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M. DiMillo and M. Ostoja-Starzewski

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PAPER STRENGTH: STATISTICS AND CORRELATION STRUCTURE

M. DiMillo

*Hercules Incorporated, Paper Technology Division
411 Hercules Ave, P.O. Box 118, Kalamazoo, MI 49004, U.S.A.
e-mail: mdimillo@herc.com fax: 616-388-6587*

M. Ostoja-Starzewski

*Institute of Paper Science and Technology, and Georgia Institute of Technology
Atlanta, GA 30318-5794, U.S.A.
e-mail: martin.ostoja@ipst.edu fax: 404-894-4778*

Abstract. Due to the presence of a multiscale, disordered, fibrous microstructure in paper, its mechanical characteristics are not universal material properties but, rather, emerge as functions of specimen size and shape, as well as loading conditions. In order to grasp the statistical variability and spatial correlation structure of elasticity and strength parameters, a random field model is introduced. One- and two-point specifications of the model are exemplified by an array of 7" × 1" specimens subjected to tensile tests.

1. Random field of mechanical properties. The degree of variability in the stress-strain response of paper is displayed in Fig. 1, which shows typical results of ten tensile tests. All the specimens are cut out of a large (several meters wide and many kilometers long) paper web. Thus, at every point, given a chosen size and shape of a test specimen, the four conventionally measured parameters (elastic modulus E , breaking strength σ_{max} , strain to failure ϵ_{max} , and tensile energy absorption TEA) are random variables. Clearly, these variables are functions of position, and so, they constitute a four-component vector random field \vec{u}

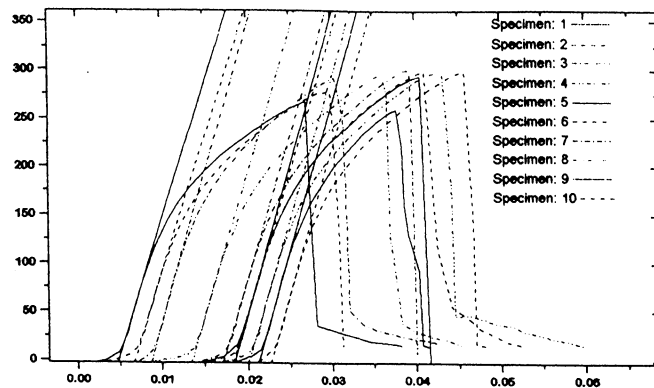


Fig. 1. Results of ten strength tests showing scatter in mechanical properties.

$$\left\{ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \right\} \equiv \left\{ \begin{array}{c} E \\ \sigma_{max} \\ \varepsilon_{max} \\ TEA \end{array} \right\} : B \times \Omega \rightarrow \mathbf{R}^4 \quad B \subset \mathbf{R}^2 \quad (1)$$

Here B is the body domain of the paper web in the x, y -plane, while Ω is the probability space (or space of elementary events ω).

The physical meaning of the space Ω is clarified by considering any single event ω . The occurrence of this ω - and hence, of specific values of field \vec{u} that correspond to it - represents an uncertain phenomenon, which can only be described statistically. Our problem formulation is analogous to the one encountered in turbulence theories (Frisch, 1995; Lumley, 1970), where one cannot predict exactly (i.e., deterministically) the velocity flow field and has to resort to probability tools. Indeed, our random field of paper properties has its origin in the turbulent settling of disordered fiber suspensions on the wire, and is interpreted as a final result of a "frozen-in turbulence." The field \vec{u} is parametrized by the size $L_x \times L_y$ of specimens - a concept again analogous to the turbulence theory, where the sampling of values of the velocity field is conducted over certain finite volumes corresponding to the resolution of a given instrument, e.g., radar, or laser Doppler.

Here, x and y correspond to the machine and cross directions, respectively. In the following, we focus on the $7'' \times 1''$ ($0.1778m \times 0.0254m$) specimen sizes, which represent the most common paper industry standard (TAPPI, 1994-95). The unnotched specimens are subjected to quasi-static tensile tests in the machine (x) direction. They are cut from a $\sim 79''$ ($2m$) wide roll of paper provided by Champion Intl. Corp. The paper has basis weight of $\sim 21 \text{ lb}/1000 \text{ ft}^2$ ($\sim 34 \text{ g}/m^2$) and a caliper of $\sim 2.6 \text{ mils}$ ($\sim 6.6 \cdot 10^{-5} m$); the sheet contains some mechanical pulp.

On the theoretical side, the random field \vec{u} is, most fundamentally, specified by a family of all the finite-dimensional m -point distribution functions ($x \equiv (x, y)$)

$$P\{U_1(x_1) \leq u_{11}, \dots, U_1(x_m) \leq u_{1m}, \dots, U_4(x_1) \leq u_{41}, \dots, U_4(x_m) \leq u_{4m}\} \quad (2)$$

Now, in view of the tremendous amount of information that would need to be collected to obtain (2), we content ourselves with an assessment of the one-point statistics and of the second-order correlation structure. These are obtained through

tensile strength tests on an 8×25 array of our $7'' \times 1''$ specimens. Figures 2(a)-(d) display the results of these tests for E , σ_{max} , ϵ_{max} , and TEA , respectively.

2. Statistics and correlation structure. The one-point statistics of E , σ_{max} , ϵ_{max} , and TEA are shown in Fig. 3 in a probability paper format with a logarithmic axis for the cumulative probability set in such a way as to result in a straight fit through the data points should they be Gaussian. While these data show only small departures from Gaussianity, Beta turns out to be the probability distribution of the best overall goodness-of-fit for a wide range of specimen sizes and other types of paper (to be discussed in a follow-up report). Beta is shown as a broken line.

The means of E , σ_{max} , ϵ_{max} , and TEA are, respectively, 3980, 34.9, 1.9, and 0.46, while their coefficients of variation are 0.04, 0.05, 0.1 and 0.14. It is most interesting to note here that the COV of E is on the same order as that of σ_{max} . This also holds for other specimen sizes and other types of paper, as will be demonstrated in a subsequent report.

The fundamental two-point, second-order information on the relation of two components u_i and u_j of the underlying vector field \vec{u} is specified by a *correlation coefficient*

$$\rho_{ij}(x_1, x_2) = [\langle C_i(x_1)C_j(x_2) \rangle - \langle C_i(x_1) \rangle \langle C_j(x_2) \rangle] / (\sigma_i \sigma_j) \quad (3)$$

which takes values between 1 and -1. These two extreme values are called, respectively, full positive and full negative correlatedness, while $\rho_{ij} = 0$ represents zero-correlatedness. The case $i = j$ is termed the *auto-correlation*, while $\rho_{12}, \rho_{13}, \dots$, are called *cross-correlation*.

There are, clearly, four auto-correlations and six cross-correlations, all of which can be put into a symmetric 4×4 array whose lower tridiagonal only is shown below (recall definition (1))

$$\rho_{ij}(x_1, x_2) \equiv \begin{bmatrix} \rho_{E,E} \\ \rho_{E,\sigma_{max}} & \rho_{\sigma_{max}\sigma_{max}} \\ \rho_{E,\epsilon_{max}} & \rho_{\sigma_{max}\epsilon_{max}} & \rho_{\epsilon_{max}\epsilon_{max}} \\ \rho_{E,TEA} & \rho_{\sigma_{max}TEA} & \rho_{\epsilon_{max},TEA} & \rho_{TEA,TEA} \end{bmatrix} \quad (4)$$

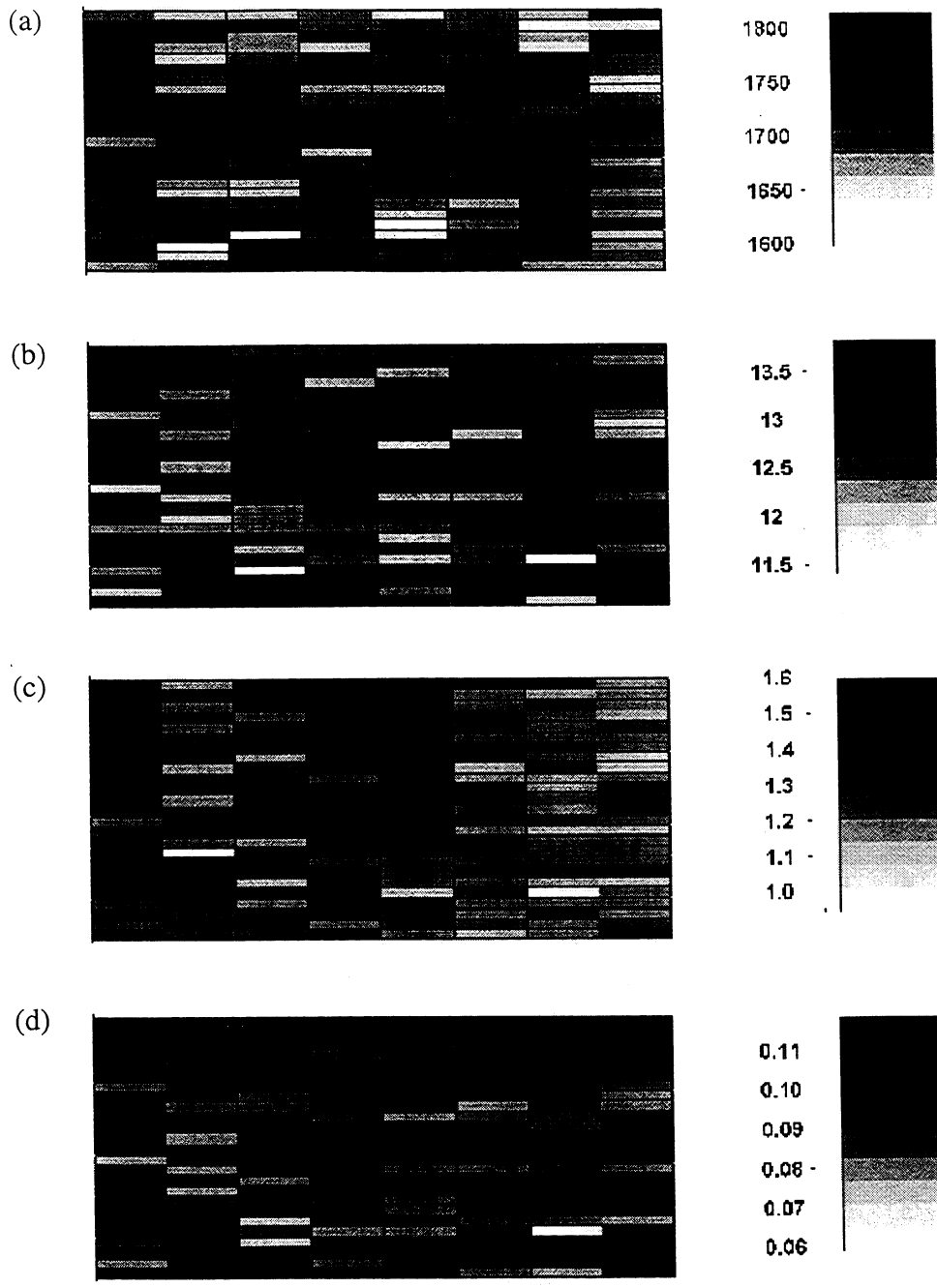


Fig. 2. A grey-scale plot of: (a) elastic modulus E lb/in ; (b) breaking strength σ_{max} in lb/in ; (c) strain to failure ϵ_{max} in %; and (d) tensile energy absorption TEA lb/in . All data are for an 8×25 array of $7'' \times 1''$ specimens. The ranges and assignments of values are shown in the respective insets.

Thus, when we consider correlations between the nearest neighbors (contiguous specimens) - i.e., when $\underline{x}_2 = \underline{x}_1 \pm L_x$ - we cannot expect the diagonal entries to equal unity. This is the setup which begins to shed light on the spatial correlation structure. We can do it with our data at hand (Fig. 2) providing we assume wide-sense stationarity of the \vec{u} field, that is $\rho_{ij}(\underline{x}_1, \underline{x}_2) = \rho_{ij}(r)$ for $\forall r = \underline{x}_1 - \underline{x}_2$. The ρ_{ij} 's for the nearest neighbors in the machine (x) and cross (y) directions are

$$\rho_{ij}(L_x) = \begin{bmatrix} 0.24 \\ 0.21 & 0.27 \\ 0.00 & 0.10 & 0.21 \\ 0.14 & 0.23 & 0.15 & 0.22 \end{bmatrix} \quad \rho_{ij}(L_y) = \begin{bmatrix} 0.33 \\ 0.18 & 0.13 \\ 0.09 & 0.02 & 0.26 \\ 0.05 & 0.04 & 0.04 & 0.04 \end{bmatrix} \quad (5)$$

The actual numbers in (5) depend on various factors - type of paper, specimen size and shape, fiber, fiber-fiber bonding, floc and streak structure, etc. - but it is already apparent that \vec{u} is a quasi-isotropic random field (e.g., Ostoja-Starzewski, 1998).

Next, focusing on ρ_{ij} functions at a point - i.e., when $\underline{x}_1 = \underline{x}_2$ - we find

$$\rho_{ij} = \begin{bmatrix} 1 \\ \rho_{12} & 1 \\ \rho_{13} & \rho_{23} & 1 \\ \rho_{14} & \rho_{24} & \rho_{34} & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.43 & 1 \\ 0.10 & 0.56 & 1 \\ 0.14 & 0.90 & 0.69 & 1 \end{bmatrix} \quad (6)$$

where $\rho_{ij} = \rho_{ij}(\underline{x}_1, \underline{x}_1)$. From this we observe that:

- i) Cross-correlations between E and inelastic parameters σ_{max} , ε_{max} , and TEA are weak, although we note that $\rho_{E, \sigma_{max}}$ is greater than $\rho_{E, \varepsilon_{max}}$ or $\rho_{E, TEA}$.
- ii) Three cross-correlations between σ_{max} , ε_{max} , and TEA are about the same.

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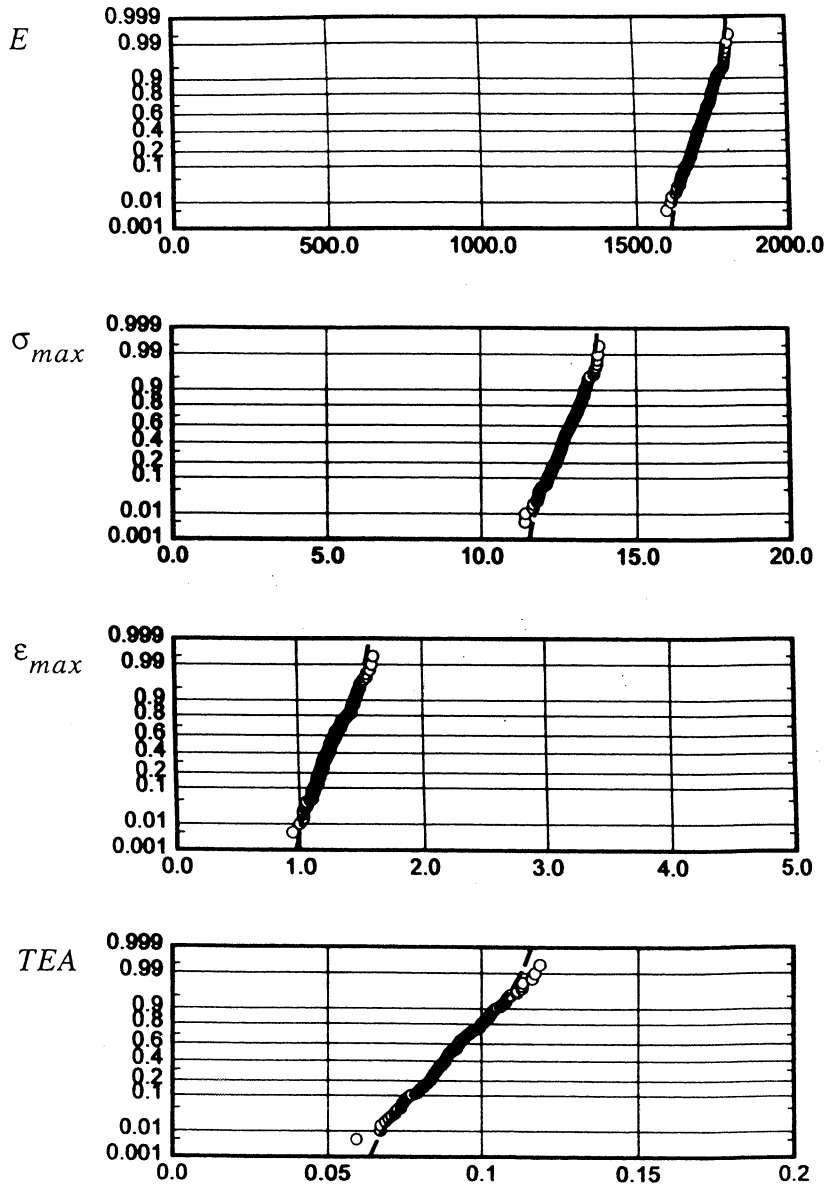


Fig. 3. Top to bottom: empirical histograms and Beta probability distribution fits of four mechanical properties.

