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THE DESIGN OF A SHOCK ABSORBER TO IMPROVE RIDE COMFORT BY REDUCING JERK

A THESIS

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THE DESIGN OF A SHOCK ABSORBER TO IMPROVE RIDE COMFORT BY REDUCING JERK

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NOMENCLATURE

A	Effective area of shock absorber piston; maximum acceleration.
a	$\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}$; amplitude.
A_c	Area of fixed opening in shock absorber piston.
A_{ci}	Area of fixed opening in inertial valve shock absorber piston.
A_i	Effective area of piston in inertial valve shock absorber.
A_v	Area of variable opening in piston.
A_2	$\frac{K_t\ s_t}{m_2}$
b	Linear damping coefficient in one degree of freedom model.
b_i	Linear damping coefficient in schematic drawing of jerk measuring device.
C	Per Cent of total kinetic energy lost by the fluid when passing through the shock absorber opening; linear damping coefficient of inertially controlled valve.
c	Linear damping coefficient of shock absorber.
C_b	$C\ A\ \rho/2$
C_{bc}	C_b for conventional shock absorber.
C_{bi}	C_b for inertial valve shock absorber.
C_v	$32\ A\ \mu\ L/D^2$
C_{vc}	C_v for conventional shock absorber.
C_{vi}	C_v for inertial valve shock absorber.

C.I.	Comfort index.
C_1	$0.72\sqrt{(K_t - k)/m_2}$
C_2	$\dfrac{f_v A_2^2 C_1^2}{4 M \omega_n^2}$
C_3	$\dfrac{-8 \zeta C_2}{\omega_n} - \dfrac{f_v A_2^2 C_1}{M \omega_n^2}$
C_4	$\dfrac{f_v A_2^2}{M \omega_n^2} - \dfrac{12 C_2}{\omega_n^2} - \dfrac{6 \zeta C_3}{\omega_n}$
C_5	$\dfrac{-A_2 C_1}{\omega_n^2} - \dfrac{4 C_4 \zeta}{\omega_n} - \dfrac{6 C_3}{\omega_n^2}$
C_6	$\dfrac{A_2}{\omega_n^2} - \dfrac{2 C_4}{\omega_n^2} - \dfrac{2 \zeta C_5}{\omega_n}$
C_7	$- C_6$
C_8	$\dfrac{\zeta C_7}{\sqrt{1 - \zeta^2}} - \dfrac{C_5}{\omega_n\sqrt{1 - \zeta^2}}$
D	Diameter of opening in shock absorber piston.
d	Amplitude of sinusoidal input.
F	Total force transmitted by shock absorber.
f	Force acting on mass in one degree of freedom model; frequency, cps.
F_b	Force transmitted by shock absorber due to the Bernoulli effect.
f_i	Initial spring force on the inertially controlled valve.

F_t Total force transmitted to upper mass.

F_v Friction force transmitted by shock absorber due to viscous friction.

f_v Constant, proportional to the effect of velocity on the opening of the inertially controlled valve.

F_1 Force transmitted by the rear shock absorber in the four degree of freedom model.

F_2 Force transmitted by the front shock absorber in the four degree of freedom model.

I Moment of inertia of top mass about its center of gravity in the four degree of freedom model

g Acceleration of gravity.

K Linear spring constant of inertially controlled valve.

k Linear spring constant in one, two, and four degree of freedom models.

k_i Linear spring constant in schematic drawing of jerk measuring device.

k_t Linear spring constant of lower spring in two and four degree of freedom models.

L Length of opening in shock absorber piston; length of wheel base in four degree of freedom model.

M Mass of inertially controlled valve.

m Ratio of effective area of piston to total opening in piston; mass in one degree of freedom model.

m_c m for conventional shock absorber.

m_i Mass in schematic drawing of jerk measuring device.

m_1 Mass of upper mass in two and four degree of freedom model.

m_2 Mass of lower mass in two and four degree of freedom model.

P Ratio of the variable opening to the distance the valve has opened.

Q $\left[\frac{R + 1}{2}\right]\left[C_{vc}\ m_c\ \pi + 8\ C_{bc}\ m_c^2\ d/3\right]$

q	Ratio of actual displacement to static displacement.
R	Ratio of damping in compression direction to damping in rebound direction.
r	Opening of the inertially controlled valve.
r_i	Initial compression of valve spring.
r_{max}	Maximum opening of inertially controlled valve.
r_{max_t}	Maximum deflection of inertially controlled valve spring from its free length.
s	Differential operator.
s_t	Height of step input.
t	Time.
V	$\frac{dy}{dt} - \frac{dx}{dt}$
V_1	$\frac{dy_1}{dt} - \frac{dx}{dt} + \frac{L}{2}\frac{d\theta}{dt}$
V_2	$\frac{dy_2}{dt} - \frac{dx}{dt} - \frac{L}{2}\frac{d\theta}{dt}$
x	Displacement of upper mass in one, two, and four degree of freedom models; displacement of mass in schematic drawing of jerk measuring device.
X_o	Amplitude of displacement in schematic drawing of jerk measuring instrument.
$\dot{x}$	$\frac{dx}{dt}$
y	Displacement of lower mass in two degree of freedom model.
y_1	Displacement of lower rear mass in four degree of freedom model.
y_2	Displacement of lower front mass in four degree of freedom model.
z	Displacement of input in two degree of freedom model.
z_1	Input to lower rear spring in four degree of freedom model.
z_2	Input to lower front spring in four degree of freedom model.

ζ	$c/2\sqrt{K\ M}$
θ	Rotation of top mass in four degree of freedom model.
μ	Viscosity of oil in shock absorber.
ρ	Mass density of shock absorber fluid.
ϕ	Phase angle between the input and output.
ω	Forcing frequency, rad/sec.
ω_n	Natural frequency of inertially controlled valve, $\sqrt{K/M}$
ω_1	$\sqrt{k/m_1}$
ω_2	$\sqrt{K_t/m_2}$

SUMMARY

The primary objective of the reported research was to investigate analytically and experimentally the feasibility of using a shock absorber of original design with inertially controlled valves to reduce jerk in an automobile-type suspension system.

A shock absorber was designed on the basis of data obtained from the study. The design incorporates either one or two inertially controlled valves, depending upon the application. These valves, which control the amount of damping of the shock absorber, are opened by the acceleration of the unsprung mass (wheels and other masses moving with them). In this way, the damping of the shock absorber is a function of the acceleration of the unsprung mass and dependent upon the road condition.

The mathematical model that was used had two degrees of freedom. In effect, one corner of the automobile was used as the model. One degree of freedom represents the motion of the automobile body, while the other degree of freedom represents the motion of the wheel. A linear spring and a shock absorber in parallel connects the mass representing the automobile body to the mass representing the wheel. A second linear spring representing the automobile tire is placed between the mass of the wheel and the road surface and transmits the excitation to the entire system.

Equations of motion were written for the complete system and solved numerically on the digital computer for step, ramp, and sinusoidal inputs. A comparison between the dynamics of the system using

a conventional type of nonlinear shock absorber and the inertial valve shock absorber shows that a significant reduction of jerk and acceleration of the top mass is obtained by using the latter.

The mathematical model was duplicated in the laboratory with a size reduction of approximately 20 with respect to a typical automobile. The system was excited by an approximate ramp input from an hydraulic piston. For each ramp input the displacement, acceleration, and jerk of the top mass were measured directly along with the displacement of the hydraulic piston. A Stratham strain gage type of accelerometer was used to measure the acceleration and an instrument designed in the laboratory was used to directly measure jerk. Tests were run using both the inertial valve shock absorber and a conventional type. Experimental results verified the mathematical analysis showing that a smaller value of jerk is transmitted to the top mass, which represents the automobile body, when the inertial valve shock absorber is used.

Design equations were derived for the inertial valve shock absorber to assist the engineer in approaching an optimum design. In order to generate enough analytical data from the response curves of many systems having different parameters, it was necessary to derive a closed-form solution. The time interval in which the solution is accurate is short, but it is long enough to include the period of high jerk transmittal after a sudden force change is experienced.

An investigation was also conducted to determine whether jerk is a factor in riding comfort. Subjects were seated in a typical wooden office chair and subjected to two types of vertical motion. One type of motion had a considerably higher value of jerk while the maximum velocity, maximum acceleration, frequency, and amplitude of the two types of motions were very nearly the same. A high percentage of the participating people felt that the motion that had the smaller value of jerk was the more comfortable. This test only attempted to show that jerk is a factor in riding comfort and that a ride with less jerk is more comfortable.

From the data taken in the laboratory and the results of the analytical investigation, it appears very promising that a shock absorber with inertially controlled valves could improve the riding comfort of an automobile without sacrificing any loss of control or stability characteristics.

CHAPTER I

INTRODUCTION

Problem Statement

This work is concerned with the investigation and design of a hydraulic shock absorber that will reduce the amount of jerk that is transmitted by the shock absorber to the upper mass in an automobile-type suspension system. "Jerk" is defined as the third derivative of displacement with respect to time or simply as the first derivative of acceleration. Physically, it represents the rate of change of force. The reason that jerk was used as the criterion of riding comfort will be discussed in the next section.

Figure 1 shows a two degree of freedom system representing the mass and suspension system of an automobile. The top mass (m_1) represents the mass of the automobile body, while the lower mass (m_2) represents the unsprung mass (the wheel and attached mass). A linear spring with a spring constant, k, and a linear shock absorber with a damping coefficient, c, connects the two masses. A linear spring of constant K_t represents the tire. Applying Newton's second law and differentiating the resulting expression, an equation for the jerk of the top mass can be written as follows:

$$\text{Jerk} = \frac{d^3x}{dt^3} = \frac{k}{m_1}\left[\frac{dy}{dt} - \frac{dx}{dt}\right] + \frac{c}{m_1}\left[\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}\right] \qquad (1.1)$$

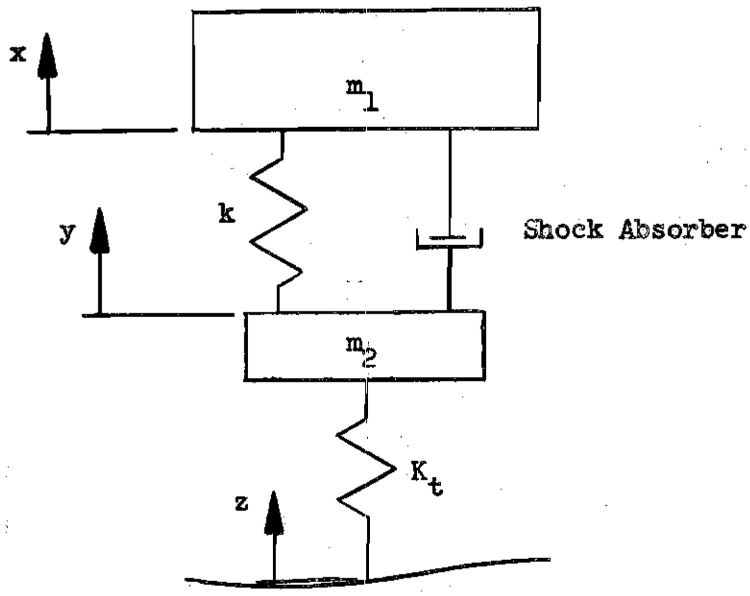


Figure 1. Mathematical Model.

By using typical values found on a representative automobile in this expression, it can be calculated that for a step input to the lower spring approximately 85 per cent of the maximum jerk that reaches the top mass is transmitted by the shock absorber. The remaining 15 per cent is transmitted by the spring. Approximately 90 per cent of the jerk transmitted by the shock absorber is the result of the product of the acceleration of the unsprung mass and the value of the linear damping coefficient of the shock absorber.

It is then obvious that to reduce the jerk transmitted by a shock absorber, one must be built such that its damping coefficient decreases when the acceleration of the unsprung mass increases. The investigation of a shock absorber that will reduce jerk in this manner is the basis of this work.

If the problem were only that of reducing jerk, it would be a trivial one, since the jerk of the top mass of an automobile suspension system that has no energy absorbing device is very small. A constraint must be imposed on the shock absorber so that it will perform in a certain manner to add stability to the automobile.

An ideal shock absorber should give a small damping force at low wheel velocity movements so that the harshness that results from riding over small road irregularities will not be transmitted to the car body. Yet, the ideal shock absorber should provide enough resistance at low wheel velocities to eliminate floating when the car is traveling at high speeds. The damping at high wheel velocities should be great enough to eliminate excessive overshoot and force the oscillations to die out quickly, but an increase in damping can only be achieved at the expense of greater transmission of force from the unsprung mass. Present shock absorbers usually have much less damping in the compression direction than in the rebound direction. This eliminates a great deal of the harshness that results when the wheel strikes a sharp bump, but also a great deal of stability and control is lost due to the decrease in damping in the compression direction. Also, most modern shock absorbers have a blow-off valve which opens when the velocity reaches a prescribed point. This reduces the damping at high velocities, and thus the harshness that results from wheel movements of high velocities is attenuated.

Historical Background

The amount of jerk that a person experiences is not usually recognized as a criterion of riding comfort. Jerk is much more difficult to measure and to control than either acceleration or velocity and can change almost instantaneously and reach very high values in a conventional automobile-type suspension system. For this reason most of the studies of riding comfort have made use of sinusoidal vertical motion where the pertinent parameters are frequency, acceleration, velocity, and amplitude. From these parameters, a mathematical expression for the relative degree of comfort, called by many the "comfort index," has been proposed. The following are examples of work that has been done in this field. In all cases a low index implies a comfortable ride.

Jacklin and Liddell (1) defined the comfort index by the equation

$$C.I. = A\,e^{0.6f}$$

where "A" is the maximum acceleration and "f" is the frequency of vibration.

Janeway (2) found the comfort index to be:

$$C.I. = a\,f^{x}$$

where,

a is the amplitude.

f is the frequency.

x is an exponent varying from 1 to 3.

For high frequency vibrations of 20 to 60 cps, $x = 1$, the comfort index is proportional to the maximum velocity. In the 6 to 20 cps range, x = 2, the comfort index is proportional to the maximum acceleration. For low frequency vibrations of 1 to 6 cps, x = 3, which means the comfort index is proportional to the maximum time derivative of acceleration.

Sperling (3) calculated a more complicated comfort index. It is given by the following equation:

$$C.I. = \sqrt[10]{A^3/f}$$

where,

A = acceleration amplitude.

f = frequency.

One can see from these three typical examples that each comfort index is quite different. For equal values of acceleration and frequency, the various comfort indices may yield widely differing index values which do not offer a reliable indication of the degree of comfort, and even more important, they do not give a good indication of what parameters are important for a comfortable ride.

A criterion that has been mentioned less frequently concerning riding comfort is the time derivative of acceleration, commonly called jerk. Den Hartog (4) considered jerk a criterion of comfort and claimed that steady acceleration is not uncomfortable and for small values cannot be felt, but that the change in acceleration produces uncomfortable sensations.

Bogdanoff and Kozin of the Land Locomotion Laboratory (5) used the "variance of vertical motion acceleration" to calculate the roughness of the ride. The smoothness of devices such as an automatic transmission is measured by the amount of jerk that results from the shifting of the transmission. The elevator companies are concerned with the maximum value of jerk that reaches the passenger. They have done no experimental work concerning the effect of jerk on human comfort, but at least two leading elevator manufacturers are both aware that a high value of jerk is uncomfortable. Both attempt to keep the maximum rate of change in acceleration below 30 feet/second^3 and if possible below 20 feet/second^3.

A paper, by Richard Fine (6) of the University of Wisconsin, claimed that the rate of change of vertical acceleration is a good indication of riding comfort. He reached this decision after measuring the actual vertical acceleration in several late model automobiles that ranged from a compact car to a high-priced luxury automobile. It was found that the maximum acceleration felt in the compact car was not greatly different than the maximum acceleration of the luxury automobile, but the slope of the acceleration-time curve, which is jerk, was much greater for the compact car.

Suspension engineers have not attempted to design suspension systems from data furnished by investigators, such as previously mentioned, who have done work on riding comfort. For the most part, they have followed two avenues of approach. One has been to use softer springs and tires to lower the natural frequency of the vehicle. A limitation exists in tire softness because of the energy absorbed in a soft tire. The spring stiffness of an automobile also has a lower limit because of overturning movement of the automobile when rounding curves and also because of the limited travel available between the wheel and the frame. The other avenue of approach has been in the direction of better energy absorbing devices. The first types of damping devices were of the friction type. They were later replaced by the cam or lever type which offered more flexibility of control. Due mainly to economic reasons, these cam or lever types were replaced by the present direct-acting type.

Thus, after the geometry and weight of the car are determined, the springs and tires made as soft as possible, and the friction in the suspension eliminated, the only other important variable that can improve the riding quality is the shock absorber.

Investigation Procedure

The first portion of the investigation involved the analytical investigation of the inertial valve and conventional type of shock absorbers on the two degree of freedom system. The analytical portion also included the derivation of equations that will assist the engineer in designing the inertial valve shock absorber.

The experimental investigation was primarily directed toward verifying the analytical portion of the work. Tests were conducted on the two degree of freedom system for various ramp inputs. Tests were also conducted to determine the force transmitted by a shock absorber. To verify that the amount of damping on the two degree of freedom system was equivalent to the actual damping of automobiles, tests were conducted that determined the damping on several late model automobiles.

CHAPTER II

ANALYTICAL INVESTIGATION

Mathematical Models

Two Degree of Freedom Model

In analyzing the performance of a shock absorber, it is necessary to use a mathematical model that will adequately represent the actual system, but it is also necessary to use a model that is simple enough to allow the dynamics of the shock absorber to be singled out and analyzed. An automobile has seven degrees of freedom. This includes the vertical motion of all four wheels plus the vertical motion of the center of gravity and the pitch and roll of the body. One could even go further by considering the elasticity of the frame and the motion of passengers on the seats. This type of system is very complex to work with and the contribution of the shock absorbers to the dynamics of the system is not easily recognized. Also, it is desirable to have a mathematical model that can be duplicated in the laboratory. For these reasons, the simple two degree of freedom model of Figure 1 was used. It can be considered as one wheel and an effective mass of an automobile.

Using the symbol "F," to represent the positive upward force transmitted by the shock absorber, the equations of motion for the mathematical model can be written as follows:

$$m_1 \frac{d^2x}{dt^2} - F + k(x - y) = 0 \qquad (2.1)$$

$$m_2 \frac{d^2y}{dt^2} + F + k(y - x) = K_t(z - y) \qquad (2.2)$$

Equations necessary to evaluate the force transmitted by the shock absorber are developed in the following section.

Theoretical Force Equations of Shock Absorbers

The hydraulic shock absorber is an irreversible energy absorbing device. It absorbs energy when fluid is forced through openings in the shock absorber piston. When the fluid is forced through the small openings, the velocity first increases and then decreases. Some of the kinetic energy that the fluid has when passing through the small openings is not recovered and is changed into thermal energy. The remaining energy is absorbed in viscous friction as the fluid passes through the openings.

Figure 2 shows a simple shock absorber with the piston moving through an incompressible fluid. Assuming that the inertial forces due to the mass of the fluid in the openings and to the mass of the piston are both small compared to the damping forces, the force needed to move the piston for a given shock absorber is a function of the velocity only. As in the case of the energy absorbed, some of the force needed to move the piston is due to the viscous friction and the remaining resisting force can be attributed to the pressure difference required to accelerate the fluid through the opening. The viscous friction term is usually associated with Hagen-Poiseuille type of flow which is assumed to be fully developed and laminar through a circular opening and has a parabolic velocity distribution. The flow through the shock absorber opening

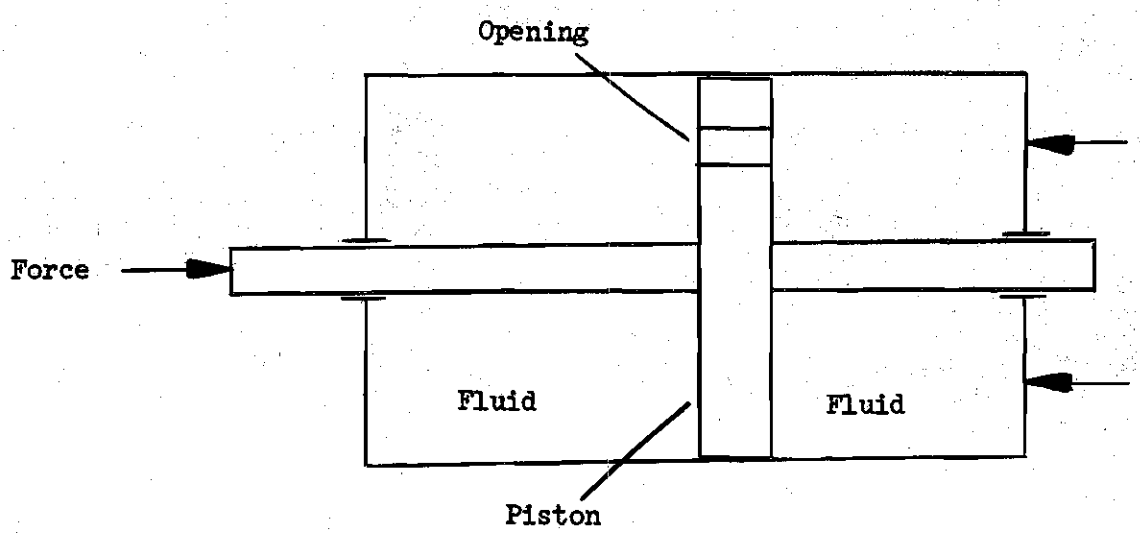


Figure 2. Shock Absorber.

never becomes fully developed, but the Hagen-Poiseuille equation still serves as a good approximation. The Reynolds number for flow in a typical shock absorber is much less than 2000, which would indicate laminar flow. The portion of the force attributed to the viscous friction can be written as follows:

$$F_v = \frac{32 A^2 \mu L}{D^2 A_c} V = C_v m V \qquad (2.3)$$

where,

A = Area of the piston minus the area of the rod (effective area of the piston).

A_c = Total area of the opening in the shock absorber piston.

μ = Viscosity of the fluid.

L = Length of the opening in the piston.

D = Diameter of the opening.

V = Velocity of the piston.

$$C_v = 32 \frac{A\mu L}{D^2}$$

$$m = \frac{A}{A_c}$$

The portion of the resisting force that is the result of accelerating the fluid can be found in Bernoulli's equation. It can be written as follows:

$$F_b = \frac{C\, A^3\, \rho\, V^2\, \text{sign}(V)}{2\, A_c^2} = C_b\, m^2\, V^2\, \text{sign}(V) \qquad (2.4)$$

where,

C = Constant, the per cent of the total kinetic energy lost by the fluid when passing through the opening.

ρ = Mass density of the fluid.

$$C_b = \frac{C\, A\, \rho}{2}$$

The sum of the two forces given by Equations (2.3) and (2.4) gives the total resisting force which is in a direction opposing the velocity,

$$F = F_v + F_b = C_v\, m\, V + C_b\, m^2\, V^2\, \text{sign}(V) \qquad (2.5)$$

Equation (2.5) was found to be valid for the inertial valve shock absorber that was built and analyzed. For this shock absorber, the value of m (the ratio of the effective area of the piston to the total area of the openings) is not a constant, but is dependent upon the opening of the inertially controlled valve. The value of m is given by the equation

$$m = \frac{A}{A_c + A_v} \tag{2.6}$$

where,

A_c = Total area of fixed opening in the piston.

A_v = Area of the variable opening in the piston.

The value of A_v is also dependent upon the opening of the inertially controlled valve. A_v can be expressed as follows:

$$A_v = P\,r \qquad \text{for } 0 \le r \le r_{max} \tag{2.7}$$

$$A_v = P\,r_{max} \qquad \text{for } r > r_{max}$$

where,

P = Constant (ratio of the variable opening to the distance the valve has opened).

r = Opening of valve, see Figure 3.

r_{max} = Point of opening of the valve where further opening yields no increase in flow area.

The opening of the valve can be found in the solution of the following differential equation:

$$M \frac{d^2r}{dt^2} + C \frac{dr}{dt} + K\, r \;=\; M \frac{d^2y}{dt^2} - f_i + M\, g + f_v\, V^2\, \mathrm{sign}(V)$$

$$r \geq 0 \qquad (2.8)$$

where,

M = Mass of the valve.

C = Linear damping coefficient of the valve.

K = Spring constant of the valve spring.

f_i = Initial spring force on the valve (r_i K).

g = Acceleration of gravity.

f_v = Constant.

$V = \frac{dy}{dt} - \frac{dx}{dt}$

In writing Equation (2.8), it was assumed that the damping on the valve was linear and was estimated to be 25 per cent of critical damping. It is also noted that a force proportional to the square of the velocity is included; however, this force is much smaller than the inertial force.

Figure 3 is a drawing of the inertial valve and piston assembly. It can be seen from Equation (2.8) that the opening of the valve is dependent upon the acceleration of the lower mass and for large values in the upward direction, the valve opens and reduces the damping ability of the shock absorber. Thus, according to the requirement stated in Chapter I, this is the general design of a shock absorber in which the damping coefficient decreases when the acceleration of the unsprung mass increases.

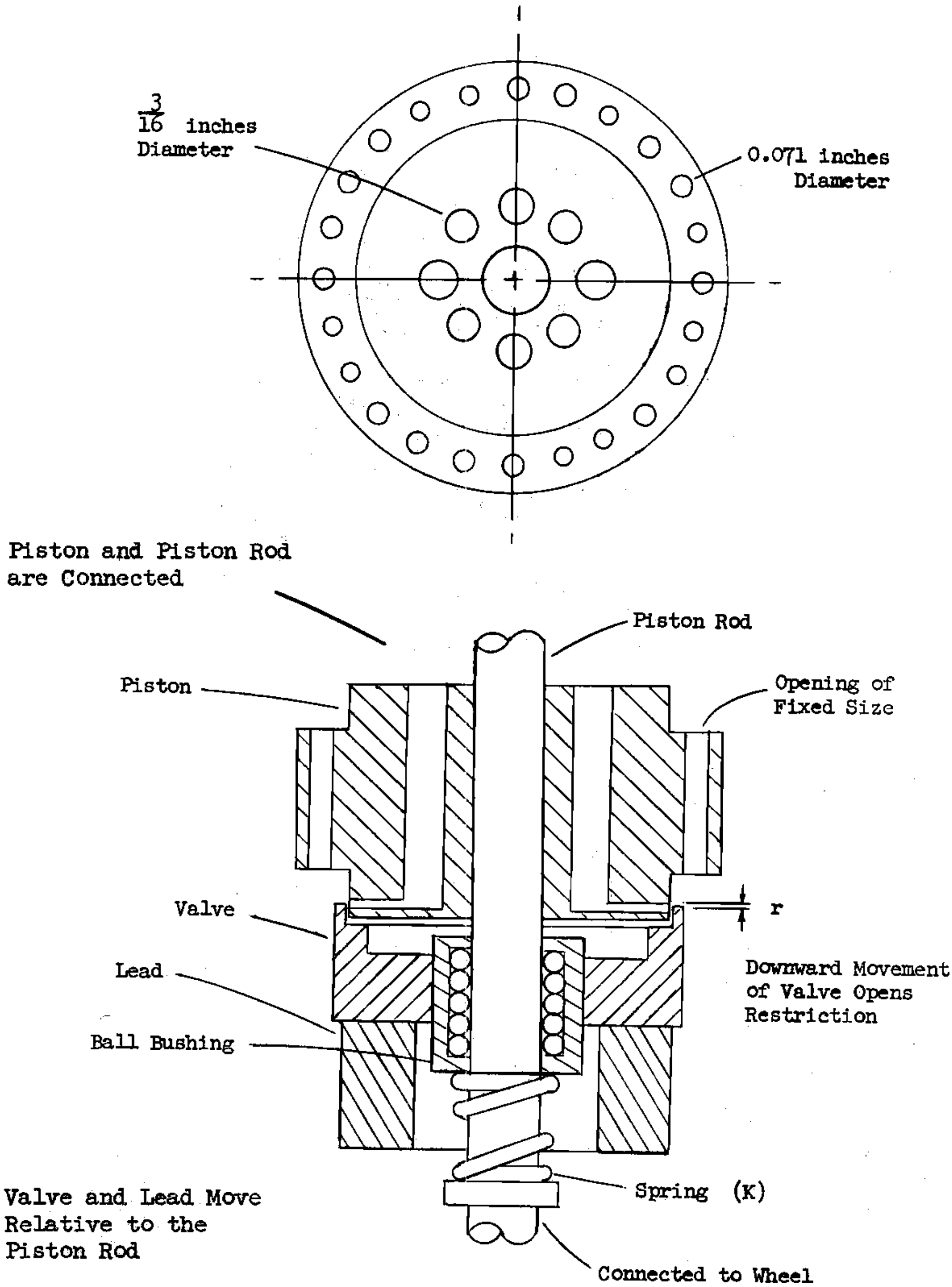


Figure 3. Actual Shock Absorber Piston and Inertial Valve.

System Equations and Solutions

By combining the equations of motion of the mathematical model with the equations for the operation of the inertial valve shock absorber, a block diagram (Figure 4) can be constructed that represents the entire mathematical model. The blocks signify mathematical operations rather than the usual transfer functions used for linear models.

The system of equations was solved numerically using the fourth-order Runge-Kutta method on a Burroughs B-5500 digital computer. A time increment of approximately 1/300 of the natural period of the valve was used. Using a smaller time increment was found to only add to the computing time without changing the final results.

The value of jerk can be calculated by the following equation:

$$\text{Jerk} = \frac{d^3x}{dt^3} = \frac{1}{m_1}\left[k\,V + C_v\left(m\,a + V\,\frac{dm}{dt}\right) + 2\,C_b\,m\,V\left(V\,\frac{dm}{dt} + m\,a\right)\,\text{sign}(V)\right] \quad (2.9)$$

where,

$$a = \frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}$$

for $0 \leq r < r_{max}$

then,

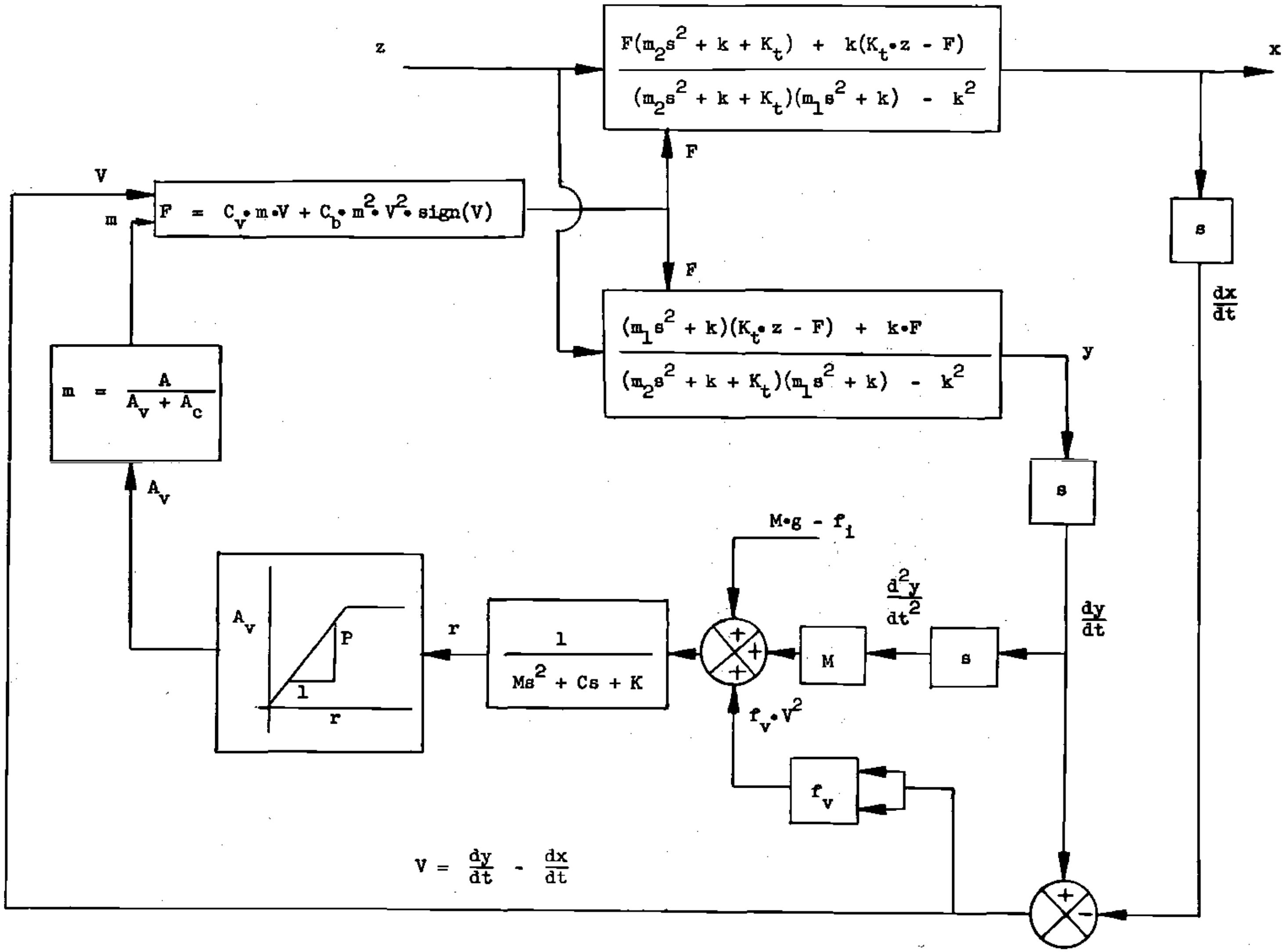


Figure 4. Block Diagram of Complete System.

$$m = \frac{A}{P\,r + A_c}$$

$$\frac{dm}{dt} = \frac{-A\,P}{(P\,r + A_c)^2}\,\frac{dr}{dt}$$

for $r \geq r_{max}$

then,

$$\frac{dm}{dt} = 0$$

$$m = \frac{A}{P\,r_{max} + A_c}$$

The value of jerk can also be calculated by subtracting the previous value of acceleration that was calculated by the Runge-Kutta method from the present value and dividing this difference by the time increment. Both methods were used, but the latter was preferred because of the saving in computer time.

The mathematical description of the system using a conventional type of shock absorber can best be described by writing the equations of motion (2.10, 2.11). In this type of shock absorber, the total opening in the piston is a constant.

$$m_1 \frac{d^2x}{dt^2} - C_v\,m\,V - C_b\,m^2\,V^2\,\text{sign}(V) + k(x - y) = 0 \qquad (2.10)$$

$$m_2 \frac{d^2y}{dt^2} + C_v\ m\ V + C_b\ m^2\ V^2\ \text{sign}(V) + k(y - x) = K_t(z - y) \qquad (2.11)$$

The expression for the jerk is much simpler for the conventional type of shock absorber and can be obtained by differentiating Equation (2.10). The expression is written as follows:

$$\text{Jerk} = \frac{d^3x}{dt^3} = \frac{1}{m_1} [k\ V + C_v\ m\ a + 2\ C_b\ m^2\ V\ a\ \text{sign}(V)] \qquad (2.12)$$

Step Input. Figures 5 and 6 show curves of displacement, velocity, acceleration, and jerk that are the result of a unit step input, for three types of shock absorbers. Shock absorber number 1 has an inertially controlled valve. Number 2 is a conventional type with the same damping in each direction. Shock absorber number 3 is of the conventional type with a reduction in damping of one half in the compression direction. In order to compare the shock absorbers, all three were designed to allow a 50 per cent overshoot. The parameters that were used in the computer solution are shown below.

The values of the spring constants (k and K_t) and the values of the masses (m_1 and m_2) were given by Reference (7) to be the average values of a medium size American automobile.

m_1 = 3.0 lb-sec^2/in	m_2 = 0.3 lb-sec^2/in
k = 100 lb/in	K_t = 1200 lb/in
A = 1.0 in^2	C_v = 0.1 lb-sec/in
C_b = 0.0001 lb-sec^2/in^2	A_c = 0.0183 in^2 (number 1 only)

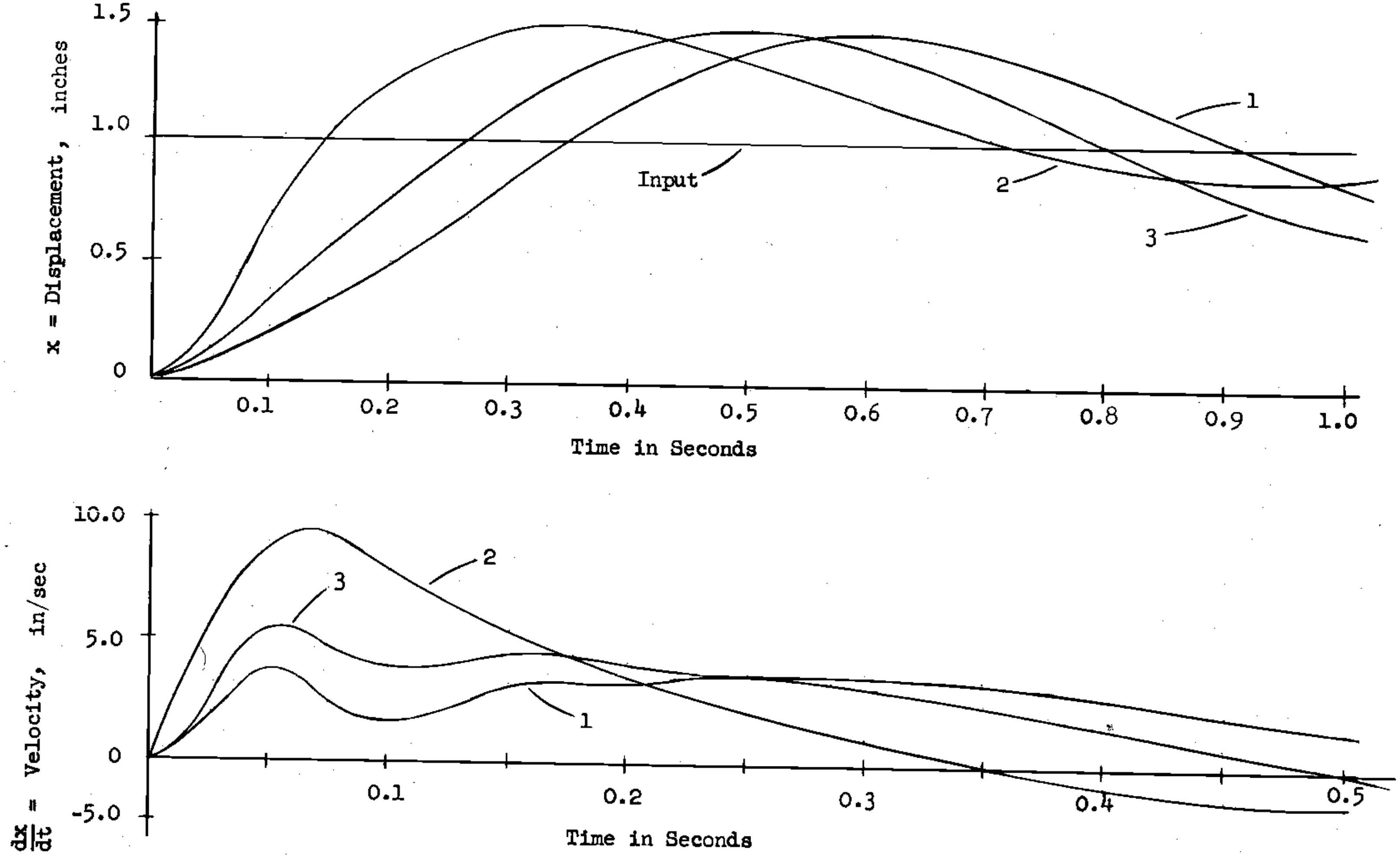


Figure 5. Computer Results for Displacement and Velocity for a Unit Step Input.

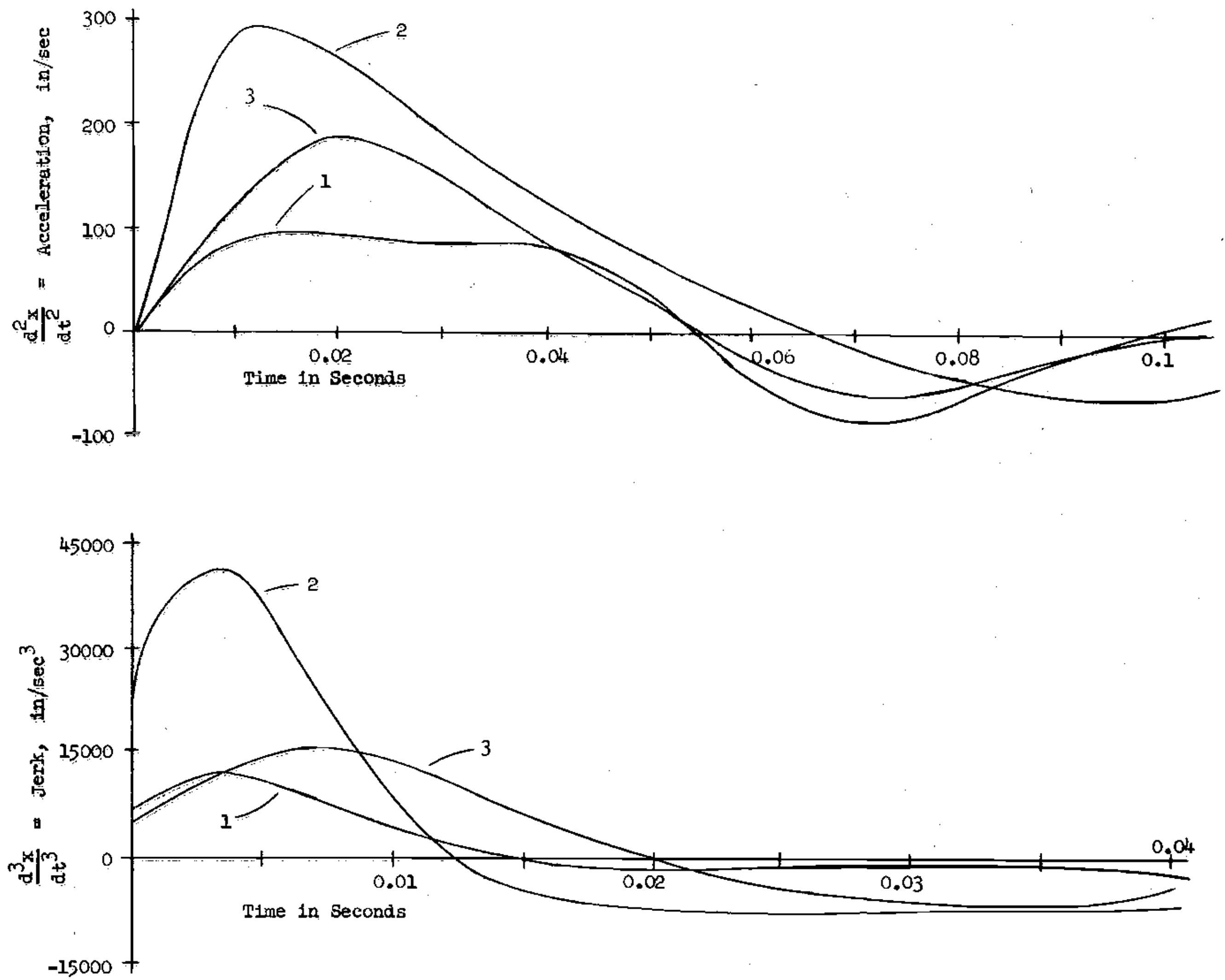


Figure 6. Computer Results for Acceleration and Jerk for a Unit Step Input.

A_c = 0.008 in^2 (number 2 only) A_c = 0.014 in^2 (number 3 only)

$A_{v_{max}}$ = 0.0458 in^2 $M\ g = f_i$

f_v = 0.00001 lb-sec^2/in^2 M = 0.000237 lb-sec^2/in

K = 1.0 lb/in   C = 0.077 lb-sec/in

Sinusoidal Input. Using the same parameters, a sinusoidal input of unity amplitude, ($z = \sin \omega t$) was used as a forcing function. When the system reached steady state after approximately five cycles, the maximum values of displacement, acceleration, jerk, and the energy absorbed by the shock absorber during the last cycle were recorded. The results for the different forcing frequencies for shock absorbers 1 and 3 are shown in Figures 7 and 8. It should be noted that the difference between the two shock absorbers is very small for low values of the forcing frequency, but at the higher frequencies, the valve opens and a reduction of jerk and acceleration occurs. It should also be noted that even though the inertial valve shock absorber allowed lower values of acceleration and jerk, it absorbed more energy than the conventional shock absorber. This would indicate that the inertial type shock absorber provided more stability and control for the automobile.

Inertial Valve Shock Absorber Design Equations

Introduction

Since many parameters affect the characteristics of the inertial valve shock absorber, it is obvious that certain combinations of these parameters will give better performance than others. It is desirable to find an expression relating the best parameters of the shock absorber to the parameters of the system in which the shock absorber is used. In

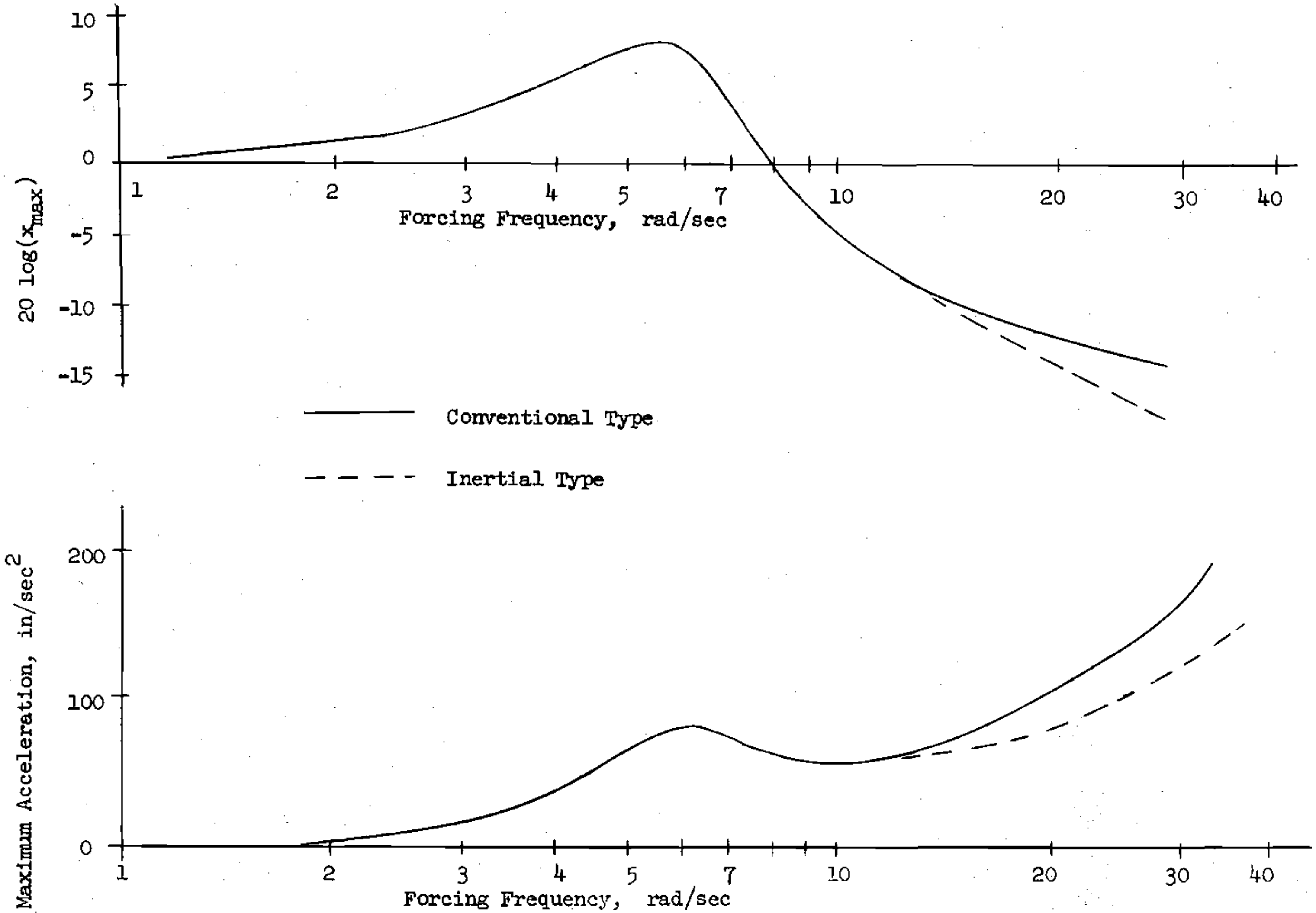


Figure 7. Frequency Response for Displacement and Acceleration.

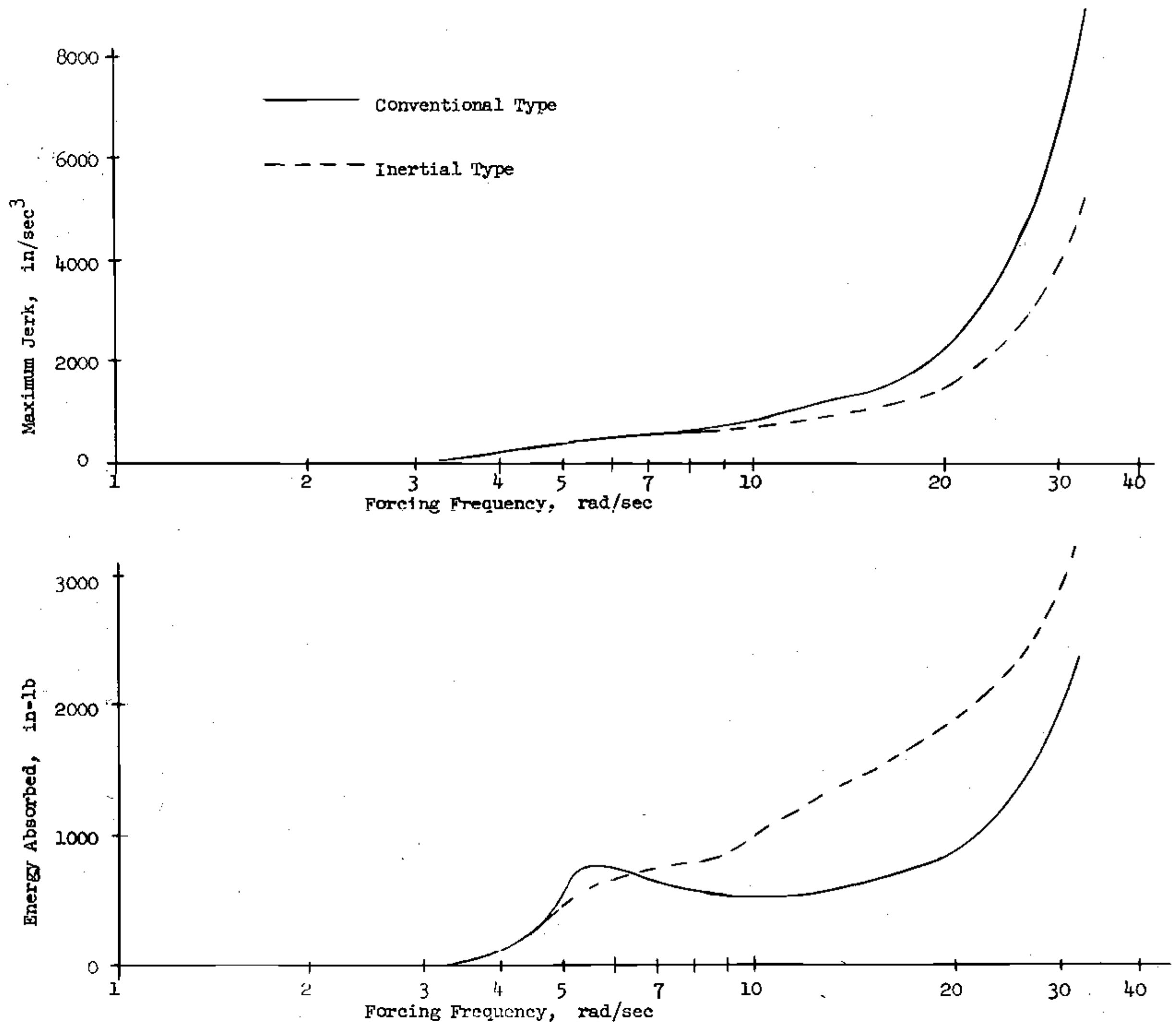


Figure 8. Frequency Response for Jerk and Energy Absorbed.

other words, if such items as the weight of the automobile, the spring and tire stiffness, and the amount of damping desired are given, an expression is needed to determine the values of the natural frequency of the valve, the amount of fixed opening, and the maximum amount of the variable opening.

The problem of finding the "best" parameter values is essentially one of finding several coefficients of derivative terms and the constants in a system of nonlinear differential equations that will give the best performance. The best shock absorber is defined as one in which, for a given overshoot from a step input, the jerk is a minimum. But it is also a requirement that the damping force in the compression direction be great enough to add stability to the vehicle. For example, if there were no damping force in the compression direction, the jerk and the overshoot due to a step input would be small, but the shock absorber would add little to the stability of the vehicle. It is for the above reasons that no direct mathematical method could be found to provide exact expressions for the best parameters of the shock absorber.

Area of Fixed Opening

The first parameter that will be considered is the area of the fixed opening in the shock absorber piston. It can be related to the parameters of an equivalent conventional shock absorber by Equation (2.13). This equation is derived by equating the energy absorbed by the conventional shock absorber to the energy absorbed by the inertial valve shock absorber, when the shock absorbers are subject to a sinusoidal input. Since the greatest amount of damping is needed when the system is excited at the natural frequency of the automobile spring-mass system, this frequency is used in calculating the area of the fixed opening in the piston of the inertial valve shock absorber. The equation is as follows:

$$A_{ci} = \frac{(16/3)\ C_{bi}\ A_i\ \omega_1\ d}{\sqrt{\pi^2\ C_{vi}^2 + (32/3)\ Q\ C_{bi}\ d\omega_1} - C_{vi}\ \pi} \qquad (2.13)$$

where,

A_{ci} = Desired area of fixed opening in inertial valve shock absorber piston.

A_i = Effective area of piston of inertial valve shock absorber.

C_{vi} = C_v for inertial valve shock absorber.

C_{bi} = C_b for inertial valve shock absorber.

d = Amplitude of sinusoidal input.

ω_1 = Natural frequency of automobile body, $\sqrt{k/m_1}$

$Q = \left[\frac{R+1}{2}\right]\left[C_{vc}\ m_c\ \pi + (8/3)\ C_{bc}\ m_c^2\ d\ \omega_1\right]$

C_{vc} = C_v for conventional shock absorber.

C_{bc} = C_b for conventional shock absorber.

R = Ratio of the damping in the compression direction to the damping in the rebound direction of the conventional shock absorber.

m_c = m for conventional shock absorber.

Area of Variable Opening

The second parameter that is considered is the desired maximum area of the variable opening in the piston of the inertial valve shock absorber. Many curves, such as Figure 9, were drawn which show the ratio of the maximum opening area of the valve to the area of the fixed opening, plotted against the maximum jerk that occurs for a unit step input. For the curve shown, the shock absorber allowed an overshoot of 50 per cent (± 1 1/2 per cent). The curve shows that the value of jerk is not substantially reduced after the ratio of the areas reaches approximately 2.5. The ratio of 2.5 also gave good results for sinusoidal and ramp inputs of various frequencies and amplitudes. For these reasons, the ratio of the maximum area of the variable opening to the fixed area was chosen to be 2.5.

Natural Frequency of Valve

The parameter that is the most difficult to obtain is the desired natural frequency of the valve, which is an indication of the speed of opening. The speed of opening of the valve is very important to the reduction of jerk, but has very little to do with the amount of damping. A step input to the lower spring is the most abrupt form of input and will give the value the greatest velocity. For this reason, a step input is used to judge the performance of the shock absorbers.

A comparison of the jerk-time curves for a unit step input for three different valve natural frequencies can be made by examining Figure 10. If the natural frequency is too high (120 rad/sec), the valve will open too fast and cause a high second peak of jerk. When the natural frequency is too low (20 rad/sec), the action of the valve is too slow and a high first peak of jerk occurs. An intermediate value (60 rad/sec) will give the best results.

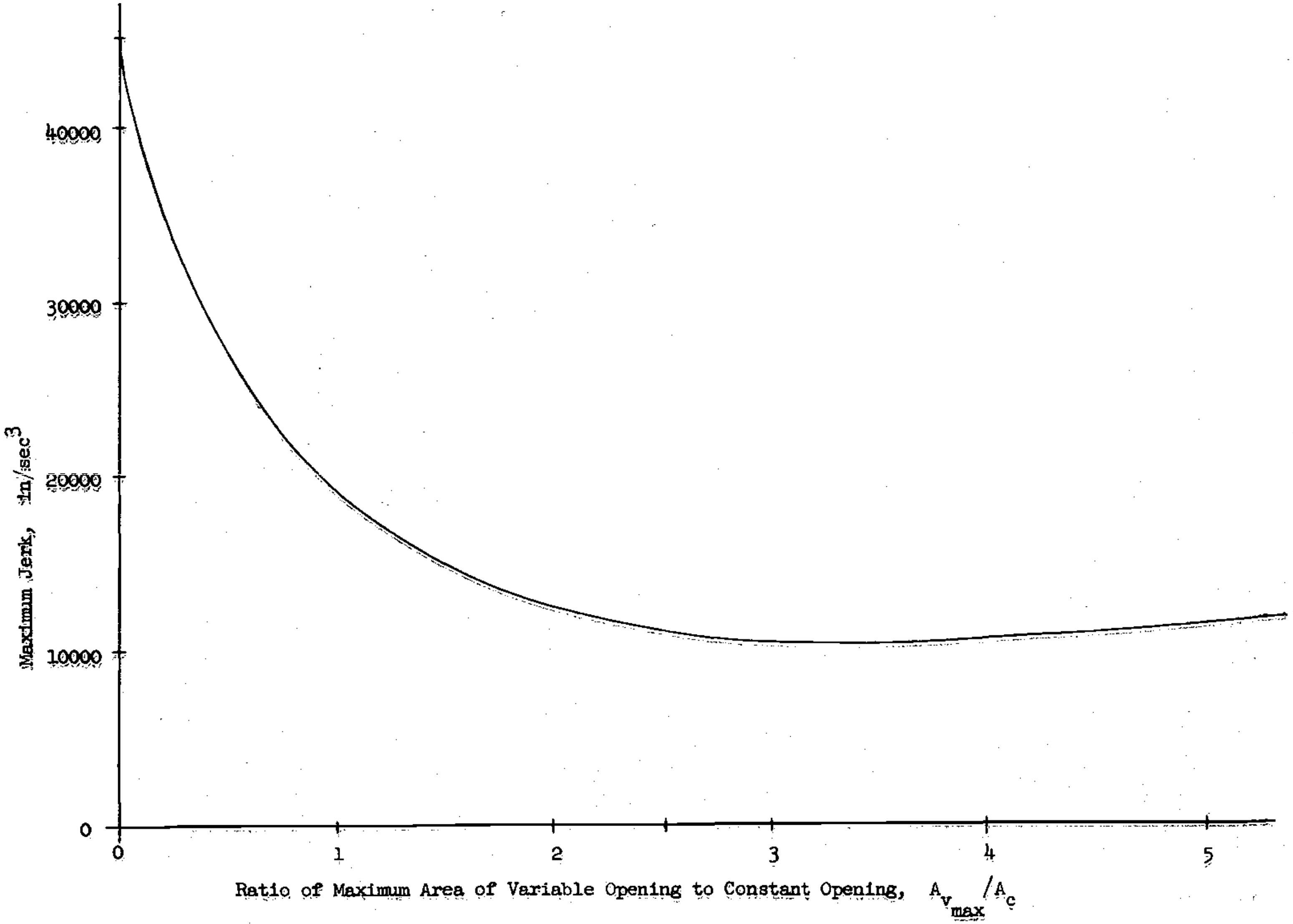


Figure 9. Plot of Maximum Jerk Versus Ratio of Area of Variable Opening to Constant Opening.

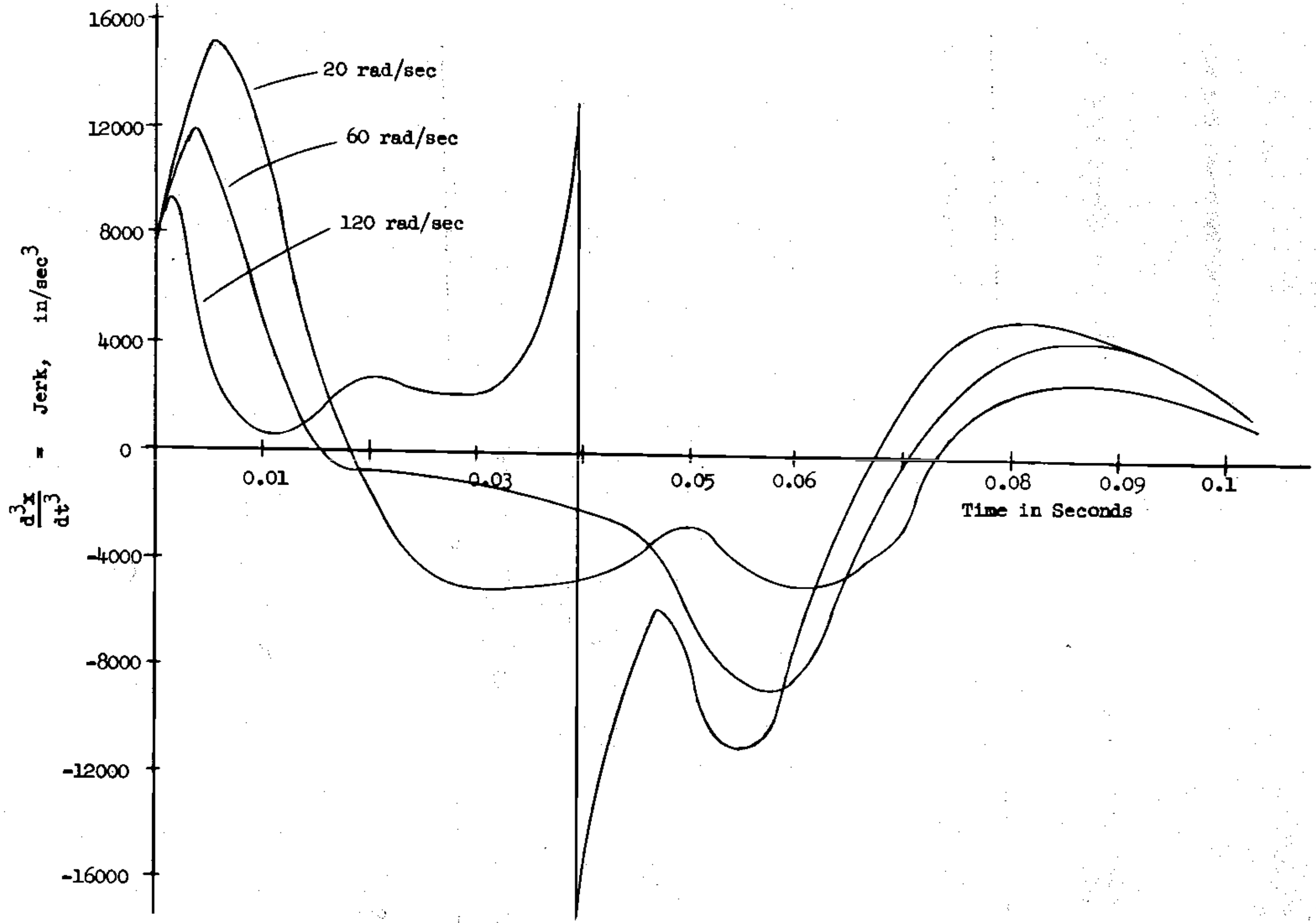


Figure 10. Jerk-Time Curves for Different Natural Frequencies.

The value of the desired natural frequency could depend upon many parameters. Among these are the weight of the vehicle, the stiffness of the tires and springs, the weight of the unsprung mass, and the amount of damping desired. Since no direct mathematical method could be found to relate these parameters to the desired natural frequency, an indirect method was used which consists of numerically solving the system for many combinations of parameters and many values of natural frequencies and then choosing the best value of the natural frequency for each set of parameters. An expression was then derived to relate the best value of the natural frequency to the chosen parameters. It was found that the computing time needed to solve the mathematical model with enough combinations of parameters and enough values of natural frequencies to give meaningful results was prohibitive.

To reduce the computing time, a closed form solution (see Appendix C) was derived that is accurate for approximately 0.035 seconds after system activation. In this time interval, it is possible to predict the performance of the shock absorber. If the value of jerk does not become negative after the first peak and stay negative for approximately one half the time width of the first peak, then the natural frequency is too high and a large second peak of jerk can be expected. The natural frequency should be reduced until this condition is met.

A total of 669 different combinations of parameters were investigated. For each combination, nine values of natural frequencies were considered and a best value was chosen for each combination. These best values of natural frequencies were related to the parameters of the system by various polynomials, some up to 28 terms. A program using the least-squares method was used on the Burroughs B-5500 computer to find the coefficients of the terms of the numerous polynomials that were tried. The parameters were m_1, m_2, k, K_t, A, A_c, C_c, and C_v, and of these, only m_2, K_t, A, and A_c were primarily effective in determining the value of the natural frequency. The polynomial that best fits the data is shown by the following equation:

$$\omega_n = 163.9 - 199.6 \sqrt{A/A_c} + A/A_c [31.76 - 0.08513 A/A_c] + \omega_2 [3.69$$

$$+ 3.897 \sqrt{A/A_c} - 0.862 A/A_c + 0.002685 (A/A_c)^2] + \omega_2^2 [-0.1475$$

$$+ 0.005566 A/A_c - 2.181 \times 10^{-5} (A/A_c)^2] + \omega_2^4 [1.335 \times 10^{-5}$$

$$- 3.336 \times 10^{-6} \sqrt{A/A_c} + 1.442 \times 10^{-7} A/A_c] \qquad (2.14)$$

where,

ω_n = Natural Frequency of Valve, $\sqrt{K/M}$

$\omega_2 = \sqrt{K_t/m_2}$

The range of the parameters that can be used in Equation (2.14) is shown below.

$0.005 \geqq A_c \geqq 0.09$		(in^2)
$0.375 \geqq A \geqq 1.6$		(in^2)
$600 \geqq K_t \geqq 1850$		(lb/in)
$0.2 \geqq m_2 \geqq 0.55$		$(lb\text{-}sec^2/in)$

Accuracy of better than 5 per cent can be expected for any combination of the variables in the range shown above. For certain combinations of the parameters, the value of the calculated natural frequency may be slightly less than 20 rad/sec. This is an inaccuracy in the polynomial, and the natural frequency used should be 20 rad/sec.

Initial Compression of the Valve Spring

The last parameter needed is the initial compression of the valve spring of the inertial valve shock absorber. The value of the initial compression of the valve spring determines at which point the valve begins to open. Since the valve is opened by the acceleration of the unsprung mass, the initial compression of the valve spring can be written in terms of the acceleration of the unsprung mass. This acceleration, which is the amount of steady acceleration needed to completely open the valve, can be written in terms of the tire stiffness, the value of the unsprung mass, and the height of step input to the tire. The equation is as follows:

$$A_2 = \frac{K_t \, s_t}{m_2} \qquad (2.15)$$

where,

A_2 = Initial acceleration of unsprung mass, also the steady acceleration needed to completely open the valve.

s_t = Height of step input.

By considering the steady state solution to Equation (2.8), the maximum deflection of the valve spring from its free length can be expressed by the following equation.

$$r_{max_t} = \frac{A_2 + g}{\omega_n^2} \qquad (2.16)$$

The value of the step input that will provide the unsprung mass with a steady acceleration great enough to completely open the valve, is entirely dependent upon the designer and the application. If the shock absorber is to be used on a vehicle in which a soft ride is very important, such as a luxury car, then the value of the step needed to open the valve should be small, approximately one inch. If the shock absorber is used on a vehicle such as a truck, the value of the step needed to open the valve would be much larger than for the luxury car. It should be understood that the value of the step used in the calculations is only used to calculate the amount of steady acceleration needed to completely open the valve. In actual operation, the valve is only opened approximately 75 to 85 per cent of the maximum by the step input. The reason is that the acceleration of the unsprung mass is not constant, but decreases during the opening of the valve.

The initial compression of the valve spring is given as follows:

$$r_i = r_{max_t} - r_{max} \qquad (2.17)$$

where,

r_i = Initial compression of valve spring.

r_{max} = Maximum opening of the valve that is physically feasible.

In order that the valve be closed in the rest position, expression (2.18) must hold:

$$r_i \geq \frac{M\,g}{K} = \frac{g}{\omega_n^2} \qquad (2.18)$$

It would be desirable if the initial compression of the valve spring would be just great enough to balance the weight of the valve. This would allow the valve to open as soon as the wheel begins to accelerate upward, which would give the best performance, but the maximum distance the valve would have to move to open completely would usually be too great for the physical dimensions of the shock absorber. Therefore, to approach this condition the designer has to allow the valve a movement as large as physically possible.

For the type of inertial valve such as shown in Figure 3, in which the valve can open to a point where further movement will not increase the flow area, a stop should be placed on the valve to limit its travel to the minimum required amount.

Design Procedure Summary

In summary, to calculate the parameters of the inertial valve shock absorber, it is first necessary to use Equation (2.13) to calculate the area of the fixed opening in the shock absorber piston. The maximum value of the variable opening in the piston should be two and one half times the area of the fixed opening. Then Equation (2.14) is used to calculate the natural frequency of the valve. A decision must then be made on how large a step is needed to open the valve. This value depends entirely upon the application of the shock absorber and should be used in Equation (2.15) to calculate the value of A_2. The final portion of the design is to determine the maximum amount the valve can physically open and use this value in Equation (2.17) to calculate the initial compression of the spring.

CHAPTER III

EXPERIMENTAL INVESTIGATION

General Procedure

The first portion of the experimental investigation was to verify the Equation (2.5) of the force transmitted by a conventional type of shock absorber. The shock absorber was mounted in a holding device and driven by the head of a Gould & Eberhardt shaper. The force transmitted by the shock absorber was measured by using strain gages and recorded on a Sanborn two-channel brush recorder.

The mathematical model of the two degree of freedom system was duplicated in the laboratory and the values of the parameters used in this laboratory model were approximately 1/20 of the values found on a medium size car. The system was excited by a hydraulic piston which gave an approximate ramp input. The displacement, acceleration, and jerk of the top mass and also the displacement of the input of the hydraulic piston were measured directly. Identical tests were conducted using the inertial valve and conventional type of shock absorbers for different combinations of two different fixed openings in the shock absorber piston and five different fluids in the shock absorbers. In this manner a fair comparison could be made between the performance of the two types of shock absorbers under various conditions.

Laboratory Models

Inertial Valve Shock Absorber

Construction Details. The design of the inertial valve shock absorber that was built and tested can best be described by reference to Figures 3 and 11. The piston, valve, and ends were machined from brass stock, and the cylinder was cut from a seamless thick-wall steel tube. A circular piece of lead was fastened to the valve to add the needed mass. A Thompson linear motion ball bearing was used to reduce the friction between the valve and the piston rod to a minimum. The actual measured force needed to overcome the static friction between the valve and the piston rod was only 0.0095 pounds.

The diameter of the piston was 1.500 inches and the piston rod diameter was 0.250 inches. This gave an effective area of 1.72 in^2. The diametrical clearance between the piston and cylinder was 0.004 inches. Twenty-four holes, 0.071 inches in diameter, provided the fixed opening in the shock absorber piston for one set of runs. The number of holes was reduced to 18 for a second set of runs. The ratio of the length of each hole to its diameter (L/D) was approximately seven. The maximum area of the opening of the inertial valve was 0.196 in^2. This made the ratio of the maximum area of opening of the inertial valve to the area of the fixed opening approximately 2.1 for the first set of runs and 2.8 for the second set.

By allowing generous clearances at all places where friction was possible, the Coulomb friction was kept to a value below 1 1/2 pounds, depending slightly upon the oil that was used.

The shock absorber was converted to a conventional type by fixing

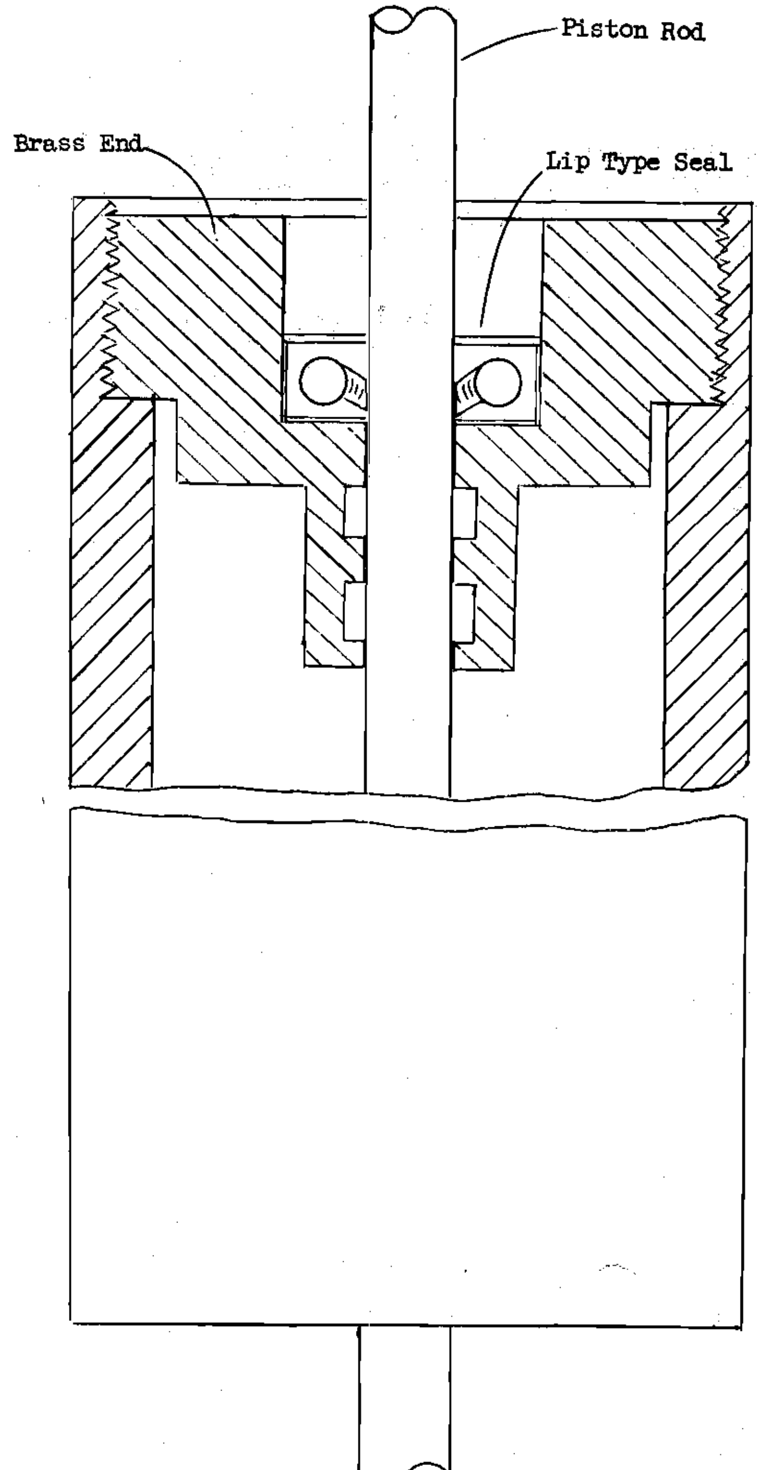


Figure 11. Shock Absorber Seals and Outer Casing.

the inertially controlled valve.

Force Velocity Equation. To determine the force transmitted by a shock absorber, it was necessary to mount the shock absorber in a holding device such as shown in Figure 12. The circular shock absorber was constrained by the device to move in a straight line in the horizontal direction. A rectangular piece of steel 0.26 inches wide and 0.06 inches thick, which had a strain gage mounted on each side, was fastened between the shock absorber and the end mount. The usual type of four-leg bridge composed of two active and two temperature compensating strain gages was connected to a Sanborn horizontal two-channel brush recorder. The force measurement apparatus was calibrated by tilting the device to a vertical position and hanging weights of known amounts on the shock absorber and recording the strain.

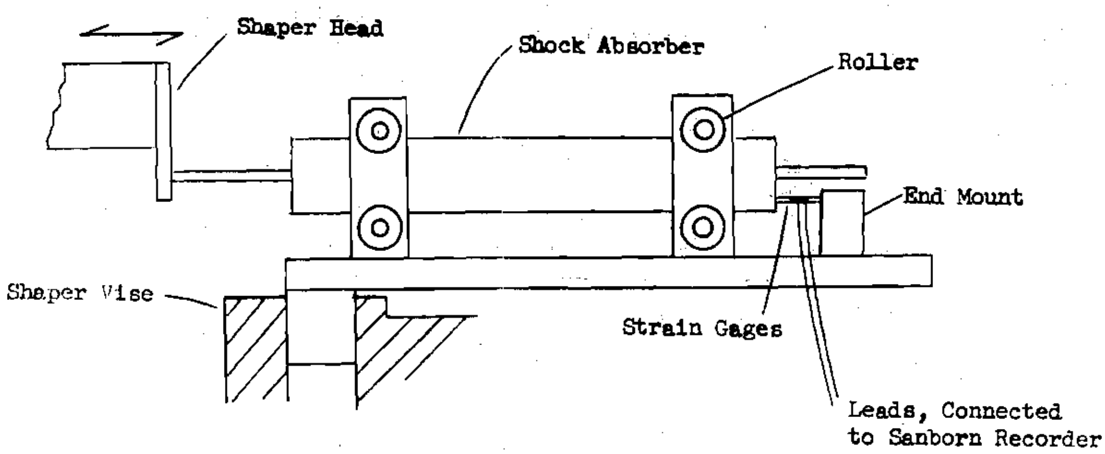


Figure 12. Holding Device.

Exact expressions were derived to relate the displacement and velocity of the shaper head to the position and speed of the rotating driving gear. It was assumed that the gear had constant angular velocity.

Sinclair Opaline motor oil of weights SAE 10w, SAE 20 and 20W, SAE 30, and SAE 40 were used in the test. Viscosity-temperature charts that were furnished by the Sinclair Refining Company were used to determine the viscosity of the oil. Five different shaper speeds were used for each type of oil in which the maximum value of the speeds ranged from 19.85 in/sec to 42.1 in/sec. The shaper was allowed to run at a constant speed until the temperature of the oil and the maximum force transmitted reached steady state. This usually took about 15 minutes depending upon the speed of the shaper and the viscosity of the oil used. The strain was then recorded and another run was made using a different speed or different oil. A two-inch stroke was used for all of the tests.

The maximum force transmitted by the shock absorber occurred at the point of the maximum velocity of the shaper head. This maximum force was recorded, along with the maximum velocity of the shaper head. From the five shaper speeds, five curves, such as Figure 13, were drawn, which are plots of the maximum force transmitted versus the maximum velocity of the shaper head. The theoretical slope can be obtained from Equation (2.3) as $32\ A^2\ L\ V/D^2\ A_c$. The ratios of the slopes of the experimental curves to the theoretical slopes ranged from 0.89 to 0.95 with an average value of 0.92. This would indicate that the actual force transmitted due to viscous friction, which is proportional to the first

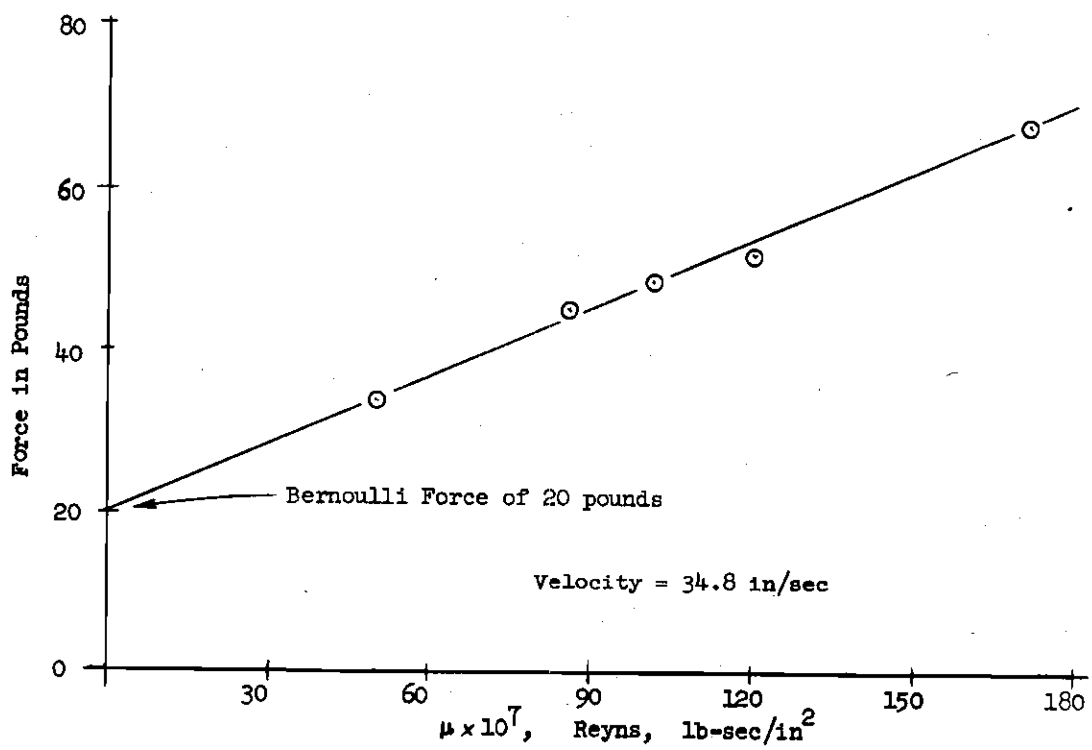


Figure 13. Representative Plot of Maximum Force Versus Viscosity.

power of the velocity and viscosity, is slightly less than the calculated force. By projecting the force-viscosity curve to the point where the viscosity is zero, the value of the force due to the Bernoulli effect can be read. The theoretical value of the Bernoulli force is taken from Equation (2.4) as $C\ A^3\ \rho\ V^2/2\ A_c^2$. The ratios of the actual Bernoulli forces to the calculated Bernoulli forces ranged for the five velocities from 0.74 to 0.85 with an average value of 0.79. This is for "C" being unity. By using the average values of the ratios, the experimental expression can be written as follows:

$$F = 0.92\ C_v\ m\ V + 0.79\ C_b\ m^2\ V^2\ \mathrm{sign}(V) \qquad (3.1)$$

The same expression holds for the inertial valve shock absorber as long as the acceleration of the piston remains below a value that does not appreciably open the inertially controlled valve. The value of this threshold acceleration is equal to the initial compression of the valve spring times the square of the natural frequency of the valve ($r_i\ \omega_n^2$).

Figure 14 shows a plot of Equation (3.1) and the points of the actual experimental data.

Two Degree of Freedom Model

Construction Details. The mathematical model of the two degree of freedom system was duplicated in the laboratory as shown by the schematic drawing of Figure 15. The mechanism for controlling the motion of the masses is in reality a four-bar linkage. The long pieces of channel iron served as the two sides of the linkage and the shock absorber rod served as the variable length third side. Extra stiffening members had to be fastened to the top beam to prevent rotation of the top mass with the flexing of the beams. The motion of the masses is not straight line as assumed in the mathematical investigation, but the deviation is not enough to cause any difficulty.

The values of the top and lower masses are 0.1505 lb-sec^2/in and 0.044 lb-sec^2/in, respectively. This includes the equivalent mass of the beams. The lower spring had a measured spring constant of 70 lb/in, while the upper spring had a constant of 15.5 lb/in.

A five-horsepower Denison hydraulic power package operating at 1150 psig supplied fluid to a 1.687-inch hydraulic cylinder which was connected to the lower spring and served as the excitation to the system.

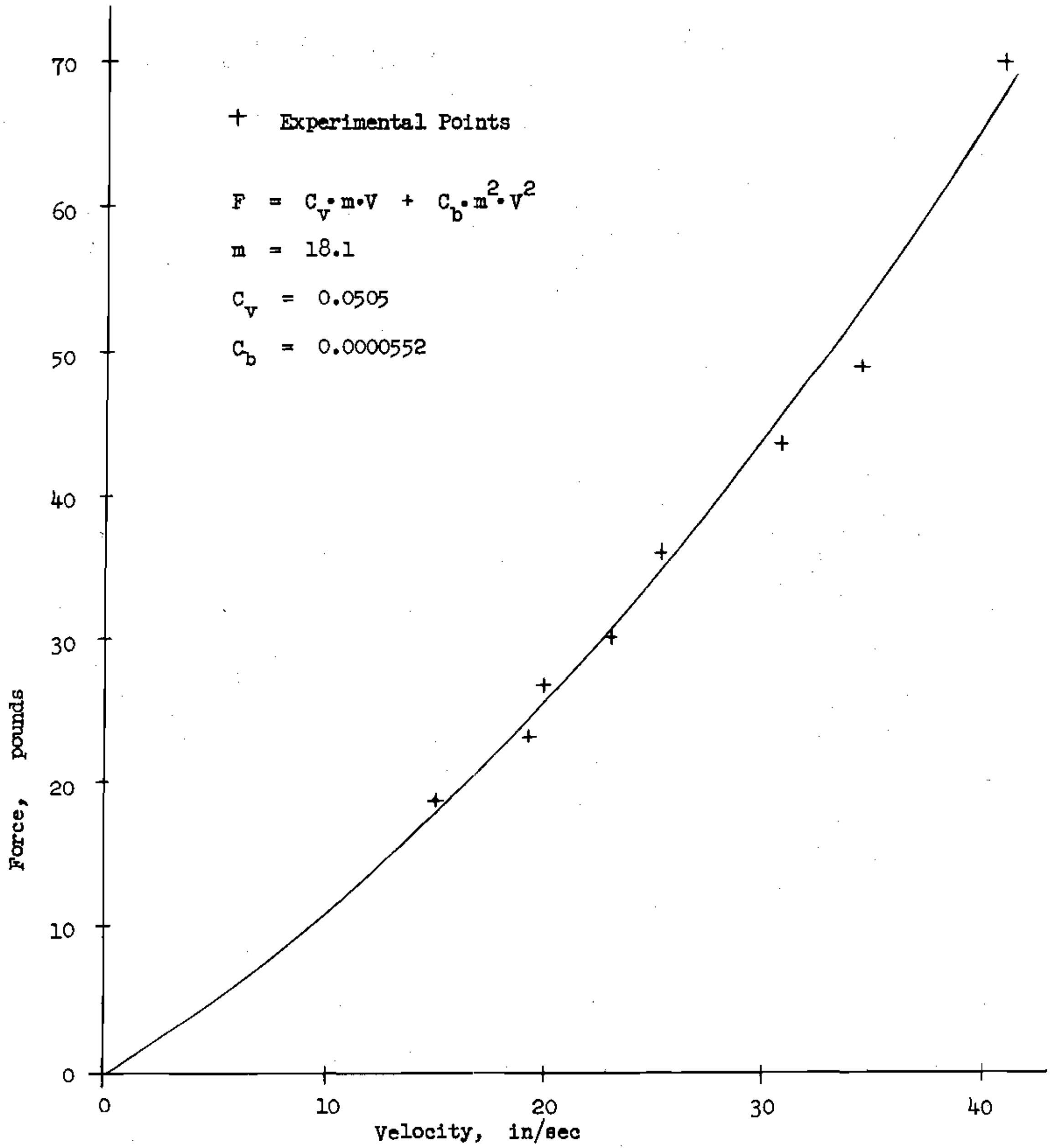


Figure 14. Experimental Points and Plot of Force Versus Velocity.

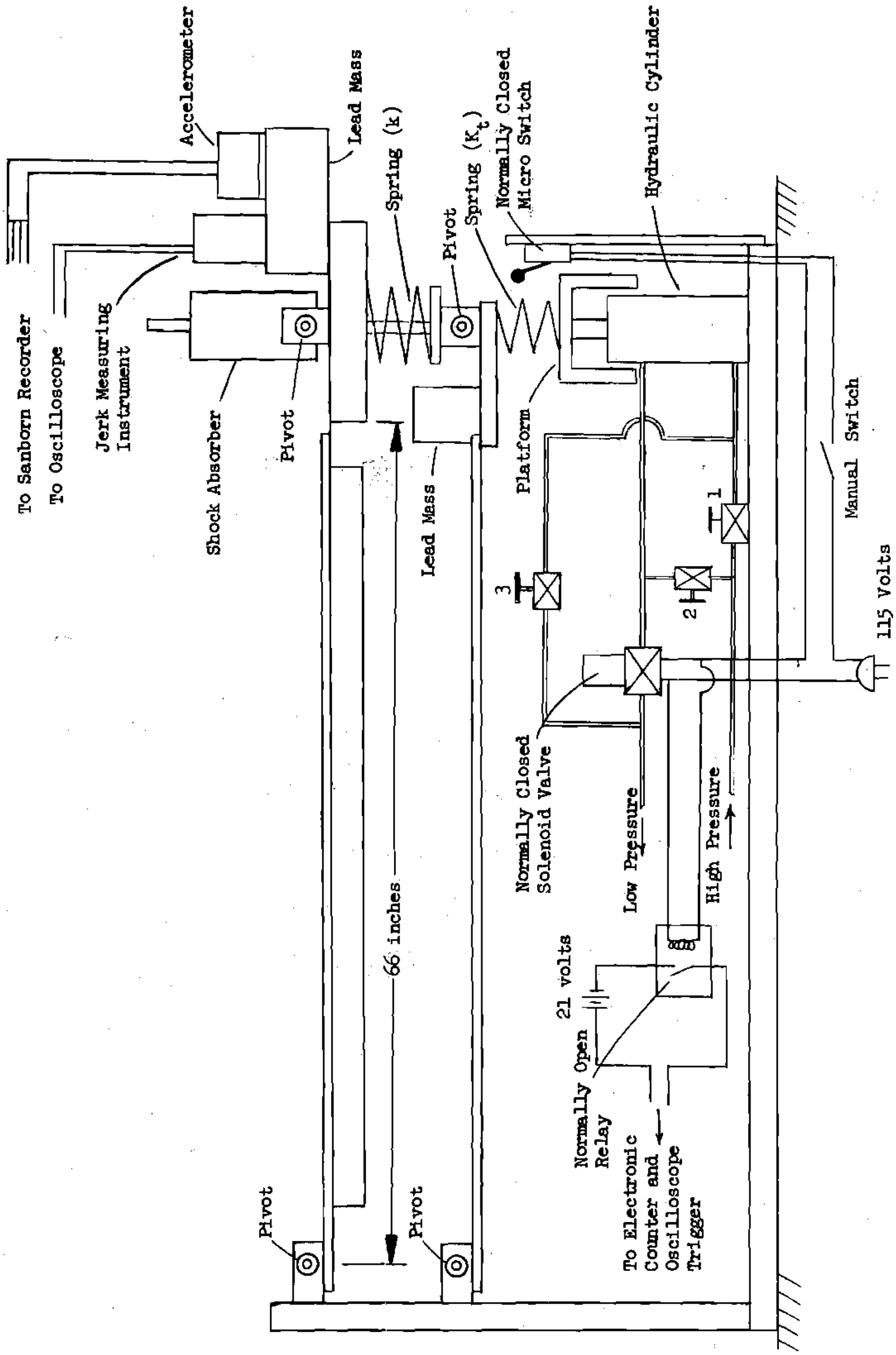


Figure 15. Schematic Drawing of Two Degree of Freedom System.

A normally closed solenoid valve controlled the flow of fluid which raised the piston and a combination of three manual valves served to lower the piston of the hydraulic cylinder.

Instrumentation. An instrument that was designed and built in the laboratory was used to measure the jerk of the top mass. Because of the mechanical noise of the system (vibrations of very high frequencies), a low-pass filter was needed to attenuate the signal from the instrument which was due to the mechanical noise. Two different filters were used for this purpose. The first filter had an attenuation factor of approximately ten for high frequency vibrations. Using this filter and a beam sweep of 20 cm/sec on the Tektronix Type 564 Storage Oscilloscope, a fairly smooth curve could be recorded on the oscilloscope. Since this filter attenuated the first peak of jerk too much, a second filter having less high frequency attenuation was used to record more accurately the first peak of jerk at a beam sweep speed of 100 cm/sec. The oscilloscope trace obtained when using this filter was not nearly as smooth as the one obtained when using the first filter because of the high frequency vibration superimposed on the true jerk signal (see Figures 35 and 36 in Appendix E). A calibration curve and a detailed description for the instrument can be found in Appendix A.

A Statham +4g to -2g strain gage type of accelerometer was used to measure the acceleration of the upper mass. A Sanborn brush recorder provided the constant voltage needed for the accelerometer and also recorded the acceleration signal at a recording speed of 10 cm/sec. The accelerometer was calibrated by mounting it on the top mass of the two degree of freedom model, removing the shock absorber, restraining the lower mass from moving, and allowing the top mass to vibrate freely. In effect, the two degree of freedom system was reduced to a system of one degree of freedom. Assuming the motion of the top mass sinusoidal, the acceleration and jerk can be calculated by knowing the frequency and amplitude of vibration. The jerk measuring instrument was checked in this manner, and the results of this test match very closely to the data taken from the more complete slider-crank calibration. The calibration using the slider-crank mechanism is described in Appendix A.

A linear potentiometer was used a sensing device for the displacements of the top mass and of the hydraulic piston. A Sanborn recorder provided the required constant voltage and also recorded the signal from the potentiometer.

A normally open electrical relay was connected in parallel with the solenoid valve. The closing and opening of the relay activated a Hewlett Packhard electronic counter and also triggered the sweep of the oscilliscope beam. In this manner, the time that current flowed to the solenoid valve could be measured to the nearest millisecond. This gave a good indication of the time during which the solenoid valve allowed flow to the hydraulic culinder.

Laboratory Tests on Two Degree of Freedom Model

Twenty tests were conducted using the two degree of freedom model. The excitation for all of these tests was an approximate ramp input from a hydraulic piston (see Figure 34 in Appendix E). The heights of the inputs ranged from 0.56 inches to 1.21 inches with the velocity being approximately 15.4 in/sec and independent of the height of the ramp. Sinclair Opaline motor oil of weights SAE 10w, SAE 20 and 20w, SAE 30, SAE 40, and Texaco Aircraft Hydraulic Oil AA (petroleum base, viscosity similar to SAE 5w oil) were used in the shock absorbers. Twenty-four holes of 0.071 inches in diameter provided a fixed opening of 0.095 in^2 in the shock absorber piston for one set of runs. The number of holes was reduced to 18, which gave a fixed opening area of 0.0713 in^2, for a second set of runs.

Identical tests were conducted with the inertial valve first inoperative and then operative. In this manner, a fair comparison could be made between the performances of the two types of damping devices. For each test, two identical runs were made. In the first run, the filter having the greater high frequency attenuation factor was used. For the second run, that filter was replaced by one which allowed the first peak of jerk to be recorded more accurately.

In many cases the experimental investigation showed that the maximum absolute value of jerk was in negative direction. This might indicate that it would be desirable to have a second inertially controlled valve that would be opened by the negative (downward) acceleration of the wheel to reduce the large negative value of jerk. In actual operation on an automobile, the input of a tire striking a square bump would be much more severe than provided by the hydraulic piston in the laboratory. For an average size tire striking a one-inch high square corner at 20 miles per hour, the duration of the input to the tire would be 0.015 seconds compared to 0.065 seconds in the laboratory. The more severe input of the tire striking the square corner would greatly increase the first positive peak of jerk but would not significantly increase the negative peak of jerk. This would make the positive peak of jerk much larger than the absolute value of the negative peak and would indicate that the greatest reduction of jerk is possible in the positive direction and a second valve would be unnecessary.

The complete tabulated results of the tests made with the two degree of freedom mathematical model are shown in Tables 1 and 2. Table 3 gives the values of the viscosities of the oils that were used in the tests.

Test Data Compared to Computer Solution

Figures 16-18 are the plotted results of the experimental data taken from a run that was excited by a one-inch ramp input and using a conventional shock absorber having 18 holes with SAE 10w oil. Also shown on the same figures are the digital computer solutions of the same system. Figures 19-21 are the similar results using the inertial valve shock absorber.

The experimental data shows that it takes a slightly longer time to reach the peaks of acceleration and jerk compared to the computer solution. The difference in time is due partly to the dynamic response of the accelerometer and jerk measuring device and also to the inexact representation of the shock absorber in the computer solution. Also, the secondary effects, such as damping due to wind resistance, friction in the pivots, and the secondary vibration of the beams were not included in the computer solution. A measured value of Coulomb friction equal to one pound was used in the computer solution. An error in the measurement of Coulomb friction and its representation in the computer solution

Table 1. Experimental Data for Ramp Input, 24 Holes

Height of Ramp (Inches)	Type of Shock	Type Oil	Per Cent Overshoot	Maximum Acceleration (in/sec^2)	Minimum Acceleration (in/sec^2)	Maximum Jerk (in/sec^3)	Minimum Jerk (in/sec^3)
0.6	Conventional	10w	71	131	-74	3160	-2680
0.69	Inertial	10w	63	116	-78	2050	-2500
1.21	Conventional	10w	80	163	-100	3170	-3410
1.21	Inertial	10w	72	156	-106	2320	-3050
0.95	Conventional	20w	49	159	-123	3720	-3310
0.65	Inertial	20w	40	111	-97	2880	-2750
1.0	Conventional	30w	53	197	-165	4400	-3900
1.0	Inertial	30w	36	145	-138	3450	-3950
1.21	Conventional	30w	53	181	-179	4300	-4800
1.21	Inertial	30w	31	144	-139	3450	-3880
0.58	Conventional	40w	51	205	-133	4350	-3220
0.56	Inertial	40w	31	132	-115	3720	-3050
0.99	Conventional	40w	56	205	-190	4200	-4400
1.03	Inertial	40w	30	162	-172	2850	-4200
1.44	Conventional	40w	59	209	-208	4390	-6420
1.43	Inertial	40w	30	156	-174	3350	-5500

Table 2. Experimental Data for Ramp Input, 18 Holes

Height of Ramp (Inches)	Type of Shock	Type Oil	Per Cent Overshoot	Maximum Acceleration (in/sec^2)	Minimum Acceleration (in/sec^2)	Maximum Jerk (in/sec^3)	Minimum Jerk (in/sec^3)
1.14	Conventional	Hydraulic	50	157	-106	3160	-3410
1.07	Inertial	Hydraulic	42	143	-99	1860	-3320
1.0	Conventional	10w	53	173	-122	3350	-3600
1.0	Inertial	10w	27	146	-91	2300	-3580

Table 3. Viscosity of Shock Absorber Oil

Viscosity	Hydraulic Fluid	SAE 10w	SAE 20 & 20w	SAE 30	SAE 40
Saybolt Universal Seconds at 80 degrees Fahrenheit	89	285	610	920	1580
Absolute Viscosity, Reyns, lb-sec/in^2	2.23×10^{-6}	7.94×10^{-6}	16.9×10^{-6}	26.6×10^{-6}	44.1×10^{-6}

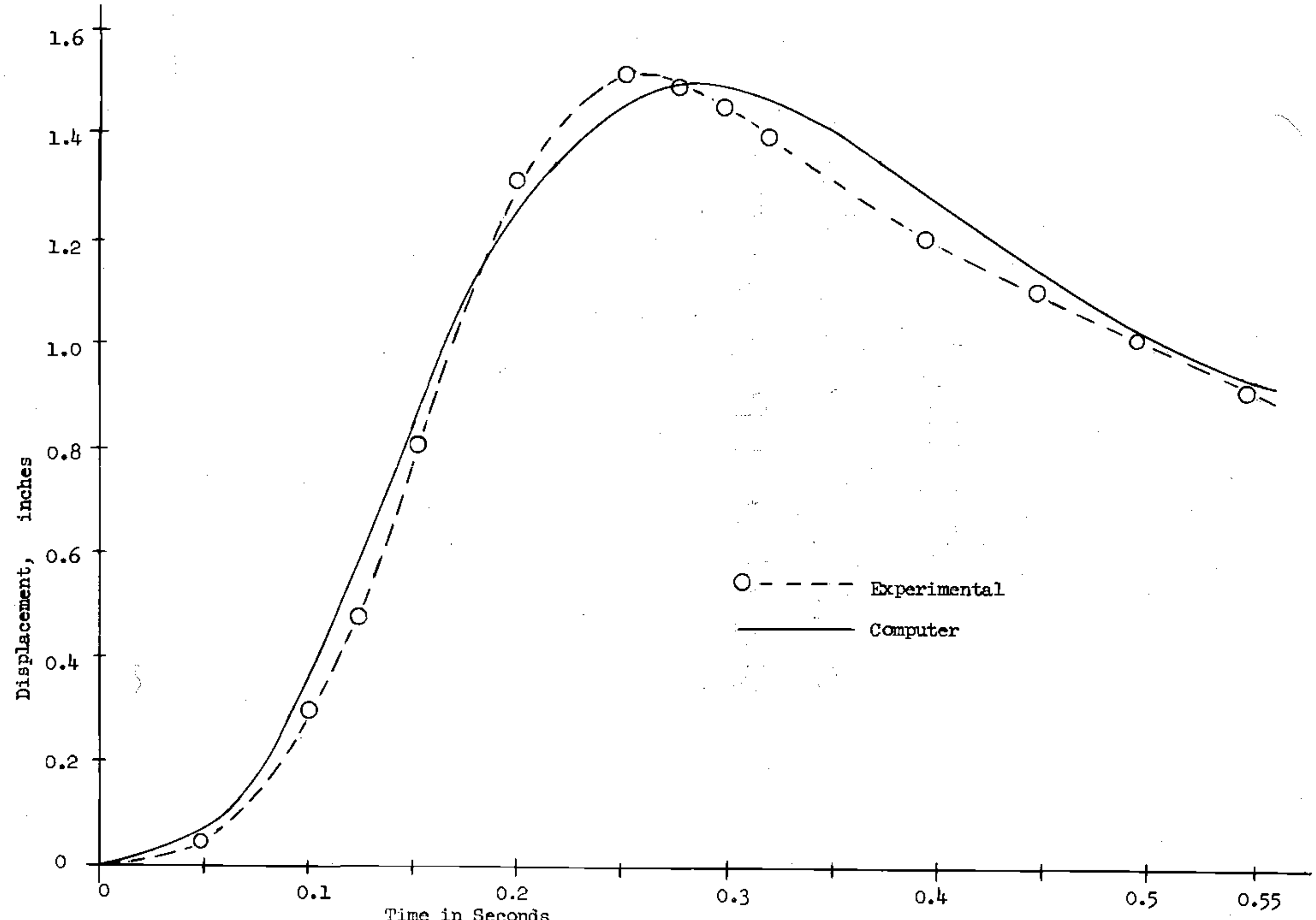


Figure 16. Displacement-Time Curves for Experimental and Computer Data, Conventional Type Shock Absorber.

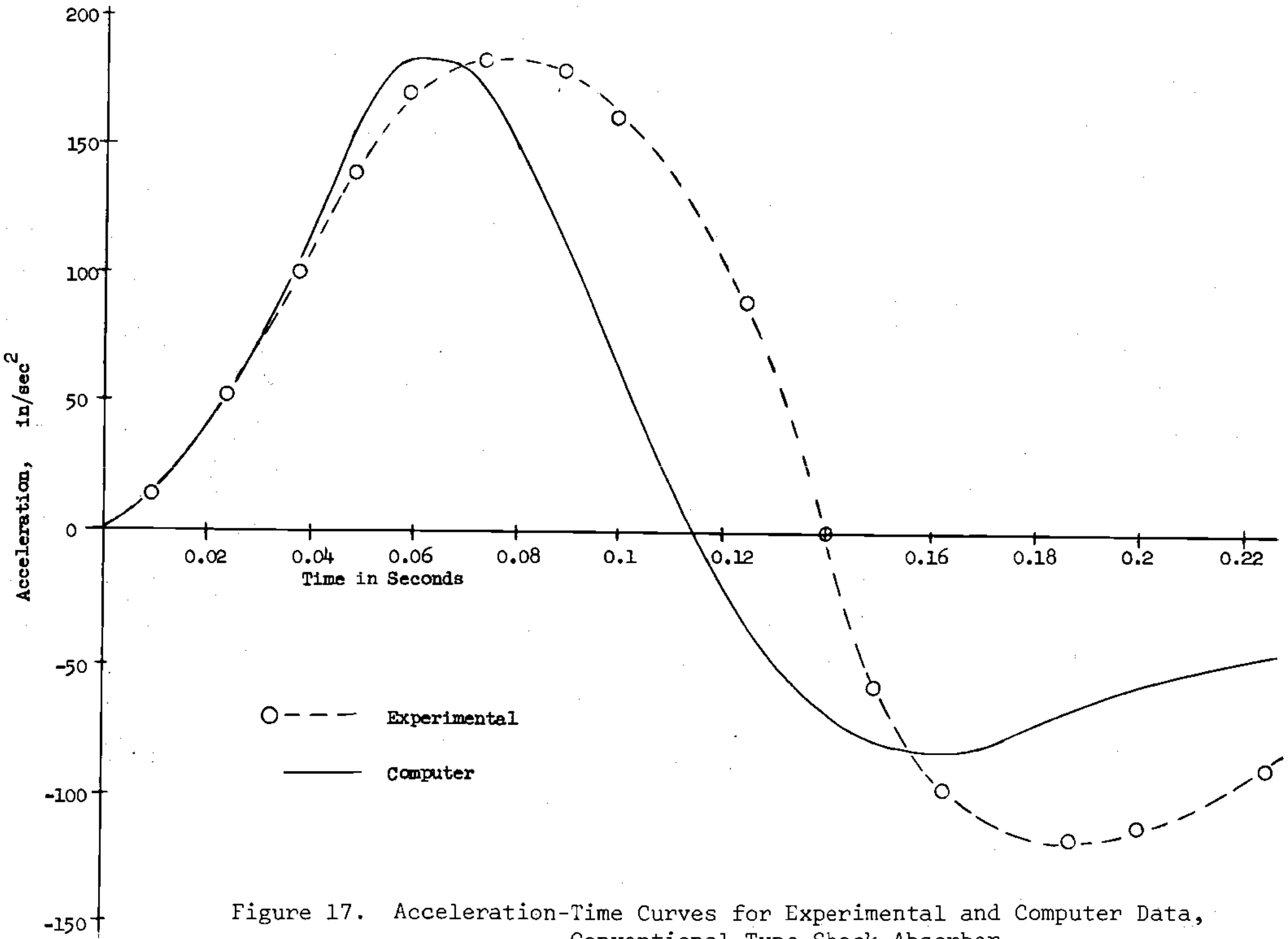


Figure 17. Acceleration-Time Curves for Experimental and Computer Data, Conventional Type Shock Absorber.

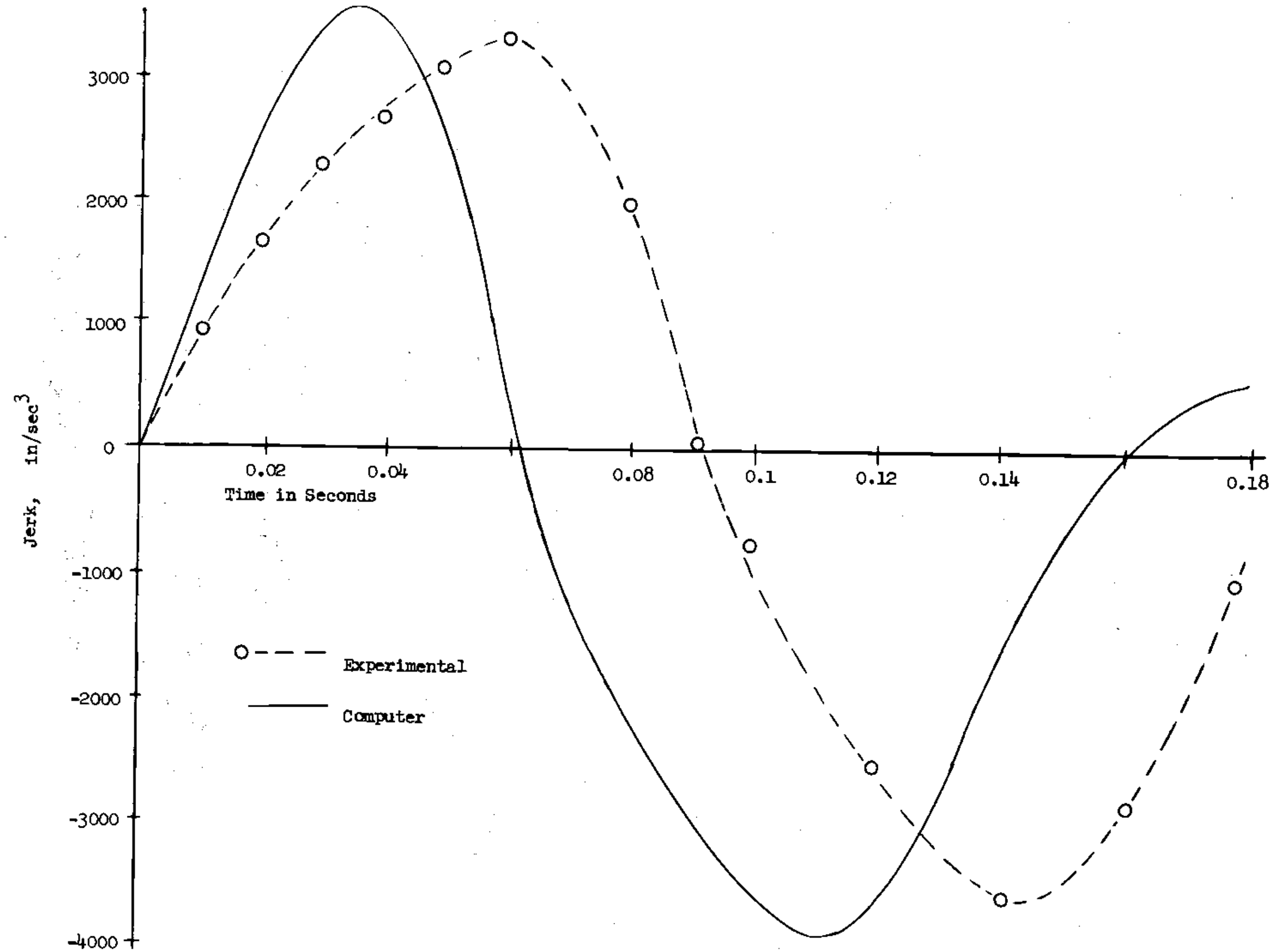


Figure 18. Jerk-Time Curves for Experimental and Computer Data, Conventional Type Shock Absorber.

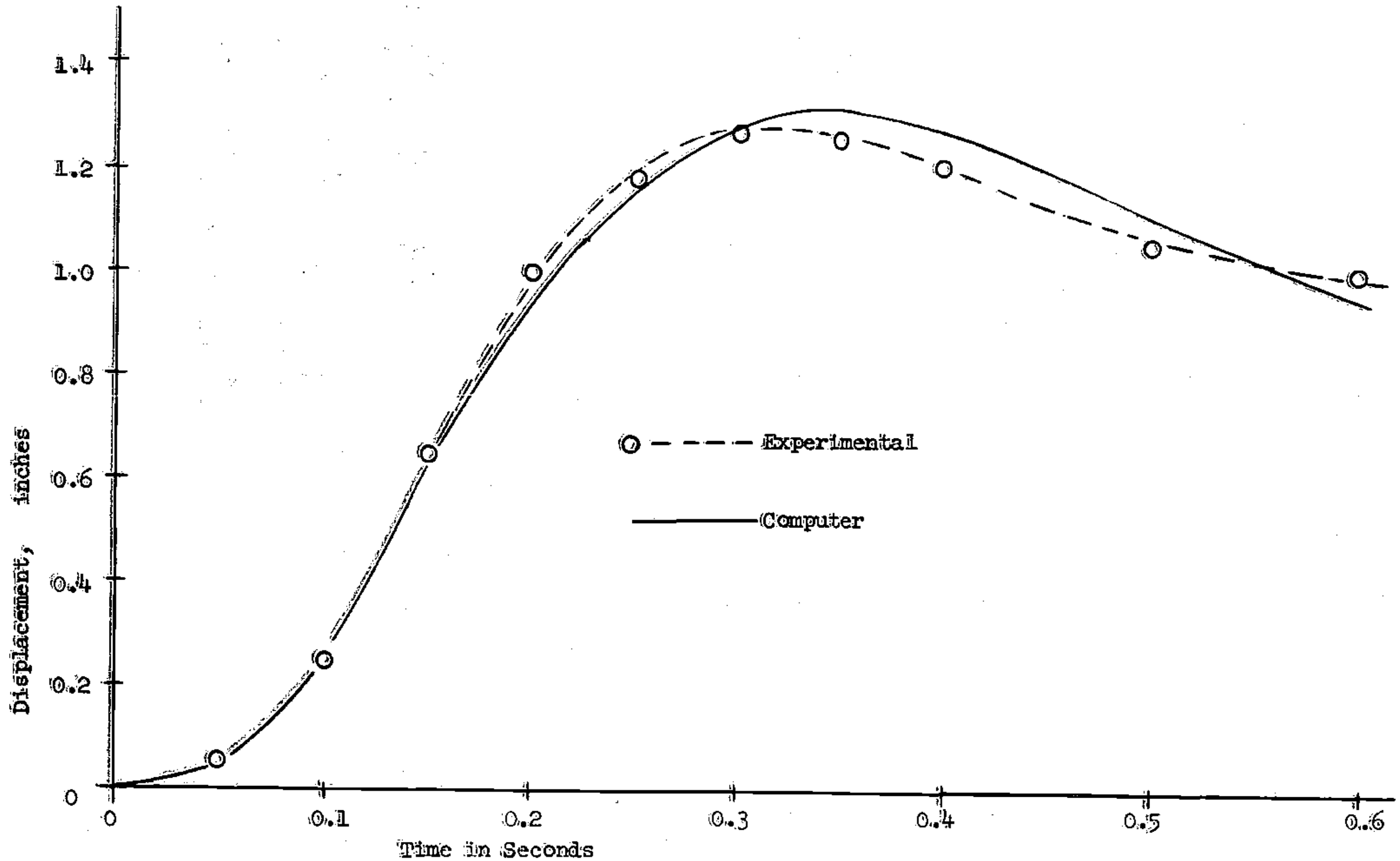


Figure 19. Displacement-Time Curves for Experimental and Computer Data for Inertial Valve Shock Absorber.

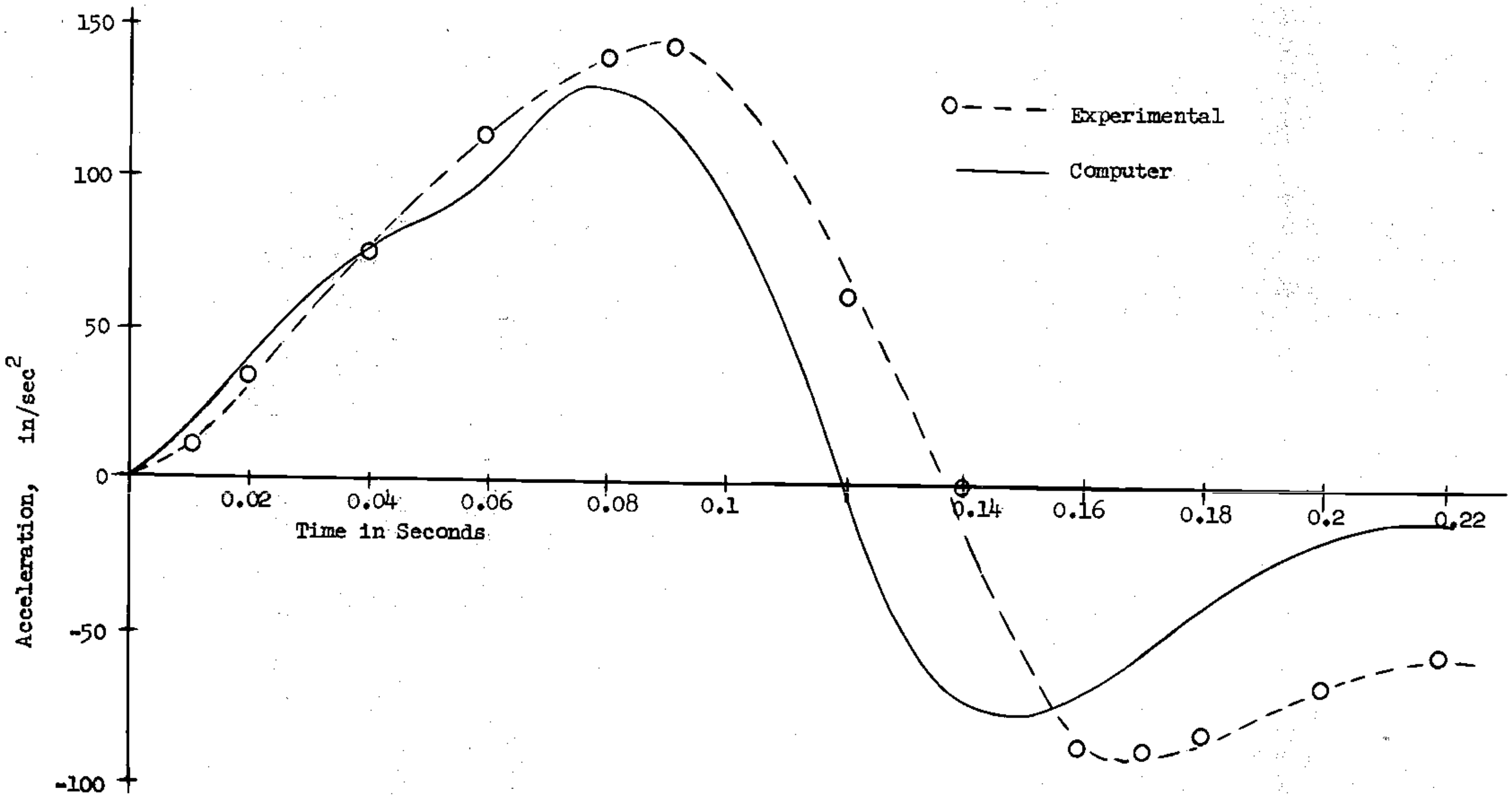


Figure 20. Acceleration-Time Curves for Experimental and Computer Data, Inertial Valve Shock Absorber.

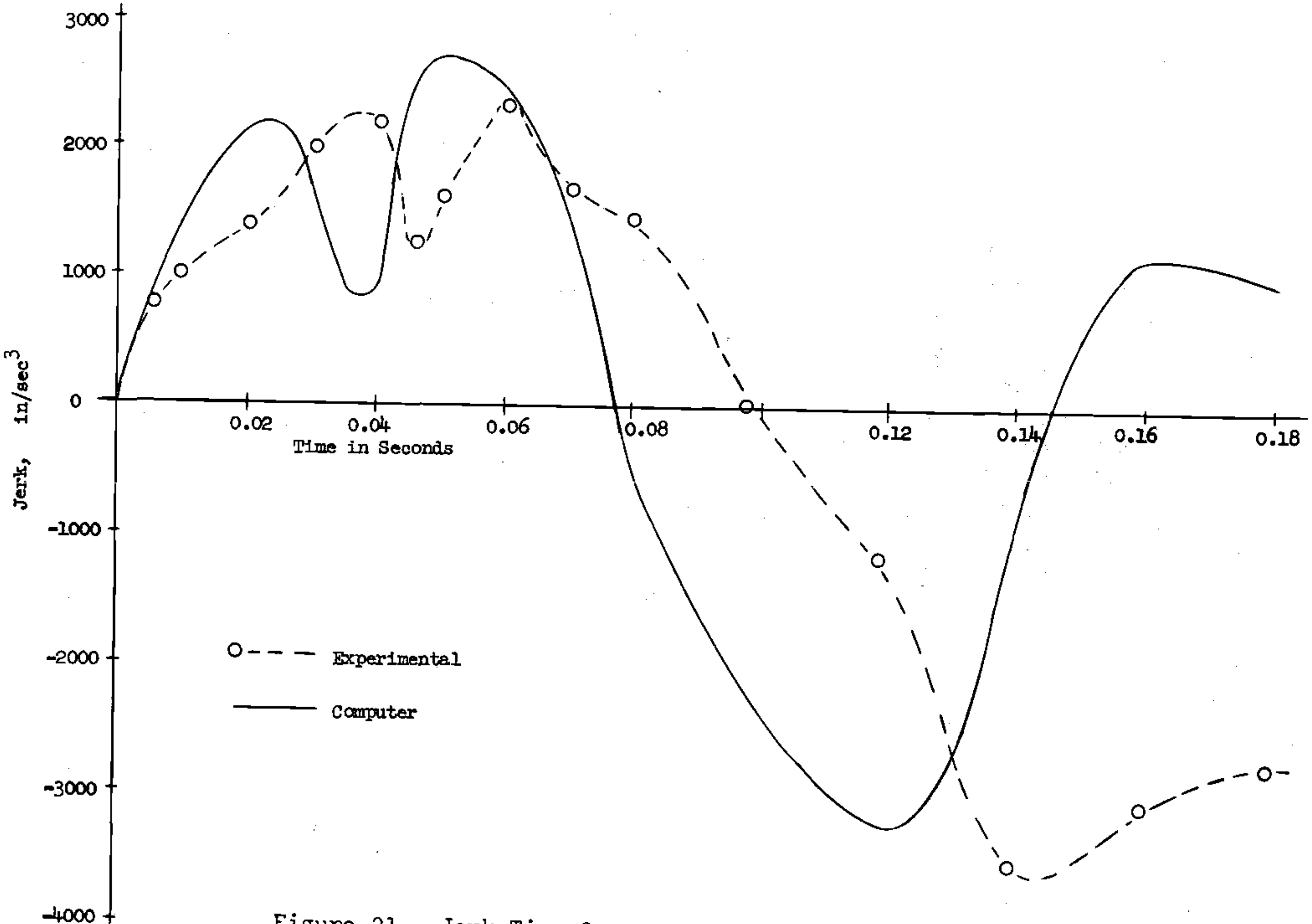


Figure 21. Jerk-Time Curves for Experimental and Computer Data, Inertial Valve Shock Absorber.

could affect the computer results.

The purpose of the experimental work with the two degree of freedom model was not to test the design of a particular type of shock absorber, but to verify the computer solution for the system. It would be very difficult to make the input in the laboratory as severe or as varied as would be experienced by an actual automobile. It was then necessary to verify the computer solutions by experimental data so that more confidence can be placed in computer solutions to systems that are not experimentally investigated.

Step Displacement Tests

Tests were conducted on several late model automobiles that consisted of releasing one end of an automobile displaced from its equilibrium position and recording the magnitude of the first overshoot in both the rebound and compression directions. The same type of test was conducted on the two degree of freedom model. The top mass was released from a displaced position and the amount of overshoot was observed.

The purpose of these tests was to compare the damping of an actual automobile with the damping provided by a shock absorber used on the two degree of freedom laboratory model and in the computer solution. In this way, a prediction can be made on the amount of jerk that can be reduced by using the inertial valve shock absorber.

The amount of the overshoot of the six automobiles that were tested ranged from 8 to 20 per cent in the rebound direction and from 13 to 40 per cent in the compression direction with the average of 12 per cent in the rebound direction and 25 per cent in the compression direction. The tabulated results for the step displacement on the two degree of freedom model are shown in Table 4.

Table 4. Experimental Data for Step Displacement

Displacements (Inches)	Per Cent Overshoot	Type Oil	Number of Fixed Holes
0.86	40	Hydraulic Fluid	18
1.55	43	Hydraulic Fluid	18
0.86	15	10w	18
1.81	15	10w	18
1.0	43	10w	24
1.98	50	10w	24
0.69	25	20w	24
1.98	27	20w	24
1.50	6	30w	24
1.88	7	30w	24
1.50	0	40w	24

A comparison between the average overshoot of the automobiles and of the laboratory model showed that the damping on an average automobile is approximately equivalent to the damping provided by the shock absorber having 18 holes and using SAE 10w weight oil. For a ramp input in the laboratory a 29 per cent reduction of jerk and a 19 per cent reduction of acceleration was obtained by using the inertial valve shock absorber in comparison to the equivalent conventional double-acting type. For a unit step input, the computer solution showed approximately a 360 per cent reduction in jerk and a 250 per cent reduction in acceleration.

CHAPTER IV

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Conclusions

The computer solution for the derived automobile suspension system model incorporating an inertial valve shock absorber showed that a significant reduction of jerk and acceleration can be obtained by using this type of shock absorber. This was shown to be true for both a step input and for sinusoidal inputs of frequencies greater than approximately 18 rad/sec. This frequency is fairly low. It corresponds to an automobile traveling at 30 miles per hour over a washboard type of road which has a peak only once every 15.4 feet.

A conventional type of shock absorber having a reduced damping rate in the compression direction and a high pressure blow-off valve was simulated on the digital computer with the two degree of freedom system. A sizeable reduction of jerk can be obtained when using this shock absorber in comparison to the double-acting type (see Figures 35 and 36), but the reduction of the damping force in the compression direction that is needed to allow a reduction of jerk comparable to that offered by the inertial valve type, is such that the shock absorber provides very little stability and control to the car. It offers little low velocity damping in the compression direction which is needed to keep an automobile level when rounding curves, stopping or starting. The inertial type provides essentially the same amount of damping in each direction and is similar in that respect to the conventional double-acting type.

As mentioned above, the conventional type of shock absorber usually has a high pressure blow-off valve. Such valves reduce the damping of shock absorbers when the transmitted force reaches a predetermined value. They reduce the harshness due to high wheel velocities but will not affect the first peak of jerk that results when the wheel strikes a sharp bump. The analytical investigation showed that this type of valve could be used in conjunction with the inertially controlled valve to provide better ride characteristics under all conditions.

The primary purpose of the experimental investigation was to verify the computer solution for the inertial valve type of shock absorber. The reasonably close correlation of the experimental results to the computer solution for both the conventional double-acting type and the inertial valve type gave good reason to have confidence in computer solutions to systems that were not experimentally investigated. Since it is much easier to simulate a system on a computer than to actually build the system in the laboratory, many more systems were investigated analytically than experimentally.

The investigation of the step displacement showed that the amount of damping on actual automobiles is equivalent to that used in the laboratory and simulated on the computer. This gave more reason to believe that the reduction of jerk, found in the laboratory and from solutions to systems simulated on the digital computer, could be duplicated on an actual automobile.

Suggestions for Further Research

One of the most important, but unanswered questions related to the investigation was the quantitative effect of jerk on riding comfort. The experimental work described in Appendix B only attempted to show that with everything else equal, a ride with a larger amount of jerk was more uncomfortable than a ride with a smaller amount of jerk. No attempt was made to make a quantitative investigation to determine how much jerk made a ride uncomfortable. There is reason to believe that the direction of jerk as well as the magnitude is important to riding comfort. Many of the people participating in the riding comfort investigation commented (without knowing the type of rides they were subjected to) that the ride with the large amount of jerk had a "bump" or "jerk" at the bottom of the vertical motion. Even though there was the same amount of jerk at the top of the motion, no one mentioned that they felt any discomfort at the top of the stroke. Elevator companies feel that a higher value of jerk in the upward direction is not as uncomfortable as the same amount in the negative direction. They believe that jerk in the downward direction gives the uncomfortable feeling of falling. For these above reasons, it would seem that the results of a complete and thorough investigation to determine the exact effect that jerk has on riding comfort would be very helpful in designing better ride control devices.

An automobile should be instrumented with accelerometers and jerk measuring devices and equipped with inertial valve shock absorbers and carefully driven over a test surface. This type of research is similar to that which was done by Fine (6). A real comparison could then be made between the ride with the conventional and the inertial type shock absorbers. By equipping an automobile with the inertial valve shock absorber a check could be made upon the validity and accuracy of the design equations given in Chapter III.

APPENDIX

APPENDIX A

JERK MEASURING DEVICE

General Description

The instrument that was used to measure jerk consisted of a mass connected to a frame by a spring and damper as shown by the schematic drawing in Figure 22. The relative velocity between the mass and the frame will be shown to be a measure of jerk. Certain types of accelerometers use the same general components but measure the displacement of the mass relative to the frame as an indication of acceleration.

A pictorial drawing of the instrument that was built and tested is shown in Figure 23. The cantilever beam is 0.210 inches thick and 0.80 inches wide, and the distance from the center of the magnet to the support is 6.25 inches. The Rayleigh method was used to include the weight of the beam in calculating its natural frequency of 643 rad/sec. The beam was machined from cast iron because of the inherent damping property of cast iron. Even by using the cast iron beam, the amount of damping of the beam was very small, approximately 4 per cent of critical damping. A better design would certainly include more damping in the system, at least 20 per cent of critical.

An Alnico U-shape permanent magnet was fastened to the end of the beam. A coil of approximately 400 turns was situated on the base in such a way that the magnet could pass through it. The voltage generated by the coil is proportional to the velocity of the magnet relative to the coil and is also proportional to the jerk of the frame.

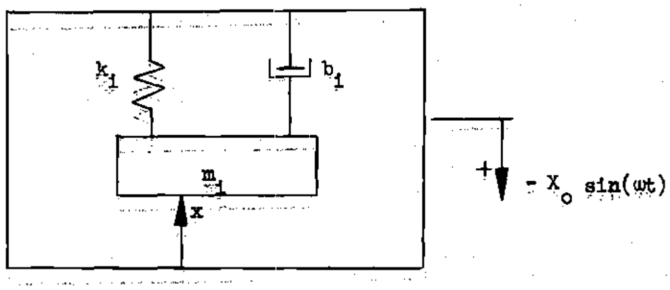


Figure 22. Schematic Drawing of Jerk Measuring Device.

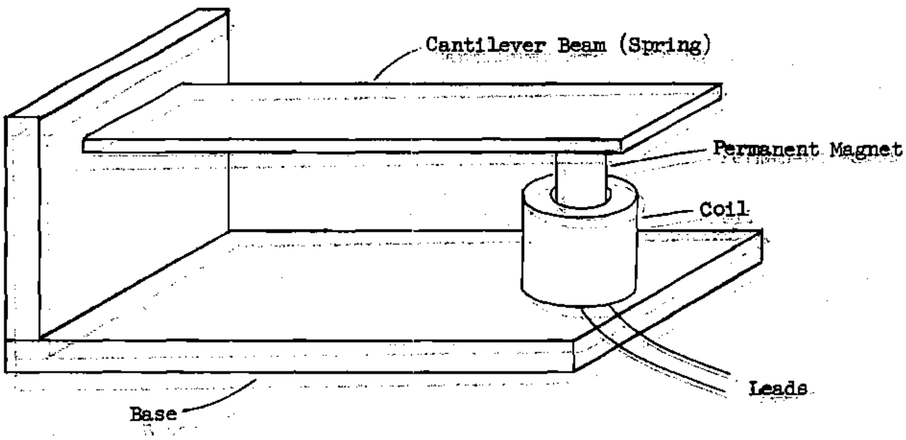


Figure 23. Jerk Measuring Device.

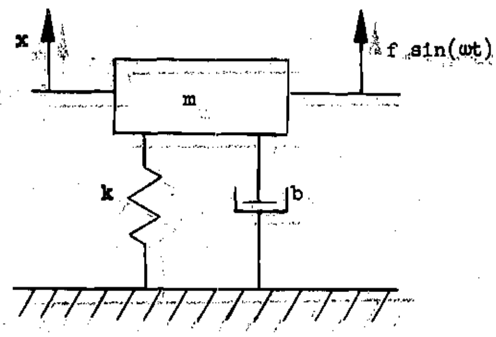


Figure 24. One Degree of Freedom System.

Mathematical Theory

For the linear system shown in Figure 24, the equation of motion can be written as follows:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + k x = f \sin(\omega t) \qquad \text{(A.1)}$$

Since the system is linear, the expression giving the steady state response can be written as follows:

$$x = q \frac{f}{k} \sin(\omega t + \phi) \qquad \text{(A.2)}$$

where,

$f \sin(\omega t)$ = Force acting on the mass.

q = The ratio of the actual displacement to the static displacement.

ϕ = The phase angle between the input and the output.

By examining a Bode plot of a second order spring-mass system, it can be seen that for forcing frequencies less than 1/10 of the natural frequency of the system there is a very small phase lag between the input and output, and also that the ratio of actual displacement to static displacement (q) is approximately one. This means that the expression for the motion of the mass can be written as follows:

$$x = \frac{f}{k} \sin(\omega t) \qquad \text{(A.3)}$$

The equation of motion for the jerk measuring device shown in Figure 22 is similar to Equation (A.1) and can be written as follows:

$$m_i \frac{d^2x}{dt^2} + b_i \frac{dx}{dt} + k_i x = \omega^2 X_o m_i \sin(\omega t) \qquad (A.4)$$

(-) $X_o \sin(\omega t)$ is the displacement of the frame and $\omega^2 X_o \sin(\omega t)$ is the absolute acceleration of the frame in the positive direction as shown in Figure 22. For values of ω that are less than 1/10 of the natural frequency ($\sqrt{k_i/m_i}$), the displacement of the mass relative to the frame can be written similar to Equation (A.3) as follows:

$$x = \omega^2 \frac{m_i X_o}{k_i} \sin(\omega t) \qquad (A.5)$$

By taking one derivative, the velocity of the mass relative to the frame can be written as follows:

$$\frac{dx}{dt} = \omega^3 \frac{m_i X_o}{k_i} \cos(\omega t) \qquad (A.6)$$

By taking three successive derivatives of the displacement of the frame, the jerk of the frame can be expressed as follows:

$$\frac{d^3x}{dt^3} = \omega^3 X_o \cos(\omega t) \qquad (A.7)$$

By comparing Equation (A.6) which represents the velocity of the mass relative to the frame and Equation (A.7) which represents the jerk of the frame, it can be seen that the velocity of the mass relative to the frame is proportional to the jerk of the frame. Thus, if the natural frequency of the instrument is at least ten times the frequency of the input signal, the relative velocity of the mass to the frame is a good measure of the jerk of the frame.

Calibration of Instrument

The jerk measuring instrument was calibrated by placing it on a carefully constructed slider-crank mechanism and connecting the leads of the coil to a low-pass filter which had a single break point at 14.2 rad/sec. The filter was connected to a Tektronix oscilloscope. The slider-crank had a 22.2-inch connecting rod and a 2.45-inch crank. The speed of the crank ranged from 34 rpm to 213 rpm which gave values of jerk that ranged from 119 in/sec^3 to 29,300 in/sec^3.

The low-pass filter was needed to attenuate the high frequency chatter of the slider. The attenuation of the filter at the different frequencies was used in conjunction with the voltage measured by the oscilliscope to obtain the data needed to construct the graph shown in Figure 25.

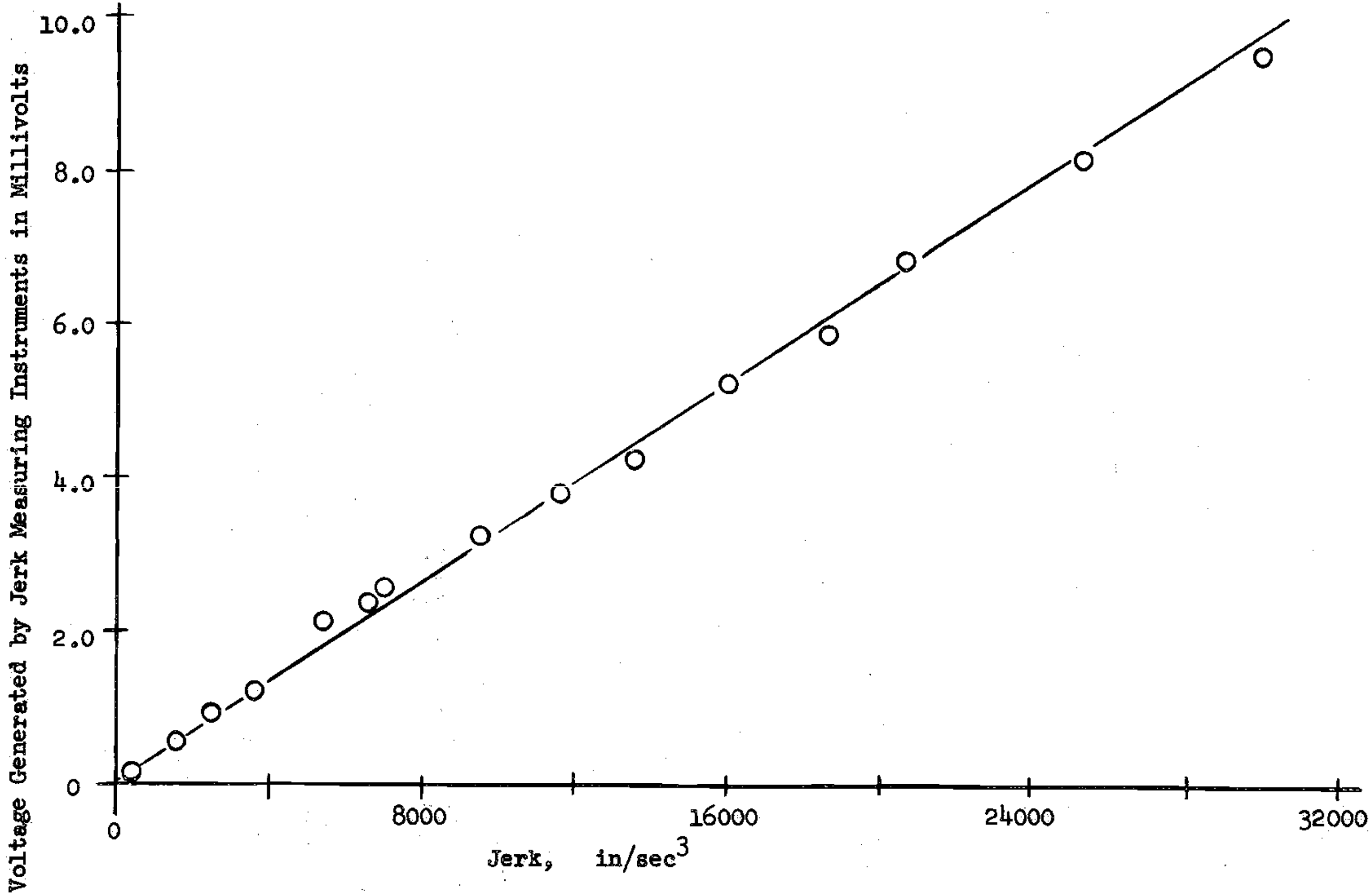


Figure 25. Calibration Curve for Jerk Measuring Device.

APPENDIX B

EFFECT OF JERK ON RIDING COMFORT

Apparatus

The device that was used to give the two types of vertical motion to people participating in the test is shown schematically in Figure 27.

The parabolic and cycloidal cams that drive the mechanism allow the same rise and nearly the same maximum acceleration and velocity, but the parabolic cam gives a larger value of jerk. The acceleration-rotation curves of both cams are shown in Figure 26.

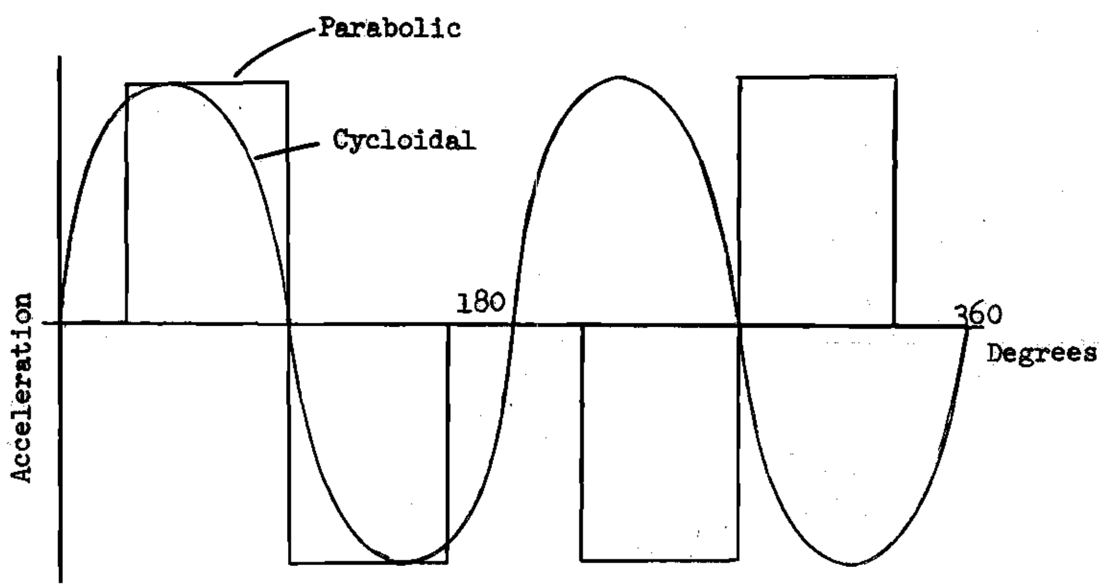


Figure 26. Acceleration-Rotation Curves.

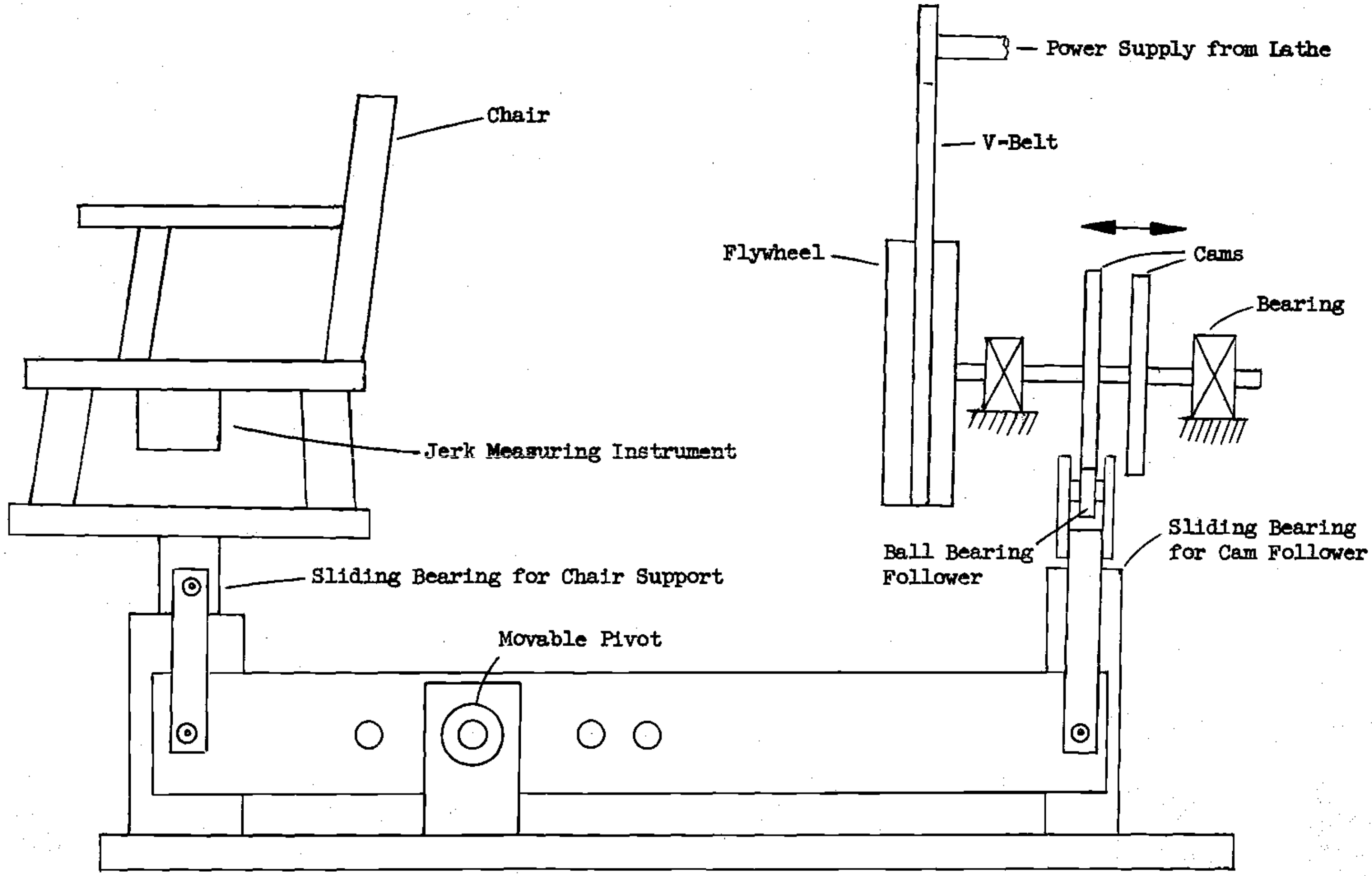


Figure 27. Ride Comfort Device.

At the points on the parabolic cam where there is a step change in acceleration, the theoretical value of jerk is infinite while the theoretical value of jerk for the cycloidal cam always remains finite.

The actual measured jerk of the chair for the cycloidal cam agreed to the calculated value within 20 per cent. The measured jerk for the parabolic cam had peaks at the points of the step change in acceleration. These peaks were approximately 2 1/2 times the maximum value of the jerk that was recorded using the cycloidal cam.

Procedure

The speed of the lathe which supplied power to the apparatus was first adjusted to a predetermined value. The person participating in the test was seated in the chair and told that he would be asked to distinguish a difference in comfort between two types of rides that he would soon be subjected to. He was also told that either ride would be repeated if he so desired. The device was then started and allowed to run for 12 seconds. It was then stopped and in approximately five seconds the second cam was put in a position to drive the mechanism. It was again started and allowed to run for another 12 seconds. The person was then asked if he felt any difference in comfort between the two types of rides, and if he did, which was the more comfortable. If the participating person chose either one of the rides to be the more comfortable, he was asked to describe the difference of the two rides by one of the following statements:

1. The ride was considerably more comfortable.
2. The ride was more comfortable.

3. The ride was only slightly more comfortable.

A random process was used to determine which ride the person was first subjected to.

Results

An attempt was made to correlate the reaction of the people participating in the test with the values of jerk, acceleration, frequency, and displacement. Since every attempt to present the results of the test in this manner was unsuccessful, the complete results were simply put into tabulated form as shown by Table 5.

With reference to Table 5, Column A gives the number of people that chose the ride with the larger amount of jerk to be "only slightly more comfortable." Column B indicates the number of people who could detect no difference in comfort between the two rides. The number of people who thought the ride with the smaller amount of jerk was "only slightly more comfortable" is shown in Column C. Column D gives the number of people who chose the ride of the smaller amount of jerk to be "more comfortable." The number of people who said the ride with the smaller amount of jerk was "considerably more comfortable" is listed in Column E.

A total of 72 people took part in the tests. Approximately 80 per cent were male students and five were women. A total of 127 tests was conducted which means that most of the people participated in two tests. The ride with the smaller value of jerk was chosen to be more comfortable to some extent over 75 per cent of the time. The ride with the larger value of jerk was considered to be "only slightly more comfortable" only 15 per cent of the time. It should be noted that 92 per cent of the people subjected to a ride where the cycloidal cam provided a maximum value of jerk equal to or greater than 750 in/sec^3, chose the ride with the smaller value of jerk to be the more comfortable. This seems to indicate that jerk can be felt and that a reduction of jerk gives a more comfortable ride.

Table 5. Results of Riding Comfort Investigation

Frequency cpm	Height of Stroke (Inches)	Maximum Jerk of Cycloidal Cam (in/sec^3)*	Maximum Acceleration (in/sec^2)	Maximum Velocity (in/sec)	A	B	C	D	E
50	0.30	55.6	5.2	0.99	0	2	8	0	0
50	0.56	106	9.9	1.88	2	1	7	0	0
60	0.56	178	14.2	2.26	1	3	5	1	0
72	0.30	162	10.8	1.43	5	1	4	0	0
72	0.56	307	20.4	2.72	1	0	1	0	0
72	0.80	435	28.9	3.84	3	1	3	2	2
72	1.25	680	45.3	6.0	5	2	13	2	2
105	1.25	2110	96.2	8.75	0	0	3	3	1
120	0.30	750	29.8	2.35	1	1	6	1	1
120	0.56	1420	56.7	4.52	0	1	7	2	0
120	0.80	2010	80.2	6.39	0	0	5	4	1
144	1.25	5440	181	12.0	0	0	2	1	2
150	0.30	750	29.8	2.38	0	0	1	0	1
175	0.30	2320	633	3.47	1	0	5	0	0
					19	12	70	16	10

* The maximum measured value of jerk for the Parabolic cam is approximately 2.5 times the jerk of the Cycloidal cam.

APPENDIX C

CLOSED FORM SOLUTION OF SYSTEM EQUATIONS

General Description

The closed form solution of the system equations was used in the process described in Chapter II to reduce the computing time in finding the best value of the natural frequency of the inertially controlled valve. It will give the values of jerk, acceleration, velocity, and displacement of the mathematical model described in Figures 1 and 4. The solution is only good for a step input and is accurate for only a short time interval, but the time interval is long enough to include the first peak of jerk. The final expression is complicated, but it takes much less calculating time than would be needed to solve the complete set of system equations by numerical methods.

For convenience, Figure 1, showing the mathematical model, is repeated.

Mathematical Derivation

For a short time interval, the acceleration of the lower mass can be approximated by the equation

$$\ddot{y} = A_2(1 - C_1 t) \qquad (C.1)$$

where,

$$A_2 = \frac{s_t K_t}{m_2} \tag{2.15}$$

s_t = Height of step input.

t = Time.

C_1 = An empirical constant determined from the computer data and equal to

$$0.72\sqrt{\frac{K_t - k}{m_2}}$$

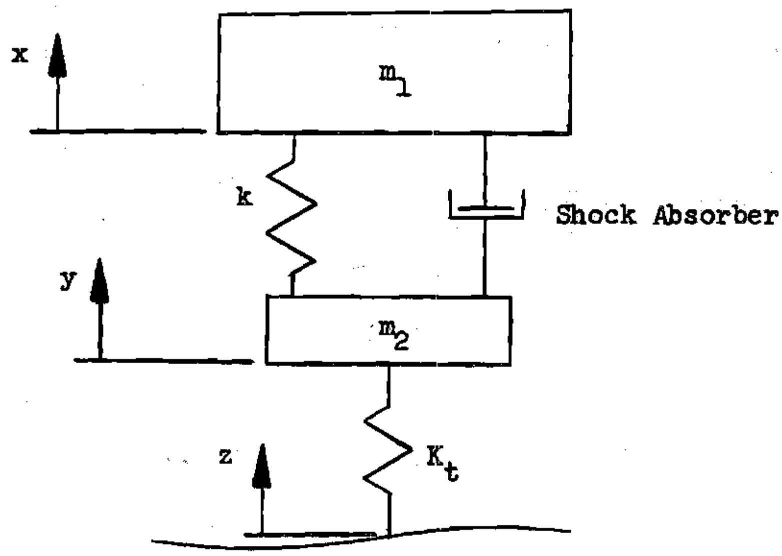


Figure 1. Mathematical Model.

The velocity can be found by integrating the expression for acceleration.

$$\dot{y} = \int_o^t \ddot{y}\, dt = A_2 \left[t - \frac{C_1 t^2}{2} \right] \qquad (C.2)$$

By assuming the velocity of the top mass to be negligible compared to the velocity of the lower mass, letting the weight of the valve equal the initial spring force, and using the above expressions for the velocity and acceleration, Equation (2.8) which gives the opening of the valve, can be solved using ordinary methods. Equation (2.8) is repeated below.

$$M \frac{d^2 r}{dt^2} + C \frac{dr}{dt} + K r = M \frac{d^2 y}{dt^2} - f_i + M g + f_v V^2 \,\text{sign}(V) \qquad (2.8)$$

The solution for Equation (2.8) for an underdamped valve using the above assumptions is as follows:

$$r = C_2 t^4 + C_3 t^3 + C_4 t^2 + C_5 t + C_6 \qquad (C.3)$$

$$+ e^{-\zeta \omega_n t}[C_7 \cos(\omega_n \sqrt{1 - \zeta^2}\; t) + C_8 \sin(\omega_n \sqrt{1 - \zeta^2}\; t)]$$

where,

$$C_2 = \frac{f_v A_2^2 C_1^2}{4 M \omega_n^2}$$

$$C_3 = \frac{-8 \zeta C_2}{\omega_n} - \frac{f_v A_2^2 C_1}{M \omega_n^2}$$

$$C_4 = \frac{f_v A_2^2}{M \omega_n^2} - \frac{12 C_2}{\omega_n^2} - \frac{6 \zeta C_3}{\omega_n}$$

$$C_5 = \frac{-A_2 C_1}{\omega_n^2} - \frac{4 C_4 \zeta}{\omega_n} - \frac{6 C_3}{\omega_n^2}$$

$$C_6 = \frac{A_2}{\omega_n^2} - \frac{2 C_4}{\omega_n^2} - \frac{2 \zeta C_5}{\omega_n}$$

$$C_7 = - C_6$$

$$C_8 = \frac{\zeta C_7}{\sqrt{1 - \zeta^2}} - \frac{C_5}{\omega_n \sqrt{1 - \zeta^2}}$$

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$\zeta = \frac{C}{2 \sqrt{KM}}$$

By taking a derivative of Equation (C.3) with respect to time, the velocity of the valve can be obtained.

$$\dot{r} = 4C_2\,t^2 + 3C_3\,t^2 + 2C_4\,t + C_5 \tag{C.4}$$

$$+ e^{-\zeta\omega_n t}\left[-C_7 \sin(\omega_n \sqrt{1-\zeta^2}\,t) + C_8 \cos(\omega_n \sqrt{1-\zeta^2}\,t)\right]\sqrt{1-\zeta^2}\,\omega_n$$

$$- \zeta\omega_n\, e^{-\zeta\omega_n t}\left[C_7 \cos(\omega_n\sqrt{1-\zeta^2}\,t) + C_8 \sin(\omega_n\sqrt{1-\zeta^2}\,t)\right]$$

By assuming the lower mass moves with constant acceleration, the first approximation for its displacement can be given by the equation

$$y = \frac{A_2\,t^2}{2} \tag{C.5}$$

By summing forces on the lower mass and neglecting the force of the shock absorber and the upper spring, an approximation can be made for the acceleration of the lower mass which is

$$\ddot{y} = \left[S_t - \frac{A_2\,t^2}{2}\right]\frac{K_t}{m_2} = A_2\left[1 - \frac{K_t}{m_2}\,\frac{t^2}{2}\right] \tag{C.6}$$

The velocity of the lower mass can be found by integrating Equation (C.6) to obtain

$$\dot{y} = \int_0^t \ddot{y}\,dt = A_2\left[t - \frac{K_t}{6\,m_2}\,t^3\right] \tag{C.7}$$

By integrating the velocity, the displacement can be found as

$$y = \int_o^t \dot{y}\, dt = A_2 \left[\frac{t^2}{2} - \frac{K_t}{24\, m_2}\, t^4 \right] \tag{C.8}$$

Using the equation of the force transmitted by the shock absorber (2.5) and also including the force transmitted by the upper spring, the total force that the lower mass exerts on the upper mass can be written as follows, neglecting the velocity and displacement of the top mass:

$$F_t = C_v\, m\, \dot{y} + C_b\, m^2\, \dot{y}^2 + k\, y \tag{C.9}$$

where,

$$m = \frac{A}{A_c + P\, r}$$

By summing the forces on the top mass, the expression for its acceleration can be written as follows:

$$\ddot{x} = \frac{F_t}{m_1} \tag{C.10}$$

Integrating Equation (C.10) with respect to time and assuming, for simplicity, the value of m to be a constant during the integration, an expression for the velocity of the top mass can be obtained according to the following equation:

$$\dot{x} = \frac{1}{m_1}\int_o^t F_t\,dt = \frac{1}{m_1}\left[C_v\, m\, A_2\left[\frac{t^2}{2} - \frac{K_t}{m_2}\frac{t^4}{24}\right]\right. \qquad (C.11)$$

$$\left. + m^2\, C_b\, A_2^2\left[\frac{t^3}{3} - \frac{K_t}{m_2}\frac{t^5}{15} + \frac{K_t^2}{m_2^2}\frac{t^7}{252}\right] + k\, A_2\left[\frac{t^3}{6} - \frac{K_t}{m_2}\frac{t^5}{120}\right]\right]$$

By using Equations (C.6) and (C.9), an expression for the acceleration of the lower mass can be written that includes the force of the shock absorber and upper spring as

$$\ddot{y} = A_2\left[1 - \frac{K_t}{m_2}t^2\right] - \frac{F_t}{m_2} \qquad (C.12)$$

By integrating Equation (C.12), the velocity of the lower mass can be more accurately described by

$$\dot{y} = A_2\left[t - \frac{K_t}{m_2}\frac{t^3}{3}\right] - \frac{1}{m_2}\left[C_v\, m\, A_2\left[\frac{t^2}{2} - \frac{K_t}{m_2}\frac{t^4}{24}\right]\right. \qquad (C.13)$$

$$\left. + m^2\, C_b\, A_2^2\left[\frac{t^3}{3} - \frac{K_t}{m_2}\frac{t^5}{15} + \frac{K_t^2}{m_2^2}\frac{t^7}{252}\right] + k\, A_2\left[\frac{t^3}{6} - \frac{K_t}{m_2}\frac{t^5}{120}\right]\right]$$

By integrating Equations (C.12) and (C.13), the displacements of the top and lower masses can be found as

$$x = \frac{1}{m_1}\left[C_v\, m\, A_2\left[\frac{t^3}{6} - \frac{K_t}{m_2}\frac{t^5}{120}\right] + m^2\, C_b\, A_2^2\left[\frac{t^4}{12}\right.\right. \tag{C.14}$$

$$\left.\left. - \frac{K_t}{m_2}\frac{t^6}{90} + \frac{K_t^2}{m_2^2}\frac{t^8}{2016}\right] + k\, A_2\left[\frac{t^4}{24} - \frac{K_t}{m_2}\frac{t^6}{720}\right]\right]$$

$$y = A_2\left[\frac{t^2}{2} - \frac{K_t}{m_2}\frac{t^4}{12}\right] - \frac{1}{m_2}\left[C_v\, m\, A_2\left[\frac{t^3}{6} - \frac{K_t}{m_2}\frac{t^5}{120}\right]\right. \tag{C.15}$$

$$\left. + m^2\, C_b\, A_2^2\left[\frac{t^4}{12} - \frac{K_t}{m_2}\frac{t^6}{90} + \frac{K_t^2}{m_2^2}\frac{t^8}{2016}\right] + k\, A_2\left[\frac{t^4}{24} - \frac{K_t}{m_2}\frac{t^6}{720}\right]\right]$$

Using the values of x, $\dot{x}$, y, and $\dot{y}$ that were calculated by Equations (C.11, C.13, C.14, and C.15), a more accurate expression for the total force transmitted between the two masses can be written as follows:

$$F_t = C_v\, m\, V + C_D\, m^2\, V^2 + k(y - x) \tag{C.16}$$

where,

$$V = \dot{y} - \dot{x}$$

Using this value of the total force and Equations (C.14) and (C.15) for the displacement of the top and lower masses, better expressions can be written for the accelerations of the two masses:

$$\ddot{x} = \frac{F_t}{m_1} \qquad (C.17)$$

$$\ddot{y} = (s_t - y)\frac{K_t}{m_2} - \frac{F_t}{m_2} \qquad (C.18)$$

Finally, the equation for jerk of the top mass (2.9) can be written as

$$\dddot{x} = \frac{1}{m_1}\left[k\ V + C_v(m\ a + V\ \dot{m}) + 2C_b\ m\ V\ (V\ \dot{m} + m\ a)\right] \qquad (2.9)$$

where,

$$a = \ddot{y} - \ddot{x}$$

$$\dot{m} = \frac{-A\ P\ \dot{r}}{(P\ r + A_c)^2}$$

Results

A comparison can be made between the closed form solution and the numerical Runge-Kutta computer solution for two different values of the natural frequency of the inertially controlled valve by examining Figure 28. The closed-form solution of jerk for the valve having a natural frequency of 20 rad/sec showed the greatest deviation from the numerical computer solution of all the combinations of parameters that were tried. The numerical solution using the Runge-Kutta method took approximately 20 times more calculating time than the closed-form solution. A comparison between the two types of solutions was not shown for acceleration, velocity, and displacement. The difference in the two solutions for these variables is much less than the difference in the values of jerk.

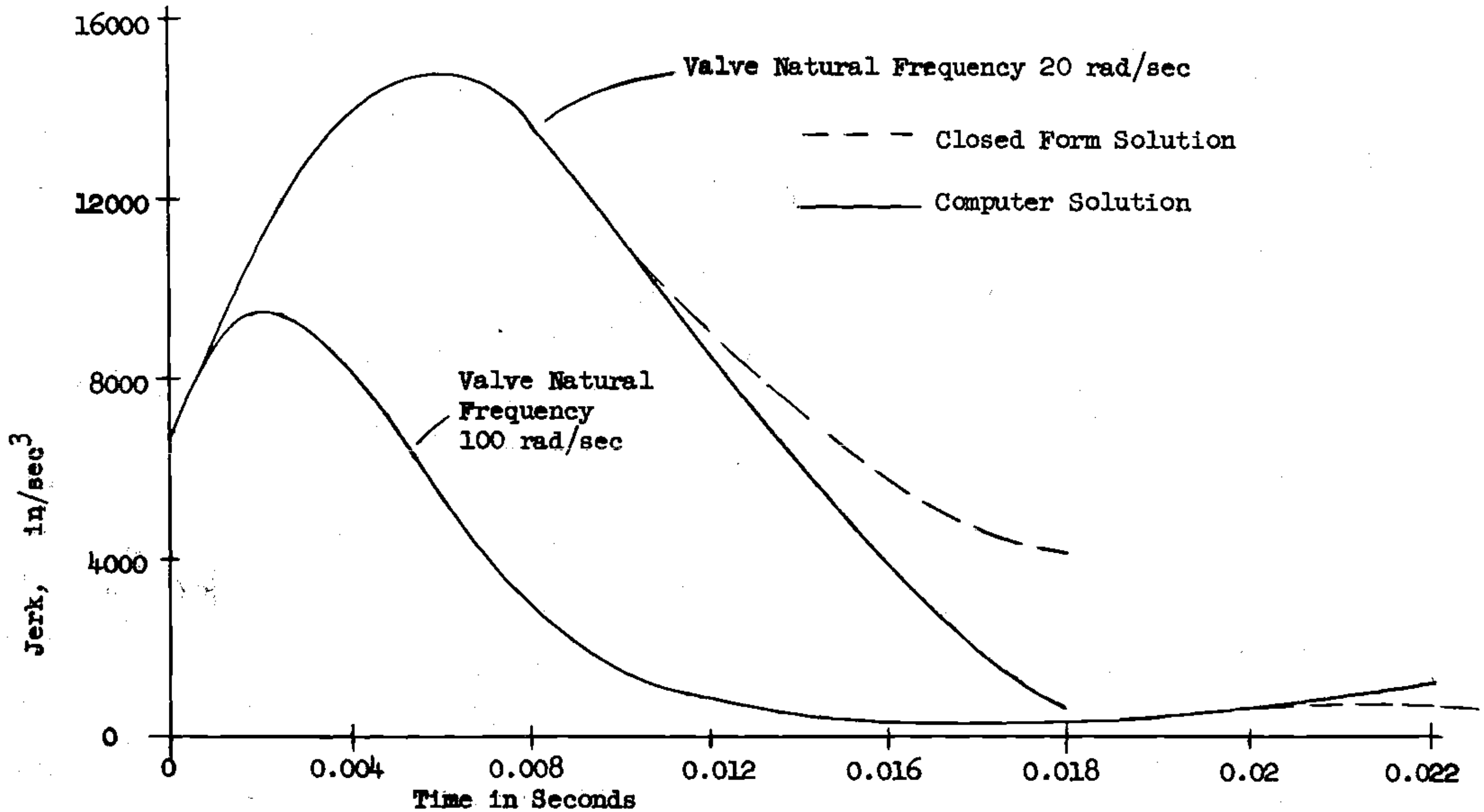


Figure 28. Comparison of Closed Form and Computer Solutions.

APPENDIX D

AN AUTOMOBILE REPRESENTED BY A FOUR DEGREE OF FREEDOM SYSTEM

General Description and Mathematical Model

An automobile was represented by a four degree of freedom system which was solved on the digital computer for both the conventional and inertial valve shock absorber. The input is equivalent to an automobile striking a perpendicular rise at 30 miles per hour. The vertical motion of the center of gravity, the angular pitch of the body, and the vertical motion of the front and rear wheels described the motion of the automobile. In effect, the automobile was split in half. The mathematical model is shown in Figure 29.

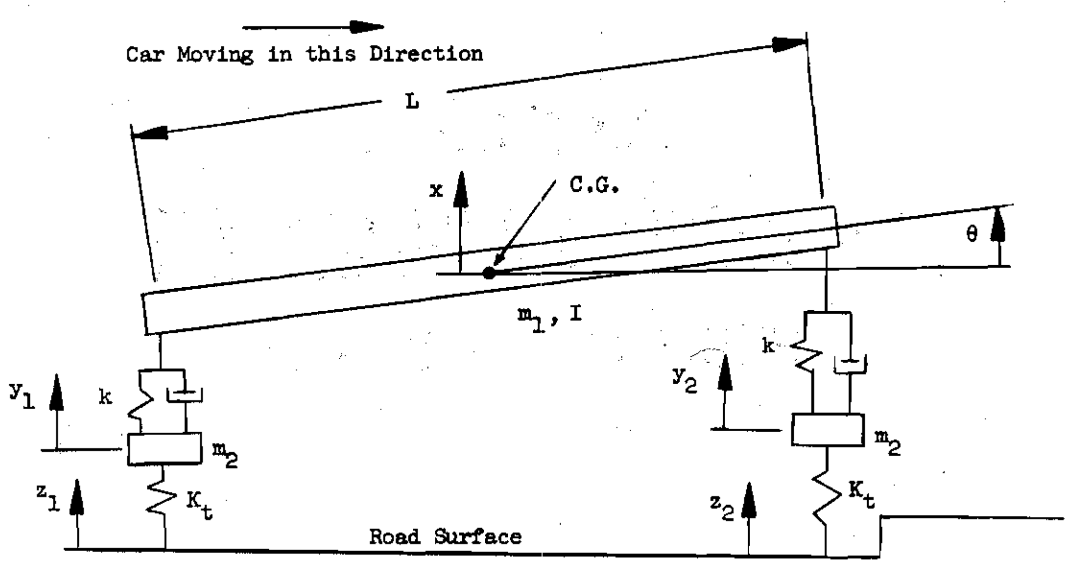


Figure 29. Four Degree of Freedom Mathematical Model.

Equations of Motion

$$m_1 \ddot{x} + k (2x - y_1 - y_2) - F_1 - F_2 = 0 \qquad (D.1)$$

$$m_2 \ddot{y}_1 + k (y_1 + \frac{L}{2} \theta - x) + F_1 = K_t (z_1 - y_1) \qquad (D.2)$$

$$m_2 \ddot{y}_2 + k (y_2 - \frac{L}{2} \theta - x) + F_2 = K_t (z_2 - y_2) \qquad (D.3)$$

$$2 \frac{I}{L} \ddot{\theta} + k (y_1 - y_2 + L \theta) + F_1 - F_2 = 0 \qquad (D.4)$$

where,

$$F_1 = C_v m V_1 + C_b m^2 V_1^2 \operatorname{sign}(V_1)$$

$$F_2 = C_v m V_2 + C_b m^2 V_2^2 \operatorname{sign}(V_2)$$

$$V_1 = \frac{dy_1}{dt} - \frac{dx}{dt} + \frac{L}{2} \frac{d\theta}{dt}$$

$$V_2 = \frac{dy_2}{dt} - \frac{dx}{dt} - \frac{L}{2} \frac{d\theta}{dt}$$

The value of m is a constant for the conventional type shock absorber, and for the inertial valve shock absorber the value of m is obtained as described in Chapter II.

The values of the parameters that were used in the computer solution are as follows:

$m_1 = 6.0$ lb-sec^2/in $\qquad$ $m_2 = 0.3$ lb-sec^2/in

$I = 20{,}000$ lb-in-sec^2 $\qquad$ $L = 126$ inches

$K_t = 1200$ lb/in $\qquad$ $k = 100$ lb/in

$A = 1.0$ in^2 $\qquad$ $A_c = 0.015$ in^2

$C_v = 0.1$ lb-sec/in $\qquad$ $C_b = 0.0001$ lb-sec^2/in^2

$M = 0.000237$ lb-sec^2/in $\qquad$ $K = 1.0$ lb/in

$f_v = 0.00001$ lb-sec^2/in^2 $\qquad$ $M\,g = f_i$

Natural Frequency of Valve = 65 rad/sec.

Results

Figure 30 shows curves for the displacement of the center of gravity and the rotation about the center of gravity of an automobile body for both types of shock absorbers. Curves of jerk and the third derivative of rotation are shown in Figure 31.

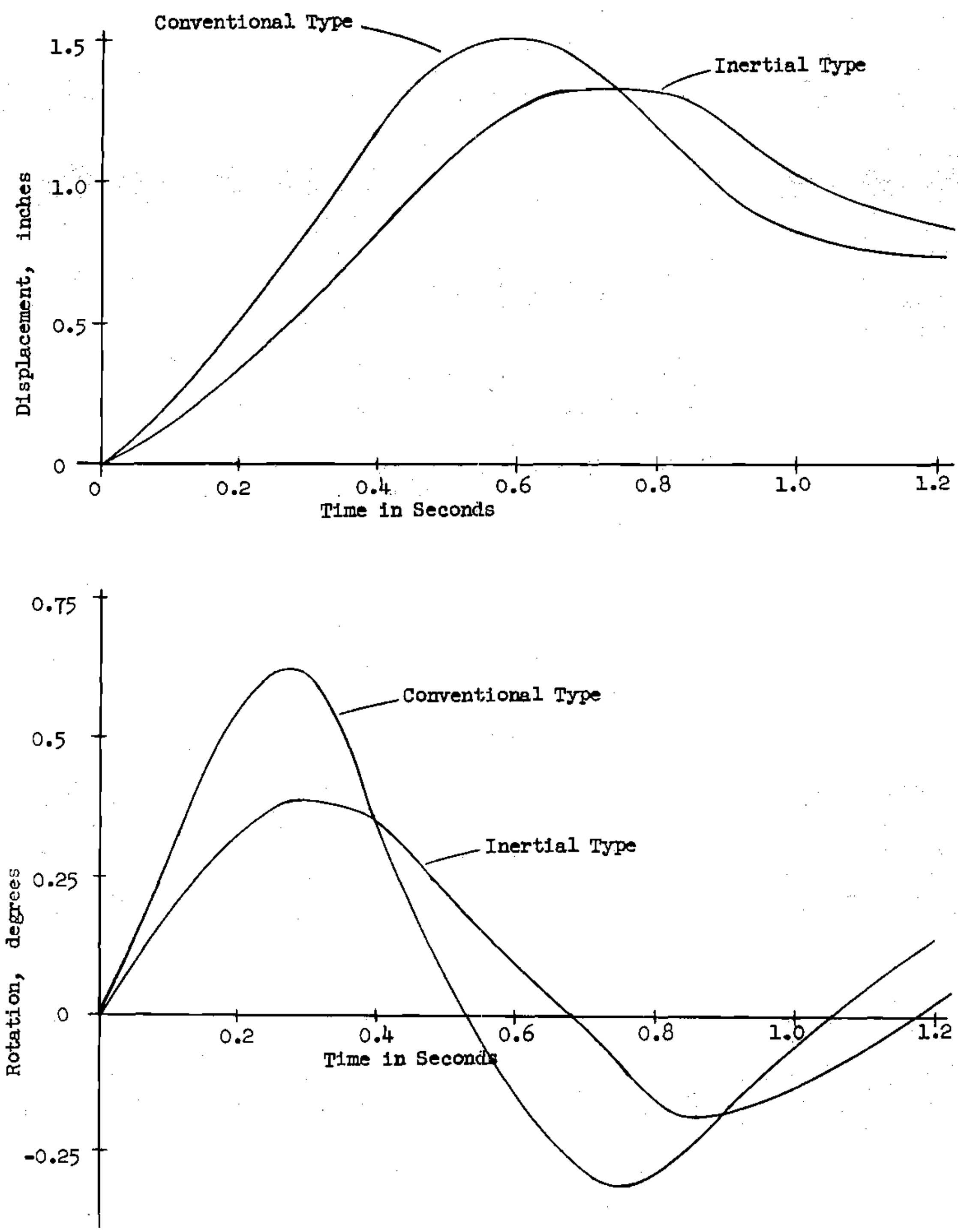


Figure 30. Displacement and Rotation Curves for Four Degree of Freedom System.

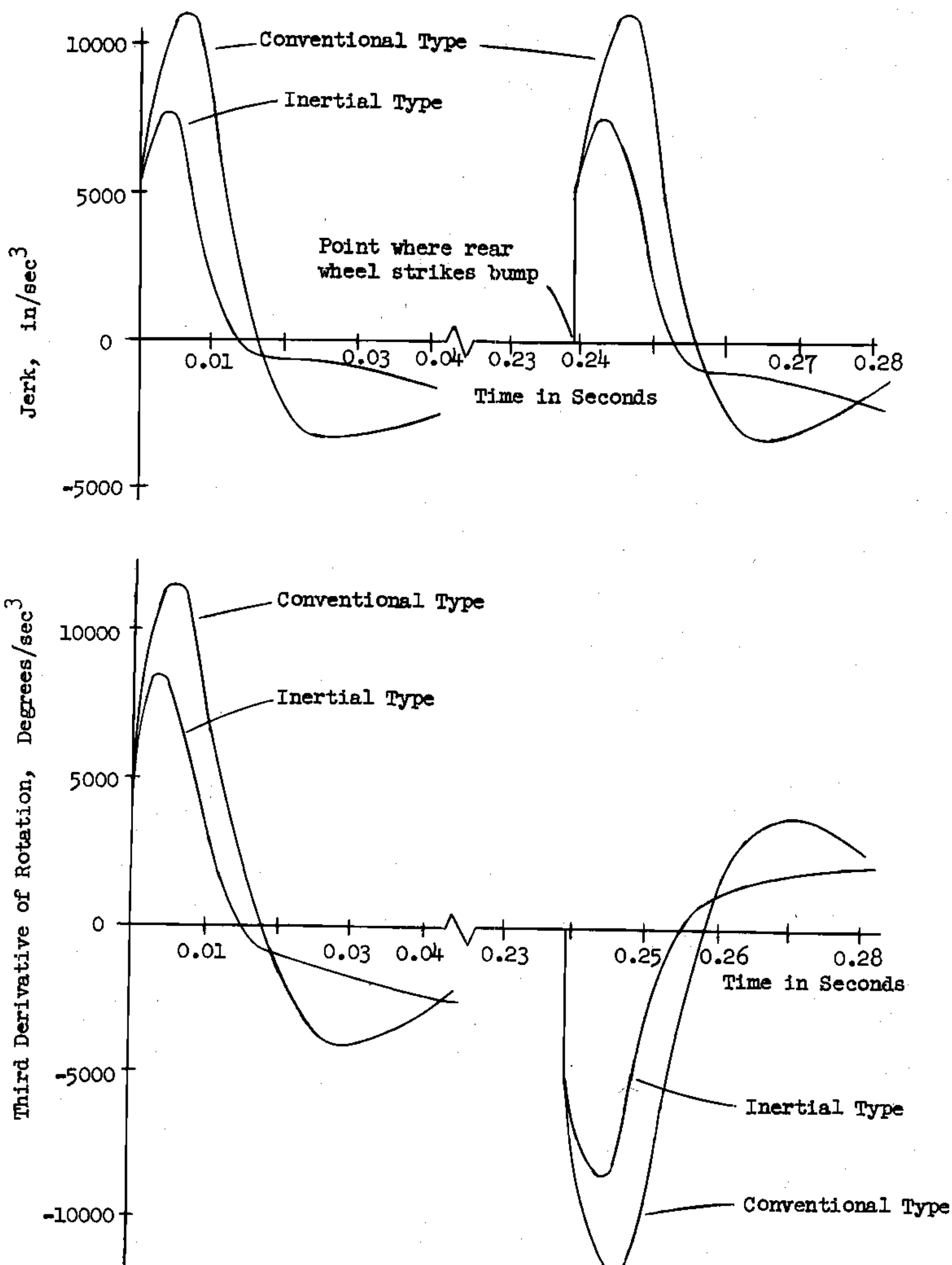


Figure 31. Jerk and Third Derivative of Rotation Curves for Four Degree of Freedom System.

APPENDIX E

REPRESENTATIVE EXPERIMENTAL DATA

The experimental data shown was taken from a run using SAE 30 weight oil and having a ramp input of 1.21 inches for the conventional shock absorber and 1.16 inches for the inertial valve shock absorber.

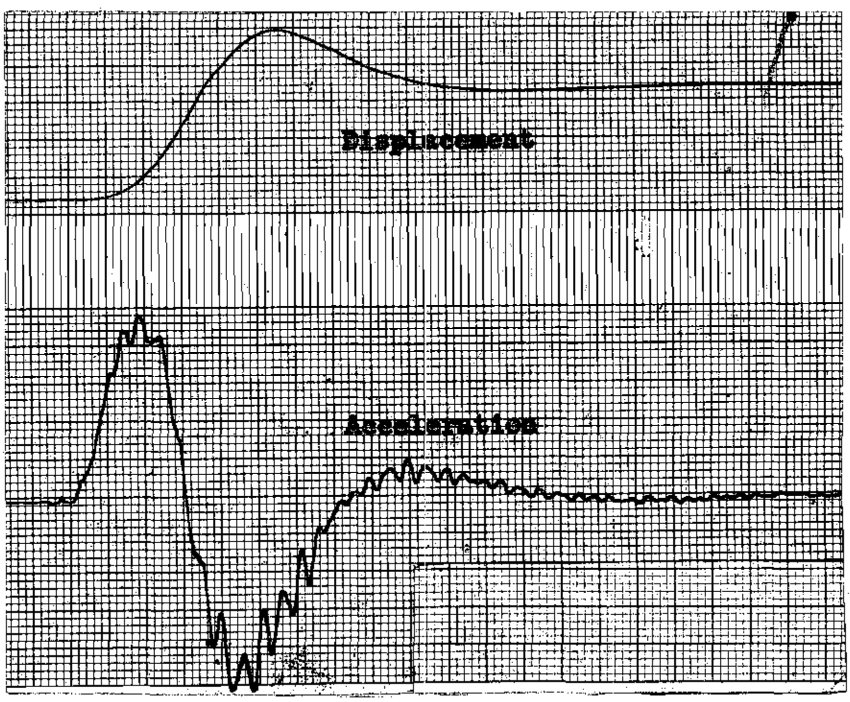


Figure 32. Displacement and Acceleration Curves for a Conventional Type of Shock Absorber using a Recorder Speed of 10 cm/sec

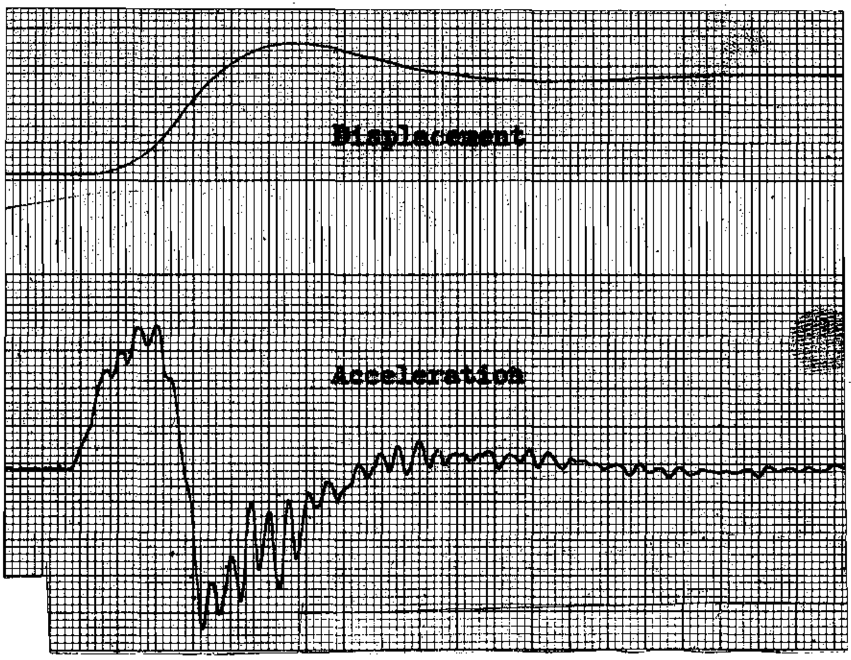


Figure 33. Displacement and Acceleration Curves for Inertial Valve Shock Absorber using a Recorder Speed of 10 cm/sec

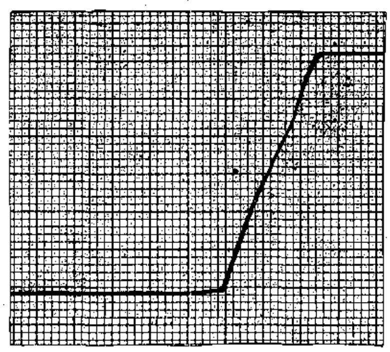
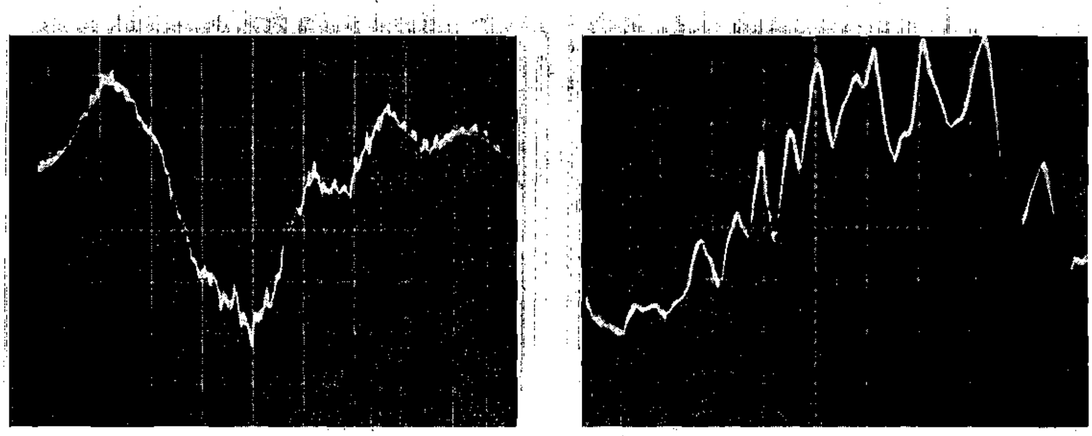
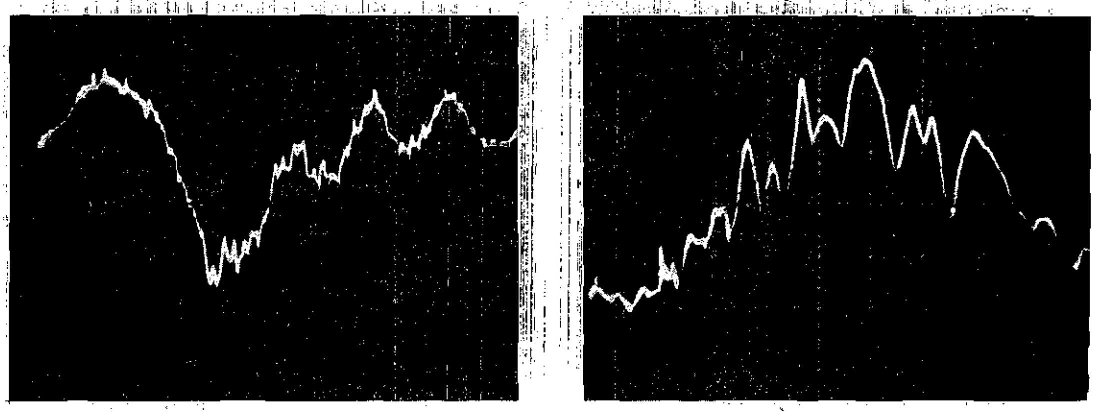


Figure 34. Displacement of Input from a Hydraulic Piston using a Recorder Speed of 10 cm/sec



20 cm/sec Recording Speed 100 cm/sec Recording Speed

Figure 35. Experimental Jerk-Time Curves for Conventional Type of Shock Absorber



20 cm/sec Recording Speed 100 cm/sec Recording Speed

Figure 36. Experimental Jerk-Time Curves for Inertial Valve Shock Absorber

APPENDIX F

REPRESENTATIVE COMPUTER PROGRAMS

```
   CURVEFIT PROGRAM -  THIS PROGRAM FINDS BY THE METHOD OF LEAST
SQUARES THE BEST VALUES OF THE COEFFICIENTS OF THE 12 TERMS OF THE
GIVEN POLYNOMIAL FOR THE SEVEN INDEPENDENT VARIABLES
BEGIN INTEGER  I, J, K ,KK  ,ZZ , F ,  H                              ;
      REAL SUM,T,Y, X1, X2, X3,X4,X5,X6,X7,RNTIM  ,P ,Q1 ,G,SS  ,AV,SD,
JLB, LAB                                        ;
LABEL L1, L2                                                          ;
ARRAY X[1:30,1:672], A[1:29] , C[1:34]  ,  B[1:250]    , M[1:250]     ;
INTEGER ARRAY Q[1:300]     ;
FILE  FIL1  (1,10), FIL2  1(1,15)                                     ;
FORMAT OUT FMT1 ("A[",I2,"]",X2,E12.5),
           FMT4 ("C[",I2,"]",X2,E12.5),
FMT5 ("B[",I3,"]",E12.5,X10,"Q[",I3,"]",I6,X10,"M[",I3,"]",E12.5),
         FMT2 ("NUMBER OF DATA POINT",I6/
"SQUARED DEVIATION",E12.5/"DEVIATION",E12.5/"ABSOLUTE DEVIATION",E12.5/
"SUM OF THE SQUARE OF THE CALCULATED Y",E12.5/
"NUMBER OF POINTS GTEATER THAN THE STANDARD",I6/
"SUM OF THE AVERAGE DEVIATION",E12.5),
         FMT3("RUN TIME",F10.4,"SECONDS")                             ;
```

```
                                     PROCEDURE CURVEFIT(N,M,X,A); VALUE N,M;
      INTEGER N, M                                                       ;
ARRAY X[1,1],A[1]; BEGIN INTEGER I,J,K,L; REAL D; ARRAY Z[1:85,1:85];FOR
  I←1 STEP 1 UNTIL M DO FOR J←1 STEP 1 UNTIL M+1 DO BEGIN Z[I,J]←0; FOR
K←1 STEP 1 UNTIL N DO Z[I,J]←Z[I,J]+X[I,K]×X[J,K] END; FOR K←M+1 STEP -1
 UNTIL 1 DO BEGIN D←0; FOR I←2 STEP 1 UNTIL K DO IF ABS(Z[I-1,1])>D THEN
 BEGIN L←I-1; D←ABS(Z[L,1]) END; IF (L-1)≠0 THEN FOR J←1 STEP 1 UNTIL K
DO BEGIN D←Z[L,J]; Z[L,J]←Z[1,J]; Z[1,J]←D END; FOR I←1 STEP 1 UNTIL M D
O A[I]←Z[I,1]; FOR J←2 STEP 1 UNTIL K DO BEGIN D←Z[1,J]/A[1]; FOR I←2 ST
EP 1 UNTIL M DO Z[I-1,J-1]←Z[I,J]-A[I]×D; Z[M,J-1]←D END END
END CURVEFIT                                                             ;
       RNTIM ← TIME(2)                                                   ;
KK  ← 12   ;  NUMBER OF TERMS IN POLYNOMIAL
       I ← 0                                                             ;
L1:    READ (FIL1,/,Y,X1, X2, X3, X4, X5, X6, X7 )[L2]                   ;
       I ←  I + 1                                                        ;
  THE FOLLOWING 12 LINES ARE THE 12 TERMS OF THE POLYNOMIAL
X[1,I]        ←   1.0                ;
X[2,I]        ←   X1                 ;
```

```
X[3,I]        ← X2              ;
X[4,I]        ← X3              ;
X[5,I]        ← X4              ;
X[6,I]        ← X5              ;
X[7,I]        ← X6              ;
X[8,I]        ← X7              ;
X[9,I]        ← X1×X2           ;
X[10,I]   ←     X3×X3           ;
X[11,I]   ←     X5×X6           ;
X[12,I]   ←     X1×X1           ;
X[13,I]     ←      Y    ;
       GO TO L1           ;
L2:    CLOSE(FIL1,RELEASE)          ;
CURVEFIT (I,KK,X,A)      ;
WRITE (FIL2, FMT1, FOR J ← 1 STEP 1 UNTIL KK DO [J,A[J]])          ;
ZZ ← 0       ;
JLB ← 0.0      ;
   F ← 0    ;     H ← 0      ;
SS ← 0.0      ;              P ← 0.0       ;
```

```
SD ← 0.0 ;
        SUM ← 0 ;
        FOR J ← 1 STEP 1 UNTIL I DO BEGIN
T ← X[KK+ 1,J] ;
  G ← 0.0 ;
FOR K ← 1 STEP 1 UNTIL KK  DO BEGIN
  Q1← X[K,J]×A[K] ;
   G ← G + Q1   END ;
JLB ← JLB + ABS((T - G)/T) ;
LAB ← (T - G)/T ;
IF ABS(LAB) > 0.25 THEN BEGIN F ← F + 1 ; Q[F] ← J ; B[F] ← LAB ;
M[F] ← T   END ;
  T ← T - G ;
IF T < 0 THEN ZZ ← ZZ + 1 ;
  SS ← SS + T ;   P ← P + ABS(T) ;
SD ← SD + G×G ;
        SUM ←  SUM +  T×T    END ;
        WRITE (FIL2, FMT2, I, SUM,SS ,P,SD,ZZ,JLB) ;
A[KK + 1] ← 1.0 ;
```

```
FOR K ← 1 STEP 1 UNTIL (KK + 1) DO        BEGIN
AV ← 0.0        ;
FOR J ← 1 STEP 1 UNTIL I DO
AV ← AV + X[K,J]                 ;
C[K] ← AV×A[K]/I       END        ;
WRITE (FIL2, FMT4, FOR J ← 1 STEP 1 UNTIL (KK + 1) DO [J,C[J]])      ;
WRITE (FIL2, FMT5, FOR J ← 1 STEP 1 UNTIL F DO [J,B[J],J,Q[J],J,M[J]]) ;
        RNTIM  ← (TIME(2) - RNTIM)/60                                  ;
        WRITE (FIL2, FMT3, RNTIM)                                      ;
        END.
930
 LABEL    00000000FIL1   001     01
0.1, 0.0, 0.0, 0.0, 2.0, 1.0, 1.5, 0.0,
1.0, 1.0, 1.0, 1.0, 1.0, 1.0,      1.0, 1.0,
2.0, 2.0, 1.0, 2.0, 1.0, 1.0, 2.0, 1.0,
3.0, 3.0, 1.0, 2.0, 1.0, 0.0, 0.0, 0.0,
10.0, 10.0, 0.0, 1.2, 2.0, 3.0, 4.1, 5.0,
10.5, 11.0, 1.0, 2.0, 0.0, 3.0, 2.0, -1.0,
-2.0, -1.0, -3.0, -2.0, -1.0, -3.0, -5.0, -1.0,
```

-10.0, -8.0, 3.0, -2.0, -4.0, -0.5, -0.5, 0.0,

-0.1,-0.1, -2.0, 0.0, -0.01, 2.0, 1.0, 0.0,

The above nine lines is the data

```
 SOLUTION OF THE SYSTEM REPRESENTING THE FOUR DEGREE OF FREEDOM MODEL
SHOWN IN APPENDIX D USING THE INERTIAL VALVE SHOCK ABSORBER
BEGIN INTEGER I, K, N                                                    ;
      REAL T, DT, M, FO, K1, KT, STP, M1, M2, KK, CC, A1, AA2, AA, AV,
         PC, XMAX, AREA, JJ, AAV, AI, BI, KJJ, MKJJ, KA1, MKA1,INFF,
         KX2, KAV, AAS, RAT, COP, JPC,MI, P,QQ,FF,RNTIM, NF, JJJ,Q,TT,
  A,  W,  VDD, AVD,AVDD, MD, MDD, AAA,  FBAR,  V,  F,  VD,N1,  AVD1,N2,
D, EX, Z, ZD, ZDD, DR, DVD, FHP, FH, G, XD, Y, YD, IFB, IIF, FD,
   C1, C2, C3, C4, C5, ZZ, ZZD,   CFH, SFH, C6, C7,C8,C9,P1,WNV,MKX2 ,
F1 , F2,  V1,  V2, ST2, ST1 ,  II ,  LL ,            AAA1 ,M3    ,
PC1 , AV1, ML,AA1,FF1          ;
ARRAY X[0:30], D[0:30,1:4]        ;
LABEL   LO, L1, L2, L3, JUDY, DAN, FHS, JANE,SUE  , MMM   ;
      FILE FIL2 4 (1,15)            ;
FORMAT OUT FMT(3E35.7),
           FMT2("RUN TIME", X3, F12.4, " SECONDS")                      ;
         RNTIM ← TIME(2)                                                 ;
N  ← 12    ; NUMBER OF FIRST ORDER DIFFERENTIAL EQUATIONS
M  ←  1.3   ;    MAXIMUM VALUE OF THE INDEPENDENT VARIABLE  (T)
```

```
   THE FOLLOWING 15 LINES ARE CONSTANTS THAT WILL BE USED IN THE
CALCULATIONS
TT   ←   10.5/44.0                                                  ;
ST2  ← 1.0                                                          ;
AA   ← 0.015          ;
NF   ← 70.0      ;
RAT   ← 2.5      ;
     KK ← NF×NF       ;    CC ← 0.5×SQRT(KK)            ;
KT   ← 1200.0     ;    M2   ← 0.3      ;
      XMAX ←  (KT)/(KK×M2)                                          ;
      COP ← (AA×M2×KK×RAT)/KT                                       ;
       Q ← 4.0                     ;
    JPC ← (Q×KK)/62500.0      ;
M1 ← 1.0/6.0 ;  M2 ← 1.0/0.3    ;  II ← 63.0/20000.0  ;  KT ← 1200.0  ;
K1 ← 100.0 ;M3 ← M2    ;         LL  ←  63.0                        ;
AI ← 0.0001                      ;      BI    ←  0.1                ;
T  ← 0.0    ;
DT  ← 0.00007   ;  TIME INCREMENT OF THE INDEPENDENT VARIABLE
      FOR I ← 1 STEP 1 UNTIL N DO
```

```
        X[I] ← 0                                                          ;   54
LO:   FOR K ← 1 STEP 1 UNTIL 4 DO                                     BEGIN   67
   THE FOLLOWING 32 LINES ARE THE EQUATIONS OF THE PROBLEM
V1 ←  X[4] − X[2] + LL×X[8]                                               ;
V2 ←  X[6] −X[2] − LL×X[8]                                                ;
IF X[11] > XMAX THEN BEGIN X[11]  ← XMAX   ;   X[12]   ← 0.0   END        ;
         IF (X[9] > XMAX) THEN BEGIN X[9] ← XMAX  ; X[10]  ← 0.0   END    ;
IF X[11] < 0.0 THEN BEGIN X[11]  ← 0.0   ;  X[12]  ← 0.0   END          ;
         IF (X[9] < 0.0) THEN BEGIN X[9] ← 0.0  ; X[10]  ← 0.0  END       ;
IF V1 > 0.0 THEN BEGIN PC   ← JPC   ;   GO TO DAN    END          ;
         PC ← 200.0×JPC                                                   ;
DAN:     AV ←  COP×X[9]                                                   ;
IF V2 > 0.0 THEN BEGIN PC1 ←   JPC    ;   GO TO  MMM    END     ;
PC1  ← 200.0×JPC             ;
MMM:   AV1   ← COP×X[11]                                                  ;
ST1 ← 0.0  ;   IF T > TT THEN ST1 ← 1.0                                   ;
MI   ← 1.0/(AA  + AV)     ;     ML   ← 1.0/(AA + AV1)       ;
F1 ←  (BI + MI×AI×V1×SIGN(V1))×MI×V1                                      ;
F2 ←  (BI + ML×AI×V2×SIGN(V2))×ML×V2                                      ;
```

```
D[1,K] ← DT×X[2]                                                    ;
D[2,K] ←DT×M1×(K1×(X[5] + X[3] −2.0×X[1])  + F1 + F2)                ;
D[3,K] ← DT×X[4]                                                    ;
D[4,K] ← DT×M2×(KT×(ST1 − X[3]) + K1×(X[1] − X[3] − LL×X[7])  −F1)   ;
D[5,K] ← DT×X[6]                                                    ;
D[6,K] ← DT×M3×(KT×(ST2 − X[5]) + K1×(X[1] − X[5] + LL×X[7])  −F2)   ;
D[7,K] ← DT×X[8]                                                    ;
D[8,K] ← DT×II×(K1×(X[5] − X[1] − LL×X[7])  +F2  −F1  −K1×(X[3] −
      X[1] + LL×X[7]))                                              ;
AA1   ← D[4,4]/DT      ;
AA2   ← D[6,4]/DT      ;
FF   ← PC×V1×V1×SIGN(V1)           ;    FF1   ← PC1×V2×V2×SIGN(V2)   ;
D[9,K]    ←  DT×X[10]           ;
D[10,K]   ← DT×(AA1  + FF  − CC×X[10]  − KK×X[9])          ;
D[11,K]   ←  DT×X[12]           ;
D[12,K]   ←  DT×(AA2  + FF1  − CC×X[12]  − KK×X[11])   ;
   THE FOLLOWING 17 LINES IS THE RUNGE−KUTTA METHOD
         IF K = 4                                          THEN
              GO TO L3                                      ELSE
```

```
                IF K = 2                                              THEN
                    GO TO L1                                                     ;
            T ← T + 0.5×DT                                                       ;
L1:         P ← 0.5                                                              ;
            IF K = 3                                                  THEN
                P ← 1.0                                                          ;
L2:         FOR I ← 1 STEP 1 UNTIL N DO
                X[I] ← X[I] + D[I,K]×P                                           ;
            IF K > 1                                                  THEN
                FOR I ← 1 STEP 1 UNTIL N DO
                    X[I] ← X[I] - 0.5×D[I,K-1]                                   ;
L3:                                                                        END   ;
        FOR I ← 1 STEP 1 UNTIL N DO
            X[I] ← X[I] + (D[I,1] + 2×D[I,2] + 2×D[I,3] + D[I,4])/6.0
                                    - D[I,3]                                     ;
A1   ←  D[2,4]/DT                                                                ;
        JJ ←  (A1 - AAV)/DT             ;          AAV ← A1                      ;
AAA   ← (D[8,4]/DT)×57.29578                                                     ;
JJJ   ← (AAA - AAA1)/DT        ;          AAA1  ←  AAA                           ;
```

```
IF T > 0.002  THEN DT ← 0.0002            ;
IF T > 0.095 THEN DT ← 0.0005          ;
MD    ← X[7]×57.29578      ;
WRITE (FIL2,FMT,T,X[1],X[2],A1,JJ,MD,AAA,JJJ)      ;
WRITE (FIL2,FMT,T,X[1],X[2])     ;
IF T < M THEN GO TO LO       ;
JUDY:     RNTIM ← (TIME(2) - RNTIM)/60                              ;
          WRITE (FIL2, FMT2, RNTIM)                                 ;
          END .
```

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