Secondary Pulse Tubes and Regenerators for Coupling to Room-Temperature Phase Shifters in Multistage Pulse Tube Cryocoolers

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ABSTRACT

Multistage pulse tube cryocoolers require separate phase shifters for each stage. For sufficiently high frequency and acoustic power, the inertance tube is commonly used for such phase shifting. For Stirling-type, multistage pulse tube cryocoolers, the warm end of the coldest pulse tube is often heat sunk to the cold end of a warmer stage rather than at room temperature to improve the figure of merit for the pulse tube and/or to achieve a larger phase shift with a cold inertance tube. The use of a secondary pulse tube or regenerator between the main pulse tube and a phase shifter allows the phase shifter to operate at room temperature where space is more readily available. It also allows for the use of commercially available pressure oscillators as expanders. The secondary regenerator amplifies the acoustic power, so a room temperature inertance tube may perform as well as a cold one. A secondary pulse tube transfers acoustic power to room temperature without amplification, so a rather small warm expander or displacer can provide the optimum phase shift even in a low-power cryocooler. In this paper, we model the behavior of these secondary pulse tubes and regenerators using REGEN3.3 and present results to assist in selecting the optimum geometry and the optimum characteristics for the expander. We show that acoustic power flows from cold to hot in such systems can be modeled with REGEN3.3 by changing the flow phase by 180 degrees.

INTRODUCTION

Small 4 K cryocoolers for the cooling of low temperature superconducting (LTS) electronic systems are necessary for broader commercial, military, or space applications of such devices. Typically these cryocoolers have been either Gifford-McMahon (GM) cryocoolers or GM-type pulse tube cryocoolers that operate at frequencies of about 1 Hz.[1] The efficiency of these cryocoolers ranges from 0.5 to 1.0 % of Carnot, whereas 80 K cryocoolers often achieve efficiencies of about 15 % of Carnot. The low efficiency of 4 K cryocoolers causes these cryocoolers to have large, noisy compressors with high input powers. The low operating frequency of the GM and GM-type pulse tubes also leads to large temperature oscillations at the cold end at the operating frequency of the cryocooler. The amplitude of the temperature oscillation decreases inversely with the cryocooler operating frequency. Higher frequencies also allow the use of Stirling cryocoolers or Stirling-type pulse tube cryocoolers, which have much higher efficiencies in converting electrical

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power to PV power. These frequencies are typically in the range of 30 to 60 Hz. The linear Stirling-type compressors (pressure oscillators) often use flexure bearings that eliminate rubbing contact and operate almost silently. However, these higher frequencies generally lead to greater losses in the 4 K regenerator unless the operating parameters are near optimum conditions. Recent regenerator modeling efforts have shown that the phase angle between flow and pressure at the cold end has a strong effect on the 4 K regenerator second law efficiency.[2] In order to achieve an optimum phase of about -30° (flow lagging pressure) at the cold end, a phase of about -60° at the pulse tube warm end is required. Inertance tubes are typically used for phase shifting, but with the small refrigeration powers of interest for electronics cooling, phase shifts of only a few degrees are possible at 30 Hz, even with the inertance tube and reservoir at a low temperature of 30 K.[3] A double inlet configuration with a secondary orifice between the regenerator and pulse tube warm ends can only provide a practical phase shift of about 30° before the lost work in the secondary orifice greatly reduces the overall efficiency. The double inlet approach also introduces the possibility of DC flow, which can reduce the efficiency.[4]

Larger phase shifts with small acoustic powers can be achieved by the use of a warm expander or warm displacer at the warm end of the pulse tube.[5-8] For single-stage pulse tube cryocoolers or for two-stage pulse tube cryocoolers operating at about 1 Hz (GM-type), the warm end of the pulse tube operates at ambient temperature. Thus, commercially available pressure oscillators can be used for the warm expander. A 4 K pulse tube may need to have the warm end at 30 K or lower to keep the efficiency of the pulse tube component high, at least for a high frequency of about 30 Hz. It would then be necessary to develop an expander that can operate at about 30 K. In order to make use of a room-temperature expander, even for a 4 K pulse tube, we examine here the use of a secondary regenerator or secondary pulse tube to couple the 30 K pulse tube warm end to a room-temperature expander.

EFFECT OF PHASE ON 4 K REGENERATOR PERFORMANCE

Regenerative cryocooler losses

The coefficient of performance (COP) of a regenerator is given by

$$\mathrm{COP} = \frac{\dot{Q}_{net}}{\langle P\dot{V}\rangle_h} \tag{1}$$

where $\dot{Q}_{net}$ is the net refrigeration power at the cold end, and $\langle P\dot{V}\rangle_h$ is the time-averaged acoustic or PV power at the hot end of the regenerator. For an ideal gas and a perfect regenerator the ideal COP for a regenerator is given by (T_c/T_h), where we assume that the reversible expansion work at the cold end is not being fed back to the hot end of the regenerator. Thus, the thermodynamic second-law efficiency of the regenerator is given by

$$\eta = (T_h/T_c)\mathrm{COP} \tag{2}$$

Calculations of the COP and efficiency of 4 K regenerators at 30 Hz were carried out by Radebaugh *et al.*[2] using the NIST software REGEN3.3. The losses considered in calculating the COP were the real-gas effects, the regenerator ineffectiveness, and conduction in the matrix. No pulse tube losses were considered, but in practice they might be approximately 20 % to 30 % of the gross refrigeration power available at the cold end. They found that the phase angle f_c between the flow and pressure at the cold end has a strong effect on the regenerator efficiency, as shown in Fig. 1. In this figure a positive phase angle means flow leads the pressure. The parameters used in these calculations were optimized for 30 Hz operation with ^{3}He working gas. An efficiency of at least 0.10 to 0.15 would be required to overcome any losses within the pulse tube. As shown in Fig. 1, it would be very difficult to reach 4 K with ^{4}He when the pressure ratio is 1.5 and the hot end is hotter than about 20 K, even with an optimum phase angle of about -30°. Pressure ratios above 1.5 can increase the efficiency some[2], but such high pressure ratios usually cause the pressure oscillator to operate far from resonance conditions. The use of ^{3}He working gas yields considerably higher second-law efficiencies for a 4 K regenerator, as shown in Fig. 1. However, even with ^{3}He, the ideal

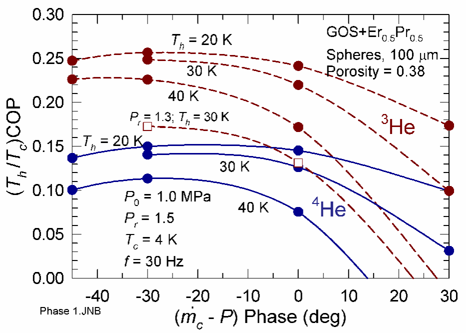

Figure 1. Calculated effect of cold-end phase on 4 K regenerators

phase angle should be about -30°, and no higher than about 0° to achieve reasonable overall efficiency at 4 K when the pulse tube losses are taken into account. A phase angle of about -30° at the cold end gives rise to a 0° phase near the regenerator midpoint. Such a phase angle provides the minimum flow amplitude for a given acoustic power. The regenerator losses are proportional to the flow amplitude, so the amplitude should be minimized to achieve high efficiency. A phase of -30° at the cold end is difficult to achieve with small acoustic powers at 30 Hz. For 4 K superconducting electronic applications, net refrigeration powers of about 0.1 W are required, which can be provided with about 1 W of acoustic power at the cold end.

PHASE SHIFT MECHANISMS

Fixed elements (orifices and inertance tubes)

Figure 2 shows schematics of the three common phase shift mechanisms used for pulse tube cryocoolers. The orifice,[9] shown in the top drawing, is a purely resistive element, so the flow is in phase with the pressure at the orifice. Thus, it provides no phase shift. As mentioned in the introduction, such a phase will result in the phase at the cold end being about +30°. Such a phase leads to large regenerator losses and a low efficiency for the 4 K regenerator.

The second drawing in Fig. 2 shows a schematic of the double inlet method.[10] In this approach the flow through the primary orifice is the sum (real and imaginary parts) of the flow through the pulse tube and the secondary orifice. Flow through the secondary orifice is in phase with the pressure drop across the regenerator, which, in turn, is approximately in phase with the regenerator flow at its midpoint. With the secondary orifice nearly closed, the regenerator midpoint flow and the secondary orifice flow will lead the pressure by about 40° to 50°. The pulse tube flow is then forced to lag the pressure to keep the flow through the primary orifice in phase with the pressure. However, as the secondary orifice flow is increased, additional compressor PV power is required to provide the extra flow. At some point the extra compressor power cancels the beneficial effect of a more favorable phase in the regenerator. Our analyses show that the overall efficiency peaks when the pulse tube warm-end phase is about -30°, which gives a cold end phase of about 0. The second-

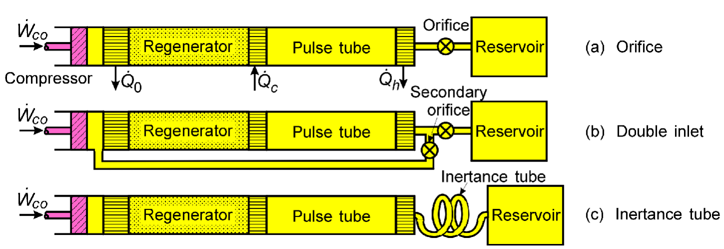

Figure 2. Schematics of three common phase shifting methods for pulse tube cryocoolers

ary orifice is generally made with two opposing needle valves to provide an asymmetric flow impedance that eliminates DC flow.

If the pulse tube warm end is at 30 K, then the double inlet normally must be at that temperature. The use of two needle valves at 30 K greatly complicates the experimental procedure. The secondary orifice could be located at room temperature if a small secondary regenerator is placed between it and the pulse tube warm end at 30 K. The other side of the secondary orifice would be connected to the transfer line at room temperature between the compressor and the aftercooler. Because a secondary regenerator has never been tried before, we modeled it, as discussed here, in an effort to optimize it. The use of a secondary regenerator is not an ideal solution, because the added gas volume reduces the possible phase shift. The flow impedance of the secondary regenerator could be made high enough to provide most of the impedance, and the room-temperature needle valves would be used only to provide a small amount of adjustment to the overall impedance.

Often the primary orifice in a double inlet configuration is replaced with an inertance tube, even when it provides only a few degrees of phase shift. These few degrees add to the phase shift the double inlet can provide compared with the primary being a simple orifice.

The inertance tube, as shown schematically in Fig. 2(c), is the most common method for phase shifting in Stirling-type pulse tube cryocoolers. For one-stage pulse tube cryocoolers, the acoustic power entering the inertance tube is often high enough to provide the ideal phase shift of about -60° at the entrance to the inertance tube.[3] For multiple-stage pulse tube cryocoolers, the acoustic power flow in the colder stages is significantly less, which in many cases is insufficient to provide the desired phase shift with inertance tubes when used at room temperature. By placing the inertance tube and reservoir at a lower temperature, the higher gas density allows for a greater phase shift in the inertance tube. A transmission line model[3] was used here to calculate the maximum phase shift possible in a 30 K inertance tube driven at a frequency of 30 Hz, an average pressure of 1.0 MPa, and a pressure ratio of 1.5. These operating conditions were found to be near optimum for a 4 K regenerator. Figure 3 shows the results of these calculations for both an adiabatic model and an isothermal model using ^{3}He and ^{4}He. For small acoustic powers (near 0.1 W) the radius of the inertance tube can become comparable to the thermal penetration depth (81 mm) , in which case the isothermal model is more accurate. At 1 W of acoustic power, the ratio of inertance tube radius to thermal penetration depth is 4.3, in which case the phase shift will be close to that predicted by the adiabatic model. From Fig. 3, we see that the maximum phase shift for ^{3}He with 1 W of acoustic power at 30 K is only about 5°, rather than the desired 60°.

Mechanical Phase Shifters

Figure 4 shows schematics for various mechanical phase shift mechanisms that are used in regenerative cryocoolers. The first method shown is the displacer, which is used in Stirling or Gifford-McMahon cryocoolers. Any desired phase shift can be obtained with such a device when it is driven mechanically or electrically. The back side of a displacer has a small gas volume and is connected to the warm end of the regenerator to feed back the recovered expansion work. Alterna-

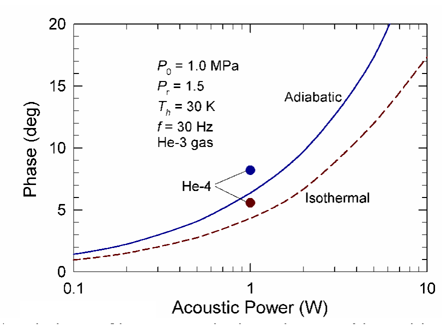

Figure 3. Calculated phase of inertance tube impedance at 30 K with a large reservoir.

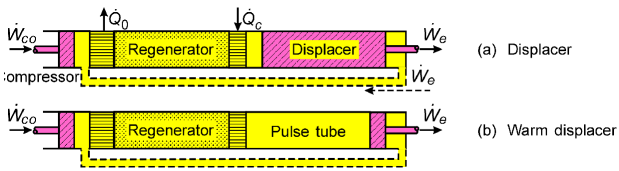

Figure 4. Schematics of mechanical phase shift mechanisms used in regenerative cryocoolers.

tively, a piston could be used at the cold end with a large backside volume at the average pressure. The recovered work could be fed electrically or mechanically to room temperature where it can be dissipated as heat, but with some reduction in system efficiency because of the lost work. Such a displacer or expander requires a moving part at the cold end.

With the second method shown in Fig. 4, a pulse tube is inserted between the cold end and the displacer or expander at the warm end. The acoustic power entering the cold end of the pulse tube is transmitted through the pulse tube with no change (ideally) to provide expansion work at the pulse tube warm end. Ideally, the cooling power at the cold end is the same whether the displacer or expander is at the cold end or the warm end. With a warm displacer the backside is connected to the regenerator warm end to recover the work. With a warm expander there is no connection to the regenerator warm end, and the work is generally dissipated at room temperature in the form of heat. This second method still requires a moving part in the cold head, but it is at the warm end of the pulse tube. For a single-stage cryocooler, it would be operating at room temperature. For a multiple-stage cryocooler, the warm end of the lower stages may be at the cold temperature of the preceding stage.

Ideally, we would like to place an expander at the warm end of the 4 K pulse tube. The expansion work could be used to drive a linear alternator whose electrical output power is either fed to room temperature to be dissipated as heat or is used to provide electrical power to drive low power superconducting electronics at 4 K. The later technique eliminates the conduction loss in electrical leads at the higher stages. The low electrical resistivity of copper at 30 K also means that the Joule heating in the alternator would be very small compared to the recovered mechanical power. Such an expander and alternator could simply be a commercial pressure oscillator run in reverse to provide power instead of supplying it with power. Unfortunately, most commercial pressure oscillators are not designed to operate at cryogenic temperatures. A specially-designed expander would need to be developed for use at about 30 K to use it at the warm end of a 4 K pulse tube. A second, and much easier option, is to use a commercial pressure oscillator as an expander at room temperature, but couple it to the 30 K pulse tube warm end by a secondary regenerator or a secondary pulse tube. A commercial pressure oscillator can be controlled electrically to provide any phase shift within the bounds of its swept volume and maximum current. The linear motor can generate electric power from the recovered PV power, or electric power input may be required if the expander is operating far from resonance and the Joule heating is larger than the generated power.

SECONDARY REGENERATORS AND PULSE TUBES

Operating Procedure

Figure 5 shows schematics for secondary regenerators and pulse tubes and how they might be incorporated into a multiple stage cryocooler to reach 4 K. In the figure, a Gifford McMahon cryocooler is shown for the precooling to about 30 K, but pulse tube or Stirling cryocoolers could also be used. The purpose of both the secondary regenerator and the secondary pulse tube is to transmit acoustic power from the cold end to the warm end with a minimum pressure drop. Any pressure drop in either of these components would represent a resistive element with flow in phase with the pressure drop. Such a pressure drop would diminish the phase shift possible with the expander. Other parameters used in the optimization are the gas volume in the element and the enthalpy flow. As the gas volume is increased, the flow amplitude at the expander is increased,

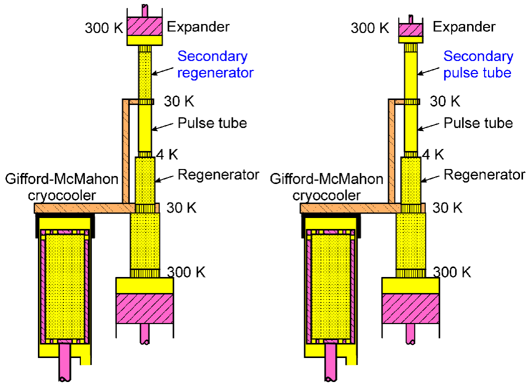

Figure 5. Schematics that show the use of a secondary regenerator or a secondary pulse tube to couple a room-temperature expander to the warm end of a 4 K pulse tube cryocooler

which requires a greater swept volume. Time-averaged enthalpy flow toward the cold end would generate heat in the heat exchanger at the warm end of the primary pulse tube. That heat then needs to be removed by the precooling stage. Ideally, we would like the enthalpy flow to be from the 30 K end to ambient temperature and be as large as possible. It was not obvious before our modeling that a secondary regenerator has an enthalpy flow toward the cold end, even though the acoustic power flow is toward the warm end. However, in a secondary pulse tube the enthalpy flow can easily be toward the hot end. If that enthalpy flow is the same as that in the primary pulse tube, then no heat needs to be absorbed at the 30 K heat exchanger. In principle that case would not require any heat exchanger, and the two pulse tubes become a single pulse tube that is connected between 4 K and ambient temperature. Usually a single pulse tube will be less efficient and not be able to transmit as much enthalpy flow from the 4 K cold end.

A fundamental difference between the secondary regenerator and the secondary pulse tube is that the regenerator behaves nearly like an isothermal element, which amplifies acoustic power proportional to the temperature. Thus, the volume flow rate also increases with temperature and a larger expander is required at room temperature compared with one that might operate at 30 K. The secondary pulse tube operates nearly like an adiabatic element, which transmits acoustic power from cold to hot with no amplification. Therefore, a secondary pulse tube is preferred, because a smaller swept volume is required of the expander.

Modeling Procedure

The NIST software REGEN3.3 was used to model both the secondary regenerator and the secondary pulse tube. It uses a finite difference technique to evaluate the four conservation equations in a regenerator. It was designed to model a normal cryocooler regenerator in which the acoustic power flow is from the hot end to the cold end.[2] We found that it is equally good at modeling regenerators with the power flow in the opposite direction. The only change required in the input conditions is to add 180° to the phase of the cold-end mass flow with respect to the pressure. That change causes the acoustic power flow to travel from the cold to the hot end of the regenerator.

We have not used REGEN3.3 in the past to model pulse tubes, because it was not designed for that task. However, with the ability to have acoustic power travel from the cold end to the hot end, we decided to try modeling the secondary pulse tube. The friction factor and heat transfer coefficient are calculated at each time increment and at each grid point in the regenerator from the steady-state correlations of Kays and London.[11] Such correlations should be good for oscillating flow in regenerators where the amplitude of gas motion is much larger than the hydraulic diameter and the hydraulic diameter is less than the viscous penetration depth. The latter condition means the Valensi number is less than 1. Those conditions usually do not hold in pulse tubes. The Valensi number for the pulse tubes of interest here are on the order of 100. The Valensi number Va is approximately the squared ratio of the tube inner radius to the viscous penetration depth, as given by

$$\mathrm{Va} = \frac{r^2 \rho \omega}{\mu} \tag{3}$$

where r is the inner radius, ρ is the gas density, ω is the angular frequency, and μ is the dynamic viscosity. For such high Valensi numbers, the friction factor and the heat transfer coefficient should be higher than those determined from steady state correlations.[12] Because the pressure drop in the pulse tube is so small, the difference has no significant effect on most of the modeling performed here. The higher heat transfer coefficient may affect the calculation of the enthalpy flow within the pulse tube. We use the enthalpy results here to understand general trends, but we do not rely heavily on the absolute values.

The parameters used for the modeling discussed here are given in Table 1. All of the calculations are with ^{4}He working fluid. Because of the relatively high temperature (30 K to 300 K) and the low pressure (1.0 MPa), real-gas effects should be small. Thus, we don't expect any significant difference if ^{4}He were to be replaced with ^{3}He. For the secondary regenerator we modeled a 6 mm diameter stainless steel tube that was filled with various mesh sizes of stainless steel screen to achieve different hydraulic diameters. Hydraulic diameters greater than about 100 µm are not practical for actual regenerators, but values up to the tube diameter were used in the calculations to observe the effect of hydraulic diameter. The porosity was kept constant at 0.68, and the cold-end mass flow rate was held constant at 0.32 g/s for all values of hydraulic diameter. For the secondary pulse tube modeling, we varied the tube diameter and the flow in such a manner that the ratio of cross-sectional area to the cold end mass flow remained constant. The relative penetration of the gas at the cold end varied from about 0.18 to 0.25. The porosity was set at 0.91 to account for a thin wall.

Modeling Results

Figure 6 shows the results of the REGEN3.3 calculations for the ratio of the swept volume at the warm ends of secondary regenerators and pulse tubes to that at the cold ends. The regenerators were filled with stainless steel screens of various hydraulic diameters of porosity 0.68. The pulse tube diameters (equal to the hydraulic diameter) were varied but with a constant porosity of 0.91 to account for heat transfer to a thin wall. Secondary regenerators with hydraulic diameters less than about 100 µm (typical of good regenerators) show a rather high swept volume ratio of about 14,

Table 1. Parameters for the secondary regenerator and pulse tube used for the modeling discussed here.

Secondary element	T_c (K)	T_h (K)	P_0 (MPa)	P_r	m_c (g/s)	ϕ_c (deg)	D (mm)	L (mm)	A_g/m_c (cm^2-s/g)
Regenerator	30	300	1.0	1.3	0.32	-60	6.0	50	0.62
Pulse tube	30	300	1.0	1.3	-	-60	0.5-6.0	50	0.79

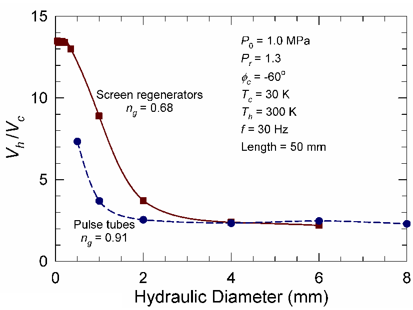

Figure 6. Ratio of hot-to-cold swept volumes in secondary regenerators and pulse tubes.

whereas the secondary pulse tubes have a ratio of about 2.5 for diameters of 2 mm and larger. This low swept volume ratio shows the advantage of using a secondary pulse tube compared to a secondary regenerator to couple to a warm expander at room temperature. The amount of PV power that the expander needs to extract or input to the gas can be determined from the ratio of the warm PV power to the cold PV power shown in Fig. 7. Because PV power must be input to the gas at the cold end to drive the acoustic power toward the warm end, the sign of this cold PV power is considered negative. For that reason, we use the absolute value of the cold-end PV power in the denominator, so the ratio reflects the sign of the warm end PV power. A positive value for this ratio then means that power must be extracted from the gas at the warm end. Ideally we would expect power to be extracted in all cases, but we see in Fig. 7 that there are some cases where the ratio is negative and power must be input at the warm end. Further studies are needed to clarify the reasons for this behavior.

An important parameter of the secondary regenerator or pulse tube is the heat load or heat lift it imposes upon the primary pulse tube warm end. The heat load is given by the sum of the time-averaged enthalpy flow and the thermal conduction in the secondary element. In analyses of entire pulse tube cryocoolers, a positive enthalpy flow is generally meant to be a flow from the compressor to the expander. We maintain that convention here and consider a positive enthalpy and conduction flow to be from the cold end to the warm end of the secondary regenerator or pulse tube. A positive value then means a cooling effect. Figure 8 shows the calculated enthalpy plus conduction divided by the absolute value of the cold-end power flow for both the secondary regenerator and the secondary pulse tube. The energy flow (enthalpy plus conduction) is negative for most cases, which means a heat load to the 30 K heat exchanger. For a typical secondary regenerator configuration with a small hydraulic diameter, the heat load as shown in Fig. 8 is fairly small. For larger hydraulic diameters, the heat load becomes quite large until the hydraulic diameter becomes much larger than the thermal penetration depth, at which point it begins to behave more like an adiabatic element and converge with the secondary pulse tube behavior.

The calculated temperature profile for a secondary regenerator with 64 mm hydraulic diameter (#325 mesh) and a 4.0 mm diameter secondary pulse tube are shown in Fig. 9. The large phase angles between flow and pressure in these elements give rise to the upward bending temperature profile. This behavior suggests that heat sinking either element at approximately the midpoint to an 80 K first stage could significantly reduce the heat load at 30 K, and maybe even result in a cooling effect at 30 K with the secondary pulse tube when there is a heating effect without the heat sink impedance matching to room temperature expander

Linear compressor modeling

For small 4 K refrigeration powers considered in this paper, a small linear compressor would be able to provide the function of a linear expander. An important property of the compressor is that

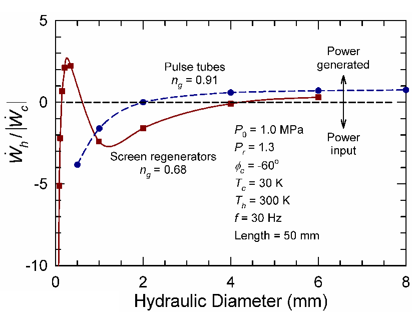

Figure 7. Ratio of hot-to-cold PV powers in secondary regenerators and pulse tubes.

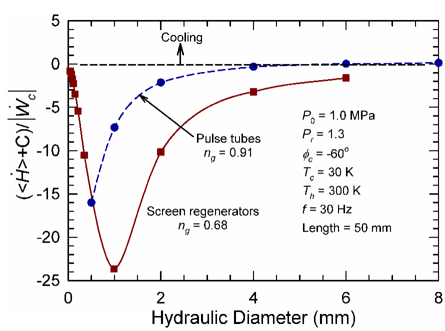

Figure 8. Calculated ratio of enthalpy plus conduction flow to the absolute value of cold-end acoustic power.

its swept volume should be a close match to the required swept volume to eliminate excessive void volume, which requires a larger swept volume to extract the same amount of PV power. The behavior of a linear compressor can be modeled by constructing a force balance, where the motor force must balance the forces due to the mechanical spring, pressure, damping, and inertia. Figure 10 shows a general phasor diagram for such a force balance. All of the forces, except the motor force, are shown as the negative of the actual forces generated by the mechanical spring, gas pressure, damping, and inertia. Their sum is shown equal to the required motor force. The highest compressor efficiency is achieved for a given pressure phasor when the motor phasor is parallel to the velocity (q_m = 90°). That condition, known as resonance, provides a given PV power with the minimum current or Joule heating. High efficiency in an expander is not so important, because the PV power needs to be dissipated in the form of heat. With an inefficient expander that dissipation occurs within the motor coil rather than in an external resistor. Because the extracted PV power is only about 1 W for a low-power 4 K cryocooler, there is very little to be gained by feeding that back into the aftercooler where several hundred watts of PV power are being fed into the system by the main compressor.

Linear expander modeling

For the example considered here, we use the smallest commercially available linear compressor as the expander for the analysis. Table 2 gives the parameters of this linear compressor needed for modeling it as an expander. Figure 11 shows the force balance for a typical case where the following conditions apply: Average pressure = 1.0 MPa; pressure ratio = 1.3; frequency = 30 Hz; PV power extracted = 1.0 W; phase between mass flow and pressure = -75° (q_m = -15°). Because the expander is operating far from its resonance condition, a fairly large motor current is required. The resulting Joule heat of 2.0 W and damping power of 0.15 W exceeds the extracted 1 W of PV power, so 1.15 W of electrical power must be applied to the expander. With this example the swept volume is 72 % of the 0.567 cm^3 maximum. A PV power of 1 W at 30 K with flow lagging pressure by 60° requires a swept volume of 0.31 cm^3.

CONCLUSIONS

Stirling-type pulse tube cryocoolers for operation at 4 K require the flow at the cold end to lag the pressure by about 30° to provide the maximum COP for the 4 K regenerator and to enable the cryocooler to operate reasonably efficient. An inertance tube at the 30 K warm end of the 4 K stage cannot provide sufficient phase shift when the operating frequency is about 30 Hz. Thus, a warm expander is required to provide the ideal phase shift. Commercial linear compressors can be used as the expander if they can operate at such low temperatures. We have shown that such an expander can also be used at room temperature to provide the required phase shift, but then a secondary pulse tube or secondary regenerator must be placed between the warm end (at about 30 K) of the 4 K pulse tube component and the room temperature expander. A smaller expander swept volume is

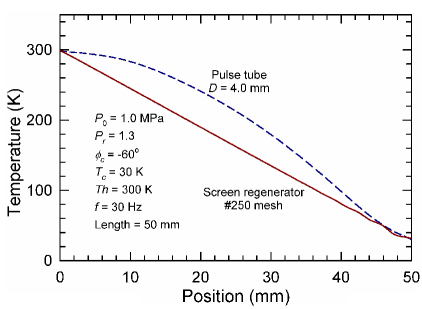

Figure 9. Calculated temperature profiles for a typical secondary regenerator and pulse tube.

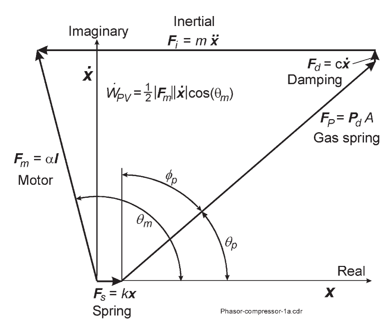

Figure 10. Linear compressor phasor diagram.

Table 1. The two tables list the results of two and three grid convergence tests. Shown are grid convergence tests and grid convergence index (GCI) and convergence order (p) results for the acoustic power (AP) and the phase shift (PS) for the k - ε and k - ω models, reservoir function used in Flake and Razani.[4]

Two grid convergence tests and results

Reservoir	Model	Time steps	GCI, p=.5 PS	GCI, p=1 PS	GCI, p=2 PS	GCI, p=4 PS	GCI, p=.5 AP	GCI, p=1 AP	GCI, p=2 AP	GCI, p=4 AP
0 cm^3	$k-\varepsilon$	1956	.0347	.0144	.0048	9.6e-4	.2083	.0863	.0288	.0058
0cm^3-mixed	$k-\omega$	4297	.0017	7.1e-4	2.4e-4	4.7e-5	.0178	.0074	.0025	4.9e-4
1 cm^3	$k-\omega$	2082	.0895	.0371	.0124	.0025	.5217	.2161	.0720	.0144
1 cm^3	Br-$k-\omega$	3188	.0431	.0179	.0060	.0012	.2657	.1100	.0367	.0073
30 cm^3	Br-$k-\omega$	1501	.1331	.0551	.0184	.0037	.0033	.0014	4.6e-4	9.1e-5
30 cm^3	$k-\varepsilon$	1451	.0895	.0371	.0124	.0025	.5217	.2161	.0720	.0144
83 cm^3	Br-$k-\omega$	1516	.0503	.0208	.0069	.0014	.0405	.0168	.0056	.0011
334 cm^3	$k-\varepsilon$	1504	.0229	.0095	.0032	6.3e-4	.0463	.0192	.0064	.0013
334 cm^3	Br-$k-\omega$	1646	.0210	.0087	.0029	5.8e-4	.0720	.0298	.0099	.0020

Three grid convergence tests and results

Reservoir	Model	Time steps	GCI-PS/JFE	GCI-AP/JFE	Order-PS/JFE	Order-AP/JFE
0 cm^3	$k-\omega$	4801	.0616/.0906	.5092/.7461	.1844/.1278	.1428/.0990
30 cm^3	$k-\omega$	4801	.0170/.0267	.0077/.012	.7655/.5306	.643/.4457
30 cm^3	Br-$k-\omega$	1501	.0492/.0756	1.9e-5/3.7e-4	.5525/.3830	4.93/1.34
334 cm^3	$k-\omega$	4801	.0029/.0048	8.4e-5/.032	1/.6931	4.53/.0822

calculated for any simulation values (like mass flow rate, pressure, temperature, etc…), including the integrated quantities like the acoustic power and phase shift (which are integrated or Fourier transformed). Thus the values for GCI and order for these integral quantities can imply that there are issues with the raw data manipulation techniques.

The metrics chosen for determining whether k - ε or k - ω are better is not readily apparent in this study. In the previous studies it was found that mass conservation values are closer to what is desired for k - ω, but without an understanding of the grid convergence issues for either technique, it is nearly impossible to identify whether one technique predicts values better than the other. One of these techniques may be better at validating the mass flow rate and the other at validating the pressure for an experiment. Determining whether the validation of the simulation is correct needs to be less subjective and in the above example, should be based on numerical analysis techniques applied to the simulation solution methods. It would be incorrect to state that this study validated the simulation results at 30 cm^3 with the experiment, while recognizing the large and small reservoir results don't correlate as well to the experiment. It could purely be coincidence that the 30 cm^3 simulation results were good and incorrect to expect the large reservoir correlation to be as good as the 30 cm^3 results. In the case of the large reservoir, the error bars for the simulation and experiment do not overlap, so if the GCI is good (or small) then there are only a few options for trying to get a better simulation correlation. These include trying different solvers, trying different time step size (and thus mesh sizes), different iteration criteria, different data manipulation techniques, different turbulence solvers and turbulence or heat transfer boundary conditions. With each of these issues, grid convergence techniques will at least yield an ability to quantify error bars and solution convergence orders.

Based on Eqn. (4) in the previous study,[8] a sensitivity analysis was performed on the three coefficients, a_1, D_1 and D_2. The sensitivity analysis verified that a_1 was insensitive, while the other two coefficients were much more sensitive to change. Sensitivity analysis was done for the small 30 cm^3 reservoir and for the large 334 cm^3 reservoir. The results showed that for both of these reservoir sizes that various combinations of D_1 and D_2 would not yield results correlated to experiment for both of the values of acoustic power and phase shift.

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