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KINEMATIC DESIGN AS APPLIED TO PLANAR LINKAGES,
ROTATING SHAFTS, AND TRANSLATING TABLES

A THESIS

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by

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LINKAGES, ROTATING SHAFTS, AND
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Chs

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SUMMARY

Designers of mechanisms often fail to give due consideration to degrees of freedom and degrees of constraint offered by kinematic pairs. The result is that mechanisms are frequently designed to be more susceptible to wear than necessary and to require excessively exacting geometrical relationships to be present for satisfactory performance.

"Kinematic design" principles are modified to consist of the use of commercially available pairs and are then utilized to eliminate the need for special geometrical relationships between pairs in the following types of mechanisms:

1. planar linkages,
2. rotating shafts on multiple bearings,
3. translating tables on multiple supports.

The method is based on the use of the general spatial mobility criterion (i.e., zero general constraints); resultant mechanisms are termed "self-aligning" indicative of their nature.

Results of the three enumerated developments are condensed in the final chapter in an attempt to make them useful to the designer. Applications and extensions of the work are also discussed there.

Added features are the catalogues of planar linkages in an appendix including all-revolute eight-bars and revolute-prism six-bars.

GLOSSARY OF ABBREVIATIONS

C	cylinder pair
C _p	cylinder-plane pair
F	number of degrees of mobility of a kinematic chain
f	number of freedoms allowed by a pair
H	helix (screw) pair
m	family number; number of general constraints
n	number of links in a kinematic chain
P	prism pair
P _L	plane pair
P _f	pair class symbol; number of pairs in a kinematic chain with f degrees of freedom
R	revolute pair
R-type	containing only R pairs
RP-type	containing both R and P pairs
S	sphere pair
S _g	sphere-groove pair
S _{gh}	sphere-grooved helix pair
S _p	sphere-plane pair
S _s	sphere-slotted cylinder pair
S _{sh}	sphere-slotted helix pair
T	torus pair
T _h	torus-helix pair
u	number of constraints provided by a pair

CHAPTER I

INTRODUCTION

Kinematic Design

Mechanisms are often designed in such a way that they are more complex, more expensive, or more susceptible to wear and inaccuracies than they would be if the concepts of degrees of freedom and degrees of constraint were applied correctly in their designs. Although they should be familiar with these concepts, many designers fail to apply them effectively in their work.

There are two distinct kinds of parts in any mechanism, the links and the pairs which connect the links together. Common design procedures are concentrated for the most part on the design of the links, which is as it should be; but the design of the pairs is often slighted, the designer being content to use only the three simple lower pairs-- the revolute, the prism, and the helix, or screw. This practice almost always produces redundant constraints demanding that there be special geometrical relationships between the various pair axes of a mechanism in order for that mechanism to "fit together," and even more special geometrical relationships must be met for the mechanism to be movable. The expression "fit together" is meant not to concern the relative length, or dimensions, of the links as does the Grashof Criterion for planar linkages, but rather to concern the relations of the pairs to one another. Here, link dimensions are always assumed to be adequate to

permit assembly of a mechanism.

Special geometrical relationships can never be met exactly in a physical mechanism, but can only be approximated within certain tolerances. To compensate for the deviations from the ideal geometrical relationships, clearances must be designed into the pairs of a mechanism larger than those required to just permit relative motion of the pair elements and lubrication; and as well, the movement of a mechanism is often dependent on the ability of its links to flex, a condition always accompanied by unnecessary stresses and indeterminate forces. The designer's relative degree of reliance on clearances versus flexure for mobility varies from one design problem to the next, and there are no generally accepted criteria available for determining what clearances must be allowed in mechanism design.

Production of accurately functioning mechanisms by common design methods depends on high precision in manufacture. The cost of precision work increases much more rapidly as the requirements are raised than does the improvement in the precision. There is definitely an economic limit to the precision which can be maintained in a given production situation; and consequently, there is also a limit to the accuracy with which the mechanism being produced can fulfill its objective function. These shortcomings of the common design methods, causing mechanisms to be more complex, more expensive, and more susceptible to wear and inaccuracies than necessary, may be at least partially overcome through proper application of the concepts of degrees of freedom and degrees of constraint.

A philosophy of design based on these concepts has existed for

many years, and is known as kinematic, or geometric, design. The latter name better describes the philosophy perhaps, but the name "kinematic design" seems to have been adopted more widely.

A kinematic design is defined as one in which each part of a mechanism is given the theoretically minimum number of constraints, in the form of point contacts, consistent with the motion desired of the part. This effectively means that there are no special geometrical relationships to be met in order to assure that the mechanism fits together, with regard to the pairs, and is smoothly operable. This "principle of kinematic design" applied to mechanism design results in the following main advantages over conventional design methods, according to A. F. C. Pollard [1]*:

1. It reduces elastic strains in the coupled links to a minimum.
2. It eliminates many of the troublesome and costly limits on functional surfaces which would otherwise be necessary and substitutes guiding surfaces of precision in relation to form but not in relation to size.
3. Adjustments are readily provided both for functional operation and wear.
4. Localization of the forces facilitates design for exacting conditions of equilibrium.

It should be added that any errors in the functioning of a kinematically designed mechanism result mainly from wear and are consistent rather than erratic.

The most important disadvantage of kinematic design is that it depends on point contacts which inherently cause kinematically designed

* Numbers in [] refer to references listed in the Bibliography.

mechanisms to have very limited load-carrying capacities. It is not surprising then that kinematic design has found its most enthusiastic advocates among the designers and manufacturers of instruments. They have built instruments by this principle which have a degree of precision that, in relation to cost, is unattainable by the usual design methods.

The early literature on kinematic design was pointed toward only instrument design. T. N. Whitehead [2] in 1934 stressed the application of kinematic design to instruments and other very light-duty mechanisms. He summed up the earlier developments in the field thoroughly, and also presented many ideas of his own. Pollard [1] was also prominent in kinematic design in the 1930's. In 1940, W. Steeds [3] produced the first significant attempt at tying together kinematics of machinery and kinematic design. His work also was mainly concerned with light-duty mechanisms.

Later references on the subject are scarcer than are the early ones. J. S. Beggs [4] gave limited mention to kinematic design as a "special topic" in his 1955 text, but most modern kinematics texts fail even to mention it. M. F. Spotts [5] and R. J. Herbert [6] have since published papers on the subject, the latter being the latest (1957) comprehensive work.

Most designers are not interested in only light-duty mechanisms, however. The principle of kinematic design as presented thus far is therefore of little use to them. Several attempts at modification of the principle are notable as helping to apply it to heavier mechanisms. Whitehead [2] was the first to suggest such modifications, producing

what he called "semikinematic designs." Pollard [1] gave strong arguments for the need of such modifications, but did not produce any himself. Steeds [3] later offered suggestions on the use of "geometric" (kinematic) design for "heavy loads." The two latest authors, Spotts [5] and Herbert [6], both gave the matter light consideration.

It is worthwhile to quote Steeds [3], pp. 58-59, at this point. His ideas are the basis of the work presented in this thesis.

Geometric [kinematic] design, involving as it does contact at points, is suitable only for instruments, etc., where the weights of the parts and the forces acting are not large, but the principle that redundant constraint should be avoided finds a wider application. Thus if redundant constraint is avoided by using suitable connexions, each of which destroys a definite number of degrees of freedom, then relative movements between the points of support of a body will not distort the body.

He went on to suggest the use of commercially available "connexions," or pairs--namely, ball-and-socket joints, ball-and-socket slides, self-aligning ball bearings, etc.--in mechanisms which conventionally contain only the three simple lower pairs.

If these commercially available pairs are of some degree of precision, as they always are, they can approximate the point contacts imposed by a true kinematic design. The higher the degree of precision, the better is the approximation. Mechanisms so designed have the advantages of kinematic design without the big disadvantage of very limited load capacity. The precision work involved is not left to the mechanism designer, either, but is up to the manufacturers of the pairs. Precision pairs are much easier to obtain than is precision in the fabrication of links, and much cheaper, too, in many cases.

Problem Description

Countless numbers of mechanisms containing redundant constraints exist. Three of the most common examples, all pertinent to this work, are as shown in Figs. 1, 2, and 3. Each of the three possesses redundant constraints requiring special geometrical relationships to be met if the mechanism is to be operable. In the planar four-bar linkage of Fig. 1, it is necessary that the axes of the four revolute pairs (R) be parallel. It is also necessary for movability that the pairs be located on their respective links in such a way that, after three pairs have been assembled, the two halves of the fourth pair "match" in the axial direction enabling the mechanism to be assembled completely. In the common two-bearing shaft (Fig. 2), the special geometrical conditions are that the R pairs be coaxial, and that the axial distances between the two pairs on the shaft and on the frame be equal. Figure 3 shows a two-support translating table in which the two prism pairs (P) must meet exacting conditions; they must be parallel and equally spaced on the table and the frame.

Thus in each of these three examples, first the assembly and then the normal operation of the simple device depends either upon excessively precise manufacture, or else, given normal tolerances, upon flexure of the members and play in the joints.

The object of the work presented here is to eliminate the need for the types of special geometrical conditions which must be present in the mechanisms of Figs. 1, 2 and 3 by following the proposals of Steeds [3] cited above. Essentially, a mechanism which contains "redundant constraints" in the form of special geometrical relations

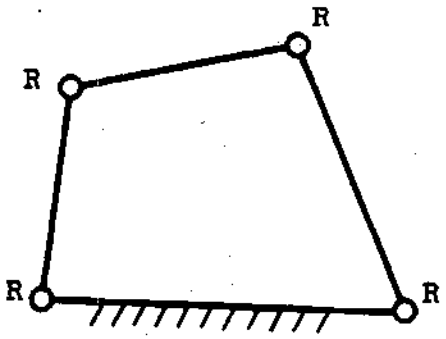


Figure 1. Planar Four-bar, Four-revolute Mechanism

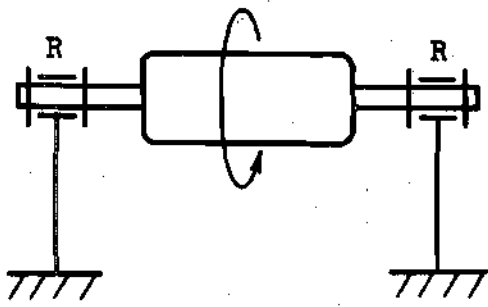


Figure 2. Rotating Shaft on Two Revolute Bearings

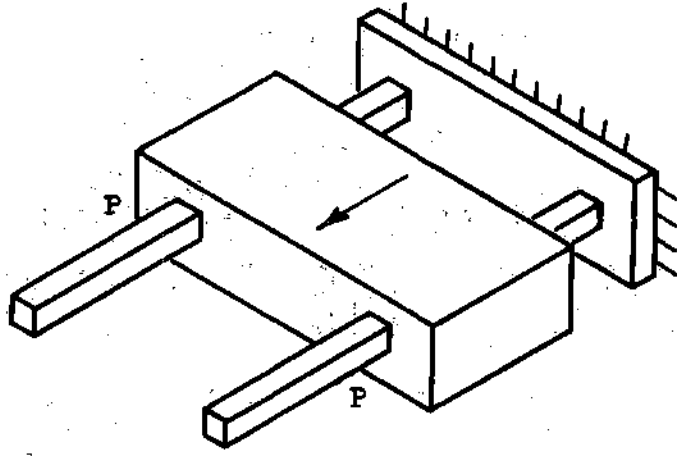


Figure 3. Translating Table on Two Prism Supports

is thereby transformed into an "equivalent" mechanism devoid of such redundant constraints.

Self-aligning Design

The "equivalent" mechanisms developed according to Steeds' suggestions are referred to as self-aligning mechanisms, and their development is termed the self-alignment process or self-aligning design. A self-aligning mechanism must be capable of essentially the same motion as its non-self-aligning counterpart, and it will be seen that a strong physical resemblance exists between the two.

Mechanisms designed according to the strict process of kinematic design contain only point-contact joints, while self-aligning mechanisms use commercially available pairs at their joints. In effect, the difference between the two is that the theoretically zero-area point contacts of kinematic design are expanded into finite-area contacts in self-aligning design. The latter design process is the more practical of the two since a practical theory of design must use only practical and commercially available components.

Problems to be solved here include the self-alignment of mechanisms of the three types shown above. Those three have been chosen because they are the simplest representative examples of their particular types. Chapter IV is concerned with the development of self-aligning equivalents for planar linkages with four and more links. Rotating shafts on two, and more, bearings are treated in Chapter V, while Chapter VI concerns translating tables on two, and more, supports.

Application of the principle of self-alignment to medium- and heavy-duty mechanisms is of prime concern since light-duty mechanisms

are served well by kinematic design with its point contacts. Let it be stressed, however, that the process of self-alignment is also applicable to light-duty mechanisms.

Hopefully, the results presented will help to give designers a valuable insight into methods which can be used to avoid undesirable redundant constraints. Only if these research results can be understood and applied by the designer, are they of practical value.

CHAPTER II

PRELIMINARY CONSIDERATIONS

Basic Kinematic TerminologyLinks and Pairs

A rigid body with two or more elements is a link, also called member or bar. If one element of one link is connected to an element of another link, the two connected elements are called a pair (of elements), or joint.

Links may be classified according to the number of elements they possess. A binary link has two elements; a ternary link, three elements; a quaternary link, four elements; etc.

Classification of pairs can be accomplished by a distinction between lower and higher pairs. In lower pairs, the two elements have a surface contact with each other, while the two elements of a higher pair have only line or point contact. A further classification of pairs depends upon their closure properties. Form closure in a pair means that one element surrounds the other physically so that the pair is held together in all possible positions. If an external force is necessary to keep the elements of a pair together, the pair has force closure. Nearly all lower pairs have form closure, while few higher pairs have that property. Most higher pairs must have force closure to be effective.

A pair, as defined above, connects two links together. It is

also possible to connect more than two links together with a "pair"-- a multiple pair. The originally defined pair, sometimes called simple or single, is used exclusively in this thesis unless otherwise specified.

Kinematic Chains

If links are joined together with pairs, the assemblage is called a kinematic chain. The chain is said to be closed if each link is joined to at least two other links; otherwise, it is an open chain.

A mobile closed kinematic chain in which one link has been fixed to a frame or reference is a mechanism. Mobility is defined in a succeeding section. If all of the pairs in a mechanism are lower, form-closed pairs, the mechanism is known as a linkage. The chain from which a linkage is formed by fixing one link is called a lower-pair chain.

Degrees of Freedom, Degrees of Constraint

The free motion of one rigid body with respect to a second body comprises six degrees of freedom, i.e., six independent variables are required to describe a relative displacement. The six variables are most easily considered to be the three linear displacements in the directions of the mutually perpendicular axes of an x-y-z coordinate system fixed to one body, and the three rotations about those same axes. Each variable which is left undetermined by a pair connecting the two bodies is called a degree of freedom, or simply a freedom, f , of that pair. Similarly, each variable which is determined by a pair is called a degree of constraint, or a constraint, u (unfreedom), of that pair. A constraint may also exist in a pair as an equation relating one variable to another, i.e., one variable is determined by the value of

another variable. It is clear that the sum of the degrees of freedom and the degrees of constraint of a pair connecting two links is equal to six, the total number of variables.

Pairs may be classified, as has been done by L. Harrisberger [7], according to the number of freedoms they allow. Class I includes only those pairs with one degree of freedom; Class II, two degrees of freedom; etc. Each class has a class symbol: p_1 , p_2 , p_3 , p_4 , and p_5 , respectively. The "known physically realizable pairs" have been identified and tabulated by Harrisberger [7]. His table is partially reproduced here as Table 1. Each type of pair within a class is given a type symbol and a name representative of the pair of geometric shapes which defines the practical construction of the pair. The terminology of Table 1 has been adopted for use here.

Degrees of Mobility

A closed kinematic chain is also referred to as having a certain number of degrees of freedom. This number is identical to the number of independent variables which must be specified to define completely the relative positions of the links of the chain, regardless of the link dimensions.

The term "degree of freedom" lacks clarity since the two meanings exist, one introduced in the previous section concerning pairs and this one with reference to chains. To alleviate this problem, the term degree of mobility, F , is used here, and not without precedence, to refer to the movability of chains.

The degree of mobility, or simply the mobility, of a chain is identical to the mobility of a mechanism derived from that chain by

Table 1. Classification of Physically Realizable Pairs

Class	Degrees of Freedom	Class Symbol	Type Symbol	Name
I	1	P_1	R	Revolute
			P	Prism
			H	Helix
II	2	P_2	T	Torus
			C	Cylinder
			T_h	Torus-helix
III	3	P_3	S	Sphere
			S_s	Sphere-slotted cylinder
			S_{sh}	Sphere-slotted helix
			P_L	Plane
IV	4	P_4	S_g	Sphere-groove
			S_{gh}	Sphere-grooved helix
			C_p	Cylinder-plane
V	5	P_5	S_p	Sphere-plane

fixing one link of the chain to the frame of reference. As specified earlier, mobility is independent of link dimensions, except for singular cases, and it is determined by only the number of links, the number and types of pairs, and the number of general constraints in the mechanism, as is discussed later in this chapter.

Planar Chains vs. Spatial Chains

Any kinematic chain, in which the actual paths of motion of all

points in the chain can be shown in one plane, is a planar kinematic chain. Most chains in practice are intended to be planar. However, it is impossible to build an exactly planar chain, so reasonable approximations are called planar. Chains which are not planar are called spatial.

As has already been described, six independent relative freedoms exist between any two unconnected bodies in space. Kinematic chains in which all six are independently present are called general spatial chains. Spatial chains also exist having fewer than six freedoms available, as is discussed later in this chapter.

Three independent freedoms exist in a plane--two perpendicular translations and one rotation about an axis normal to the two translations. A planar chain in which all three freedoms are present independently might be called "general planar," but it is ordinarily understood that a planar chain has all three available unless otherwise designated.

General Constraints

Constraints as discussed thus far are imposed only by the kinematic pairs of a chain. It is common for there also to be constraints due to special geometric conditions existing in the chain. The most common example of this is the planar four-bar, four-revolute chain discussed in Chapter I (see Fig. 1). There are, of course, an infinite number of different all-revolute planar chains all having the same special geometrical relations. As well, there are many other known types of chains with special geometry; e.g., the spherical chains, the Bennett four-bar chain, the Sarrus six-bar chain, the Bricard six-bar chain, and many others. The special geometry in these

chains is not necessarily simply defined, and it may even be as yet undefined. Chains for which the special geometry is undefined have caused much difficulty to kinematicians since it is very difficult to determine the exact number of constraints supplied by the geometry. Many researchers have devoted large amounts of time to overcoming this problem.

Two Russian authors, I. I. Artobolevski [9] and W. W. Dobrovolski [10], introduced the term general constraints as a name for the special geometrical relationships which must be present in so many chains. L. Harrisberger and A. H. Soni [11] have given a simple English-language explanation of the concept. The later sections of this chapter, on families of chains and mobility criteria, make use of the idea.

Families of Chains

Kinematic chains have been grouped into families, m , according to the number of general constraints present [9, 10]. Those chains which have no general constraints are of the family $m = 0$; those with one general constraint are of the family $m = 1$; families with up to five general constraints are conceivable. The families and some of their representative chains are listed in Table 2.

Mobility Criteria

A mobility criterion is an equation or set of conditions which, if satisfied, predict the degree of mobility of a kinematic chain. Most kinematic chains which are in use today are planar. This type of chain was also the first to be analyzed by the early kinematicians, although spatial chains certainly existed at that time. It is not

Table 2. Families of Kinematic Chains

Family m	General Constraints	Representative Chains
0	0	General spatial
1	1	Sarrus six-bar, Ericard six-bar
2	2	Five parallel screws, four non-coplanar prisms
3	3	Planar, spherical, Bennett four-bar
4	4	Three coplanar prisms, three coaxial screws
5	5	Rotating shaft on multiple R bearings, translating table on multiple P supports

surprising then that the first mobility criteria were established for planar chains. The earliest was by Chebyshev in 1870 [12, 13]. In the symbology of this thesis, Chebyshev's formula had the form

$$3n - 2p_1 - 4 = 0 \quad (1)$$

where n is the number of links in the chain. If a planar ($m = 3$) chain satisfies Eq. (1), it has one degree of mobility ($F = 1$).

Grübler and Grashof later developed the same relation, independently.

A. P. Malytcheff is credited with having developed the first

general spatial mobility criterion in 1923 [11]:

$$F = 6(n - 1) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - p_5. \quad (2)$$

K. Kutzbach produced the same relationship a few years later. This criterion, being general, does not explain the existence of the "paradoxical" chains which have special geometrical relationships, i.e., general constraints.

Equation (2) yields an important fact about spatial mechanisms. Consider a single-loop $F = 1$ chain. By Eq. (2),

$$1 = 6(n - 1) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - p_5.$$

The number of links, however, in a single closed loop is the same as the number of pairs, i.e.,

$$n = p_1 + p_2 + p_3 + p_4 + p_5.$$

These two equations combined give

$$p_1 + 2p_2 + 3p_3 + 4p_4 + 5p_5 = 7.$$

But the sum of the freedoms of the pairs in the chain is

$$\sum f = p_1 + 2p_2 + 3p_3 + 4p_4 + 5p_5.$$

Hence, whatever pairs are used and regardless of the number of pairs (whether 3, 4, 5, 6, or 7) the total sum of their freedoms is

$$\sum f = 7$$

in a single closed loop. In a multiple-loop chain, a loop may receive constraints by virtue of its sharing pairs with other loops, in which case it must contain more than seven freedoms among its pairs. In short, the pairs of each loop of an $F = 1$ general spatial chain must contain at least seven freedoms.

Artobolevski [9] and Dobrovolski [10] have modified Eq. (2) to include the number of general constraints, m (also the family number), so that

$$F = (6 - m)(n - 1) - \sum_{i=1}^{5-m} (6 - m - i)p_i. \quad (3)$$

If the number of general constraints in a chain is known, then Eq. (3) gives the mobility.

The kinematic chains which are to be developed here, however, are to possess no general constraints, i.e., $m = 0$, because the object is to eliminate the need for special geometry. If $m = 0$, both Eqs. (2) and (3) take the form

$$F = 6(n - 1) - \sum_{i=1}^5 (6 - i)p_i, \quad (4)$$

the general spatial mobility criterion to be used here.

CHAPTER III

GENERAL PROVISIONS

Procedure

In Chapter I, the object of this thesis has been stated to be the development of self-aligning mechanisms of three types--planar linkages, rotating shafts on multiple bearings, and translating tables on multiple supports. Some principles basic to two, or to all three, of these developments are presented in this chapter.

The first step toward developing a self-aligning mechanism is to choose a particular type of mechanism containing general constraints and then analyze it according to the mobility criterion given by Eq. (3). Such an analysis presupposes that the number of general constraints in the mechanism is known, as it must be for any mechanism which one intends to render self-aligning. The resultant mobility from the analysis just mentioned is also to be the mobility of the equivalent self-aligning mechanism, calculated by the general spatial criterion of Eq. (4), in which the general constraints have been disallowed. Transition from the first mobility criterion, Eq. (3), to the most general one, Eq. (4), is accomplished essentially by adding enough select freedoms to certain pairs of the mechanism in order to eliminate the need for special geometric relationships, i.e., erasing the general constraints. Succeeding sections describe these select freedoms, the forms the modified pairs may take, and the particular freedoms which may be added to pairs in a given mechanism.

Commercially Available Pairs

It is necessary to set down some criteria for the pairs which may exist in self-aligning mechanisms. Lower and higher pairs have been defined in Chapter II, along with the closure properties of pairs. Table 1 has presented the 14 physically realizable pairs, many of which are commercially available as units in themselves. Commercial availability is a prerequisite for any pair which would be used here in a self-aligning mechanism, since only such pairs can be used economically in practice. Also, it is advantageous to have form closure in the pairs of a self-aligning mechanism since this allows the pairs to operate effectively in any physical orientation. Earlier, it was stated that normally only lower pairs have form closure. Many commercially available, form-closed pairs, however, are not lower pairs but rather are assemblages of several higher pairs united as one pair, e.g., ball bearings, roller bearings, ball bushings, etc. These assemblages can be considered as lower pairs since they have the overall effect of lower pairs when joining two links, and indeed they are classified here as lower pairs. Recalling from Chapter I that the precision of self-aligning mechanisms is dependent upon the degree of precision in pairs, it is necessary that all pairs possess some degree of precision; in fact, pairs can be built with almost no limit to their degree of precision. Self-aligning mechanisms may be of any quality precision desired and still be smoothly mobile since they do not depend on precise manufacture (except within the pairs) for their mobility.

Those pairs which meet the criteria above are listed in Table 3,

Table 3. Commercially Available Pairs

Type Symbol	Degrees of Freedom	Examples
R	1	Journal, deep-groove ball, cylindrical roller, and thrust ball bearings; hinge; pin joint
P	1	Spline, ball spline, dovetail slide, keyed shaft-and-bushing
H	1	Nut-and-screw, ball nut-and-screw
T	2	Cross-and-roller universal joint
C	2	Ball bushing, piston-in-cylinder, simple shaft-and-bushing
S	3	Ball-and-socket joint; spherically seated ball, self-aligning roller bearings
S _g	4	Spherically seated ball bushing, self-aligning pillow block on ladder bearings, ball-and-socket slide

along with at least one physical example in each case. Seven of the 14 physically realizable pairs which were listed in Table 1 are not listed in Table 3, as they do not satisfy the criteria. This is not to say that they can never do so. New forms of a pair may be developed in the future satisfying the criteria. Only those pairs listed in Table 3, however, are to be used here in the design of self-aligning mechanisms.

The commercially available pairs listed in Table 3 are not sufficient to meet all needs. For instance, there is no Class V pair,

i.e., one allowing five degrees of freedom. Such a pair will be needed in the translating table problem (see Chapter VI). Two pairs whose freedoms total five, however, may be joined by a binary link to form what will be called here a compound pair. This compound pair allows the five freedoms desired, providing no two of the freedoms are identical. The most common example of this in practice occurs when two ball-and-socket joints (3 pairs) are joined by a coupling member. Unless the possible rotation of the member about its own axis is objectionable, the assembly effectively allows five degrees of freedom when used as a pair in itself to connect two bodies, restricting only the relative translational freedom which would allow the bodies to move toward, or away from, one another. Compound pairs may also be improvised in the form of three pairs and an intermediate ternary member. This type of compound pair is to find use in the rotating shaft problem of Chapter V. Compound pairs are not used, however, unless absolutely necessary to produce the simplest, and therefore usually the most economical, self-aligning version of a particular mechanism.

Selection of Eligible Pairs

Converting a mechanism containing general constraints into a self-aligning mechanism primarily involves the addition of one or more freedoms to one or more of the pairs of the mechanism so that the resulting equivalent mechanism satisfies the general spatial mobility criterion given by Eq. (4). Freedoms cannot be indiscriminately added to pairs, however; certain precautions must be taken.

Appendix A consists of an example problem which demonstrates the need for these precautions. From the development there, a simple rule

can be stated for the purposes of this thesis.

Rule: Only freedoms non-existent in a mechanism having general constraints may be added to the pairs of that mechanism in order to produce an equivalent self-aligning mechanism.

The expression "freedoms non-existent in a mechanism" requires some explanation. In the case of a planar mechanism, the freedoms which are existent are two orthogonal translations and the rotation about the axis normal to the two translations. In a rotating shaft or a translating table, there is only one freedom existent--a rotation or translation, respectively. All others are non-existent.

Any pair which would be eligible to exist in a self-aligning mechanism must not only be derived from a pair in the equivalent mechanism containing general constraints according to the rule, but must also be commercially available according to Table 3.

It is important to realize that parallel translations within a mechanism of the three types discussed here really represent only one translational freedom, and similarly, rotations about parallel axes represent only one rotational freedom. Translations added to the pairs of a mechanism should be approximately perpendicular to the translations already present in the system; likewise, rotational freedoms added to a system should be about axes roughly normal to the rotational axes already present.

Unfortunately, a rule is not stated as easily for most of the other systems of mechanisms in the five families possessing general constraints (see Table 2). It is not clear in many of these what is meant by "freedoms non-existent in a mechanism" since, for instance, the rotational freedoms present at one configuration of a mechanism

may not be the same as those present at a subsequent one. A much more involved development would be necessary to self-align a mechanism of this type than is used here for planar linkages, rotating shafts and translating tables.

In addition to the limitations set down by the rule above, further restrictions regarding eligible pairs are made where needed in the interest of practicality and economy.

These first three chapters have presented all concepts necessary to the comprehension of the developments to follow. From this point, Chapters IV, V and VI may be taken in any order without loss of continuity.

CHAPTER IV

SELF-ALIGNING PLANAR LINKAGES

Planar Linkages

A planar mechanism, as defined by Hartenberg and Denavit [14], is a mechanism in which "the true paths of all particles of all links may be shown in one plane, 'the plane of the paper.'" With this definition, planar mechanisms may exist in the families $m = 3$, $m = 4$, and $m = 5$. This is not to say, however, that all mechanisms of these three families are planar; e.g., spherical mechanisms are also of the family $m = 3$, but are certainly not planar.

The meaning of "planar" as used here, however, is restricted to the family $m = 3$, which may be called the "general" planar case because it has the least general constraints (three). Mechanisms of families $m = 4$ and $m = 5$ possess additional general constraints.

The three general constraints can be met exactly only in the "ideal planar mechanisms"-- those which exist only theoretically. The physical approximations to these mechanisms are "real planar mechanisms." When no distinction is necessary between the ideal and real, the common term "planar mechanism" is used.

Attention throughout this thesis is limited to mechanisms containing only simple, lower pairs. Multiple pairs do occur quite frequently in planar mechanism, however, and the procedures can be accommodated by recognizing that (1) a multiple R pair occurs when one or more sides of a polygon representing a link are reduced to zero

length so that two or more R pairs coincide, and (2) a multiple P pair occurs when two or more P pairs on the same polygonal link have the same direction of travel.

Only two kinds of (lower, form-closed) pairs exist in planar linkages, the revolute R and prism P. Each allows one degree of relative freedom between its elements. Accordingly, there are two distinct groups of planar linkages. Let those which contain only R pairs be designated R-type, and those containing both R and P pairs, RP-type.

Planar Linkages of Mobility $F = 1$

The number of pairs, p_1 , in a planar linkage is determined by the mobility of the linkage, F , and the number of links, n . The relation is

$$p_1 = \frac{1}{2} [3(n - 1) - F]$$

as may be directly obtained from Eq. (3) where $m = 3$. For a mobility $F = 1$, this simplifies to

$$p_1 = \frac{1}{2} (3n - 4) \quad (5)$$

Notice that an odd number of links fails to give an integer for p_1 in Eq. (5)--a meaningless result. The number of links then must be even in an $F = 1$ planar linkage. Substituting various even values for n into Eq. (5), it is readily seen that $F = 1$ planar four-bar, six-bar, eight-bar, . . . linkages have four, seven, ten, . . . pairs, respectively.

Planar linkages of both the R- and RP-types are catalogued in

Appendix B. Illustrations there show lower-pair kinematic chains, not linkages; i.e., no link is designated as fixed. If one link of one of these chains is fixed, that chain becomes a linkage. Each different linkage formed by fixing various links is called an inversion of the chain.

Note that four-, six-, and eight-bar R-type chains are catalogued (see Figs. 23, 24 and 25). R-type $F = 1$ chains with more links are not needed. Attempts to catalogue the ten-bar chains by Alt (see Crossley [15]) and Crossley [16] have resulted in the catalogue of 230 chains by Davies and Crossley [17]. Woo [18] has also produced the same catalogue. It provides a good foundation for an extension of this work. No complete catalogue exists for a chain with more than ten links.

Four- and six-bar RP-type $F = 1$ chains are catalogued in Appendix B (see Figs. 26, 27 and 28). As evidenced by the increase from three four-bars to 50 six-bars, the number of chains with eight, or more, links would be extremely large. They are out of the scope of this thesis.

Planar Linkages of Mobility $F > 1$

In Appendix B, the R-type five-bar $F = 2$ chain and the four seven-bars are illustrated (see Figs. 29 and 30). Larger chains and higher degrees of mobility are discussed, but not catalogued. Only the multiple-mobility chains shown there, with the exception of the one with fractionated mobility (Fig. 30d), are to be of importance here.

No RP-type chains with $F > 1$ are shown in Appendix B. They are similar to the R-type chains, but the presence of the two types

of pairs in them multiplies the number of possible chains. Self-alignment of any of these chains would be quite voluminous.

Self-alignment of Planar Linkages

The task is to apply the principle of self-alignment to planar linkages in order to produce self-aligning "planar" linkages. A planar linkage moves in a three-freedom space (a plane) and possesses three general constraints. Its mobility is given by Eq. (3) with $m = 3$. Self-aligning linkages, on the other hand, move in six-freedom space, having zero general constraints ($m = 0$), and their mobilities are according to Eq. (4).

The self-aligning linkages here are referred to as "planar" since they are intended to resemble their planar counterparts in both appearance and function. Thus, it is meaningful to refer to the plane of the paper as the "plane of the linkage" for both planar linkages and their self-aligning equivalents, even though in a self-aligning linkage there is no need for exact parallelism of R axes and perpendicularity of R and P pairs in order to guarantee mobility. In fact, the words, "parallel" and "perpendicular" (and their synonyms) are used in discussing self-aligning mechanisms only as aids in describing approximate relations between pairs.

Eligible Pairs

Before the self-alignment process can proceed, it is necessary to determine which pairs are eligible to appear in the self-aligning linkages. The rule of Chapter III, commercial availability (see Table 3), and practical considerations are the bases. Each of the pairs which may exist in planar linkages (i.e., R and P) must be changed

into other pairs according to the rule and then the resultant pairs must be tested according to the other criteria.

Considering the revolute R, it is obvious that most simply it can remain an R in the self-aligning linkage, having had no freedoms added to it. If one rotational freedom is added, a T pair results; two rotations produce an S pair. Addition of the one (axial) translation to an R yields a C pair. One rotation and the translation being added does not give a pair listed in Table 3. The only other possible way to add freedoms is to add all three, thereby creating an S_g pair. Thus according to the rule, an R of a planar linkage must appear in the equivalent self-aligning linkage as one of the following:

1. an R coincident with the original R, or
2. a T with one rotation coincident with the original R, or
3. a C with axis coincident with the original R, or
4. an S with center on axis of the original R, or
5. an S_g with translation parallel to, and center located on, axis of the original R.

To replace an R pair in some of the ways just listed is not always feasible. For example, if an R pair in a planar linkage is required to allow complete revolutions; the T pair in its common form is not able to perform such a maneuver and therefore cannot serve in the capacity of the R. This shortcoming of the T is reason enough to eliminate it from consideration as an eligible pair in this development of self-aligning planar linkages.

Similarly, a C pair cannot always replace an R pair. The translation of the C pair is normal to the plane of the linkage. The lengths

of the links connected by such a pair are usually much larger than the desirable axial length of the pair itself, judging from common practice in planar linkages. A simple force analysis of the pair shows that either the pair must have very low sliding friction or it must have excessive length; otherwise, smooth action is unlikely. The low friction alternative is quite expensive since then a C pair with both good rotational and good translational capabilities is required. The latter alternative, a longer pair, is undesirable from the standpoints of cost and space. Consequently, a C is not to replace an R in the development of self-aligning planar linkages.

S pairs are very plentiful and versatile, seeming ideally suited for the proposed application. The S_g pair, on the contrary, is eliminated by the same arguments as presented above for the C pair.

Now turning to the prism pair P, it is again possible that the pair remain a P throughout the self-alignment process. The only other possible form for it to take, according to the rule and Table 3, involves the addition of a rotational freedom to form a C pair. Thus, a P pair in a planar linkage appears in the equivalent self-aligning linkage, subject to further restrictions, as either:

1. a P coincident with the original P, or
2. a C with axis parallel to the translation of the original P.

There are reasons for not allowing a P pair to remain a P pair in the self-aligning linkage. True P pairs are rare in practice. Where they could occur, the pair is most often a C; e.g., the common hydraulic cylinder. Two factors are primarily responsible--the partial self-alignment introduced and the lower cost of the C compared with

the P. For these reasons (1) C pairs may occur in place of P pairs in self-aligning planar linkages, and (2) all P pairs in planar linkages are to be converted to C pairs in the process of self-alignment, unless doing so introduces an excess of additional freedoms into the linkage.

It has been established, therefore, that in the self-alignment of planar linkages an R pair must either remain an R or become an S; likewise, a P must become a C pair, except in the event that too many freedoms are added by so doing. The arguments expressed leading to these limitations are admittedly debatable, but they are practical and are considered valid for the work presented here. Without these, or similar, limitations, the task at hand would be practically insurmountable. It is certain that some, or all, of the pairs eliminated could be used advantageously in the design of self-aligning linkages. The general procedure is the same regardless of the eligible pair assumptions.

R-type Linkages of Mobility $F = 1$

Four-bar. The self-aligning equivalent to the four-bar planar linkage of Fig. 23 may contain only R and S pairs. Equation (4) takes the form

$$1 = 6(4 - 1) - 5p_1 - 3p_3,$$

$$5p_1 + 3p_3 = 17,$$

and since there are to be only four pairs in all,

$$p_1 + p_3 = 4.$$

There is no simultaneous solution of positive integral values to these two equations. It is obvious that a negative or non-integer solution is useless since neither a negative number of pairs nor a fraction of a pair may exist in a closed kinematic chain.

At this point, it is necessary to realize that a binary link between two S pairs possesses a "spin" property in that it is free to rotate about the axis through the two S pairs. Thus, a linkage containing a binary between two S pairs has an additional mobility. Such a binary spin is harmless when the binary is simply a connecting rod between two other members. On the other hand, the spin is not acceptable if a certain point on the binary (not on the spin axis) is intended to trace some curve in the plane of the linkage as is required of coupler points in many planar linkages. Hence, binary spins are acceptable in some cases and are not acceptable in others.

It is assumed here that the spins are not objectionable. If a self-aligning chain contains a spin, only those inversions in which the spin does not disrupt the performance of the linkage are considered to be valid.

If the mobility of the four-bar is increased to $F = 2$ by allowing a binary spin, the two simultaneous equations are changed to:

$$5p_1 + 3p_3 = 16,$$

$$p_1 + p_3 = 4.$$

These equations are satisfied by $p_1 = p_3 = 2$. Thus, a self-aligning equivalent linkage is possible if it contains two R pairs and two S

pairs. The two possible kinematic chains are shown in Fig. 4. Spin freedoms are indicated by curved arrows. The chain (a) possesses a binary spin as described above, while in (b) two binaries are allowed to spin with respect to the other two. This latter condition is usually very undesirable since the spin is really a "folding" of the chain, and in fact disqualifies the chain from being called "self-aligning planar" since the motion of such a chain need not resemble the motion of the corresponding planar chain. Folding is thus not to be allowed.

Hence, there is only one valid self-aligning equivalent to the R-type four-bar planar chain--that of Fig. 4a. This chain can become a linkage by fixing any link, except that the spin link understandably cannot be fixed. If the spin link were fixed, the plane of the linkage would be indeterminate. The inversion in which the R-R member is fixed is the most interesting since it pivots the links which commonly serve

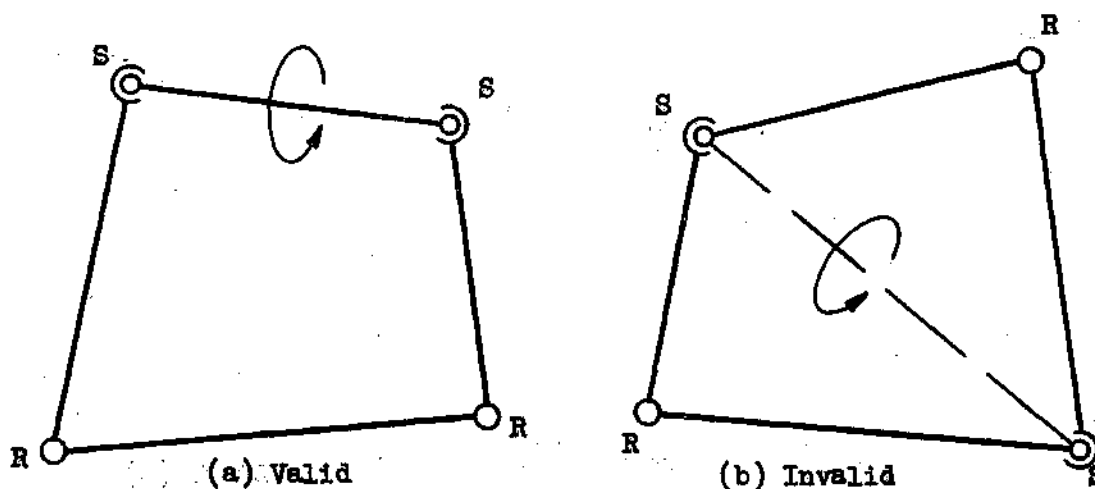


Figure 4. Self-aligning Equivalents to the R-type Planar Four-bar Chain

as input and output links on the one degree-of-freedom R pairs rather than on the S pairs.

It has been shown in Chapter II that each loop of a multiple-loop $F = 1$ self-aligning chain must have at least seven freedoms among its pairs. In contrast, each loop of a planar ($m = 3$) chain must have at least four freedoms. Consequently, it is deduced that each such loop must receive at least three freedoms in the transition of the planar chain into the self-aligning equivalent. These three freedoms correspond to the elimination of the three general constraints. It is apparent that in order for at least three freedoms to be added to an R-type planar loop, two R pairs must be replaced by S pairs since each such replacement adds only two freedoms to a loop. This action produces a spin in some cases and does not in others, dependent on whether or not one or both of the S pairs are shared with other loops. These technicalities are enlarged upon in the later developments of the six- and eight-bar chains, which are all multiple-looped.

Six-bar. R-type planar six-bar linkages each contain seven R pairs; therefore, the two equations for a self-aligning equivalent are

$$F = 6(6 - 1) - 5p_1 - 3p_3 = 1,$$

which reduces to

$$5p_1 + 3p_3 = 29,$$

and

$$p_1 + p_3 = 7.$$

A simultaneous solution of positive integral values is possible in this case-- $p_1 = 4$, $p_3 = 3$. Note that this solution is arrived at using $F = 4$, i.e., there is to be no binary spin present.

Two different series of six-bar planar linkages exist, the Watt and the Stephenson (see Fig. 24). Each has two independent loops, say the interior loops, and one dependent loop, the peripheral loop. Each loop of a general spatial linkage must have at least seven freedoms among its pairs if the loop is to be mobile. This applies to all loops, both independent and dependent, in a linkage when developing a self-aligning counterpart. The dependent loop serves as a check on the other loops. (It is, in fact, possible to "turn a linkage inside out" so that what was the peripheral loop is found in the interior, and a formerly interior loop comprises the periphery.)

Each loop of the self-aligning equivalent to a planar Watt linkage must have two S pairs and there are to be only three S pairs in all. Each of the three loops then must hold one S in common with each of the other loops. The connection between the two ternary links is then an S pair since this is the only pair common to the two interior loops. Allowing for kinematic symmetry, the other two S pairs may be situated in any of the four ways shown in Fig. 5.

Frequently in practice, a ternary member has the form of a bar with its three pairs being in a line. If the ternary with three S pairs in (a) has its pairs on a line, then the member is free to spin similar to the way binary members may spin. This ternary spin is very objectionable and must be avoided. An S pair replacing an R pair in a linkage can be placed anywhere along the R axis and retain the

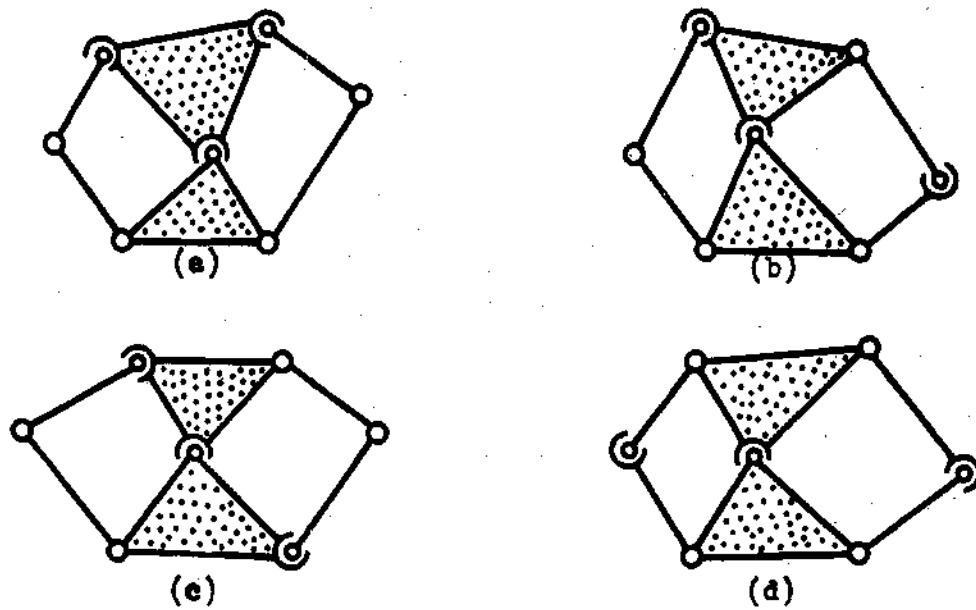


Figure 5. Self-aligning Equivalents to the R-type Planar Watt Six-bar Chain

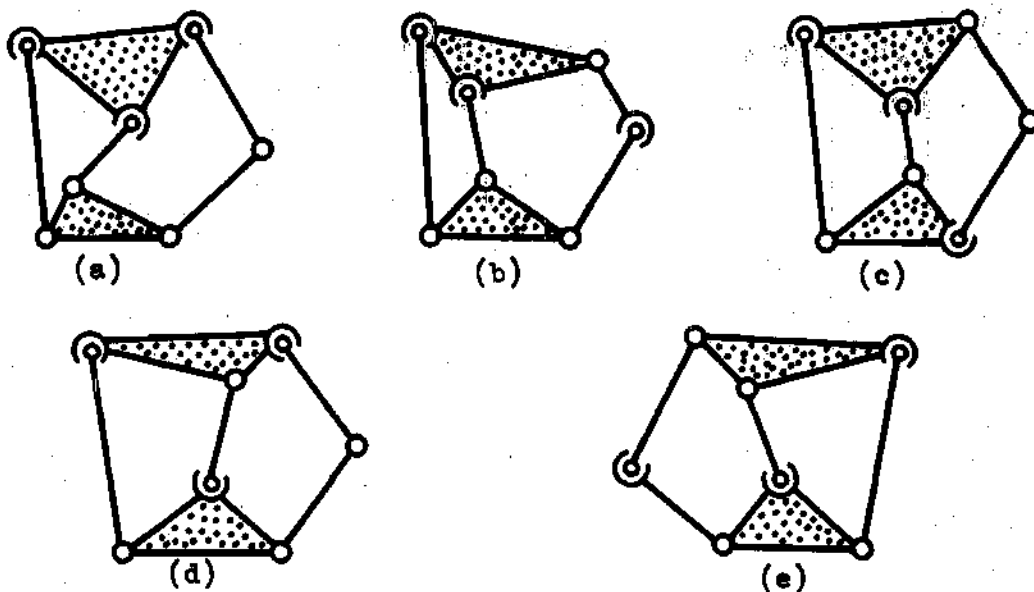


Figure 6. Self-aligning Equivalents to the R-type Planar Stephenson Six-bar Chain

rotation of the R. The spin of the one ternary in (a) can be avoided by simply constructing that link so that the three S pairs are not collinear, and preferably not even nearly collinear.

Linkages formed from the chains of (b), (c) and (d) suffer from a liability to fold when all three S pairs happen to be collinear in some position of the linkage. It is impossible to formulate a simple general rule which will assure that a folding action is not possible in chains like these since the S pairs are not all on one link. The form shown in Fig. 5a is for this reason the "preferred" self-aligning equivalent to the R-type Watt planar chain.

Turning now to the Stephenson six-bar, there are five distinct arrangements for the placement of the S pairs as shown in Fig. 6. In (a), the ternary with three S pairs may spin (like in Fig. 5a) if the link has the S pairs all in line. This is easily avoided, however, by purposely constructing the link with the S pairs not collinear. In (b), (c), (d) and (e) there is again possible a folding action if the S pairs ever become collinear. No simple general rule can be made to guarantee that to be impossible. The chain of Fig. 6a is preferred over the others.

For both the Watt and the Stephenson, the preferred arrangement of the three S pairs is then that they be all located on one ternary link with the reservation that the three S pairs must not lie on a line. There are no spins present in such a chain; therefore, an $F = 1$ linkage is formed by fixing any link.

Eight-bar. Eight-bar R-type planar linkages each contain ten R pairs. The simultaneous equations for the self-aligning equivalents

with $F = 1$ and $n = 8$ are

$$5p_1 + 3p_3 = 41,$$

and

$$p_1 + p_3 = 10.$$

These have no solution of positive integral numbers, just as there was none in the case of the four-bar linkage earlier.

Allowing there to be one binary spin present in the linkage, the equations are

$$5p_1 + 3p_3 = 40,$$

$$p_1 + p_3 = 10.$$

These are satisfied by $p_1 = p_3 = 5$. Thus, a self-aligning R-type eight-bar has five R pairs and five S pairs, with one binary spin present. The linkage has effectively only one degree of mobility if the binary spin is unimportant.

Keeping in mind that every loop in a general spatial kinematic chain must contain at least seven freedoms and that each self-aligning planar eight-bar chain is to possess one binary spin, it is simple to arrange the five S pairs in each of the 16 varieties of planar eight-bars listed in Appendix B (see Fig. 25). Due to the large number of varieties, it is judged sufficient to display only those chains similar to those called "preferred" in the six-bar development; i.e., those

in which spins of polygonal links and folds can be prevented. One polygonal link always has S pairs exclusively.

The self-aligning equivalents of the Series 1 chains, as shown in Fig. 7, are developed by placing four S pairs on one quaternary link and then placing the fifth S so that a binary spin is present. It can be placed in any of three equivalent places in chain (i); there are two distinct arrangements for (ii). The fifth S pair has no effect on there being two S pairs in every loop. The quaternary link with only S pairs must be designed so that the four S pairs are not collinear; otherwise, the quaternary may spin.

Series 2 and 3 kinematic chains are shown in Figs. 8 and 9, respectively. The development here has placed three S pairs on one ternary and then placed the other two in such a position that a binary spin occurs somewhere and also that each loop has at least two S pairs. The ternary with the three S pairs must be designed with its pairs non-collinear.

Figure 10 shows the different arrangements for the Series 4 chains. Here, one group of chains is formed by placing four S pairs on the quaternary and then placing the other S where it is needed to produce the spin and to put two S pairs in each loop. Another group of chains is formed by placing three S pairs on a ternary and then placing the other two S pairs where they are required to produce the spin and give each loop at least two S pairs. The polygonal link containing only S pairs must be constructed with those pairs non-collinear.

There is only one preferred self-aligning R-type planar four-

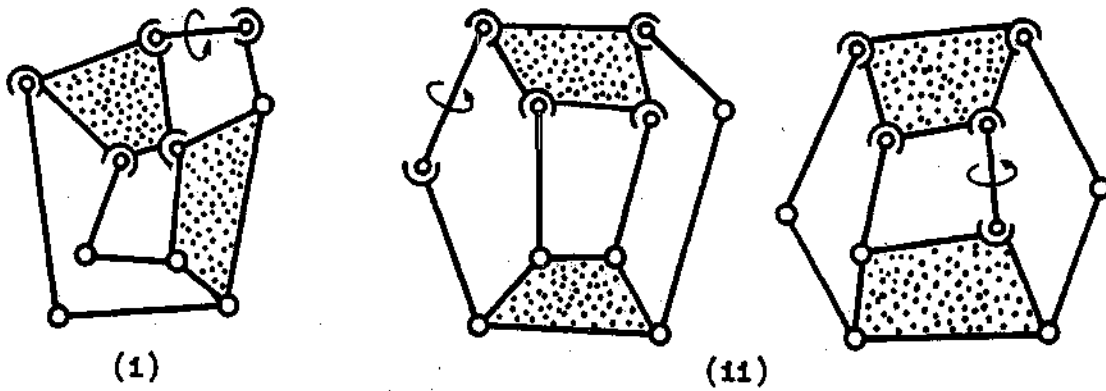


Figure 7. Self-aligning Equivalents to the R-type Planar Eight-bar Chains of Series 1

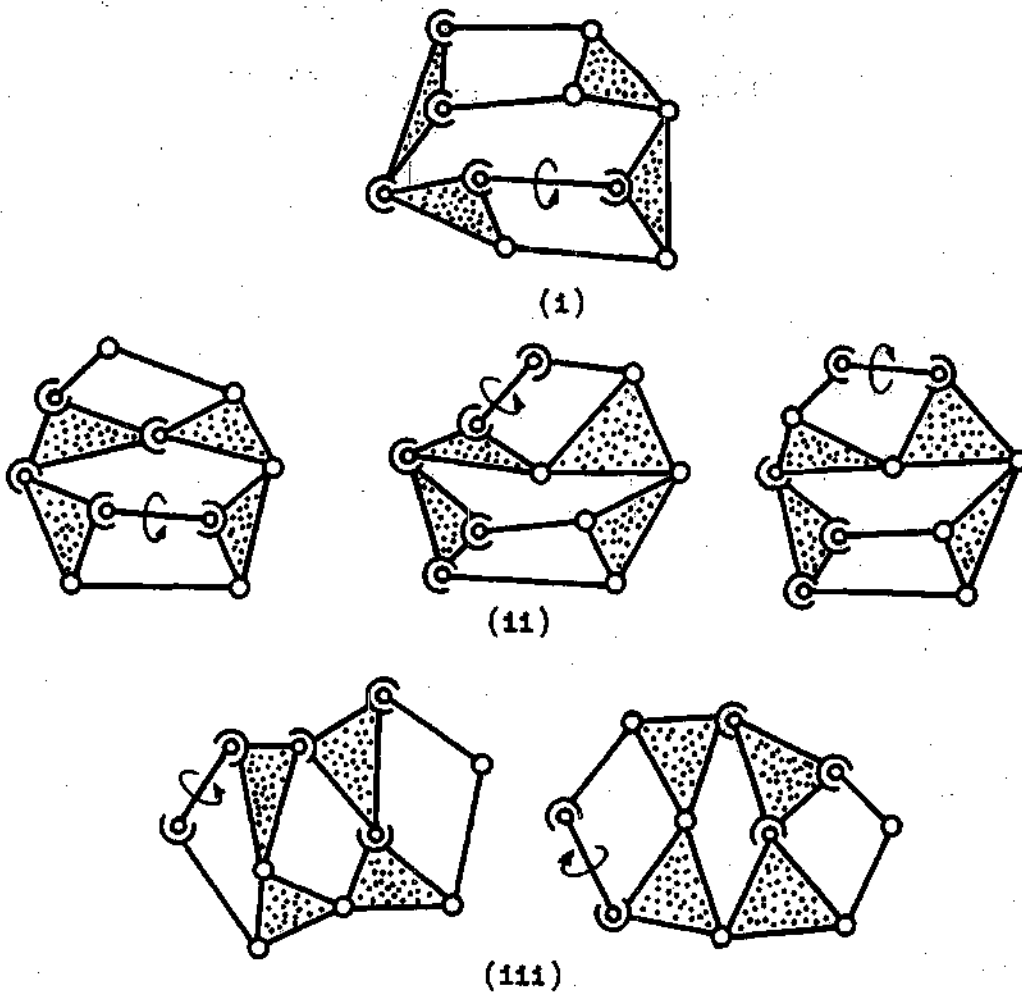


Figure 8. Self-aligning Equivalents to the R-type Planar Eight-bar Chains of Series 2

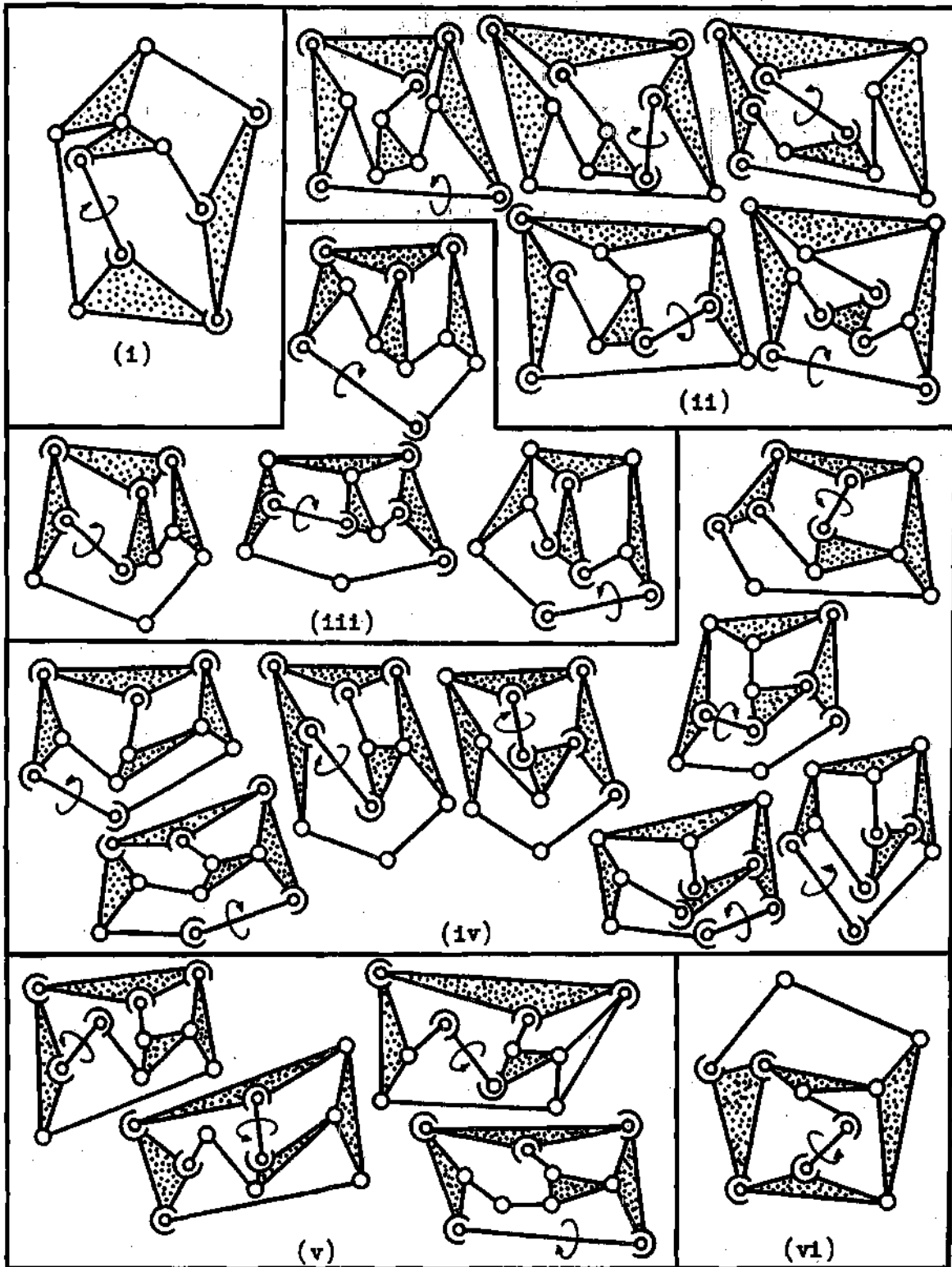


Figure 9. Self-aligning Equivalents to the R-type Planar Eight-bar Chains of Series 3

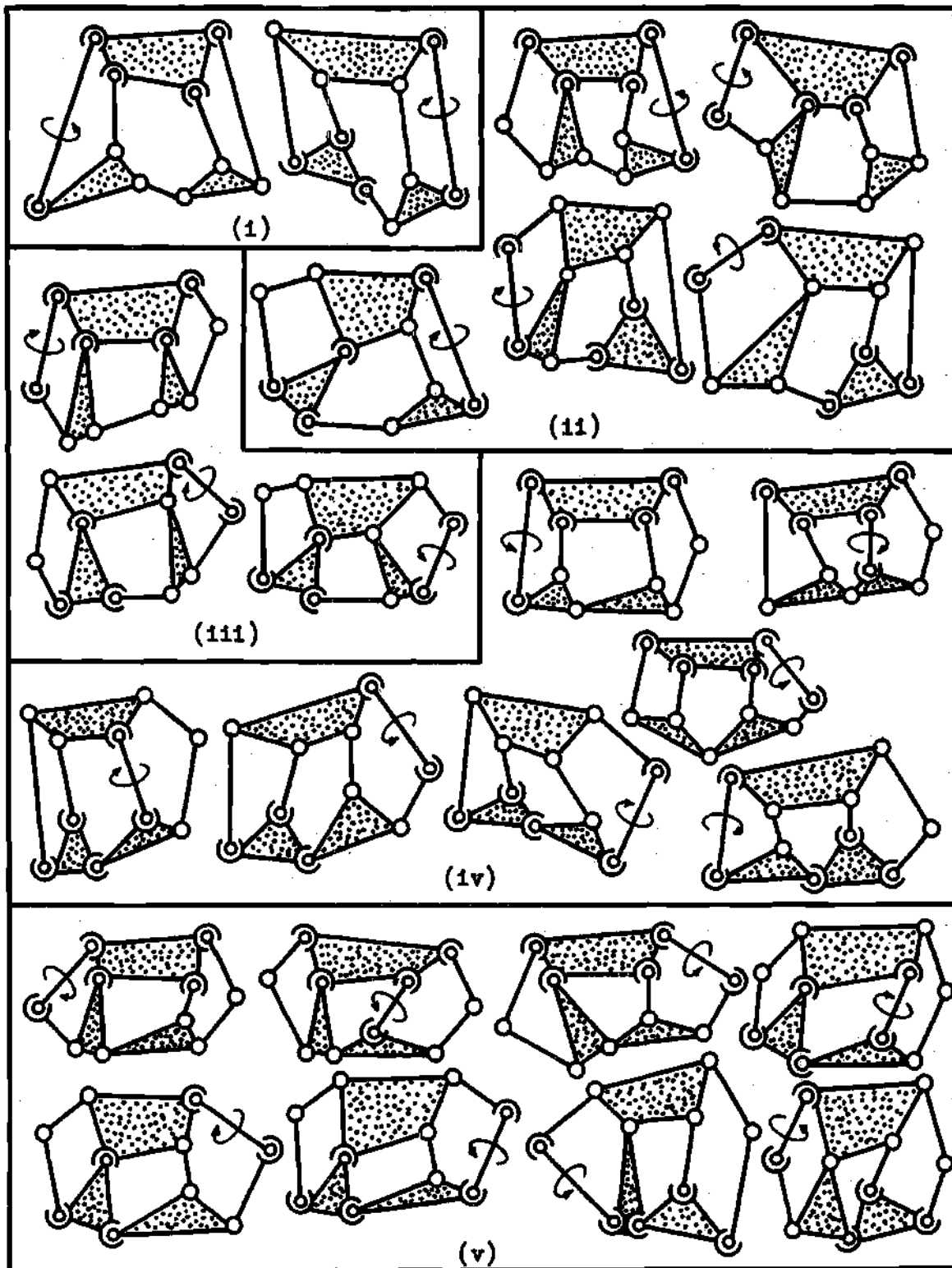


Figure 10. Self-aligning Equivalents to the R-type Planar Eight-bar Chains of Series 4

bar chain and only two six-bars; in contrast, there are 57 preferred eight-bar-chains. There should be a linkage among the numerous inversions of these chains to satisfy every need for a self-aligning R-type eight-bar linkage.

More than Eight-Bars. It is simple matter to determine the number of S pairs necessary to form the self-aligning equivalents to any more complex R-type planar $F = 1$ linkage. If the two simultaneous equations are solved for such linkages, the resulting numbers of R and S pairs are as given in Table 4. Note that linkages whose number of links is evenly divisible by four possess a binary spin. Only a superficial investigation has been carried out with regard to the validity of the values given in Table 4 beyond $n = 10$, but there is no apparent reason to doubt their validity and that of those inferred by the generalized expressions at the end of the table.

A procedure similar to that used on the six- and eight-bar linkages is suggested for use on the R-type linkages with 10, 12, 14, . . . members. These planar linkages are very rare in practice, possibly due in part to the difficulties in obtaining smooth action from an $m = 3$ mechanism with so many members. There are obviously multitudes of possible self-aligning linkages derivable from each.

RP-type Linkages of Mobility $F = 1$

Planar linkages containing both R and P pairs have been designated RP-type linkages. The procedure used here to convert such a linkage to a self-aligning form is (1) to replace all P pairs by C pairs, and (2) to replace enough R pairs by S pairs so that the linkage satisfies the general spatial mobility criterion. Other rules are

Table 4. Pairs in Self-aligning R-type $F = 1$ Planar Linkages

Number of Links n	Number of R Pairs P_1	Number of S Pairs P_3	Binary Spin Present*
4	2	2	Yes
6	4	3	No
8	5	5	Yes
10	7	6	No
12	8	8	Yes
14	10	9	No
.	.	.	.
.	.	.	.
.	.	.	.
$4k$	$2 + 3(k - 1)$	$2 + 3(k - 1)$	Yes
$4k + 2$	$4 + 3(k - 1)$	$3 + 3(k - 1)$	No

($k = 1, 2, 3, \dots$)

* In addition to the mobility $F = 1$ of the linkage

conceivable.

Four-bar. RP-type planar four-bar chains may possess either one or two P pairs (see Fig. 26). Since all the P pairs are to be replaced by C pairs, the self-aligning chains will have p_2 pairs as well as p_1 and/or p_3 pairs. The mobility criterion, Eq. (4), requires that

$$F = 6(4 - 1) - 5p_1 - 4p_2 - 3p_3 = 1,$$

from which

$$5p_1 + 4p_2 + 3p_3 = 17.$$

Also, the number of pairs in the self-aligning chain is

$$p_1 + p_2 + p_3 = 4,$$

in which p_2 is equal to the number of P pairs in the planar chain.

There are three RP-type four-bar chains, conveniently described as being R-R-R-P, R-R-P-P, and R-P-R-P with reference to the types of pairs and the order of the pairs in the chain. In the first, only one P exists, and therefore $p_2 = 1$. The solution of the equations for the R-R-R-P is then $p_1 = 2$, $p_2 = p_3 = 1$, where the p_1 pairs are both R pairs, the p_2 is a C, and the p_3 is an S. The C pair must be oriented with its axis parallel to the original P pair, and two of the R pairs of the planar chain are not changed. The third R is replaced by the S pair (see Fig. 11a). It is not clear at this point which R is the best to replace.

Consider the problem which would arise in the chain just described if the S joint should come to lie on the C axis at some position of the chain. An unexpected folding action or spin could take place in such a configuration rendering such a chain invalid as a self-aligning chain. There is no all-encompassing rule here to guarantee that a chain is valuable in all positions, except that the S must never lie on the C axis. This is a geometric restriction, dependent on the designer for application. Most effectively, it is met by requiring the link dimensions to be such that the S can never come even close to the C axis. The two chains of Fig. 11a are thus self-aligning equivalents to the R-R-R-P planar chain subject to this geometrical restriction.

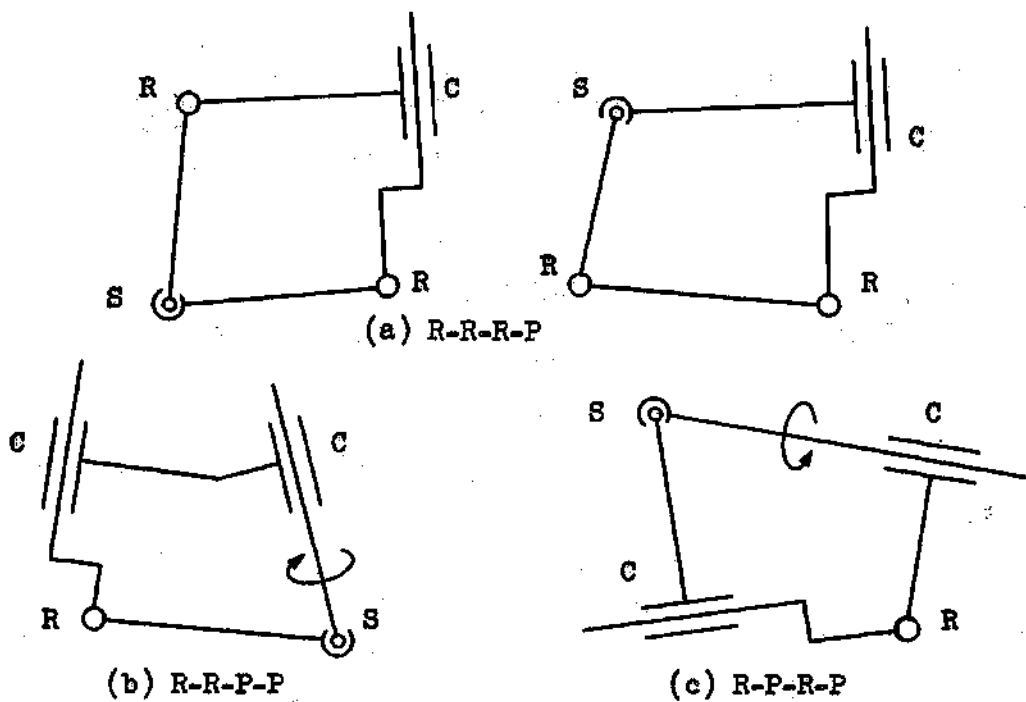


Figure 11. Self-aligning Equivalents to the RP-type Planar Four-bar Chains

The two four-bar planar chains having two P pairs each present a different problem from that just covered. The two simultaneous equations take a different form. In order to obtain a positive integer solution, it is necessary to allow a spin to be present, i.e., to let $F = 2$. The two equations are then

$$5p_1 + 4p_2 + 3p_3 = 16,$$

$$p_1 + p_2 + p_3 = 4,$$

where $p_2 = 2$. The solution is $p_1 = p_3 = 1$. Just as for the R-type chains, the extra mobility has been added here with the idea of allowing a spin. It is readily apparent, however, that there are not two S pairs present. Hence, any spin must occur on a member between a C and an S. This is practical only if the S is placed on the C axis; otherwise, the spin would have a very disconcerting effect on the motion of the chain. The chains derived from the R-R-P-P and R-P-R-P planar chains are shown in Figs. 11b and 11c, respectively.

Note that if the two C pairs in (c) were to come into a position such that they were parallel, then two binaries could translate at random with respect to the other two. This type of action, like the folds discussed earlier, is not to be allowed in any case. Some means must be employed by the designer to assure that such a translation cannot occur. Similarly, the S in both (b) and (c) must never lie on the axis of both C pairs simultaneously.

Six-bar. The six-bar planar RP-type chains each possess one, two, three, or four P pairs as shown in Figs. 27 and 28 in Appendix B.

In the process of self-alignment, the P pairs are all replaced by C pairs. The equivalent self-aligning chains may have R, C, and S pairs, i.e., p_1 , p_2 , and p_3 ; therefore, the mobility criterion, Eq. (4), becomes

$$F = 6(6 - 1) - 5p_1 - 4p_2 - 3p_3,$$

$$5p_1 + 4p_2 + 3p_3 = 30 - F.$$

The second of the usual two simultaneous equations is

$$p_1 + p_2 + p_3 = 7.$$

Unless a spin must be allowed in order to obtain a solution to these equations, the mobility is $F = 1$. Allowing one spin, it is $F = 2$.

The two equations for $F = 1$ are

$$5p_1 + 4p_2 + 3p_3 = 29,$$

$$p_1 + p_2 + p_3 = 7;$$

and for $F = 2$, they are

$$5p_1 + 4p_2 + 3p_3 = 28,$$

$$p_1 + p_2 + p_3 = 7.$$

Referring to the discussion presented earlier concerning multiple-loop chains, each loop of the six-bar chains here must receive at least three new freedoms in the self-alignment process. This fact

is helpful in distributing pairs throughout the chains.

For the planar chains containing only one P pair, the $F = 1$ equations above do not give a positive integer solution. It is necessary to introduce a spin. The applicable equations then are

$$5p_1 + 3p_3 = 24,$$

$$p_1 + p_3 = 6$$

since it is already known that $p_2 = 1$. The solution is $p_1 = p_3 = 3$.

For the case of two P pairs existing in the planar chain, the equations are ($F = 1$)

$$5p_1 + 3p_3 = 21,$$

$$p_1 + p_3 = 5.$$

The corresponding solution is $p_1 = 3, p_2 = p_3 = 2$.

When three P pairs belong to the planar chain, the resulting equations are

$$5p_1 + 3p_3 = 16,$$

$$p_1 + p_3 = 4,$$

where the mobility has been taken to be $F = 2$. The solution to these equations is $p_1 = p_3 = 2, p_2 = 3$.

If there are four P pairs in the planar chain, it is necessary to solve simultaneously the equations for $F = 1$ in the form

$$5p_1 + 3p_3 = 13,$$

$$p_1 + p_3 = 3.$$

That solution is $p_1 = 2$, $p_2 = 4$, $p_3 = 1$.

As has been seen in the four-bar development, there are more obstacles to self-alignment of RP-type chains than there are to self-alignment of R-type chains. One of the obstacles, of course, is the binary spin between two S pairs. Another is the C-S axial spin of a binary member as described just above in the four-bar development. These two conditions may be useful at times, but it must be remembered that they can be allowed to be present only if they have been taken into account in the mobility relation.

Three other complications are never acceptable. They all involve two binaries in series between one of the following combinations:

1. two S pairs,
2. one S and one C,
3. two C pairs.

The first allows the linkage to fold in the manner described earlier. The second produces another type of fold. The last condition listed represents a translation of the two binaries with respect to the remainder of the chain. In addition, there are higher-order conditions to be avoided, e.g., the alignment of three S pairs across a chain causing an even more serious fold than those already mentioned. They are too involved and too many in number to be summed up nearly as simply as has been the case for even the eight-bar R-type chains.

In Appendix B, there is shown a total of 50 RP-type planar six-bar chains, compared to the two R-type six-bar chains. It has been established that three kinds of pairs may be present in self-aligning RP-type chains--namely, the R, C, and S. Consequently, it is obvious that a complete catalogue of the RP-type self-aligning chains would be quite large. Moreover, the lack of a general method of avoiding the problems discussed in the previous paragraph makes a complete catalogue very difficult to obtain. It must suffice to show only the few examples of Fig. 12. Any number of self-aligning chains can be developed by the designer from the four sets of equations and their solutions above, providing he is careful to stay within the guidelines discussed in this section.

More than Six Bars. It is possible, of course, to apply the self-alignment principle to RP-type constrained chains with more than six links. The sketchy development of the six-bars, however, suggests that one might meet even greater obstacles in larger chains. As concluded for the six-bars, it would be necessary to consider one particular mechanism in order to arrive at a flawless solution.

Any self-aligning RP-type chain shown here can be made into several different linkages by inversion. The fixed link must not be a link which is free to spin.

Linkages of Mobility $F > 1$

Development of self-aligning linkages in this chapter has so far been limited to linkages with $F = 1$. Consider now briefly the procedures involved in the development of self-aligning planar linkages with $F > 1$. The procedures are similar to those already used for

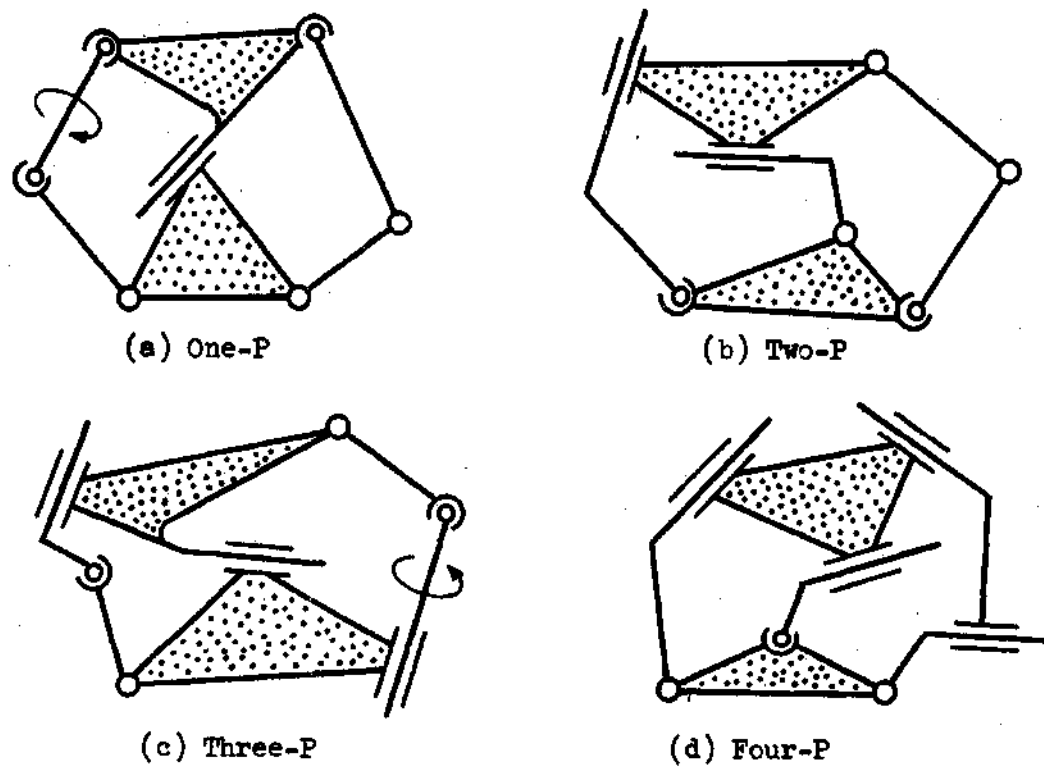


Figure 12. Examples of Self-aligning Equivalents to the RP-type Planar Six-bar Chains

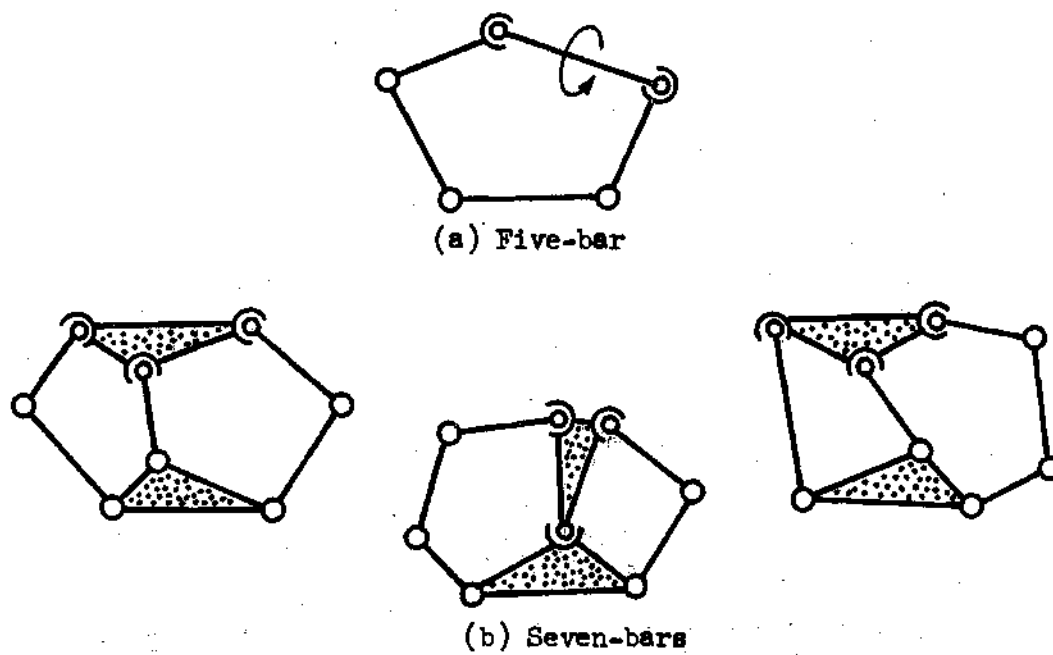


Figure 13. Examples of Self-aligning Equivalents to $F = 2$ Planar Chains

the $F = 1$ linkages. The only difference is that here the mobility equation contains a larger value of F . Whenever a spin must be allowed, the mobility is increased by one just as before.

R-types Linkages. In the case of R-type chains, it is necessary to add enough S pairs (replacing R pairs) so that the general mobility criterion is satisfied, and so that each loop gains at least three freedoms. Let the five-bar $F = 2$ chain (see Fig. 29 in Appendix B) be investigated first regarding self-alignment. The mobility equation reduces to

$$5p_1 + 3p_3 = 22,$$

while the number of pairs present is

$$p_1 + p_3 = 5.$$

There is no positive integer solution; therefore, the first equation becomes, for $F = 3$ (including a spin),

$$5p_1 + 3p_3 = 21,$$

with the second equation remaining the same. The solution is $p_1 = 3$, $p_3 = 2$. Only one self-aligning five-bar exists--the one shown in Fig. 13a. Note the similarity to the $F = 1$ four-bar of Fig. 4a.

Seven-bar $F = 2$ chains (see Fig. 30) are also easily made self-aligning. The appropriate equations are

$$5p_1 + 3p_3 = 34,$$

$$p_1 + p_3 = 8.$$

These equations have the solution $p_1 = 5$, $p_3 = 3$. Using the arguments presented for the "preferred" $F = 1$ six-bars, the three self-aligning seven-bars are as shown in Fig. 13b. Recall that the preferred six-bars are those with all three S pairs on one ternary and with that ternary constructed so that the three S pairs are not on a line.

Naturally, larger chains can be made self-aligning, and larger mobilities handled, in the same way as have been the five- and seven-bar chains here.

RP-type Linkages. Considering the difficulties involved in developing self-aligning RP-type chains of mobility $F = 1$ in a previous section, it is certain that similar difficulties would be present here with the chains of multiple mobility. No attempt is made to handle any chains of this sort.

A different viewpoint may be helpful in self-aligning multiple mobility chains of both the R and the RP types. If the mobility of a planar chain is $F_0 > 1$, one can reduce the chain to an $F = 1$ chain by shrinking $(F_0 - 1)$ binary links to zero length, provided each binary to be shrunk is connected to its neighbors only by R pairs. Reversing the process, the expansion of an R pair in a planar chain into two R pairs joined by a binary link produces a chain with one mobility more than that of the original chain. This phenomenon also holds true for self-aligning planar chains as is made clear by a comparison of the $F = 1$ self-aligning chains of Figs. 4a, 5a, 6a with the $F = 2$ chains

of Fig. 13. Observe that these $F = 2$ chains each differ from at least one of the $F = 1$ chains by one binary and one R pair. Similar steps could be taken to form self-aligning chains of any mobility.

This chapter has been concerned with the self-alignment of planar ($m = 3$) linkages. Lower-pair kinematic chains have been used in the illustrations, exclusively. Any one of the chains shown could be made into a linkage by merely fixing one link which is not being allowed to "spin". Chapter VII discusses the application of these developments to design problems.

CHAPTER V

SELF-ALIGNING SUPPORTS FOR ROTATING SHAFTS

Rotating Shafts

Two bodies connected by one revolute pair form one of the simplest (open) kinematic chains. If, however, the bodies are joined by two R pairs, the chain becomes closed. This closed, one-loop chain is obviously immobile unless the two R pairs are coaxial, a very special geometrical relationship signifying the presence of general constraints. As an illustration, the two bodies might be a household door and its door jamb, the two being joined by two hinges. It is most readily considered, however, as a rotating shaft on two bearings, a quite common mechanism in practice (see Fig. 2 in Chapter I). Just as there are two-bearing shafts, there are also shafts with three, four, and more, bearings. In this chapter, the principle of self-alignment will be applied to such multi-bearing shafts in order to produce self-aligning equivalent mechanisms in which the only intended mobility is a rotation of one member relative to the other.

Table 2 (see Chapter II) shows that rotating shafts on multiple R bearings possess five general constraints ($m = 5$). Each bearing is, in fact, a portion of one extended revolute pair, e.g., consider a piano hinge. These five general constraints are to be removed in the ensuing development.

It is by coincidence and chance if a real mechanism meets

exactly restrictions imposed by general constraints. Designers have attempted to remedy this problem in rotating shafts in several ways, among them (1) by specifying larger clearances in journal bearings than are conducive to long life and well-constrained motion, and (2) by substituting C, S, and S_g pairs for the R pairs acting as bearings. These attempts have not always been successful. There is a need to analyze the problem and to apply the principle of self-alignment in order to determine the various combinations of commercially available pairs which properly eliminate the general constraints when shafts are mounted on two, three, and four bearings.

Self-aligning Rotating Shafts

Eligible Pairs

As discussed in Chapter III, not all pairs are necessarily well-suited to replace other pairs in the development of self-aligning mechanisms. From that discussion, the eligible pairs here are derived from the R pair by the addition of any of the five remaining freedoms. Some further restrictions may be made due to practical considerations.

A bearing of a self-aligning rotating shaft might, therefore, remain an R pair, or be a T, C, S, or S_g derived from the R. If an R pair, or a C pair, acts as a bearing in a self-aligning rotating shaft mechanism, no relaxation is obtained in the chief problem, that of making the rotational axes of the bearings coincide. It would be impossible, without general constraints, for any other pair to join the frame to some other point on the shaft if an R or a C were to be one of the shaft bearings. In other words, the use of an R or a C inevitably causes overconstraint in the mechanism.

The commercially available versions of the T pair are not particularly suited for use as bearings since the rotations of the usual T are not designed to be continuous. In contrast, an S pair may serve very well as a bearing; the only stipulation needed is that its center lie on the rotational axis of the shaft.

Basic to the present development is the premise that a shaft bearing should allow no relative translation between its two elements perpendicular to the rotational axis of the shaft. This premise allows a rotating shaft mechanism to be oriented independently of the force field present, whether gravitational, magnetic, or whatever. The S_g pair may therefore be used here if it is oriented with its translational direction parallel to the rotational axis of the shaft. Its center must lie on the shaft axis.

It is concluded that the pairs eligible to serve as bearings in a self-aligning multiple-bearing rotating shaft are limited to two--the S and the S_g . Both supply the rotational freedom of the shaft about its axis and also prevent translations of the shaft normal to that axis.

Two-bearing Shafts

It is possible to develop self-aligning two-bearing rotating shaft mechanisms which satisfy the general spatial mobility criterion as expressed by Eq. (4). For the equivalent self-aligning mechanism containing only S and S_g pairs, the mobility is $F = 1$; the number of members is $n = 2$; and $p_1 = p_2 = p_5 = 0$ since no one, two, or five degree-of-freedom pairs are eligible. The mobility criterion takes the form

$$F = 6(2 - 1) - 3p_3 - 2p_4 = 1,$$

$$3p_3 + 2p_4 = 5.$$

In addition, the total number of pairs in the mechanism must be

$$p_3 + p_4 = 2.$$

The only solution to the above two equations is $p_3 = p_4 = 1$. The bearings of a two-bearing self-aligning rotating shaft must thus be one S pair and one S_g pair, as illustrated in Fig. 14. This particular shaft design is commonly in use.

Three-bearing Shafts

It is often desirable, or even necessary, to support a rotating shaft on more than two bearings. In case of three bearings, the simultaneous equations to be solved are

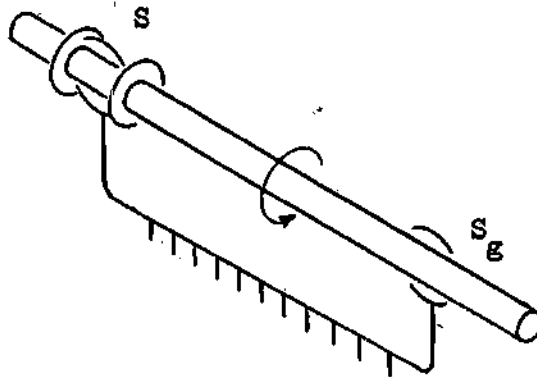


Figure 14. Self-aligning Rotating Shaft on Two Bearings

$$3p_3 + 2p_4 = 5,$$

$$p_3 + p_4 = 3.$$

But a positive integer solution does not exist. From Eq. (4), it is obvious that if a p_5 pair, or pairs, were allowed in this mechanism, there might be a solution to the two resultant equations which would each differ from the last two above by a p_5 term on the left-hand side. No such pair could serve as a bearing, however, even if it were available, because it could not prevent both translations perpendicular to the shaft axis.

It becomes desirable to employ what are called "compound pairs" in order to develop self-aligning three-bearing shaft designs. Let a compound pair consist of a binary member and two pairs, one of them joining to the shaft and one to the frame. If this is inserted in place of one bearing, it can clearly offer no more freedoms than the S_g pair and still prevent all translations normal to the shaft axis, and therefore it offers no improvement. Compound pairs can also take the form of a ternary member and three pairs, as, for example, when two of the three shaft bearings connect the shaft to an intermediate ternary which is in turn connected to the frame by means of its third pair. The mechanism is in this way converted to one having three members ($n = 3$) and four pairs.

Some restrictions must be placed on the selection of the pairs which can be used in the various locations in such a mechanism. The three shaft bearings must still be one or the other of the two eligible

forms--S and S_g . The bearings which connect the shaft to the intermediate ternary could be (1) as determined earlier for the two-bearing shafts, i.e., an S and S_g , or (2) both S_g pairs, relying on the third shaft bearing to restrict axial translation of the shaft. This latter case requires the third bearing to be an S, while the former case allows it to be either an S or an S_g . The only restriction concerning the pair connecting the intermediate ternary to the frame is that it must not allow translations normal to the shaft axis since these translations would be passed on to the bearings of the shaft.

The equations for the three-bearing shaft with a ternary intermediate member are then

$$5p_1 + 4p_2 + 3p_3 + 2p_4 = 11,$$

$$p_1 + p_2 + p_3 + p_4 = 4.$$

In the preceding paragraph, it has been pointed out that at least one bearing must be an S_g and at least one must be an S, with the third being either an S or an S_g . This can be put into mathematical terms as

$$p_3 \geq 1, p_4 \geq 1, p_3 + p_4 \geq 3.$$

The solution to these equations and conditions are as follows:

<u>Solution</u>	<u>p_1</u>	<u>p_2</u>	<u>p_3</u>	<u>p_4</u>
A	0	1	1	2
B	0	0	3	1

Since a bearing must be either an S or an S_g , the pair between the ternary and the frame in A is a p_2 , i.e., either a T or a C. These two forms are shown in Figs. 15a and 15b. The ternary-frame pair in B must be a p_3 (S) pair as shown in Fig. 15c.

It should be noted that, in (a) and in (b), the S pair could be located at any of the bearings, but it is shown in probably the most

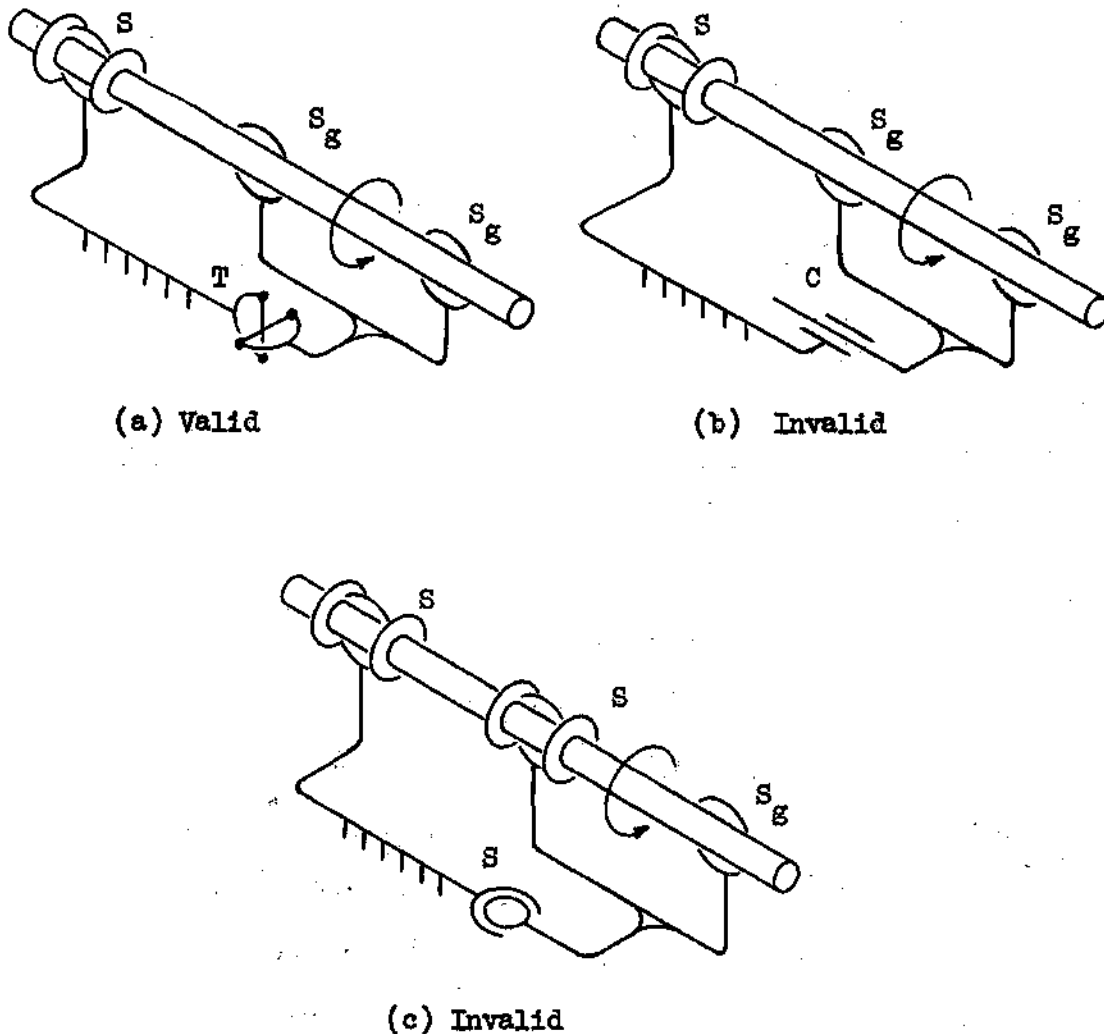


Figure 15. Self-aligning Rotating Shafts on Three Bearings

practical place as the bearing joined directly to the frame. Another point is that the two bearings on the ternary link need not be adjacent to one another with respect to the shaft; i.e., the third bearing could be between them, but this would be an unlikely configuration.

Two of these three designs are however impractical. The arrangement of (b) is invalid because the C pair allows a translation parallel to the shaft axis. As shown in the figure, the intermediate ternary is not prevented from sliding off the shaft and thus disconnecting the mechanism. Simultaneously, it does not provide for adjustment rotation about the vertical axis. The shaft design of (c) is likewise not acceptable since the entire mechanism can fold about the axis through the centers of the two S pairs on the frame.

The mechanism of Fig. 15a, however, is considered to be the valid design since it does not display obvious drawbacks.

Four-bearing Shafts

Carrying the present development one step further, a four-bearing shaft mechanism is now considered. The two equations to be solved simultaneously, assuming no compound pairs, are

$$3p_3 + 2p_4 = 5,$$

$$p_3 + p_4 = 4.$$

But again, there is no positive integer solution. If one compound pair is added as has sufficed for the three-bearing problem, still no solution exists. Two such compound pairs must be added, producing the equations

$$5P_1 + 4P_2 + 3P_3 + 2P_4 = 17,$$

$$P_1 + P_2 + P_3 + P_4 = 6.$$

For reasons similar to those used for the three-bearing shafts, it is also necessary that

$$P_3 \geq 1, P_4 \geq 2, P_3 + P_4 \geq 4.$$

Three solutions exist to this set of two equations and three inequality conditions. They are as follows:

<u>Solution</u>	<u>P₁</u>	<u>P₂</u>	<u>P₃</u>	<u>P₄</u>
A	0	1	3	2
B	0	2	1	3
C	1	0	2	3

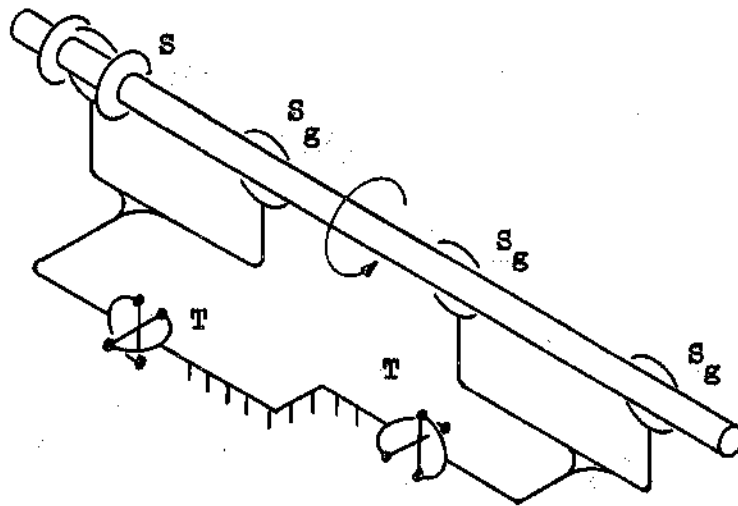


Figure 16. Self-aligning Rotating Shaft on Four Bearings

In solution A, one of the ternary intermediate members is joined to the frame with an S pair, while the other is joined with either a T or a C. Solution B requires the ternaries to be paired to the frame with two T pairs, or two C pairs, or one of each. One of the pairs is an R, a P, or an H in solution C, while the others can be S or S_g pairs. By essentially the same arguments as presented for the three-bearing shaft, the only valid design here is that of B with two T pairs and no C pairs. This design is shown in Fig. 16. Note the similarity to the valid three-bearing design of Fig. 15a.

The self-aligning rotating shaft mechanisms have thus been developed, pictured schematically, and discussed. Those with three or four bearings may appear somewhat impractical, but in Chapter VII some practical and useful variations of these mechanisms are discussed for the designer.

CHAPTER VI

SELF-ALIGNING SUPPORTS FOR TRANSLATING TABLES

Translating Tables

A common design problem is that of joining two bodies so that they may move relative to one another with only one translational freedom and with no rotational freedom. Examples of such one-translation machines are an overhead crane and its rails, and the table and bed of a planing machine.

If the two members are joined by only one pair, necessarily a P pair, the kinematic chain is open and thus does not require special geometrical relationships (general constraints) in order for the desired motion to take place. General constraints are present, however, in a mobile chain of this type if the two members are joined by two, or more, P pairs.

If one member of a chain of the type described above is fixed to the frame of reference, a mechanism is formed. It is convenient to label such a mechanism a "translating table" just as the typical one-rotation mechanism has been labeled a rotating shaft in Chapter V (see Fig. 3 in Chapter I).

Normally, P pairs in practice are effected as dovetail slides, splines, tongue slides, etc. In each of these, the surfaces of the two members are ideally in contact over their entire common areas, allowing a uniform finite clearance. This is a practical impossibility since it would require perfectly planar surfaces arranged in an exact

manner. However, it is often necessary to produce a very precisely linear motion, such as in a planing machine; and in order to do so, high penalties must be paid both in the form of money and of time to produce by conventional methods the highly precise mating pair required. Even if the sliding pair can be made quite precisely, there are so many redundant constraints present that it is difficult to predict exactly what motion will take place as the slider is actuated and is confronted with the irregularities which are unavoidably present.

It is therefore obviously advantageous to use something other than P pairs in the design of one-translation mechanisms, especially large ones.

Translating tables on multiple P bearings, as stated above, possess general constraints; namely, five in number according to Table 2. Multiple-bearing translating table mechanisms of the two-, three-, and four-bearing types are now treated with the principle of self-alignment.

Self-aligning Translating Tables

Eligible Pairs

Certain pairs can be eliminated from consideration as bearings in translating table mechanisms. Any pair which does not allow an independent translational freedom is naturally ineligible. This eliminates the R, H, T, and S pairs as translational bearings but does not exclude them from occurring in compound pairs, however, if compound pairs are required at some time during the developments. The eligible pairs would seem then to be the remaining three commercially available pairs (P, C, and S_g), especially as they all satisfy the general rule established in Chapter III.

The P can be eliminated from consideration as a bearing in a multiple-bearing table mechanism, however, except as it may occur in a compound pair since a P acting as a bearing would determine in itself the plane and line of motion of the table. No other pair could join the table and frame without introducing general constraints. Further, the P pair is not allowed to be a translational bearing as a part of a compound pair since the prime purpose of these developments is to eliminate the need for such bearings. The P may occur here only as an "adjustment" pair in a compound pair.

Only the C and S_g pairs, then, are eligible to serve as translational bearings in the development of self-aligning tables.

Two-bearing Tables

The general spatial motility criterion, Eq. (4), subject to the mobility of a rotating shaft, $F = 1$, and with two members and two pairs, takes the form

$$4p_2 + 2p_4 = 5.$$

Only p_2 and p_4 pairs are eligible to serve as bearings; therefore,

$$p_2 + p_4 = 2.$$

No positive integer solution exists to these two equations.

It is therefore necessary, even in this simplest two-bearing case, to introduce a compound pair. Adding one pair and one member, the two equations take the form

$$5p_1 + 4p_2 + 3p_3 + 2p_4 = 11,$$

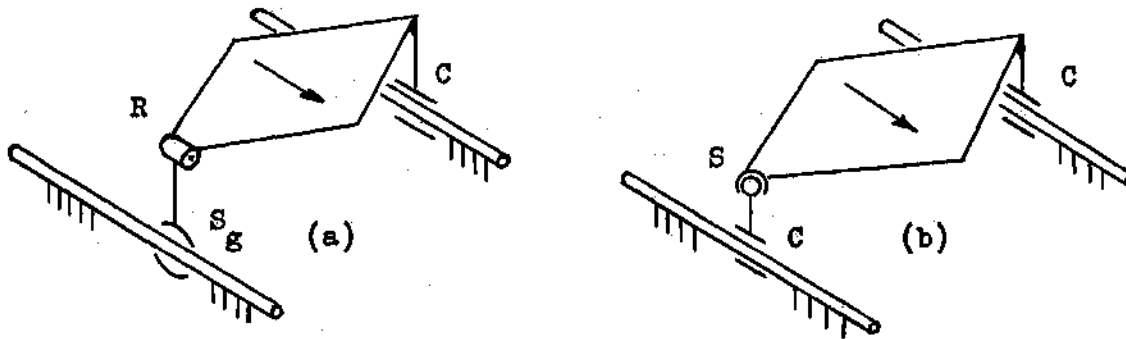


Figure 17. Self-aligning Translating Tables on Two Supports

$$P_1 + P_2 + P_3 + P_4 = 3$$

since any class of pair may exist in the compound pair. The two pairs serving as bearings must be either C or S_g pairs; consequently, the inequality

$$P_2 + P_4 \geq 2.$$

The only solutions to the two equations and one inequality condition are:

Solution	P_1	P_2	P_3	P_4
A	1	1	0	1
B	0	2	1	0

In A, the bearings are one C and one S_g , while the pair joining the intermediate member to the table is an R, a P, or an H. The last possibility, the H, is very unlikely for obvious reasons. The P is somewhat more likely, its translation being normal to the intended translation of the table and normal to the load force being exerted

on the table. The most likely configuration, using an R, is shown in Fig. 17a. Note that the R axis is roughly parallel to the intended translation of the table. The C and S_g pairs could be interchanged in the figure without causing difficulties.

Solution B requires the two bearings to be C pairs and the third pair to be an S. This configuration is shown in Fig. 17b.

Note that both valid two-bearing designs require two separate bearing surfaces, i.e., the rods shown in Fig. 17. If the rods were made to coincide, the table would tend to spin on the rod.

Three-bearing Tables

Supporting a table on three bearings seems to be most logical, much the same as a three-legged stool is the most stable configuration for a stool.

The simultaneous equations, without compound pairs being used, are

$$4p_2 + 2p_4 = 5,$$

$$p_2 + p_4 = 3.$$

Again, no positive integer solution exists.

Adding one member and one pair to form a compound pair with one of the pairs already present, the equations take the form

$$5p_1 + 4p_2 + 3p_3 + 2p_4 = 11,$$

$$p_1 + p_2 + p_3 + p_4 = 4.$$

The condition

$$p_2 + p_4 \geq 3$$

is also applicable since the three bearings should be C or S_g pairs. Two solutions to the two equations and one condition exist, and are as follows:

<u>Solution</u>	<u>p_1</u>	<u>p_2</u>	<u>p_3</u>	<u>p_4</u>
A	1	0	0	3
B	0	1	1	2

One form of solution A is shown in Fig. 18a. The R pair could be placed at any corner of the triangular table shown with like effect, provided that its axis is not normal to the intended translation of the table. An H or a P could also be used in place of the R, but would generally be less desirable.

Solution B is illustrated in Fig. 18b. The S pair must be used in conjunction with the C pair, rather than an S_g , since an S- S_g compound pair would have an intermediate member capable of spinning. That mobility would require general constraint to be present elsewhere in the mechanism. The C-S compound pair could be placed at any corner of the table with like results.

Note that each of the three-bearing tables has two S_g pairs and a five-freedom compound pair. Observe also that both tables shown in Fig. 18 have two bearing surfaces, the rods pointing in the intended translational direction. There could just as well be three rods in

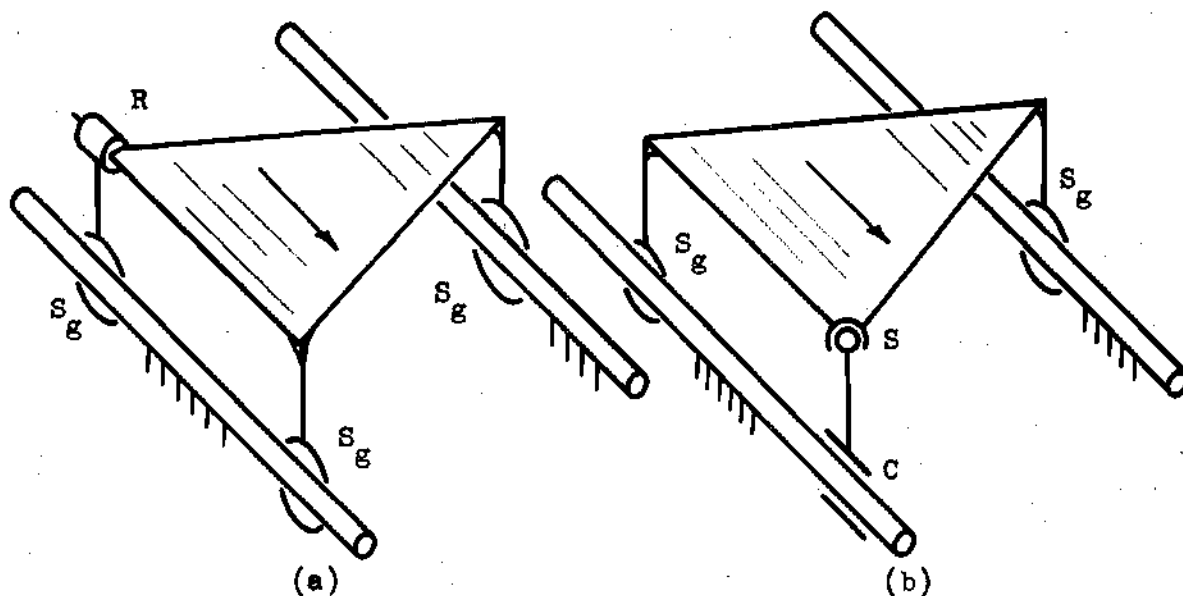


Figure 18. Self-aligning Translating Tables on Three Supports

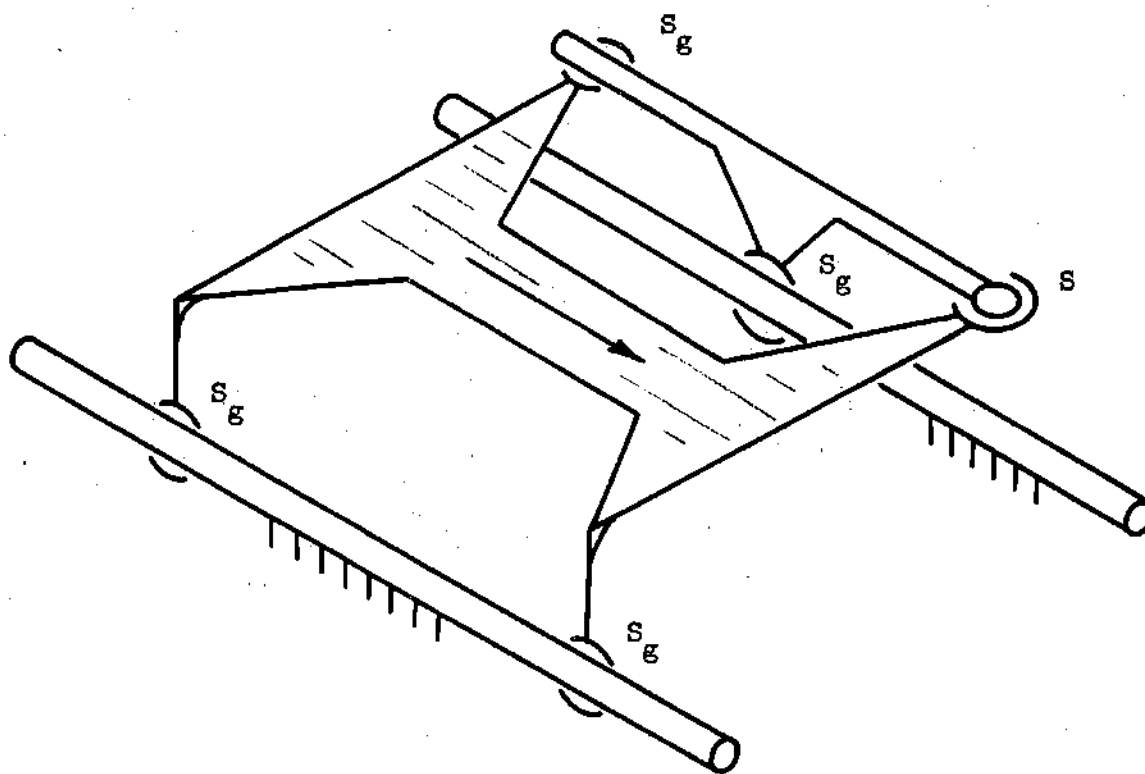


Figure 19. Self-aligning Translating Table on Four Supports

each mechanism if for some reason that were preferred.

Four-bearing Tables

It may be desired to provide support to a translating table at four points, e.g., at each corner of a rectangular table. Without a compound pair, the two simultaneous equations are

$$4p_2 + 2p_4 = 5,$$

$$p_2 + p_4 = 4.$$

There is no positive integer solution.

If a link and a pair are added to make one bearing compound, as has been done for the three-bearing tables, the resulting equations are

$$5p_1 + 4p_2 + 3p_3 + 2p_4 = 11,$$

$$p_1 + p_2 + p_3 + p_4 = 5.$$

The appropriate inequality, in view of the fact that all bearings must be either C or S_g pairs, is

$$p_2 + p_4 \geq 4.$$

Only one solution exists for these two equations and the inequality: $p_1 = p_2 = 0$, $p_3 = 1$, $p_4 = 4$. This implies that there is a compound pair with seven freedoms, there being an S on one end of the intermediate member and an S_g on the other end. That member would be capable of spinning and thus another solution is necessary.

Considering the other kind of compound pair produces a valid

result. Connecting the table to an intermediate member at two pairs and then joining the intermediate member to the frame involves the last two equations above, while the constraint is changed to

$$p_2 + p_4 \geq 3.$$

The solution is again the same— $p_1 = p_2 = 0$, $p_3 = 1$, $p_4 = 4$. Such an arrangement is shown in Fig. 19. The intermediate member can join any two corners of the table, and there can be three bearing rods instead of the two shown.

Note that the term "four supports" is actually a misnomer here. There are only three translational bearings in the mechanism. Another possibility exists in which there are four supports. This is the case if two corners of the table are connected to the frame each through its own compound pair. Such a mechanism, however, contains a general constraint and must depend on flexure of the table if all supports are to carry a part of the load.

It is suggested that a table can be supported at any number of places by simply cascading intermediate members as has been done here in going from the mechanism of Fig. 18a to that of Fig. 19. The three-bearing table is the basically stable one as related earlier in analogy with the three-legged stool.

Chapter VII presents some discussion about the development of this chapter especially important to the designer.

CHAPTER VII

USE OF RESULTS FOR DESIGN PURPOSES

In order to condense the previous chapters into the form most useful to the designer, this chapter presents a brief synopsis of the purposes of this thesis and the principles involved, proposes applications for the results, and suggests extensions to the work presented earlier.

Reflection on Basics

The primary purpose of this thesis is to assist the designer in more effective application of the concepts of degrees of freedom and degrees of constraint in the design of mechanisms, without which designs are more susceptible to wear and inaccuracies than otherwise. The principle of kinematic design is the primary basis. It essentially involves the avoidance of special geometrical relationships (general constraints) by giving each member of a mechanism the theoretically minimum number of constraints consistent with the motion desired. Strictly, kinematic design imposes constraints only at point contacts. This impractical property is replaced here by the use of commercially available pairs at joints, thereby allowing mechanisms to be handled regardless of their load sizes. Point contacts limit loading very severely even though they must in reality be small finite areas. Mechanisms designed by the principle of kinematic design, but with commercially available pairs instead of point contacts, are termed

"self-aligning."

There are several distinct advantages to kinematic design as opposed to conventional design, all of which are also held by self-aligning mechanisms. First, elastic strains in the links are minimized. Many of the close tolerances needed in conventional design are eliminated; small inaccuracies in manufacture do not affect smooth motion of the links, providing the pairs used are adequately free-moving, a condition which is the responsibility of the pair manufacturer. Wear is more even in self-aligning mechanisms, and there is less wear since forces are inherently smaller. The forces throughout a self-aligning mechanism, and the locations of its members, may be determined or predicted accurately in any position of the mechanism, disregarding elastic and plastic deformations. One of the most important advantages is that a self-aligning mechanism does not bind even after plastic straining of its links. This is particularly advantageous in mechanisms which are subjected irregular impact loads.

Three main problems have been attacked in this thesis--the self-alignment of planar linkages, multiple-bearing rotating shafts (as the typical one-rotation mechanisms), and multiple-support translating tables (as the typical one-translation mechanisms). These three problems, covered in Chapters IV, V, and VI, respectively, are three of the most common in practice and often present problems due to their needs for special geometrical relationships.

Assumptions have necessarily been made in the development of self-aligning mechanisms. In the interest of practicality, the pairs allowed to be present in self-aligning mechanisms have been limited

to those which are commercially available (see Table 3). A rule has been established in Chapter III for the selection of those commercially available pairs eligible to appear in the various types of self-aligning mechanisms considered in this thesis. Each of the three chapters devoted to the solution of one of the main problems contains a discussion of the pairs eligible for use in that chapter. Every mechanism considered herein has been assumed to have link lengths adequate to allow assembly of the mechanism. It has also been stipulated that only those commercially available pairs which are available as form-closed are to be used. This precludes the need for force-closure and makes mechanisms more versatile in that changes of the force field do not tend to disassemble the mechanism. This might be important in space travel or in use on any moving vehicle.

Applications

As has been already stated, the work presented here is applicable to the design of a mechanism of any weight, from the lightest instrument to the heaviest construction equipment. The smooth, sure motion of self-aligning mechanisms makes them most attractive as high-precision machines whether light-, medium-, or heavy-duty. Self-alignment allows the component parts of such machines to be lightened to the minimum because of reduced stresses and assures accurate knowledge of the positions of the various parts.

A review of the most important results of the previous developments and a discussion of the applications of self-aligning mechanisms follows.

Planar Linkages

Chapter IV has developed the self-aligning planar linkages. The R-type $F = 1$ chains are shown in Figs. 4a, 5a, 6a, 7, 8, 9, and 10. Their possibilities for application are innumerable. Anywhere a planar linkage serves, the equivalent self-aligning linkage would serve better, although it might not be economically justifiable in all cases.

S pairs must be substituted for R pairs as described in Table 4 in the formation of self-aligning $F = 1$ R-type linkages. It is important that the S pairs be arranged so that in no position of the linkage is a folding or an undesirable spin present. This is most handily accomplished in all but the four-bar by placing a maximum number of S pairs on a polygonal link in such a way that they are definitely not collinear.

RP-type linkages present a much more difficult situation. For the purposes here, it has been decided to replace all P pairs by C pairs and then replace enough R pairs by S pairs so that all of the general constraints are removed.

One particular situation especially deserves comment. The four-bar R-R-R-P chain most often serves in the slider-crank inversion, e.g., in reciprocating engines and compressors. The self-aligning forms (see Fig. 11a) do not lend themselves well to such usage, especially if the mechanism is to perform at high speeds and/or with high forces involved. This is because the S must be located in a way that never allows it to lie on the C axis. This unsymmetrical configuration may produce inadmissible force and moment relations.

Many practical considerations are involved in the design of a

self-aligning linkage. One of these is that, in order to avoid interference, the links of a planar linkage must be in parallel planes and must either project from one another at an R pair as cantilevers or they must be forked, with two coaxial R bearings serving as an R pair. This affects the placement of S pairs. An S pair cannot be split in a forked fashion. Usually, if split joints are used in a mechanism, they are the joints which connect to the frame, e.g., the main bearings of a reciprocating engine crankshaft.

The advantages of self-alignment make it extremely attractive in the limited production of large linkages. For example, if one were to attempt to drill parallel holes at the extremities of large links into which pins would be inserted to form R pairs, the operation would require a large, expensive radial drill. This would be impractical if only a few pieces of that size were needed. Much less refined drilling would be required if self-aligning bearings were used in the manner described in Chapter IV. Similarly, self-alignment is well-suited for use in situations which are expected to require field maintenance and modification. Even bent parts can be used if a mechanism is self-aligning.

Rotating Shafts

Principally, self-alignment should be applied to rotating shafts where it is necessary to have smooth rotation and determinate bearing reaction forces. To help motivate the designer toward the use of self-aligning three- and four-bearing rotating shafts, several possible purposes for application can be suggested. The critical speeds of a shaft are generally increased by the addition of a bearing. The larger

the number of bearings, the smaller are the deflections of the shaft, lessening the chances of interference. Bearing loads and shaft shear forces and bending moments are all decreased; this is most important when loads are large. If two separately designed shafts are required to be rigidly joined together, they may be supported effectively as a self-aligning multiple-bearing shaft.

These possible reasons for applications are not always what they may seem, however. For example, the critical speed of a particular self-aligning shaft may be raised by increasing the number of bearings, but the overall dynamic properties of the mechanism may be worsened since the intermediate member introduced becomes a part of the oscillating system.

The two-bearing self-aligning rotating shaft of Fig. 14 is quite often used in practice, but no present use is known of the self-aligning three- and four-bearing models of Figs. 15a and 16, respectively. Perhaps a more realistic drawing of a three-bearing shaft (see Fig. 20) may give an insight into possible applications. Figure 20 also suggests the appearance of the four-bearing shaft since the two are quite similar. Note that the T pair has been transformed into two R pairs, roughly intersecting each other and the shaft axis. The one R has been separated into two sections (one section has been removed in the figure to reveal the remainder of the mechanism) and would best take the form of a two-bearing shaft itself, pivoting on an S and S_g . The small rotation involved, however, and the relative stiffness of the intermediate members would probably allow the use of an R and a C as shown.

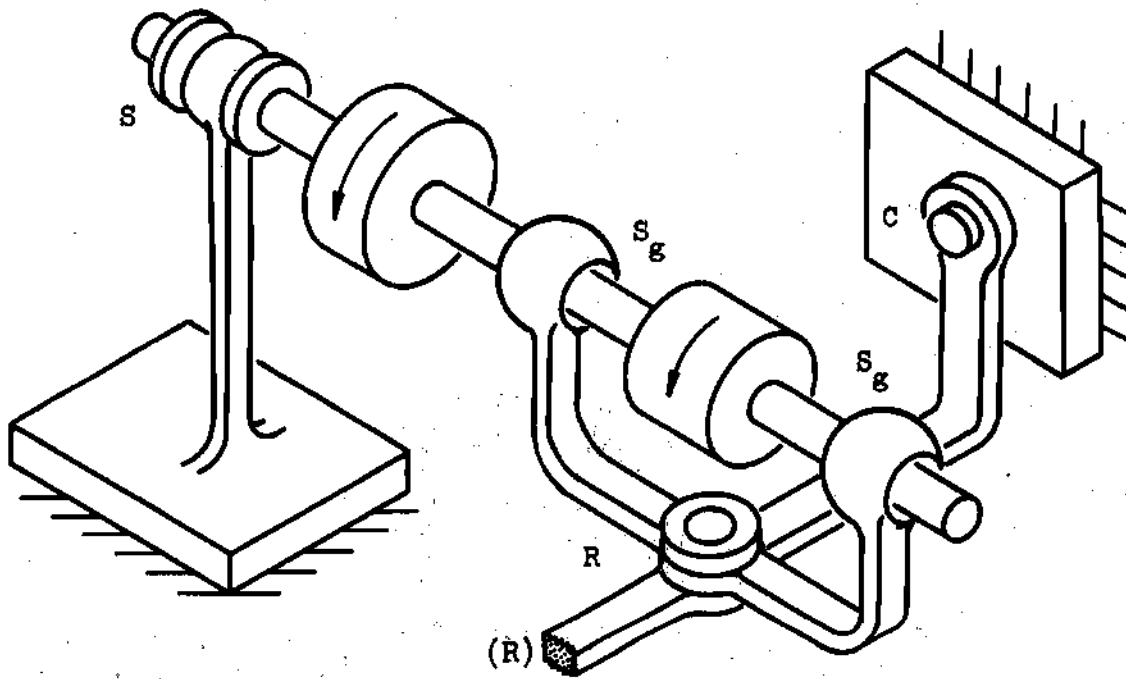


Figure 20. Pictorial View of a Self-aligning Rotating Shaft on Three Bearings

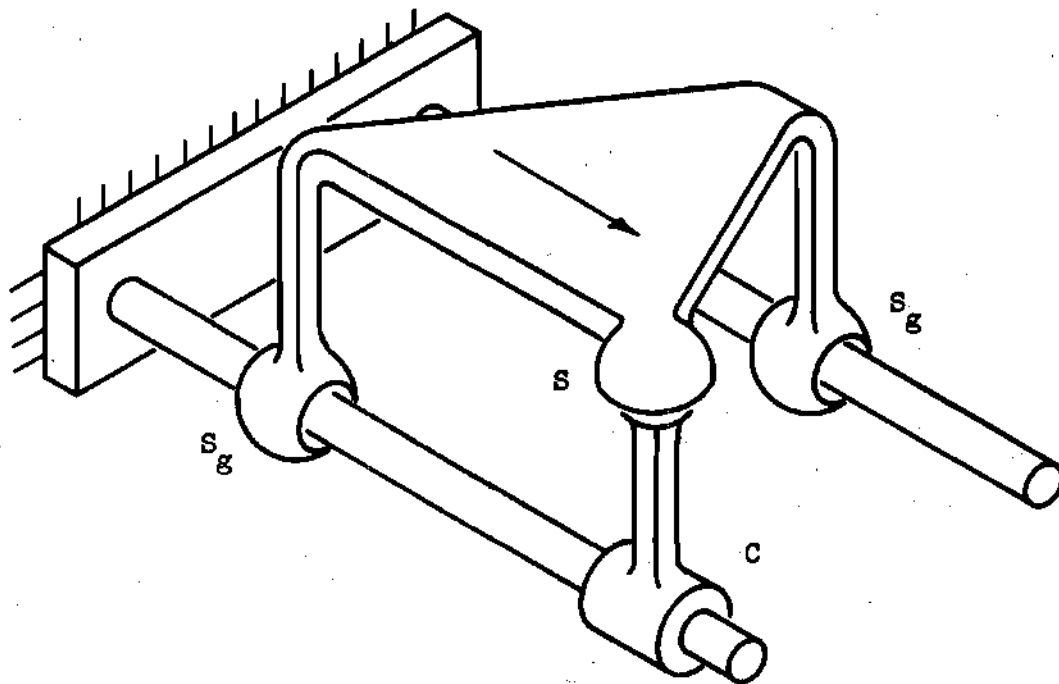


Figure 21. Pictorial View of a Self-aligning Translating Table on Three Supports

An interesting arrangement for an S_g connecting a shaft to the frame exists in the spherically seated pillow block on "ladder" (translational) bearings available from at least one manufacturer.

Translating Tables

Self-alignment of translating tables is perhaps more strongly motivated than that of rotating shafts. Large prism pairs, e.g., dovetail slides and tongue slides, tend to be either extremely expensive and heavy, or inaccurate and high in friction. In addition, such pairs are very vulnerable to damage. For one thing, the lubrication needed in many such pairs attracts foreign debris which is harmful to the pair. The three-bearing tables of Fig. 18 are those most likely to find application. Figure 21 shows a more pictorial view of the mechanism of Fig. 18b.

Highly finished circular steel rods are available for use as the rails at reasonable cost. It would probably be economically feasible to simply replace the rods whenever they become distorted or worn. "Zero-clearance" low-friction translational bearings of both the C and the S_g varieties are also available to be paired with the steel rods.

One particular possibility for the tables is that the C pair of Fig. 21, for example, could be changed to a screw, or helix, H. In that way, the power to drive the table could be transmitted through the screw. The C-S member, then an H-S member, would preferably rest on its own threaded rod while the two S_g pairs would rest on one smooth rod together or on two separate rods.

A desirable condition in the use of self-aligning tables is

that the support rails should be parallel; otherwise, the table has a helical motion instead of pure translation. Non-parallel rails do not affect the smoothness of the motion, however. The rails may be easily attached to the frame of reference in such a way that they may be adjusted for parallelism.

Another problem is that self-aligning tables as proposed here may not be capable of possessing enough rigidity to perform satisfactorily in the presence of large impulsive forces. Such loading sometimes demands a conventional table design because of its larger mass.

It should be remembered that whatever has been developed for a rotating shaft or translating table also holds for any one-rotation or one-translation mechanism, respectively. An interesting concept here is that a rotating shaft on multiple bearings could actually be a single R pair in a larger mechanism. Similarly, a translating table could be a single P pair.

Proposed Extensions

Plentiful opportunities exist for extensions of the work presented here. Many can be imagined based on simply changing some of the basic assumptions used, e.g., by establishing that some other pair has been made available in addition to those listed as commercially available in Table 3.

The presence of the T pair could be allowed in self-aligning planar linkages if it were available in a form in which one of its rotations could be complete or if that were not considered to be a necessary ability. There is one particularly interesting catalogue of

the forms of commercially available linkage joints [20] which may be helpful in extending the self-alignment of linkages. Perhaps the C pair could be allowed in R-type linkages. It might be practical, too, for P pairs to be present in certain RP-type linkages.

The self-aligning planar linkages could be developed endlessly by considering chains with a larger number of links and/or with larger mobilities than those discussed here.

A minor extension might be the consideration of the inversions (one each) of the rotating shaft and translating table, i.e., where the shaft or table is fixed and the "frame" moves. The problems of coupled shafts and coupled tables could also be investigated.

Extensions might also reach into different families of mechanisms (see Table 2) or into other members of the family $m = 3$, e.g., spherical mechanisms. Most of these possibilities would be more difficult than the work presented here since the rule for eligible pairs as stated herein is meaningless unless the rotations present are about mutually perpendicular axes. Each sub-family of mechanisms would probably require a different rule for eligible pairs, this rule being one of the necessary bases of the self-aligning process.

There are countless other possibilities for the development of self-aligning designs. Only a few of the simplest design problems have been discussed here, but they are probably the most important and the most likely to find application.

APPENDIX A

PRECAUTIONS IN ADDING FREEDOMS TO PAIRS

In Chapter III it is mentioned that precautions must be taken in the addition of freedoms to pairs in the self-alignment process. The need for such precautions is perhaps best illustrated by a simple example concerning the familiar four-bar, slider-crank planar linkage, which contains three R pairs and one P (see Fig. 22). Being planar, it is of the family $m = 3$ (see Table 2). A mobility analysis of the planar mechanism by Eq. (3) yields

$$\begin{aligned} F &= 3(n - 1) - 2p_1 - p_2 \\ &= 3(3) - 2(4) - 0 = 1 \end{aligned}$$

since the number of links is $n = 4$, the number of one-freedom pairs is $p_1 = 4$, and the number of two-freedom pairs is $p_2 = 0$. Requiring the mobility given by Eq. (4) to be the same gives the compatibility

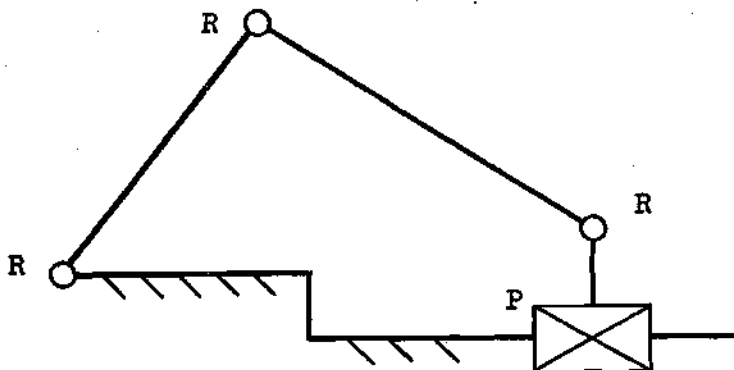


Figure 22. Planar Slider-crank Mechanism

relation for the freedoms of the pairs of the self-aligning mechanism.

Here again, $n = 4$, and

$$F = 1 = 6(3) - 5p_1 - 4p_2 - 3p_3 - 2p_4 - p_5,$$

which simplifies to

$$5p_1 + 4p_2 + 3p_3 + 2p_4 + p_5 = 17.$$

It is also apparent that there is a total of four pairs in the self-aligning mechanism, just as there is in the planar linkage, and thus

$$p_1 + p_2 + p_3 + p_4 + p_5 = 4.$$

All pairs in the self-aligning mechanism must be among the types listed in Table 3 as being commercially available, unless there is no solution to the problem using only those pairs in which case compound pairs must be resorted to. Since no pairs allowing five degrees of freedom are listed, $p_5 = 0$. There are then three solutions to the two equations above. The values of p_1, p_2, p_3, p_4 are, respectively, as follows: (1) 3, 0, 0, 1; (2) 2, 1, 1, 0; and (3) 1, 3, 0, 0. Compound pairs are therefore not required. Let the first solution of the equations be investigated, namely, $p_1 = 3, p_2 = p_3 = 0$, and $p_4 = 1$. This indicates that a possible self-aligning mechanism contains three p_1 pairs (each an R, a P, or an H) and one S_g pair (the only p_4 pair commercially available). Let the S_g pair be derived arbitrarily from the P pair of the original slider-crank, meaning that three rotational freedoms have been added to the original P pair. The direction of the translational

freedom of the S_g must be the same as that of the P if the S_g is to serve in the capacity of the P. The three p_1 pairs must then be R pairs since they are to have been derived from the original three R pairs, with the possibility of additional freedoms being allotted to each, but all of the added freedoms are being absorbed by the P pair as it transforms into the S_g .

The proposed self-aligning mechanism, then, consists of the four original links connected by the three original R pairs and an S_g pair, the last serving in the capacity of the former P pair. One of the principal purposes of the conversion of the mechanism to the self-aligning state is to avoid the need for special geometrical relationships between the pairs, e.g., allowing the R pair axes to be only roughly parallel here, not exactly parallel as they must be in a truly planar mechanism. It would certainly be self-defeating if, by chance or by design, the R axes were very nearly parallel, and consequently caused the mechanism to have a mobility different from that anticipated in the design. Such is the case with the R-R-R- S_g four-bar now under consideration. This is easily realized by comparing the original linkage with the proposed one. The former has one translational freedom (the P pair) and one rotational freedom about each of three parallel axes (the three R pairs), with the translation oriented perpendicular to the three R axes. Now, if the three R axes in the latter are parallel to one another and perpendicular to the translational freedom of the S_g , the four freedoms which exist in the former mechanism also are present in the latter; furthermore, the S_g pair has a rotational freedom about each of any three mutually perpendicular axes, and there-

fore, if one of these axes is chosen parallel to the axes of the three R pairs, the proposed self-aligning mechanism possesses two degrees of mobility--one more than desired or expected. The excess mobility is obviously an undesirable consequence of the particular way in which the three freedoms have been distributed among the pairs. Perhaps converting an R pair into the S_g , or using one of the other solution sets of the two equations, would provide satisfactory results in producing a self-aligning mechanism.

This example illustrates well the situation to be avoided in distributing the added freedoms among the pairs of a mechanism in order to make it self-aligning. A rule is stated in Chapter III to summarize the necessary precautions which must be taken as illustrated here.

APPENDIX B

CATALOGUE OF PLANAR KINEMATIC CHAINS

In connection with Chapter IV, the planar kinematic chains of both the R-type and the RP-type are partially catalogued here. Four-, six-, and eight-bar $F = 1$ chains of the R-type are completely catalogued, while the RP-type is limited to four- and six-bars. Planar chains with $F > 1$ are also discussed briefly, the only catalogues of such chains given here being those of the R-type $F = 2$ five- and seven-bars. Each chain shown can be made into a linkage by fixing any link with respect to a frame of reference.

Planar Chains of Mobility $F = 1$ R-type Chains

The number of R pairs in an $F = 1$ R-type chain is as determined by Eq. (5). This section catalogues the $F = 1$ R-type planar kinematic chains with four, six, and eight links.

Four-bar. There is only one four-bar planar kinematic chain. It is composed of four binary links and four R pairs, as is shown in Fig. 23.

Six-bar. Six-bar R-type planar chains each contain two ternary links, four binary links, and seven R pairs. Two different chains exist, as shown in Fig. 24--the Watt having its two ternaries directly connected, and the Stephenson in which the ternaries are not directly connected.

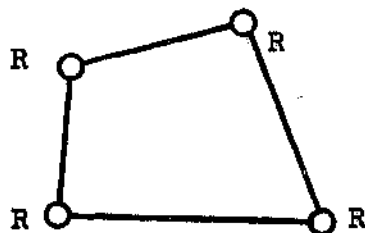


Figure 23. Four-bar R-type Planar Chain

Eight-bar. Sixteen varieties of R-type eight-bar planar kinematic chains exist. They may be grouped for convenience into four series as follows:

- Series 1 chains composed of two quaternary links and six binary links (two varieties)
- Series 2 chains composed of four ternary links and four binary links with each ternary connected to only two of the other ternaries, either directly or through one or two binaries (three varieties)
- Series 3 chains composed of four ternary links and four binary links with each ternary connected to each other ternary, either directly or through one or two binaries (six varieties)
- Series 4 chains composed of one quaternary link, two ternary links, and five binary links (five varieties)

These series and their chains are shown in Fig. 25.

RP-type Chains

The planar linkages containing both R pairs and P pairs have been labeled RP-type planar linkages. Hain [8], pp. 19-20, sets down three conditions which must be met in a planar RP-type chain:

1. No link of the chain may contain only prismatic pairs whose directions of motion are parallel to each other.
2. Binary links of the chain which have only prismatic

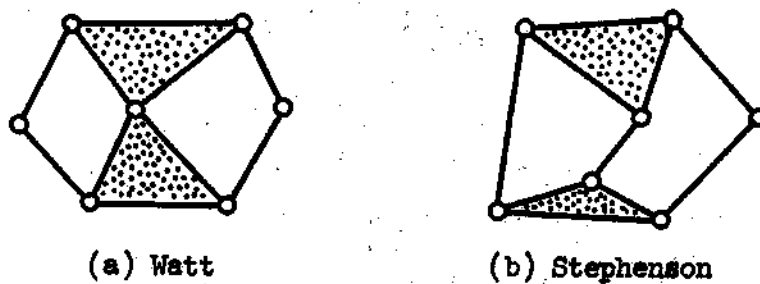


Figure 24. Six-bar R-type Planar Chains

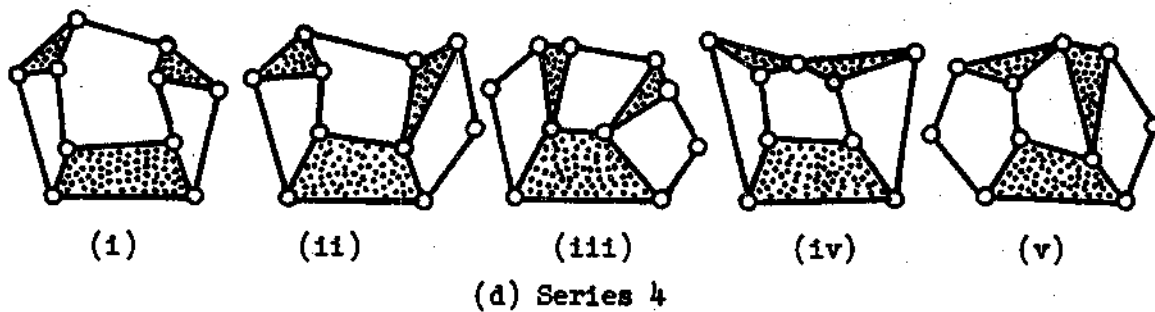
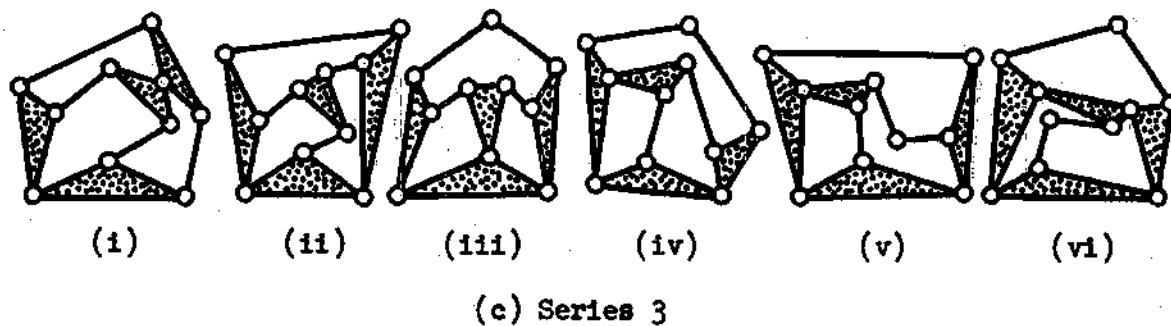
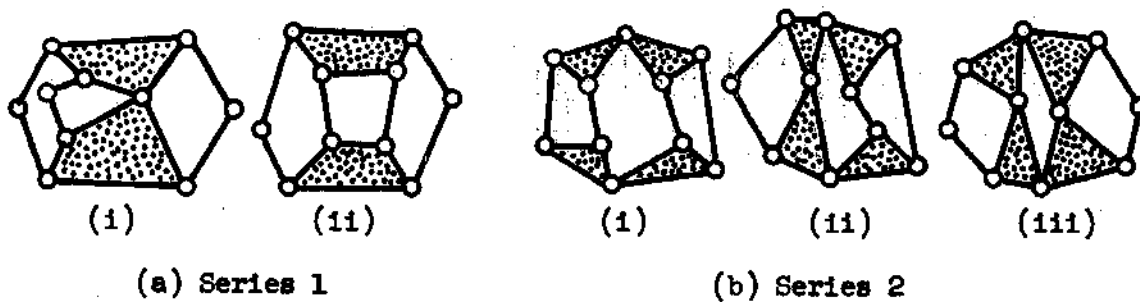


Figure 25. Eight-bar R-type Planar Chains by Series

pair elements may not be directly connected with each other.

3. No closed link polygon in the chain may have less than two turning [revolute] pairs.

These restrictions on the placement of P pairs in planar $F = 1$ linkages have been strictly adhered to in the formation of the catalogues to follow. The first condition is really a matter of dimensions and is assumed to be met in each chain presented. The designer of a particular mechanism must see to it that that dimensional condition is met. Hain [8] clarifies the necessity of his restrictions with illustrations.

The third condition implies that there is no planar ($m = 3$) chain which is made up entirely of P pairs. The "three-wedge" chain (three coplanar P pairs) is planar in the sense that "the true paths of all particles of all links may be shown in one plane"; but it has one additional general constraint in that there is no rotation present and is of the family $m = 4$ [9, 11].

Four-bar. Three varieties of RP-type $F = 1$ planar four-bar

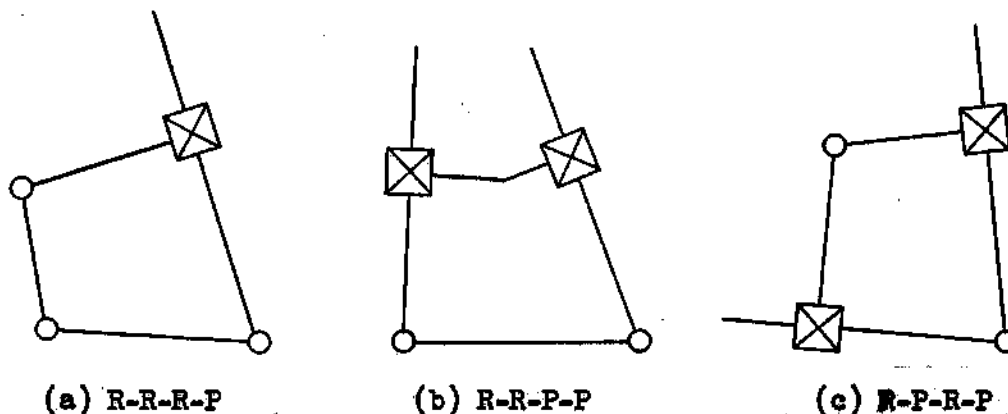
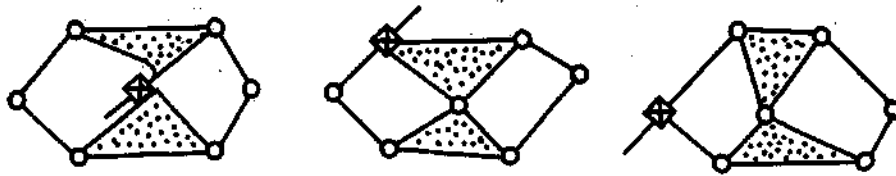
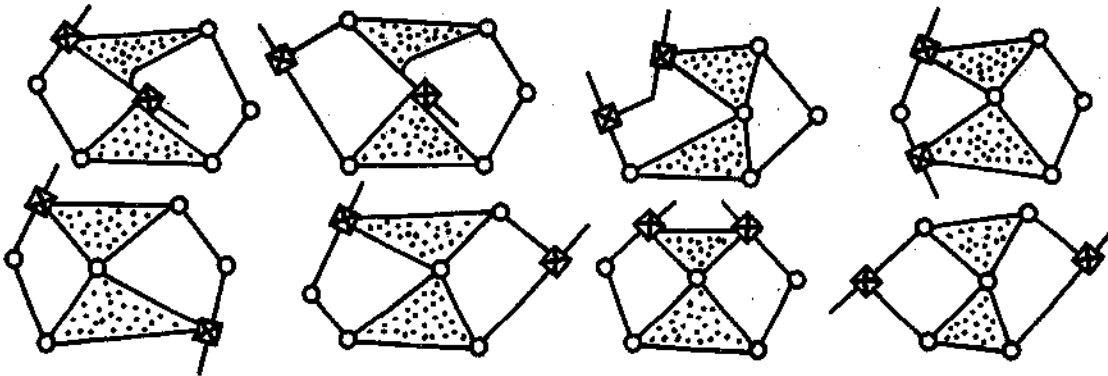


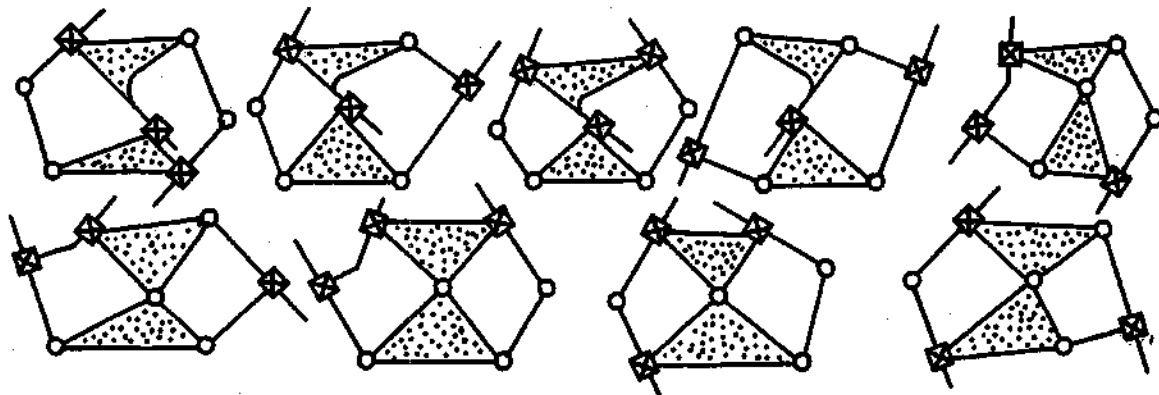
Figure 26. Four-bar RP-type Planar Chains



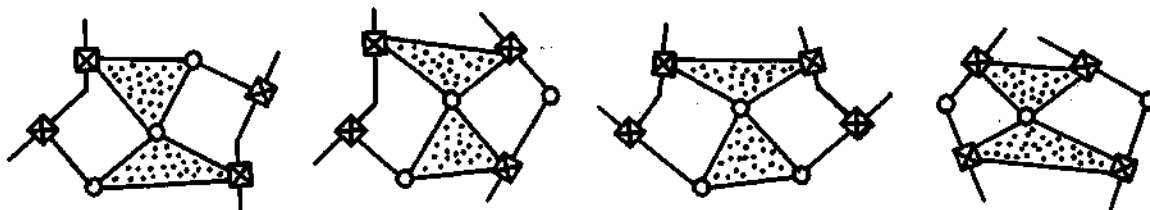
(a) One P pair



(b) Two P pairs

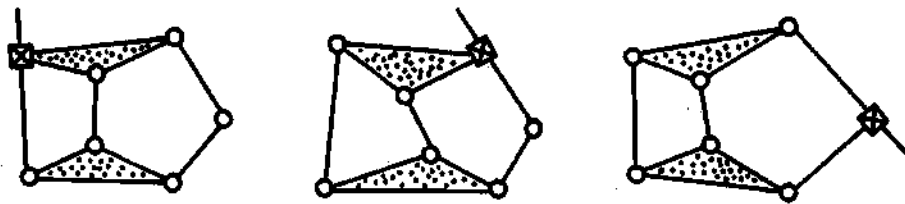


(c) Three P pairs

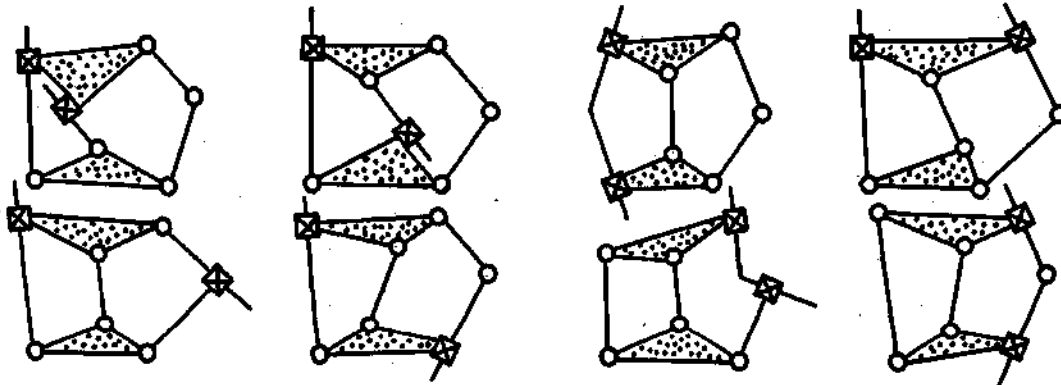


(d) Four P pairs

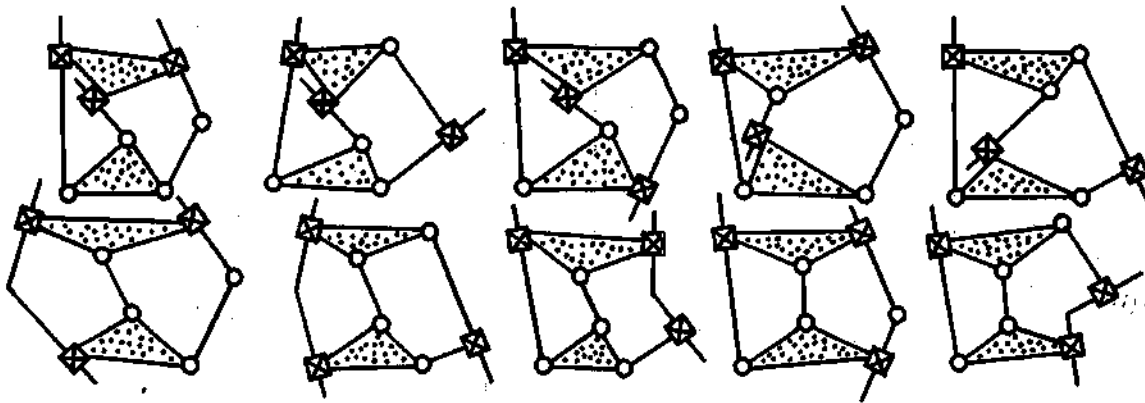
Figure 27. Watt Six-bar RP-type Planar Chains



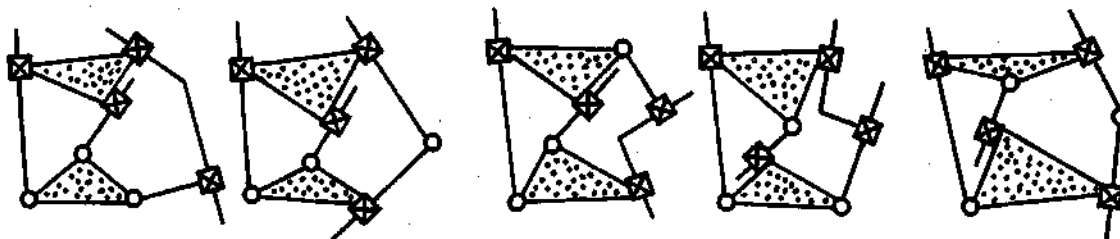
(a) One P pair



(b) Two P pairs



(c) Three P pairs



(d) Four P pairs

Figure 28. Stephenson Six-bar RP-type Planar Chains

chains exist. One has only one P pair, while the others have two P pairs each (see Fig. 26).

Six-bar. There appear to be 50 six-bar RP-type kinematic chains --24 of the Watt type and 26 of the Stephenson type. Each chain has from one to four P pairs, four being the maximum number in compliance with Hain's restrictions [8]. He has catalogued them only recently [21]. The Watt chains are shown in Fig. 27, and the Stephenson chains in Fig. 28.

Planar Chains of Mobility $F > 1$

R-type Chains

The simplest $F = 2$ closed planar chain is made up of five binary links and five R pairs connected in one loop, the result of adding one link and one pair to the constrained four-bar-R-type chain in Fig. 23. The next simplest $F = 2$ chains are made by adding a binary and R to a constrained R-type six-bar chain, thus producing seven-bar chains with eight pairs each. Another seven-bar $F = 2$ chain exists in the form of one quaternary link and six binary links, identical to the chain formed by giving two constrained four-bars a common link. The $F = 2$ R-type five-bar and seven-bar chains are illustrated in Figs. 29 and 30, respectively.

A chain of the form of Fig. 30d is said to have "fractionated mobility", a condition which is not to be considered here. It is obvious, however, that the separate loops of a fractionated mobility chain may be considered individually; e.g., the two constrained four-bar loops in the chain just discussed.

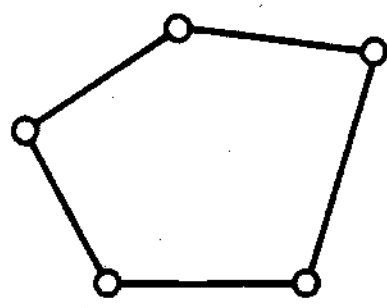


Figure 29. Five-bar R-type Planar Chain ($F = 2$)

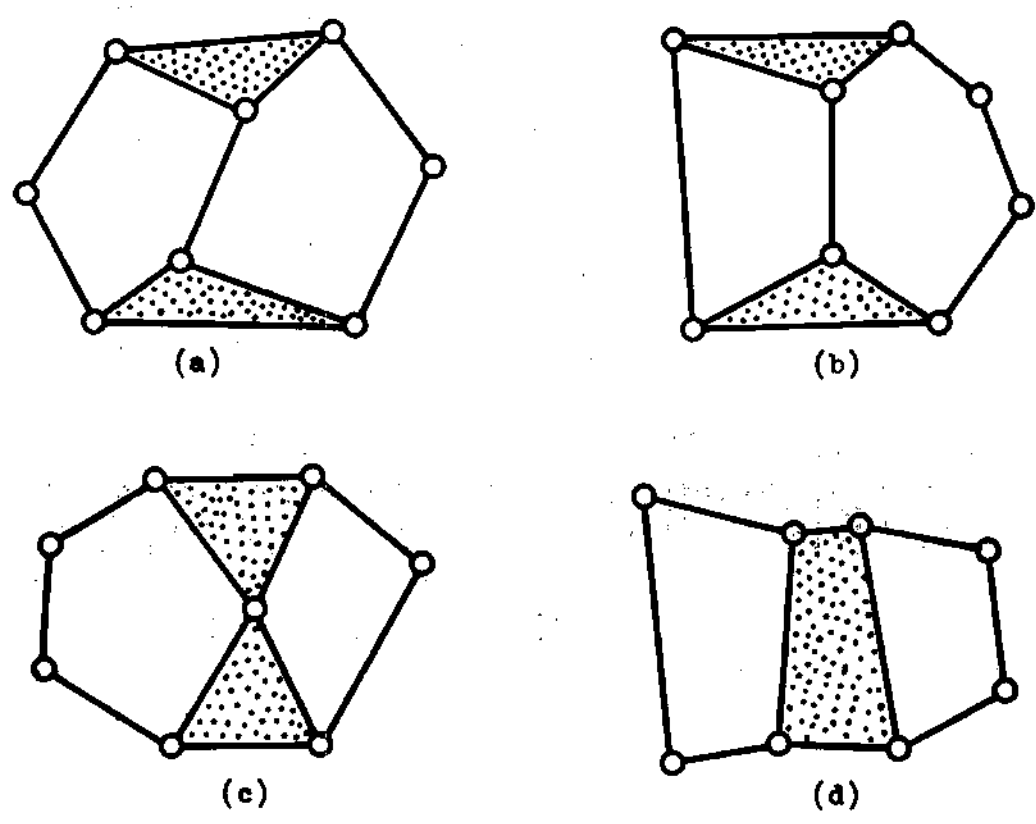


Figure 30. Seven-bar R-type Planar Chains ($F = 2$)

Nine-bar $F = 2$ chains can be formed from the eight-bars in a manner similar to that used for the smaller chains, and so on for larger $F = 2$ chains. Davies and Crossley [17] and Manolescu [19] have catalogued the R-type $F = 2$ nine-bars and the former have laid the foundation for a catalogue of the eleven-bars. No larger $F = 2$ chains are known to have been catalogued to any extent.

The simplest $F = 3$ closed chain is a six-bar with six R pairs, formed from the $F = 2$ five-bar by adding a binary and R. Others can be formed similarly from seven-bars, nine-bars, etc. There is no need to illustrate any of the $F = 3$ chains here because they are out of the scope of this paper, except as they may be mentioned as possibilities for expansion of the work presented here; likewise for chains with $F > 3$.

RP-type Chains

There are also RP-type kinematic chains with $F > 1$. These can be formed in a manner very similar to the formation of the R-type chains. There, a binary and an R pair are added to a chain to increase its mobility by one. Here, a binary and a pair, either an R or a P, is added, on the condition that the resulting chain does not violate any of Hain's three restrictions.

BIBLIOGRAPHY

1. Pollard, A. F. C., "Kinematic Design in Engineering," Proceedings of Institution of Mechanical Engineers, vol. 125, 1933, pp. 143-195.
2. Whitehead, T. N., Design and Use of Instruments and Accurate Mechanism, New York: Macmillan, 1934.
3. Steeds, W., Mechanism and the Kinematics of Machines, London: Longmans, Green and Co., 1940.
4. Beggs, J. S., Mechanism, New York: McGraw-Hill, 1955.
5. Spotts, M. F., "Kinematic Concepts that Simplify Design," Product Engineering, vol. 26, no. 5, May 1955, pp. 152-154.
6. Herbert, R. J., "Kinematic Design," Junior Institution of Engineers Journal, vol. 67, pt. 11, August 1957, pp. 320-336.
7. Harrisberger, L., "A Number Synthesis Survey of Three-Dimensional Mechanisms," Journal of Engineering for Industry, Trans. ASME, Series B, vol. 87, no. 5, May 1965, pp. 213-220.
8. Hain, K., Applied Kinematics, 2nd ed., New York: McGraw-Hill, 1967.
9. Artobolevski, I. I., Teoria Mehanizmov i Mashin, Moscow: Gosudarstv. Izdatl Tehn-teori. Lit, 1953.
10. Dobrovol'ski, W. W., Teoria Mechanizmov, Moscow: Mashgis, 1953.
11. Harrisberger, L., and A. H. Soni, "A Survey of Three-Dimensional Mechanisms with One General Constraint," ASME Paper No. 66-Mech-44, 1966.
12. Markoff, M. A., and N. Sonin, Oeuvres de P. L. Tchebychef, vol. II, New York: Chelsea Publishing Co., 1961.
13. Manolescu, N., and V. Manafu, "Sur la Détermination du Degré de Mobilité des Mécanismes," Buletinul Institutului Politehnic Bucuresti, vol. 25, no. 5, Sep-Oct 1963, pp. 45-66.
14. Hartenberg, R. S., and J. Denavit, Kinematic Synthesis of Linkages, New York: McGraw-Hill, 1964.

15. Crossley, F. E., "On an Unpublished Work of Alt," Journal of Mechanisms, vol. 1, no. 2, 1966, pp. 165-170.
16. Crossley, F. E., "The Permutations of Ten-link Plane Kinematic Chains," Antriebstechnik, vol. 3, no. 5, 1964, pp. 181-185.
17. Davies, T. H., and F. E. Crossley, "Structural Analysis of Plane Linkages by Franke's Condensed Notation," Journal of Mechanisms, vol. 1, no. 2, 1966, pp. 171-183.
18. Woo, L. S., "Type Synthesis of Plane Linkages," ASME Paper No. 66-Mech-12, 1966.
19. Manolescu, N. I., "Une Méthode Unitaire pour la Formation des Chaines Cinématiques et des Mécanismes Plans Articulés avec Différents Degrés de Liberté et Mobilité," Mécanique Appliquée, vol. 9, no. 6, 1964, pp. 1263-1313.
20. Chow, H., "Commercially Available Linkage Joints," Mechanisms, Linkages, and Mechanical Controls (N. P. Chironis, ed.), New York: McGraw-Hill, 1965, pp. 348-349.
21. Hain, K., "Systematik sechsgliedriger kinematischer Ketten," Maschinenmarkt, vol. 74, no. 38, 10 May 1968, pp. 717-723.