

THE ABANDONMENT OPTION IN SEQUENTIAL CAPITAL  
RATIONING DECISIONS

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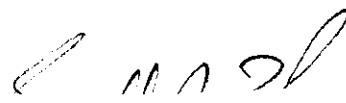
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THE ABANDONMENT OPTION IN SEQUENTIAL CAPITAL  
RATIONING DECISIONS

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## SUMMARY

This research is primarily oriented towards the investisation of abandonment as an integral part of a sequential capital rationing process. The investment situation covered by the study is one where investment decisions are made on a regular, periodic basis and the objective is to maximize the total accumulated wealth of a hypothetical firm at some predetermined horizon time. It is assumed that the outcome of the proposals is stochastic and that there is no information about the investment proposals that may become available in the future.

The probabilistic nature of the investment proposals determines the need to derive an abandonment rule capable of dealing explicitly with risk. Since the decision to hold or to abandon a project may be viewed as a decision to accept one of two mutually exclusive alternatives, three methods commonly used in decision making under risk are analyzed: the Probability of Loss Method, the Coefficient of Variation Method, and the Expected Gain Confidence Limit Method. The latter proves to be the most advantageous for the purposes of this research, and it is upon its theoretical basis that the abandonment rule is derived.

A simulation model is developed to generate a number of investment situations, where the decision making process is performed. In each situation, a comparison is made between the results obtained for three capital rationing criteria. The analysis is also extended to cover the cases where perfect information about the investment proposals is available.

The results obtained demonstrate that under most circumstances, when capital rationing decisions are made on a sequential periodic manner, the effect of making abandonment considerations in the evaluation of the proposals is to increase the firm's wealth with no significant changes in its variability. In addition, the results show that abandonment has a dramatic effect on the accumulated wealth of the firm when greater knowledge about the proposals is available.

## CHAPTER 1

### INTRODUCTION

#### 1.1. Definition of the Problem.

In recent years, businessmen have been paying increasing attention to what might be called a more "scientific" approach to capital expenditure decisions. In this, they have been aided by a growing body of writers who have sought to use methods of financial and economic analysis in solving the problems that confront decision-makers in this difficult area. One class of problem that has engendered widespread interest in business and academic communities is capital budgeting.

The term capital budgeting refers to a multifaceted investment activity that deals with the effective utilization

of capital resources. Basically, it involves the administration and organization of capital expenditure programs, the development of new investment opportunities, the estimation of the proposal's future disbursements and receipts, the evaluation and selection of the projects based upon an acceptance criterion, and, finally, the continual reevaluation of the projects after their implementation. In other words, the problem that confronts the decision-maker is one of determining how much capital is budgeted or should be budgeted for investment at the decision times, and of determining how the budgeted capital should be allocated among the investment proposals that are competing for these funds [49].

In this context, the capital budgeting procedure traditionally followed by most investment firms has assumed that the funds invested in a project are committed over its entire estimated life. Proposals have, in fact, been evaluated as though all outlays were sunk. This view of the decision making process has led those firms to an incomplete appraisal of the economic worth of investment opportunities because it disregards the possibility of abandoning projects before their estimated life is over. In practice, it is possible to find projects that have substantial abandonment or disposal values which may have a significant effect upon the above mentioned process. Specifically, the recognition of the

abandonment possibility may enhance the project's expected return as well as lower its risk over what would be the case if abandonment were not considered.

Of particular interest for this research is the investigation of abandonment as an integral part of a capital budgeting process where investment decisions are made on a sequential, periodic manner. In such a situation, abandonment may not only improve the outcome of the decision making process because it recognizes the complete economic worth of the proposals but, also because it increases the flexibility of the firm's operations.

#### 1.2. Objectives of the Study.

This study is primarily concerned with the estimation of the effect that abandonment considerations have in the solution of sequential capital budgeting problems, where the outcome of the investment proposals is not known with certainty. In order to accomplish this objective, a two step plan is followed,

1. Since the outcome of the investment proposals is assumed to be of probabilistic nature, an abandonment rule that takes into account the important

characteristic of risk behavior is derived. This rule is referred to as the EGCL Abandonment Rule because it is built upon the theoretical basis of the Expected Gain Confidence Limit (EGCL) concept.

2. A simulation model is developed to generate a number of investment situations where the decision making process takes place. In each situation, a comparison is made between the result obtained with and without abandonment for three capital rationing criteria. Also, a comparison is made between the outcome of the different criteria when abandonment is considered. In addition to these results, the analysis is extended to the cases where perfect information about the future investment opportunities is available.

### 1.3. Plan of Study.

Chapter II contains a review of the literature on the topics related to this research, i.e., abandonment analysis and capital rationing. From that review, two main observations are drawn. One concerns the lack of an abandonment rule that properly accounts for the decision-maker's risk-return preferences. The other refers to the existence of rather limited applications of the abandonment concept.

Chapter III discusses the abandonment rules currently available in the literature. It also investigates three methods for decision making under risk, one of which is used to develop the EGCL Abandonment Rule, a rule that makes an explicit consideration of risk in the decision making process.

Chapter IV describes the sequential capital rationing process and presents different models often used to make the capital allocation decision.

Chapter V describes the features and assumptions of the simulation model used to estimate the effect of making abandonment considerations in the outcome of a regular periodic investment decision process. The input parameters, assumed probability distributions, and other elements of the simulation are discussed.

Chapter VI defines the five investment situations covered by the study and analyzes the results obtained in each case. In addition, this chapter contains the sensitivity analysis of a parameter relevant to the generation of the abandonment values.

Chapter VII contains a brief summary of results, conclusions, and recommendations for further research.

## CHAPTER II

### REVIEW OF THE LITERATURE ON ABANDONMENT ANALYSIS AND OTHER TOPICS RELATED TO CAPITAL RATIONING

Decisions regarding capital and associated expenditures are among the toughest and most important of those facing business management. These decisions are difficult because they require knowledge of the underlying process, and have to be made on the basis of probabilistic estimates and forecasts. Their importance can be attributed to several factors. First, considerable amounts of money are usually involved. Second, they imply a commitment to action over an extended time frame and thus consume other valuable corporate resources, for example, time of executives and technical personnel, which might be otherwise allocated. Third, the decisions have a high visibility or what some authors term a "brick-and-mortar permanence" [45] (recall projects like Ford's Edsel), that

serves as a constant reminder of poor past decisions.

The importance of the investment decision making process is confirmed by the extensive effort that has been devoted towards its development. Countless articles have addressed the problem, and provided the necessary methodology to improve its outcome. Special attention has been given to capital rationing and accept-reject decisions, where mathematical programming plays a major role. Replacement and abandonment analysis, as well as other topics, have also received considerable attention.

Another factor of great importance in the decision making process is the incorporation of risk into the decision models. Since uncertainty may well be said to be the dominant characteristic of capital investment decisions [45], providing the analysts with procedures that explicitly take into account the variability of the alternatives has been a major breakthrough.

In this study, two of the above mentioned topics are of major concern, abandonment analysis and sequential capital rationing. Therefore, the review of the literature begins with a discussion on abandonment analysis topics. The current state of the art on abandonment decision rules is analyzed in detail. Then, section 2.1.2 examines some of the most often

used procedures for estimation of abandonment values. The second half of this chapter contains a review of the literature on sequential capital rationing models. Deterministic and non-deterministic models are discussed separately.

## 2.1 Review of the Literature on Abandonment Analysis.

Although it stands as a goal for this research to establish the importance of the abandonment option, the early work by Robichek and Van Horne [57] showed that this option should be considered in the capital budgeting process if funds are to be allocated optimally. In spite of this, very few articles have been written on the subject and most of them are just slight modifications of the work of the above mentioned authors. Therefore, the review of the literature on abandonment decision rules is presented as a critique of the original article by Robichek and Van Horne where relevant variations introduced by other authors are mentioned. Then follows a discussion about the estimation of abandonment values.

### 2.1.1 The Abandonment Decision Rules.

As pointed out above, the first article in the

literature that stressed the importance of considering abandonment in capital budgeting was Robichek and Van Horne's [57]. The basic point in their article was that: "... a project should be abandoned at that point in time when its abandonment value exceeds the net present value of the subsequent expected future cash flows discounted at the cost of capital rate." [57,pg 578].

Although the basic idea behind the article was correct, its implementation had several pitfalls. First, as Dyl and Long [18] indicated in a comment, it is not enough to know whether to abandon a project but also when to abandon it. Specifically, "... that the abandonment decision rule employed must be one which allows for an optimal "timing" of the abandonment decision." [18,pg 89]. Thus even when at first instance it occurs that the abandonment value of a project exceeds the present value of continued operations, the firm may wish to hold the project and abandon it later at a still more profitable time.

In a responding comment, Robichek and Van Horne [58] accepted Dyl and Long's main point, i. e., that the abandonment values analysis should consider the possibility that deferred abandonment may be more desirable than present abandonment. However, they criticized Dyl and Long's algorithm as cumbersome and only partially correct.

Specifically, they argued that the identification of the optimal abandonment period in the Del and Long's model would only be relevant when future operating cash flows are known with certainty.

This criticism led Robichek and Van Horne to offer a revised abandonment algorithm that captured the main point of Del and Long's, and was intended to be "... considerably less costly and time consuming..." [58,pg 97]. The revised Robichek and Van Horne formulation requires identification of only one instance where the present value of continued operations exceeds the abandonment value. If such instance exists, the project is held another period and then reevaluated, based upon the expectations at that time.

Although the revised algorithm by Robichek and Van Horne makes it appear that Del and Long's approach is in part unnecessary, in debating appropriate algorithm construction, the authors did not explicitly distinguish between two major classes of investment decisions, which have to be made using different approaches [37]. They are the accept-reject (and keep-abandon) problems, and the mutually exclusive choice (and capital rationing) problems.

The accept-reject decision associated with a proposed investment project does not necessarily require a

determination of the project's a priori optimal abandonment period nor its associated maximum net present value. All the analyst needs, to reach an accept decision, is to find one instance where the net present value is positive. This implies that there is no need for an exhaustive search abandonment algorithm, such as Del and Long's.

On the other hand, the mutually exclusive choice does require identification of the a priori optimal abandonment period. This has to be done because the existence of a positive net present value for a particular project is no longer a sufficient condition for investment in that project. Since the mutually exclusive projects under consideration are all competing for scarce resources, it is necessary to present each one in its anticipated optimal perspective. Thus, the mutually exclusive choice decision requires an exhaustive search approach.

A second pitfall in Robichek and Van Horne's original article was a statement that future abandonment values are assumed to be certain and the resultant use of constant values in their simulation example. In fact, discounting the future abandonment values back to the present using the cost of capital as the discount rate, as Robichek and Van Horne did, implies not certainty but rather that the abandonment values are mean values of distributions having characteristics

similar to the cash flows associated with continued holding of the project. Thus, as Pappas [50] indicates, in the simulation the abandonment values should be drawn from such distributions for consistency with the underlying theoretical relationships. This modification properly generalizes Robichek and Van Horne's example but does not introduce great variations in their relevant conclusions.

A different approach to the generation of the abandonment values has been proposed by Bonini [8]. This author states that the abandonment values are also dependent upon the project's cash flow realizations so, in a simulation of the decision process, such relationships should be considered. This approach has some appeal when the decision maker is confronted with a keep-abandon decision of an already owned project, where increased knowledge about this relationship is available. However, when the investment decision involves a choice among mutually exclusive alternatives, the problem of establishing realistic relationships between abandonment values and cash flows for all the alternatives, diminishes substantially that appeal.

A third pitfall in the article by Robichek and Van Horne is that it assumes that the decision-maker is risk indifferent. The abandonment rule proposed in the article is based upon expected values, therefore, it implies that in

making the decision no consideration is given to the variability of the cash flows and abandonment values. This assumption is somewhat unrealistic and also restrictive because it led the authors to the formulation of a rule which does not provide an explicit way to incorporate the important characteristic of risk behavior. In a recent article, Jarrett [36] developed an abandonment decision model in which he attempted to incorporate risk into the decision process. Unfortunately, his model is based on a faulty economic logic, and does not in general lead to optimal abandonment decisions. The integration of risk behavior into the abandonment rule is one of the topics that will be addressed in this research.

Finally, other authors that have dealt with abandonment are Herbst [29], Schwab and Lusztig [61]. Herbst article has two important contributions. Firstly, it establishes a relationship between abandonment and replacement analysis. He shows that Robichek and Van Horne's rule is analogous to the "dual" of Terborsh's MAPI method [67]. Although for the situation he analyzes these observations are correct, abandonment and replacement are intrinsically different problems, as will be discussed later in this chapter, and should be approached in a different way. Secondly, it shows the effect of considering the abandonment option in capital budgeting when the decision is made ex-ante, that is before the capital is allocated. Schwab and Lusztig [61] note that

abandonment may also be a beneficial alternative when new, more profitable investments opportunities, mutually exclusive of the currently owned project, become available. This observation is certainly valid when the initial project is underway. However, when the abandonment decision has to be made ex-ante, there is a significant problem in forecasting proposals, and the budget that might become available in the future.

#### 2.1.2 Estimation of the Abandonment Values.

Another important problem in abandonment analysis is the estimation of the abandonment values. Similar to the proposal's series of cash flows, the series of abandonment values depends on a number of factors. The type of project, the function it performs, its usage, obsolescence, and other environmental considerations play a major role in the determination of the amount to be obtained by disposal.

Dean [15], Hendrick [28], and Sackman [59] agree that there are two possible valuation bases:

1. The book value (as determined by subtracting depreciation from original cost).

## 2. Market value of the asset.

Drinkwater et al. [17] add a third and fourth possibilities based on empirical evidence. They have observed that for some assets the abandonment values throughout their lives are negligible, and in other cases they are simply a constant fraction of the initial investment. However, these are certainly exceptions.

Of the first two possibilities, Dean [15] discards the former because it is "clearly and undisputably wrong for the purpose of capital budgeting" [15, p. 166]. The book value is a concept independent of the future earnings, it represents sunk costs which has become irrevocable and over which no control can be exercised. Therefore, it is only the market value that is relevant as future earnings are concerned.

Although it is clear that the market value is the adequate bases for computing the abandonment values, from an analytical point of view the estimation problem persists. No formula exists for determining and forecasting market values. Even if the analyst could develop such a formula, prime consideration should be given to the periodic reaction of the market [28]. In fact, there is also a problem of opportunity which has a definite influence in the amount that can be obtained for the asset.

In spite of all these complications, several authors have proposed some analytical methods to estimate expected abandonment values. Terborgh [67] in a classical reference on replacement analysis assumes that the series of abandonment values follow a linear decay function. Eilon et al. [20] and Gringer [27] utilizing empirical evidence on lift trucks suggest that the series be computed using an exponential decay function. Finally, Friberg [25] uses an equation which relates the initial investment with the salvage values through several parameters that can be adjusted to represent different patterns. He also proposes a method based on the properties of the Beta distribution which allows the decision-maker to obtain estimates of the abandonment values when little information is available.

## 2.2 Review of the Literature on Sequential Capital Rationing.

The capital rationing decision is the phase of capital budgeting that deals with the allocation of scarce resources among competing investment opportunities [78]. It is a situation where not all profitable proposals can be undertaken because of limits on funds available for investment. It is also a situation in which some projects with lower than acceptable return may be accepted if they generate funds at crucial times.

Basically, the problem that faces the decision-maker is one of determining how much of the capital budgeted for a decision time should be allocated among the available investment proposals. Although the statement of the problem is extremely simple, this phase of the capital budgeting process is conceptually difficult, and the subject of much controversy. Important conflicting points are the determination of proper discounting rates, the utilization of consistent decision criteria, and the integration of stochastic (probabilistic) considerations into the analysis. As mentioned above, the latter constitutes a major breakthrough in the field of capital investment decisions. In the case of capital rationing decisions it has posted a landmark which separates the conventional or deterministic approach, where the variables that define a proposal are assumed to be known with certainty, from the non-deterministic approach, where the knowledge about those variables is assumed to be incomplete and of probabilistic nature.

Of primary interest for this research is the problem of making capital rationing decisions on a regular, periodic basis. More specifically, the situation is one where, in every period, the decision-maker examines a set of investment proposals submitted for consideration during that period and then makes the investment decision at the end of the period. An important aspect in this sequential process is the amount

of knowledge the decision-maker has about the future investments. One view of this problem is that he has complete knowledge about the investment opportunities that are to be selected for implementation at the present and in the future. When such is the case, the investment decision problem is reduced to a computational problem. A more common and generally more practical point of view is that the decision-maker has complete information about only those investment opportunities that are to be considered in the current decision period [49]. When such is the case, he will probably try to maximize his wealth for every period independently while at the same time searching for a fast recovery of the funds committed to the proposals. Two other situations, probably more realistic ones, are similar to those described above but, in these the decision-maker's knowledge about the proposals amounts only to probabilistic information about their receipts and disbursements. When these are the cases, several approaches which consider risk may be used.

The review of the literature on this topic begins with a description of some deterministic approaches to capital rationing. Then, in section 2.2.2, the review continues with a discussion about non-deterministic methods. Finally, in section 2.2.3, an observation on the application of abandonment analysis in sequential capital rationing problems is made.

### 2.2.1 Deterministic Sequential Capital Rationing Decisions.

When complete knowledge of the investment proposals is available, two cases are of importance. One is that where the knowledge about the proposals extends to the future. In other words, all the proposals to be submitted in the present and forthcoming periods are known with certainty. The other case considers the situation where only the proposals submitted in the current decision period are known with certainty. Both cases are analyzed in the following sections.

2.2.1.1. Deterministic Case with Complete Information about the Present and the Future. The most comprehensive analysis of this problem was first made by Weinsartner [73]. He used the Lorie and Savage problem [41] as a point of departure and formulated a model which maximizes the firm's wealth in a future predetermined time, called the planning horizon (or simply horizon). Although Weinsartner's work contains some simplifying assumptions, it is considered among the outstanding publications of the 1960's, because it provided a correct theoretical formulation and treatment of constrained capital budgeting problems through the use of mathematical programming. Since then, linear, non-linear, quadratic, and dynamic programming have been extensively used in solving capital investment problems. Mathematical

programming is specially useful in capital rationing decisions when the number of proposals submitted in every period, and the number of periods within the planning horizon is relatively large. Another desirable property of mathematical programming is that it is able to handle easily interrelationships among proposals, such as mutual exclusion and interdependencies.

An extension of Weinsartner's deterministic model has been made by Bernhard [5]. He modified the basic horizon model by adding constraints on the availability of capital and other scarce resources. This model also allows for the incorporation of other (possibly non-linear) objective functions.

2.2.1.2. Deterministic Case with Complete Information about only the Proposals in the Decision Period. This is probably the most extensively discussed case in capital rationing decisions. In one of the first books that covered the subject, Dean [15] stated the problem as one of screening the proposals on the basis of their prospective rate of return. Although the rate of return is an inadequate screening measure, and Dean assumes the proposals are divisible, his work put capital rationing on an economically sound foundation and limited the degree to which persistence and persuasiveness

influenced the allocation of funds. Later, Lorie and Savase [41] proposed an approach, where the net present worth of investments is to be maximized subject to the choice of investment satisfying the condition that the initial outlays do not exceed the budget available. No investment can be undertaken at more than unit level nor less than zero level, and the discount interest rate is known a priori. Originally, Lorie and Savase set the problem as one that allowed the choice of fractional projects. This happened mainly because at that time they were not able to cope efficiently with integer constraints. Later, Weinsartner [74] extended the Lorie and Savase formulation to permit the inclusion of projects with several disbursements, and in reference [75] he provided several algorithms to solve the problem with zero-one constraints. The main difficulty in application of the Lorie-Savase model is the determination of the discounting rate that should be used for computing the net present worth of the investments. Lorie and Savase [41] and Weinsartner [73] mention the problem but do not solve it. Baumol and Quandt [3] attempted unsuccessfully to establish appropriate discounting factors internally by utilizing the dual model. Since then, a number of authors, including Carleton [13], Elton [20], Myers [46] and others, have taken Baumol and Quandt's work as their point of departure and attempted to reconcile various mathematical programming formulations with capital market theory. However, in recent papers Weinsartner

[77], and Bradley and Frey [9] have criticized those articles because of their naive view of capital rationing and of capital market behavior. Weingartner concludes that what is needed is not more on how capital rationing ties in with capital market but how capital rationing can play a role in decision and control within a firm.

Returning to the capital rationing problem, it should be mentioned that traditionally, this problem of making the allocation when only the proposals available in the current decision period may be identified, has been solved independently for each period and the effectiveness of different decision criteria tested under these circumstances. Such procedure disregards the dynamic nature of the capital allocation process. In fact, decisions are made periodically and the current decisions influence future decisions through budget constraints. Oakford and Thuesen [49] use an approach which constitutes an exception. They employ simulation to compare the Maximum Prospective Value Criterion with other decision criteria, using as a basis for comparison the accumulated wealth at the end of the planning horizon.

## 2.2.2 Non-deterministic Sequential Capital Rationing Decisions.

The situation described hereafter is one where the decision-maker has to allocate the budget available to a set of proposals which have cash flows not known with certainty. Two cases are of interest, one is that where the decision-maker can identify all the proposals that will be available within the planning horizon, but the outcome of the proposals is probabilistic. The other case is that where only the proposals that may be undertaken in the decision period are known, and this knowledge amounts only to probabilistic information about the proposals initial investment, cash flows, and life. Both cases are discussed in the following sections.

2.2.2.1. Non-deterministic Case, All Proposals within the Planning Horizon can be Identified. When the proposals available during the current and future decision can be identified, but the cash flows of those proposals are random variables, several different procedures for making the capital rationing decision have been suggested in the literature. Byrne et al. [12] use a chance constrained model which maximizes the expected net present worth of investments. In this model, a group payback restriction is included, that is,

a probabilistic payback constraint imposed to all proposals simultaneously. Bernhard [5] has shown that this model makes some extremely simplifying assumptions in order to be able to incorporate meaningful chance constraints. Mao [42] has also criticized it for its lack of realism. Another application of chance constrained programming to capital rationing decisions has been suggested by Naslund [47]. He changes Weingartner's [73] deterministic cash balance constraints into chance constraints, and solves essentially the same problem. This model has also been criticized because of its unrealistic assumptions [5].

Salazar and Sen [60] have proposed the use of stochastic linear programming to determine the expectation and variability of the return provided by a set of proposals. They use Weingartner's basic horizon model as a point of departure and explore the effects of two types of risk. The first type is the effect of future variations in significant and competitive variables which are likely to influence subsequent cash flows. The second type of risk involves the intrinsic variability of the cash flows. Using this probabilistic information, they simulate the outcome of the proposals and then apply Weingartner's model to the data. Several runs provide an insight to the distribution of returns.

Finally, Lockett and Gear [40] have extended Weinsartner's mathematical programming formulation to allow for the inclusion of proposals which demand periodic decisions through their lives. They use decision trees to represent each proposal and a form of stochastic integer programming to optimize the selection of a subset of proposals. The authors have observed that their approach is very inefficient when the number of proposals is large. In that case they recommend to use linear instead of integer programming.

2.2.2.2. Non-deterministic Case, Only the Proposals in the Decision Period can be Identified. In this case, only the proposals that may be undertaken in the current decision period are known, and this knowledge amounts only to probabilistic information about the proposal's initial investment, cash flows, and life. Several procedures to solve this problem have been proposed in the literature. Some of them are discussed in the following sections.

2.2.2.2.1 The Expected Net Present Worth Maximization. This method is an extension of the Lorie-Savase problem [41] to consider proposals with stochastic behavior. In fact, the formulation is the same, the only variation that is made is that here, expected values are used instead of presumably certain values. This formulation of the capital rationing

problem implies that the decision-maker is risk indifferent, i.e. that he gives no consideration to the variability of the proposal's outcome when allocating the funds. It also implies that the decision-maker knows the probability distribution of the relevant random variables. Using this information, he computes the expected net present worth of the investment proposals and bases his decision on those expectations. An extensive analysis of this problem has been made by Weinsgartner [73] and Fosler [24]. Petersen [54], Glover [26], Kaplan [38], Nemhauser and Ullman [48], Peterson and Luashhunn [55] have also formulated the capital rationing problem in terms of the Lorie-Savase model.

In reference [74], Weinsgartner extends the original Lorie-Savase problem to consider second-order effects derived from interrelationships among the proposals. He also provides a solution for the resulting quadratic integer programming model through Reiter's method [56].

A different approach which also uses expected values has been proposed by Cord [14]. He erroneously maximizes the proposals expected rate of return. However, his model has an interesting feature in that it introduces a constraint on the variability of the proposals.

2.2.2.2.2. The Mean Variance Criterion. This method is a direct application of Markowitz's work on portfolio selection [43]. He suggested that the decision-maker should be interested in the probability distribution of his future wealth, and that the selection of investment proposals should deal, at the very least, with some measure of dispersion of this distribution as well as with its central tendency. Markowitz's theory starts with the simple assumption that "... the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing." [44,pg 77]. Thus, the decision-maker would probably seek out a set of proposals (portfolio) which provides the maximum return for a given level of risk (dispersion or variance) or the minimum risk for a given level of return. Since each decision-maker may weigh risk and return differently, Markowitz makes no attempt to specify the exact preference of every individual [72]. Rather he attempts to find portfolios which provide maximum return for every possible level of risk and minimize the risk for every level of return. This set of portfolios is referred to as the "efficient" set.

Baumol [2] has shown that Markowitz's "efficient" set contains some portfolios which should be rejected out of hand. He states that the variance (or standard deviation) is not a good measure of risk per-se. That sometimes an investment with a relatively high variability may be safe if its expected

return is sufficiently high. He then proposes the use of the expected gain confidence limit instead of the variance, as a measure of risk and a solution for the problem.

A simplified model for portfolio selection has been developed by Sharpe [63]. He observed that Markowitz's procedure is quite time consuming, primarily due to the matrix inversion required for computing the critical line [23], and thus is not operationally feasible. Sharpe offers an alternative computational scheme, the Diagonal Model, which assumes that the comovement of the securities depends only on their response to a basic underlying factor, a single market index. Some accuracy is sacrificed, but the savings in complexity are significant [23].

A new method which combines the best features of the Markowitz's and Sharpe's models has recently been developed by Ennslin [21]. His Cycle Elimination Model solves the portfolio selection problem with the full variance-covariance matrix, as formulated by Markowitz [43] yet, the set of efficient portfolios is determined in a fraction of the time required by Sharpe's model [23].

In the application of these concepts to capital rationing decisions, the decision-maker is asked to define his risk-return preferences. Thus defining a critical line. This

information is then used along with the expectation and variance of the outcome to establish a measure of the desirability of each proposal. Finally, the budget available in the current decision period is allocated among those proposals which provide the most preferred risk-return combinations. Several model of this form are discussed by Peterson and Laughhunn [55], Weinsgartner [74], Bey and Porter [6].

#### 2.2.2.2.3. Expected Utility Maximization Criterion.

One of the most widely discussed approaches toward risk in the capital rationing literature is the Expected Utility Maximization Criterion. This approach utilizes the decision-maker's utility function to formalize his judgement of risk tradeoffs, expressing it quantitatively so that the selection of proposals with different risk profiles may be carried out [34]. It is an especially useful approach when the decision-maker presents a non-linear utility function (a linear utility function is analogous to the expected values maximization). When this is the case, given the form of the function and its parameter values, the decision-maker may search for the set of proposals which maximizes the expected value of the function. Weinsgartner [74], Peterson and Laughhunn [55] have analyzed the situation where the decision-maker is assumed to have a quadratic utility

function, and the relationship of this model with the mean variance criterion. Hillier [34] studies the generation and use of exponential utility functions. He provides a simple procedure to compute the function and in reference [33] he describes two solution procedures for seeking the combination of proposals that maximize the expected utility.

The first and most difficult requirement of this criterion is to derive and specify a meaningful utility function [51]. This task is often complicated by the existence of a group of decision-makers. A second problem is that under nearly all circumstances, finding the exact value of the expected utility function is not a straight forward task, since this requires calculating the expected value of a rather complicated function of a random variable (the net present worth) having perhaps a very complex distribution function [34]. As opposed to the first problem, which is inherent to the approach, some solutions have been proposed to the second one. Bussey and Stevens [10] have attempted to represent complex utility functions through high-order polynomials, whose expectation is easier to compute. Hillier [34] simplifies the problem assuming normality in the distribution of the proposal's present worth.

2.2.2.2.4. The Project Balance Criterion. When the

investment decisions are made on a sequential, periodic basis, a relevant factor which is not taken into account by any of the criteria described above is the rate at which a proposal recovers its initial investment. This is an important factor because it conveys information regarding the internal generation of funds for investment in the future, which is a determinant of the flexibility of the firm's investment activities [51]. A rather unique approach that takes this factor into account has been proposed by Park and Thuesen [52]. They developed the Project Balance Criterion which makes the allocation decision according to a measure of desirability that reflects three of the proposal's characteristics: profitability, variability, and flexibility. The profitability is measured by the expected value of the net present worth. The variability is measured by the standard deviation of the net present worth, and the flexibility is measured by the resolution of uncertainty through time. The measure also considers the decision-maker's attitude toward risk in making the trade off between expectation and variability.

Using simulation, stochastic programming, and Weinsartner's horizon value as a measure of effectiveness, they show that under most circumstances, the Project Balance Criterion performs significantly better than the currently used decision criterion.

### 2.2.3 Abandonment Analysis in Sequential Capital Rationing.

It is also important to mention that the article by Robichek and Van Horne [57], as well as most of the articles that have dealt with the abandonment option have reported their results on single period capital rationing decisions and on accept-reject problems. Furthermore, nowhere in the literature has the project abandonment option been regarded in an environment where capital allocation decisions are made on a regular, periodic basis, and the outcome of the proposals not known with certainty. Shore [64] analyzes the effect of budget constraints on replacement decision, which are often confounded with abandonment decisions.

At this point, a distinction should be drawn between the problem of abandoning a project and of replacing it. In a replacement decision, the project is not necessarily sold. It may be retained in progressively degraded service, so that the decision to replace does not stipulate disposal of the displaced project. Moreover, replacement implies a decision to continue the activity, whereas abandonment of a project normally means discontinuance of the activity. In a corporate firm, both replacement and abandonment involve transfer among assets of stockholder's funds, but an asset abandonment is a conversion of a specialized asset into general purchasing

power, while replacement involves a direct and narrow choice in transfer [15].

The current applications of abandonment analysis have not considered abandonment as an integral part of a sequential, periodic capital budgeting problem. Therefore it is the purpose of this study to develop a methodology that will allow one to investigate such a situation.

## CHAPTER III

## THE ABANDONMENT RULES AND RISK BEHAVIOR.

Section 2.1 of the previous chapter summarized the most important developments in the field of abandonment analysis, and its implications in capital budgeting. One of the points raised in that summary was that, despite the fact that several years have passed since the first article that stressed the importance of the abandonment concept appeared in the literature, still some of its pitfalls have not been rectified. This chapter concentrates on the analysis of one of those pitfalls. Specifically, on the development of an abandonment rule which takes into account the important characteristic of risk behavior. The chapter begins with a description of the basic assumptions used in the formulation of abandonment rules. Then follows an explanation of the Expected Value Abandonment Rule, which is basically the same

as that proposed by Dyl and Long [18]. Here, a simplification in the solution procedure is introduced with the application of the project balance concept. Thereafter, a suboptimal abandonment rule, based also on the project balance, is presented. This rule does not consider "timings" in the analysis of the proposals, but seems appropriate for application in sequential capital rationing decisions. Finally, an abandonment rule which takes into account risk behavior is developed. This rule is referred to as the EGCL Abandonment Rule.

### 3.1 The Assumptions in the Formulation of the Abandonment Rules.

This Section contains a summary of the simplifying assumptions made in the formulation of the abandonment rules currently available in the literature. The basic assumptions are,

1. The series of cash flows are assumed to be independent of the series of abandonment values. Bonini [8] studies a situation where this assumption is relaxed and solves the problem using Markov chains. The procedure requires the computation of a transition matrix for every proposal, which renders

the problem unmanageable if the number of proposals increases significantly.

2. The cash flows and abandonment values are assumed to have probability distributions with similar characteristics. In fact, in the simulation model they are assumed to be normally distributed.
3. In the formulation of the EGCL Abandonment Rule it is assumed that the investor is "risk averse", which in the present context is interpreted to mean that, as to options with equal expected outcomes that option would be preferred that had the narrower spread in returns. Supporting this assumption is the work by Wilkes [78] who states that "Apart from the obvious desire to avoid ruinous losses, it is generally accepted that investors are risk averters." [78,pg 22].
4. A meaningful estimate of the Minimum Attractive Rate of Return (MARR) is available.

### 3.2 The Expected Values Abandonment Rule.

The abandonment rule based on expected values states

that a project should be abandoned at that time when the expected abandonment value exceeds the discounted present value of the project's future expected cash flows. That is, the abandonment value is the relevant opportunity cost of continuing to hold a project at any point in time, and it is against this opportunity cost that the successive stream of expected cash flows must be compared [50]. That stream is composed of the net expected cash flows associated with holding the project for either its remaining economic life or until some future date at which the then current expected abandonment value exceeds the discounted values of all successive cash flows.

The problem of finding the optimum abandonment period may appear to be an involved and complex one, given that it requires an analysis of the abandonment option in each of the years of the remaining economic life of the project. In fact, that opinion may even persist after observing the algorithm proposed by Dyl and Long [18], which is by no means easy to follow. However, the procedure is little more complicated than a standard discounting routine, similar to the "roll-back" method used in decision tree analysis. The simplest procedure has been proposed by Pappas [50] as a four step algorithm:

1. Beginning with the last year of the proposals life, discount the expected cash flow in that year (including any salvage value) back one period.
2. Compare the resulting discounted cash flow with the expected abandonment value available at the beginning of that period.
3. Add the greater of the two figures, i.e., the larger of the discounted cash flows or the expected abandonment value, to the project's expected cash flow in the preceding period, and discount the sum back one year.
4. Repeat steps two and three until the future cash flows have been discounted back to the first period.

This procedure of beginning at the end of the project's life and discounting the expected cash flows back one period at a time, with periodic comparisons of the discounted value of the prospective cash flows and the current abandonment value ensures calculation of the cash flow stream with the largest discounted present value.

A more formal presentation of this algorithm can be

made through dynamic programming. Bonini [8] has proposed the following formulation:

Find  $f(0)$  using the recursive relationship,

$$f(L) = ECAV(L)] \quad \text{for period } L.$$

$$f(t) = \max \{ECAV(t)] ; r(ECCF(t+1)] + f(t+1))\} \quad \text{for periods } 1 \text{ to } L-1 \text{ ( to be solved backwards starting from } L-1).$$

$$f(0) = \max \{0. ; r(ECCF(1)] + f(1)) - C_0\} \quad \text{for period } 0.$$

Where,

$f(t)$  = optimal expected net present worth at period  $t$  of the prospective cash flows.

$ECAV(t)]$  = expected abandonment value at time  $t$ .

$ECCF(t)]$  = expected cash flow at time  $t$ .

$L$  = life of the project.

$r$  = discounting factor =  $1/(1+i)$  ,  $i$  = MARR.

Both of the algorithms presented in this section lead to an optimum abandonment decision for the expected values approach, and are the most advanced presented in the literature.

### 3.2.1 An Abandonment Rule Based on the Project Balance Concept.

One of the main disadvantages of the algorithm developed by Dyl and Long [18] is that it requires a

considerable amount of computation and therefore of computer time. When a single project is analyzed, this disadvantage is of little concern but, as the number of proposals increases, the problem becomes significant. In order to simplify the computational procedure and to provide a background for the rule that will be described in the following section, a rule based on the project balance concept is now introduced.

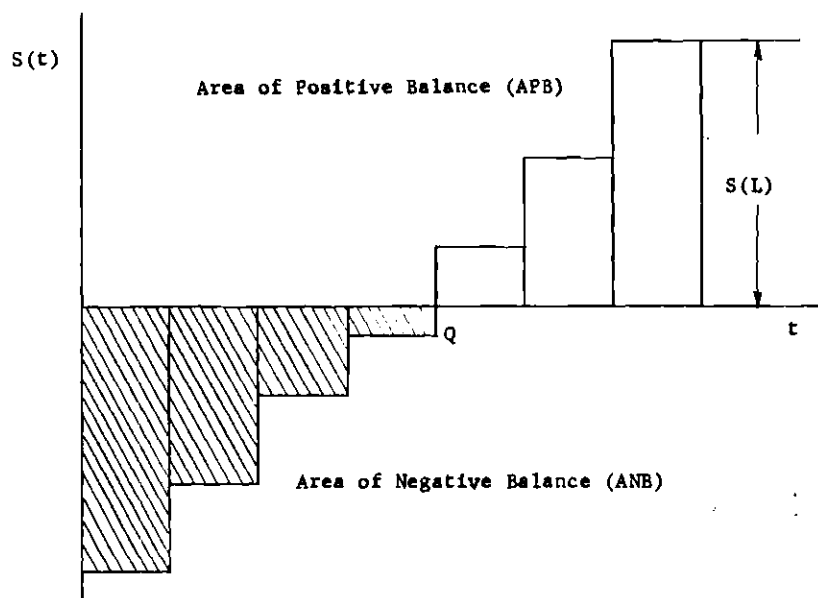


Figure 3.1. Project Balance Pattern.

The project balance is defined as the net equivalent amount a firm has invested in a project, or received from a project at the end of period  $t$ , if the outstanding balance at the end of each period  $0, 1, 2, \dots, t-1$ , is compounded at an interest rate  $i$  during the following period [51], [52]. In

mathematical notation,

$$S(0) = C_0$$

$$S(t) = (1+i) S(t-1) + EECF(t)]$$

where,

$S(t)$  = project balance in period  $t$ ,  $t = 1, 2, 3, \dots, L$ .

$i$  = MARR.

$L$  = life of the project.

$EECF(t)]$  = expected cash flow in period  $t$ .

$C_0$  = initial investment.

From this recursive relationship it follows that,

$$S(t) = C_0 (1+i)^t + EECF(1)] (1+i)^{t-1} + \dots + EECF(t)] \quad (3-1)$$

In general, when the  $S(t)$  is plotted as a function of time over the project's life, the time path of the project balance shown in Figure 3.1 can be obtained. This figure shows the four different elements of information provided by the project balance. They are, the area of negative balance (ANB), the discounted payback period (Q), the area of positive balance (APB), and the terminal profitability  $S(t)$  of the proposal. Each of these parameters can be expressed mathematically as follows,

$$ANB = \sum_{t=0}^{Q-1} S(t)$$

$Q$  = is the first period for which  $S(t) \geq 0$ .

$$APB = \sum_{t=0}^L S(t)$$

The project balance presents some very interesting properties which have been thoroughly discussed in reference [51]. Among those properties, the most important for the formulation of an abandonment rule is that the project balance reflects the amount of profits (losses) that a project has accumulated in each year of its entire life. Thus, if in period  $t$ , the corresponding abandonment value is added to the right hand side of equation 3-1, the sum represents the future worth of the project, if it were abandoned in that period.

$$E[FW(t)] = S(t) + E[AV(t)] \quad (3-2)$$

where,

$E[FW(t)]$  = future worth if the project were abandoned  
in period  $t$ .

$E[AV(t)]$  = abandonment value in period  $t$ .

Here, the question is, how can this information be used in making an abandonment decision? Assume that an investor is comparing the alternative of abandoning a project in period  $t$  with that of abandoning it in period  $t+1$ . His decision would certainly be to abandon the project in period  $t$  if the following relationship is satisfied,

$$(1+i) \{S(t) + E[AV(t)]\} \geq S(t+1) + E[AV(t+1)] \quad (3-3)$$

What this says is that if the total accumulated profit in period  $t+1$  is larger when the project is abandoned in period  $t$ , then the project should not be held for another period. Observe that in this case the decision is made only on a two period basis. When the decision involves multiple periods, the investor should extend this same rationale to cover all periods.

The general procedure to determine the optimum abandonment period can be stated as a three step algorithm,

1. Starting from period  $t=0$  compound the  $ECFW(t)]$  one period forward and compare it to  $ECFW(t+1)]$ , i.e., the comparison is between

$$ECFW'(t)] = (1+i) ECFW(t)]$$

and  $ECFW(t+1)]$ .

2. Take the largest amount of the two values above, compound it one period forward and then compare it to the  $ECFW(t)]$  in the next period.
3. Repeat step two until the whole life of the project is covered.

Because of the information carried by the project

balance, this procedure ensures calculation of the cash flow stream with the largest expected future worth at the end of the project's original life, or equivalently, with the largest expected net present worth. This is the same result given by Dyl and Long [18], which preferred for multiple choice investment decisions.

The algorithm has been proposed in a forward manner because this will prove to be most useful in the formulation of the abandonment rule of the next section. But it can also be stated in a backward manner with no change in the final solution.

### 3.3 A Suboptimal Abandonment Rule.

In sequential capital rationing decisions, where the future budgets are influenced by current decisions, it is of great importance to undertake proposals which provide a fast recovery of its initial investment. The main reason for doing so is that the faster a proposal is repaid, the sooner more budget is available for reinvestment. In this context, it seems reasonable to assume that occasionally it may not be necessary to wait until the optimum abandonment period is reached to dispose of a project. It is sometimes possible that the proposal's expected net present worth may follow a pattern similar to that shown in Figure 3.1., when plotted

against the abandonment period. In this case, it would probably be more convenient to abandon the proposal in the third period and reinvest its abandonment value in other, more profitable proposals available at that time.

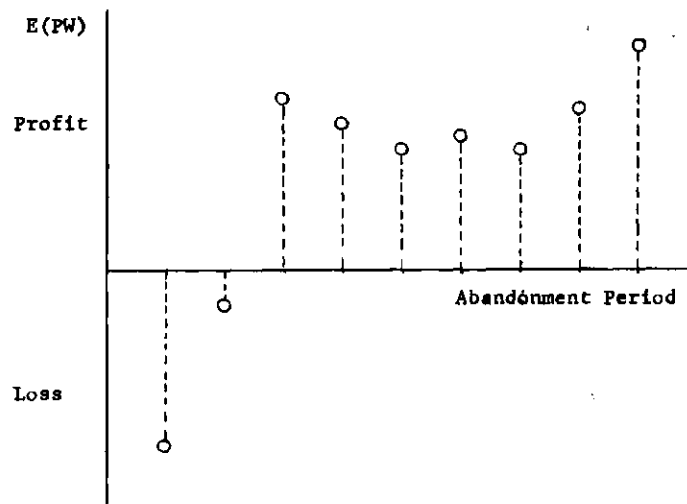


Figure 3.2 E(PW) vs. Abandonment Period.

An alternative abandonment rule which considers this possibility would indicate that a project should be abandoned at that time where the first positive local optimum for the expected present worth occurs. In figure 3.1, it would be period 3.

This rule is obviously suboptimal from an abandonment analysis point of view, because it does not abandon the proposals in that period that provides the maximum expected

present worth. However, at this point nothing can be inferred about its effects on the growth of a firm that makes investment decisions in a sequential, periodic manner.

Using the same notation as in section 3.2.1., the abandonment decision algorithm is as follows,

1. Search for the first period where  $EEFW(t)]$  is positive.

2. Compute

$$EEFW'(t)] = (1+i) EEFW(t)]$$

3. Compare this amount to  $EEFW(t+1)]$ . If  $EEFW'(t)]$  is smaller then advance one period and repeat step two. If  $EEFW'(t)]$  is larger than  $EEFW(t+1)]$  then abandon the project in period  $t$ . Do this until the whole life of the project is covered.

#### 3.4 The EGCL Abandonment Rule.

It was mentioned in the previous chapter that one of the problems that has not been addressed in the literature is related to the incorporation of the decision-maker's risk into

the abandoning rules. In fact, it can be observed that the algorithm presented in Section 3.2., uses expected values as a basis for determining the optimum abandonment period. This implies that the decision-maker does not consider the variability of the alternative outcomes. In other words, it is assumed that he will always take the abandonment option when confronted with the following situations,

Table 3.1 Three Cases on Abandonment Decisions.

	ABANDON		HOLD	
	$E[AV(k)]$	$\sigma[AV(k)]$	$E[PW(k)]$	$\sigma[PW(k)]$
Case I	5000	200	4000	100
Case II	5000	500	4000	100
Case III	4700	1000	4500	100

where,  $E[PW(k)]$  and  $\sigma[PW(k)]$  represent the expected value and standard deviation of the net present worth of the prospective series of cash flows, respectively.

In the first case, it seems reasonable to take the abandonment option because of its relatively high expected return and low variability. But in the second and third cases, where the variability of the abandonment option increases significantly, the proper action is not obvious.

The problem of deciding whether to abandon or continue to hold a project can be viewed in the same way as a decision to take one of two mutually exclusive projects. On one hand, there is the abandonment option, which provides an expected abandonment value,  $E[AU(k)]$ , with the corresponding standard deviation  $\sigma[AU(k)]$ . On the other hand, the hold option promises a certain expected value and standard deviation of the series of prospective cash flows,  $E[PW(k)]$  and  $\sigma[PW(k)]$ , respectively. The statistics related to the present worth of the prospective series cash flows can be computed using Hillier's approach [32].

$$E[PW(k)] = \sum_{J=J+1}^m E[CF(J)] / (1+i)^{(J-J)}$$

$$\sigma[PW(k)] = \left( \sum_{J=J+1}^m \sigma^2 [CF(J)] / (1+i)^{2(J-J)} \right)^{1/2}$$

where  $J$  is the current decision period and  $m$  is the current optimal abandonment period. Observe that since the cash flows are assumed to be mutually independent, the covariance terms in the expression for the standard deviation are all zero.

In order to obtain a solution to this decision problem, the next section contains a brief description of some of the methods most often used in decision making under risk. Then, an abandonment rule based on the expected gain confidence limit is described.

### 3.4.1 Methods for Decision Making Under Risk.

Decision making under risk refers to a situation where the probability distributions describing the possible outcome of future events are known or assumed [51]. In this case, the events are two, either to abandon the project taking a chance at a random disposal value, or to hold the project until the next most profitable abandonment period, taking a chance at a series of random cash flows that can be discounted and summarized into a present worth. In order to find a solution for the problem, three methods are discussed in the following sections: the probability of loss method, the coefficient of variation method, and the expected gain confidence limit method. The first method makes use of the whole probability distribution of the outcome when making the decision, while the last two use only the first two moments of the distribution. Another difference between these methods is that the expected gain confidence limit requires subjective considerations while the other two methods are based on standard norms of behavior.

3.4.1.1. The Probability of Loss Method. One of the approaches often used in making investment decisions between risky alternatives is called the probability of loss method [69,22]. This probability represents the likelihood that an

investment will yield a rate of return less than the minimum attractive rate of return, or equivalently, the likelihood that it will yield a negative net present worth. Symbolically,

$$P.L. = \int_{-\infty}^0 f(PW) dPW \quad (3-4)$$

where,

P.L. = probability of loss

PW = net present worth.

$f(PW)$  = net present worth density function

Since losses are not desirable, the decision would be to select that alternative which minimizes this probability.

The method assumes that the probability distribution of the alternative outcomes is known or may be approximated by a standard distribution function. If the outcome is measured by the expected net present worth, Millier [32] has shown that this probability distribution is approximately normal. In that case the computation of the probability of loss is straight forward.

The main advantage of this method is its simplicity of use and understanding. Only a basic knowledge of statistics is needed and, if the outcome is normally distributed, the first moments of the probability distribution is all the decision-maker needs to arrive at a decision.

Its major disadvantage rests upon the fact that when the alternatives have a very small or null probability of having losses, the discriminating ability of the method is reduced significantly. This explains why it is often used in conjunction with other screening methods [69].

3.4.1.2. The Coefficient of Variation Method. The coefficient of variation is a measure that was originally developed by Markowitz [43] as an alternative to the variance for measuring risk. Van Horne [70] has also used it to trace the uncertainty resolution of a project. In this case, the measure is assumed to summarize the decision-maker's preference in risk-return. In its most simple form, the coefficient of variation  $CV(x)$ , can be expressed as,

$$CV(x) = \sigma(x) / E(x) \quad (3-5)$$

where,  $x$  represents the outcome (Present worth or abandonment value).

In the decision making process, this method calls for acceptance of the alternative with the smallest  $CV(x)$ . The rationale for use of such method is plausible enough. The larger the expectation, the larger is an acceptable range of variation about that expectation. To illustrate the way this method operates, recall the three cases presented in section 3.4., which are repeated here.

Table 3.2 Three Cases on Abandonment Decisions.

	ABANDON		HOLD	
	$E[AV(k)]$	$\sigma[AV(k)]$	$E[FW(k)]$	$\sigma[FW(k)]$
Case I	5000	200	4000	100
Case II	5000	500	4000	100
Case III	4700	1000	4500	100

In the first case, the abandonment option with a CV of 0.04 is rejected when compared to the 0.025 of the hold option. Following a similar reasoning, the hold option is accepted in the other two cases.

The main advantage of using this method is that it concentrates all the required information in a single index, which is of great importance when a large number of decisions have to be made.

To show of the limitations the CV as a measure for selecting investment alternatives, Case I in Table 3.2 is analyzed. As shown above, if the decision-maker decides on the basis of the information provided by the coefficient of variation, then he would take the hold option because it has a lower variability. Consider now the data shown in table 3.2.

To facilitate the analysis assume that the outcome of both alternatives is normally distributed. Then, by definition there is only a 16% chance that the realized outcome will ever fall outside the interval  $E + \sigma$ , only a 2% that it will be outside the interval  $E + 2\sigma$  and no more than 0.21% that it will fall outside  $E + 3\sigma$ .

Table 3.3 Probabilistic Data for Case I.

ALTERNATIVE	ABANDON	HOLD
E	5000	4000
$\sigma$	200	100
C.V.	0.04	0.025
$E + 3\sigma$	5600	4300
$E + 2\sigma$	5400	4200
$E + \sigma$	5200	4100
E	5000	4000
$E - \sigma$	4800	3900
$E - 2\sigma$	4600	3800
$E - 3\sigma$	4400	3700

In Table 3.3, the worst anticipated outcome associated with the abandon option,  $E[AU(k)] - 3\sigma[AU(k)] = 4400$ , is better than the highest possible outcome of the hold option,  $E[PW(k)] + 3\sigma[PW(k)] = 4300$ . Now, it may well be questioned

whether anyone would still be willing to take the hold option. The difficulty with the CV is that the decision-maker is not interested in the variability as a relative magnitude, but as an actual magnitude. He would prefer the danger of coming out \$ 200 short on a \$ 5,000 expected return as against a \$ 100 shortfall below \$ 4,000 expected return. That is, what worries him is the expected upper or lower bound of an alternatives outcome, not the relative variability of the outcome.

#### 3.4.1.3. The Expected Gain Confidence Limit Method.

Another approach that uses the standard deviation as a measure of risk is the expected gain confidence limit method. This method, sometimes called the certainty equivalent method [4], is built upon a measure proposed by Baumol [2] in a modification to the mean variance criterion used in portfolio selection. The measure combines the expected value and standard deviation of the project's outcome with a parameter that expresses risk behavior. Symbolically, the relationship is,

$$EGCL[x] = E[x] - k \sigma[x] \quad (3-6)$$

where,

x = outcome

EGCL = expected gain confidence limit.

k = risk aversion coefficient.

It can be observed that equation 3-6 assumes that the decision-maker is a risk averter because, as the variability of the outcome increases, a larger expectation is required to level off its negative effects. When the investor is a risk seeker, the appropriate expression for the EGCL is,

$$EGCL[x] = E[x] + k' \sigma[x] \quad (3-7)$$

where  $k'$  represents risk seeking behavior.

In equation (3-6), if  $E[x]$  stands for the profitability of a project, then the  $EGCL[x]$  is an indicator which corrects that measure of desirability by taking into account the risk involved in the project. The result is a new measure of desirability that reflects the decision-maker's attitude towards risk.

A different way of looking at the EGCL is to conceive it as a theoretical "floor" on the outcome of a project. Where the probability that the outcome will fall under that "floor" can be computed by standard statistical methods, e. g., if the outcome is assumed to be normally distributed then, for  $k=2$  the probability that the realization of it will be lower than the EGCL is only 2%.

To employ this method, the investor must first express his risk-return preferences by choosing a specific value of  $k$ .

This can be done by intuitive selection or regression analysis [4]. Then, follows the computation of the EGCL for all the alternatives. Finally, the alternative with the largest EGCL is chosen. This procedure will lead to the selection of the alternative which provides the highest "floor on earnings" [2]. When applied to the data in Table 3.2, with  $k=1$ , this method would select the abandon option in the first two cases and the hold option in the third one.

The advantages of this method are: Firstly, as the coefficient of variation, it groups all the relevant information in a single measure. Secondly, by allowing the incorporation of the decision-maker's risk characteristics, it is a more flexible method. The decision-maker is not constrained to use conventional norms of behavior. Finally, it considers risk in its actual magnitude, avoiding the problems presented by the coefficient of variation.

The main disadvantage of this method is that the determination of the coefficient of risk aversion may become a very intricate problem. Especially complicated are the cases when the decision is not made by a single individual and when there is not enough data to compute the coefficient through regression analysis.

### 3.4.2 The Abandonment Rule.

Taking into account the advantages and disadvantages of the different methods for decision making under risk discussed in the previous sections, it has been decided to use the expected gain confidence limit in the formulation of the present abandonment rule. The EGCL provides an adequate way to incorporate risk into the decision and also simplifies the task by collecting all the relevant information in a single measure.

In general terms, the abandonment rule indicates that a project should be abandoned at that time when the EGCL of the abandonment value is larger than the EGCL of the present worth of the stream of successive cash flows. This stream is composed of the net cash flows associated with holding the proposal for its remaining economic life or until some future date at which the EGCL of the current abandonment value exceeds the EGCL of the discounted value of all successive cash flows. The process of finding the optimal abandonment period should begin at the end of the proposal's life and proceed backwards one period at a time with periodic evaluations of the abandonment option.

In algorithmic form, the procedure can be stated as follows,

1. Beginning with the last year of the proposal's life, discount the expected value and variance of the cash flow in that period back one period. Using Hillier's approach [32] this would be,

$$E[FW(L-1)] = E[CF(L)] / (1+i)$$

$$\sigma^2[FW(L-1)] = \sigma^2[CF(L)] / (1+i)^2$$

where,

$E[ ]$  = expected value operator.

$\sigma^2[ ]$  = variance operator.

$FW(L-1)$  = Net Present Worth in the next to last period of the cash flows in the last period.

$CF(L)$  = cash flows in the last period.

$i$  = MARR.

$L$  = life of the proposal.

2. Compute the EGCL for the resulting discounted cash flows and compare it to the EGCL of the abandonment value at the beginning of that period. Since the decision-maker is assumed to be a risk averter, this comparison is made at the low side of the EGCL.
3. Add the expected value and variance corresponding to the option with the largest EGCL to the expected value and variance of the proposal's cash flow in the

preceeding period, and discount these sums one period back.

4. Repeat steps two and three until the future cash flows have been discounted back to the first period.

This algorithm ensures calculation of the stream of cash flows with the largest expected gain confidence limit of the discounted present worth.

## CHAPTER IV

## DESCRIPTION OF THE SEQUENTIAL CAPITAL RATIONING PROCESS.

In Chapter II it was pointed out that one of the pitfalls in the original article by Robichek and Van Horne [51] that has not been discussed in the literature is the integration of risk into the abandonment rule. Thus, in Chapter III the problem was addressed and an alternative rule proposed. Another observation made in Chapter II is related to the currently reported applications of abandonment analysis. It is known that in order to attain optimality in the allocation of funds, the appraisal of deterministic investment proposals should consider the abandonment option. However, at present it is not possible to determine the magnitude of the effect of that option on the firm's wealth, when capital rationing decisions are made on a regular, periodic basis, and the outcome of the proposals is

non-deterministic. Since capital rationing is a dynamic problem, it is of interest to study the improvements that can be made in the rationing decision when abandonment is integrated into the analysis.

In order to understand how this integration can be made, this chapter begins with a discussion on the objectives and general investment decision process of a firm where investments are made in a sequential manner. Then, several capital rationing models for decision making under risk are described in Section 4.2. The chapter ends with a discussion on the effects of partial and complete information in the decision process.

#### 4.1 The General Investment Situation.

One of the objectives of this study is to establish the importance of including the abandonment option in sequential capital rationing decisions, when the investment proposals are not known with certainty. Specifically, the situation is one where the decision-maker knows only the proposals available in the current decision period and that knowledge is restricted to probabilistic information about the proposal's receipts, disbursements, and abandonment values. This type of investment setting is of special interest because it fits the situation most firms face in reality [7], [51].

Section 4.1.1 contains a statement of the objectives of a hypothetical firm confronted to a situation similar to that described above. Then, Section 4.1.2. gives a detailed description of the decision process followed by that firm.

#### 4.1.1 Objectives of the Firm.

When it comes to the definition of the economic objectives of a firm, there are two different, not necessarily opposed points of view. One is that of management and the other is the stockholder's. When the control of the firm is separate from its ownership, management may not always act in the best interest of the stockholders. Management people are sometimes said to be "satisfiers" rather than "maximizers", they may be content to "play it safe" and seek an acceptable level of growth, being more concerned with perpetuating their own existence than maximizing the value of the firm to its shareholders [16], [71]. The most important goal for a management of this kind may be its own survival [1]. However, it is true that in order to survive over the long run, management may have to behave in a manner which is reasonably consistent with maximizing the shareholder's wealth. Van Horne states that maximization of the value of the firm to its owners "... is an appropriate guide for how a firm should act." [71, pg. 8].

In addition to the economic goals that bear on a firm's behavior, there are non-economic objectives of which the decision-maker should also be aware. These considerations tend to add a new dimension to the strategy of a firm that usually have to do with the so-called human element [1].

The advent of the application of behavioral sciences to the study of business administration reflects the fact that economic objectives are not the unique goal for managers of business firms and therefore do not fully explain these firm's behavior. Some studies [65] indicate that non-economic considerations may sometimes dominate the decision making process of certain firms and that business objectives appear to be a combination of both economic and non-economic factors that reflect the characteristics of the individuals that run the firms.

Despite these factors, the value of the firm is still the measure by which the success of a business firm is judged [1]. This is true both from an economic and legal point of view as well as from the point of view of the society. Therefore, in this research it is assumed that the decision-maker will seek to maximize the value of the firm. This value will be measured by the total accumulated capital at the end of a predetermined planning horizon.

#### 4.1.2 Description of the Decision Process.

The general setting where this study is performed can be viewed in the following way. During each decision period, a hypothetical firm is faced with a number of investment opportunities. The decision-maker is aware of all the proposals submitted for consideration during that period, but lacks complete information about the opportunities arising in the subsequent decision periods.

The proposals submitted provide a set of possible cash flows over several periods, and may be abandoned in any of these periods, exchanging a series of prospective cash flows by the current abandonment values. The future cash flows generated by the proposals, as well as their respective abandonment values are not known with certainty but assumed to be random variables with a specific expected value, variance, and probability distribution.

Before making the capital allocation decision, the firm evaluates the possibility of modifying the investment proposals through an explicit consideration of the abandonment option. When such possibility is identified, the proposal is modified accordingly. Once the capital allocation decision is made, the budgeted funds are used to finance the accepted

proposals, and the rejected proposals are discarded from consideration in the future. This process is repeated in every decision period within the planning horizon.

The accepted proposals are held until the end of their most economic life. That is, once the modified proposals are accepted, no earlier abandonment is allowed. Supporting this assumption is the practical evidence reported in reference [15], which shows that in most firms assets are kept in business until their economic life is reached. This happens for essentially two reasons. First, some assets are especially designed for the firm's purpose; and second, there are substantial transfer costs of selling in imperfect markets.

If the proposals selected at a decision period do not exhaust the budget, the funds that remain are invested at an interest rate  $i_0$  in a highly liquid investment such as a bank account, where the funds may be withdrawn at any time. Thus, the decision-maker will have all these funds available for investment at the next decision time.

It is not known precisely what the budget will be in the future periods because those budgets are determined by the net cash received from investments made in prior periods. Funds from outside the firm are considered not available for

two reasons. First, as a matter of policy, some firms finance all their projects with funds generated internally; and second, since borrowing affects both, the solutions obtained with and without abandonment, it is expected that the results which are relevant to this study will only experience minor variations as a consequence of including this alternate source of funds in the decision making process. In addition, the borrowing option adds another dimension to the capital rationing problem and, therefore, increases significantly the amount of computer resources required to solve the problem with integer constraints. At present, this imposes a severe limitation in the scope of the study.

Finally, all proposals are assumed to be independent of each other.

#### 4.2 Sequential Capital Rationing Decisions Under Risk.

This section contains a brief description of the models that will be used to make the capital rationing decision among risky proposals. Three models have been selected from those presented in Section 2.2.2.2, the Expected Present Worth Maximization Criterion, the Mean Variance Criterion, and the Project Balance Criterion. It was decided not to include models that maximize utility functions because of the inherent

problems associated with the determination of an appropriate function carries (see Section 2.2.2.2.3.) The three models described hereafter will also be used for the case where the allocation decision disregards abandonment considerations.

#### 4.2.1 The Expected Present Worth Maximization Criterion.

As was indicated in Section 2.2.2.2.1., the Expected Present Worth Maximization model is an extension of the Lorie-Savage problem [41] which, as its name indicates, uses expected values instead of presumably certain ones. Under this decision criteria, the decision-maker must find in every period the set of proposals that maximizes the expected present worth of investments, subject to budget constraints. That set of proposals may be found solving the following integer linear programming problem,

$$\text{Model I: } \max Z = \sum_j E[FPW(J)] X(J)$$

subject to

$$\sum_j C_0(J) X(J) \leq B$$

$$X(J) = (0,1), \text{ integer, } J = 1,2,\dots,n$$

Where,

$E[FPW(J)]$  = expected present worth of proposal J  
computed at the MARR.

$C_0(J)$  = initial investment on proposal J.

B = funds budgeted for the decision period.

$X(J)$  = decision variable representing proposal J,

0 = rejected, 1 = accepted.

This formulation of the capital budgeting problem implies that the decision maker is risk indifferent, that he is able to construct the probability distribution of net present worth for each proposal, and that he can compute the mathematical expectation of each distribution.

#### 4.2.2 The Mean Variance Criterion.

The Mean Variance Criterion is an application of Markowitz's work on portfolio selection to capital budgeting. Markowitz's portfolio selection model was originally concerned with financial investments, but the model's implications for capital budgeting are now well recognized [42]. The basic idea behind the model is that the optimal portfolio for a firm is not simply any collection of good proposals but a balanced whole, providing the decision-maker with the best risk-return combination [72]. The procedure consists of successively finding that portfolio which minimizes the variance for each of a number of expected returns and does not violate the budget constraint. Formally, the decision-maker has to solve the following  $m$  integer quadratic programming problems,

$$\text{Model II: } \min \sigma_s^2 = \sum_i \sum_j X(i) \sigma[\text{PW}(i, j)] X(j) \quad i, j = 1, 2, \dots, n$$

subject to

$$\sum_i E[\text{PW}(i)] = E_s$$

$$\sum_i C_0(i) X(i) \leq B$$

$$X(i) = (0, 1) \text{ integer}$$

Where,  $\sigma_s^2$  is the variance of the  $s$ th portfolio's net present worth and  $E_s$  is its minimum acceptable expected net present worth ( $s=1, 2, \dots, m$ );  $\sigma[\text{PW}(i, j)]$  represents the covariance between the net present worth of proposal  $i$  and  $j$ ,  $i \neq j$ , and  $\sigma[\text{PW}(i, i)] = \sigma^2[\text{PW}(i)]$  are the variances.

By solving these  $m$  problems, the decision-maker obtains the set of  $(E, \sigma^2)$  efficient portfolios. That is the set which contains those portfolios that provide minimum variance  $\sigma_s^2$ , for a given return  $E_s$ , or maximum return  $E_s$  for a given variance  $\sigma_s^2$ . Then, from this set he selects that portfolio which best suits his risk-return preferences.

Another formulation which is more oriented towards capital budgeting has been proposed by Weinsartner [74] and modified by Peterson and Laushhunn [55]. In their model, the decision-maker defines his risk-return preferences through a single parameter  $w$ , and then solves the following integer linear programming problem,

$$\text{Model III: } \max E - w \sigma^2 = \sum_i E[FPW(i)] X(i) - w \sum_i \sum_j X(i) \sigma[FPW(i,j)] X(j)$$

subject to

$$\sum_i C_0(i) X(i) \leq B$$

$$x(i) = (0,1) \text{ integer, } i = 1, 2, \dots, n.$$

Where,

$E$  = expected net present worth of the portfolio.

$\sigma^2$  = variance of the portfolio's net present worth.

$w$  = coefficient of risk aversion.

$E[FPW(i)]$  = expected net present worth of proposal  $i$ .

$\sigma[FPW(i,j)]$  = covariance between the net present worth of proposals  $i$  and  $j$ .

$\sigma[FPW(i,i)] = \sigma^2[FPW(i)]$  = variance of proposal  $i$ 's net present worth.

$C_0(i)$  = initial investment of proposal  $i$ .

$B$  = funds budgeted.

$X(i)$  = decision variable representing proposal  $i$ ,

0 = rejected, 1 = accepted.

If this problem is solved for different values of  $w$ , the decision-maker obtains the same set of  $(E, \sigma^2)$  efficient portfolios as with Model II.

Peterson and Laughunn [55] indicate that this

formulation is equivalent to one which uses the following objective function,

$$\text{Model IV: } \max E - w' \sigma = \sum_i E[CPW(i)] X(i) - w' \left\{ \sum_i \sum_j X(i) \sigma[CPW(i, j)] X(j) \right\}$$

Where  $w'$  is another coefficient of risk aversion.

When all the investment proposals are independent, Model III can be simplified to,

$$\text{Model V: } \max E - w'' \sigma = \sum_i \{ E[CPW(i)] - w'' \sigma[CPW(i)] \} X(i)$$

subject to

$$\sum_i Co(i) \leq B$$

$$X(i) = (0,1) \text{ integer, } i = 1, 2, \dots, n.$$

which can be solved as a zero-one integer programming model.

#### 4.2.3 The Project Balance Criterion.

The Project Balance Criterion is a unique approach to capital rationing decisions under risk. This criterion selects the proposals on the bases of measure of desirability which combines three characteristics of a proposal, its profitability its variability, and its flexibility.

Profitability is measured by the proposals present worth (PW), the variability is measured by the standard deviation of the PW, and flexibility is measured by the normalized area of uncertainty resolution.

The steps in computing the measure of desirability are as follows: the first step involves finding an economic trade-off between the proposal's expected PW,  $E[PW(J)]$ , and standard deviation  $\sigma[PW(J)]$ , by means of the expected gain confidence limit,

$$EGCL[PW(J)] = E[PW(J)] - r1 \sigma[PW(J)] \quad (4-1)$$

where  $r1$  represents the rate of trade-off between reduction in expected value for reduction in variability. In other words, it represents the decision maker's preference for risk-return.

Once a trade-off between  $E[PW(J)]$  and  $\sigma[PW(J)]$  is obtained, the next step consists of determining a trade-off between the  $EGCL[PW(J)]$  and the measure of flexibility  $FLEX(J)$ . The latter is computed as follows,

$$FLEX(J) = AUR(J) / Co(J) \quad (4-2)$$

where  $Co(J)$  is the initial investment on proposal  $J$  and,

$$AUR(J) = \sum_t EGCL[ANB(J,t)]$$

where,  $AUR(J)$  is the area of uncertainty resolution of proposal  $J$ , and  $EGCL[ANB(J,t)]$  is the expected gain confidence limit of proposal  $J$  in period  $t$ .

The area of uncertainty resolution is normalized dividing by  $Co(J)$  to eliminate the effect introduced by the size of the initial investment.

The trade-off between  $EGCLIPW(J)$  and  $FLEX(J)$  is found by,

$$Z(J) = EGCLIPW(J) / FLEX(J)$$

Finally, the decision-maker can find the combination of proposals that maximizes  $Z(J)$  by solving the following integer linear programming problem at each decision period,

$$\text{Model VI: } \max Z = \sum_j Z(J) X(J)$$

subject to

$$\sum_j Co(J) \leq B$$

$$X(J) = (0,1) \text{ integer, } J = 1, 2, \dots, n.$$

where,

$B$  = budget.

$X(J)$  = decision variable related to proposal  $J$ ,

0 = rejected, 1 = accepted.

This model was originally formulated to solve problems where the proposals are represented by probability trees. In fact, the concept of uncertainty resolution developed by Van Horne [70] was conceived for application to proposals represented in that special manner. Since this research follows Hillier's approach [32], i. e., the proposal's cash flows are assumed to be random variables with a corresponding probability distribution, a different measure of flexibility must be derived.

#### 4.2.3.1. An Alternative Measure of Flexibility.

Before developing the measure of flexibility, it is convenient to define what this concept involves. Here, the flexibility of a firm is related to the way its investments make funds available for reinvestments in other attractive opportunities that may become available in the future. This concept is somewhat related to that of liquidity, which is defined as follows: when a proposal has a short payback period, it is said that the investment is liquid [51]. However, flexibility is a more extensive concept because it does not only consider the time it takes a proposal to repay its initial investment but also how funds become available during that time.

In this context, two of the four elements of information provided by the project balance (see Section 3.2.1.), the discounted payback period ( $Q$ ) and the area of negative balance ( $ANB$ ), are the most suitable for developing a measure of flexibility. As mentioned above,  $Q$  is more of a measure of liquidity because it fails to provide complete information about the cash flow pattern. For example, if  $Q$  were used to measure the flexibility provided two proposals with project balance patterns as shown in Figure 4.1., it would rate both projects as providing the same amount. Since Project A provides more funds in the early periods it would probably be preferred from a flexibility point of view.

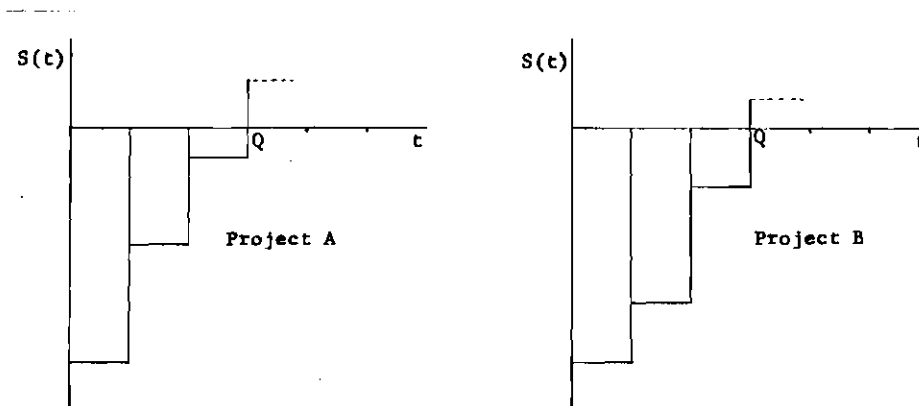


Figure 4.1. Project Balance Patterns for Projects A and B.

The  $ANB$  can be interpreted as the aggregate total amount of funds to be tied up for a particular investment activity. Thus, a measure of flexibility based upon the  $ANB$

statistics will provide valuable information regarding the generation of funds in the critical periods of an investment. In fact, the ANB summarizes the pattern of the negative project balance which, at the same time, is an indicator of how funds become available in the early periods of a proposal. The ANB also shows how uncertainty about an undesirable outcome is resolved through time. Figure 4.2. is helpful to illustrate the last point. During the first and second periods, both projects show an identical behavior. Both have tied up and released identical amounts of funds. However, as they reach the end of period 3, the first would have resolved most of the uncertainty about a prospective undesirable outcome. A fact that is reflected by a smaller area of negative balance. Therefore, other characteristics being the same, i. e., profitability and variability, a firm which makes investment decisions on a regular, period basis would probably prefer the first proposal because it makes funds available sooner. Although the ANB performed as an adequate measure of the flexibility provided by these proposals, this will not always happen because the expected ANB itself uncovers the effect of several factors. First, it is a random variable, therefore in the formulation of the index of flexibility, the ANB should be modified to take into account its variability. This may be accomplished by computing the expected gain confidence limit,

$$E[GLEANB(J)] = E[ANB(J)] + r^2 \sigma [ANB(J)] \quad (4-3)$$

where,  $r_2$  is a coefficient of risk aversion.

Since a large ANB is an undesirable event for a risk averse decision-maker, the  $EGCLEANB(J)$  takes an upper confidence limit rather than a lower confidence limit. What is of importance is to select those proposals that promise the lowest upper bound in unfavorable outcomes. In this study, unfavorable outcomes refer to slow recovery of funds and extended payback periods.

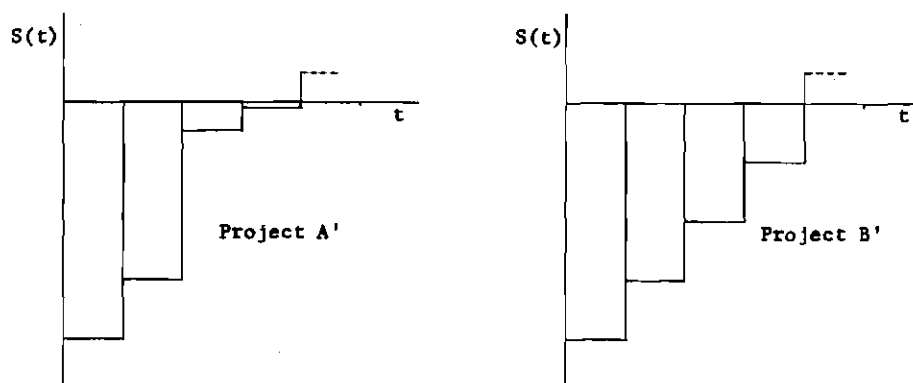


Figure 4.2. Project Balance Patterns for Projects A' and B'.

The second problem that arises by using the ANB as a measure of flexibility is that its size is influenced by the size of the initial investment, thus the index should be normalized in a similar way as it was done in equation 4-2. That is,

$$FLEX(J) = EGCLEANB(J) / Co(J) \quad (4-4)$$

Finally, since the ANB is an accumulation of the proposal's negative project balance, much of the information conveyed by the pattern of the project balance is lost. This determines the appearance of situations as that shown in Figure 4.3, where the measure of flexibility proposed above would rate both projects as providing equal amounts of flexibility (the  $\sigma [ANB(J)]$  is assumed to be the same in both cases). Park [51] states that proposal A would be preferred because it resolves the uncertainty about negative outcomes in an earlier period. However, it is the opinion of this research that the situation is not that simple because the firm might prefer to have more funds available at the end of the first period, for investment in more desirable opportunities that may be obtained at that time, than waiting until the end of the second period, when a large outcome would be secured from proposal A". Thus selecting proposal B" instead of that recommended by Park.

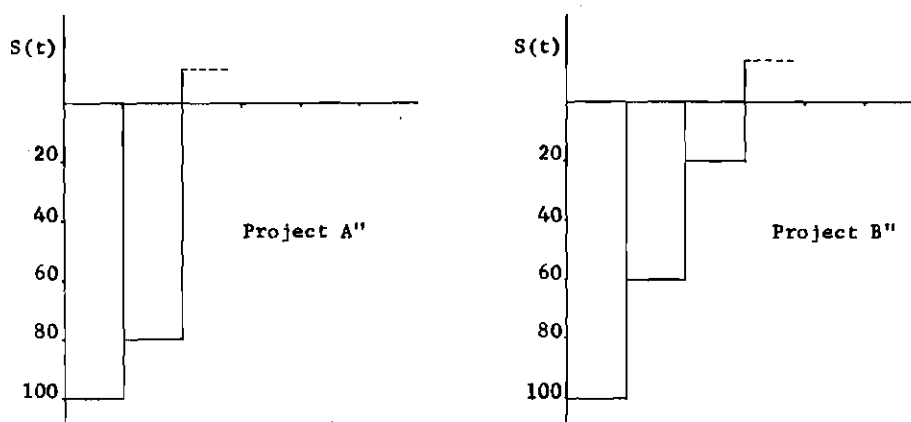


Figure 4.3. Project Balance Patterns for Projects A" and B".

One solution to this last problem would be to associate flexibility with the average rate of change of the project balance in the negative segment. However, this would require a ratio type of index, which is undesirable because it does not reveal appropriately the magnitude of the flexibility. Other possible solution procedures that take into account the shape of the project balance pattern would demand the formulation of rather complex relationships which certainly cannot be summarized by a single index. Thus, for this study the flexibility provided by a proposal is measured by  $FLEX(J)$  as formulated in equation 4-4. The project balance model remains the same, only this minor change is made on  $Z(J)$ .

#### 4.3 Sequential Capital Rationing Decisions with Partial and Complete Information.

It must be realized that when the capital rationing decision is made under risk, none of the models presented in the previous sections will guarantee the optimum selection of proposals that could be achieved if the decision-maker had known with certainty the outcome of those proposals and the availability of future investment opportunities.

Since this study is aimed at finding out the improvements that can be made in sequential capital rationing

under risk, when the abandonment option is considered, it is of interest to compare the horizon value obtained by using the models presented in the previous sections with those obtained when complete knowledge about the current and forthcoming investment opportunities is available. Two levels of knowledge are examined in the next sections, one is that where the decision-maker knows only the proposals available in the current decision period (partial information), and the other is that where he knows all the proposals available during the planning horizon (complete information).

#### 4.3.1 Sequential Capital Rationing with Partial Information.

This case assumes that the decision-maker has complete information about only those investment opportunities that are to be considered for immediate implementation. He does not have access to information about the proposals that may become available in the future. When this is the case, the decision criterion utilized is to select that set of proposals which maximizes the net present worth of investments at each decision period [51]. That set of proposals may be identified by solving the Lorie-Savase problem,

$$\text{Model VII: } \max Z = \sum_i PW(i) X(i)$$

subject to

$$\sum_i C_0(i) X(i) \leq B$$

$$X(i) = (0,1) \text{ integer, } i = 1, 2, \dots, n.$$

where,  $PW(i)$  represents the net present worth of proposal  $i$ . This model can be solved as a zero-one integer linear programming problem.

#### 4.3.2 Sequential Capital Rationing Decisions with Complete Information.

This case assumes that the decision-maker, at the time of decision, has complete knowledge about the investment opportunities that may be obtained at the present and in the future. Since all this information is available, the allocation decision is more of a programming problem. In fact, decisions are no longer made in a sequential manner but only once at the beginning of the planning horizon. Then, the overall set of optimum proposals is selected.

As it was indicated in Section 2.2.1.1, a comprehensive analysis of this problem has been made by Weinsartner [73], who proposed the "horizon model" as a solution procedure,

$$\text{Model VIII: } \max Z = \sum_j a(j) X(j) + v(H)$$

subject to

$$- \sum_j CF(1, j) X(j) + v(1) = B(1)$$

$$- \sum_J CF(t,J) X(J) - (1+i_0) v(t-1) + v(t) = B(t)$$

$$t = 2,3,\dots,H$$

$$X(J) = (0,1) \text{ integer, } J = 1,2,\dots,n.$$

$$v(t) \leq 0 \quad t = 1,2,\dots,H.$$

Where,  $a(J) = \sum_{t=H+1}^{\infty} CF(t,J) (1+g(J))^{H-t}$

= present worth in period H of the cash flows of proposal J that extend beyond the H.

CF(t,J) = cash flow in period t of proposal J

g(J) = proposal J's rate of growth.

v(t) = funds available for lending at period t.

B(t) = Funds for investment in period t attributable to operations made before the decision period.

$i_0$  = rate of interest from highly liquid investments.

This formulation differs from the basic "horizon model" in two aspects. First, since borrowing is not considered, the corresponding terms have been omitted; and second, the interest rate used to discount the cash flows that are realized beyond the horizon is the proposals rate of growth. Weinsartner [73] does not specify what rate should be used, and Bernhard [5] uses the minimum attractive rate of return. However, since the firm's objective is to maximize the accumulated capital at the horizon time, the interest rate that should be used is the respective growth rate of the proposals that terminate after the horizon [51].

## CHAPTER V

## DESCRIPTION OF THE SIMULATION MODEL.

In order to evaluate the effects of integrating the abandonment option into sequential capital rationing decisions, simulation is used to generate the environment where the decision making process takes place. This chapter contains a description of the simulation model developed for that purpose. Basically, it (the model) may be separated into three parts. Part one is the generation of the set of investment proposals (SIP) to be considered for investment in each period. The second part comprises the evaluation of the possibility of an early abandonment of the investment proposals; and, the third part consists of the application of the different capital rationing criteria, to allocate the funds budgeted for each period. In addition, this part includes the solution to the sequential capital rationing

problem when abandonment is disregarded, and the accumulation of the statistics relevant to the solution of the problems with partial and complete information.

The chapter begins with the formulation and discussion of the assumptions that have been made in the development of the model. Then follows the identification of the phases of the decision process and the description of the SIF generation. Finally, some topics related to the simulation process are discussed.

#### 5.1 Assumptions of the Simulation Process.

In addition to the assumptions implicit in the abandonment rules and in the capital rationing decision criteria, several other assumptions have to be made to allow for the simulation of the decision making process. These assumptions limit the model's representation of the actual process but reduce significantly the time and effort needed to simulate it. The assumptions are,

1. That the firm makes the capital allocation at the end of the decision periods. It selects the investment proposals from a set submitted during the period, according to some decision criteria and subject to

the budget constraints. Parra-Vasquez and Oakford [53] compare the relative effectiveness of this procedure with that of making decisions continuously as investment proposals are presented for consideration. They conclude that there are statistical differences between the two procedures and that the firm should investigate the periodic decision making procedure as an alternative to the continuous decision making procedure.

2. That the firm acts consistently. That is, if a rule based on expected values is used to make the abandonment decision, then the capital rationing decision is also made with a criterion that uses expected values. Analogously, if risk is taken into account when making the abandonment decision, then a similar consideration should be made in the capital rationing decision.
3. That the funds available for investment during the initial periods come from investments made prior to the beginning of the study period. Thus, it is necessary to estimate these funds in order to initialize the simulation process. Thuesen [68] has proposed a procedure for doing so, which is described later in Section 5.7.1.

## 5.2 Description of the Simulation Process.

The simulation process utilized in this study consists of four phases:

1. This phase comprises the generation of the SIP submitted to the decision-maker's consideration in every period within the time horizon.
2. Once the set of investment proposals has been generated, each of the proposals is redefined through an explicit consideration of the abandonment possibility. That is, if any of the proposals should be abandoned before the completion of its life, those proposals will be modified according to the procedure described in Section 5.6 before proceeding to the selection process.
3. This phase includes the regular application of the three capital allocation criteria described in Chapter V to the modified set of investment proposals. In addition, this phase contains the calculation and accumulation of the statistics relevant to this study.

4. This phase consists of the solution of the capital rationing problem with partial and complete information. The realization of the cash flows and abandonment values of the original set of investment proposals are utilized to solve the above mentioned problems. Also in this phase, the solution of the capital rationing problem with no abandonment is obtained.

### 5.3 Generation of the Set of Investment Proposals.

A set of investment proposals consists of a number of investment possibilities submitted to the consideration of the decision-maker during a decision period. The information which is normally available comprises the size of the initial investment, and expectations about the cash flows, abandonment values, rate of growth, and life of the proposals. Each one of these pieces of information has associated a certain variability, which makes it very unlikely that the investor will be confronted with the same set of investment proposals in two or more periods over a normal time horizon (20 years). Therefore, it is reasonable to visualize an investor having sets of proposals drawn from an underlying probability distribution where the size of the investment, expected pattern of cash flows and abandonment values, life, and

expected rate of growth are all random variables [51]. Although the proposals are eventually modified when the abandonment option is taken into account, the existence of that probability distribution is recognized, and assumed to be represented by the distribution of the proposal's rate of growth.

The simulation of the investment proposals begins with the estimation of the average prospective rate of growth. This parameter is computed from the distribution of the proposals rate of growth and is equivalent to the average rate of growth of the investment proposals that could be accepted by the firm if no budget constraints were present. Then, the type of proposal is determined by specifying its life, size of the initial investment, rate of growth, and expected cash flows and abandonment values patterns. These variables are controlled by assumed probability distributions. The generation of the SIP is summarized in Figure 5.1 and a detailed description of each step is given in the following Sections.

### 5.3.1 Distribution of the Proposals Rate of Growth.

As defined in the Section 5.3, the distribution of the proposals rate of growth represents the probability distribution on which the proposals submitted in every period are based. This distribution describes the average fraction

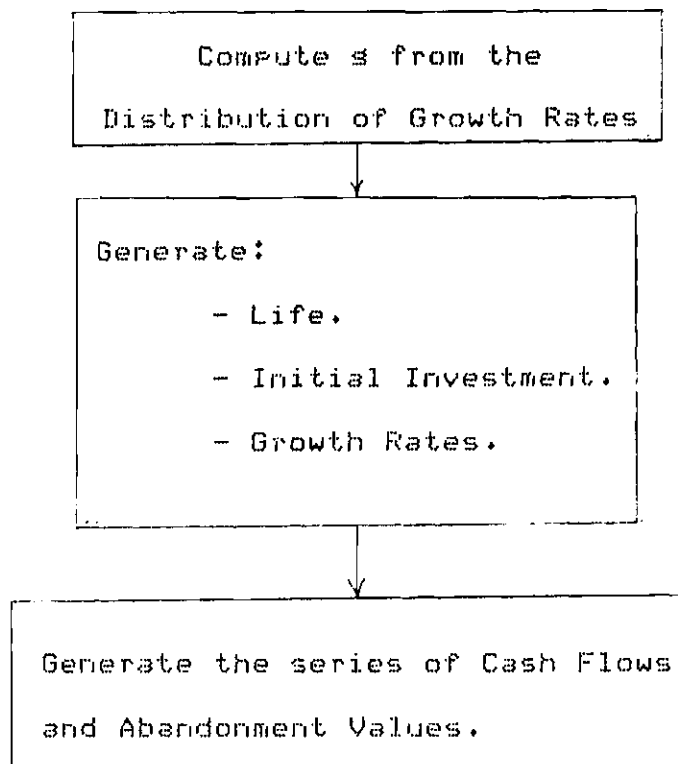


Figure 5.1. Generation of the SIP.

of proposals that are expected to provide a certain growth.

5.3.1.1. The Shape of the Distribution of the Proposals Rate of Growth. An important characteristic of the distribution of the proposals rate of growth is its shape. The importance of this characteristic rests upon the fact that it describes the average fraction of proposals that are expected to provide a certain growth rate. The following two shapes have been examined in the literature:

1. Linear shape. In this case, the proportion of proposals that are expected to be submitted with a particular rate of growth equals the proportion of proposals expected to be submitted at any other rate of growth. Figure 5.2. displays a linear shape type of distribution in which the rates of growth range from a maximum of 36% down to 6%.
  
2. Exponential shape. When the exponential shape is adopted, as Figure 5.3. shows, the proportion of high-return proposals to be submitted is lower than that of proposals with low return. If this type of distribution is compared to the previous one, the firm represented by the linear shape type of distribution has a larger proportion of investment proposals at higher rates. The relationship between the growth rates  $s$  and the cumulative probability  $F$  in Figure 5.3. is expressed as,

$$s = 0.0131 + 0.0469 e^{2(1 - F)} \quad (5-1)$$

This type of distribution has been examined by Dean [15], Thuesen [68], and Park [51].

The curve in the distribution of the proposals growth rate terminates at  $i = 6\%$ , for either shape of distribution,

because the firm can invest an unlimited amount of funds at that rate in a highly liquid investment.

Considering that the shape of the distribution of the proposals growth rate largely depends on the expectations of the firm [51], and that it is of interest to analyze the effect of a time varying exponential shape, this study will concentrate its attention on this latter shape. This is also done as a means to restrict scope of the study.

5.3.1.2. Assumptions about the Changes of the Distribution of the Proposal's Rate of Growth. Another important characteristic of the distribution of the proposals rate of growth is its ability to change overtime. Since the investor is performing in a dynamic environment, it is most probable that the above mentioned distribution will not remain constant [51]. The chances are that, as time passes, the average fraction of proposals with higher growth rates will decrease, increase, or oscillate around a certain value. The first one of these possibilities is not relevant for this study because the number of worthwhile proposals will also decrease, and most of the funds available in each period will end up invested at  $i_0$ . Thus, the effect of using different capital allocation criteria would be obscured.

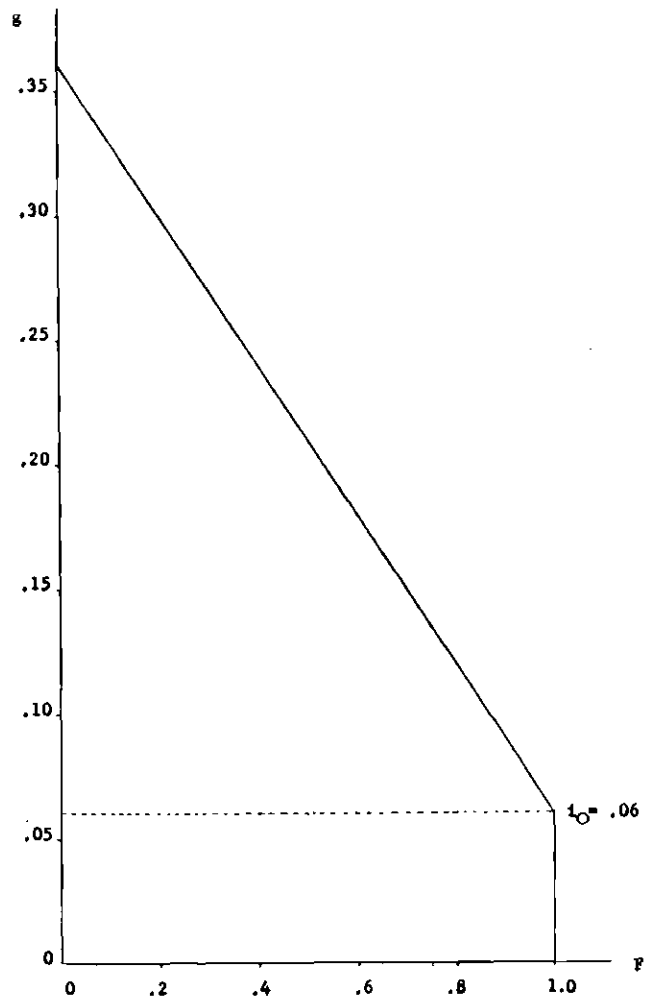


Figure 5.2. Linear Distribution of Growth Rates.

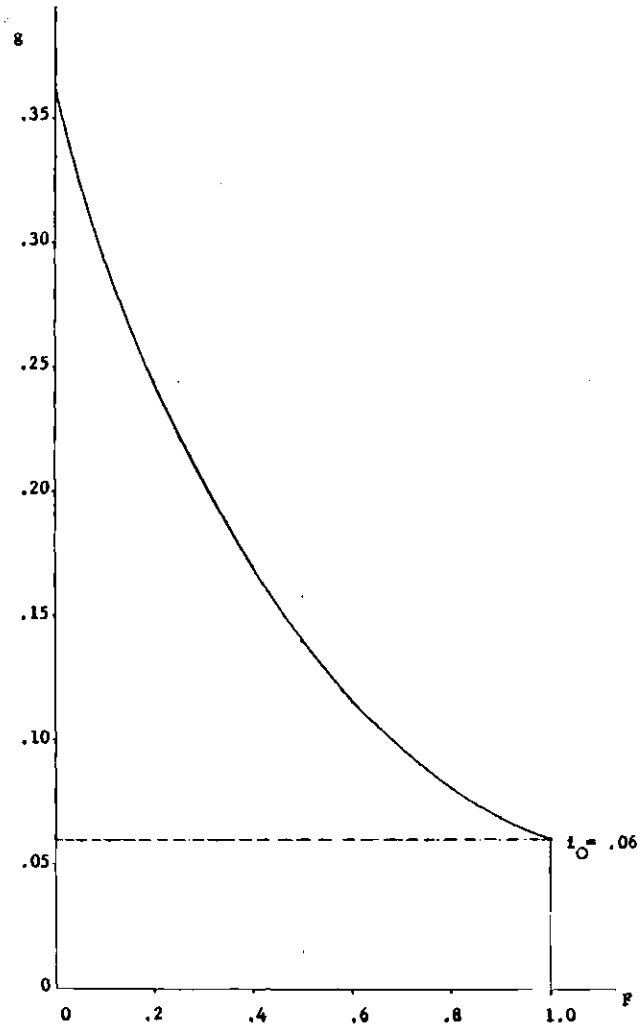


Figure 5.3. Exponential Distribution of Growth Rates.

The second possibility is related to the situation where the increase in investment opportunities with higher rates of growth are independent of the size of the firm. This situation occurs when the fraction of proposals with higher growth rates keeps growing through time. An assumption of this nature is supported by the empirical evidence reported by Hymer and Pashigian [35] and has some appeal from a practical stand point. In the case where a firm starts at time  $t=0$  with a distribution having an exponential shape as that shown in Figure 5.3, it is necessary to make some modifications in the equation of the curve to allow for the simulation of an increasing average growth rate. After several trials it was decided to use the following equation,

$$g(t) = 0.0131 + 0.0469 e^{2(1 - (F/1+\theta T))} \quad (5-2)$$

which at  $t=0$  is,

$$g(0) = 0.0131 + 0.0469 e^{2(1 - F)}$$

Figure 5.4 shows how the distribution changes through time during a horizon of 20 years. In that Figure the parameter  $\theta$  takes a value of 0.2 and the minimum rate of growth increases from a value of 0.06 to 0.245.

The third possibility, where the fraction of proposals with a certain growth rate oscillates around a constant value, and those situations where the increase or decrease in the

fraction of proposals with higher growth rates are not significant, may be approximated assuming that the distribution of the proposal's growth rate remains constant through time. This type of environment is described by Dean [15] and adopted in the simulation model used by Thuesen [68] and Park [51].

This study will concentrate its attention in the last two alternatives and disregard the first one. No loss of generality is expected as a consequence of doing so.

5.3.1.3 Computation of the Average Prospective Growth Rate ( $\bar{g}$ ) and its Use in the Simulation Process. Another assumption adopted in conjunction with Thuesen's [68] and Park's [51] work with respect to the distribution of investment proposal's growth rate is that the total dollar value of the proposals increases from period to period at a rate equivalent to the average prospective growth rate  $g$ .

As mentioned in Section 5.3.,  $\bar{g}$  corresponds to the average growth of the proposals that could be accepted by the firm, if no budget constraints were present, i.e., those proposals with a rate of growth larger than the minimum attractive rate of return. Therefore, it is possible to estimate  $g$  by constructing a function which describes the expectation of how the firm's funds are to be invested.

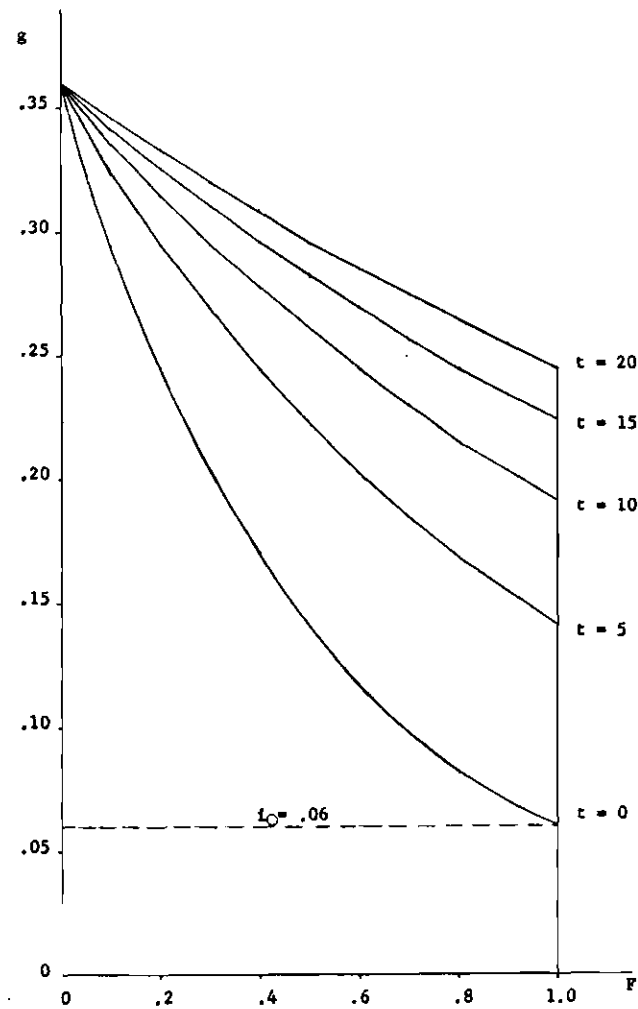


Figure 5.4. Time Varying Distribution of Growth Rates.

For the distribution of the proposals rate of growth with exponential shape, shown in Figure 5.3, the computation of  $g$  is as follows. Since  $g$  is a function of  $F$ ,

$$g = 0.0131 + 0.0469 e^{2(1 - F)}$$

$$\bar{s} = \left( \int_{F_{\text{MAX}}}^{F_{\text{MARR}}} (0.0131 + 0.0469 e^{2(1-F)}) dF \right) / F_{\text{MARR}}$$

Supposing that  $\text{MARR} = s_{\text{MARR}} = 0.15$  then,

$$\bar{s} = \left( \int_0^{0.464} (0.0131 + 0.0469 e^{2(1-F)}) dF \right) / 0.464 = 0.24$$

When the fraction of proposals with higher rates of growth keeps growing through time, the computation of  $\bar{s}$  is as follows,

from equation 5-2,

$$s(t) = 0.0131 + 0.0469 e^{2(1-(F/1+\theta t))}$$

and

$$F = (1+\theta T) - (1+\theta t/2) \ln(s-0.0131/0.0469)$$

Defining,

$$T = \ln G/\theta(2-\ln G)$$

where,

$$G = (s_{\text{MARR}} - 0.0131)/0.0469$$

Then,

$$\bar{s}(t) = \left( \int_0^{\bar{F}} s(t) dF \right) / \bar{F}$$

where,

$$F = \begin{cases} (1+\theta t) - (1+\theta t/2) \ln G & \text{if } 0 < t \leq T \\ 1 & \text{if } T < t \leq H \end{cases}$$

Thus, for  $\alpha = 0.15$  and  $\theta = 0.2$

$$s(t) = \begin{cases} 0.24 & \text{if } 0 < t \leq 6 \\ 0.0131 - 0.173(1+\theta t)(e^{-(2/(1+\theta t))} - 1) & \text{if } 6 < t \leq H \end{cases}$$

### 5.3.2 Types of Proposals.

Following the outline of the process given in Figure 5.1., the generation of the proposals is now described. For each proposal, the following characteristics are defined:

1. Initial investment cost.
2. Life.
3. Rate of growth (internal rate of return).
4. Cash Flow pattern (expected values and variances).
5. Abandonment values pattern (expected values and variances).

5.3.2.1 The Proposals Initial Investment Cost. The generation of the initial investment cost is based on an approach proposed by Thuesen [68] and used by Park [51]. The first cost of the proposal ( $C_0$ ) is generated from a distribution that is defined by a mean  $\bar{C}_0$  and six other parameters which represent three different exponential distributions. The three distributions are located along the horizontal axis so that the mean of the resulting  $C_0$  distribution is  $\bar{C}_0$ . Grafically, this relationship can be depicted as in Figure 5.5.

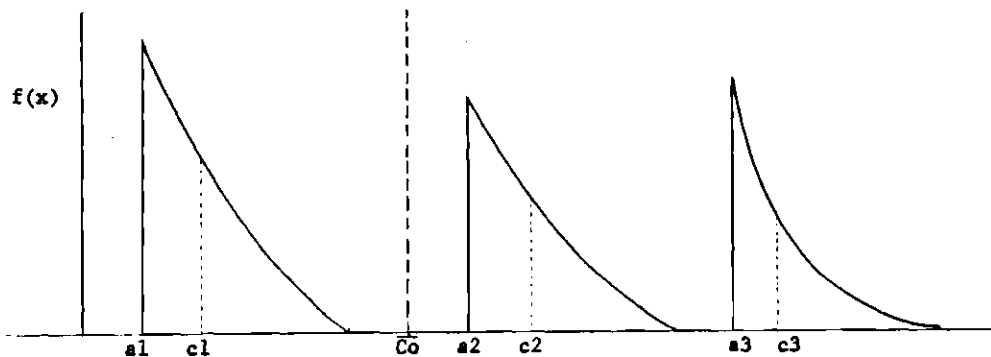


Figure 5.5 Distribution of the Initial Investment Cost

In Figure 5.5, the three exponential distributions correspond to,

$$f(x) = (1 / (c_i - a_i)) e^{-(1/(c_i - a_i))(x - a_i)}$$

where,  $x > a_i$ ,  $i = 1, 2, 3$ .

and the cumulative distribution  $F(x)$  is,

$$F(x) = 1 - e^{-(1/(c_i - a_i))(x - a_i)}$$

Thus,  $x$  can be viewed as,

$$x = a_i - (c_i - a_i) \ln(1 - F(x)) \quad (5-3)$$

Therefore, by specifying  $a_i$  and  $c_i$ , an exponential distribution can be placed anywhere on the horizontal axis. By placing three such distributions and by sampling from each one an appropriate fraction of time ( $f_i$ ), it is possible to have the expected value of all the samplings equal to  $C_0$ . The following conditions must be satisfied in order to assure that the samples drawn represent those from the  $C_0$  distribution,

$$\bar{C}_0 = f_1 c_{11} + f_2 c_{22} + f_3 c_{33}$$

$$f_1 + f_2 + f_3 = 1$$

$$c_{11} f_1 = c_{22} f_2$$

$$c_1 \leq \bar{C}_0 \leq c_2$$

The following two reasons are stated for using this rather complicated scheme for the generation of the initial investment:

1. The approximate exponential shape that results from this combination generates a greater proportion of proposals with a smaller first cost. This property is desirable because the number of smaller proposals is usually greater than the number of large proposals in most capital budgeting situations.
2. This approach makes it possible to extend the range of  $C_0$  in the sample with relative ease. Thus, it is possible to have some reasonable probability of selecting a  $C_0$  that is rather large relative to  $\overline{C_0}$ .

Another assumption made in conjunction with Thuesen's [68] and Park's [51] work is related to the variation of the proposal's initial investment through time. As the total capital of the firm being simulated grows from period to period, it is logical to expect that the funds available for investment will also grow. Thus, if the average initial investment of the proposals  $\overline{C_0}$ , and the number of proposals submitted in every period are held constant through time, the budget after some periods of operations will be large enough to undertake all the proposals that meet the minimum conditions set by each decision criterion. If this is the case, the effect of using different criteria to abandon and to accept proposals becomes obscured by the fact that the

decision rules don't have to discriminate when there is no financial constraint.

To increase the number of proposals from period to period would be an alternative to solve this problem. However, such modification would probably render the simulation of the capital budgeting process unmanageable. Therefore, it has been decided to hold the number of proposals constant and to have the proposal's initial investment increase at the average growth rate,  $\bar{g}$ . Thus, maintaining a reasonable balance between the funds available for investment and the initial investment of the proposals. Figure 5.6. depicts the algorithm used to generate the proposal's initial investment for each period.

5.3.2.2. Proposals Life. The generation of the proposals life is made using a truncated exponential distribution similar to that shown in Figure 5.7.. This distribution allows for the generation of a greater number of proposals with shorter lives, what in practice seems to occur [51]. It also permits to establish upper and lower bounds on the lives of the proposals, which is an advantage from a computational point of view. As the life of the proposals gets longer, the number of variables and statistics that have to be kept in memory increases considerably.

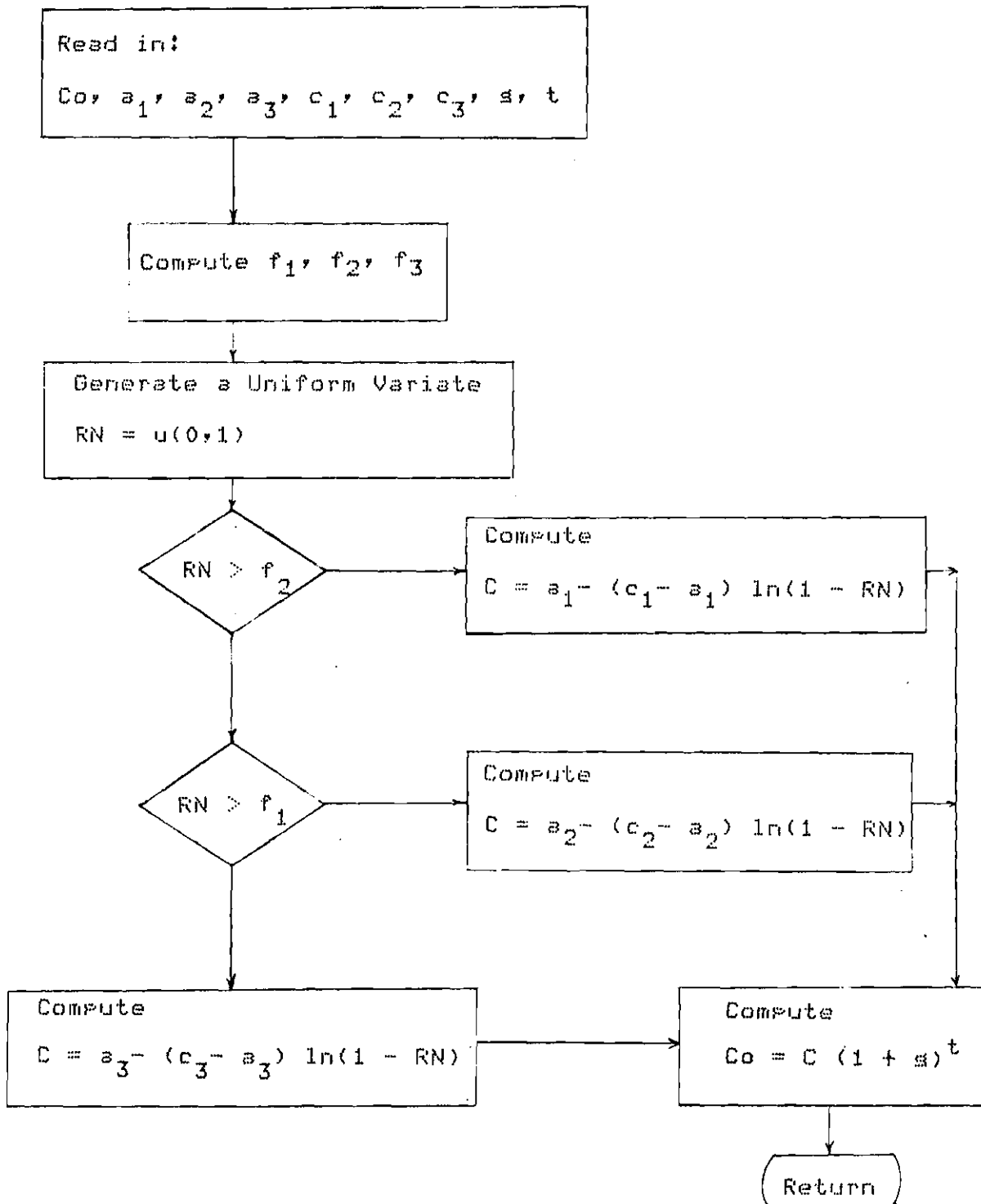


Figure 5.6. Logic to Generate the Initial Investment Cost.

The three parameters used to define the truncated probability distribution are,  $(L_{min}, L_{av}, L_{max})$ , where

$L_{min}$  = lower bound on the proposals life.

$L_{av}$  = the average proposal's life.

$L_{max}$  = the upper bound on the proposals life.

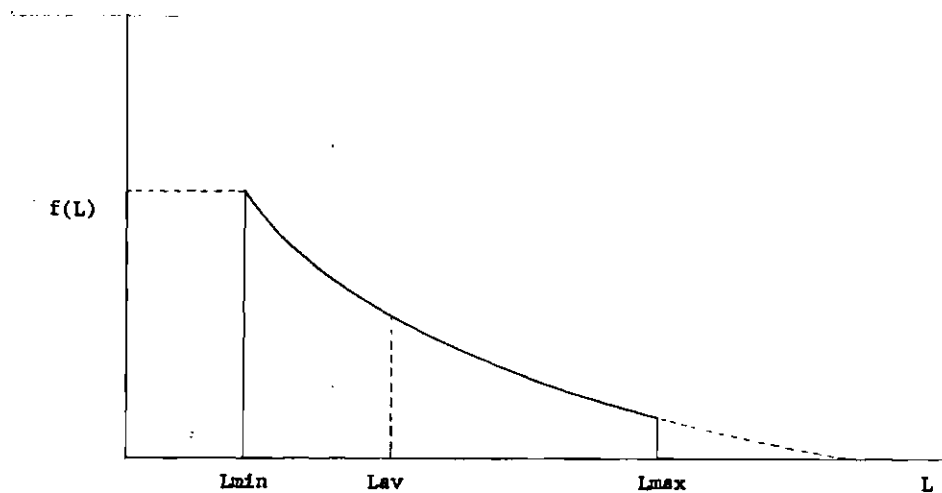


Figure 5.7 Proposal's Life Distribution.

The cumulative probability function for the truncated distribution is,

$$F(L) = (1/F(L_{max})) (1 - e^{-(1/(L_{av}-L_{min}))(L-L_{min})})$$

Thus, the proposals life is determined by,

$$L = L_{min} - (L_{av} - L_{min}) \ln (1 - F(L_{max}) F(L)) \quad (5-4)$$

Figure 5.8. shows the algorithm used to generate the proposal's life.

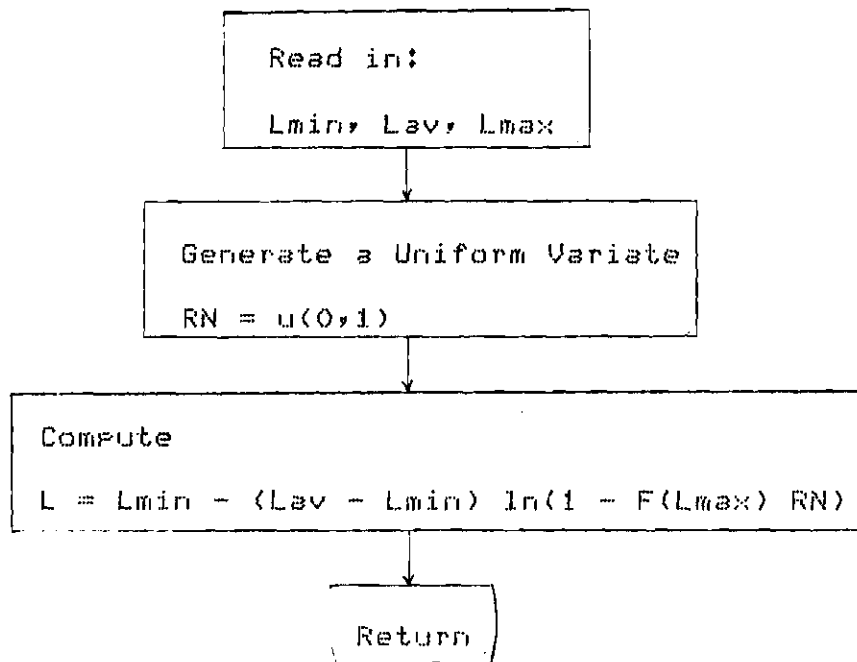


Figure 5.8. Logic to Generate the Proposal's Life.

#### 5.3.2.3. Expected Rate of Growth for the Proposals.

The generation of the proposal's rate of growth was somewhat discussed in Section 5.3.1.. There, it was observed that the characteristics of the environment where the study is held, play a major role in the determination of the rate of growth. If the fraction of proposals with a certain growth rate is held constant through time, then the following equation should be used in the Monte Carlo sampling,

$$s = 0.0131 + 0.0469 e^{2(1 - RN)} \quad (5-5)$$

where,  $s$  is the proposal's growth rate and  $RN$  is a random variate generated from the interval  $(0,1)$ .

On the other hand, if the fraction of proposals with higher rates keeps growing through time, the following

equation should be used,

$$s(t) = 0.0131 + 0.0469 e^{2(1 - (RN/1+\theta t))} \quad (5-6)$$

Where  $\theta$  is the parameter that controls the changes in the curves shape. The algorithm used to generate  $s$  is shown in Figure 5.9..

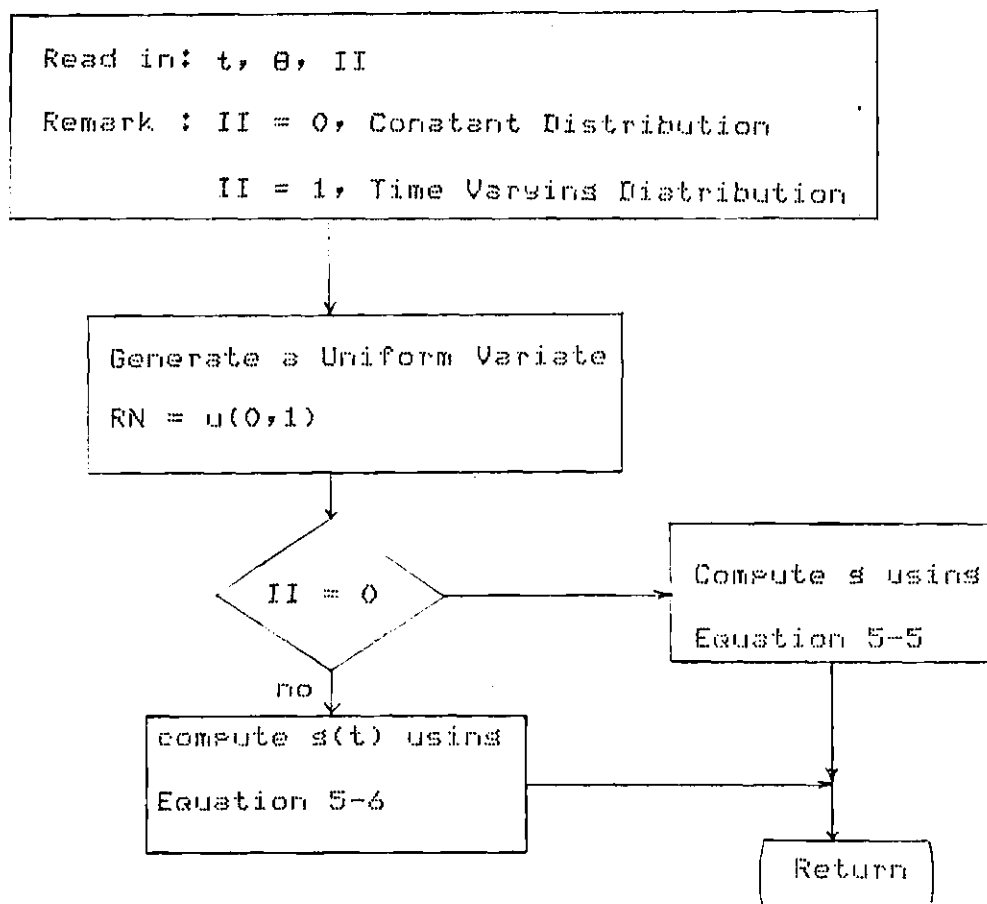


Figure 5.9. Logic to Generate the Growth Rates.

5.3.2.4. Expected Cash Flow Pattern. Once the initial

investment, the life, and the rate of growth have been generated, the only factor left to consider in the computation of the series of expected cash flows is the cash flow pattern. In this simulation, the following four basic cash flow patterns are used:

1. Single payment.
2. Uniform series.
3. Gradient series (increasing)
4. Gradient series (decreasing).

By using combinations of the gradient series patterns and the uniform series pattern, it is possible to generate an unlimited number of variations in these patterns. Figures 5.10 and 5.11 show the series of cash flows that can be obtained by combining a uniform series and a gradient series.

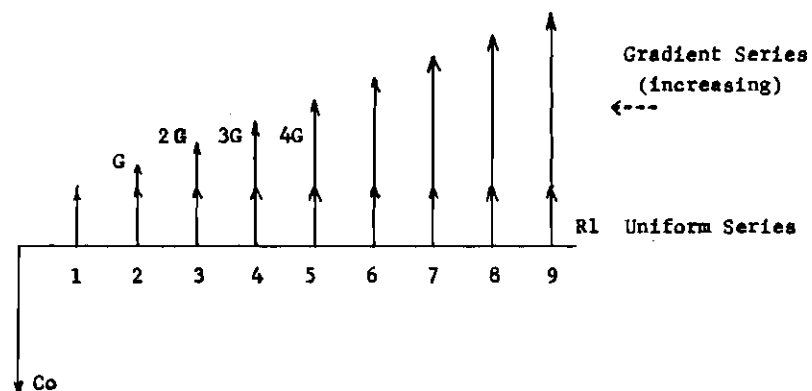


Figure 5.10 Combination of a Uniform and Increasing Gradient Series.

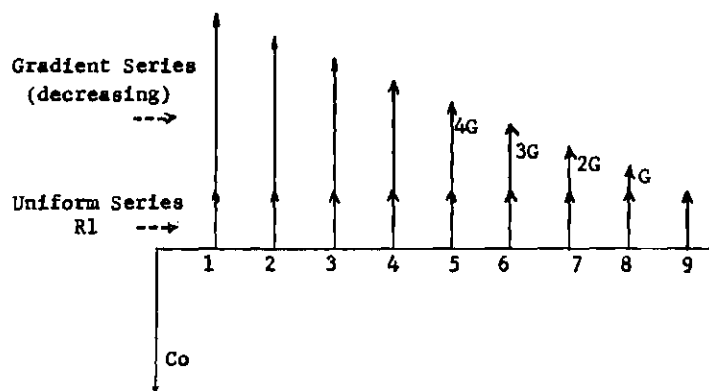


Figure 5.11 Combination of a Uniform and Decreasing Gradient Series.

An important fact to be recognized at this point is that the cash flow patterns shown in Figures 5.10 and 5.11 can be obtained from a combination of series such as those shown in Figure 5.12..

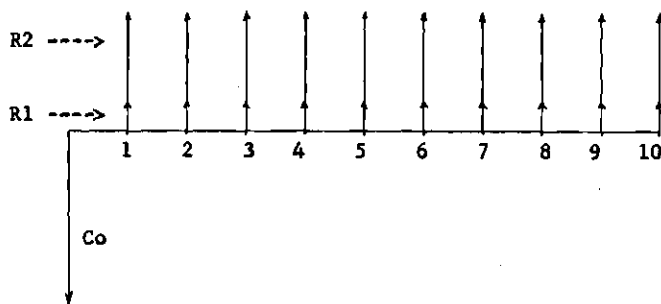


Figure 5.12 Combination of two Uniform Series.

The conversion of the R2 uniform series in Figure 5.12, to the decreasing or increasing portion of the cash flow patterns in Figure 5.10 and 5.11 is made through the following computation:

For increasing gradient series:

$$R2 = G \left( \frac{A/G, s, L}{s} \right), \quad G = R2 / \left( \frac{A/G, s, L}{s} \right)$$

where,

$$\left( \frac{A/G, s, L}{s} \right) = \frac{1}{s} + \frac{L}{1 - (1+s)^{-L}}$$

For decreasing gradient series:

$$R2 = (L - 1) G - G \left( \frac{A/G, s, L}{s} \right)$$

$$G = R2 / (L - 1 - \left( \frac{A/G, s, L}{s} \right))$$

Therefore, a variety of these combinations can be obtained by controlling the size of R2 relative to R, where

$$R = R1 + R2 = Co(J) \left( \frac{A/P, s, L}{s} \right) \quad \text{and}$$

$$\left( \frac{A/P, s, L}{s} \right) = \frac{s(1+s)^L}{(1+s)^L - 1}$$

Observing this relationship, it is evident that the larger the fraction of  $R$  corresponding to  $R_2$ , the larger the gradient portion in the cash flow series. The smaller  $R_2$ , the more the cash flow the series approaches a uniform series.

The single payment type of cash flow pattern refers to those proposals that have a single receipt at the end of their lives. Figure 5.13 depicts an example of this kind of proposal.

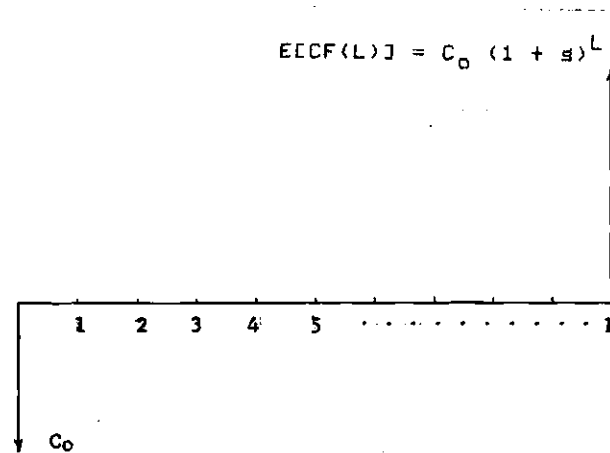


Figure 5.13 Single Payment Type of Cash Flow Pattern.

In the simulation, a particular cash flow pattern is randomly generated from a predetermined distribution of cash flow shapes. If the cash flow pattern selected is a single payment, the computation of the only receipt is straight forward (see Figure 5.13). On the other hand, if the cash flow pattern selected is a gradient series, a random choice is

made between the increasing series and the decreasing series. Then, the value of  $R_1$  and  $R_2$  are determined using the following relationship,

$$R_1 = f^* \cdot R$$

$$R_2 = R - R_1$$

$$0 \leq f^* \leq 1$$

where  $f^*$  is a random variable uniformly distributed in the interval (0,1). Thus, when  $f^*$  is selected, the following cash flow patterns result for the particular values of  $f^*$  shown below:

If  $f^* = 0$ , the resulting series is strictly a gradient series.

If  $f^* = 1$ , the resulting series is strictly a uniform series.

If  $0 < f^* < 1$ , the resulting series is a combination of a uniform and a gradient series.

The computation of the series of expected cash flows for a proposal is made using the following equations,

$$ECCF(J,t) = R_1(J) + (t-1) G(J), \text{ for increasing series}$$

$ECCF(J,t) = R_1(J) + (L(J)-1) G(J) - (t-1) G(J)$ , for decreasing series.

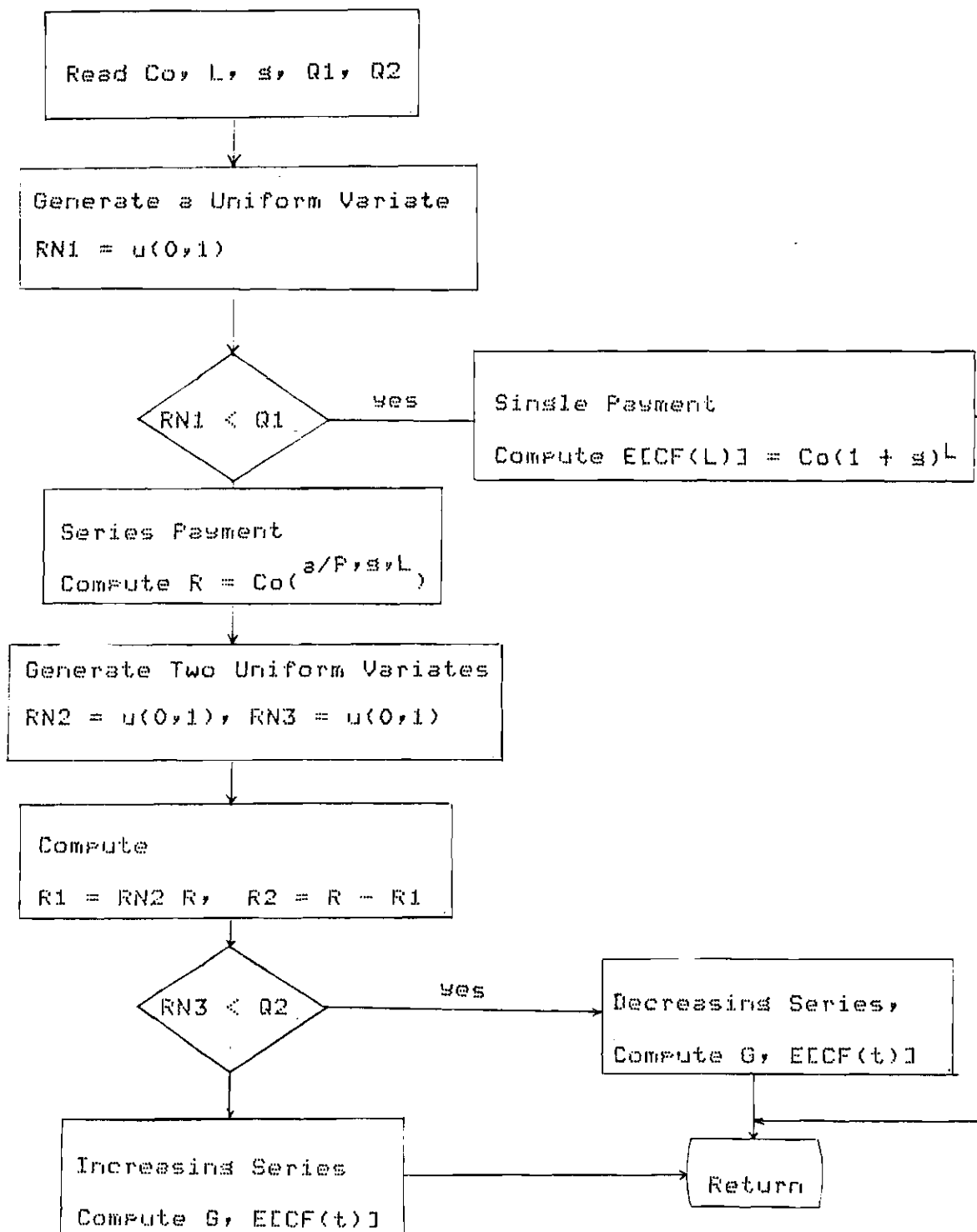


Figure 5.14. Logic to Generate the Series of Cash Flows.

The algorithm used in the computation of the expected cash flow series is shown in Figure 5.14. Where,

$Q1$  = Probability of observing a single payment type of cash flow.

$Q1'$  =  $1 - Q1$  = Probability of observing a series payment type of cash flow.

$Q2$  = Probability that a series payment type of cash flow be a decreasing series.

$Q2'$  =  $1 - Q2$  = Probability that a series payment type of cash flow be an increasing series.

5.3.2.5 Expected Abandonment Values Patterns. Similar to the cash flows, the abandonment values are a series of values that extend throughout the life of the proposal. In their simulation three basic assumptions are made,

1. That the series of abandonment values is a continuous function of the life of the proposal. Although this is not completely realistic, it simplifies considerably the simulation process.
2. That the abandonment values are directly related to the initial investment and to the final salvage value. Since the abandonment values at the beginning and at the end of a proposal's life are its initial

investment and final salvage value, respectively, it is believed that such a relationship would exist.

3. That the final salvage value is a fraction of the last expected cash flow. This expression is

$$E[AV(J,L)] = P_1 E[CF(J,L)] \quad (5-7)$$

where,

$E[AV(J,L)]$  = expected final abandonment value of proposal J

$E[CF(J,L)]$  = expected final cash flow of proposal J

$P_1$  = parameter,  $0 \leq P_1 \leq 1$

This assumption avoids a modification of the original proposal by setting abandonment values within reasonable ranges.

Another resemblance between the series of cash flows and the series of abandonment values is that both may follow different patterns. Here, two patterns are considered.

1. Linear decay, which is often used in replacement analysis [67], refers to the case where the series of abandonment values vary linearly from the initial investment, at the beginning of the proposal's life, to the final salvage value, at the end of the

proposal's life. Equation 5-8 is used in the computation of the series of abandonment values,

$$E(AV(J,t)) = C_0(J) - c \cdot t \quad (5-8)$$

where,

$$c = \{C_0(J) - E(AV(J,L))\} / L(J), \quad t = 1, 2, \dots, L(J).$$

2. Fast decay. In this case, the rate of change in the abandonment values decreases as the life of the proposals approaches its end. Eilon [19] and Gringer [27] have reported empirical evidence of a similar situation, where the abandonment values follow the function:

$$E(AV(J,t)) = C_0(J) e^{-at} \quad (5-9)$$

where,

$$a = -\frac{1}{L} \ln \left( \frac{E(AV(L))}{C_0} \right)$$

An alternative formulation of the fast decay pattern could be made with Friberg's formula,

$$E(AV(t)) = \frac{C_0 + ut^v E(AV(L))}{1 + ut^v}$$

where  $u$  and  $v$  are constant parameters which depend on the type of asset and on its life. The main advantage of using this

approach is that a wide variety of different series of abandonment values may be obtained for the same  $Co(J)$  and  $ECAV(J,L)$  (see Figure 5.15.). However, it has a major drawback in the fact that there is no easy way of obtaining the relationship between the parameters and the life of the proposals. The retention of this relationship is essential if the proposals are not to be modified.

Finally, it should be mentioned that in most situations, the abandonment values vary inversely with the life of the project [25]. Thus, the term decay is used to identify the patterns just proposed. However, this is not the case for all projects. There are cases where the abandonment values do not fall overtime. It is also conceivable to have projects where the abandonment values increase through time. These unusual situations were one of the main factors considered in the selection of the equations proposed for the computation of the abandonment values. Specifically, they can handle any of the above situations. For example, if the final salvage value is greater than or equal to the initial investment the series of abandonment values will no longer decay through time.

Figure 5.16. summarizes the algorithm used in the computation of the abandonment values, where,  
 $P2$  = Probability of observing a linear decay series of

abandonment values.

1 - P2 = Probability of observing a exponential decay series of abandonment values.

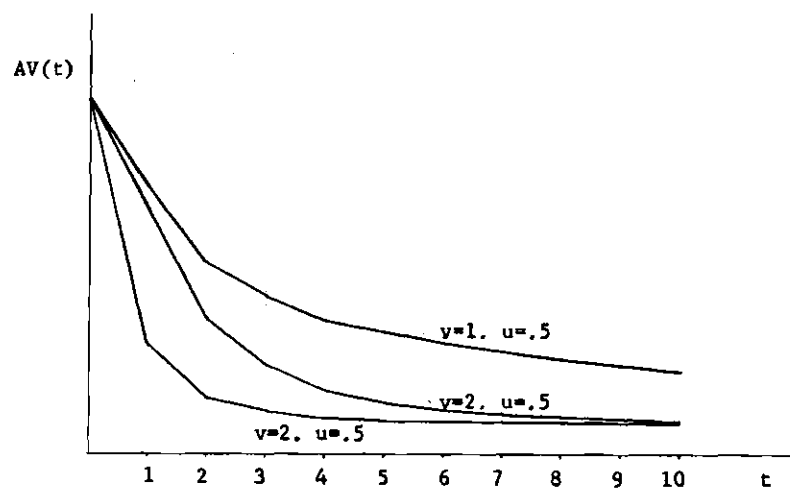


Figure 5.15. Series of Abandonment Values.

5.3.2.6. Variability of the Cash Flows and Abandonment Values. The cash flows and abandonment values generated in Sections 5.2.3.4. and 5.3.2.5, respectively, are only the expected values of the corresponding probability distributions. To specify a measure of the variability two procedures are used. The first one described by Levy [39] and adopted in the simulation model used by Park [51], defines the

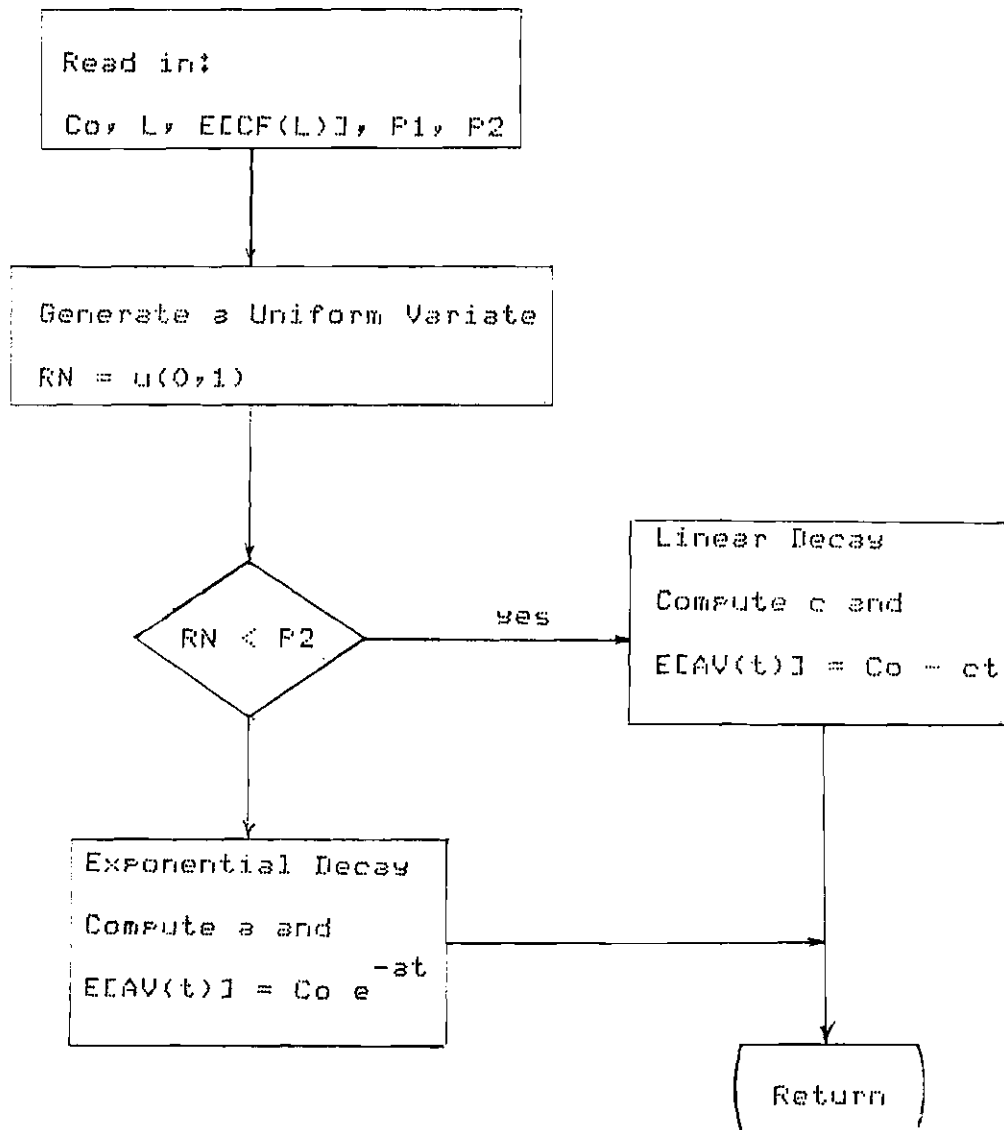


Figure 5.16. Logic to Generate the Series of Abandonment Values.

standard deviation in terms of a percentage of the expected value. It has been recognized that this approach is not completely justified. However, it would be reasonable to assume that a greater variability will often accompany larger expected values [51]. If this is the case, the use of equation 5-10, would be the simplest way to define the relationship between the expected values and the standard deviations of the cash flows and abandonment values.

$$\sigma [CF(J,t)] = b_1 \cdot E[CF(J,t)] \quad (5-10.1)$$

$$\sigma [AV(J,t)] = b_2 \cdot E[CF(J,t)] \quad (5-10.2)$$

Where,  $t = 1, 2, \dots, L(J)$ , and  $b_1$  and  $b_2$  are constant parameters.

The second procedure, proposed by this study, states that the standard deviation of the above mentioned variables is directly proportional to the size of the expected value and to a function of the time elapsed between the moment the forecast is made and the period when the cash flow or abandonment value is realized. Again, it is recognized that this approach cannot be completely justified. However, it seems reasonable to assume that a larger variability will accompany the expected values forecasted further in the future. In this case, the simplest way to express such relationship would be through the following equations.

$$\sigma [CF(J,t)] = b_1 \cdot f_1(t) \cdot E[CF(J,t)] \quad (5-10.1)$$

$$G[EAV(j,t)] = b_2 \cdot f_2(t) \cdot E[EAV(j,t)] \quad (5-10.2)$$

where  $f_1(t)$  and  $f_2(t)$  are continuous and increasing functions of  $t$ , the time elapsed between the forecasting period and the realization period.

This step completes the simulation of the set of investment proposals. Figure 5.17. shows the detailed logic used in that simulation.

### 5.3.3 Number of Proposals per Period.

In Section 5.3.2.1 it was decided to maintain a constant number of proposals to be submitted in every period, as a means of avoiding the problem of an unmanageable number of projects. In this Section, the study focuses its attention on the determination of that constant number of proposals.

From a computational point of view, it is convenient to have as few proposals per period as possible. On the other hand, too few proposals might cause the simulation to undertake all those proposals that satisfy the minimum requirements set by each decision criterion. In light of these considerations, it is necessary to determine the smallest number of proposals per period which increased will have little effect on the results.

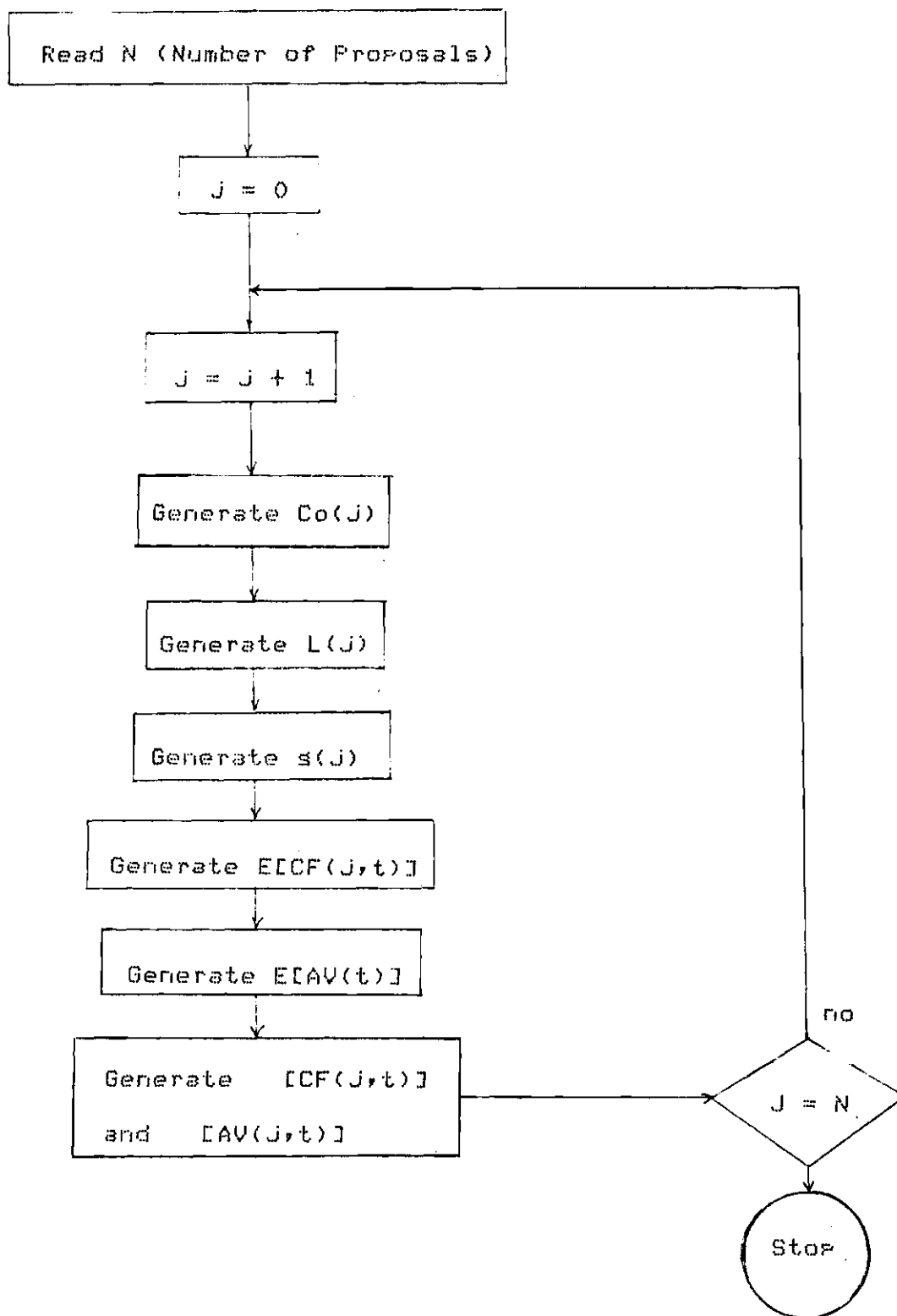


Figure 5.17. Logic for the Generation of the SIP.

Ten runs for each of four alternatives (five, ten, fifteen, and twenty proposals per period) were made to examine the effects of the number of proposals on the firm's total accumulated wealth. The results, plotted in Figure 5.18, show that there is not a great difference between using 15 or 20 proposals. In fact, the increase in the horizon value after 20 periods amounts to only a 4.8%. Therefore, it was decided to use 15 proposals per period. The results reported were obtained using the EGCL Abandonment Rule and the Project Balance Criterion, but similar results were arrived at using the other two decision criteria.

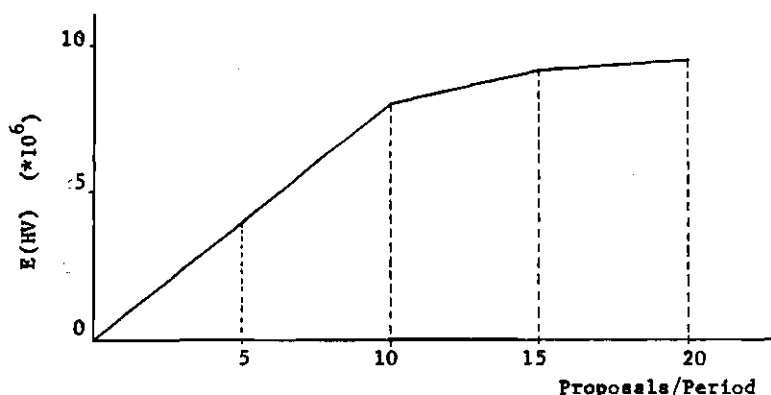


Figure 5.18  $E(HV)$  vs. Number of Proposals per Period.

#### 5.3.4 Number of Decision Times.

The number of decision times in a study period, as the number of proposals considered in every decision time, has a direct influence in the amount of time and computer memory needed to solve the problem. A study period that is too long increases significantly the data and statistics that have to be generated and kept in storage. Also the time needed to simulate the process increases considerably. On the other hand, if the study period is too short, the proposals with lives longer than the study period would not exert their full effect on the decision process [51]. Therefore, it is necessary to use a study period long enough to allow the process to stabilize and at the same time short enough to maintain the computational needs as low as possible.

In this simulation, the length of the study period is set at  $H = 20$ , which is more than three times longer than the average life of the proposals, and helps keep the solution time and the computer space required at reasonable levels.

#### 5.4 Application of the Abandonment Rules to the SIP.

Phase 2 of the simulation process comprises the application of the abandonment rules to the SIP submitted in

every period. In this phase, the firm evaluates the possibility of an early abandonment of the proposals using any of the three rules presented in Chapter III. When such possibility is recognized, the proposals are modified according to the following procedure:

If a proposal (which originally was defined as having a life  $L$ , an initial investment  $C_0$ , expected cash flows  $E[CF(t)]$ , and expected abandonment values  $E[AV(t)]$ , with standard deviations  $\sigma[CF(t)]$  and  $\sigma[AV(t)]$ , respectively), should be abandoned in a certain period  $J$  ( $0 < J < L$ ), then the proposal's life should be reduced from  $L$  to  $J$ , and the expected cash flow in period  $J$  changed from  $E[CF(J)]$  to,

$$E[CF'(J)] = E[CF(J)] + E[AV(J)]$$

and the variance, from  $\sigma^2[CF(J)]$  to

$$\sigma^2[CF'(J)] = \sigma^2[CF(J)] + \sigma^2[AV(J)]$$

The cash flows prior to period  $J$  remain unchanged and those beyond  $J$  discarded from further consideration. If the proposal should be abandoned in period zero, then the proposal as a whole is discarded.

This procedure follows the assumption that the cash flows and abandonment values are variables having probability distributions with similar characteristics [50]. Specifically, both are assumed to be normally distributed.

## 5.5 Application of the Capital Allocation Criteria to the Modified SIP.

The third phase in the simulation process comprises the regular, periodic application of the capital allocation criterion to the modified set of investment proposals. The essentials of this part of the process have already been addressed in Chapter IV. Therefore, here attention is concentrated in the discussion of the assumptions implicit in the model. Of special concern are those assumptions related to the generation of the budget available in every period and to the measure of effectiveness that will be used.

### 5.5.1 Generation of the Budget Available in the Decision Periods.

One of the assumptions made in Section 4.1.2, constrains the firm to operate only with funds generated by itself. No allowance is made for obtaining funds from external sources. Therefore, the budget available for investment in each period comes from the net cash received from investments made in prior periods. This constraint determines that in the simulation, a special provision has to be made for the estimation of the budget available in the early part of the study period. Thuesen [68], and Park [51],

in their simulation use a simple method to derive a meaningful estimate of the cash flows coming due within the study period of investments made before  $t = 0$ . The method is based on the assumption that all investments made prior to  $t = 0$  are uniform series type of investment with a life equal to the average life of the proposals in the SIP, and with a rate of growth equal to the firm's average rate of growth,  $s$ . Although the proposals in the SIP do not have this uniformity assumption, it should be noted that the combination of several different proposals can result in a series of cash flows that approaches a uniform series [51]. Based on this assumption, the equation derived to estimate  $R(t)$ , the cash receipts in period  $t$  coming from investments made before the beginning of the study period, is the following,

$$R(t) = R(0) (1+s)^{-t} \quad t = 1, 2, \dots, L_{av} \quad (5-12)$$

where,  $L_{av}$  is the average life of the proposals in the SIP and  $R(0)$  is an input parameter, which in this simulation is arbitrarily set at twice the average initial investment of the proposals.

Finally, if  $CF(i, J, t)$  is the  $i$ th cash flow of the  $J$ th proposal undertaken in period  $t$ , and  $RESB(t)$  is the residual balance in period  $t$  then the budget available in period  $t$   $B(t)$  is,

$$B(t) = R(t) + (1+i)^t RESB(t-1) + \sum_{k=1}^{t-1} \sum_{j=1}^{J(k)} CF(t-k, j, k) \quad (5-13)$$

where,

$J(k)$  = the number of proposals accepted in period  $k$ .

$i$  = interest rate on a highly liquid investment.

This equation shows that, in order to determine the size of the budget available at any decision time it is necessary to keep track of all realizations of the cash flows associated with proposals undertaken before the current decision period.

#### 5.5.2 The Horizon Value as a Measure of Effectiveness.

The basis to compare the effectiveness of the different approaches to the abandonment and selection of proposals is the firm's total accumulated wealth in a future predetermined point in time  $t = H$ , called the "horizon". The computation of the "horizon value" is analogous to the computation of the objective function in Weinsgartner's basic horizon model [73]. That is, the present worth at time  $H$  of the future receipts of investments that extend beyond  $H$  but were made on or before  $H$ , plus the funds available for lending in period  $H$ . No borrowing is considered in this study, thus the corresponding terms are omitted.

Symbolically, the "horizon value" is,

$$H.V. = \sum_{k=0}^H \sum_{J=1}^{J(k)} CF(H-k+1, J, k) (1+g(J, k))^{-(H-k+1)} + RESB(H) \quad (5-14)$$

where,

H.V. = the "horizon value".

CF(i, J, k) = i th cash flow of the J th proposal accepted in period k.

J(k) = number of proposals accepted in period k.

RESB(H) = residual balance in period H, (funds available for lending).

g(J, k) = rate of growth of the J th proposal accepted in period k.

In the simulation, the realizations of the cash flows (abandonment values included) of each proposal that extends beyond the horizon have to be stored to compute the corresponding rate of growth and the H.V..

#### 5.6 Solution of the Problem with Complete and Partial Information.

The solution of the problem with complete information requires that the realization of the series of cash flows and abandonment values be known for all the proposals available in the study period. Once this information is obtained, the proposals are modified by an explicit consideration of the abandonment option. The abandonment rule used in this case is

precisely the same as proposed by Dyl and Long [18]. A proposal is abandoned at that time which provides the largest present worth. After the proposals are modified, the capital allocation problem is solved. In Section 4.3.2, a mixed-integer linear programming formulation was proposed to solve horizon model. The currently available computer codes for this type of model are able to solve problems with as many variables (32) and constraints (20 plus 300 upper bounds) however, a significant amount of computer time is required. Since a large number of problems has to be solved to arrive at statistically significant results, it has been decided to use a linear programming code, sacrificing some accuracy in exchange of considerable savings in the utilization of the available resources.

For the solution of the problem with partial information, a similar approach is used. The main difference with the above described procedure is that the capital rationing decision is made in a sequential manner using a zero-one integer linear programming code.

#### 5.7 Solution to the Problem with no Abandonment.

The integration of abandonment analysis into capital rationing decisions has been suggested as a means of improving

the overall capital budgeting process. In order to measure the magnitude of that improvement, it is necessary to solve the capital rationing problem when no abandonment is considered. The solution to this problem is obtained assuming that the investor can only identify the proposals submitted and selected in the past and in the current decision periods. All the information that he is provided are expected values and standard deviations of the proposal's series of cash flows. He then solves the allocation problem using any of the criteria described in Section 4.2.

#### 5.8 Replication of Simulation Runs.

In order to estimate the effect of using different abandonment decision rules and capital allocation criteria, several combinations of these rules and criteria are applied to the sets of investment proposals submitted within the planning horizon. Each run of the simulation model comprises the generation and analysis of the SIP for the 20 decision periods. Thus, each run produces a single sample of the horizon value obtained by the above mentioned combinations.

Since the horizon value is a random variable, it is necessary to make several runs for each investment situation to compute significant estimates of its expected value and

standard deviation. The decision regarding the number of runs for each investment situation can be made observing the variability of the horizon value after several preliminary runs. In general, as the number of runs increases, the variability of the horizon value is expected to decrease. Therefore, preliminary runs were made with 5, 10, and 15 runs. In the average, a significant reduction in the variability (16.3%) was observed as a consequence of increasing the number of runs from 5 to 10; and only a 5.4% was obtained by increasing the number of runs from 10 to 15. Therefore, in this study, the number of runs was set at 10.

Another factor that should be taken into account in the determination of the number of runs is the utilization of computer time. The model used in this study is composed of five programs linked by the information each one provides. One program generates SIP, simulates the cash flows and abandonment values, and computes the parameters for the capital rationing models that solve the non-abandonment case. This program takes around 13 SRU (System Resource Units) to generate the data for each run. A second program makes the abandonment decision and solves the capital rationing problem for the abandonment case. It takes around 35 SRU to solve the problem with 7 different risk parameters. A third program solves the non-abandonment case. It takes 9 SRU to solve the allocation problem for 7 risk parameters. A fourth program

solves the case with partial information and prepares the input matrix for the solution of the problem with complete information. This program takes around 11 SRU. Finally, an LP code from MPOS (Multi Purpose Optimization System) [66] is used to solve the horizon model. This program takes around 20 seconds to solve the problem. The first four programs were written in FTNTS (FORTRAN Time Sharing) and the SRU correspond to those estimated by the CDC System at the Georgia Institute of Technology.

## CHAPTER VI

### THE SIMULATION RESULTS AND ANALYSIS

This chapter contains the summary and analysis of the results obtained from the application of the model described in the previous chapter to several investment situations. Since the primary concern of this study is to estimate the effect that abandonment considerations have on the outcome of the capital rationing process when decisions are made on a sequential periodic basis, these investment situations provide a good sensitivity analysis.

The chapter begins with the definition of the four basic situations that the study covers. Then follows a brief description of the measure of effectiveness used to compare the results obtained with and without abandonment. Thereafter, the analysis of the results for each investment

situation is made, along with the sensitivity analysis of the parameter that controls the size of the abandonment values. The chapter concludes with a discussion about the results obtained using the Expected Values Abandonment Rule as compared to those obtained when the Suboptimal Abandonment Rule is used.

#### 6.1 Types of Investment Situations.

The simulation model described in Chapter V permits the generation of several different investment situations, which may be used to analyze the sensitivity of the abandonment rules and capital allocation criteria proposed in Chapters III and IV, respectively.

For this study it is of importance to visualize the effect of changing three elements of the simulation model:

1. The characteristics of the distribution of the proposal's growth rate. This distribution may be constant or variable. In the constant case, the fraction of proposals to be generated with a certain growth rate remains unchanged throughout the study period. In particular, the shape of the probability

distribution is that shown in Figure 5.3. For the variable case, it is assumed that the fraction of proposals with higher rates of growth increases through time. The time path followed by the distribution is that shown in Figure 5.4.

2. The class of expected cash flow patterns. The cash flow patterns of the SIP submitted in every period may be heterogeneous or single payment. By heterogeneous it is understood that the proposals may have increasing, decreasing, uniform or single payment series of cash flows. The fraction of each type of pattern is determined by the parameters Q1 and Q2 defined in Section 5.3.2.4.. The other case is that where all the proposals submitted have a single receipt at the end of their lives.
  
3. The class of expected abandonment values pattern. According to the type of pattern of the series of abandonment values, the sets of investment proposals may be classified as heterogeneous or exponential decay. In the heterogeneous case, the proposals may have a linear or exponential decay series of abandonment values. The fraction of each type of pattern is determined by the parameter P2 defined in Section 5.3.2.5.. For the exponential decay case,

the series of abandonment values of all the proposals follow an exponential decay function.

#### 6.1.1 The Investment Situations.

Combining the different possibilities provided by the elements discussed in the previous section, eight investment situations may be generated. To simulate each of these eight situations would certainly require a considerable amount of time. Furthermore, the results obtained for some situations could likely be inferred from those obtained for other situations. Therefore, this study will concentrate its attention on four of the most appealing situations. In the following sections they are referred to as Company A, Company B, Company C, and Company D.

Company A represents an investment situation where the shape of the distribution of the proposal's rate of growth remains constant through time, and the pattern of the series of cash flows and abandonment values are of the heterogeneous type.

Company B has heterogeneous series of cash flows and abandonment values, and a time varying distribution of the proposal's growth rate.

Company C represents a situation where the fraction of proposals with higher growth rates remains constant throughout the study period, the series of cash flows are heterogeneous, and the series of abandonment values follow an exponential decay function.

Company D corresponds to a situation similar to Company A but in this case the series of cash flows of all the proposals are of the single payment type.

In addition to the four basic investment situations described above, it is of interest to examine the effect of having the standard deviation of the cash flows and abandonment values computed as a function of the time elapsed between the forecasting and realization periods, the forecasting horizon. Therefore, a fifth investment situation similar to Company A, but including that feature is added to the study. This special case is identified as Company E.

The differences among the five investment situations are reflected in the next section, where the specific parameters for each company are defined.

## 6.1.2 Definition of the Parameters for the Investment Situations.

### 6.1.2.1. Company A - Parameters.

1. Constant distribution of the proposal's growth rate (Figure 5.3).
2. Discount rate (MARR) = 15%.
3.  $i = 6\%$  (see Section 5.4.1.).
4. Average rate of growth = 24% (see Section 5.3.1.3.).
5. Size of the external funds,

$$R(0) = \$ 30,000.0 \quad R(3) = \$ 15,734.6$$

$$R(1) = \$ 24,193.5 \quad R(4) = \$ 12,689.2$$

$$R(2) = \$ 19,510.9 \quad R(5) = \$ 10,233.2$$

6. Proposals life

$$L_{\min} = 4, L_{\text{av}} = 6, L_{\max} = 9$$

7. Probability of a particular cash flow pattern (see Section 5.3.2.4.)

- a. Probability of a single payment type of cash flow,

$$Q1 = 0.30$$

- b. Probability of a series payment type of cash flow,

$$Q1' = 1 - Q1 = 0.70$$

If the proposal is a series payment type of cash flow

- i. Probability of a decreasing series

$$Q2 = 0.60$$

ii. Probability of an increasing series

$$Q2' = 1 - Q2 = 0.40$$

8. The fraction of the final cash flow corresponding to salvage value (see Section 5.3.2.5),

$$P1 = 0.50$$

9. Probability of a particular abandonment values pattern (see Section 5.3.2.5)

a. Probability of a linear decay function,

$$P2 = 0.40$$

b. Probability of an exponential decay function,

$$1 - P2 = 0.60$$

10. The probability distribution of the magnitude of the cash flows and abandonment values is Normal with the standard deviation computed as a constant fraction of the expected values (see Section 5.3.2.6.),

$$b1 = b2 = 0.15$$

11. Number of proposals per period = 15

12. Number of decisions times (H) = 20

#### 6.1.2.2. Company B - Parameters.

1. Time varying distribution of the proposals growth rate, (see Figure 5.4 and Section 5.3.2.3.),

$$g(t) = 0.0131 + 0.0469 e$$

4. Average rate of growth = 27% (see Section 5.3.1.3.)

5. Size of the external funds (see Section 5.7.1.)

$$R(0) = \$ 30,000.0 \quad R(3) = \$ 14,645.7$$

$$R(1) = \$ 23,622.0 \quad R(4) = \$ 11,532.0$$

$$R(2) = \$ 18,600.0 \quad R(5) = \$ 9,080.4$$

All other parameters are the same as for Company A.

6.1.2.3. Company C - Parameters.

9. Probability of a particular abandonment values pattern (see Section 5.3.2.5.),

a. Probability of a linear decay function,

$$P2 = 0.0$$

b. Probability of an exponential decay function,

$$1 - P2 = 1.0$$

All other parameters are the same as in Company A.

6.1.2.4. Company D - Parameters.

7. a. Probability of a single payment cash flow,

$$Q1 = 1.0$$

8. Fraction of the final cash flow corresponding to salvage value,

$$F1 = 1.0$$

All other parameters are the same as in Company A.

#### 6.1.2.5. Company E - Parameters.

10. The standard deviation is computed as a time varying fraction of the expected values (see Section 5.3.2.6.),

$$b_1 = b_2 = 0.15$$

$$f_1(t) = f_2(t) = t^{1/2}$$

All other parameters are the same as in Company A.

## 6.2 Risk-Return Analysis as a Measure of Effectiveness.

### 6.2.1 The Efficiency Concept.

In practice, most managements prefer to have investment policies that both maximize financial results over the long run and minimize uncertainty or risk. Seeking for additional returns, however, normally entails accepting additional risk [30]. Thus, the decision-maker's task is to find those investment policies that provide maximum return for a given level of risk, or minimum risk for a given level of return.

If two policies produce the same average outcome, the one that involves less risk (variability) is a more desirable or "efficient" policy. On the other hand, of two policies entailing the same variability, the one producing the higher expected outcome is a better policy.

In this study, by simulating the financial results of investments selected on the basis of a particular policy (use of a particular risk parameter), the expected value and standard deviation (measure of variability or dispersion) of the Horizon Value realized in each case indicates the "efficiency" of the sets of investment proposals selected under that policy [31]. The expected Horizon Value and its standard deviation can be plotted on a graph to show the effectiveness of any policy, and a line can then be drawn through the efficient points. This line is called the "efficiency frontier".

As pointed out by Park [51], an important characteristic of the efficiency frontier generated by simulation is its stochastic nature. Since the computation of the expected values and standard deviation of the Horizon Value is based on a fixed sample size, as the size of that sample changes the efficiency frontier is likely to change. Thus, using basic sampling concepts and information about the sample size, it is possible to determine the **confidence** limit

for the efficiency frontier (see Figure 6.1).

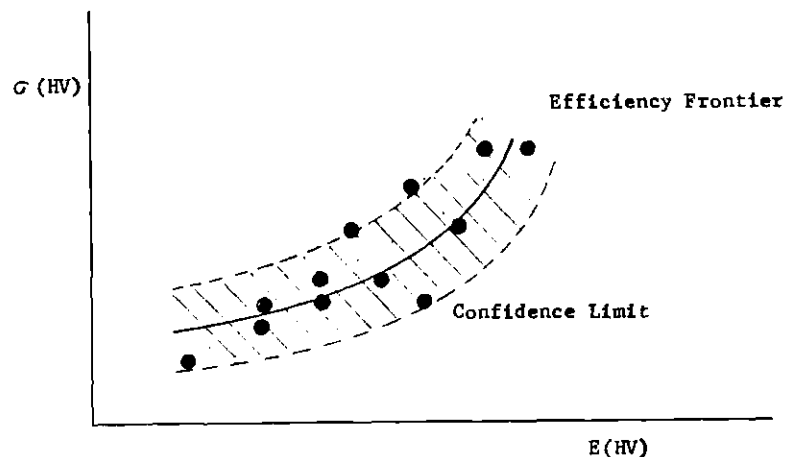


Figure 6.1. The Efficiency Frontier.

#### 6.2.2 The Risk Aversion Parameters.

Due to the fact that each decision-maker may weigh risk and return differently, this study does not attempt to specify the exact preference of every individual. Rather an attempt is made to find the efficiency frontier generated by each combination of abandonment rule and allocation criteria. Thus providing a measure to test the effectiveness of each approach.

Since the outcome of the proposals is assumed to be

normally distributed, the efficiency frontiers are generated using risk parameters that range from 0. to 3.0 (see Section 3.4.1.3). In the simulation, the same risk parameter ( $k$ ) is used in the EGCL Abandonment Rule, the Mean Variance Criterion, and the Project Balance Criterion.

### 6.2.3 Dominance.

To illustrate the concept of dominance, Figure 6.2 depicts the efficiency frontiers generated by two different approaches to the evaluation of investment proposal. In that Figure it is said that the efficiency frontier that runs from A to B is dominated by the one that runs from C to D. This implies that the approach used to generate the CD efficiency frontier can provide higher expected Horizon Values (E[HV]), without increasing  $\sigma$  [HV], the standard deviation of the Horizon Value (used as a measure of variability). Thus, the latter approach would be preferred from a risk-return point of view.

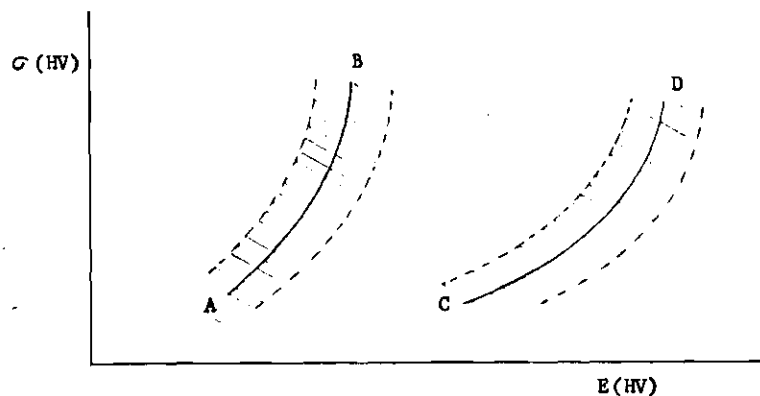


Figure 6.2. Dominance Illustration.

### 6.3 Summary and Analysis of the Simulation Results.

Following the outline given at the beginning of the Chapter, this section presents the results obtained for each of the investment situations described in Section 6.1.1.. In each case a comparison is made between the results obtained with and without abandonment, when different combinations of abandonment rules and capital rationing criteria are used. In addition, the results contain an analysis of the effect of abandonment considerations on the accumulated wealth of the firm when partial and complete information is available.

### 6.3.1 Company A.

This Company represent an investments situation where the distribution of the proposals growth rate remains constant throughout the study period. It is also a situation where the series of cash flows and abandonment values are of the heterogeneous type.

The results for this Company are plotted in Figures 6.3., 6.4., and 6.5., and the detailed statistics are summarized in Table A-1, in Appendix A.

In Figure 6.3., the dashed line represents the efficiency frontier generated by the Mean Variance Criterion when abandonment is disregarded. The solid line is the efficiency frontier generated by the same criterion but including abandonment. For the no-abandonment case the largest expected Horizon Value,  $\$ 8243.1 * 10^3$ , is obtained at  $k=0.$ , which simply maximizes the expected Present worth of investments. As the value of  $k$  increases, the ECHV] gradually decreases. This decreasing trend in ECHV] is a consequence of the fact that a smaller number of proposals meet the requirements set by the criterion and, consequently, larger amounts of funds are invested at  $i$ , a low interest rate in a highly liquid investment. For values of  $k$  larger than 3.0 the

number of proposals accepted gradually diminishes, and the expected Horizon Value approaches a minimum of \$ 324,233, which is equivalent to invest all funds available during the study period at  $i$ . Certainly this amount has no variability.

Comparing the results obtained with and without abandonment it is seen that the ECHV] are consistently higher when abandonment is taken into account. However, the importance of the abandonment considerations diminishes significantly as the value of the risk parameter increases. This is due primarily to the effect of  $k$  in the abandonment decision. As  $k$  increases, most of the original proposals are not only modified but also discarded. In fact, most of the proposals screened out in the abandonment phase are the same rejected by the capital rationing criterion when abandonment is disregarded. Thus leading to insignificant differences in the results obtained in each case.

The standard deviation of the Horizon Value is somewhat higher when abandonment is considered. Apparently this contradicts the results obtained by Robichek and Van Horne [57], who reported lower variability when abandonment is taken into account. However, this contradiction is only apparent because the difference in variability is found to be not significant, with a degree of confidence of 0.95 (F-ratio test). Therefore, although there is a slight increase in the

variability of the Horizon Value, there is not enough evidence to state that they are different. In fact, Figure 6.3 could be modified to show that there is no intersection of the efficiency frontiers (intersection means no strict dominance) but that both tend to a common curve as  $k$  sets large.

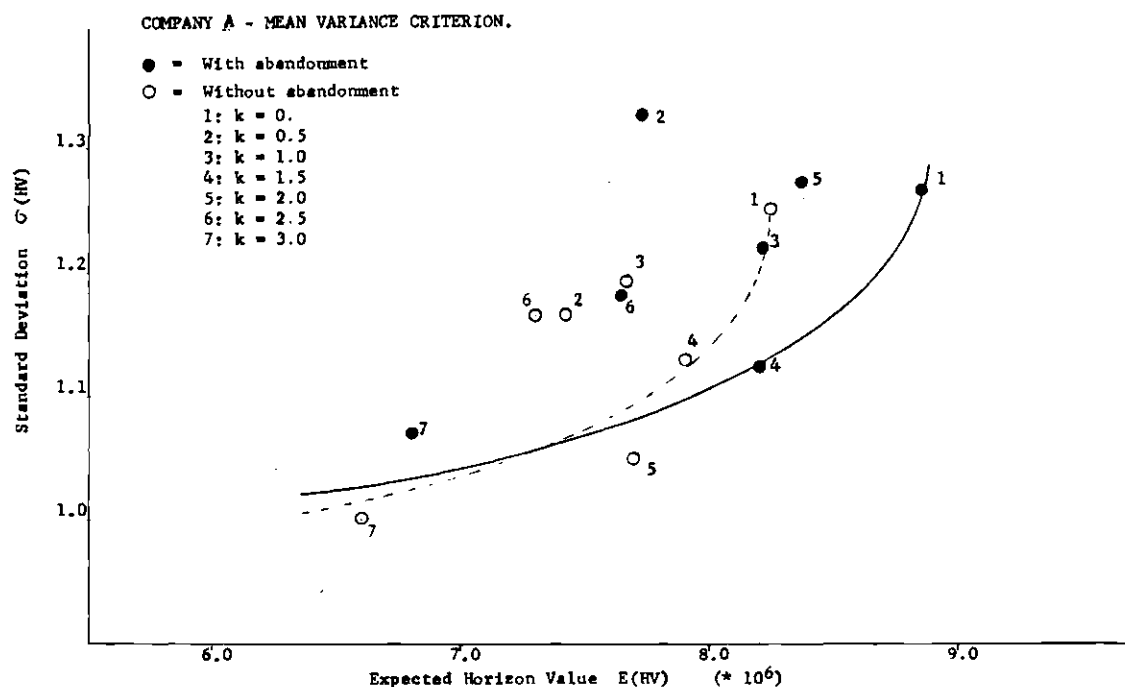


Figure 6.3. Risk-Return Chart, Company A, M-V Criterion.

Figure 6.3 also contains the statistics corresponding to the solution of the capital budgeting problem for a risk indifferent decision-maker ( $k=0.$ ). ●<sup>1</sup> represents the solution obtained using the Expected Values Abandonment Rule

and the Expected Present Worth Maximization Criterion, while  $O^1$  represents the solution obtained by this criterion when abandonment is disregarded. These statistics show that as a consequence of including the abandonment option in the decision making process there is an increment in the E[HV] of 7.5%. This improvement is statistically significant with a degree of confidence of 0.9 (paired t-test).

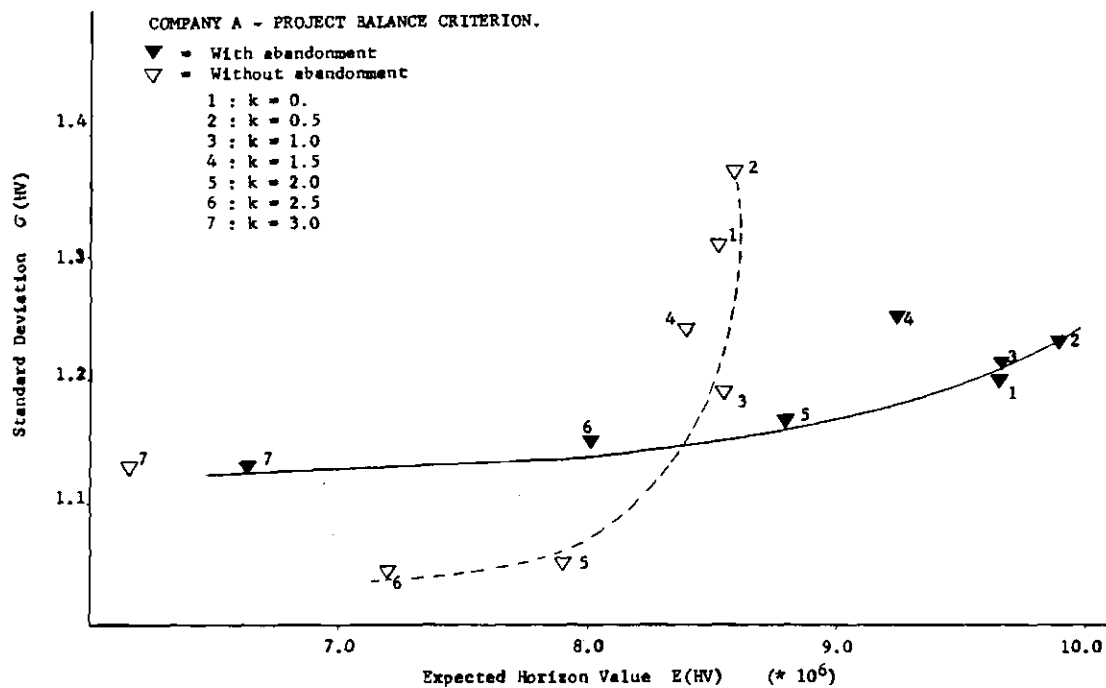


Figure 6.4. Risk-Return Chart, Company A, P. B. Criterion.

For the Project Balance Criterion, the results generated are similar to those reported above. In Figure 6.4, the dashed line that runs from  $\nabla^2$  to  $\nabla^6$  represents the

efficiency frontier for the no-abandonment case, and the solid line corresponds to the efficiency frontier for the results with abandonment. The maximum EHVJ for the latter case is realized at  $k=0.5$  and amounts to  $\$ 9888.4 \times 10^3$ , which is 15% higher than the maximum attained in the no-abandonment case.

Comparing the results for the Mean Variance Criterion (Figure 6.3.) to the results for the Project Balance Criterion (Figure 6.4.) it is seen that the relative improvement in EHVJ caused by the integration of abandonment considerations is larger for the latter criterion. This difference is statistically significant with a degree of certainty of .95 (paired t-test) for values of  $k$  less than 3.0. Since the Project Balance Criterion takes into account the resolution of the proposals uncertainty, and the integration of abandonment into the analysis of investments is a crude way of reducing that uncertainty, it was expected that the effect in the EHVJ would be larger when the Mean Variance Criterion is used. The results obtained show that in the average, the improvements made by the Mean Variance Criterion is 5.7%. If this average is compared to the 11.3% observed for the Project Balance Criterion, it may well be concluded that the above expectation was incorrect. However, as it will be seen later in Section 6.3.6, the relative importance of the abandonment option when the Mean Variance Criterion is used, as compared to the relative importance of that option when the

Project Balance Criterion is employed, is a function of  $P_1$ , the parameter that controls the size of the abandonment values. And, it turns out that for the value of  $P_1$  used in this study, the difference between the improvement made by both criteria reaches a maximum. In fact, for larger or smaller values of  $P_1$  the difference between the relative importance of the abandonment option when either criterion is used diminishes significantly.

It is also of interest to observe that for the Project Balance Criterion there is an apparent reduction in the variability of the Horizon Value when abandonment is taken into account. However, again this reduction is not statistically significant.

When partial and complete information is available, the integration of the abandonment option into the decision making process has dramatic consequences. In both cases, the EHVJ increases substantially (31% and 40% respectively), and in the case with complete information the reduction in variability is statistically significant with a degree of confidence of 0.95 (F-ratio test).

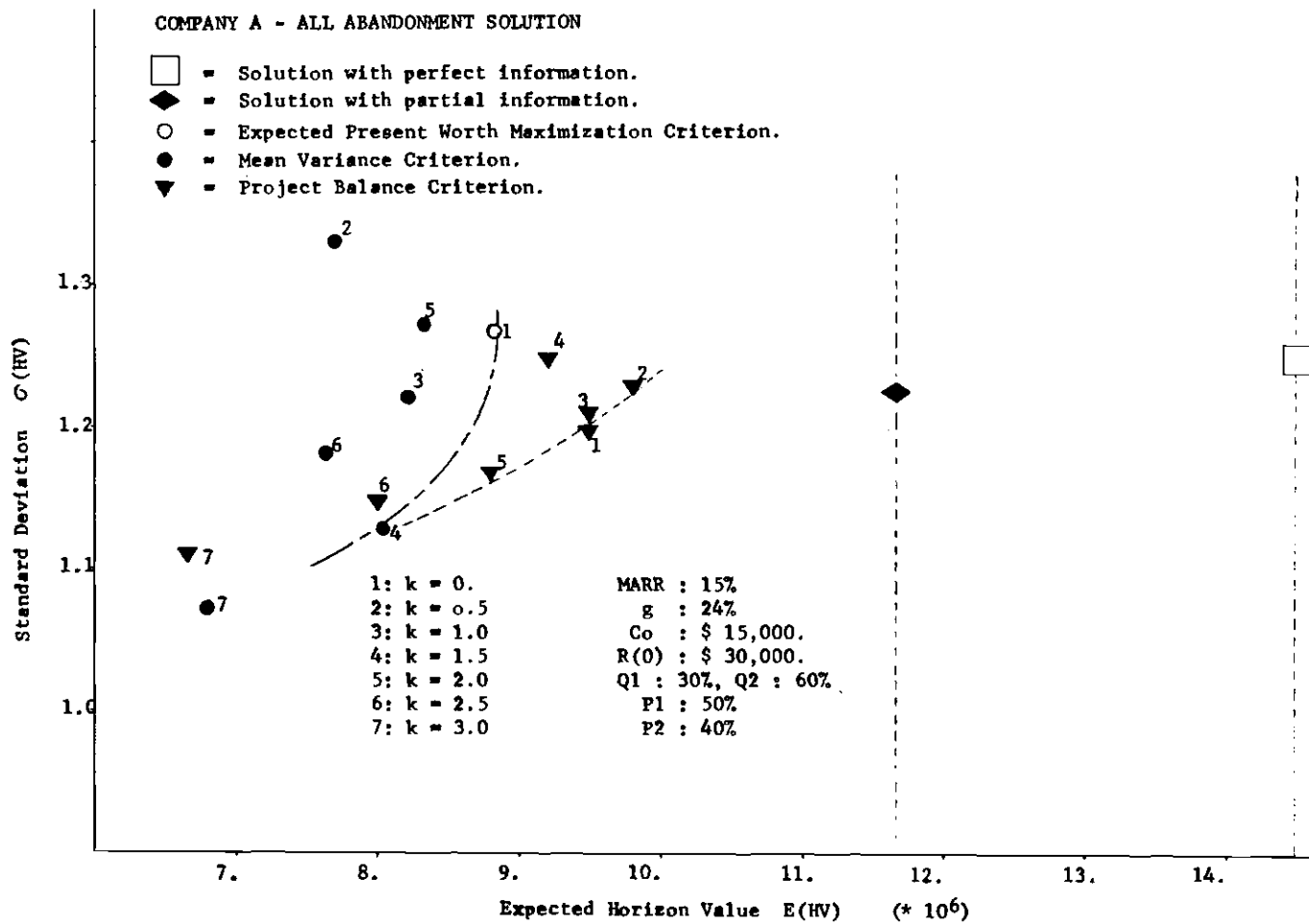


Figure 6.5. Risk-Return Chart, Company A, All Abandonment Solution.

Finally, Figure 6.5 depicts the results obtained by the different capital rationing criterion when the evaluation of investment proposals considers the abandonment option. Confirming Park's results [51], it is seen that for practical values of  $k$  (from 0. to 3.0), the efficiency frontier generated by the Project Balance Criterion dominates the efficiency frontier obtained by the Mean Variance Criterion (statistically significant in terms of ECHVJ with a degree of confidence of 95%). In addition, the maximum ECHVJ obtained by the Project Balance Criterion (realized at  $k=0.5$ ) is almost 12% higher than the maximum achieved by the Mean Variance Criterion.

When partial information is available, and abandonment is disregarded, the local optimum for the process has an ECHVJ of \$  $8528.5 \times 10^3$ . This amount is less than the maximum attained by the Project Balance Criterion with no abandonment. Therefore, when abandonment is disregarded, the availability of partial information is not important. However, when abandonment is considered, the local optimum reaches an ECHVJ of \$  $11165.4 \times 10^3$ , which is 13% higher than the maximum obtained by the Project Balance Criterion under similar circumstances.

The solution to the problem with complete information is denoted in Figure 6.5 by a **rectangle** (  $\square$  ). As it was

expected, in this case the expected Horizon Value is considerably higher than that obtained by any other criterion. However, it should be remembered that this solution was obtained using a linear programming formulation.

#### 6.3.2 Company B.

Another investment situation which is of interest for this study is that where the distribution of the Proposal's growth rate changes through time. Specifically, the situation analyzed is one where the fraction of Proposals with higher growth rates increases as time approaches the end of the planning horizon.

In order to be able to compare the results obtained in this case with those of Company A, all parameters have remained the same. The only change has been to shift from a constant distribution of investment opportunities to a time varying distribution as that shown in Figure 5.4.

The results generated for this investment situation are plotted in Figures 6.6, 6.7, and 6.8, and a more detailed summary of the statistics may be found in Table A-2 an Appendix A.

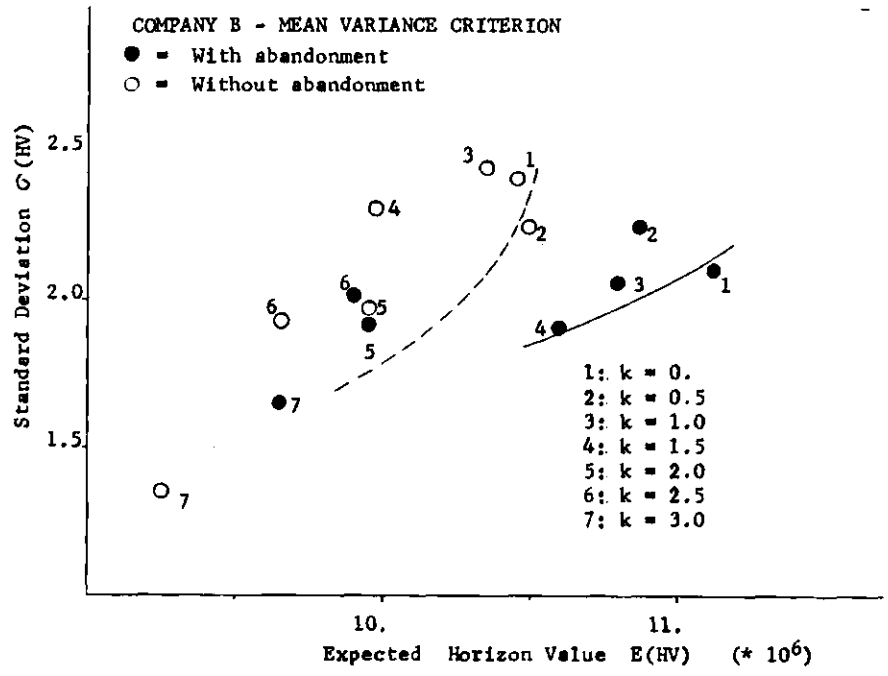


Figure 6.6. Risk-Return Chart, Company B, M-V Criterion.

The statistics for the Mean Variance Criterion (Figure 6.6) show that for this investment situation, the abandonment option also exerts an influence in the outcome of the capital rationing process. Again here, as in Company A, the efficiency frontier generated when abandonment is disregarded, which in Figure 6.6 is represented by the dashed line, is dominated by the efficiency frontier generated when abandonment is taken into account. The largest expected Horizon Value for

the case with abandonment is verified at  $k=0$ , and amounts to  $\$ 11134.9 \times 10^3$ . This quantity is 6% higher than the maximum obtained when abandonment considerations are disregarded.

Figure 6.6 also shows that when abandonment is taken into account, there is a slight reduction in the variability of the Horizon Value. However, a hypothesis supporting a statement of this nature is rejected with a .95 degree of certainty.

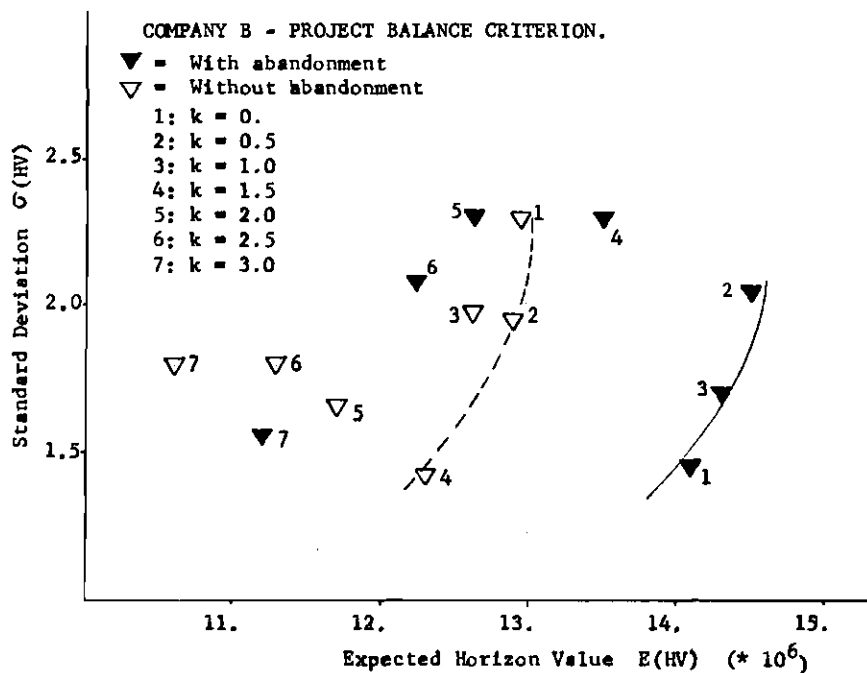


Figure 6.7. Risk-Return Chart, Company B, P. B. Criterion.

For the Project Balance Criterion, the statistics plotted in Figure 6.7 show that again in this case, the abandonment option produces a much larger effect than that observed for the Mean Variance Criterion. Especially noticeable is the difference between the values obtained for  $k=0$ , which does not only present a statistically significant difference in expectation but also a significant reduction in the variability of the Horizon Value. Both hypothesis are accepted with degrees of confidence of .95 and .99, respectively.

When partial and complete information is available, the effect is also similar to that observed in Company A. However, in the present investment situation, the reduction in variability is not significant for either level of knowledge. With partial information the ECHV obtained when abandonment is considered is  $\$ 14683.7 * 10^3$ . This amount is 25% higher than the ECHV obtained without abandonment, a difference which is statistically significant with a degree of confidence of .95. With complete information, the ECHV is almost 28% larger when abandonment considerations



Finally, Figure 6.8 shows that the efficiency frontier generated by the Project Balance Criterion dominates the efficiency frontier obtained with the Mean Variance Criterion (Expected Present Worth Maximization included). The reason the Project Balance Criterion performs so well in this type of investment settings is that it recognizes the advantages of keeping funds available for investment in attractive proposals that may be obtained in the future. Since in this case the fraction of proposals with higher growth rates keeps growing through time, the importance of making such considerations raises significantly.

When the results plotted in Figure 6.8 are compared to the results of Company A (Figure 6.5), it may be seen that the general tendency in the results generated by each criterion is to show an increase in the  $E[HV]$  and  $\sigma[HV]$ . Only the solution for the problem with partial information does not follow the general tendency and, although it shows an increase in  $E[HV]$ , the variability remains almost unchanged.

### 6.3.3 Company C.

This investment situation corresponds to the case where all the proposals show exponential decay series of abandonment

values. When this is the case, it may be observed from the data summarized in Table A-3 in Appendix A that the abandonment option has no effect in the outcome of the capital allocation process. In fact, only the results obtained for the case with partial and complete information are statistically significant with a degree of certainty of .95 (paired t-test).

Since the equation used in the generation of the abandonment values was obtained by Eilon et al. [19] from empirical evidence on lift trucks (see Section 5.3.2.5.), the results obtained for this investment situation are important because they show that a decision-maker dealing with proposals whose abandonment values show this behavior, does not stand a better chance by integrating the abandonment option into the analysis of investments. Although he should be aware of the possibility of abandoning the proposals that have already been undertaken, it is not critical to consider abandonment in the process of evaluating the proposals.

#### 6.3.4 Company D.

In reality most firms are confronted by sets of investment proposals which are defined in this study as

heterogeneous. That is, the SIFs submitted in every period contain a combination of proposals with several receipts during their lives, and also proposals with a single or scarce receipts. Thus, an investment situation as that represented by Company D, where all the proposals have a single receipt or payment at the end of their lives, is rather unusual. However, it is useful to test the effect of the abandonment option under extreme conditions. The results obtained for this investment situation are plotted in Figures 6.9, 6.10, and 6.11, and a detailed summary is provided in Table A-4 in Appendix A. In the first place, it should be mentioned that in this case the consideration of the abandonment option is only relevant for values of  $P_1$  (the parameter that controls the size of the abandonment values) greater than or equal to 0.9 (paired t-test with a degree of confidence of .95). The results reported have been generated using  $P_1=1.0$ . That means that the final and unique cash flow is assumed to be entirely the salvage value. It is recognized that this assumption cannot be completely justified, but has been made in order to arrive at meaningful results.

When an expected value approach is used in the appraisal of the investment proposals ( $O^1$  and  $\bullet^1$  in Figure 6.9), the effect of the abandonment option in the outcome of the capital budgeting process is to increase the expectation and the variability of the Horizon Value. The increase in

$E[HV]$  is approximately a 9%, a difference which is statistically significant with a degree of certainty of at least .95 (paired t-test). As in previous situations, the increase in variability is not significant (F-ratio test).

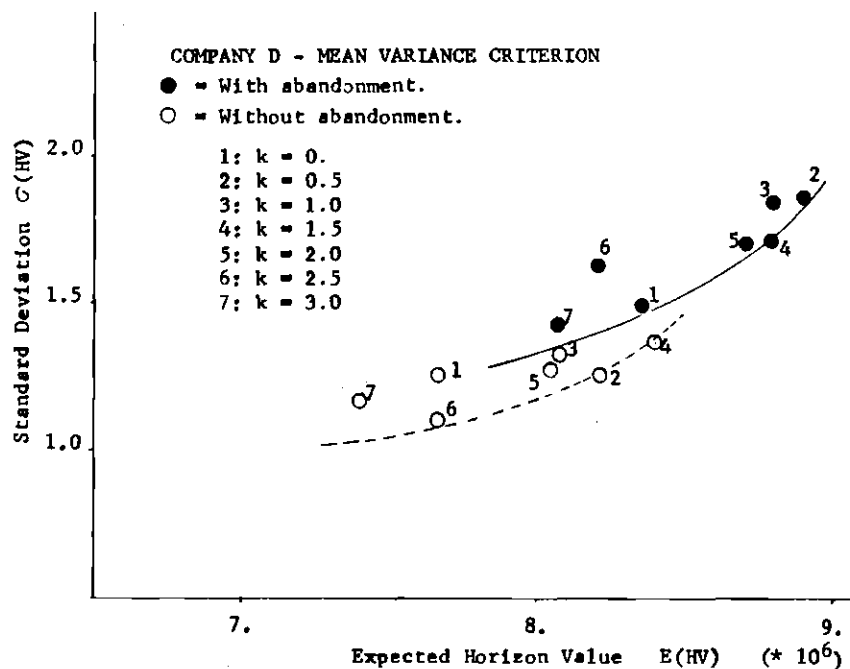


Figure 6.9. Risk-Return Chart, Company D, M-V Criterion.

When risk is taken into account in making the abandonment and allocation decisions, the results obtained with the Mean Variance and Project Balance Criteria are basically the same as those reported for Company A. There is a significant increase in the  $E[HV]$  accompanied by no significant increases in variability. In Figure 6.9 it may be observed that for the Mean Variance Criterion, the efficiency

frontier generated when abandonment is disregarded (dashed line) is not strictly dominated by the efficiency frontier generated when abandonment is considered. However, for most values of  $k$  the increase in  $E[HV]$  is more than enough to counterbalance the negative effects of an increase in  $G[HV]$ . For the Project Balance Criterion, a decreasing trend in the difference in variability between the results with and without abandonment, for increasing values of  $k$ , accompanied by significant differences in expectation for all values of the risk parameter, determine that the efficiency frontier for the no abandonment case (dashed line in Figure 6.10) be dominated by the efficiency frontier for the results with abandonment.

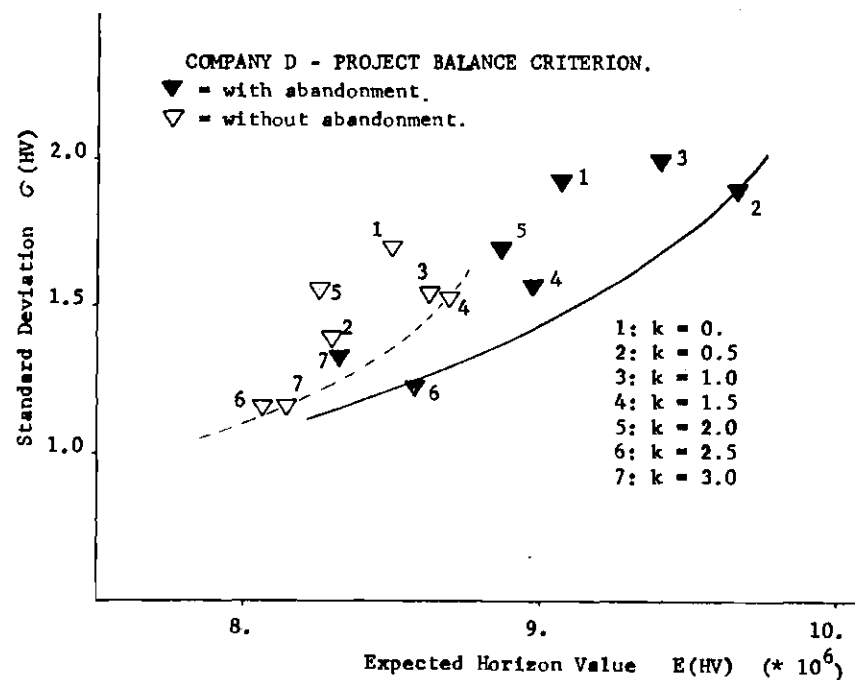


Figure 6.10. Risk-Return Chart, Company D, P. B. Criterion.



An important result obtained from this investment situation is related to the relative improvement in ECHV] made by the Mean Variance and Project Balance Criteria, as a consequence of introducing abandonment considerations into the decision process. Confirming the statement made in the analysis of a similar situation in Company A, for large values of P1 (1.0 in this case), the improvement made by both criteria is not significantly different. On the average, the increase in ECHV] made by the Mean Variance Criterion is 7.9%, and by the Project Balance Criterion is 7.5%. Although the relative improvement is lower for the latter criterion, the difference is not statistically significant (paired t-test) with a degree of confidence of 0.9.

Finally, Figure 6.11 shows that for this investment situation, the availability of partial and complete information exerts a considerable influence on the outcome of the capital rationing process. When partial information may be obtained, and abandonment is taken into account, the ECHV] generated ( $\$ 18297.9 \times 10^3$ ) is 89.4% higher than the ECHV] realized when abandonment is disregarded. This increase in expectation is accompanied by a significant increase in variability (F-ratio test with a degree of confidence of .95). When complete information is available, the abandonment option rises the ECHV] from  $\$13432.4 \times 10^3$  to  $\$26692.3 \times 10^3$  (almost 99% increase) and the  $\sigma$ [HV] from  $\$2323.1 \times 10^3$  to  $\$3412.1 \times 10^3$ .

The difference the E[HV] is statistically significant with a degree of confidence of at least .95 (paired t-test), but the difference between the  $\sigma$  [HV] is not significant (F-ratio test).

#### 6.3.5 Company E.

In Section 5.3.2.6., two approaches were proposed for the generation of the standard deviation of the cash flows and abandonment values. One approach, used so far, assumes that the standard deviation is directly proportional to the size of the expected value. Thus, the following equation was used in the simulation,

$$\sigma [X] = b E[X]$$

where b is a constant parameter.

The second approach assumes that the standard deviation is also a function of the time elapsed between the forecasting period and the realization period, the forecasting horizon. If this is the case, the equation proposed for the calculation of the standard deviations is,

$$\sigma [X] = b f(t) E[X]$$

where f(t) is a continuous function of the forecasting horizon. Assuming that a greater variability will accompany forecasts

made for periods further in the future,  $f(t)$  must be an increasing function of  $t$ . In addition, the rate at which  $C[X]$  changes may either increase, decrease or remain constant through time. That behavior depends entirely on the situation to be simulated. In this study it has been assumed that the rate of change will decrease through time. Thus, the following equation is used,

$$f(t) = t^{1/2}$$

which follows the path shown in Figure 6.12.

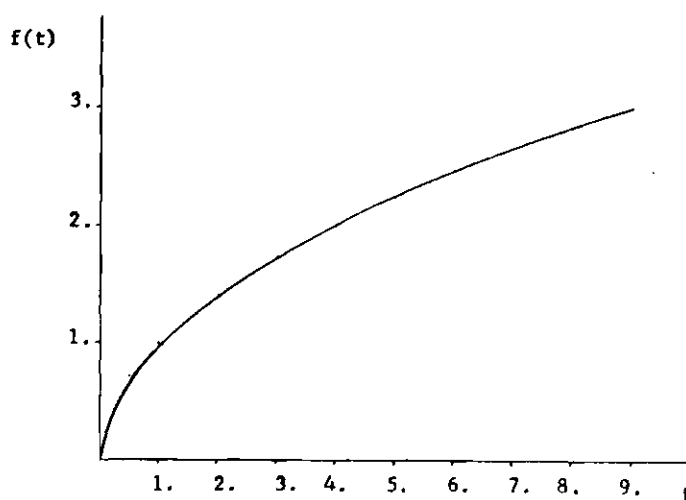


Figure 6.12. Function  $f(t) = t^{1/2}$

The results, plotted in Figures 6.13, 6.14, and 6.15, and summarized in Table A-5 in Appendix A, were obtained for an investment situation equivalent to that represented by Company A with the minor modification proposed above for the generation of the standard deviation of the cash flows and abandonment values.

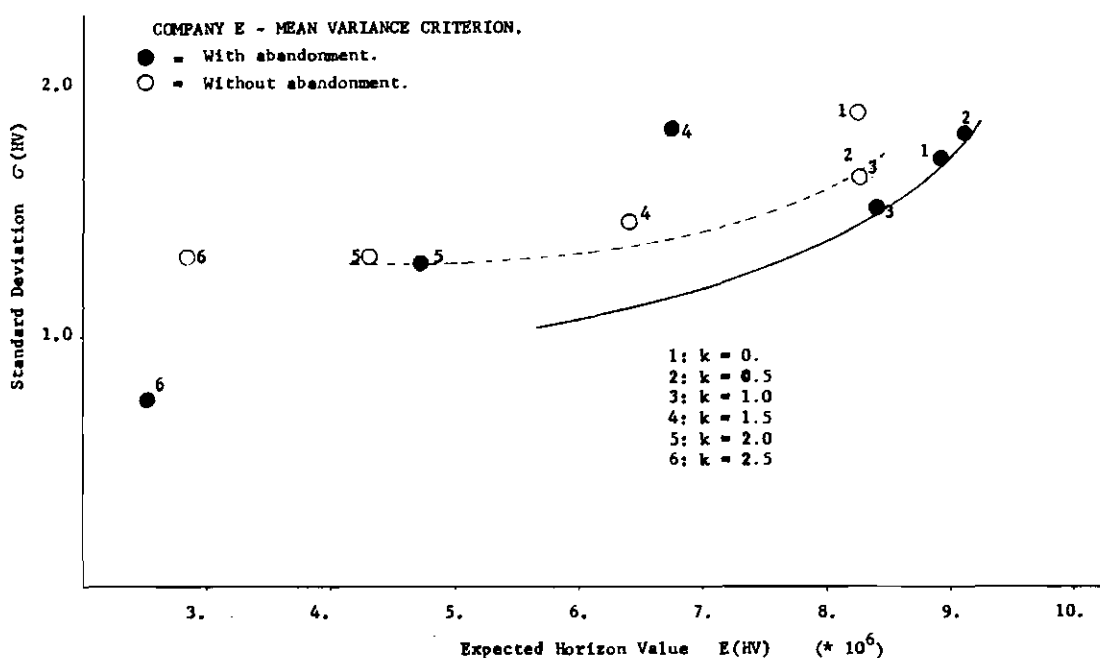


Figure 6.13 Risk-Return Chart, Company E, M-V Criterion.

When Figures 6.13 and 6.14 are compared to the corresponding figures (6.3 and 6.4) of Company A, it is seen that in this case, the effect of considering the abandonment option is only significant for the low values of  $k$ . When the Mean Variance Criterion is used to make the capital allocation, the difference between the expected Horizon Value obtained with and without abandonment is only statistically significant for values of  $k$  less than or equal to 0.5. What this says is that the integration of the abandonment option is only significant for decision-makers which are basically risk indifferent. For the Project Balance Criterion the same

difference is only significant for values of  $k$  larger than or equal to 1.0.

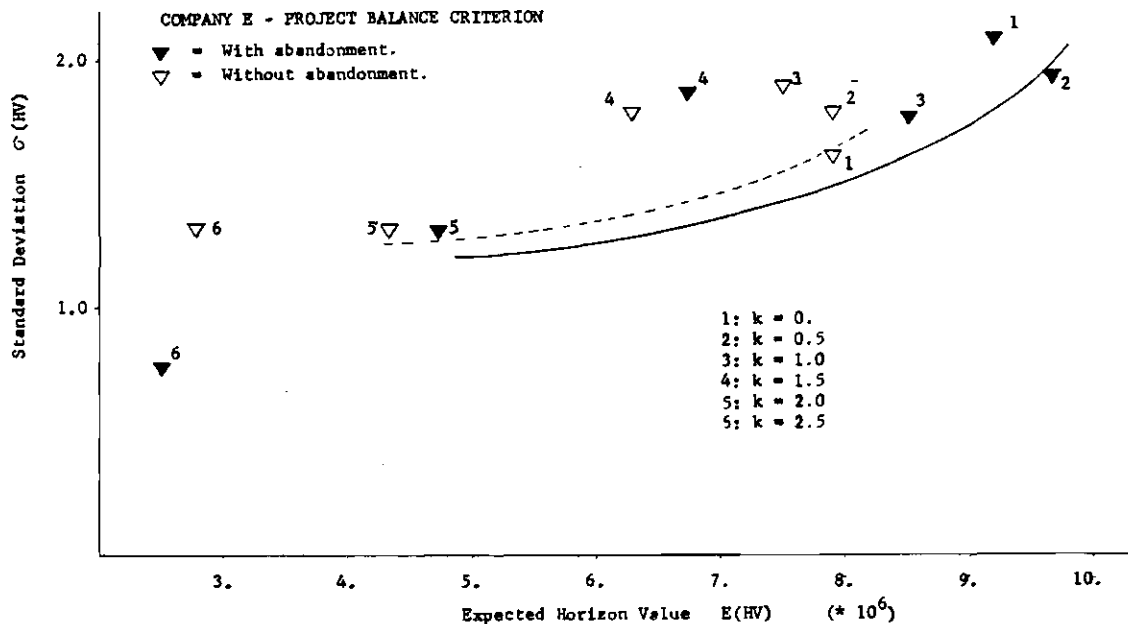


Figure 6.14. Risk-Return Chart, Company E, P. B. Criterion.

The change observed in these results, as compared to those of Company A is due primarily to the increase in the variability of the proposals. Since the standard deviations are no longer a fraction of the size of the expected values, the expected Horizon Value becomes extremely sensitive to changes in the risk parameter and, for even moderate values of  $k$ , most of the proposals are not only modified but also discarded from further consideration during the abandonment analysis phase.

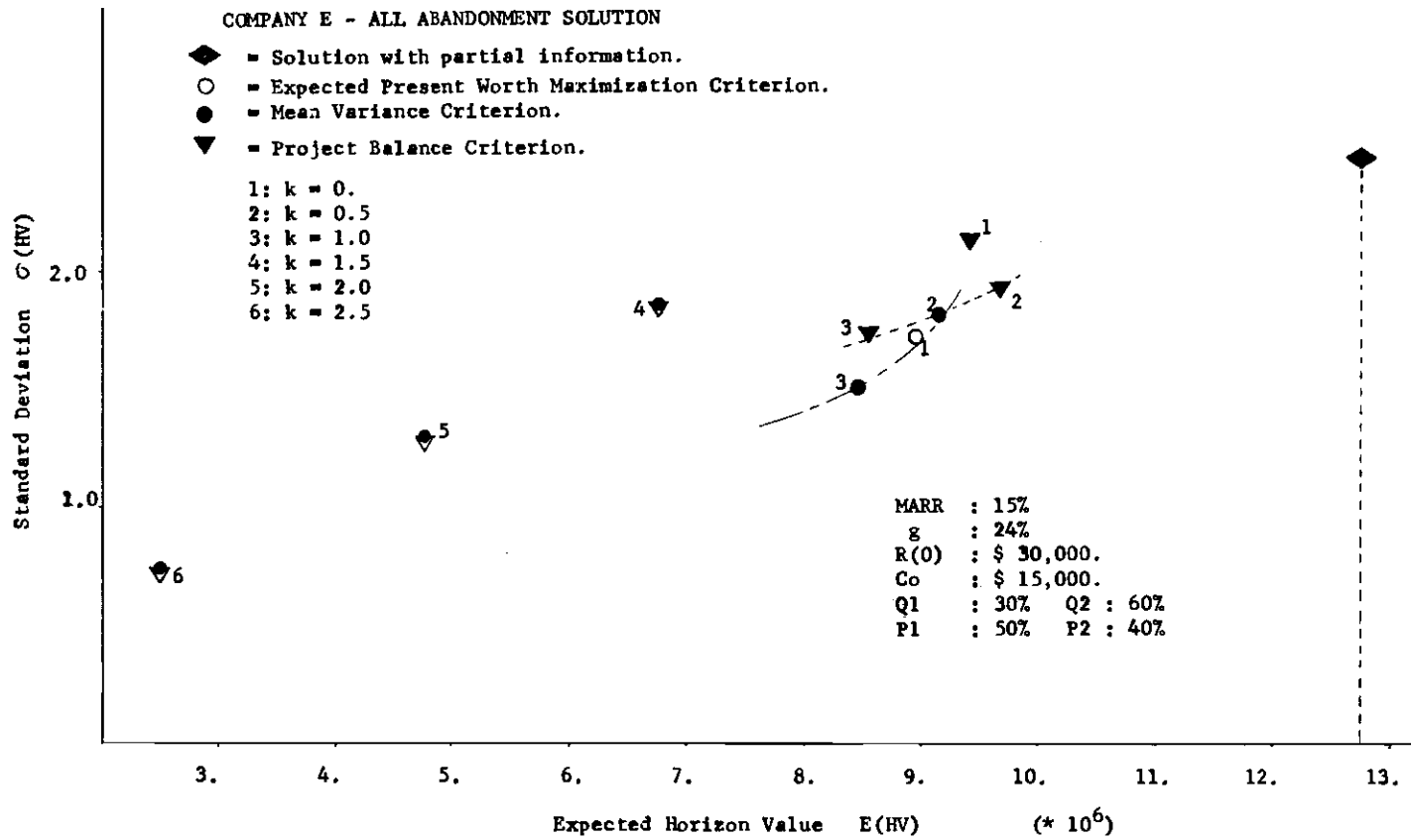


Figure 6.15. Risk-Return Chart, Company E, All Abandonment Solution.

Another interesting result obtained from this investment situation may be visualized in Figure 6.15. There it is seen that when abandonment consideration are made, the Project Balance Criterion performs better than the Mean Variance Criterion (Expected Present Worth Maximization included) only for values of  $k$  less than or equal to 0.5. These results show that when the variability of the investment proposals is a dominant factor in the decision process, the importance of taking into account the proposals uncertainty resolution diminishes substantially. In fact, when abandonment is disregarded, the difference between the expected Horizon Value obtained by the Mean Variance Criterion and Project Balance Criterion is not statistically significant for any value of the risk parameter (paired t-test with at least a .95 degree of confidence).

In Figure 6.15 the maximum ECHV] for the Project Balance Criterion occurs at  $k=0.5$  ( $\nabla^2$ ) and this value is statistically significant over the maximum ECHV] achieved by the Mean Variance Criterion ( $\bullet^2$ ) and by the Expected Present Worth Maximization Criterion ( $\circ^1$ ), with a degree of confidence of .95. Figure 6.15 also shows that in this situation the availability of partial knowledge about the outcome of the proposals is of great importance. The ECHV] obtained when such information is available ( $\blacklozenge$  in Figure 6.15) is almost 33% higher than the maximum achieved by the

Project Balance Criterion.

#### 6.3.6 The effect of changing $P_1$ .

The value of  $P_1$ , the parameter that controls the size of the abandonment values, is a factor that plays an important role in determining the improvements that can be made in capital investment decisions through abandonment considerations. Since the value of  $P_1$  used in the simulation of the results generated for most of the investment situations has remained constant at  $P_1=0.5$ , it is of interest to establish the effect that a change in  $P_1$  would have on the results.

This section contains an analysis of the results generated by different investment criteria for several values of  $P_1$ . In particular, six values of  $P_1$  ranging from 0.2 to 1.0 are selected to compute the E[HV] statistics associated with each decision criterion. Since the Project Balance Criterion, the Mean Variance Criterion, and the EGCL Abandonment Rule require the specification of the risk aversion coefficient, the value of  $k$  is set at 1.0. The rest of the parameters take the same values as in Company A.

The statistics obtained from varying the value of  $P_1$  for each decision criterion are summarized in Table A-6 in Appendix A. Figure 6.16 illustrates the relative improvements made by each criterion, as a consequence of considering the abandonment option. Before continuing with the analysis, it should be noted that in Figure 6.16, the standard against which the results for each criterion are compared is not the same. The basis of comparison for the results obtained by each combination of abandonment rule and capital rationing criterion are the results obtained by the corresponding capital rationing criterion when abandonment is disregarded.

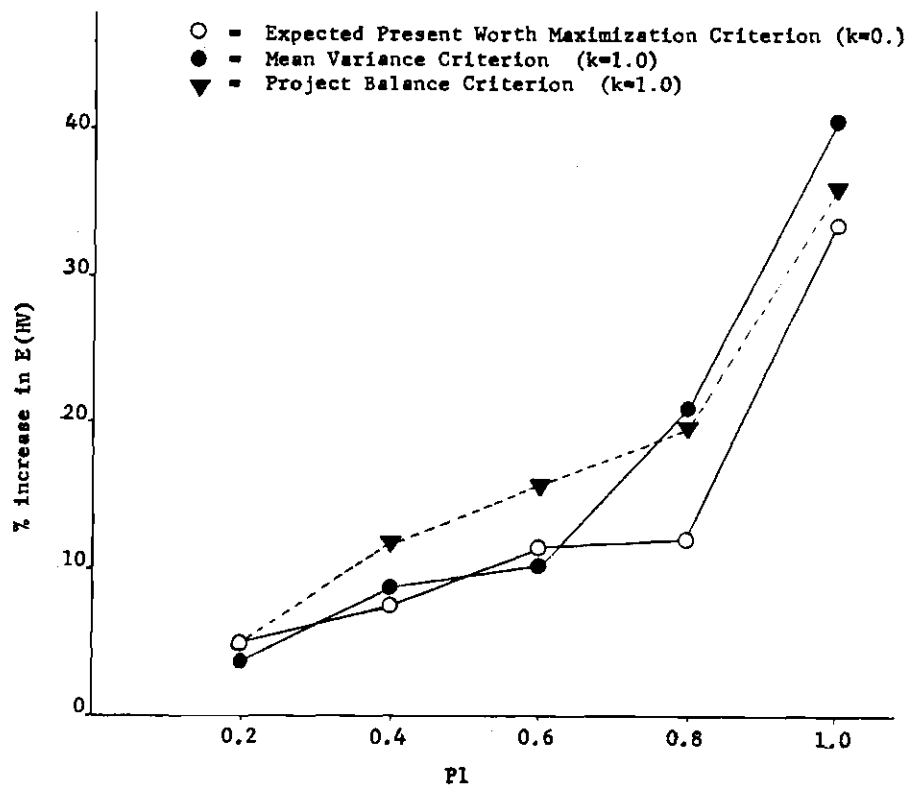


Figure 6.16. Effects in ECHVJ of changes in  $P_1$ .

The ECHVJ statistics obtained with the Expected Values Abandonment Rule and the Expected Present Worth Maximization Criterion( O ) show that the integration of abandonment is only significant for values of P1 greater than or equal to 0.4 (paired t-test with a degree of confidence of at least .95). The improvement made by this combination of abandonment rule and capital rationing criterion is rather unresponsive for values of P1 up to 0.8. From that point on, the difference between the results obtained with and without abandonment increases significantly. The maximum improvement in ECHVJ is attained at P1=1.0 and is almost 34%.

For the Mean Variance Criterion and the EGCL Abandonment Rule, the improvement in ECHVJ increases steadily but at a low rate for values of P1 smaller than 0.6. For larger values of P1, the effect of the abandonment option becomes very sensitive to changes in that parameter. The maximum improvement achieved by this combination of abandonment rule and capital rationing criterion is realized at P1=1.0 and amounts to approximately a 41%.

When the Project Balance Criterion and the EGCL Abandonment Rule are used, the improvements made in ECHVJ as compared to the no abandonment solution shows a fairly constant increasing trend for increasing values of P1. Again in this case, the maximum improvement is realized at P1=1.0,

and is almost 36%.

During the analysis of the results of Company A it was observed that, since the Project Balance Criterion takes into account the resolution of the proposals uncertainty, and the integration of abandonment considerations into the analysis of investments is a crude way of reducing that uncertainty, the effect of such an integration should be lower for this criterion than for the Mean Variance Criterion. Although, this assumption is partly incorrect because the Project Balance Criterion also selects those proposals which as a consequence of the abandonment option provide a faster recovery of its investment, Figure 6.16 shows that the difference between the improvement achieved by both **criteria** reaches a maximum in the vicinity of the value of  $P1$  used in this simulation. That difference is statistically significant with a degree of certainty of at least 0.95. For lower values of  $P1$ , the influence of abandonment in the outcome of both criteria becomes negligible and, therefore, the difference in improvement is also negligible. For higher values of  $P1$  the abandonment option fully exerts its influence as a means of reducing the uncertainty involved in the proposals. In Figure 6.16, for  $P1=1.0$  the improvement made by the Mean Variance Criterion appears to be higher than the improvement made by the Project Balance Criterion, however, the difference is not statistically significant (paired t-test).

#### 6.4 The Expected Values Abandonment Rule and the Suboptimal Abandonment Rule.

In Chapter III, a Suboptimal Abandonment Rule was suggested as an alternative to the Expected Values Abandonment Rule derived by Dyl and Long [18]. The Suboptimal Rule says that a proposal should be abandoned at that time where the first positive local optimum for the expected present worth occurs. To illustrate the point, Figure 6.17 shows a plot of the expected present worth as a function of the abandonment period. In that figure, from an abandonment point of view, the proposal should be abandoned in period 6 because it provides the maximum expected present worth. However, if the Suboptimal Abandonment rule is used, the proposal would be abandoned in period 3. This latter rule has some appeal when capital rationing decisions are made on a sequential, periodic manner because it releases faster the funds tied up in a proposal, providing more budget for investment in the future in more profitable, alternatives.

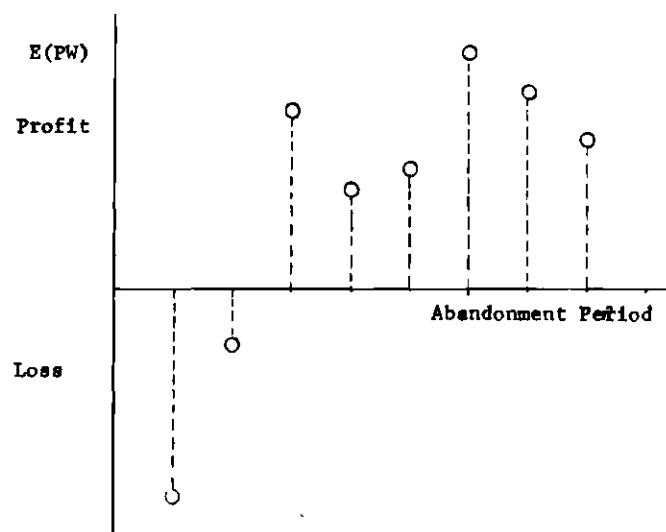


Figure 6.17. ECPW] vs. Abandonment Period.

Several simulation runs were made to compare the results obtained by these rules and in all cases the outcome was exactly the same. Far from indicating that both rules are equivalent, these results showed that for the type of proposals generated in this study it makes no difference when using either rule. The problem is that for these proposals there is a single optimum abandonment period and, therefore, the first positive local optimum detected by the Suboptimal Rule is always the global optimum (see proof in Appendix B).

Since a completely revised simulation model would be needed to generate proposals whose Expected Present Worth had

a behavior similar to that shown in Figure 6.17, it has been decided to submit the comparison of these rules as a recommendation for further research and to concentrate the study on the analysis of the effect of the abandonment option on the situations presented throughout this chapter.

## CHAPTER VII

## SUMMARY OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS.

Throughout this research, a methodology has been derived to allow for the simulation of a sequential, periodic capital budgeting process where abandonment considerations are made. In this chapter, a summary of the results and conclusions arrived at during the development of the study is presented. Section 7.1 reviews the results of the research. Thereafter, the general conclusions are formulated in Section 7.2, followed by recommendations for further research in Section 7.3.

### 7.1 Summary of Results.

This study begins with a discussion about the current state of the art of abandonment analysis. From that discussion, two main observations are drawn. One concerns the lack of an abandonment rule capable of including risk considerations in the decision process. The second refers to the limited reach of the currently reported applications of abandonment analysis. Most of the articles that have dealt with abandonment have reported their results on single period capital rationing decisions and on accept-reject problems. In view of these observations, it is decided to orient this research towards estimating the importance of making abandonment considerations in sequential capital rationing problems, where the outcome of the investment proposals is not known with certainty. Thus incorporating abandonment as an integral part of a capital budgeting process.

The first step towards attaining that goal is to derive an abandonment rule capable of dealing explicitly with risk. Since the decision to hold or to abandon a project may well be viewed as a decision to accept one of two mutually exclusive alternatives, three procedures commonly used in making decisions under risk are analyzed, the Probability of Loss Method, the Coefficient of Variation Method and the Expected

Gain Confidence Limit Method. The Probability of Loss Method proves to be an extremely weak screening procedure when the chances that the outcome of the alternatives be negative are close to zero. The Coefficient of Variation Method is based on a rationale which is plausible enough. However, it is shown that the use of the coefficient of variation as a measure for selecting alternatives has inherent limitations. By definition, that measure uses the standard deviation in its numerator and it therefore does not reflect the risk involved in the decision in its actual magnitude. It also fails to distinguish between positive and negative side variations.

Finally, the Expected Gain Confidence Limit Method provides an adequate means of overcoming the deficiencies associated with the Coefficient of Variation Method, while at the same time maintaining its main advantages such as simplicity and ease of computation. Because of the advantages shown by this last method, it is upon its theoretical bases that the abandonment rule is developed.

In addition to the abandonment rule that takes risk into account, an alternative rule for the case where the decision-maker is risk indifferent is derived. This rule, suboptimal from an abandonment point of view, has some appeal for application in sequential capital rationing processes.

The study continues with a description of the sequential capital rationing process. The investment situation suggested is one where the investment decisions are made on a regular, periodic basis and the objective is to maximize the total accumulated wealth of the firm at some predetermined horizon time. It is assumed the knowledge about the outcome of the proposals is probabilistic and that there is no information about the investment opportunities that may become available in the future.

When abandonment is included as an integral part of the capital budgeting process, the decision to hold or to abandon a proposal in a certain period may take place in two different stages of the process. One stage comprises the mutually exclusive choice among investment proposals. The other stage corresponds to the decisions made in every period after the proposals have been undertaken. In the first stage, an a priori identification of the proposals optimal abandonment period is needed. This has to be done because at this stage all the proposals are competing, therefore each one should be presented in its anticipated optimal perspective. Once the proposals have been accepted, the abandonment period need not be precisely the one forecasted during the first stage. Two factors influence that eventual change: The possibility of having more adequate estimates of the proposal's cash flows and abandonment values; and the availability of more

profitable proposals in future periods. The effect of the first factor in the decision making process is evident, the closer the realization period, the more accurate the forecast and the decision. The second factor concerns the fact that in every period all the proposals eligible for abandonment should be considered as sources of funds through disposal [15].

Although from a theoretical point of view, the integration of abandonment into capital rationing problems should consider both of the above stages, this study assumes that all the proposals are held until their a priori estimated optimal abandonment period. It is recognized that this assumption is not rigorous because a firm should be able to abandon at least some projects at intermediate periods. However, if actual data is not available, it is unrealistic to select randomly the fraction of assets eligible for abandonment in every period. Furthermore, practical evidence shows that projects are usually kept in business until their economic lives are reached. Therefore, the decision-making process followed by this study assumes that before making the capital rationing decision, the decision-maker evaluates the possibility of abandoning the proposals. When that possibility is recognized, the proposals are modified accordingly, and then submitted to the capital allocation phase. This process is repeated in every period within the planning horizon.

A simulation model is developed to generate a number of investment situations where the decision making process is performed. In each situation, a comparison is made between the results obtained with and without abandonment for three capital rationing criterion. Also a comparison is made between the outcome of the different criterion when abandonment is taken into account. In addition to these results, the analysis is extended to the cases where information about the future investment opportunities is available. Two levels of knowledge are examined. One assumes that the decision-maker has complete knowledge about the proposals submitted for implementation at the time of decision. The second assumes that complete information about the proposals being considered throughout the planning horizon is available. The analysis of the latter two approaches is intended to show the importance of having more information when abandonment is considered.

## 7.2 Conclusions.

The results reported in Chapter V show that under most circumstances, when capital rationing decisions are made on a sequential, periodic basis, the effect of making abandonment considerations in the evaluation of the proposals is to

increase the wealth of the firm, with no significant change in its variability. Exceptions to the rule are the situations described by Companies C and D and to some extent Company E.

Company C represents the situation where all the proposals submitted during the planning horizon have exponential decay series of abandonment values. It is possible to find firms confronted with investment proposals with a series of abandonment values that present this kind of behavior. In fact, the equation used to simulate the abandonment values was obtained by Eilon et al. [19] from empirical evidence. This result indicates that a firm performing in a similar investment situation will probably not realize improvements in its accumulated wealth by integrating the abandonment option in the evaluation of investment proposals. However, this does not imply that abandonment should be completely disregarded from the decision process because, eventually, some advantages could be observed if the disposal of projects in progress is viewed as a means of gathering funds for investment in more profitable proposals that may become available.

Company D corresponds to the investment situation where all the proposals have a single payment type of cash flow pattern. The results reported for this case show that abandonment considerations are only relevant for values of  $P_1$

greater than or equal to 0.9. The investment situation represented by this Company is rather unusual because in practice it is likely that most firms will be confronted by sets of heterogeneous investment proposals. However, these results are useful to point out that for this type of investment proposal the influence exerted by abandonment is relevant only under extreme conditions.

In the special case where the variability of the cash flows and abandonment values is a function of the forecasting horizon (Company E), the effect of the abandonment option in the accumulated wealth of the firm is only relevant for firms which are basically risk indifferent. When moderate and high values of the risk aversion parameter are used, the improvements made by including abandonment considerations into the analysis are not significant. Due to the large variability that accompanies the forecasts made further in the future, the possibility of reestimating the cash flows and abandonment values during the life of the proposals will probably have a greater influence in the final abandonment decision.

The integration of the abandonment option into capital budgeting processes has a substantial effect on the accumulated wealth of the firm when partial and complete information about the investment proposals is available. In

particular, when partial information may be obtained, the improvements range from 6.5 % in Company C to 89.4 % in Company D. When complete information is available, the improvements are more noticeable, and range from 18% for Company C to 99% for Company D. It is also important to observe that for this latter level of knowledge a significant reduction in the variability of the Horizon Value is realized in the investment situation represented by Company A.

Regarding the performance of the different capital rationing criteria when abandonment is taken into account, it may be concluded that the Project Balance Criterion is consistently superior than that other criteria. However, its effectiveness as a decision rule is substantially increased when the firm is facing growing investment opportunities. In only one of the situations analyzed the Project Balance Criterion does not clearly outperform the other criteria. That is Company E, where the variability of the forecasted variables is a function of the forecasting horizon.

Finally, the fact that the Project Balance Criterion takes into account the resolution of the proposals uncertainty makes it less sensitive than other criteria to changes in the parameter that controls the size of the abandonment values. As opposed to the Project Balance Criterion, the Mean Variance Criterion and the Expected Present Worth Maximization

Criterion become very sensitive to changes in that parameter when it assumes values larger than 60%.

### 7.3 Recommendations for Further Research.

This research is considered to be an initial phase in the study of abandonment as an integral part of capital budgeting processes. A second phase should consider the analysis of the possibility of making the final abandonment decision once the projects are underway. This extension would demand a thorough investigation of actual investment firms in order to gain some insight about the factors that control the disposal of investment projects. Such information will be useful to simulate the decision making process on a more realistic basis.

The nature of the proposals generated by the simulation model described in Chapter V, did not enable this research to compare the Suboptimal Abandonment Rule derived in Chapter III with the abandonment rules currently available in the literature. In order to make this comparison, an effort should be made to develop a methodology for the simulation of proposals which have series of cash flows composed of combinations of increasing and decreasing gradient series.

Such proposals are likely to present an ELPWJ function with several local optima (see Section 3.3), a condition required in order to test the effectiveness of the above mentioned rule.

APPENDICES

## APPENDIX A

## SIMULATION RESULTS

- Table A-1. Simulation Results, Company A.
- Table A-2. Simulation Results, Company B.
- Table A-3. Simulation Results, Company C.
- Table A-4. Simulation Results, Company D.
- Table A-5. Simulation Results, Company E.
- Table A-6. Simulation Results of the Sensitivity Analysis of F1.

Table A-1. Simulation Results, Company A (all in  $\$10^3$ )

## Mean Variance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.5	7728.8	1332.1	7420.3	1166.4
1.0	8216.8	1219.0	7672.6	1192.7
1.5	8205.8	1126.4	7905.1	1127.4
2.0	8377.3	1273.4	7693.5	1041.5
2.5	7647.0	1180.9	7269.3	1166.6
3.0	6811.6	1070.8	6593.4	1001.0

## Expected Present Worth Maximization Criterion.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
8863.0	1264.4	8243.1	1254.9

## Project Balance Criterion

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.0	9573.4	1199.5	8532.6	1312.3
0.5	9888.4	1231.4	8597.3	1373.0
1.0	9586.5	1216.6	8547.3	1189.9
1.5	9258.3	1257.8	8396.7	1242.4
2.0	8813.1	1166.8	7909.2	1047.4
2.5	8003.0	1148.4	7206.0	1041.6
3.0	6644.7	1115.0	6171.3	1126.6

## Partial Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
11165.4	1225.6	8528.5	1131.0

## Complete Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
14563.0	1251.4	10373.0	3452.6

Table A-2. Simulation Results, Company B (all in  $\$10^3$ )

## Mean Variance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.5	10870.6	2246.7	10492.2	2346.8
1.0	10797.9	2066.4	10350.3	2442.0
1.5	10598.7	1905.5	9884.9	2530.3
2.0	9965.2	1911.3	9982.1	2305.3
2.5	9915.5	2028.4	9661.9	1934.3
3.0	9642.2	1658.3	9248.9	1334.1

## Expected Present Worth Maximization Criterion.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
11134.9	2104.3	10466.3	2392.8

## Project Balance Criterion

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.0	14125.5	1442.4	12944.8	2313.1
0.5	14484.9	2095.7	12912.4	1959.1
1.0	14277.8	1693.0	12604.3	1983.1
1.5	13505.3	2269.9	12334.0	1405.8
2.0	12635.3	2307.6	11735.9	1679.8
2.5	12244.1	2088.6	11323.1	1801.5
3.0	11202.9	1565.1	10598.7	1800.1

## Partial Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
14683.7	1122.8	11716.1	1635.3

## Complete Information

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
24058.4	2806.0	18901.1	3283.6

Table A-3. Simulation Results, Company C (all in  $\$10^3$ )

## Mean Variance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.5	7851.4	1280.5	7790.7	1258.3
1.0	7641.9	1070.8	7567.2	1214.9
1.5	7730.1	1120.6	7620.1	1195.6
2.0	7191.7	1065.3	7095.2	1001.1
2.5	6630.5	992.4	6625.4	1083.2
3.0	6412.7	872.1	6400.8	970.1

## Expected Present Worth Maximization Criterion.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
8242.0	1148.6	8152.8	1122.6

## Project Balance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.0	8620.9	1152.5	8609.5	1064.1
0.5	8482.9	1126.9	8391.7	1141.9
1.0	8262.5	1185.2	8202.9	1224.1
1.5	7896.0	1384.6	7890.1	1396.8
2.0	7707.0	1022.6	7841.0	1129.1
2.5	7610.9	1130.7	7536.7	1064.4
3.0	6820.2	1080.1	6732.1	1035.6

## Partial Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
9071.5	1372.0	8514.2	1212.3

## Complete Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
11769.1	2650.1	9971.3	3151.2

Table A-4. Simulation Results, Company D (all in  $\$10^3$ )

## Mean Variance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.5	8898.0	1896.0	8220.1	1252.9
1.0	8807.3	1836.7	8080.1	1316.2
1.5	8796.3	1714.9	8408.0	1377.6
2.0	8716.9	1773.6	8043.5	1280.2
2.5	8216.7	1635.1	7674.6	1104.5
3.0	8073.2	1425.9	7410.7	1175.8

## Expected Present Worth Maximization Criterion.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
8365.8	1488.3	7670.9	1253.6

## Project Balance Criterion

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.0	9071.5	1915.9	8494.4	1693.6
0.5	9666.3	1901.7	8309.7	1336.6
1.0	9404.1	1976.2	8629.7	1539.7
1.5	8984.2	1568.1	8647.4	1559.1
2.0	8876.1	1688.4	8265.9	1550.4
2.5	8581.1	1216.3	8073.5	1140.2
3.0	8321.6	1315.5	8101.4	1160.2

## Partial Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
18297.9	3431.2	9660.9	1552.1

## Complete Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
26692.3	3412.1	13432.4	2323.1

Table A-5. Simulation Results, Company E (all in  $\$10^3$ )

## Mean Variance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.5	9132.5	1819.9	8373.3	1650.8
1.0	8407.0	1512.8	8342.5	1664.0
1.5	6776.3	1845.0	6408.7	1472.5
2.0	4725.1	1295.5	4326.0	1311.9
2.5	2512.2	740.7	2838.5	1319.9

## Expected Present Worth Maximization Criterion.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
8902.4	1725.3	8242.4	1907.5

## Project Balance Criterion.

k	With Abandonment		Without Abandonment	
	E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
0.0	9390.5	2089.4	7904.0	1590.6
0.5	9604.8	1941.8	7894.8	1782.0
1.0	8539.3	1776.5	7847.3	1876.7
1.5	6740.6	1879.6	6249.7	1786.7
2.0	4761.7	1310.6	4362.2	1314.4
2.5	2504.9	737.3	2838.5	1319.0

## Partial Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
12733.9	2498.0	9637.7	1637.4

## Complete Information.

With Abandonment		Without Abandonment	
E[HV]	$\sigma$ [HV]	E[HV]	$\sigma$ [HV]
16988.1	2622.1	13840.4	3081.7

Table A-6. Simulation Results of the Sensitivity Analysis  
of P1(all EHVJ in \$103)

Capital Rationing Criterion.

		Expected P.W. Max.	Mean Variance	Project Balance
P1	1.0	10643.2	11317.5	11797.9
	0.8	8942.9	9786.9	10382.9
	0.6	8812.8	8878.9	10029.6
	0.4	8655.7	8801.5	9701.8
	0.2	8347.6	8364.3	9579.7
No abandonment solution.				
		7963.8	8060.2	8666.0

## APPENDIX B.

## UNIQUENESS OF THE OPTIMUM ABANDONMENT PERIOD.

This appendix shows that when the pattern of the series of cash flows and abandonment values follow a linear function, there is a unique optimum abandonment period which is the global optimum.

Let,

$$E[CF(t)] = A + b t \quad t = 1, 2, \dots, L.$$

be the equation that generates the series of expected cash flows, where A and b are constants. Also, let,

$$E[AU(t)] = C_0 - c t \quad t = 1, 2, \dots, L.$$

be the equation that generates the series of expected abandonment values; where

$$c = \{C_0 - E[CF(L)]\}/L$$

but,

$$E[CF(L)] = A + b L$$

therefore,

$$E[AU(t)] = C_0 - (C_0 - A - bL/L) t$$

If a proposal is abandoned in period T, the expected present worth may be computed as follows,

$$\begin{aligned} E[PW(T)] &= -C_0 + \sum_{t=1}^T E[CF(t)](1+i)^{-t} + E[AU(T)](1+i)^{-T} = \\ &= -C_0 + \sum_{t=1}^T (A+bt)(1+i)^{-t} + (C_0 - cT)(1+i)^{-T} \end{aligned}$$

In order to find that  $T$  which maximizes  $ECPW(T)$ , the first derivative with respect to  $T$  is computed and equated to zero,

$$\frac{d ECPW(t)}{dT} = b(1+i)^{-T} + (A+bT)(-T)(1+i)^{-T-1} - c(1+i)^{-T} - T(C_0-cT)(1+i)^{-T-1} = 0$$

Simplifying,

$$-(b-c)rT^2 - (A+C_0)rT + (b-c) = 0$$

where,  $r = 1/(1+i)$ . The solution for this expression is,

$$T = \frac{(A+C_0)r \pm \sqrt{(A+C_0)^2 r^2 + 4(b-c)^2 r}}{-2(b-c) r}$$

Since the expression under the square root is always greater than or equal to  $(A+C_0)r$ , there is only one positive value of  $T$  ( $T^*$ ), which satisfies that equation. And  $ECPW(T^*)$  is a maximum if the following relationship is satisfied,

$$2(c-b)T^* - (A+C_0) > 0$$

Observe that nowhere in this development has the sign of  $b$  been constrained to be **positive** or negative. In fact, the results are valid for decreasing, increasing, or uniform series of cash flows.

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