In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

3/17/65
b

AN INVESTIGATION OF THE SEDIMENT ENTRAINMENT MECHANISM

A THESIS

Presented to

The Faculty of the Graduate Division

by

James Ivan Dangar

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

Georgia Institute of Technology

September, 1966

AN INVESTIGATION OF THE SEDIMENT ENTRAINMENT MECHANISM

Approved:

Chairman

Date approved by Chairman:

ACKNOWLEDGMENTS

The author would like to express his sincere appreciation to all those individuals who have contributed to this work by their help and encouragement. A special note of thanks is extended to Dr. M. R. Carstens, faculty adviser on this project, for suggesting the research topic and aiding throughout its development. Appreciation is extended to Dr. G. M. Slaughter and Dr. N. H. Wade for their time spent in reviewing this work.

TABLE OF CONTENTS

LIST OF TABLES

Table Page

LIST OF ILLUSTRATIONS

SUMMARY

The object of this study was to derive a relationship between the diameter of a particle lying on a bed of material in a moving stream of fluid and the velocity of the fluid required to begin movement of the particle.

Heretofore, most relationships involving an incipient velocity and the particle diameter have involved the bottom velocity - a quantity which is often difficult to determine. It is contended in this study that the boundary of most channels formed of granular material is hydraulically rough and that the velocity of the fluid acting on the particles will be very nearly equal to the mean velocity of flow. Consequently, there should be rather good correlation between the mean velocity of the stream and the particle diameter at incipient-motion.

Incipient mean velocities were measured for motion out of a constant geometry scour hole with different sediments using air as the fluid. These data are presented along with the data of another investigation with the same scour hole with water as the fluid. The empirical relation

$$V = K\left(\sqrt{(s-1)gd_m}\right)^{1.11}$$

where K is a numerical coefficient, V is the mean velocity, d_m is the mean particle diameter, and s is the ratio of the density of the sediment to the density of the fluid is derived for incipient motion. The above relationship is also shown to apply very closely to a flat bed

scouring situation in which the boundary was hydraulically rough.

The dimensionless quantity N_s where

$$N_s = \frac{V_b}{\sqrt{(s-1)gd_m}}$$

in which V_b is the bottom velocity, is shown not to be a constant at incipient-motion for all sizes of particles as has been contended by many previous investigators. This phenomenon is shown to be the result of a changing Reynolds number, R, with increasing particle size such that

$$N_s = 1.53\ R^{0.0375}$$

CHAPTER I

INTRODUCTION

The object of this study is to derive a relationship between the diameter of a particle lying on a bed of material in a moving stream of fluid and the velocity of the fluid required to begin movement of the particle.

The ability of a fluid to transport solid sediment particles from place to place has attracted the attention of engineers for many years. The prevention of damaging scour at the base of bridge piers and abutments and at the base of spillways has been an important part of hydraulic designs in recent years. In the design of irrigation canals, the engineer has always been interested in the maximum velocities of flow attainable in the canal without eroding material from the channel walls. A phenomenon of great interest to the agricultural engineer is the maximum wind velocity which may occur over a field before movement of the soil particles begins.

Thus, engineers have continually been searching for some parameter that would describe the instant of entrainment of a particle into a moving fluid. In recent years, many investigators have been shifting to the use of a parameter termed the critical tractive force - force resulting from shear - to describe incipient-motion. These investigators have felt that the use of the fluid velocity to describe incipient motion was not sufficient. In particular, they have avoided the use of the average stream velocity - a quantity which is usually easiest to obtain. These investigators have argued that the particles on the bed of a stream are immersed in a boundary layer and were not being acted upon by the velocity of the fluid in the mainstream. Consequently, any significant relationship must involve the fluid velocity just above the bed particles.

History of the Literature

The velocities required to initiate the movement of grains on a bed of material have been called the critical, competent, threshold, or incipient velocities. All of the terms are found in the literature and all have the same meaning.

Perhaps the earliest to derive a relationship between the competent velocity and the particle size was Brahms (1). Brahms deduced that

$$V_{cr} = KW^{1/6}$$

in which V_{cr} is the pick-up bottom velocity, K is a coefficient, and W is the submerged weight of the particle. Since

$$W \sim d^3$$

where d is the diameter of the particle, substitution and simplification of Brahms' equation yields

$$V_{cr} = K_2 d^{1/2}$$

Fortier and Scobey (2) presented the first major work in the twentieth century on the relation of critical velocities and the diameter of particles. They obtained their data by questioning experienced irrigation engineers as to the maximum canal velocity permissible without eroding certain classifications of soil. However, they did not arrive at a direct relationship between the critical canal velocity and the diameter of the moveable soil particle; instead, they presented their data in tabular form.

Mavis (3), using laboratory data and purely empirical methods, derived the expression

$$V_b = \frac{1}{2} d^{4/9} (s-1)^{1/2}$$

in which V_b is the competent bottom velocity in feet per second, s is the ratio of the specific gravity of the sediment to the specific gravity of the fluid, and d is the particle diameter in millimeters. This expression tended to be near the lower limit of the data obtained by several other investigators. Later Mavis (4) suggested that the exponent on the diameter be changed from 4/9 to 1/2. He reasoned that this would simplify computations but would not change the basic equation appreciably.

Jeffreys (5) investigated the hydrodynamic problem of lift on a cylinder lying on a horizontal plane in a moving fluid through purely mathematical techniques using potential functions. He showed that a cylinder will be lifted if

$$V > 0.59\sqrt{(s-1)gd}$$

Jeffreys concluded that for a sphere, where there will be some flow under the solid, the lift will be less, and the velocity required for movement will be somewhat higher. Nevertheless, the form of the equation will remain the same with only the numerical coefficient varying.

Chepil (6), using velocity measurements in the open field and in a wind tunnel, investigated the initiation of soil movement during wind erosion. He found that the threshold velocity varied greatly with the diameter of the grain and that the most readily eroded material ranged from 0.1 millimeters to 0.15 millimeters in diameter. He found that below this range the threshold velocity increased as the diameter decreased, and that above this range the threshold velocity was proportional to the square-root of the diameter.

Hjulstrom (7) prepared a curve of mean critical velocity as a function of mean sediment size for quartz sediment (specific gravity = 2.65) in water using the data of several workers. These data, appearing in Figure 1, show that he, like Chepil, found that there was a range of particle diameters which was most susceptible to movement. However, his range of most active movement was between 0.2 millimeters and 0.3 millimeters in diameter. But most important, Hjulstrom showed that there was some relationship between the critical <u>average</u> velocity of a stream and the particle diameter.

Most of the previous work has essentially reached the same conclusion; namely,

$$V = \Phi(d^{1/2})$$

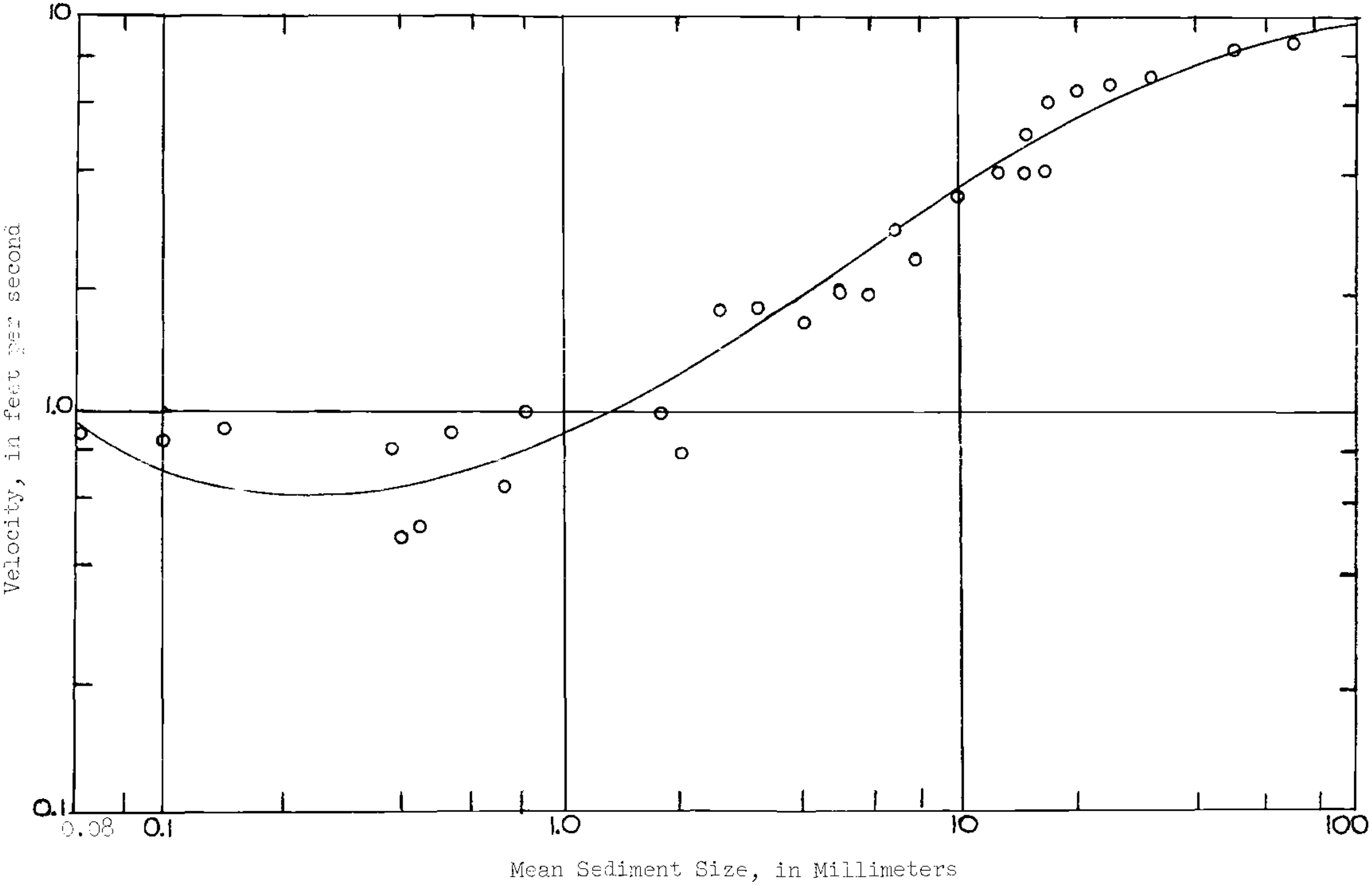

Figure 1. Critical Water Velocity as Function of Grain Size after Hjulstrom

where Φ denotes some function. However, as stated previously the velocity used has usually been the bottom velocity - a quantity which is often difficult to determine.

It is contended in this study that the boundary of most channels formed of granular material is hydraulically rough. As a result, the velocity distribution in the stream depends only upon the relative roughness including the bed-form geometry. Under these conditions, the velocity acting on the particles will be more nearly equal to the mean velocity of flow than if the particles were submerged in a laminar sublayer. Consequently, there should be rather good correlation between the mean velocity of the stream and the particle diameter at incipient motion.

CHAPTER II

EXPERIMENTAL APPARATUS

The object of this study was to derive some relationship between the incipient velocity and the diameter of a particle lying on a bed in some moving fluid. In this study, the experimental bed is the bottom of a scour hole which was constructed in the hydraulics laboratory at Georgia Tech and was used in a previous study by A. R. LeFeuvre (8).

In this test section the geometry of the scour hole remained constant. The scour hole boundaries were formed of plexiglass with only the bottom of the scour hole being moveable. The scour hole was formed by the intersection of a two-inch diameter tube with a three-inch diameter plexiglass tube carrying the main flow. A plastic wedge with a 60-degree slope was fastened to the upstream side of the two-inch diameter tube to form the defined scour area as shown in Figure 2. The bottom of the scour hole was formed of sediment which was kept at a constant level, 2.5 inches below the inside wall of the three-inch tube, by a screw-piston which forced the sediment up into the bottom of the scour hole.

The scouring flow of fluid was produced by air blowing through the three-inch diameter plexiglass tube. This air was supplied to the test section through a six-inch diameter pipe through an elliptical contraction. The six-inch pipe was in turn connected to a centrifugal blower. This blower was driven by an electric motor with a variable speed transmission between the motor and the blower. The flow leaving

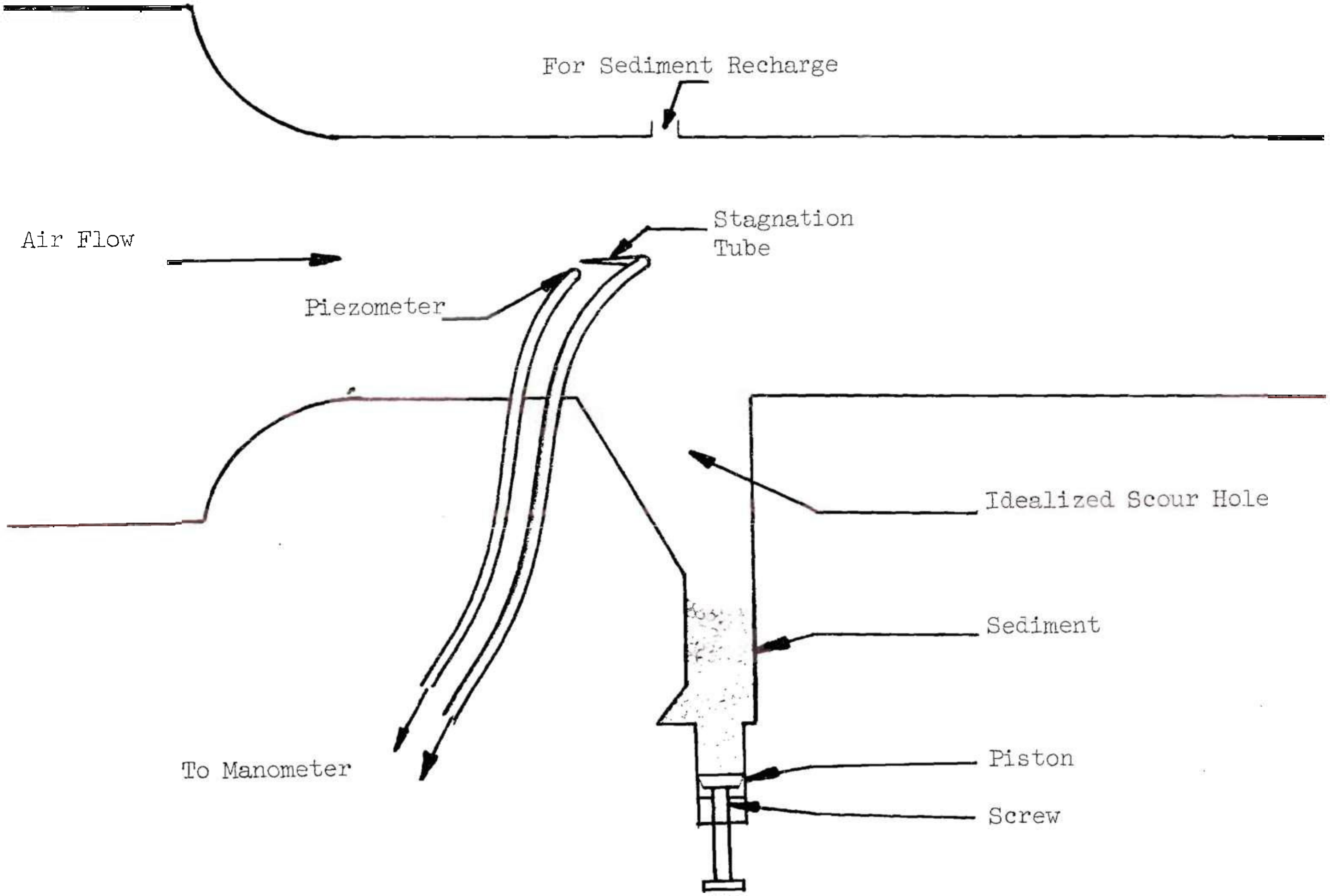

Figure 2. Sketch of the Test Section

the test section entered a fabric bag which acted as a strainer to remove sediment particles from the airstream. Figure 3 is a diagram of the arrangement of the experimental apparatus.

Velocity measurements were performed by means of a stagnation tube which was located on the centerline of the three-inch flow section and a wall piezometer located in the plane of the tip of the stagnation tube. Only the velocities at the centerline were measured. Pressure differences were measured with a displacement manometer which could be read to ±0.001 inches of water. The accuracy with which the velocity could be computed using this manometer was far greater than the accuracy with which the incipient condition would be judged.

Since only the effect of particle weight against movement was desired, any form of interlocking between particles was to be hopefully avoided during the tests. Therefore, several sizes of the very uniform, nearly spherical, glass beads and one size of well-rounded Ottawa sand were selected as the test materials. The different properties of the material appear in Table 3 in the Appendix.

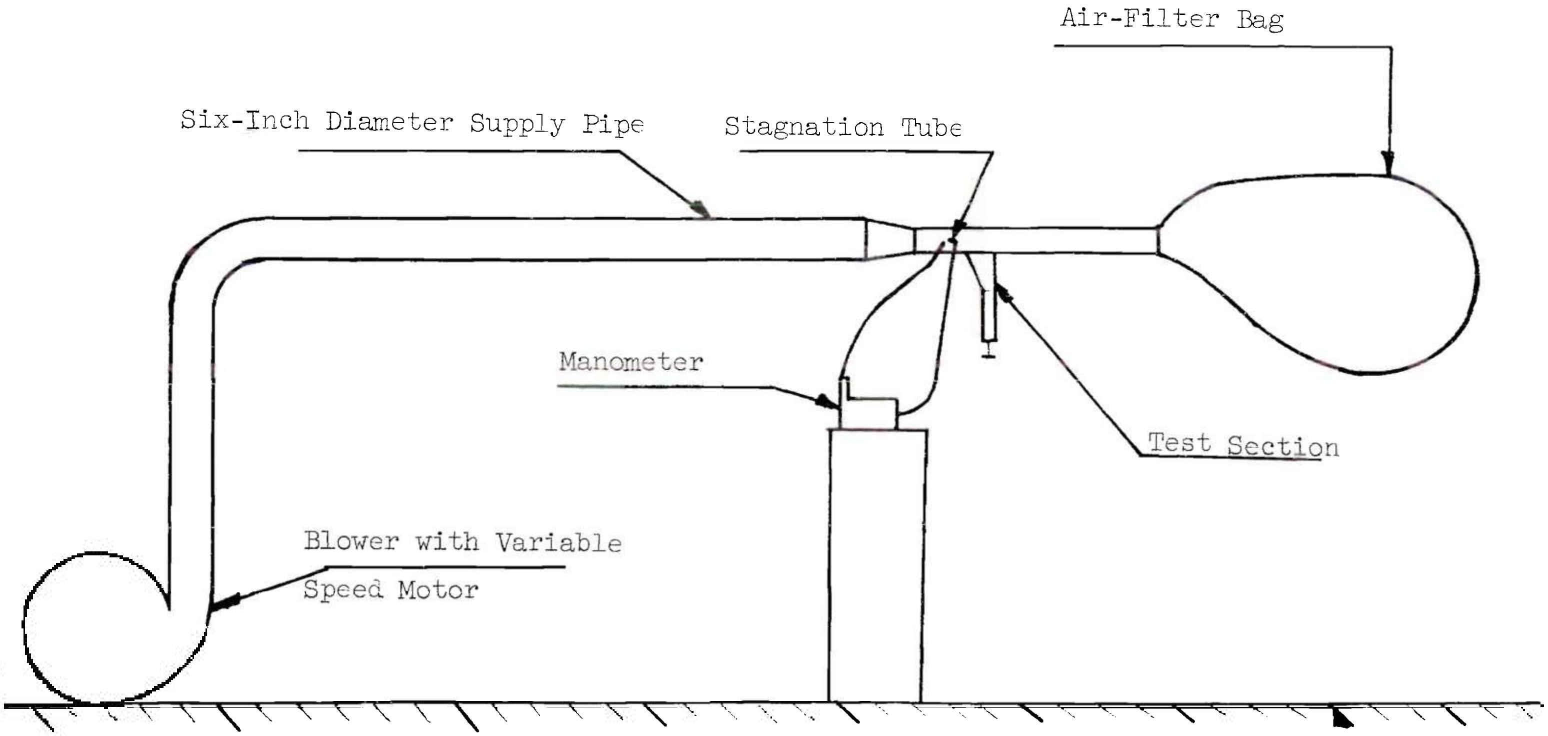

Figure 3. Arrangement of Experimental Apparatus

CHAPTER III

EXPERIMENTAL PROCEDURE

The object of the experimental work was to determine the velocity of the flow in the three-inch diameter pipe which would pick up the first particles and throw them upward from the scour hole and into the main flow.

The sediment level was first adjusted to a reference mark in the scour hole. The air blower was then turned on at a very low speed. The flow was gradually increased by adjusting the blower speed through the use of the speed selector connected to the motor. At each successive velocity increase, the bed of the scour hole was examined for movement. Usually the first few velocity increments caused no movement. As the velocity of the main flow was increased, there was an occasional rocking of some isolated particles. As the velocity was increased further, the bed of the scour hole began to appear vibrated and movement occurred within the hole. Particles would make sporadic jumps about the bottom of the hole; but none would leave the scour hole. At still higher velocities, there was an occasional particle transported up into the main flow. This is the point at which incipient-motion was defined as occurring. Any further increase in velocity would only increase the number of sediment particles leaving per unit of time. The manometer reading was then taken, the barometric pressure and temperature were recorded, and the velocity was calculated.

The point defined as incipient motion was so chosen because, if given time, the bed of the scour hole would be eroded away, even at this slow rate of transport. Because of the uniformity of the bed material, there would be no sorting of the grains and coating of the bottom with larger pieces, as is often the case with the naturally occurring river sediment in scour holes. The velocities for the incipient motion tests are tabulated in Tables 4-7 in the Appendix.

The mean diameters of the sediment materials were determined by first performing a sieve analysis on a material using the U. S. Standard Sieves which follow a progression of the fourth-root of two. The percent-finer-than was plotted versus the diameter on probability-log paper as in Figure 10 in the Appendix. The mean diameter of a material is the diameter at the point of fifty percent-finer-than on a plot of this type. The standard deviation of each material about its mean was also easily obtained from these plots.

The tests for specific gravity of the glass beads were performed according to the procedure stated in the latest ASTM Designation D 854. The specific gravity of the Ottawa Sand had been determined in an earlier study, and its value was accepted for use in this study.

CHAPTER IV

DISCUSSION OF RESULTS

Incipient-motion of bed particles is obviously dependent upon forces acting upon the particles. These forces, in turn, are dependent upon the velocity of the moving fluid near the particles. When the particles are at the bottom of a scour hole, the incipient-motion will then be dependent on the velocity of flow in the bottom of the scour hole.

Upon examining an idealized scour hole, it may be observed that as the flow of the mainstream passes over the scour hole, shear forces between the fluid in the mainstream and fluid in the scour hole sets the fluid in the hole in motion. A spiral roller is developed as shown in Figure 4. The velocity of this roller is proportional to the velocity of the mainstream. Thus, the incipient velocity of the sediment particles at the bottom of the scour hole will be proportional to the mainstream velocity.

Using the theory of many previous researchers that the velocity is a function of the square-root of the particle diameter, the velocities

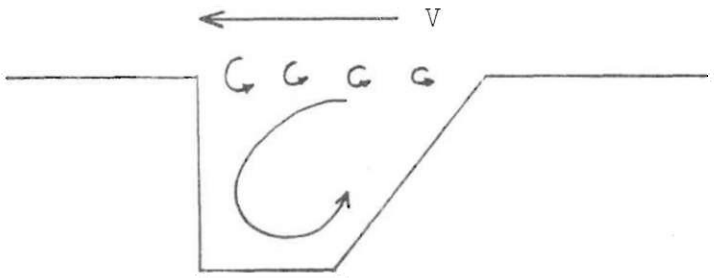

Figure 4. Spiral Roller in Scour Hole

required to produce incipient motion as measured in this study (Figure 2) are plotted against the particle diameters, as shown in Figure 5. The experimental results tend to verify the trend obtained by Chepil (6), shown by the dashed line. However, Figure 5 does not reveal that the velocity is proportional to the square-root of the diameter for the larger particles as was found by Chepil.

To extend the range of variables to cover both air and water as the fluid, the incipient-motion data obtained by LeFeuvre (8) are also included in the analysis. LeFeuvre made incipient-motion measurements for different sediments with water as the fluid, using the same test-section, Figure 2, as used in this experiment. He measured discharge by means of a triangular weir. LeFeuvre used the mean velocity through the test-section in his presentation.

In addition, LeFeuvre measured the velocity in the test-section over the scour hole using air as the working fluid. The velocity profile along a vertical diameter of the horizontal test-section was determined for two different discharges as shown in Figure 11. LeFeuvre found that the velocity profile near the boundary could be compared to the laminar boundary layer on a flat plate by using an equivalent distance X from the leading edge of the plate. He computed a value for X of 0.41 feet. Using the displacement thickness as defined by Schlichting (9), LeFeuvre showed that

$$\delta^* = 1.10 \sqrt{\frac{\nu}{V}} \qquad (1)$$

where δ^* is the boundary layer displacement thickness, ν is the kinematic viscosity of the fluid, and V is the mean velocity of the fluid.

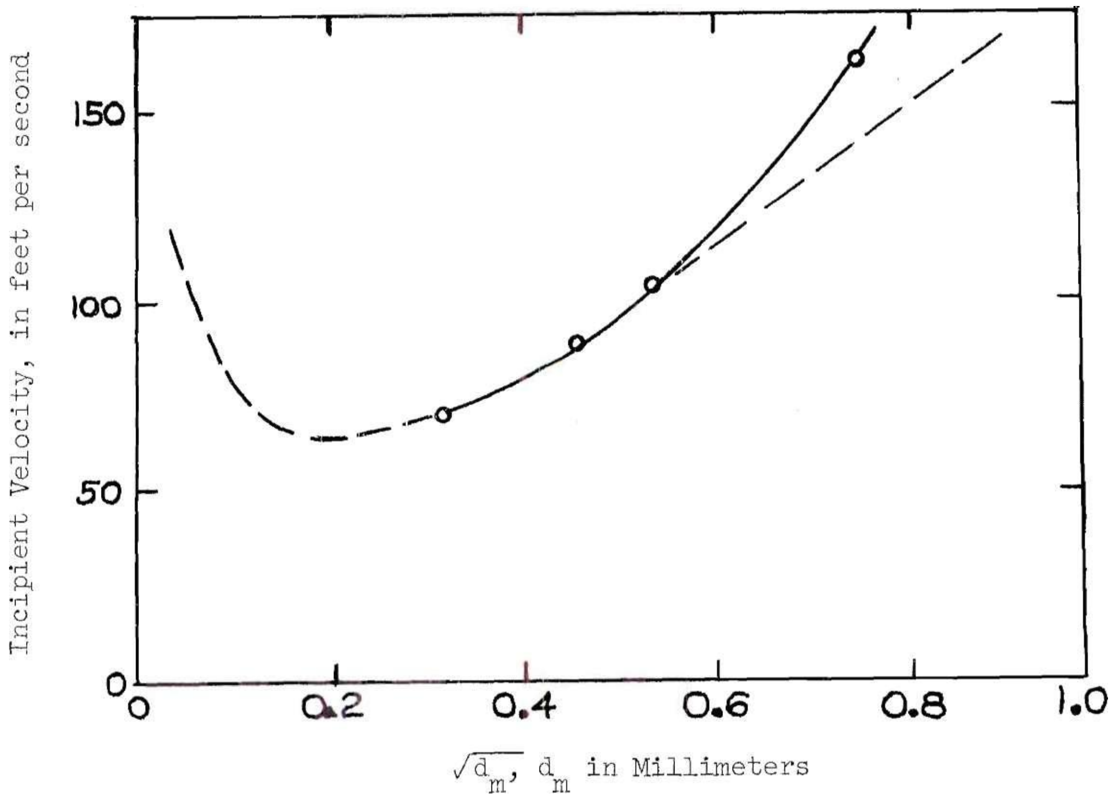

Figure 5. Air Velocity as a Function of Particle Diameter for Scour Hole Motion

Equation (1) may be combined with the equation of continuity to yield

$$\frac{V}{V_{max}} = (1.0 - 8.8 \sqrt{\frac{\nu}{V}})^2 \qquad (2)$$

where V_{max} is the maximum velocity of the mainstream.

Table 1 shows the data from the present study along with the calculated mean velocities from equation (2) while Table 2 shows LeFeuvre's incipient-motion data.

Mavis (4) has shown empirically and Jeffreys (5) has shown theoretically that at incipient-motion

$$V \propto \sqrt{(s-1)gd_m} \qquad (3)$$

where s is the ratio of the density of the sediment to the density of the fluid, g is the acceleration of gravity, and d_m is the mean sediment diameter. The data of the present study and LeFeuvre's incipient motion data are presented in the form of equation (3) in Figure 6, with all dimensions being in the English system of units. By using empirical methods, this variation could be very closely approximated by the function

$$V = 10.3 \left(\sqrt{(s-1)gd_m}\right)^{1.11} \qquad (4)$$

This particular function will apply only to a scour hole with the dimensions and geometry of the one used in this investigation. However, the general equation

$$V = K \left(\sqrt{(s-1)gd_m}\right)^{1.11} \qquad (5)$$

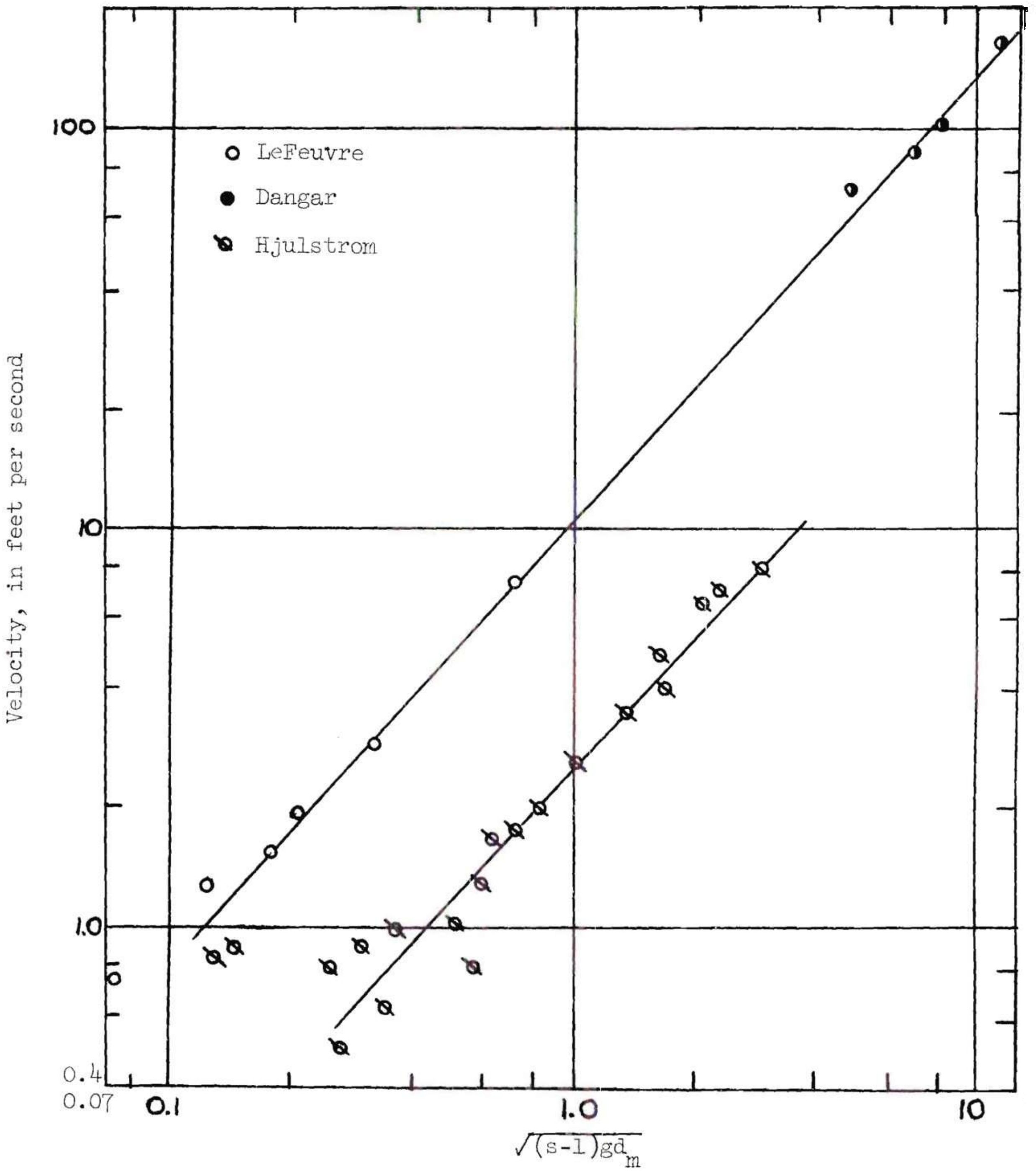

Figure 6. Variation of V with $\sqrt{(s-1)gd_m}$

Table 1. Incipient Motion Data

Material	Mean Diameter (mm)	Specific Gravity	Critical* Maximum Velocity fps	Computed* Mean Velocity fps
Glass	0.102	2.47	72.0	70.1
Glass	0.215	2.47	88.8	86.5
Glass	0.294	2.47	103.7	101.2
Ottawa Sand	0.565	2.60	160.9	157.9

Table 2. LeFeuvre's Incipient Motion Data

Material	Mean Diameter (mm)	Specific Gravity	Critical* Mean Velocity (fps)
Nickel	0.570	8.75	7.00
Sand	0.585	2.62	2.75
Sand	0.185	2.63	1.45
Glass	0.297	2.47	1.83
Glass	0.106	2.46	1.23
Lucite	0.250	1.20	0.64

where K is a numerical coefficient depending on the dimensions and geometry of the scour area, should apply to any scouring situation over a hydraulically rough boundary such as would be on the upstream face of dunes found on the bed of rivers. Hjulstrom's data from the Fyris River are shown in Figure 1. The variation of these values with the critical velocities also appears in Figure 6. The form of equation (5) appears to be a very close approximation for incipient-motion from a flat bed where the diameter of the particle is equal to 0.3 millimeters or greater.

---

* In the three-inch diameter pipe above the scour hole.

An empirical equation for Hjulstrom's data is

$$V = 2.33\ (\sqrt{(s-1)gd_m})^{1.11} \qquad (6)$$

Since some of Hjulstrom's data was obtained with beds of exaggerated roughness, the mean velocity would be nearly equal to the velocity just above the particles. Thus the equation for a flat bed should be applicable to the flow situation at the bed of the scour hole itself. If so, the critical velocity as computed by equation (6) for a diameter particle would be the velocity of the fluid in the scour hole just above the bed particles. The ratio of equation (5), for the specified scour hole, to equation (6) would then be approximately equal to the ratio of the mean stream velocity, V, to the velocity, V_b, in the bottom of the scour hole. For the situation under investigation

$$\frac{V}{V_b} \cong 4.4 \qquad (7)$$

When V is plotted against the parameter $\sqrt{(s-1)gd_m}$, a variation similar to Figure 7 is obtained. The straight-line function occurs when the boundary is hydraulically rough. Deviation from the straight-line function occurs when the particles become submerged in a laminar sublayer. Greater mean velocity is required for particles in a laminar sublayer because the velocity just above the particles is considerably less than the mean velocity. Thus as the particle diameters become smaller, the velocities required to move them becomes larger.

In sediment transport mechanics, a useful criteria to describe incipient-motion on a hydraulically rough boundary is the dimensionless sediment number, N_s, where

$$N_s = \frac{V_b}{\sqrt{(s-1)gd_m}} \tag{8}$$

Substituting equation (5) into equation (8) and simplifying

$$N_s = K\left(\sqrt{(s-1)gd_m}\right)^{0.11} \tag{9}$$

The values of N_s necessary for incipient-motion at the bottom of the scour hole were obtained by substituting equation (7) into equation (8) and performing the computations with the data shown in Table 1 and Table 2. Plotting these values of N_s along with those for Hjulstrom's data as a function of $\sqrt{(s-1)gd_m}$ in Figure 8 reveals that the form of equation (9) is a close approximation for the variation.

Figure 8 reveals that N_s is not constant but increases somewhat with particle size. A rational explanation for this changing value of N_s may be obtained by examining some of the common parameters of fluid mechanics.

LeFeuvre showed by a force analysis that at the instant of incipient motion

$$N_s^{\,2} = \frac{K_c}{C_d} \tag{10}$$

where C_d is the coefficient of drag acting on the particle and K_c is a numerical coefficient. He approximated the value of K_c as being 0.265. Examination of equation (10) shows that in order for N_s to be a constant, the value of C_d must be constant for all sizes of particles.

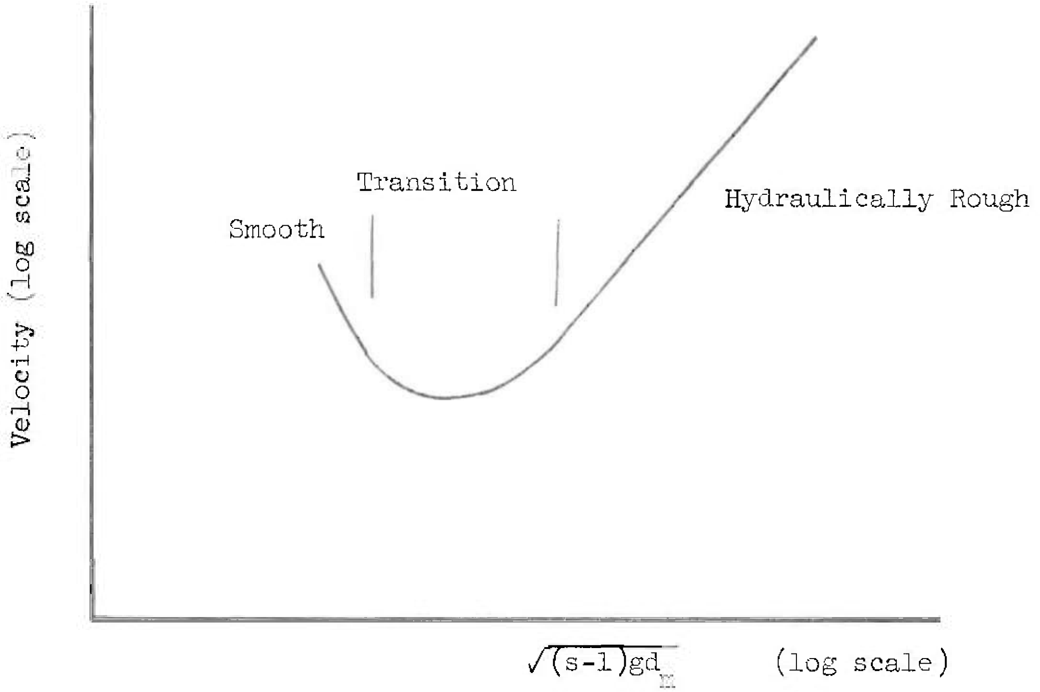

Figure 7. Form of the Incipient-Motion Curve

The relationship between C_d and the Reynolds number R for a spherical particle in an infinite fluid is well known [Rouse (10)]. Reynolds number is defined as

$$R = \frac{V\,d_m}{\nu} \tag{11}$$

in which V is the average velocity of the fluid approaching the particle, d_m is the particle diameter, ν is the kinematic viscosity of the fluid, and the units of all parameters are consistent. The functional relationship between C_d and R for flow over a bed of particles and for a single particle in an infinite fluid should be similar. As has been stated, equation (6) applies at incipient-motion on a hydraulically rough bed. Using equation (6) with water as the scouring fluid and sediment with s equal to 2.65, the values of R for Hjulstrom's data (0.1 mm < d_m < 60 mm)

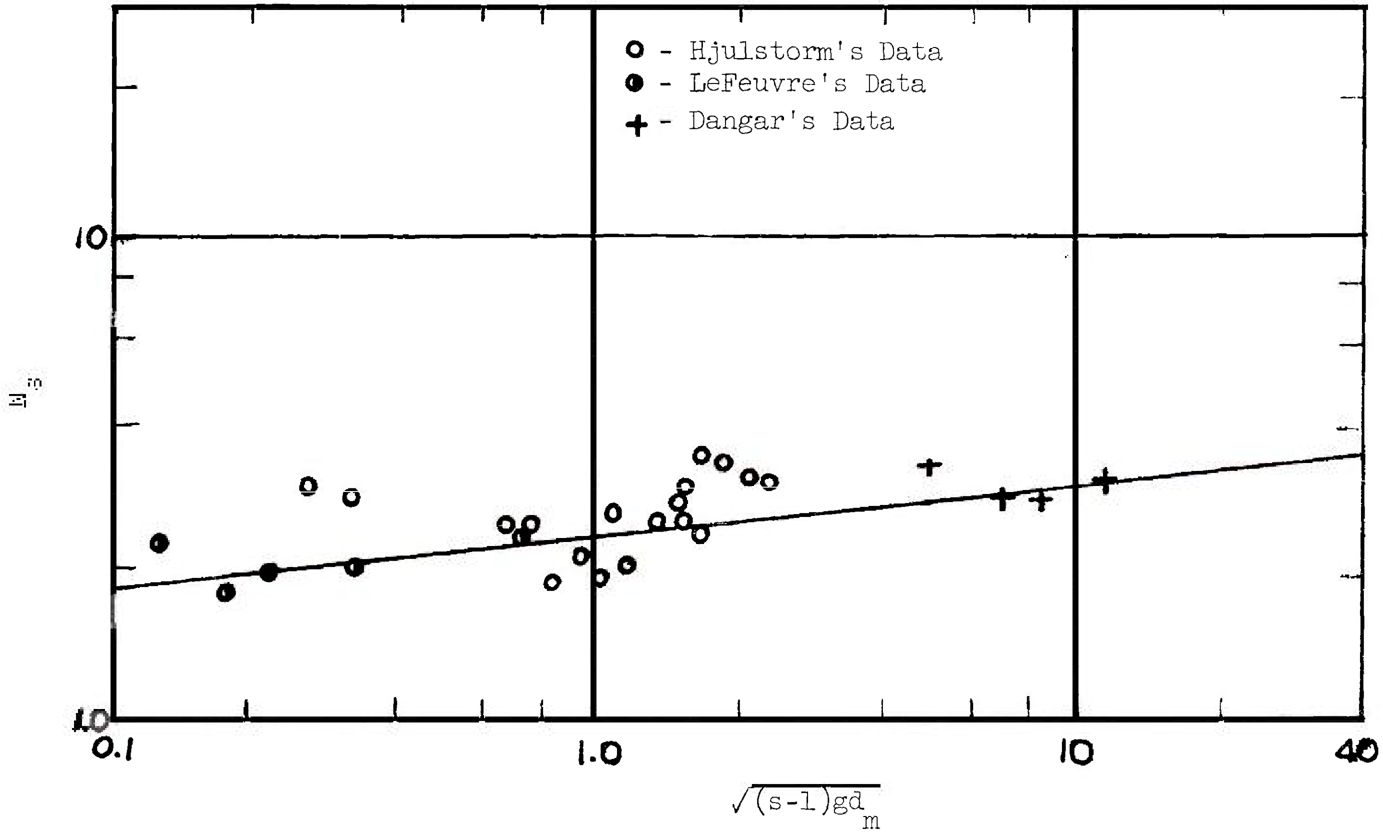

Figure 8. Variation of N_s with $\sqrt{(s-1)gd_m}$

were computed. Again applying equation (7) to all the scour hole data, the values of R at incipient-motion from the scour hole were computed. Values of C_d for incipient-motion were computed from equation (11) and plotted against the values of R. This variation, shown in Figure 9, reveals that

$$C_d = \frac{0.113}{R^{0.075}} \tag{12}$$

For these same values of R, the values of C_d were taken from Rouse (10) for a sphere falling in an infinite fluid. This plot, also in Figure 9, shows that

$$C_d = \frac{1.36}{R^{0.075}} \tag{13}$$

is a reasonable approximation for the curve. While the two equations differ by approximately one order of magnitude, the form of the equations

$$C_d \propto \frac{1}{R^{0.075}} \tag{14}$$

is the same. Substitution of equation (12) into equation (10) reveals that

$$N_s = 1.53\ R^{0.0375} \tag{15}$$

in which the Reynolds number, R, is computed using the fluid velocity just above the particles. Substituting equation (11) and equation (5) into equation (15) shows that it is identical to equation (9); namely,

$$N_s \propto d_m^{0.055} \tag{16}$$

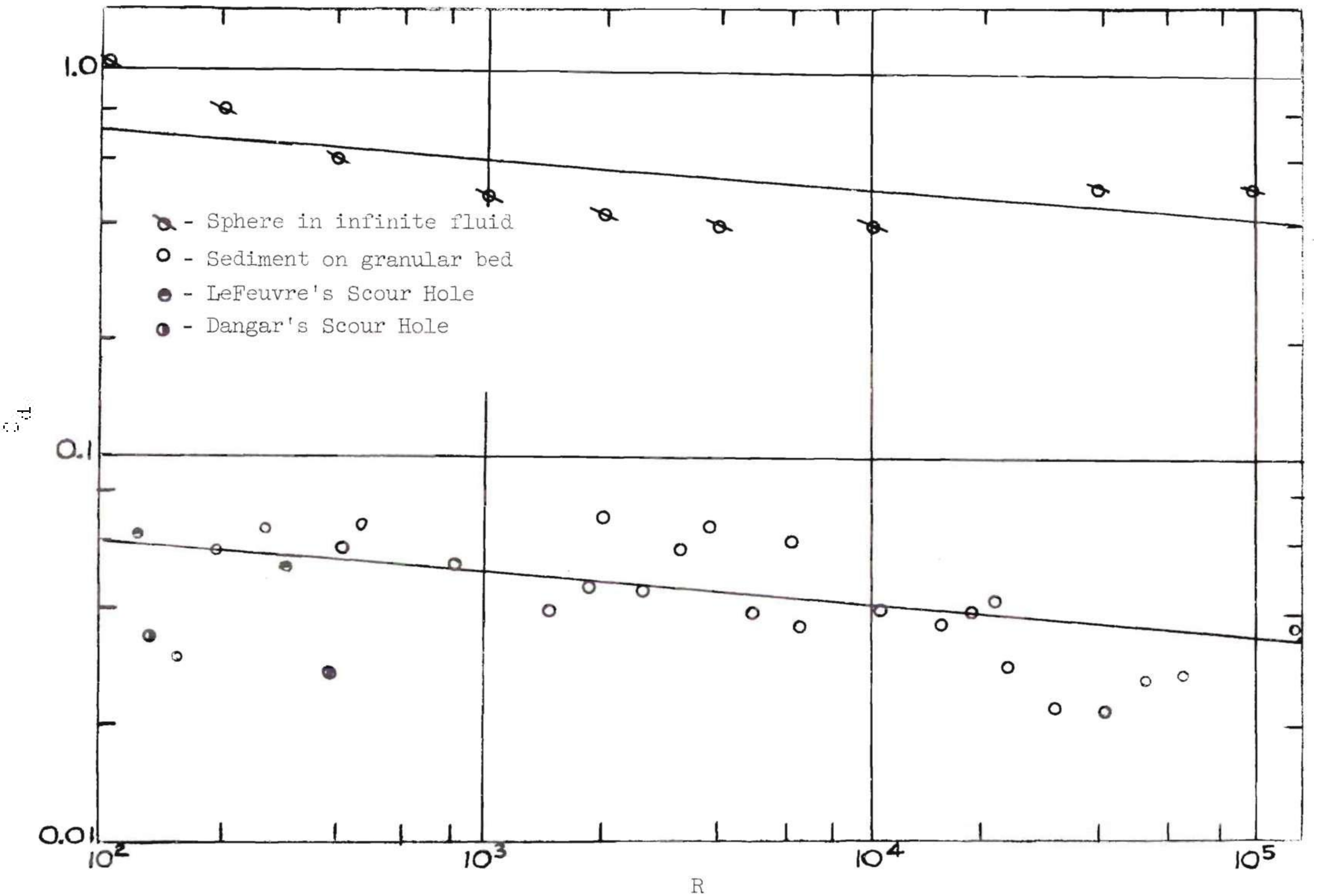

Figure 9. Variation of C_d with R

Thus, N_s cannot be a constant but must increase as the coefficient of drag decreases. If N_s is increasing, equation (8) reveals that V must increase at a faster rate than does the parameter $\sqrt{(s-1)gd_m}$. Therefore, the slope of the line in Figure 6 must be greater than one.

CHAPTER V

CONCLUSIONS

The significant conclusion obtained in this investigation is that the value of N_s at incipient-motion is

$$N_s = 1.53\ R^{0.0375}$$

for particles lying on a hydraulically rough boundary. The velocity used in evaluating the Reynolds number, R, is the velocity just above the particles.

APPENDIX

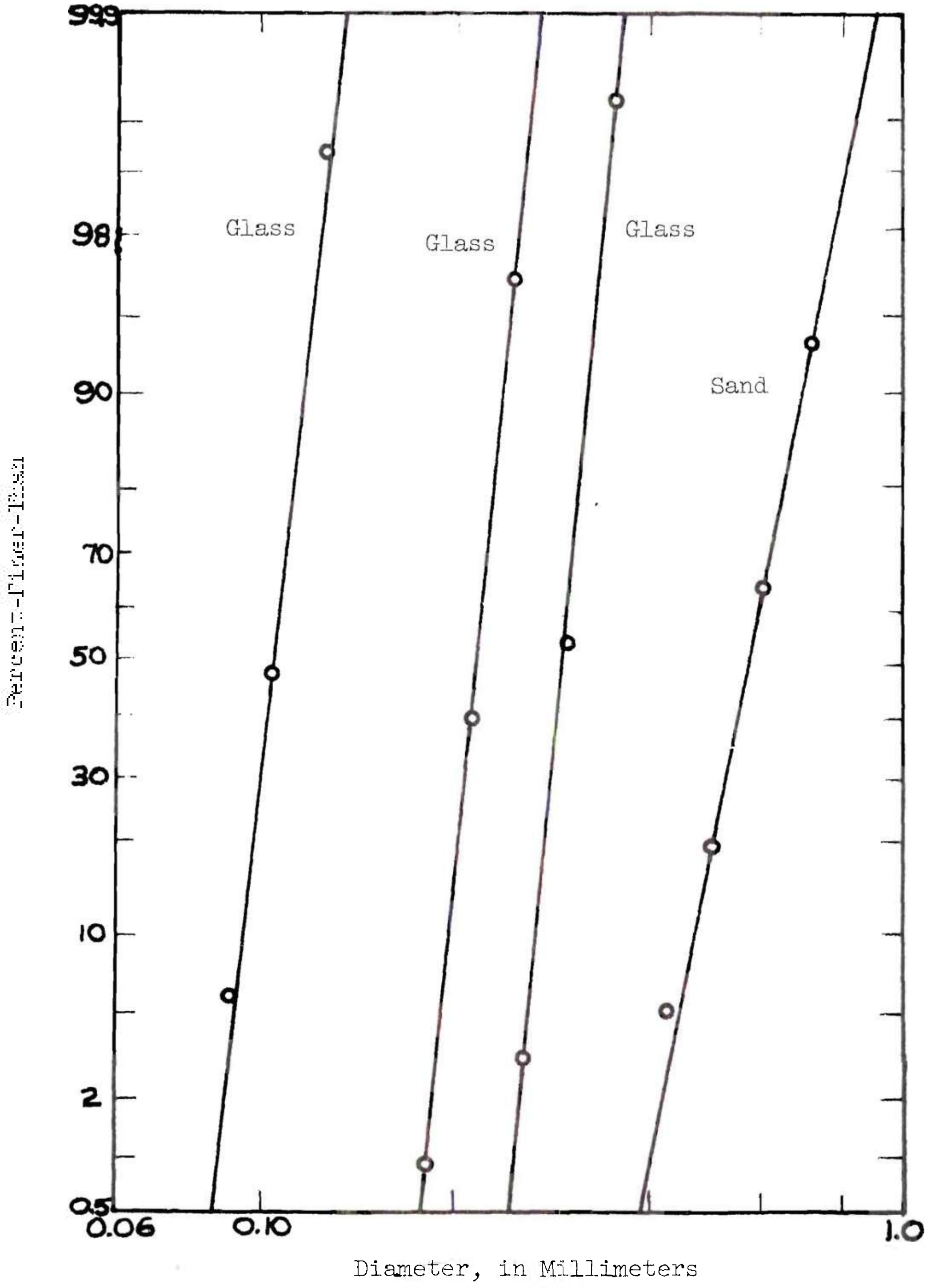

Figure 10. Sieve Analysis

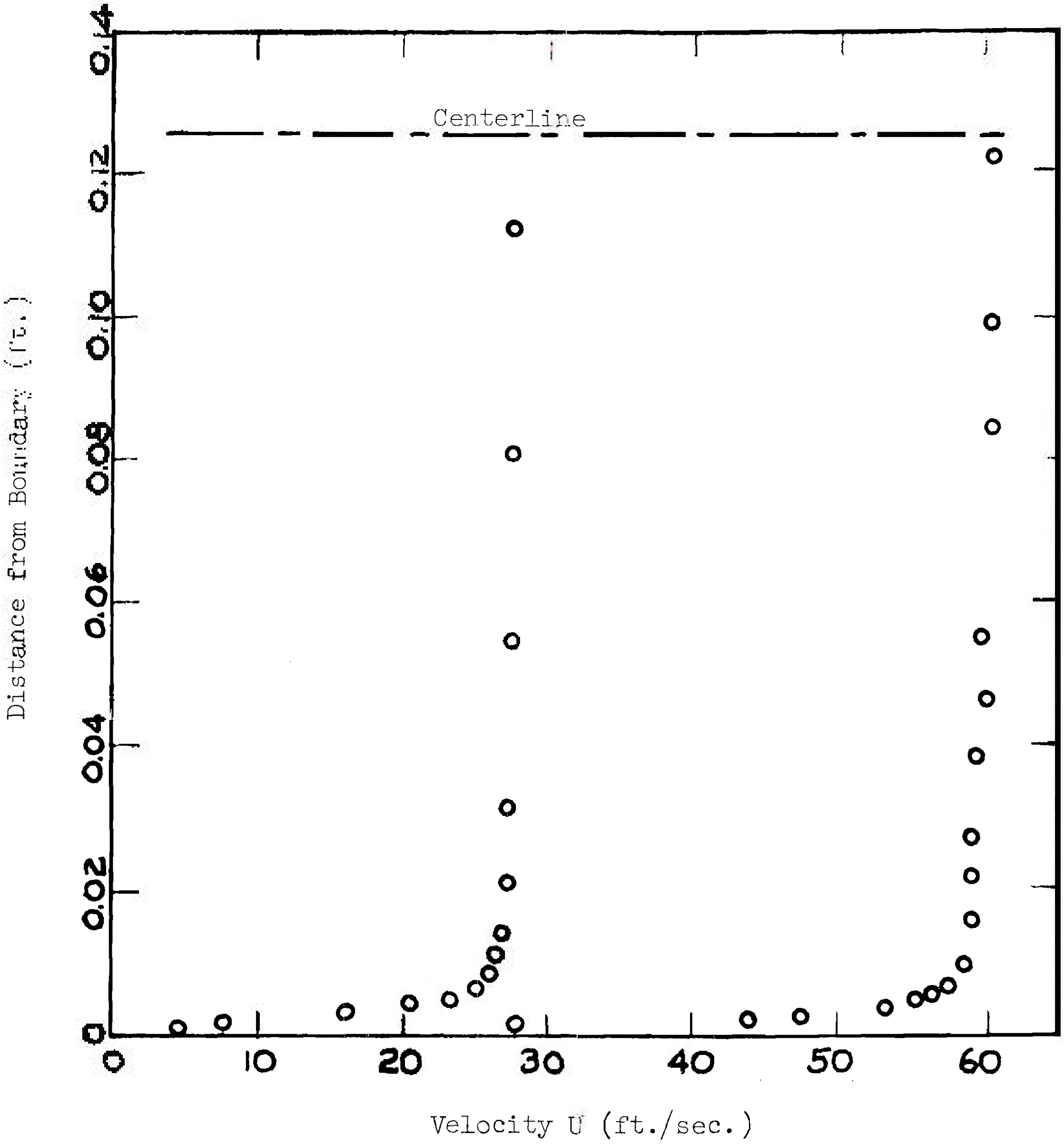

Figure 11. Velocity Profiles

Table 3. Sediment Properties

Material	Mean Diameter (mm)	Geometric Standard Deviation	Specific Gravity
Glass	0.102	1.09	2.47
Glass	0.215	1.09	2.47
Glass	0.294	1.08	2.47
Ottawa sand	0.565	1.16	2.60

Table 4. Incipient Motion Data for Glass Beads (Diam. = 0.102 mm)

Run No.	Incipient Velocity	Sediment Number
1	77.0	15.8
2	76.3	15.7
3	73.0	15.0
4	72.0	14.8
5	71.0	14.7
6	71.0	14.7
7	71.5	14.7
8	72.4	14.8
9	72.0	14.8

Table 5. Incipient Motion Data for Glass Beads
(Diam. = 0.215 mm)

Run No.	Incipient Velocity	Sediment Number
1	86.4	12.3
2	92.2	13.1
3	86.0	12.2
4	94.0	13.4
5	91.4	13.0
6	89.5	12.7
7	92.0	13.1
8	92.2	13.3
9	86.0	12.4
10	85.2	12.2
11	84.0	12.1
12	89.0	12.7
13	88.6	12.7
14	89.7	12.9
15	86.0	12.3

Table 6. Incipient Motion Data for Glass Beads
(Diam. = 0.294 mm)

Run No.	Incipient Velocity	Sediment Number
1	102.9	12.5
2	101.3	12.3
3	100.0	12.2
4	105.4	12.8
5	103.0	12.6
6	103.0	12.6
7	104.0	12.7
8	103.0	12.6
9	105.2	12.9
10	109.0	13.3

Table 7. Incipient Motion Data for Ottawa Sand
(Diam. = 0.565 mm)

Run No.	Incipient Velocity	Sediment Number
1	161.0	13.8
2	157.8	13.5
3	164.0	14.1
4	159.5	13.7
5	162.0	13.9

BIBLIOGRAPHY

1. Brahms, A., Anfangsgrunde der Deich-u Wasserbaukunst, 1753, as referenced in An Introduction to Fluvial Hydraulics, Leliavsky, S, Constable and Company Ltd, London, 1955, p. 34.

2. Fortier, S., and Scobey, F. C., "Permissible Canal Velocities," Transactions, American Society of Civil Engineers, Vol 89, 1926, pp 940-984.

3. Mavis, F. T., "The Transportation of Detritus by Flowing Water - I," Bulletin 5, University of Iowa Studies in Engineering, Iowa City, 1935.

4. Mavis, F. T., and Laushey, L. M., "Formula for Velocity at Beginning of Bed-Load Movement is Reappraised," Civil Engineering, Vol 19, No 1, January, 1949, pp 38-39.

5. Jeffreys, H., "On the Transport of Sediments in Streams," Proceedings, Cambridge Philosophical Society, Vol 25, 1929, pp 272-276.

6. Chepil, W. S., "Dynamics of Wind Erosion: II. Initiation of Soil Movement," Soil Science, Vol 60, 1945, pp 397-411.

7. Hjulstrom, Felip, "Studies of the Morphological Activity of Rivers as Illustrated by the River Fyris," Bulletin, Geological Institute of Upsala, Vol XXV as referenced in "Sediment Transportation Mechanics: Initiation of Motion," Proceedings, American Society of Civil Engineers, HY 2, March, 1966.

8. LeFeuvre, A. R., Sediment Transport Functions with Special Emphasis on Localized Scour, Ph.D. Dissertation, Georgia Institute of Technology, Atlanta, Georgia, 1965.

9. Schlichting, H., Boundary Layer Theory, McGraw-Hill, 4th Ed., New York, 1960, p. 123.

10. Rouse, Hunter, Engineering Hydraulics, John Wiley and Sons, Inc, New York, 1950, p. 122.