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AN INVESTIGATION OF THE VISCOSITY OF STEAM  
AT HIGH PRESSURES

A THESIS

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the Faculty of the Graduate Division

by

Samuel C. Barnett

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AN INVESTIGATION OF THE VISCOSITY OF STEAM  
AT HIGH PRESSURES

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## SUMMARY

An experimental program to determine the viscosity of high pressure steam by means of an annulus-type transpiration viscosimeter is described. A unique method of measuring small pressure drops at very high static pressures was developed to measure the pressure drop along the annular capillary. High pressure experimental data up to 10,000 psi were obtained with the annular viscometer. These data are presented in relation to existing data on the viscosity of steam.

The estimated error in the data is  $\pm 3$  per cent.

## CHAPTER I

## INTRODUCTION

Modern investigations of viscosity began with experiments by Hagen and Poiseuille who studied the flow of water through small tubes and found that the rate of flow was proportional to the pressure drop, the fourth power of the tube diameter, and the reciprocal of the tube length. They also found that the factor of proportionality was a function of the particular fluid being used.

Research on steam viscosity at high pressures and temperatures began about 1925 when Speyerer (1,2) built a capillary viscosimeter having a length of 2.4 meters. Speyerer's experimental data were in the sub-critical region. The first supercritical studies were reported by Hawkins, Solberg and Potter (3) in 1935. They used a Lawaczeck viscometer, which is a nonflow type and which uses a falling weight to produce the pressure difference. In 1936, Sigwart (2,4) presented viscosity data which he obtained with a capillary viscometer. Sigwart's measurements did not extend beyond the critical point. However, he extrapolated his data into the region of supercritical temperatures and apparently did this incorrectly. Timroth (5) in 1940 used a procedure almost identical to that used by Sigwart and obtained data which were in relative agreement with Sigwart's data below the critical temperature. Above the critical temperature Sigwart's results do not agree with the higher temperature data of Timroth. Timroth's data were apparently modified to some extent in the tabulation of the Russian steam tables (6). An excellent summary of the status of viscosity

measurements as of July 1959 is presented in reference 7 by Nowak and Grosh.

The kinematic viscosity of steam reported on herein was measured by the use of an annulus-type viscosimeter similar to that used by Thomas and Jackson (8). Values of viscosity were measured using the following equation relating kinematic viscosity to various quantities

$$v = \frac{\pi \Delta P}{8 WL} \left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right] \quad (1)$$

where

$v$  = Kinematic viscosity

$\Delta P$  = pressure drop across a given length of the annulus

$L$  = length of the annulus

$W$  = mass rate of flow

$b$  = inside radius of the outer tube of the annulus, and

$a$  = outside radius of the inner tube of the annulus.

It is evident from the above equation that if  $\Delta P$ ,  $W$ , and the dimensions of the system were known, the viscosity could be calculated. If it were possible to manufacture and measure the dimensions of the annulus with sufficient accuracy, viscosity could be calculated directly when the other factors were known. However, the allowable tolerances for the use of an annulus in direct viscosity measurements are so severe that it is usually necessary to calibrate the annulus with a fluid whose viscosity is known very accurately, such as nitrogen.

The difficulties of measuring small pressure drops across the test section at a high pressure level are self evident. Most pressure gauges in the range of 0-15,000 psi are accurate to only 0.1% of the full-scale

deflection. For instance, the dead weight tester which was utilized in determining the pressures in this experimental program was only accurate to  $\pm 10$  psi. If the total pressure can be measured to within 10 psi of the true value, then the difficulty of measuring  $\Delta P$ , a pressure drop of a few millimeters of mercury, is self evident. This difficulty was overcome in the present project by a unique manometer which made use of a standard optical absorption cell (9) and the flexibility of 1/4" O. D. high-pressure stainless steel tubing. This manometer is discussed further in the appendix.

## CHAPTER II

## DESCRIPTION OF EQUIPMENT

The equipment used in this investigation is shown schematically in Figure 1.\* Water was deaerated by boiling before it entered the pump which raised the pressure to the desired value. From the pump the water flowed through surge chambers into a preheater and then into a pressure vessel in which the annulus was located. After passing through the annulus the water or steam flowed through a series of condensers and valves to the condensate collecting flask. As indicated in Figure 1 the pressure drop across the annulus was measured with a high pressure mercury manometer made of a standard optical absorption cell (9) which is available commercially. The optical absorption cell has a one centimeter clearance between the quartz windows and is guaranteed to withstand pressures up to 50,000 psia. The manometer made use of the flexibility of high pressure 1/4" stainless steel tubing which permitted the level of the mercury to be centrally located in the quartz window in the high pressure cell. By taking a zero reading on the mercury surface with a cathetometer and comparing it to the reading at a given flow setting the  $\Delta P$  across the annulus test section could be determined. It is possible to use another optical absorption cell and thereby determine accurately both

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\*A second system very similar to the system herein described was also constructed and used to measure viscosities. The second system employed an improved manometer system. It is described by Whitesides (12). Data taken with this second set of equipment are presented in this report and are identified as "annulus #2."

levels of the mercury in the manometer. This was believed, however, to be unnecessary since the volume in the 1/4-inch flexible stainless steel tube should not have changed appreciably with the slight flexing involved in making readings.

The high-pressure autoclave which surrounded the annulus was wrapped in a 1/4"-thick copper sheet and placed into an automatically controlled electric furnace. A strip chart recorder indicated that at steady state conditions the temporal variation of temperature inside the furnace was less than one degree Fahrenheit.

Water legs were maintained over the mercury in the manometer by means of dams and a condenser which were placed at strategic positions on the horizontal portions of the manometer leads (see Appendix A). Some difficulty was experienced in maintaining steady state in the initial shakedown of the equipment. However, this was found to be due to minute quantities of air which were trapped in various sections of the apparatus. The air was eliminated by utilizing vents at the top of the system, filling the system completely with deaerated water, and periodically bleeding the vent lines.

The condensate which was collected during a run was weighed on a laboratory analytical balance. System pressure was measured by means of a dead-weight tester manufactured by the American Instrument Company.

The annulus used in this investigation was made in a manner similar to those constructed by Thomas and Jackson (8). It had a test section four inches in length. The outer diameter of the inner tube measured approximately 0.305 inch, and the inside diameter of the outer tube was 0.3125 inch. The outer tube was made from standard 9/16-inch O. D. by 5/16-inch

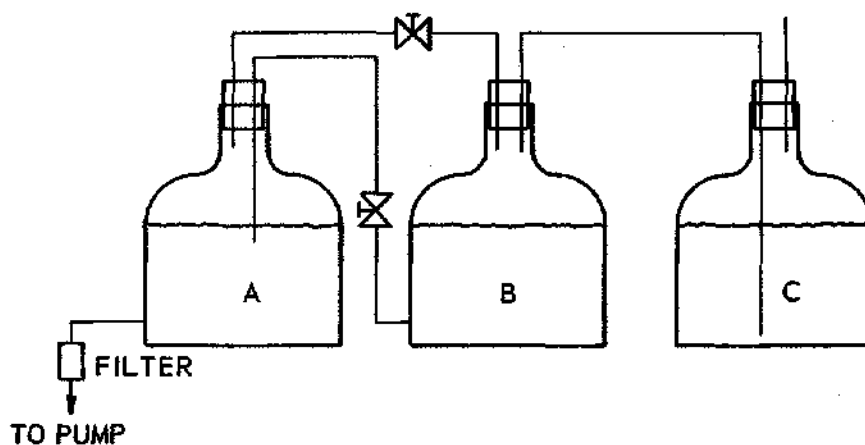
I. D. Type 304 stainless steel tubing. The inner tube was made from standard 3/8-inch O. D. by 1/8-inch I. D. Type 304 stainless steel tubing. The outside tube was ballized with a 0.3125 inch diameter tungsten-carbide ball and polished with jeweler's rouge. The inner tube was machined to size and polished with jeweler's rouge. Surface roughness was checked with a Surfindicator and the measured variance was between 5.5 and 7.5 microinches. A schematic diagram of the annulus is shown in Figure 2. It is apparent from this drawing that no corrections had to be made for entrance or exit effects. The relatively long calming sections eliminated this need.

## CHAPTER III

## TEST PROCEDURE

Measurement of Viscosity.--Because of the large heat capacities of the autoclaves and other equipment it was necessary to run the equipment for several hours before any experimental data were taken at a given temperature.

In preparing the system for a series of tests the preheater and the furnace controls were placed in operation and adjusted to the desired settings. Then a long warm-up period was started. The inlet water, which was triple distilled, was deaerated by boiling prior to being introduced into the system by means of the pump. Triple-distilled water was used throughout the experiment. It was essential that this water be completely deaerated before being introduced into the system. This deaeration was accomplished by using three glass jars connected as shown below.



Jars A and B were boiled vigorously for at least thirty minutes prior to starting the pump. Any entrained air was thus vented back through jar C. To fill jars A and B, a cold damp cloth was placed over the upper part of A and B, thus producing a vacuum to fill these jars. The line connecting the bottom of B with jar A was then valved off until the liquid level in A dropped to a low point. This system worked very successfully. It should be mentioned that attempts to use a single jar for the deaeration were not nearly as successful as the three-jar arrangement.

After passing through the pump, the water then passed through a 5-micron filter manufactured by American Instrument Company. It next entered an unheated surge chamber and then entered a heated surge chamber. The object of these surge chambers was to eliminate surges in pressure caused by the reciprocating pump.

A preheat tube connected the hot surge chamber with the annulus test section. This tube was fitted with three electrical resistance heaters wired through separate rheostats. The temperature of the water leaving the preheat tube was sensitive to small voltage changes, and it was therefore necessary to use a voltage stabilizer on the rheostat power supply.\* In establishing temperature equilibrium, two of the resistance heaters were set at a constant voltage and the voltage was varied on the third heater to obtain the desired constant temperature value.

After passing through the annulus test section the water entered a series of throttling valves in which the pressure was reduced to atmospheric pressure. Adjusting these valves to achieve a constant flow rate

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\*The system using annulus No. 2 employed a voltage regulator to supply constant voltage power to the rheostats.

proved to be perhaps the most difficult part of the experiment. This problem was overcome by installing a pressure gauge in front of the valve nearest the collecting beaker. This outlet valve was then partially closed until a flow rate prevailed that was slightly higher than the desired flow rate. After noting the pressure gauge reading the valve nearest the annulus was gradually closed until the pressure gauge indicated a predetermined pressure drop across the valve. This procedure was continued until the pressure drop across each valve was approximately the same as the pressure drop across any other valve.

During the latter part of the experimental program a capillary tube was used as a throttling orifice. This method of controlling flow rates worked exceptionally well and is worthy of the following detailed discussion.

The tube length required for a specified pressure drop and flow rate was estimated by using the well known expression for head loss in a tube:

$$h_1 = f \frac{L}{D} \frac{V^2}{2g}$$

where

$$f = \frac{64}{N_{Re}} \text{ for laminar flow}$$

L = length of capillary

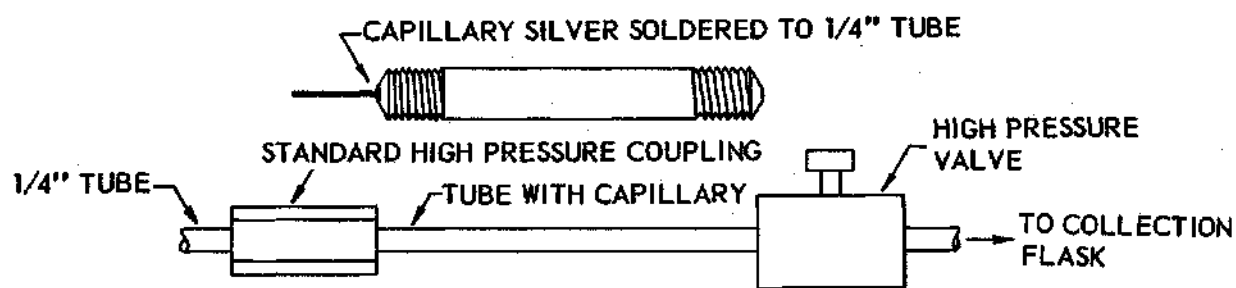
D = diameter of capillary

V = velocity of fluid in the capillary

g = gravitational constant

The capillary tube used had an inside diameter of 0.004 inch and

an outside diameter of about 0.012 inch. This tubing is suitable for pressures up to 15,000 psi at room temperature. The tubing was installed as shown in the sketch below. The throttling valve at the end of the tube was used to absorb only a very small amount of the total pressure drop. By using this arrangement the amount of water collected during successive 5-minute runs did not vary by more than about 0.05 gram, which is approximately the mass of one drop of water.



Some difficulty was experienced in cutting the fine capillary tube without pinching the ends. The most satisfactory method of doing this proved to be that of gripping the tube with two pair of pliers and cold working the tube until it parted, giving a clean break.

Data required to determine the kinematic viscosity are the flow rate of condensate and the pressure drop across the annulus. The system pressure and the system temperature were also measured so that the viscosity would be known as a function of these thermodynamic properties. Of these variables, the most difficult to measure accurately was the pressure drop across the annulus. The unique system developed for this measurement has been described and is shown in Figure 1. The actual mercury level in the optical absorption cell was measured by means of a cathetometer.

Further details of this manometer are given in the appendix.

The usual duration of a run was from 5 to 20 minutes. Prior to the beginning of each run a preliminary set of data was obtained. These data were checked again before starting a test. At the beginning of a test a collecting flask was placed under the discharge from the system and a timer was started. The pressure drop, the necessary temperatures, and the system pressure were recorded during the interval of time the condensate was being collected. The system pressure was determined by means of gages and a dead-weight tester. The run was terminated when the collected condensate was removed from the system discharge.

Calibration of Annulus.—The simplified equation used to calculate kinematic viscosity was:

$$v = C C_t \frac{\Delta P}{W} = \frac{\mu}{\rho} \quad (2)$$

The coefficient,  $C$ , which appears in this equation is a constant which is dependent upon the dimensions of the annulus and on the units by which the pressure drop and the volume flow rate are expressed (see equation 1).  $C_t$  is the temperature correction factor to account for the dimensional changes of the annulus with temperature. It should be mentioned that there would normally be a correction factor for pressure; however, since the radial pressure forces across the annulus were eliminated by encasing the entire annulus in a pressure vessel, no pressure correction was necessary. To determine the constant,  $C$ , for the annulus requires that the dimensions of the annulus be known to an extreme degree of accuracy or that the annulus be calibrated against a fluid whose viscosity is known quite accurately. In the case reported on herein the annulus

constant was determined by calibration with dry nitrogen (99.998% purity) using viscosity data published by the U. S. Bureau of Standards (10).

The annulus constant,  $C$ , in the above equation is proportional to the annulus dimensions as follows:

$$C = C_1 \frac{\left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right]}{L} = C_1 C_2$$

where

$$C_2 \equiv \frac{\left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right]}{L} .$$

The constant,  $C_2$ , was determined by calibration with nitrogen to be  $22.20 \times 10^{-13}$  ft.<sup>3</sup> and to be independent of Reynolds number within the experimental accuracy of the results. In the calibration the maximum deviation of any point from the above value was less than 0.5 per cent up to Reynolds number of 72. Details of the nitrogen calibration are included in Appendix B.

The temperature correction factor was determined by theoretical calculations and was checked experimentally with nitrogen to within  $\pm 0.5\%$ . On annulus No. 2, the experimentally determined temperature correction factor was consistently higher than the theoretical value by about 0.3%. Consequently, on annulus No. 2 the experimental value of  $C_T$  was used in all calculations.

## CHAPTER IV

## COMPARISON OF RESULTS WITH EXISTING VISCOSITY DATA

Results obtained in this investigation are compared with existing viscosity data in Figures 3, 4, 5, and 6.

Figures 3 and 4 compare values of kinematic viscosity with the data of Whitelaw (11) and the Russian Steam Tables (6), respectively. The comparison of kinematic viscosities is probably more meaningful than a comparison of dynamic viscosity values because the calculation of dynamic viscosity from the measured value of kinematic viscosity involves the density of steam thus requiring that different experimenters use the same steam density data in computing the dynamic viscosity.

It is noted that values of kinematic viscosity determined in the present study are slightly lower than Whitelaw's data. Since Whitelaw's supercritical data are lower than the Russian values, it follows that the Russian values appear to be too large.

Figures 5 and 6 compare values of dynamic viscosity with the data of Whitelaw (11) and Thomas and Jackson (8), respectively. The data from the present study are consistently somewhat lower than Whitelaw's data. Part of this deviation may be due to the use of different density values used in converting from kinematic viscosity to dynamic viscosity.

A comparison with the data of Thomas and Jackson indicates reasonably good agreement, below 800 °F. Above 800 °F Thomas and Jackson apparently under-corrected for the pressure effect.

Since the data from the present study and Whitelaw's data are in

close agreement, and since these data represent two completely different systems, it is felt that data from the present study and the data of Whitelaw are the most accurate data on steam viscosity at supercritical conditions.

## CHAPTER V

## SOURCES OF ERROR

Temperature Measurement.--The temperature of the steam flowing through the annulus was measured by a 28-gauge Chromel-Alumel thermocouple which was calibrated against a Platinum-Platinum and 10%-Rhodium thermocouple which had been calibrated by the U. S. Bureau of Standards.

After the experimental data had been taken, the Chromel-Alumel thermocouple was recalibrated and was found to agree with the original calibration. Details of the calibration procedure are given in Appendix D.

It is estimated that the temperature was measured accurately to within one degree Fahrenheit.

Pressure Measurement.--Pressures were measured by means of a 10,000 psi Crosby test gauge and a 20,000 psi dead-weight tester manufactured by the American Instrument Company. The dead-weight tester was guaranteed to be accurate to  $\pm 10$  psi. It was not possible to keep the dead-weight tester in exact balance at all times during a run. However, it was kept sufficiently well balanced that addition or subtraction of a 10 psi weight would swing the balance from one extreme to the other. It is therefore assumed that the pressure measurements should be correct to within  $\pm 20$  psi and that they are probably accurate to within  $\pm 15$  psi.

Mass Flow Rate.--The mass of condensate collected was weighed on an analytical balance to the nearest 0.0001 gram. Since the smallest sample collected weighed approximately 7 grams, the error in mass measurement was

considered negligible.

The length of a run was from 5 to 20 minutes; this time period was measured with a stop watch which was calibrated against an electric timer. The stop watch was graduated to read accurately within 0.005 minute.

In view of the above it is estimated that the maximum mass flow rate error is less than 0.2 per cent.

Pressure Drop Across Annulus.--Pressure drop across the annulus was measured by visually sighting on a mercury level, which was visible in the optical absorption cell, with a cathetometer. The cathetometer was equipped with a vernier graduated to 0.005 centimeter; however, it was possible to estimate the pressure drop to the nearest 0.0025 centimeter. Since most pressure drops were in excess of 0.4 centimeter, it is estimated that the error in measuring the pressure drop was less than one per cent.

Temperature and pressure effects on the density of the water and mercury in the manometer legs were taken into consideration. These effects are discussed in Appendix C.

Overall Accuracy.--Considering that

$$v = C C_t \frac{\Delta P}{W}$$

it follows that the kinematic viscosity may be in error by  $\pm 2.2$  per cent, since

C may be in error by 0.5 per cent

$C_t$  may be in error by 0.5 per cent

$\Delta P$  may be in error by 1.0 per cent, and

W may be in error in 0.2 per cent

It is believed that the overall accuracy is better than  $\pm 3$  per cent.

## CHAPTER VI

## THEORETICAL APPROACHES TO THE CALCULATION OF VISCOSITY

General.--The difficulties associated with making reliable calculations of the viscosity of gases are reflected in the following statement by Professor J. Kestin of Brown University: "The calculation of viscosity of gases has so far met with only limited success, extensive experimental determinations still forming the basis for practical applications." (13)

In spite of these difficulties, however, a large amount of very capable attention has been given to this problem. A very exhaustive summary of the various theoretical approaches is contained in reference 14. These authors have concluded that the Enskog theory represents the best approach to the calculation of viscosity values for dense gases. The U. S. Bureau of Standards has patterned an equation based on Enskog's theory to fit the existing data on the viscosity of steam (10).

Dilute Gases.--Before proceeding to a discussion of the viscosity of dense gases, a brief discussion of techniques for computing the viscosity of dilute (low pressure) gases is in order.

Certain physical properties of gases, such as the equation of state and the viscosity are closely connected with the laws of force between the individual molecules. The general relation between the transport properties of gases and the intermolecular forces has been known for some time, the basic theory having been developed by Chapman, Enskog, and others. Chapman and Cowling (15) have expressed the viscosity and other transport properties in terms of a set of integrals, which are called

$\Omega^{(1,n)}$ . Once these integrals are evaluated for a molecular model in which the laws of interaction are realistic, it is possible to determine values of physical properties under conditions for which experimental data do not exist.

A reasonable model for polar gases, such as water, may be composed of a spherically symmetric Leonard-Jones potential but including a point dipole at its center. Such a model is due to Stockmayer (16) and may be expressed as follows:

$$E(r) = 4\epsilon \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right] - \frac{\mu^2}{r^3} \alpha$$

where

- $E(r)$  = energy of interaction
- $r$  = separation between the molecules
- $\epsilon$  = maximum energy of attraction
- $r_0$  = low velocity collision diameter, i.e. the separation for which the energy of interaction is zero
- $\mu$  = dipole moment of the molecule
- $\alpha$  = an orientation factor
- $= 2 \cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \varphi$
- $\theta_1$  and  $\theta_2$  = angles of inclination of the two dipole axes to the intermolecular axis
- $\varphi$  = azimuthal angle between the dipole axes.

It should be noted that  $r_0$  and  $\epsilon$  are the collision diameter and the maximum energy of interaction, respectively, if dipole forces are not present.

In order to avoid the task of evaluating the Chapman and Cowling

collision integrals for the above potential expression, Krieger (16) put  $\alpha = 2$ ; that is, he assumed that the dipoles are in an attractive end-on position and maintain their maximum attractive energy value of  $-\frac{2\mu^2}{r^3}$  throughout their encounter. This results in

$$E(r) = 4\epsilon \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 - \delta \left( \frac{r_0}{r} \right)^3 \right]$$

where  $\delta$  is a pure number, which Rowlinson (16) called the "reduced dipole energy," and is defined by the relation

$$\delta = \frac{\mu^2}{2\epsilon r_0^3}$$

Hirschfelder, Curtiss, and Bird (14) have called this simplified expression for  $E(r)$  the Krieger potential.

Using this potential function, Krieger has evaluated the Chapman and Cowling integral for  $\ell = 1$ ,  $n = 2$ . He has also evaluated  $\epsilon$ ,  $r_0$ , and  $\delta$  from existing experimental low temperature viscosity data. By this procedure he has computed the viscosities of several gases over a temperature range from 298.16° K to 3000° K at a nominal pressure of 1 atmosphere. The results for water are almost identical with the experimental values given by the U. S. Bureau of Standards (10). However, it is most important to note that these theoretical calculations apply only at conditions of low pressure and that they are completely invalid for high pressures.

Dense Gases.—In the rigorous kinetic theory of dilute gases it is assumed that there are only two-body collisions and that the molecular diameter is negligibly small in comparison with the average distance between molecules.

Enskog's theory is based on a gas composed of rigid spherical molecules having a diameter  $\sigma$ . For this special model there are no three-body and higher order collisions. By thus considering only two-body

collisions and by taking into account the finite size of the molecules he succeeded in grafting a theory of dense gases onto his earlier theory of dilute gases. Enskog also noted that in dense gases there is a change in the rate of collisions between molecules because of two competing factors: (1) the frequency of collisions tends to become greater because  $\sigma$  is not negligibly small compared with the average distance between molecules, and (2) the frequency of collisions tends to decrease because the molecules are so close together that they shield each other from collisions.

The result of the Enskog theory for calculating dynamic viscosity values has been written by Chapman and Cowling (15) as:

$$\frac{\mu}{\mu'} = b\rho \left[ \frac{1}{b\rho\psi} + \frac{4}{5} + 0.7614 b\rho\psi \right]$$

where

$$b = \frac{2}{3} \pi \frac{\sigma^3}{m}$$

$m$  = mass of a molecule, gm

$\rho$  = mass density of the gas, gm/cc

$\psi$  = a factor greater than unity in the case of a dense gas, accounting for the reduction of space available for the motion of a molecule ( $\psi = 1$  for a dilute gas)

$\sigma$  = radius of molecule, cm

$\mu$  = dynamic viscosity of the gas at specified pressure and temperature, poise

$\mu'$  = dynamic viscosity of the gas at specified temperature and low pressure, poise

If  $\psi$  is put equal to unity, this equation can be written:

$$\frac{\mu}{\mu'} = 1 + \frac{4}{5} (bp) + 0.7614 (bp)^2$$

The U. S. Bureau of Standards (10) has used the results of Hilsenrath and Touloukian (17) who wrote the Enskog equation in the form

$$\frac{\mu}{\mu'} = 1 + A (bp) + B (bp)^2$$

where

$$A = 0.175$$

$$B = 0.8651$$

$$b = \frac{1.783}{M^{1/4}} \left( \frac{\sqrt{T}}{\mu'} \right)^{3/2}$$

$$M = \text{molecular weight} = 18.016 \frac{\text{gm}}{\text{mole}}$$

T = Kelvin temperature

The constants A and B were evaluated from existing experimental values of the viscosity of steam. The equation is reported to predict the viscosity of steam within  $\pm 10\%$  at pressures not exceeding 300 atmospheres. A comparison of this equation with the results of the present study is made in Figure 8. The Rich Electronic Computer Center at Georgia Tech fitted the experimental data obtained in this project to a third degree polynomial of the form  $\frac{\mu}{\mu'} = A + B(bp) + C(bp)^2 + D(bp)^3$ . Using values of (bp) ranging from 0.3 to 2.89, the resulting equation was

$$\frac{\mu}{\mu'} = 0.790 + 0.817(bp) - 0.0751(bp)^2 + 0.0937(bp)^3$$

The above equation should be valid between values of  $bp$  from 0.3 to 2.89

Principle of Corresponding States.—The principle of corresponding states is often very useful in predicting viscosity values, provided viscosity data and  $P, V, T$  data are available for a similar substance. It was suggested by van der Waals that the critical point is a point of "corresponding states" and that the properties of various substances should be compared where the variables are the same multiples of the variables at the critical point. This leads to an equation of the form

$$\frac{\mu}{\mu_c} = \mu_r(P_r, T_r)$$

where

- $\mu$  = viscosity at any temperature and pressure
- $\mu_c$  = viscosity at the critical point
- $P_r$  = absolute pressure divided by critical pressure
- $T_r$  = absolute temperature divided by critical temperature
- $\mu_r$  = indicates functional notation

This equation forms the basis of the generalized viscosity charts of Hougen and Watson (18). Comings and Egly (19,20) also used this approach in an attempt to correlate existing viscosity data for various gases. Uyehara and Watson (21) found that viscosity values for more than 50 gases could be predicted within 10% by means of this correlation. They found, however, that predictions for water and some other polar substances differed from experimental values by more than 100%. This prompted the suggestion, which now seems to be widely accepted, that reduced property

viscosity charts for non-polar substances are almost completely worthless when applied to polar substances. Reliable high-pressure viscosity data on polar substances is so scarce that it seems impractical at the present time to attempt the construction of a reduced property viscosity chart for polar substances. Reference (22) gives a limited amount of high-pressure data for ammonia, but there is such a wide disagreement in experimental values that the reliability of the data seems questionable.

## CHAPTER VII

## CONCLUSIONS AND RECOMMENDATIONS

The parameter ( $bp$ ) used by the U. S. Bureau of Standards (10), appears to be very suitable for correlating the viscosity of steam at pressures up to 10,000 psia. The Russian Steam Tables (6) appear to be the most complete set of data on steam viscosity for pressures up to 300 kg per square centimeter. However, it appears that these viscosity values are much too large at supercritical temperatures. It is suggested that Figure 8 or the following equation be used for determining viscosity values:

$$\frac{\mu}{\mu'} = 0.790 + 0.817 (bp) - 0.0751 (bp)^2 + 0.0937 (bp)^3$$

where the various symbols have meanings as previously defined.

It is suggested that as more data become available on the viscosity of polar substances, such as ammonia, an attempt be made to develop a reduced-property viscosity chart for polar substances.

It is also suggested that data from this study be used to supplement the existing theoretical methods for computing the viscosity of dense gases. In the limit, it might be possible to develop theoretical equations for the viscosity of liquids, a field in which the theory is badly in need of extension.

The annulus appears to be an excellent tool for making viscosity measurements. It is especially well adapted for measuring the viscosity

of gases which do not condense in the manometer legs. It is also well adapted to measurements at supercritical pressures so that there is no problem regarding the location of a liquid-vapor interface in the manometer leads. However, as used in this experiment, there was a problem of "surging" in the manometer at subcritical conditions; this was thought to be due to sudden flashing of liquid into the vapor state at some point in the manometer leads.

The manometer which was designed specifically for use in this study served unusually well. However, the possible error in pressure drop measurements could be minimized considerably by attaching a wideview microscope, with a cross hair, directly to the optical absorption cell. This improvement was made in building a second set of equipment used with annulus #2. Whitesides (12) has described the details of the improved manometer.

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Table 1. Experimental Data for Determination of Annulus Constant.

Barometer In. Hg.	Annulus Temp. °F	Meter Temp. °F	Inlet Press. In. Hg. gauge	$\Delta P$ Cm. H <sub>2</sub> O	Meter Flow Rate, cc/sec.	Annulus Constant ft. <sup>3</sup> x 10 <sup>13</sup>
29.23	95.2	87	34.55	28.340	8.4126	22.26
29.23	95.2	87	34.55	28.360	8.4202	22.23
29.23	95.2	87	34.55	28.400	8.4277	22.23
29.23	93.5	88.5	34.56	13.850	4.1545	22.14
22.23	93.5	88.5	34.56	13.850	4.1545	22.14
29.29	95.3	87.5	34.95	14.880	4.4587	22.20
29.29	95.5	88	20.08	30.780	7.0757	22.26
29.30	93.0	84	32.53	57.580	16.302	22.14
29.29	94.3	84.5	34.41	61.580	17.945	22.20

Table 2. Experimental Data - 3750 psia. Annulus #1.

Point No.	Temperature °C	Avg. flow rate gm/min	$\Delta P$ cm. Hg & H <sub>2</sub> O	$v$ $\frac{\text{cm}^2}{\text{sec}} \times 10^3$
7	371.7	3.794	0.218	1.067
74	380.2	11.239	0.651	1.079
73	381.3	10.466	0.602	1.070
75	385.4	11.791	0.683	1.078
9	394.3	9.062	0.670	1.377
8	395.2	9.119	0.673	1.375
10	396.1	3.583	0.294	1.492
2	403.6	7.257	0.624	1.602
6	403.7	7.326	0.626	1.592
4	405.8	10.647	0.944	1.649
5	406.4	10.500	0.940	1.665
76	420.3	9.749	1.019	1.949
1	434.1	5.984	0.681	2.124
60	436.6	9.780	1.127	2.151
77	448.5	9.859	1.269	2.404
78	448.8	9.866	1.276	2.416
82	449.4	6.245	0.793	2.373
79	474.7	10.989	1.575	2.680
81	486.9	4.840	0.758	2.933
80	495.4	11.233	1.855	3.093

Table 3. Experimental Data - 5000 psia. Annulus #1.

Point No.	Temperature °C	Avg. flow rate gm/min	$\Delta P$ cm. Hg & H <sub>2</sub> O	$\frac{v}{\text{sec}} \times 10^3$
21	395.9	3.627	0.212	1.087
20	396.1	3.243	0.190	1.089
19	403.8	3.612	0.210	1.083
71	413.9	8.005	0.498	1.160
72	427.1	10.580	0.725	1.279
53	435.3	7.427	0.539	1.354
51	435.6	13.926	1.060	1.421
52	436.6	12.886	0.992	1.437
83	450.1	7.237	0.610	1.574
84	459.7	7.605	0.698	1.715
85	468.7	7.686	0.740	1.800
86	479.9	7.679	0.810	1.970

Table 4. Experiment Data - 7500 psia. Annulus #1.

Point No.	Temperature °C	Avg. flow rate gm/min	$\Delta P$ cm Hg & H <sub>2</sub> O	$\frac{v}{\text{sec}} \times 10^3$
13	367.5	3.041	0.172	1.051
65	379.0	8.669	0.518	1.115
64	386.2	6.617	0.387	1.089
63	394.7	3.085	0.169	1.023
15	395.2	7.220	0.402	1.032
14	395.7	7.362	0.408	1.036
66	420.8	7.798	0.468	1.104
68	424.6	8.316	0.490	1.102
61	435.2	9.976	0.601	1.125
58	435.7	3.540	0.215	1.135
54	435.9	4.864	0.302	1.141
59	436.6	3.490	0.213	1.144
67	453.8	8.758	0.539	1.150
92	468.3	7.312	0.481	1.230

Table 5. Experimental Data - 10,000 psia. Annulus #1.

Point No.	Temperature °C	Avg. flow rate gm/min	$\Delta P$ cm Hg & H <sub>2</sub> O	$\frac{\text{cm}^2}{\text{sec}} \times 10^3$
23	368.4	2.672	0.150	1.051
22	369.1	2.660	0.150	1.047
55	435.6	9.778	0.576	1.099
70	449.4	9.582	0.573	1.117
57	469.8	10.533	0.639	1.136
56	470.4	10.435	0.638	1.144

Table 6. Experimental Data. Annulus #2.

Point No.	Pressure Psia	Temperature °C	Avg. flow rate gm/min	$\Delta P$ in Hg & H <sub>2</sub> O	$\frac{cm^2}{sec} \times 10^3$
1	3750	384.2	4.757	0.05062	1.049
2	3750	414.2	4.771	0.08603	1.786
3	3750	447.5	4.791	0.1143	2.364
4	3750	474.6	4.802	0.1343	2.775
5	3750	488.9	4.880	0.1471	2.995
6	5000	383.3	6.188	0.07034	1.122
7	5000	413.9	6.178	0.07488	1.199
8	5000	440.3	5.843	0.09018	1.523
9	7500	388.9	7.011	0.07613	1.073
10	7500	426.7	6.931	0.07820	1.118
11	7500	453.9	6.918	0.08440	1.210
12	7500	491.7	6.852	0.09984	1.449

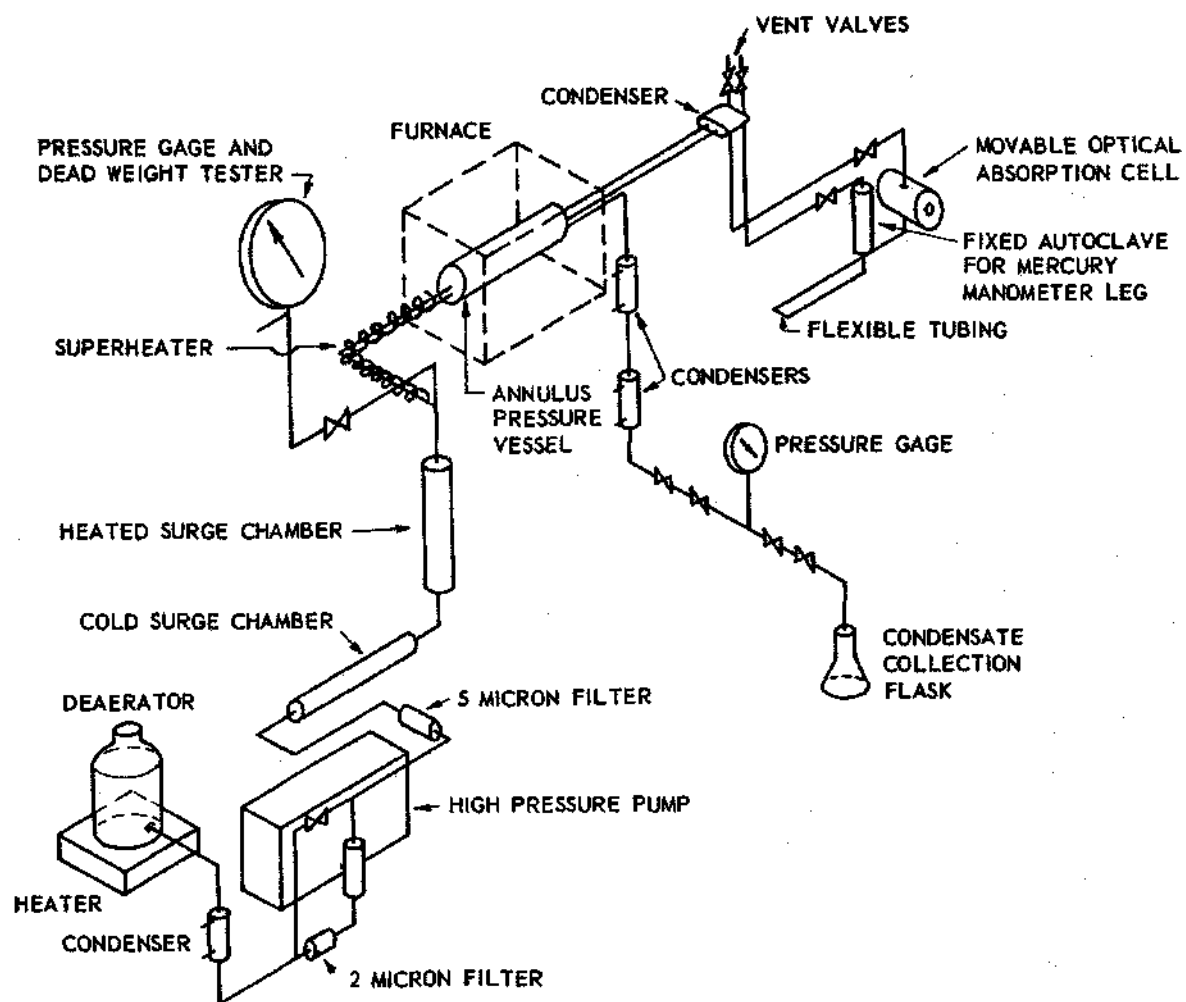


Figure 1. Schematic Drawing of Annular Viscometer and Associated Equipment.

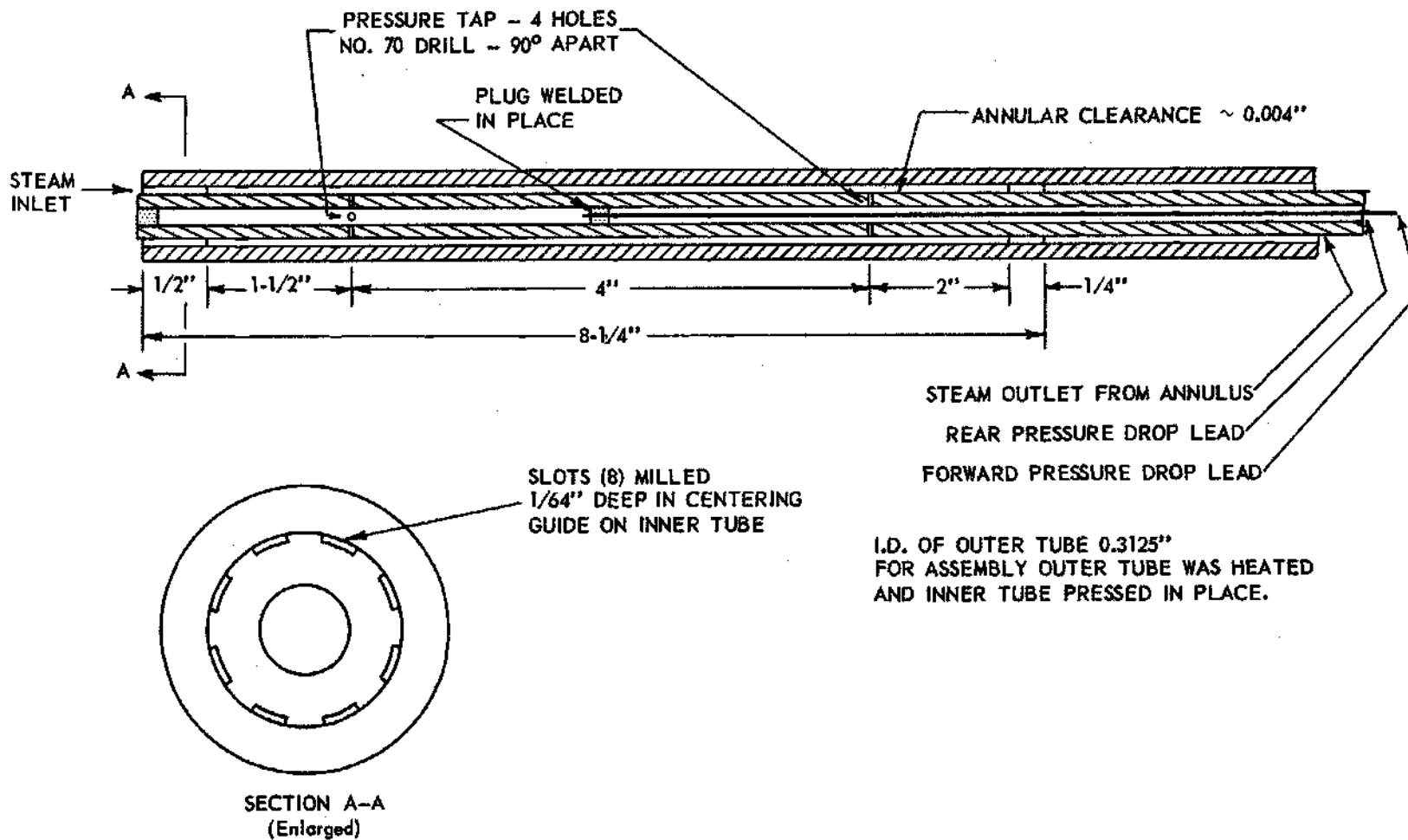


Figure 2. Schematic Drawing of Annulus Assembly.

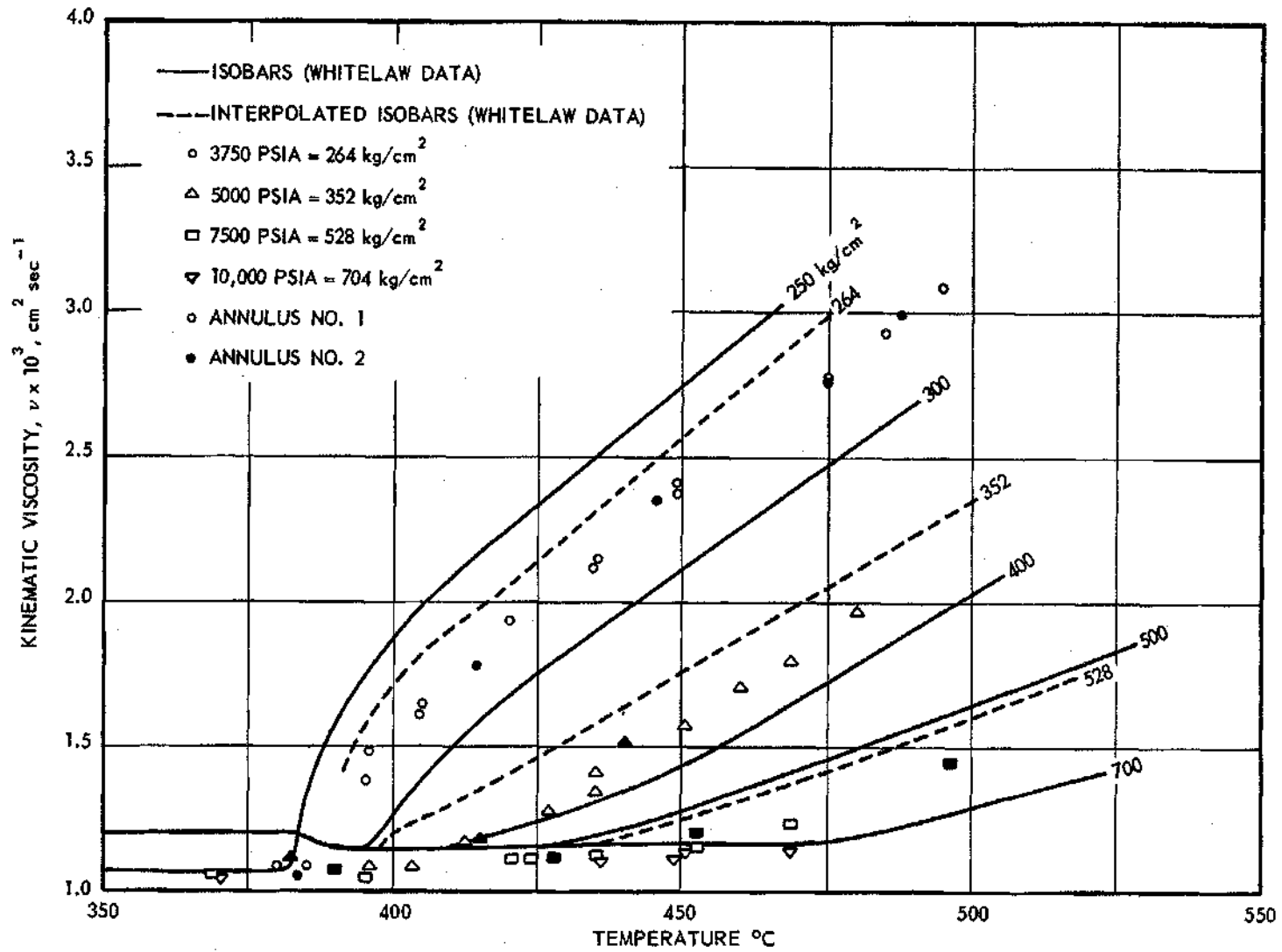


Figure 3. Comparison with Whitelaw Data (11), Kinematic Viscosity.

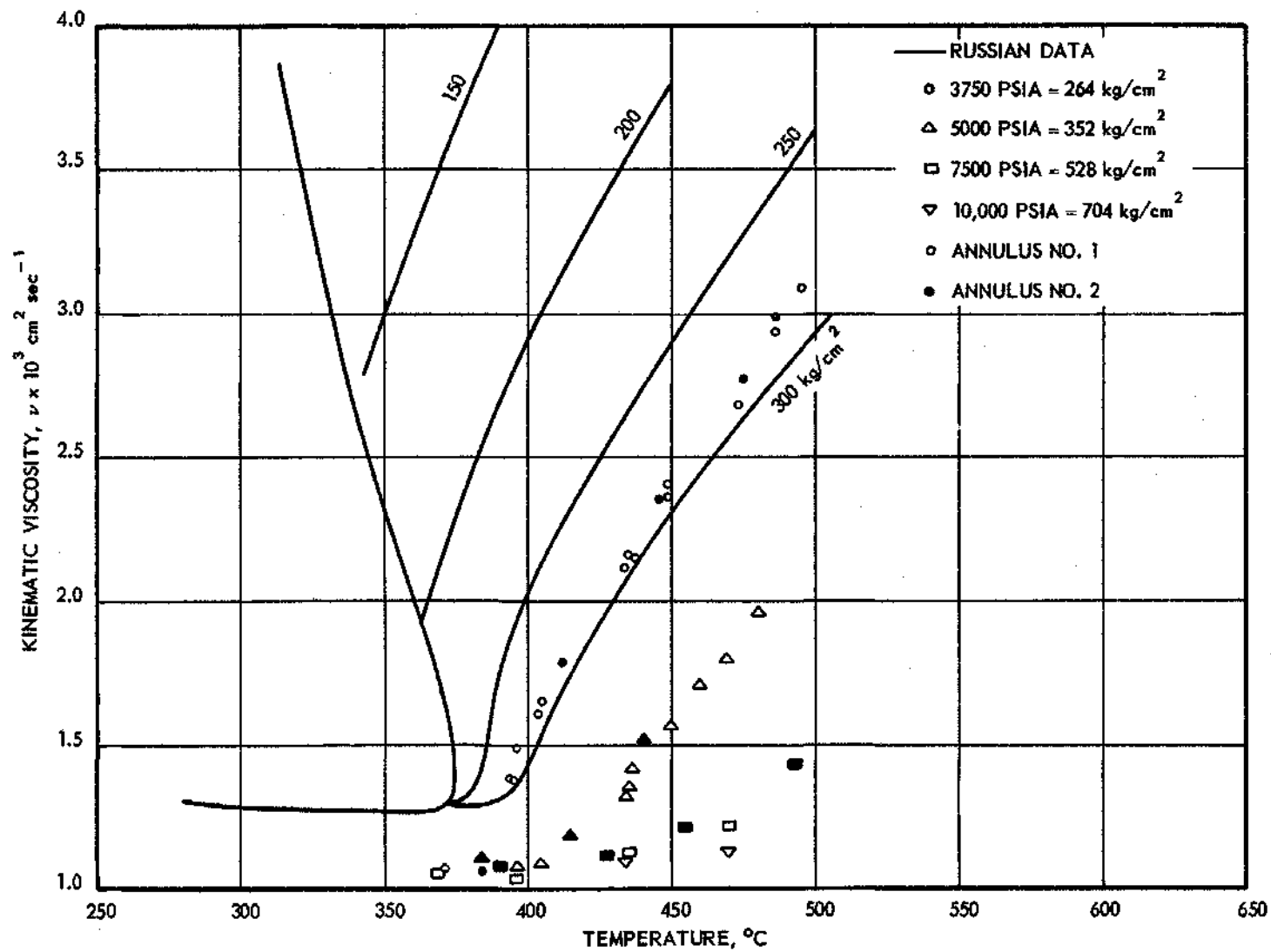


Figure 4. Comparison with Russian Data (6), Kinematic Viscosity.



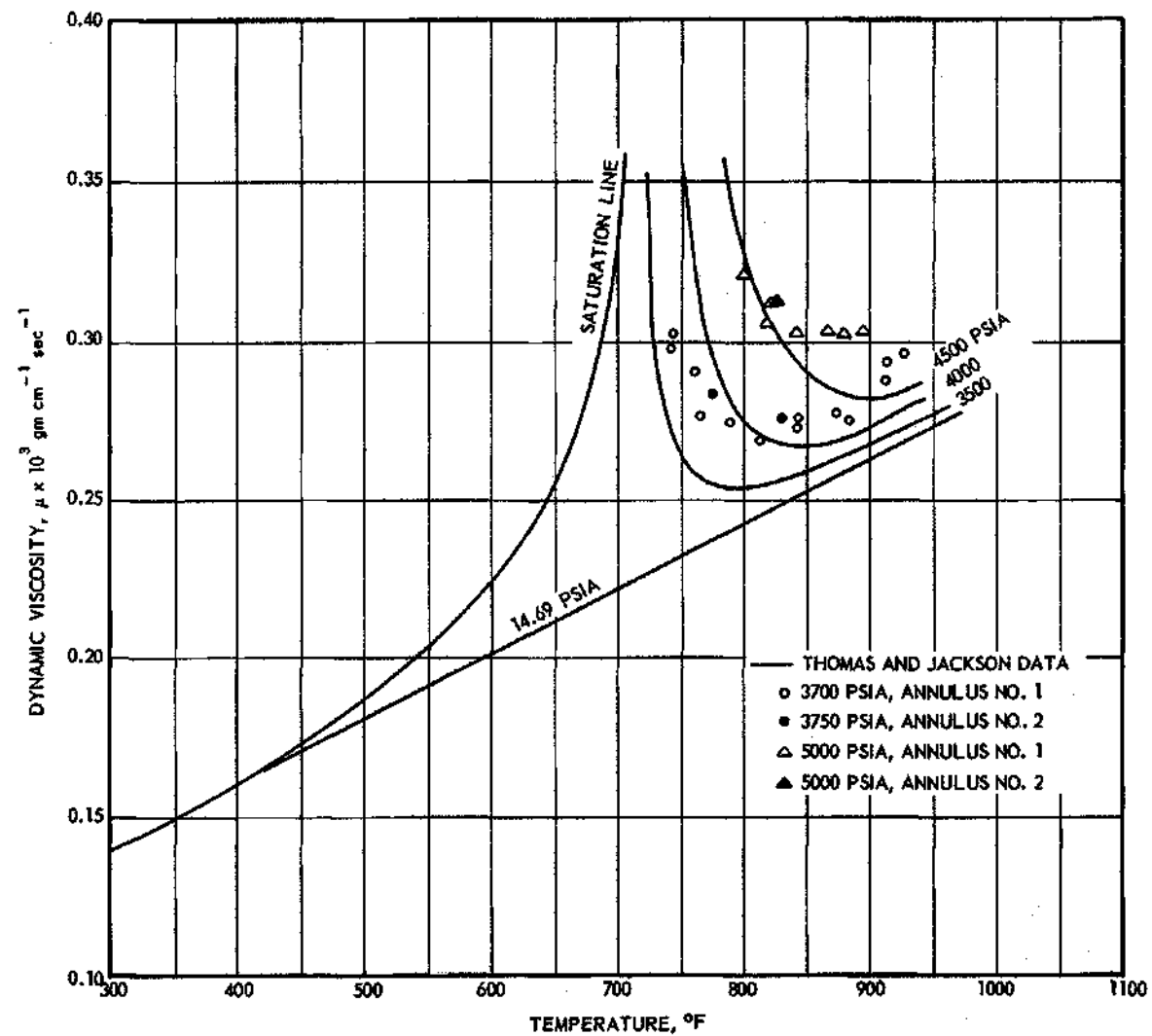


Figure 6. Comparison with Thomas and Jackson Data (8), Dynamic Viscosity.

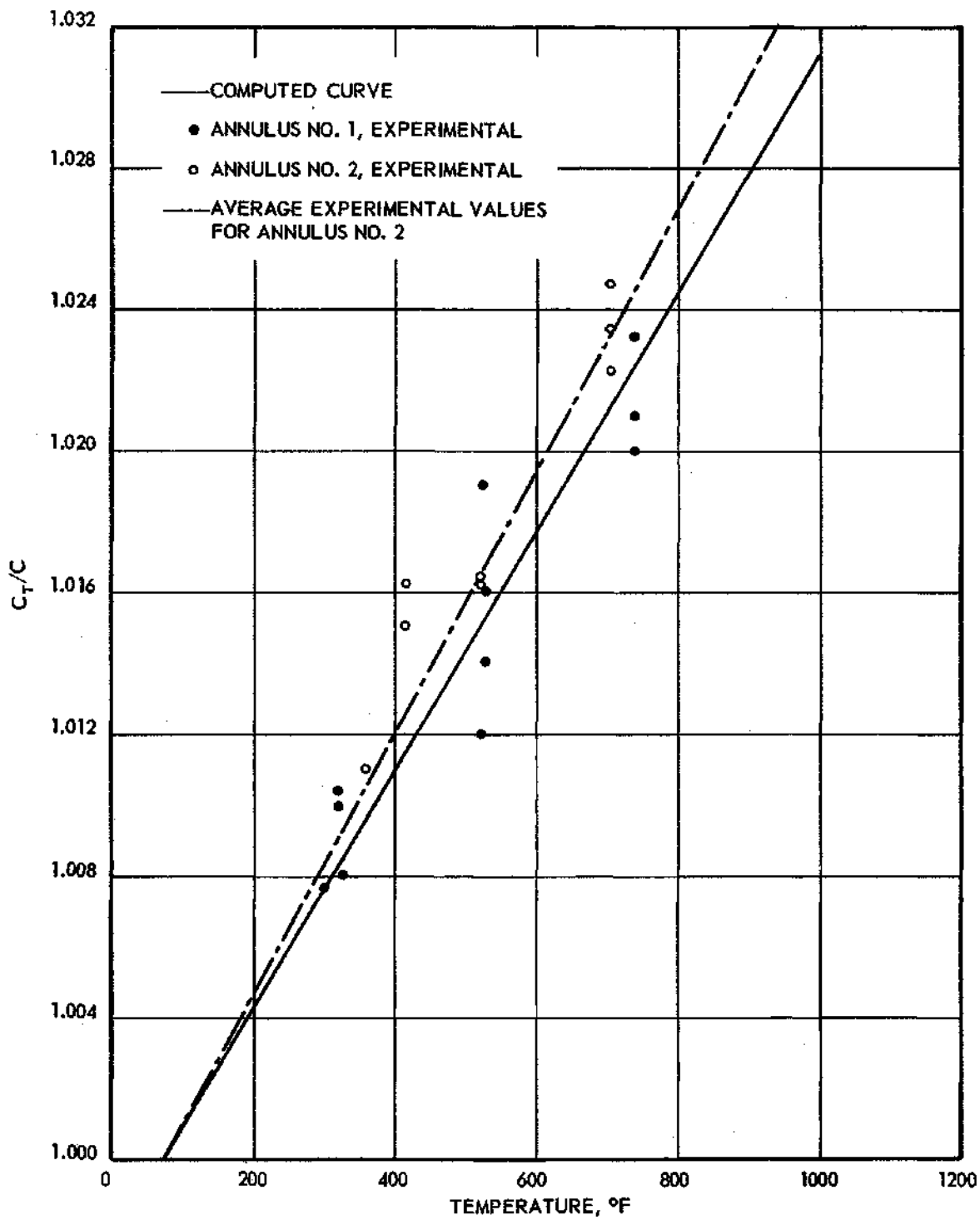


Figure 7. Temperature Correction Factor for Annulus Constant.

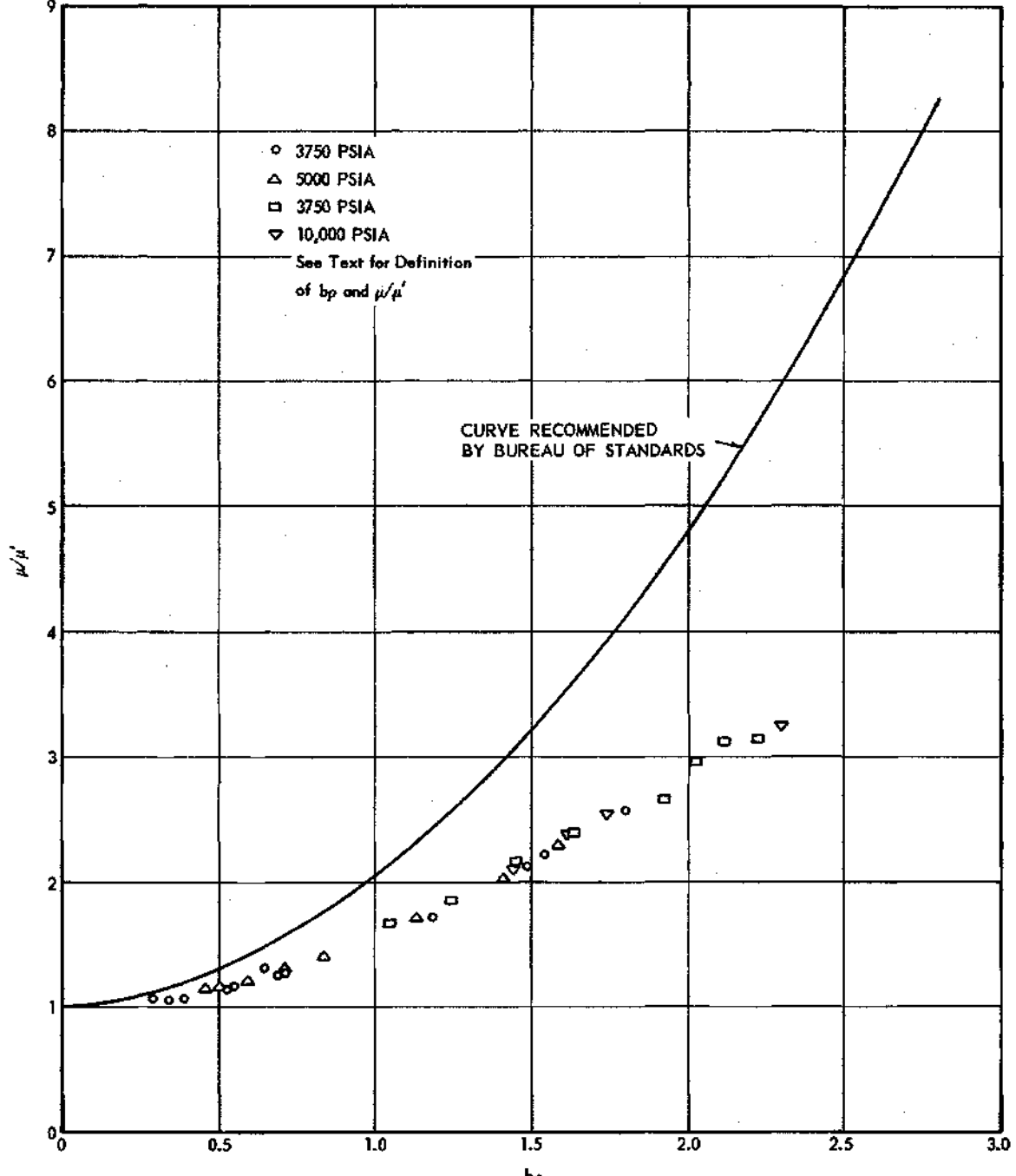


Figure 8. Comparison with U.S. Bureau of Standards Correlation.

## APPENDIX A

## DETAILS OF HIGH PRESSURE MANOMETER\*

A new type of high pressure manometer was designed and constructed for use in this experiment. It is an unusual manometer in that it can be used to measure very small pressure drops at high static pressures while observing the mercury surface at all times. Furthermore, the instrument requires no calibration. A schematic diagram of the manometer is shown in Figure A.

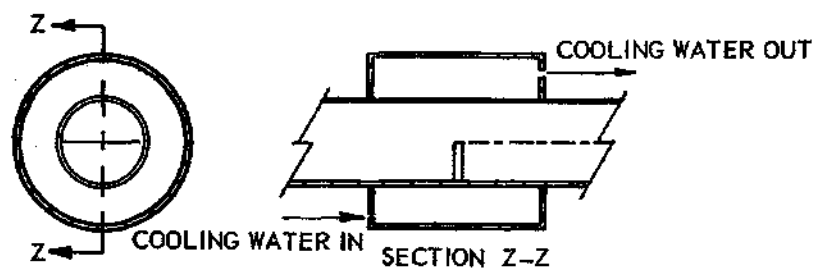
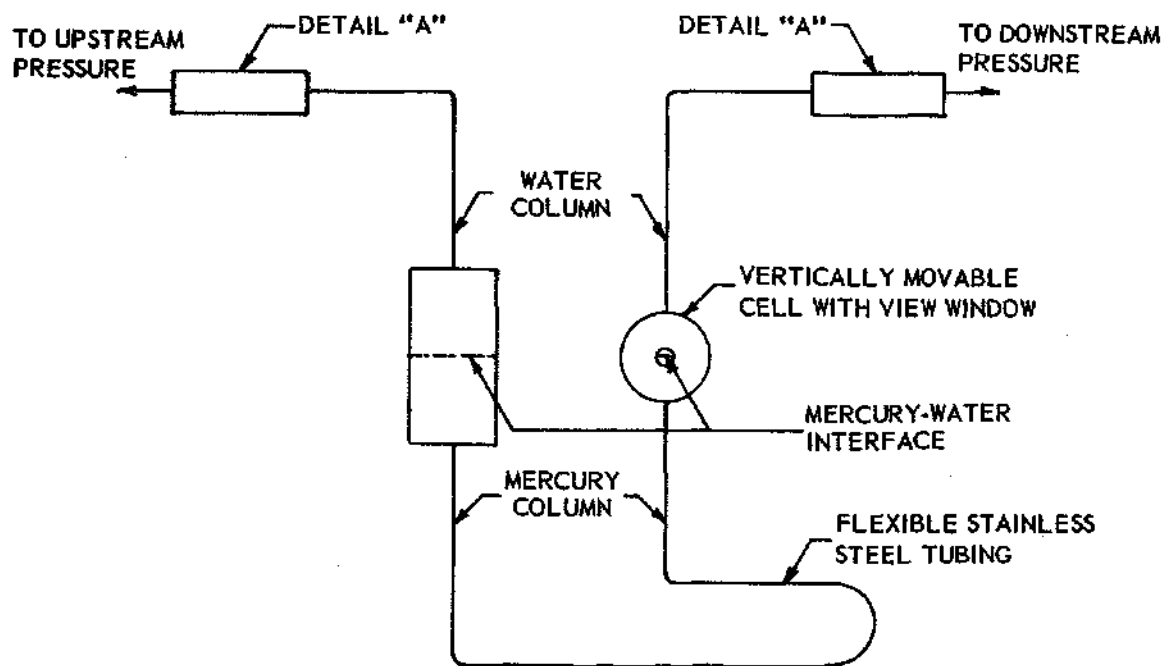
The principle of operation is as follows. With no flow through the system, the elevation of the movable cell is adjusted until the mercury surface coincides with a reference hairline on the movable cell. When fluid flows through the annulus the mercury level rises in the movable cell. The movable cell is elevated until the mercury surface is again coincident with the reference hairline on the movable cell. This procedure also restores the mercury level in the fixed tube to its initial position. A cathetometer is used to measure the change in elevation of the mercury surface in the movable cell. This difference in elevation is the pressure drop across the annulus.

A brief study of Figure B and the following equations will explain the details of operation. Note that  $P_1 = P_2$  under no-flow conditions, and  $P_2 < P_1$  when flow occurs through the annulus test section.

Since there is no flow through the manometer, the principles of fluid statics may be applied. This yields:

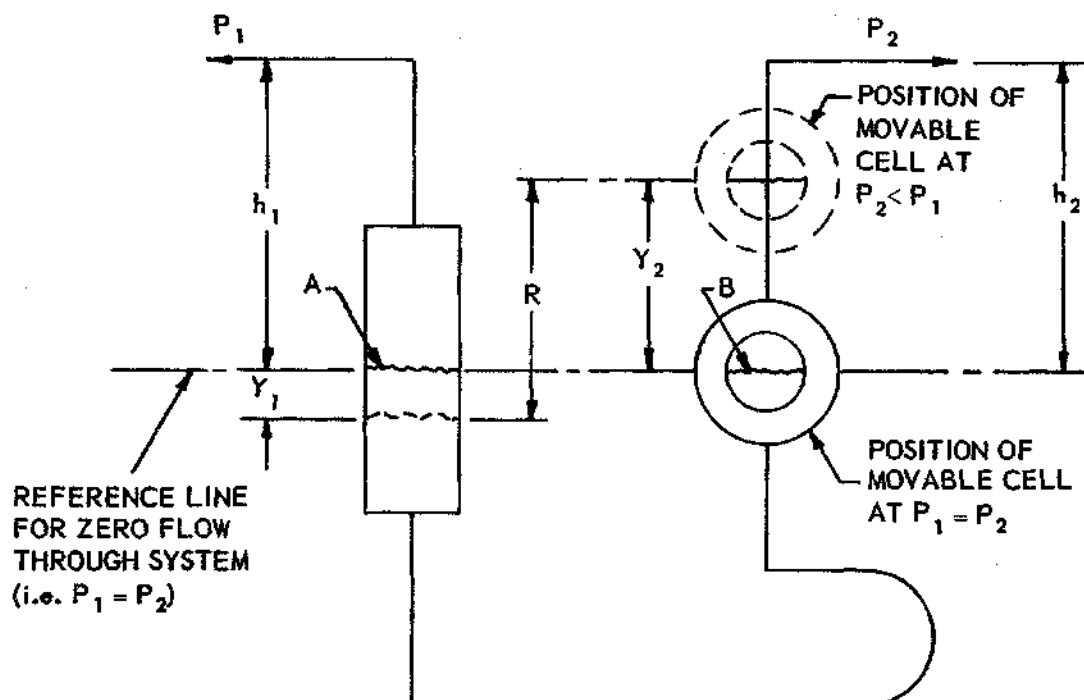
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\*This description applies to the manometer used with annulus #1. See Reference (12) for a description of improvements made before operating annulus #2.



DETAIL A: WEIR IN MANOMETER LEADS

Figure A. Schematic Diagram of Manometer.



**EXPLANATION OF SYMBOLS:**

- $Y_1$  = Deviation of Mercury Surface from Reference Line
- $Y_2$  = Deviation of Mercury Surface from Reference Line
- $R$  =  $Y_1 + Y_2$
- $h_1$  = Distance from Reference Line to Liquid Surface
- $h_2$  = Distance from Reference Line to Liquid Surface
- A & B = Indicate Location of Mercury Surface Relative to the Fixed Tube and Movable Cell, Respectively
- $P_1$  = Pressure at Upstream End of Annulus Test Section
- $P_2$  = Pressure at Downstream End of Annulus Test Section
- $\gamma_a$  = Specific Weight of Fluid in Manometer Leads
- $\gamma_b$  = Specific Weight of Fluid in Flexible Stainless Steel Tube

Figure B. Schematic Diagram of Manometer Operating Principle.

$$P_1 + \gamma_a (h_1 + Y_1) - \gamma_b (R) - \gamma_a (h_2 - Y_2) = P_2 \quad (1)$$

It should now be noted that if liquid water flows through a horizontal annulus,  $h_1 = h_2$ . If steam is flowing through the annulus then some form of condenser must be arranged to insure that the free surface of liquid water is at the same elevation in each manometer lead. Detail "A" in Figure A shows how this was accomplished in the present work. By putting  $h_1 = h_2$  in Equation (1) and also noting that  $R = Y_1 + Y_2$ , the equation becomes

$$P_1 + \gamma_a Y_1 - \gamma_b (Y_1 + Y_2) + \gamma_a Y_2 = P_2 \quad (2)$$

Now note that the volume between Point A and Point B is constant if the mercury level in the movable cell is always read at the same reference hairline on the cell and if it is assumed that flexing of the stainless steel tube produces no change in volume. It follows from this assumption of constant volume that  $Y_1$  must be identically zero. Equation (2) then becomes:

$$P_1 - \gamma_b Y_2 + \gamma_a Y_2 = P_2 \quad (3)$$

or

$$P_1 - P_2 = Y_2 (\gamma_b - \gamma_a) \quad (4)$$

Equation (4) justifies the use of the manometer as described herein. It should be noted that high pressures might cause a change in tube volume between points A and B in Figure B; it is therefore necessary that

the zero reading of the manometer be made at the same static pressure that prevails under conditions of flow through the annulus.\*

The cathetometer used to measure the pressure drop was equipped with a vernier graduated to 0.005 cm; however, the cathetometer reading could be estimated to the nearest 0.0025 cm. The movable cell was mounted on a micrometer screw, and the cell elevation could be read accurately to 0.0002 inch. Comparisons between micrometer screw readings and cathetometer readings consistently indicated agreement within 0.005 cm. This agreement suggests that this type of manometer could be used without a cathetometer; however, some kind of telescopic device with a reference hairline would still be required in order to locate the mercury surface at the proper position relative to the movable cell.

It should be noted that the flexible stainless steel tube containing mercury must be absolutely free of air in order for this manometer to measure small pressure drops accurately.

Both mercury and water are slightly compressible. However, the difference in densities is  $12.54 \pm 0.02$  gram per cc over the pressure range from atmospheric pressure to 10,000 psia and over the room temperature range from 60° to 80° F. Consequently, the pressure drop expressed as grams per square centimeter is obtained by multiplying the pressure drop

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\* All questions regarding changes in volume for any reason can be resolved by using two optical absorption cells and sighting the mercury level in each cell with a cathetometer. If a single cathetometer is used for this purpose it will be necessary to rotate the cathetometer in a horizontal plane and to re-level the instrument before taking each reading. This procedure could introduce sizable percentage errors when measuring very small pressure drops. It is therefore recommended that two cathetometers be used when two optical absorption cells are used to measure pressure drops in the order of a few millimeters.

indicated by the water-mercury manometer (expressed as centimeters) by 12.54. This can be converted to pounds per square inch by multiplying by 0.01421. Or, more simply, the pressure drop expressed as psi, for the water-mercury manometer, is given by

$$\Delta P = 0.1782 Y_2$$

where  $Y_2 = \text{cm}$

$\Delta P = \text{psi}$

## APPENDIX B

## NITROGEN CALIBRATION PROCEDURE

Under conditions of steady laminar flow through an annulus, the following equation relates the variables:

$$\mu = \frac{\pi \Delta P}{8QL} \left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right]$$

where

- $\mu$  = dynamic viscosity of the fluid
- $\Delta P$  = pressure drop across the annulus
- $Q$  = volume rate of flow
- $L$  = length of the annulus
- $b$  = inside radius of the outer tube
- $a$  = outside radius of the inner tube

If it were feasible to manufacture an annulus to exact dimensions, the bracket term could be calculated and the annulus could be used as an absolute viscometer. However, for an annulus such as was used in the present work, an error of  $\pm 0.0005$  inch in  $a$  or  $b$  could result in changing the value of the bracket term by more than 200%. The annulus, therefore, appears to be impractical as an absolute viscometer. However, if a fluid of known viscosity is passed through the annulus under conditions of steady laminar flow, the bracket term can be evaluated experimentally without knowing the annulus dimensions.

It is convenient to write the annulus equation in the form:

$$\mu = \frac{\Delta P}{QL} C'$$

or, putting  $\frac{C'}{L} = C$ , the equation becomes:

$$\mu = \frac{\Delta P}{Q} (C)$$

In the present experiment,  $C$  was evaluated by establishing steady laminar flow of nitrogen through the annulus and measuring  $\Delta P$  and  $Q$ . Values of  $\mu$  were obtained from data published by the U. S. Bureau of Standards (10).

The calibration procedure was started by establishing a steady laminar flow of nitrogen gas (99.998% purity) through the annulus. Nitrogen pressure was measured by a mercury column attached to the upstream pressure lead on the manometer. Pressure drop across the annulus was measured by a U-tube water manometer and a cathetometer. Annulus temperature was measured by a 28-gauge chromel-alumel thermocouple. Volume flow rate of nitrogen was measured by a Precision Scientific Wet Test gas meter. This meter was calibrated against a gas burette tube.

The pressure of the dry nitrogen in the wet test gas meter was determined by subtracting the vapor pressure of water from the total pressure of the wet gas mixture in the meter. The volume flow rate of dry nitrogen at annulus conditions was obtained from the relation:

$$\frac{P_A V_A}{T_A} = \frac{P' V_M}{T_M}$$

where

- $P_A$  = pressure at center of annulus  
 = inlet pressure -  $1/2 (\Delta P)$   
 $V_A$  = volume flow rate of dry nitrogen at annulus conditions  
 $T_A$  = absolute annulus temperature  
 $P'$  = pressure of dry nitrogen in the gas meter  
 $V_M$  = volume of flow rate of wet mixture through wet test gas  
 meter  
 $T_M$  = absolute temperature in wet test gas meter

By measuring  $\Delta P$ ,  $P$ ,  $Q$ ,  $T_A$ , and  $T_M$  in this manner, the "effective" value of  $C$  was determined.

Experimental values of the annulus constant at room temperature were plotted against Reynolds number and  $C$  was found to be independent of Reynolds number. The Reynolds number was computed using the hydraulic diameter = 0.000633 ft. and the cross-sectional area for flow =  $2.56 \times 10^{-5}$  ft<sup>2</sup>. The experimental points did not deviate from the constant value of  $22.20 \times 10^{-13}$  ft<sup>3</sup> by more than 0.5%.

Experimental apparatus limited the range of possible Reynolds numbers in the nitrogen calibration. However, the validity of the calibration constant was extended by measuring the viscosity of steam (at fixed pressure and temperature) at a Reynolds number of 70 and repeating the measurement at Reynolds numbers up to 205. The variation in viscosity with Reynolds number was less than the possible error in the experimental determination of the viscosity.

The constant  $C$  is proportional to 
$$\frac{[b^4 - a^4 - (b^2 - a^2)^2 / \ln \frac{b}{a}]}{L}$$
.

Since  $b$ ,  $a$ , and  $L$  will change with temperature, it follows that  $C$  will vary with temperature. Obviously, if  $C$  is evaluated at some reference

temperature, such as 70 °F, the theoretical value of C at some different temperature will be given by the relation

$$C_T = C [1 + \beta (t - 70)]^3$$

where

$C_T$  = annulus constant at t °F

C = annulus constant at 70 °F

$\beta$  = linear coefficient of expansion of type 304 stainless steel

Using  $\beta = 11.12 \times 10^{-6}$  per °F (23), the following values of  $C_T/C$  were computed.

t, °F	$\beta (t - 70)$	$\frac{C_T}{C}$
70	0	1
300	0.00256	1.0077
600	0.00589	1.0177
1000	0.0103	1.0312

These values of  $C_T/C$  are plotted against temperature in Figure 7.

Experimental values of  $C_T/C$  are also shown on this figure.

## APPENDIX C

## EFFECTS OF COMPRESSIBILITY ON MANOMETER READINGS

The density of liquid mercury at 80° F increases by less than 0.3% as the pressure is increased from 1 atmosphere to 10,000 psia (24). In contrast to this, the density of water at 80° F increases by almost 3% for the same change in pressure (25). This suggests that it might be necessary to account for compressibility effects when a water-mercury manometer is used at high static pressures. An investigation of compressibility effects, however, yields the following results:

<u>Density of mercury minus Density of water(cm/cc)</u>		
<u>Pressure</u> <u>psia</u>	<u>60° F</u>	<u>80° F</u>
900	12.5585	12.5335
1200	12.5592	12.5344
3750	12.5596	12.5351
5000	12.5608	12.5356
7500	12.5620	12.5371
10000	12.5646	12.5408

It is therefore evident that if  $h$  (cm) is the distance between the two mercury surfaces, then the pressure difference on a water-mercury manometer is given by  $(12.54 \pm 0.02) \text{ gm/cm}^3 (h) = \Delta P \text{ gm/cm}^2$  if the manometer is used in a region where room temperatures are between 60° F and 80° F.

## APPENDIX D

## TEMPERATURE MEASUREMENT

The importance of accurate temperature measurement in determining viscosity values can scarcely be over-emphasized. It is evident that isobars on a plot of kinematic viscosity versus temperature will appear lower than the true value if too high a temperature is read experimentally. Conversely, the isobars will read higher than the true values if the thermocouple used to determine the temperatures reads low.

This error due to inaccurate temperature measurement is compounded when a plot of dynamic viscosity versus temperature is made, because a high temperature reading not only leads to an apparently lower value of kinematic viscosity but also leads to a low value of density which still further reduces the value of the dynamic viscosity. Conversely if the thermocouple reads low there is a double compounding of error in dynamic viscosity values in the upward direction.

In order to insure accurate temperature measurements, the 28-gauge chromel-alumel thermocouple used in this experiment was calibrated against a platinum and platinum-10% rhodium thermocouple which had been calibrated by the U. S. Bureau of Standards. During the calibration procedure, the reference thermocouple and the chromel-alumel thermocouple were placed adjacent to each other inside an electric furnace. The thermocouple leads were connected to a double-pole-double-throw switch inside an isothermal box. The temperature at this junction point was read on an ASTM mercury thermometer graduated in tenths of a degree Fahrenheit. The switch was

connected to a Leeds and Northrup, Type K-3 potentiometer by copper wires cut from the same spool. The emf generated by each thermocouple was read to the nearest 0.1 microvolt.

The emf generated by the platinum-platinum rhodium thermocouple, corrected for cold junction temperature, was used to obtain the true temperature from the calibration data furnished by the Bureau of Standards. This value of true temperature was then plotted versus the emf generated by the chromel-alumel thermocouple.

After taking the experimental data for viscosity measurements, the chromel-alumel thermocouple was re-calibrated in the same manner, and the second calibration agreed with the original.

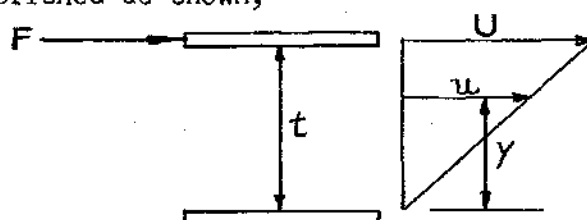
Temperatures listed in this report were measured using the calibrated 28-gauge chromel-alumel thermocouple and a Leeds and Northrup portable potentiometer. The emf of the thermocouple was read to the nearest microvolt; temperatures obtained in this manner were then rounded to the nearest 0.1° F. It is felt that the maximum error in temperature measurement does not exceed 1° F.

## APPENDIX E

## DEVELOPMENT OF THE EQUATION FOR VOLUME FLOW RATE IN AN ANNULUS

An expression for the volume flow rate,  $Q$ , through an annulus can be developed by determining the velocity profile and integrating the expression,  $dQ = u dA$  to obtain  $Q = \int_A u dA$ . It is first necessary to define the dynamic viscosity for laminar flow.

Consider two parallel flat plates, one located above the other, and let the space between the plates be filled with some fluid. If the lower plate is held stationary and the upper plate is moved to the right with a steady velocity,  $U$ , as shown, a steady state velocity gradient will be established as shown,

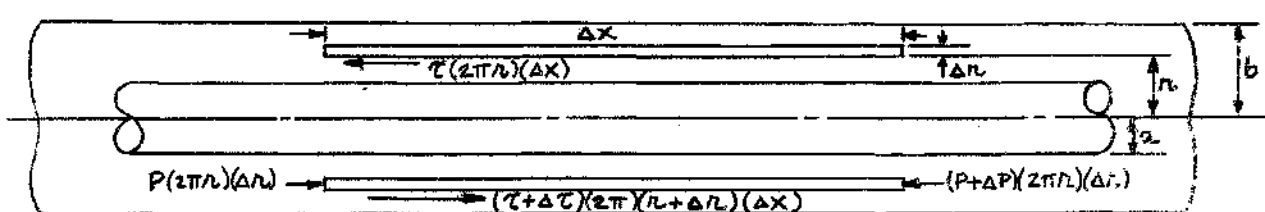


In this figure,  $F$  is the force required to maintain steady motion of the plate,  $t$  is the distance between the plates,  $y$  is the distance from the top surface of the lower plate to any arbitrary point in the fluid, and  $u$  is the velocity of the fluid at this arbitrary point.

Experiment shows that the fluid in immediate contact with a solid boundary does not move relative to the boundary. Therefore, the velocity profile for a Newtonian fluid such as water would be as represented. (For a non-Newtonian fluid in which the rate of angular deformation is not a linear function of the applied shear stress the local fluid velocity would not vary linearly with  $y$ ). Experiment also shows that the force

required to maintain steady motion of the upper plate is proportional to the area of the plate, the plate velocity, and the reciprocal of the distance between the plates. Mathematically:  $F = \frac{\mu AU}{t}$ , where  $\mu$  is a proportionality constant, different for different fluids, defined as the viscosity of the fluid. A more general expression for  $F$  can be obtained by replacing  $\frac{U}{t}$  by its equivalent  $\frac{du}{dy}$ . This derivative can be visualized as the rate at which one layer of fluid moves relative to an adjacent layer when laminar flow prevails. This leads to an expression  $F = \mu A \frac{du}{dy}$ . It is often convenient to divide both sides of this equation by  $A$  and write  $\tau \equiv \frac{F}{A} = \mu \frac{du}{dy}$ , in which  $\tau$  is called the shear stress. Dimensional homogeneity requires that  $\mu$  have the dimensions\*  $\frac{\text{Force-Time}}{(\text{Length})^2}$ .

Consider now the case of fully developed laminar flow in a horizontal annular passage as shown below. The forces acting on the cylindrical shell



of fluid at general position,  $r$ , and having a thickness  $\Delta r$ , will be as shown.

\*It is also customary to express  $\mu$  as  $\frac{\text{Mass}}{\text{Length-time}}$ . This conversion is made by using the relation between force, mass, length, and time as given by Newton's second law of motion. Thus, in the cgs system the dynamic viscosity is usually expressed in units of  $\frac{\text{gm}}{\text{cm sec}}$ . In this system of units the viscosity is expressed in poises, or centipoises, the poise being defined by  $1 \text{ poise} = \frac{1 \text{ gm}}{\text{cm sec}}$ . The centipoise is 0.01 poise. The relation between the engineering system of units and the cgs system of units is given by:  $479 \text{ poise} = 1 \text{ lb}_f \text{ sec} / \text{ft}^2$ . Viscosity values are often expressed as the ratio of dynamic viscosity to the density of the substance, i.e.,  $\nu = \frac{\mu}{\rho}$ . Thus, in the engineering system  $\nu$  is expressed as  $\text{ft}^2/\text{sec}$  and in the cgs system  $\nu$  is expressed as  $\text{cm}^2/\text{sec}$ .

Since the flow is fully developed, there is no acceleration in the x direction and Newton's Second Law requires  $\Sigma F_x = 0$ .

Therefore:

$$P(2\pi r\Delta r) - (P + \Delta P)(2\pi r\Delta r) - \tau(2\pi r\Delta x) + 2\pi (r + \Delta r)(\tau + \Delta\tau)(\Delta x) = 0$$

After expanding and simplifying:

$$-\Delta P(2\pi r\Delta r) + 2\pi\Delta x[\tau\Delta r + r\Delta\tau + \Delta r + \Delta\tau] = 0$$

Dividing by the volume of the cylindrical shell ( $=2\pi r\Delta r\Delta x$ ):

$$-\frac{\Delta P}{\Delta x} + \frac{1}{r\Delta r}[\tau\Delta r + r\Delta\tau + \Delta r\Delta\tau] = 0$$

or:

$$-\frac{\Delta P}{\Delta x} + \frac{\tau}{r} + \frac{\Delta\tau}{\Delta r} + \frac{\Delta\tau}{r} = 0$$

If  $\Delta x$  and  $\Delta r$  approach zero, it follows that  $\Delta\tau$  and  $\Delta P$  also approach zero. The ratio of infinitesimals will then become derivatives, and the equation can be written:

$$-\frac{dp}{dx} + \frac{\tau}{r} + \frac{d\tau}{dr} = 0$$

It is now noted that the last two terms can be combined by using the identity,

$$\frac{d(\tau r)}{dr} = \tau + r\frac{d\tau}{dr}$$

This results in

$$-\frac{dP}{dx} + \frac{1}{r} \frac{d(\tau r)}{dr} = 0$$

Noting that  $-\frac{dP}{dx}$  is a constant\*, this can be integrated to give:

$$-\frac{dP}{dx} \int r dr + \tau r = A_1$$

where  $A_1$  is a constant of integration.

Substitute for  $\tau$  as previously defined:  $\tau = \mu \frac{du}{dr}$ , where  $u$  is the local velocity in the axial direction.

This gives

$$-\frac{dP}{dx} \int r dr + r\mu \frac{du}{dr} = A_1$$

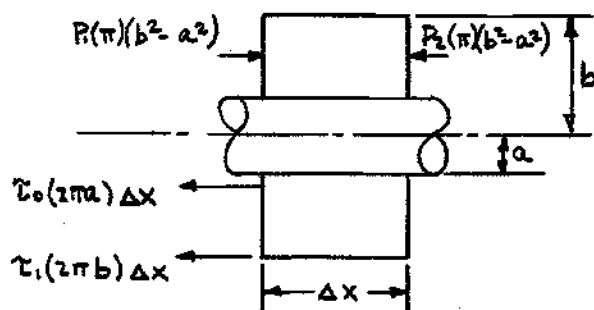
or

$$-\frac{dP}{dx} \frac{r^2}{2} + r\mu \frac{du}{dr} = A$$

or

$$-\frac{dP}{dx} \frac{r}{2} + \mu \frac{du}{dr} = \frac{A}{r}$$

\*For fully developed steady flow in an annulus, a free body of an annular section of fluid is as follows:



For any  $\Delta x$ , finite or infinitesimal, the velocity distribution is the same at any point along the  $x$  axis. Then, for any two arbitrary sections,  $\tau_1$  and  $\tau_0$  will not change in moving from one  $x$  value to another. Therefore  $\frac{P_1 - P_2}{\Delta x}$  is constant; or  $\frac{dP}{dx}$  is constant.

Multiply by  $dr$  and integrate:

$$-\frac{dP}{dx} \frac{r^2}{4} + \mu u = A \ln r + B$$

where  $B =$  constant of integration

Impose the boundary conditions at the annulus surfaces to evaluate

$A$  and  $B$ . That is, put:

$$u = 0 \text{ when } r = a$$

$$u = 0 \text{ when } r = b$$

This results in an expression for the local velocity at any point:

$$u = -\frac{dP}{dx} \left( \frac{1}{4\mu} \right) \left[ b^2 - r^2 + \frac{b^2 - a^2}{\ln \frac{a}{b}} \ln \frac{b}{r} \right]$$

The volume flow rate through a cylindrical shell at radius  $r$  and local fluid velocity  $u$  can be written:

$$dQ = u (2\pi r) dr$$

The total volume flow rate can be found by substituting for  $u$  and integrating this expression from  $r = a$  to  $r = b$ . This gives:

$$Q = -2\pi \frac{dP}{dx} \left( \frac{1}{4\mu} \right) \int_a^b \left[ b^2 - r^2 + \frac{b^2 - a^2}{\ln \frac{a}{b}} \ln \frac{b}{r} \right] r dr$$

After a rather tedious integration, the result is

$$Q = -\frac{\pi}{8\mu} \frac{dP}{dx} \left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right]$$

As previously shown,  $\frac{dP}{dx}$  could be written as  $\frac{\Delta P}{L}$ . Since  $\Delta P$  is negative, it is customary to write this expression without the minus sign.

The resulting expression is

$$Q = \frac{\pi \Delta P}{8 \mu L} \left[ b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right]$$

where  $\Delta P$  is inherently positive.

## VITA

Samuel Clarence Barnett was born in Chatsworth, Georgia on May 10, 1922. He attended public schools in Fort Meade, Florida, and graduated from Fort Meade High School in 1940.

He served an apprenticeship as an office machine mechanic during his high school years and also learned Morse telegraphy as a hobby. He was employed in both of these trades from 1940 until late 1942, at which time he entered the military service.

After two years of military service he entered Georgia School of Technology in March, 1945. He was graduated in 1948 with the degree of Bachelor of Industrial Engineering.

After four years of industrial experience, he joined Georgia Institute of Technology as an assistant research engineer and part-time graduate student in Mechanical Engineering. He was awarded the Master of Science degree in 1956, at which time he was appointed Assistant Professor of Mechanical Engineering.

He is a member of the American Society of Mechanical Engineers, Phi Eta Sigma, Tau Beta Pi, Phi Kappa Phi, and the Society of Sigma Xi.