5	1	0	1				
34.5-36.5	1	0	1				
36.5-38.5	0	0	0				
38.5-40.5	0	0	0				
40.5-42.5	1	0	1				
Total	74	74	74	Total	74	74	74

Table 7. Cumulative Frequency Distributions of Normalized Cycle Times From Study C (Unadjusted Times with Variables Excluded)

Cycle Times (in Frames)	Normalized Cycle Time	Frequency	Cumulative Frequency	Cumulative Per Cent	Upper Band- Boundary	Lower Band- Boundary
233	-1.44	1	1	1.3	17.1	0
234	-1.39					
235	-1.34	1	2	2.7	18.5	0
236	-1.29	2	4	5.4	21.2	0
237	-1.24	2	6	8.1	23.9	0
238	-1.19	1	7	9.4	25.2	0
239	-1.14					
240	-1.09	4	11	14.8	30.6	0
241	-1.04	1	12	16.2	32.0	0.4
242	-0.99	2	14	18.9	33.7	2.1
243	-0.94					
244	-0.89	1	15	20.2	36.0	4.4
245	-0.84					
246	-0.79					
247	-0.74					
248	-0.69	2	17	22.9	38.7	7.1
249	-0.64	2	19	25.6	41.4	9.8
250	-0.59	1	20	27.0	42.8	11.2
251	-0.54	4	24	32.4	48.2	16.6
252	-0.49	2	26	35.1	50.9	19.3
253	-0.44	1	27	36.4	52.2	20.6
254	-0.39					
255	-0.34	3	30	40.5	56.3	24.7
256	-0.29	1	31	41.8	57.6	26.0
257	-0.24	6	37	50.0	65.8	34.2
258	-0.19					
259	-0.14	4	41	55.4	71.2	39.6
260	-0.09	4	45	60.8	76.6	45.0
261	-0.04	2	47	63.5	79.3	47.7
262	0.00	3	50	67.5	83.3	51.7
263	0.05					
264	0.10					
265	0.15	2	52	70.2	86.0	54.4
266	0.20					
267	0.25					
268	0.30	1	53	71.6	87.4	55.8
269	0.35	3	56	75.6	91.4	59.8
270	0.40					

Table 7. Cumulative Frequency Distributions of Normalized
Cycle Times From Study C (Unadjusted Times with Variables Excluded)

(continued)

Cycle Times (in Frames)	Normalized Cycle Time	Cumulative Frequency	Cumulative Frequency	Cumulative Per Cent	Upper Band- Boundary	Lower Band- Boundary
271	0.45					
272	0.50					
273	0.55	1	57	77.0	92.8	61.2
274	0.60	1	58	78.3	94.1	62.5
275	0.65					
276	0.70					
277	0.75	2	60	81.0	96.8	65.2
278	0.80	1	61	82.4	98.2	66.6
279	0.85	1	62	83.7	99.5	67.9
280	0.90	1	63	85.1	100.	69.3
281	0.95	1	64	86.4	100.	70.6
282	1.00					
294	1.60	1	65	87.8	100.	72.0
295	1.65	2	67	90.5	100.	74.7
296	1.70	1	68	91.8	100.	76.0
297	1.75	1	69	93.2	100.	77.4
298	1.80					
299	1.85	1	70	94.5	100.	78.7
300	1.90	1	71	95.9	100.	80.1
310	2.40	1	72	97.2	100.	81.4
315	2.64	1	73	98.6	100.	82.8
329	3.34	1	74	100.	100.	84.2

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

Element 2

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
16				
17	-2.01	1	1	1.3
18		0		
19		0		
20	-1.40	3	4	5.4
21	-1.20	2	6	8.1
22	-1.00	5	11	14.8
23	-0.80	11	22	29.7
24	-0.60	4	26	35.1
25	-0.40	11	37	50.0
26	-0.20	1	38	51.3
27	0.0	7	45	60.8
28	0.20	4	49	66.2
29	0.40	8	57	77.0
30	0.60	2	59	79.7
31	0.80	3	62	83.7
32	1.00	1	63	85.1
33		0		
34	1.40	2	65	87.8
35	1.61	3	68	91.8
36	1.81	3	71	95.9
37	2.01	1	72	97.2
38		0		
39	2.41	1	73	98.6
40		0		
41	2.81	1	74	100.0
42				

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 3

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
12				
13	-1.25	3	3	4.0
14	-1.07	4	7	9.4
15	-0.89	13	20	27.0
16	-0.71	7	27	36.4
17	-0.53	4	31	41.8
18	-0.35	4	35	47.2
19	-0.17	4	39	52.7
20	0.01	10	49	66.2
21	0.19	4	53	71.6
22	0.37	1	54	72.9
23	0.55	1	55	74.3
24	0.73	1	56	75.6
25	0.91	3	59	79.7
26	1.09	1	60	81.0
27	1.27	8	68	91.8
28	1.46	2	70	94.5
29				
30	1.82	1	71	95.9
31				
32				
33	2.36	1	72	97.2
34				
35	2.72	1	73	98.6
36				
37				
38	3.26	1	74	100.0
39				

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element h

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
29	-1.20	1	1	1.3
30	-1.10	1	2	2.7
31	-0.99	6	8	10.8
32	-0.88	4	12	16.2
33	-0.78	3	15	20.2
34	-0.67	5	20	27.0
35	-0.57	3	23	31.0
36	-0.46	6	29	39.1
37	-0.36	6	35	47.2
38	-0.25	5	40	54.0
39	-0.14	6	46	62.1
40	-0.04	6	52	70.2
41	0.07	3	55	74.3
42	0.17	3	58	78.3
43				
44	0.38	2	60	81.0
45	0.49	2	62	83.7
46				
47	0.70	1	63	85.1
48	0.81	2	65	87.8
54	1.44	1	66	89.1
55	1.55	1	67	90.5
60	2.08	1	68	91.8
61	2.18	1	69	93.2
62				
63	2.39	2	71	95.9
66	2.72	1	72	97.2
67				
68	2.92	1	73	98.6
72	3.35	1	74	100.0

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 5

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
9				
10	-1.25	2	2	2.7
11	-1.05	5	7	9.4
12	-0.85	9	16	21.6
13	-0.64	9	25	33.7
14	-0.44	8	33	44.5
15	-0.24	10	43	58.1
16	-0.04	5	48	64.8
17	0.16	4	52	70.2
18	0.36	5	57	77.0
19	0.56	3	60	81.0
20	0.76	2	62	83.7
21	0.97	3	65	87.8
22	1.17	2	67	90.5
23	1.37	3	70	94.5
24	1.57	1	71	95.9
25				
26	1.97	1	72	97.2
27				
30	2.78	1	73	98.6
39	4.59	1	74	100.0

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 6

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
33	-1.49	1	1	1.3
36	-1.21	2	3	4.0
37	-1.12	3	6	8.1
38	-1.03	3	9	12.1
39	-0.93	4	13	17.5
40	-0.84	4	17	22.9
41	-0.75	3	20	27.0
42	-0.66	6	26	35.1
43	-0.56	4	30	40.5
44	-0.47	3	33	44.5
45	-0.38	3	36	48.6
46	-0.28	3	39	52.7
47				
48	-0.10	1	40	54.0
49	-0.01	4	44	59.4
50	0.08	3	47	63.5
51	0.18	3	50	67.5
52	0.27	1	51	68.9
53	0.36	1	52	70.2
54	0.46	2	54	72.9
55	0.55	2	56	75.6
56				
57	0.73	3	59	79.7
58	0.83	1	60	81.0
59	0.92	2	62	83.7
60	1.01	1	63	85.1
63	1.29	3	66	89.1
64	1.38	1	67	90.5
65				
66	1.57	1	68	91.8
69	1.85	1	69	93.2
70	1.94	1	70	94.5

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 6

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
75	2.40	2	72	97.2
76				
77	2.59	1	73	98.6
85	3.33	1	74	100.0

Table 8. Cumulative Frequency Distributions
of Normalized Element Times from Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 7

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
11				
12	-1.27	3	3	4.0
13	-1.05	6	9	12.1
14	-0.82	9	18	24.3
15	-0.60	7	25	33.7
16	-0.38	10	35	47.2
17	-0.15	9	44	59.4
18	0.07	7	51	68.7
19	0.29	3	54	72.9
20	0.52	7	61	82.4
21	0.74	2	63	85.1
22	0.96	2	65	87.8
23				
24	1.41	2	67	90.5
25	1.63	2	69	93.2
26				
27	2.08	2	71	95.9
28	2.30	1	72	97.2
29				
30				
31				
32	3.20	1	73	98.6
33	3.42	1	74	100.0
34				

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 8

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
16				
17	-1.31	2	2	2.7
18	-1.14	1	3	4.0
19	-0.97	1	4	5.4
20	-0.80	1	5	6.7
21	-0.63	10	15	20.2
22	-0.46	7	22	29.7
23	-0.29	12	34	45.9
24	-0.12	10	44	59.4
25	0.05	7	51	68.9
26	0.22	6	57	77.0
27	0.39	4	61	82.4
28	0.56	2	63	85.1
29	0.73	3	66	89.1
30				
31	1.07	5	71	95.9
32				
33				
34	1.58	1	72	97.2
35				
38				
39	2.42	1	73	98.6
40	2.59	1	74	100.0

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 9

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
3				
4	-2.35	1	1	1.3
5	-1.80	2	3	4.0
6	-1.26	6	9	12.1
7	-0.72	15	24	32.4
8	-0.17	25	49	66.2
9	0.37	8	57	77.0
10	0.91	11	68	91.8
11	1.46	2	70	94.5
12	2.00	1	71	95.9
13	2.54	1	72	97.2
14	3.09	2	74	100.0
15				

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 10

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
17				
18	-1.44	1	1	1.3
19				
20	-0.91	6	7	9.4
21	-0.64	11	18	24.3
22	-0.38	19	37	50.0
23	-0.12	12	49	66.2
24	0.15	9	58	78.3
25	0.41	4	62	83.7
26	0.68	5	67	90.5
27				
28	1.21	3	70	94.5
29				
32				
33	2.53	1	71	95.9
34	2.79	1	72	97.2
35				
36				
37				
38	3.85	1	73	98.6
39				
40				
41	4.64	1	74	100.0
42				

Table 8. Cumulative Frequency Distributions
of Normalized Element Times From Study C (Unadjusted
Times with Variables Excluded)

(continued)

Element 11

Element Times (in Frames)	Normalized Element Times	Frequency	Cumulative Frequency	Cumulative Per Cent
21				
22	-1.60	1	1	1.3
23				
24				
25				
26	-0.98	2	3	4.0
27	-0.83	8	11	14.8
28	-0.67	6	17	22.1
29	-0.52	9	26	35.1
30	-0.37	13	39	52.7
31	-0.21	6	45	60.8
32	-0.06	4	49	66.2
33	0.10	4	53	71.6
34	0.25	2	55	74.3
35	0.40	2	57	77.0
36	0.56	1	58	78.3
37	0.71	2	60	81.0
38	0.86	1	61	82.4
39	1.02	2	63	85.1
40	1.17	5	68	91.8
41				
42	1.48	3	71	95.9
43				
46				
47	2.25	1	72	97.2
48				
52				
53	3.18	1	73	98.6
54				
62				
63	4.72	1	74	100.0
64				

Table 9. Cumulative Frequency Distributions of
Normalized Cycle Times From Study D (Leveled Times)

Cycle Time (in Frames)	Normalized Cycle Time	Frequency	Cumulative Frequency	Cumulative Per Cent	Upper Band Boundary	Lower Band Boundary
234	-1.51	1	1	1.3	17.1	0.0
235	-1.46	1	2	2.7	18.5	0.0
236	-1.41					
237	-1.35					
238	-1.30	2	4	5.4	21.2	0.0
239	-1.25					
240	-1.20					
241	-1.15	3	7	9.4	25.2	0.0
242	-1.10	4	11	14.8	30.6	0.0
243	-1.05					
244	-1.00	1	12	16.2	32.0	0.4
245	-0.95	1	13	17.5	33.3	1.7
246	-0.90					
247	-0.85	3	16	21.6	37.4	5.8
248	-0.80	2	18	24.3	40.1	8.5
249	-0.75					
250	-0.70	1	19	25.6	41.4	9.8
251	-0.64	2	21	28.3	44.1	12.5
252	-0.59	5	26	35.1	50.9	19.3
253	-0.54	4	30	40.5	56.3	24.7
254	-0.49					
255	-0.44	2	32	43.2	59.0	27.4
256	-0.39	2	34	45.9	61.7	30.1
257	-0.34					
258	-0.29	3	37	50.0	65.8	34.2
259	-0.24	1	38	51.3	67.1	35.5
260	-0.19	3	41	55.4	71.2	39.6
261	-0.14	3	44	59.4	75.2	43.6
262	-0.09	2	46	62.1	77.9	46.3
263	-0.03					
264	0.02	2	48	64.8	80.6	49.0
265	0.06	2	50	67.5	83.3	51.7
266	0.12	2	52	70.2	86.0	54.4
267	0.17					
268	0.22	2	54	72.9	88.7	57.1
269	0.27	1	55	74.3	90.1	58.5
270	0.32	2	57	77.0	92.8	61.2

Table 9. Cumulative Frequency Distributions of
Normalized Cycle Times From Study D (Leveled Times)

(continued)

Cycle Time (in Frames)	Normalized Cycle Time	Cumulative Frequency	Cumulative Frequency	Cumulative Per Cent	Upper Band Boundary	Lower Band Boundary
271	0.37	1	58	78.3	94.1	62.5
272	0.42	2	60	81.0	96.8	65.2
278	0.72	1	61	82.4	98.2	66.6
279	0.77					
280	0.83	1	62	83.7	99.5	67.9
281	0.88	1	63	85.1	100.	69.3
282	0.93	1	64	86.4	100.	70.6
283	0.98					
284	1.03					
285	1.08	1	65	87.8	100.	72.0
286	1.13	1	66	89.1	100.	73.3
287	1.18					
288	1.23	1	67	90.5	100.	74.7
289	1.29					
290	1.34	1	68	91.8	100.	76.0
295	1.59	1	69	93.2	100.	77.4
296	1.64					
297	1.69	1	70	94.5	100.	78.7
308	2.25	1	71	95.9	100.	80.1
311	2.40	1	72	97.2	100.	81.4
316	2.65	1	73	98.6	100.	82.8
332	3.47	1	74	100.0	100.	84.2

**Table 10. Cumulative Frequency Distributions of Normalized
Element Times From Study D (Leveled Times)**

Element 2

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
18	-1.82	1	1	1.3
19				
20				
21	-1.21	2	3	4.0
22	-1.01	11	14	18.9
23	-0.81	4	18	24.3
24	-0.61	9	27	36.4
25	-0.41	3	30	40.5
26	-0.21	12	42	56.7
27	-0.01	1	43	58.1
28	0.19	5	48	64.9
29	0.39	1	49	66.2
30	0.59	13	62	83.7
31				
32	0.99	1	63	85.1
33				
34	1.40	3	66	89.1
35	1.60	3	69	93.2
36	1.80	2	71	95.9
37	2.00	1	72	97.2
38	2.20	1	73	98.6
39				
40				
41				
42	3.00	1	74	100.0
43				

Table 10. Cumulative Frequency Distributions of Normalized
Element Times From Study D (Leveled Times)

(continued)

Element 3

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
12	-1.66	1	1	1.3
13	-1.45	1	2	2.7
14	-1.25	4	6	8.1
15	-1.04	4	10	13.5
16	-0.84	6	16	21.6
17	-0.63	12	28	37.8
18	-0.42	11	39	52.7
19	-0.22	2	41	55.4
20	-0.01	3	44	59.4
21	0.19	5	49	66.2
22	0.40	4	53	71.6
23	0.60	5	58	78.3
24	0.81	4	62	83.7
25	1.02	5	67	90.5
26				
27	1.43	2	69	93.2
28	1.63	1	70	94.5
29				
30				
31	2.25	1	71	95.9
32				
33	2.66	1	72	97.2
34	2.87	1	73	98.6
35				
36	3.28	1	74	100.0

Table 10. Cumulative Frequency Distributions of Normalized
Element Times From Study D (Leveled Times)

(continued)

Element 4

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
28	-1.38	3	3	4.0
29	-1.27	1	4	5.4
30	-1.16			
31	-1.05	1	5	6.7
32	-0.94	6	11	14.8
33	-0.83	4	15	20.2
34	-0.72	4	19	25.6
35	-0.61	2	21	28.3
36	-0.50	6	27	36.4
37	-0.39	8	35	47.2
38	-0.28	5	40	54.0
39	-0.17	2	42	56.7
40	-0.06	7	49	66.2
41	0.05	4	53	71.6
42	0.16	2	55	74.3
43	0.27	3	58	78.3
44	0.38	2	60	81.0
45	0.49	1	61	82.4
46	0.60			
47	0.71	2	63	85.1
50	1.04	1	64	86.4
51	1.14	2	66	89.1
55	1.58	1	67	90.5
56	1.69			
57	1.80	1	68	91.8
58	1.91	1	69	93.2
59	2.02			
60	2.13	1	70	94.5
64	2.57	1	71	95.9
65	2.68			
66	2.79	1	72	97.2
67	2.90	1	73	98.6
68	3.01			
69	3.12	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Labeled Times)

(continued)

Element 5

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
9	-1.55	2	2	2.7
10	-1.33	1	3	4.0
11	-1.11	2	5	6.7
12	-0.90	8	13	17.5
13	-0.68	8	21	28.3
14	-0.47	12	33	44.5
15	-0.25	10	43	58.1
16	-0.04	7	50	67.5
17	0.18	3	53	71.6
18	0.39	3	56	75.6
19	0.61	4	60	81.0
20	0.82	2	62	83.7
21	1.04	3	65	87.8
22	1.26	4	69	93.2
23	1.47	2	71	95.9
24				
25				
26				
27	2.33	2	73	98.6
28				
35				
36				
37	4.27	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 6

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
33	-1.46	1	1	1.3
34				
35	-1.28	1	2	2.7
36	-1.19	1	3	4.0
37	-1.10	2	5	6.7
38	-1.01	2	7	9.4
39	-0.92	2	9	12.1
40	-0.83	9	18	24.3
41	-0.74	6	24	32.4
42	-0.65	2	26	35.1
43	-0.56	4	30	40.5
44	-0.47	2	32	43.2
45	-0.38	2	34	45.9
46	-0.29	3	37	50.0
47	-0.20	1	38	51.3
48	-0.11	2	40	54.0
49	-0.02	3	43	58.1
50	0.07	5	48	64.8
51	0.16			
52	0.25	2	50	67.5
53	0.34	3	53	71.6
54	0.43	2	55	74.3
55	0.52	1	56	75.6
56	0.61	1	57	77.0
57	0.70	1	58	78.3
58	0.79	1	59	79.7
59	0.88	3	62	83.7
62	1.16	3	65	87.8
63	1.24	2	67	90.5
66	1.52	1	68	91.8
69	1.79	1	69	93.2

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 6

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
72	2.06	1	70	94.5
73	2.15			
74	2.24	1	71	95.9
75	2.33	1	72	97.2
77	2.51	1	73	98.6
83	3.05	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 7

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
11	-1.79	1	1	1.3
12	-1.33	2	3	4.0
13	-1.09	6	9	12.1
14	-0.86	10	19	25.6
15	-0.63	6	25	33.7
16	-0.39	9	34	45.9
17	-0.16	7	41	55.4
18	0.07	8	49	66.2
19	0.30	3	52	70.2
20	0.54	7	59	79.7
21	0.77	3	62	83.7
22	1.00	4	66	89.1
23	1.24	1	67	90.5
24	1.47	2	69	93.2
25	1.70	1	70	94.5
26	1.94	1	71	95.9
27	2.17	1	72	97.2
28	2.40			
29	2.64			
30	2.87			
31	3.10			
32	3.34	1	73	98.6
33	3.57	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Levelsd Times)

(continued)

Element 8

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
16	-2.13	1	1	1.3
17	-1.88			
18	-1.64	1	2	2.7
19	-1.39	2	4	5.4
20	-1.15	3	7	9.4
21	-0.90	9	16	21.6
22	-0.66	5	21	28.3
23	-0.41	10	31	41.8
24	-0.17	11	42	56.7
25	0.08	8	50	67.5
26	0.32	8	58	78.3
27	0.57	3	61	82.4
28	0.81	2	63	85.1
29	1.06	2	65	87.8
30	1.30	3	68	91.8
31	1.55	3	71	95.9
32	1.79	1	72	97.2
33	2.04			
34	2.28			
35	2.53			
36	2.77			
37	3.02			
38	3.26			
39	3.51	2	74	100.0
40	3.75			

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 9

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
4.1	-2.34	1	1	1.3
4.9	-1.88	1	2	2.7
5.5	-1.54	1	3	4.0
5.6	-1.48	2	5	6.7
5.9	-1.31	1	6	8.1
6.3	-1.08	1	7	9.4
6.5	-0.97	3	10	13.5
6.6	-0.91	1	11	14.8
6.9	-0.74	5	16	21.6
7.1	-0.63	1	17	22.9
7.5	-0.40	8	25	33.7
7.6	-0.34	4	29	39.1
7.9	-0.17	7	36	48.6
8.1	-0.06	5	41	55.4
8.3	0.06	1	42	56.7
8.5	0.17	9	51	68.9
8.9	0.40	1	52	70.2
9.1	0.51	1	53	71.6
9.5	0.74	5	58	78.3
9.6	0.80	1	59	79.7
9.9	0.97	4	63	85.1
10.1	1.08	3	66	89.1
10.3	1.20	1	67	90.5
10.5	1.31	3	70	94.5
10.9	1.54	1	71	95.9
11.6	1.94	1	72	97.2
13.6	3.08	1	73	98.6
13.9	3.26	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 10

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
19	-1.24	5	5	6.7
20	-0.97	5	10	13.5
21	-0.70	9	19	25.6
22	-0.43	13	32	43.2
23	-0.16	17	49	66.2
24	0.11	8	57	77.0
25	0.38	7	64	86.4
26	0.65	1	65	87.8
27	0.92	4	69	93.2
28	1.19			
29	1.46	1	70	94.5
30	1.73			
31	2.00	1	71	95.9
32	2.27			
33	2.54	1	72	97.2
34	2.81			
35	3.08			
36	3.36	1	73	98.6
37	3.63			
38	3.90			
39	4.17			
40	4.44			
41	4.71	1	74	100.0

Table 10. Cumulative Frequency Distribution of Normalized
Element Time From Study D (Leveled Times)

(continued)

Element 11

Element Time (in Frames)	Normalized Element Time	Frequency	Cumulative Frequency	Cumulative Per Cent
24	-1.39	1	1	1.3
25	-1.23	1	2	2.7
26	-1.07	5	7	9.4
27	-0.90	2	9	12.1
28	-0.74	7	16	21.6
29	-0.58	11	27	36.4
30	-0.42	6	33	44.5
31	-0.26	8	41	55.4
32	-0.09	8	49	66.2
33	0.07	3	52	70.2
34	0.23	4	56	75.6
35	0.39	2	58	78.3
36	0.56	2	60	81.0
37	0.72	2	62	83.7
38	0.88	4	66	89.1
39	1.04			
40	1.20	1	67	90.5
41	1.36			
42	1.53	3	70	94.5
43	1.69			
44	1.85	2	72	97.2
49	2.67			
50	2.83	1	73	98.6
51	2.99			
61	4.62			
62	4.78	1	74	100.0
63	4.94			

Analysis Sheet for T-600 Ball Point Pen

Operator Carrie Time 12:05A Cycle 1 Film No. 50

Analyst WL Date of Analysis 6 May 53

Time Unit K

	Frame	Subtracted		Frame	Remarks
	No.	Time		No.	
	LH	LH	RH	RH	
	962			962	
Get Bbl-TE,ST & G	937	25	14	948	
Place Bbl-TL,P,A & RL	905	32	39	909	
Get Unit-TE, ST & G	883	22	19	890	
Place Unit-TL, P, A&RL	835	48	51	839	
Get Dr. Nut-TE, ST & G	813	22	18	821	
Place Dr. Nut-TL,P,A&RL	748	65	74	757	
Get Ferrule-TE,ST, & G	732	16	17	740	
Place Ferrule-TL,P,A&RL	693	39	20	720	
Get Comp. Unit-TE & G	684	9	8	712	
Place Comp. Unit-DA,TL&P	645	39	30	682	
Stake-A, H&DA	614	31	68	614	
Aside-TL & RL	608				Transferred pens to LH

Figure 21. Sample Film Analysis Sheet

$$(1) \quad A_s = \frac{t_1 + t_2 + t_3 \dots + t_n}{n}$$

$$(2) \quad A_g = \frac{A_{s1} + A_{s2} + A_{s3} \dots + A_{s8}}{N}$$

$$(3) \quad E_t = t + (A_g - A_s)$$

where A_s is "shot" average, A_g is grand average, t_1, t_2, \dots, t_n are element times, n is "shot" sample size, N is grand sample size, and E_t is the leveled element time.

(1) Element 2
Left hand
Shot 1

$$\frac{31 + 26 + 22 + 43 + 35 + 21 + 32 + 26 + 24 + 27}{10} = 28.7$$

(2) Element 2
Left Hand

$$\frac{28.7 + 28.5 + 29.9 + 25.0 + 25.5 + 27.9 + 25.0 + 23.9}{74} = 27.0$$

(3) Element 2
Left Hand
Shot 1 - Cycle 1

$$31 + (-28.7 + 27.0) = 29.3$$

Figure 22. Sample Calculations and Formulae Used for Leveling of Element Times

Element 2

Left Hand

Study C (Unadjusted Times)

Class Interval	Class Mark	f	d	df	d ² f	d ³ f	d ⁴ f	(d+1) ⁴ f	(x- \bar{X}) ² f
13.5-16.5	15								
16.5-19.5	18	1	-3	-3	9	-27	81	16	8.60
19.5-22.5	21	11	-2	-22	44	-88	176	11	61.54
22.5-25.5	24	24	-1	-24	24	-24	24	0	62.28
25.5-28.5	27	20	0	0	0	0	0	20	8.10
28.5-31.5	30	5	1	5	5	5	5	80	17.02
31.5-34.5	33	8	2	16	32	64	128	64.8	51.24
34.5-37.5	36	3	3	9	27	81	243	768	28.22
37.5-40.5	39	1	4	4	16	64	256	625	12.40
40.5-43.5	42	1	5	5	25	125	625	1296	15.40
43.5-46.5	45								
Total		74		-10	182	200	1538	3464	264.80

$$1/N (\text{sums}) \quad \quad \quad -0.135 \quad 2.459 \quad 2.703 \quad 20.784 \quad \quad \quad 3.578$$

$$\bar{X} = -0.135 \quad \quad \quad \bar{X}^2 = 0.0182 \quad \quad \quad \bar{X}^3 = -0.0024$$

$$\text{Charlier's Check: } 3464 = 1538 + 4(200) + 6(182) - 4(10) + 74 = 3464$$

$$\text{Mean: } \bar{X} = c\bar{d} + X_0 = 3(-0.135) + 27 = \underline{26.645}$$

Variance and Standard Deviation:

$$\mu_2 = V_2 - \bar{X}^2 = 2.459 - 0.0182 = \underline{2.441}$$

$$\sigma_x = \sqrt{\mu_2} = \sqrt{2.441} = \underline{1.562}$$

Skewness:

$$\mu_3 = V_3 - 3V_2\bar{X} + 2\bar{X}^3 = 2.703 - 3(2.459)(-0.135) + (-0.0024)2 = \underline{3.694}$$

$$\gamma_1 = \frac{\mu_3}{\sigma_x^3} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{3.694}{3.811} = \underline{0.969}$$

Peakedness:

$$\alpha = \frac{(X - \bar{X})^3}{N} \cdot \frac{1}{\sigma_x} = \frac{3.578}{4.686} = \underline{0.764}$$

Figure 23. Sample Calculations and Formulas for Determination of Moments

Element 2

Study D (Leveled Times)

Normal Curve

Upper Class Limit	t	Cumulative Area	Interval Area	Normal Frequency	Observed Frequency	$\frac{(f - F)^2}{F}$
$-\infty$						
19.5	-1.52	.0643	.0643	4.8	1	3.0091
22.5	-.91	.1814	.1171	8.7	13	2.1252
25.5	-.31	.3783	.1969	14.6	20	1.9972
28.5	.29	.6141	.2358	17.4	14	.6643
31.5	.89	.8133	.1992	14.7	14	.0333
34.5	1.50	.9332	.1199	8.9	3	3.9112
∞		1.000	.0668	4.9	9	3.4306
Total			1.0000	74.0	74	15.1709

 $\bar{X} = 27.05$ $\sigma_x = 4.98$ Degrees of freedom = $7 - 3 = 4$ Probability Level = 0.004

Figure 24. Sample Calculations for Fitting the Normal Curve to the Experimental Distributions

Element 2

Study C (Unadjusted Times)

Log Normal

Upper Class Limit	Log X	t	Cumulative Area	Interval Area	Log Normal Frequency	Observed Frequency	$\frac{(f-F)^2}{F}$
$-\infty$							
22.5	1.3522	-.94	.1736	.1736	12.8	11	.253
25.5	1.4065	-.23	.4090	.2354	17.4	26	4.250
28.5	1.4548	.40	.6554	.2464	18.2	12	2.112
31.5	1.4983	.96	.8315	.1761	13.0	13	.000
34.5	1.5378	1.48	.9306	.0991	7.3	3	2.533
∞			1.000	.0694	5.0	9	3.200
Totals				1.000	73.7	74	12.348

 $\overline{\log X} = 1.4243$ $S_{\log X} = 0.0769$

Degrees of freedom = 3

Probability Level = 0.066

Figure 25. Sample Calculations for Fitting the Log Normal Curve to the Experimental Data

Element 2 Study C (Unadjusted Times) Pearson Type III

Upper Class Limit	t	Cumulative Area	Interval Area	Pearson Type III Frequency	Observed Frequency	$\frac{(f-F)^2}{F}$
- ∞						
22.5	-.90	.1840	.1840	13.6	11	.4970
25.5	-.30	.4294	.2454	18.2	26	3.3428
28.5	.30	.6632	.2338	17.3	12	1.6236
31.5	.90	.8265	.1633	12.1	13	.0669
34.5	1.51	.9208	.0943	7.0	3	2.2857
∞		1.000	.0792	5.9	9	1.6288
Totals			1.000	74.1	74	9.4448

$$\bar{X} = 27.00$$

$$\sigma_x = 4.980$$

$$\chi^2_1 = .833$$

Probability Level = 0.022

Degrees of Freedom = 6 - 3 = 3

Figure 26. Sample Calculations for Fitting the Pearson Type III Curve to the Experimental Distributions

$$(1) \quad z_c = \frac{t_c - \bar{X}_c}{\sigma_c}$$

$$(2) \quad z_e = \frac{t_e - \bar{X}_e}{\sigma_e}$$

where z_c and z_e are the normalized times for the cycle and element times, t_c and t_e are the cycle and element times, \bar{X}_c and \bar{X}_e are the mean cycle and element times, and σ_c and σ_e are the standard deviation of the cycle and element time distributions.

$$(1) \quad z_c = \frac{233 - 261.9}{20.07}$$

= -1.44, for the cycle of Study C

$$(2) \quad z_e = \frac{17 - 27}{4.98}$$

= -2.01, for element 2 of Study C

Figure 27. Sample Calculations for Determination of Normalized Cycle and Element Times

Study C
(Unadjusted Element Times)

$$F = \frac{s_e^2}{s_c^2} = \frac{400.25}{402.80} = 0.994$$

Study D
(Leveled Element Times)

$$F = \frac{365.61}{388.09} = 0.941$$

95 per cent confidence limits: $1.46 > F > \frac{1}{1.46}$

$$1.46 > F > .685$$

Degrees of Freedom = 73

where F is the Fisher statistic, s_e^2 is the sum of the element variances, s_c^2 is the cycle variance.

Figure 28. Calculations for Testing Significant Difference
Between Cycle Variance and Sum of Element Variances.

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