THE CONTROL OF AGGREGATE INVENTORY LEVELS IN A MULTI-ITEM INVENTORY SYSTEM

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THE CONTROL OF AGGREGATE INVENTORY LEVELS IN A MULTI-ITEM INVENTORY SYSTEM

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To my Joey

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SUMMARY

This thesis is directed at a control procedure for minimizing aggregate inventory level fluctuations in a multi-item inventory system. The control procedure, COPEAK, is designed to reduce these inventory level fluctuations while maintaining minimal levels of operating costs.

A theoretical multi-item inventory system is examined when it operates under the specially designed control procedure. Various heuristic ordering rules designed around an "optimal reordering range" for each item within the system in lieu of an "optimal reorder point" are examined and tested through a computer simulation model. Results using the control procedure are statistically compared to a similar system having no control procedure but which instead uses individually derived optimal reorder points for each item.

Analysis of the data obtained from the simulation indicates that the control procedure derived in this research effectively reduces the fluctuation of aggregate inventory levels without imposing severe cost penalties on the system.

CHAPTER I

INTRODUCTION

Extensive work in the field of analyzing inventory systems has been undertaken in recent years for the purpose of investigating a multitude of inventory related problems. Research has proposed solutions for solving complex mathematical problems related to how much to order and when to order in deterministic as well as stochastic inventory systems. Systems constrained by items such as warehouse capacity, budget limitations, and other managerial requirements have been studied resulting in the development of many different operating policies.

In a multi-item inventory environment, deriving an independently optimal inventory policy for each item may not always be the complete answer to the development of an efficient inventory control procedure. Oftentimes, in sophisticated inventory systems concurrent arrivals of outstanding orders place a strain on the system causing fluctuations of aggregate on-hand inventory amounts. Even though constraints such as warehouse capacity or dollar investment may be met, monetary losses attributable to severe fluctuations in inventory levels create less than optimal conditions. Most merchandizing firms receive economic discounts by an expeditious payment for goods received. If one such company is operating under a stringent budget, then depending on the number of different stock deliveries that arrive in any short span of time, it may not be able to effectively take advantage of all the price discounts possible at that time. Also, efficient utilization of warehouse personnel can be affected by the fluctuations of aggregate on-hand inventory. When the inventory level becomes too large, plant managers may have to hire part-time personnel to accommodate the untimely shipments and rent more warehousing equipment (i.e., forklifts, etc.) to assist in the receiving operations. Conversely, during periods when aggregate inventory levels are at a minimum, warehouse personnel may incur idle time which can lead to morale problems and create worker dissatisfaction. Another consideration is the fact that during times of extreme fluctuations of on-hand inventory, the time required and space available for offloading may force plant managers to rent additional space to accommodate the extra shipments. This latter case is exemplified in the payment of railroad demurrage costs by companies. For some firms, when rail shipments are received they can not be offloaded in one day; thus, the company in effect is using the side cars as additional warehouse space and pays "rent" for this service.

If every item in the inventory system had a completely deterministic demand, then these fluctuations in the aggregate inventory level could be minimized. It is only when demands follow a stochastic pattern that inventory fluctuations present a severe problem. This is due to the fact that each item no longer has a fixed "cycle" of reordering. Thus, at some point in time orders for items could arrive simultaneously; this of course assumes that an independently optimal reorder policy is being followed for each item. This may indeed be the fallacy in operating such a multi-item inventory system. In a realistic system, it may be advisable to operate such that independently optimal policies are followed as nearly as possible, but some deviations in the ordering policy are permitted when an aggregate inventory peaking is anticipated.

This discussion and examples of potential monetary losses due to severe fluctuations of aggregate inventory demonstrate a need for policies to reduce erratic deviations about the average aggregate on-hand inventory.

Definition of the Research Area

The objective of this research is to develop a control procedure encompassing a new ordering approach for use in a multi-item inventory system. The goal of this procedure is to reduce the fluctuations around the average aggregate on-hand inventory level (AOHI). The effectiveness of this procedure will be determined by any resulting reduction in fluctuation as compared with an uncontrolled system utilizing independently optimal reordering policies for each individual item. The following measure of aggregate inventory dispersion (fluctuation) was selected as the performance criterion to gauge this effectiveness:

$$(MD)^2 = \sum_{j=1}^{n} (I_j - \overline{AOHI})^2/(n - 1)$$

where I_j is the aggregate inventory level on day j (i.e., at the end of day j), n is the number of days in the planning horizon, and $\overline{AOHI}$ is the average aggregate on-hand inventory over the planning horizon. This measure of dispersion is calculated in both the controlled and uncontrolled systems each having similar average AOHI amounts. An alternative criterion for evaluating the derived procedure was selected as the rise in conventional operating costs compared to similar costs resulting from the uncontrolled system.

Research Procedure

To accomplish the objective of developing an effective control procedure as described above, an analysis of the performance of a multi-item inventory system with independently derived ordering policies for each item was first conducted. A simulation model, written in GASP II, a Fortran based simulation language, was used to furnish total annual operating costs and average AOHI information for such a system. Next, the control procedure was designed and tested using the same simulation model, modified to accommodate the revised ordering procedure. Statistical comparative analysis was used to determine the effectiveness of the new procedure.

Survey of the Literature

A myriad of research projects have been conducted in the field of inventory management. Whitin (1) and Plossal (2) present brief chronologies of the early work performed in this area. Results of any significance were first published in 1915 with the basic concept of the economic lot size model presented in 1934 by R. H. Wilson. From the mid 1930's to the late 40's and early 50's research endeavors in inventory management were slowed due to an economic depression which saw firms abandoning management techniques to "stay alive." Then followed World War II. Shortly after the end of the war, expanded emphasis was placed on Operations Research and scientific management techniques in which inventory analysts began studying multi-item systems as opposed to single-item systems.

Many researchers have contributed to analyzing the multi-item inventory system. Churchman, Ackoff, and Arnoff (3) in 1957, using the Economic Lot Size model, discussed procedures for obtaining optimal reorder sizes. Through the use of the Lagrangian Multiplier technique they presented methods for the solution to constrained multi-item inventory problems. Since Churchman's publication, a vast number of articles have been written on this subject, each addressing a different facet pertinent to some real world situation. Evans (4) developed a dynamic programming model in an effort to minimize total operating costs in an inventory system permitting lost sales. Gordon P. Wright (5) researched optimal policies for a multi-item system with "negotiable" lead times and backorders permitted. In his research, he permitted "emergency orders" having lead times different from normal orders. Holt (6) presented the results of his analysis of an Economic Lot Size model in an constrained environment. He was not concerned whether the constraint was monetary, or related to storage capacity or production limitations. In his research Holt developed a methodology which used quadratic approximations to the Economic Lot Size cost functions to obtain general solutions for determining the optimal lot sizes for individual items when a constraint had been placed on the aggregate inventory.

Another multi-item inventory model of interest was proposed by Balintfy (7) in 1964. It was called the "Random Joint-Order Policy." Whenever the stock of a given item reached its reorder point, the inventory levels of the other remaining items were checked to determine whether any of them had also reached a pre-established "can-order" point. The items whose stocks were below their"can-order" points were ordered concurrently along with the item whose on-hand level had reached its reorder point. His research in developing "can-order" points was centered around comparing classes of multi-item inventory problems where joint order of several items could save a part of the ordering setup cost.

Most of the literature surveyed dealt with constrained inventory systems, each having their own peculiarity. Many of these systems proposed solution techniques involving the use of the Lagrangian Multipliers or related concepts since most of them centered around either a budget or warehouse capacity constraint imposed on the AOHI. In 1958 Baumes (8) conducted a study in which he attempted to highlight the problems generally encountered in the area of constrained inventory problems. He wrote

> While some companies rent additional storage space to provide for their peaking requirements, most companies are trying to schedule their deliveries to provide for more frequent shipments from the vendor.... These companies protect their volume price by placing a firm order for a six month's or a year's supply and giving the vendor a shipping schedule.

Although the problem of inventory fluctuations was mentioned here, no attempts were made to pursue any solution techniques.

In researching the literature related to constrained multi-item systems, it was found that most of the efforts have been directed at a maximum constraint on capacity. Little if anything is said about operating policies to reduce day-to-day variations in the aggregate inventory level. Aside from the brief acknowledgement by Baumes, no research was located which addressed the problem of developing an effective approach to reduce inventory fluctuations in an multi-item inventory system.

CHAPTER II

DESCRIPTION OF THE CONTROL PROCEDURE, "COPEAK"

This chapter provides an orientation into the algorithmic control procedure developed in this research. In addition, the various parameters inherent to this procedure along with rules used in establishing ordering priority among items are discussed.

General Description

This control procedure for reducing peakings of aggregate on-hand inventory levels, COPEAK, was designed using properties analogous to both the Wilson (Q,r) model and the periodic review (S,s) system. Daily, the inventory level of each item is checked to determine which items are within a specified time frame away from their reorder level as determined by the Wilson reorder point, r. A prediction matrix is computed from forecasts of the inventory levels of each item for a fixed planning horizon. Various techniques specifically developed for this procedure use these forecasts to estimate the aggregate inventory level for the expected arrival data of a particular order. Next, priority ordering procedures are used to regulate arrivals of orders around this expected arrival date.

The Inventory Level Prediction Matrix and Associated Ordering Procedures

COPEAK functions around the use of a matrix (reference Figure 1) capable of forecasting the daily inventory levels over a fixed planning

Days in the Planning Horizon

Row Information:	1	2	...	n-1	n
Expected daily demand					
Orders due-in					
Probable due-in					
Estimate of the daily AOHI					

Figure 1. The Prediction Matrix.

horizon. Row 1 of this matrix contains daily demand information expected to be imposed on each item in the system. This information is derived from results of historical demand data. Row 2 contains outstanding orders which are expected to arrive on a given date. Row 3 contains a projection of "probable" due-ins. Based on the arrival date of an outstanding order of a complete or fractional quantity of Q_i^*, COPEAK determines the projected arrival date of the next anticipated order (Q_i^*) and designates this as a "probable" due-in. Then using expected cycle lengths, reorder quantities, Q_i^*, are loaded in the matrix on their appropriate due-in dates starting with the first probable due-in. Through the use of an accounting procedure and also by considering current on-hand quantities, estimates of anticipated inventory levels for any day of the planning horizon are acquired and appropriately stored in Row 4 of the prediction matrix. After selecting an item for ordering using one of several ordering priority rules developed for this control procedure, COPEAK then uses the average procurement lead time for the item (obtained from historical data) to acquire its expected lead time. From the data contained in the prediction matrix for this expected arrival date, a determination is made as to what quantity of an item could arrive without forcing the estimated inventory level for that day to exceed a desirable limit. The design of this procedure permits the ordering of a variable amount of the optimal Q_i^*; thus, for this research a parameter, AMTQ, was introduced to permit the system to order a percentage of Q_i^* which is greater than or equal to AMTQ. If the AOHI level in relationship to a predetermined ceiling limit is such as to preclude the ordering of an item, it will be considered in the next daily review. Figure 2 contains a general flow chart of the control procedure. It is noted that due to the stochastic nature of the demand pattern, entries contained in the prediction matrix are only estimates and as such will not always constrain the AOHI level at the desired maximum limit.

Summary of the Parameters Used in the Control Procedure

The following is a discussion of the basic input parameters which are a part of COPEAK:

1. "Days-to-Order": This parameter permits COPEAK to examine each item a fixed number of days before its inventory level reaches its optimal Wilson reorder point, r. If the inventory level of an item is such that the expected number of days left till reordering is less than or equal to "Days-to-Order," this item is placed in the ordering queue for consideration by COPEAK.

2. AMTQ (Amount of Q_i^*): As mentioned earlier, this parameter sets the minimum limit on the percentage amount of the Wilson Q_i^* which

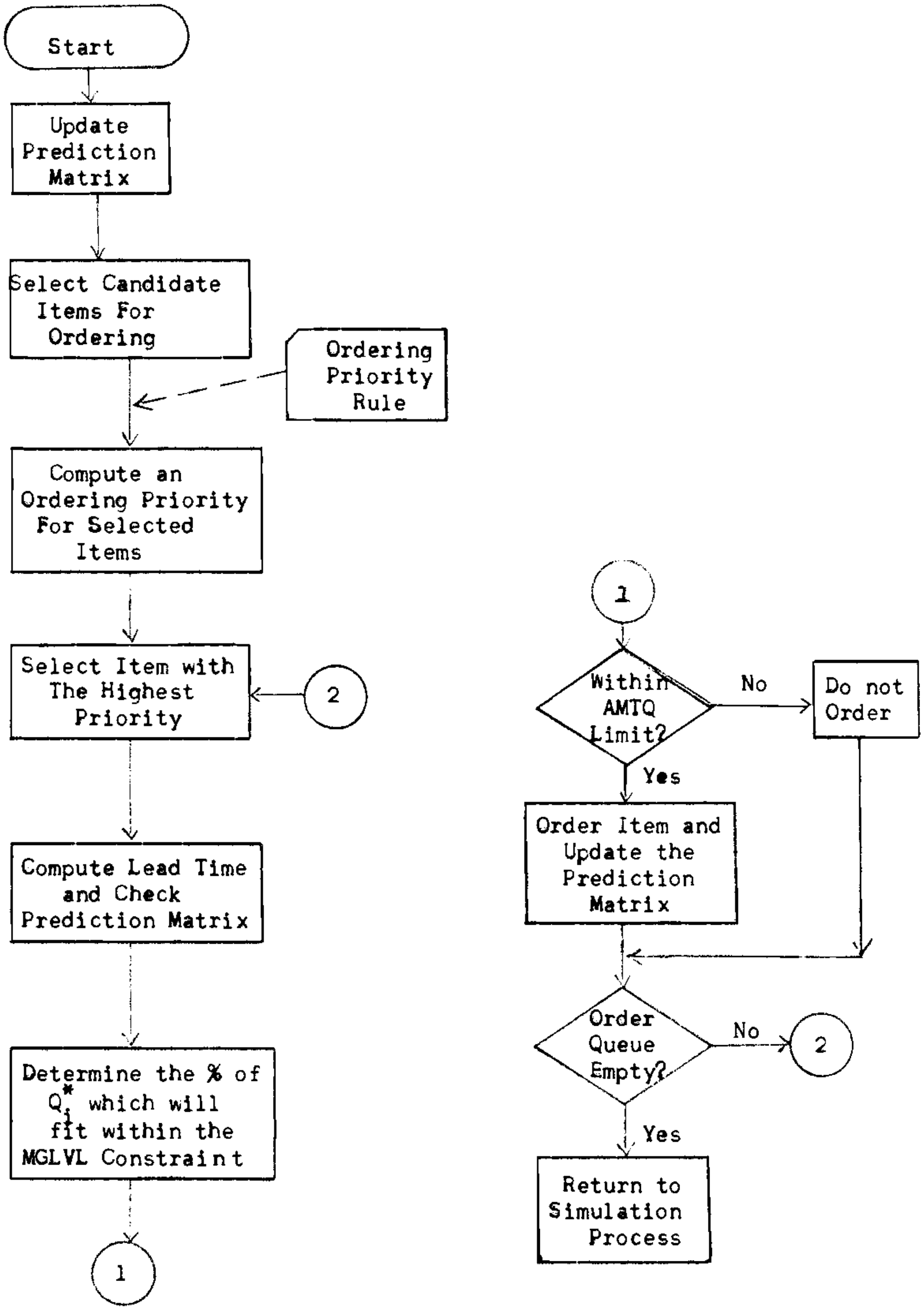

Figure 2. General Flow Chart of the Control Procedure COPEAK.

can be ordered for item i. The use of this parameter thus permits the ordering of fractional values of Q_i^* and as such plays an integral part in the design of COPEAK.

3. AMTPIN (Amount Projected In): This parameter is used by Row 4 of the prediction matrix in its determination of the system's expected AOHI for any given day within the fixed planning horizon. AMTPIN specifies the percentage level of Row 2, "probable" due-ins, which is to be considered in the calculations necessary to compute the information for Row 4. A value of 0% would negate Row 2 completely, whereas a value of 100% would cause all of Row 2 entries to be used in compiling the data for Row 4.

4. MGLVL (Management Level): This is a predetermined ceiling limit on the anticipated AOHI amount for any given day.

5. ILOOK: This parameter performs a "smoothing" effect on the AOHI for a set number of days beyond the expected arrival date of an order. The value of ILOOK stands for the number of days beyond the expected arrival date that are to be included in the ordering decision process. This insures that the arrival of a shipment does not cause the AOHI to exceed the MGLVL for any one of "ILOOK" days <u>beyond</u> the due-in date.

The Ordering Priority Rules

Daily under COPEAK the time required for each item to reach its reorder point, r_i^*, is estimated using daily demand rates obtained from historical data. Similar to Balintfy's "can-order" points COPEAK uses the input parameter, "Days-to-Order," which establishes an ordering range ahead of an item's optimal reorder point during which it is permissible to order this commodity. If an item is within this range, it is selected as a candidate for ordering. All of those selected are then subjected to a ranking scheme which assigns ordering priorities. Those items are first ordered which would contribute the least cost in terms of holding and backorder costs but would keep the AOHI level as close to the MGLVL as possible. Consequently, to satisfy this design criterion four heuristic rules were developed for assigning priorities to those items in the ordering candidate queue.

Rule 1:

It seemed logical that the first axiom would be related solely to the Wilson Economic Lot Size theory which holds that it is optimal to reorder an item when its stock level reaches its r_i^*. Thus, ranking the items merely on the premise that the one with the smallest "Days-to-Order" should be the first ordered was the initial policy tested.

Rule 2:

The rationale for this rule focused around creating a ratio based on unit daily holding and backorder costs. Thus, for example, an item having a high holding cost and low backorder cost would have a high ranking and ordered last whereas the reverse situation would place an opposite ranking causing an item with a high backorder cost to be ordered first. Although the reasoning behind this rule fundamentally remained the same, some added refinement to this concept resulted in the final version being:

$$\frac{(\text{constant} + \text{"Days-to-Order}_i\text{"})(\text{unit daily holding cost}_i)(\text{avg }\#_i\text{ held to date})}{(\text{unit daily backorder cost}_i)(\text{avg }\#_i\text{ backordered to date})}$$

This rule accommodates for the stochastic demand situation in utilizing actual prior information concerning the average number of item i held and average number of item i backordered. The constant, DDAY, is used to keep an item with a slightly negative "Days-to-Order" from overwhelming an item with a positive "Days-to-Order" value but extremely high backorder cost and which conceivably should be ordered first. For this research, a value of 25 was heuristically chosen for this constant.

Rule 3:

The third rule was designed similarly to rule 2. For items having a negative "Days-to-Order" value the ratio was a negative inventory holding cost_i to a factor equal to the backorder cost_i multiplied by "Days-to-Order_i." The "Days-to-Order" factor in the denominator weights the rule such that a smaller "Days-to-Order" value can favor an item resulting in a prime ranking priority. The negative constant allows for the possibility of a high ranking of an item which, although already late in being ordered, may have such a low backorder cost as to not be desirable for ordering. Not ordering such an item may permit space within the MGLVL constraint for the subsequent ordering of an item possibly having a higher backorder cost. For items with a zero "Days-to-Order" value, a ratio of inventory holding cost_i to backorder cost_i was formulated. Lastly, for the positive "Days-to-Order" case, the ratio used was one of inventory holding cost_i multiplied by "Days-to-Order_i" divided by backorder cost_i.

Rule 4:

The last rule devised was an additive function relating an item's "Days-to-Order" value, backorder cost, and inventory holding cost. Weighting constants are introduced to yield the following rule:

$$\text{Priority} = (\text{"Days-to-Order}_i\text{"}) + C_1(\text{backorder cost}_i) + C_2(\text{inventory holding cost}_i) \; .$$

For the main part of this research constants C_1 and C_2 were fixed at -.1 and +.1 respectively. No attempts were made to ascertain optimal values for them; however, as discussed in Chapter IV, in order to evaluate the sensitivity of the system to these parameters, C_1 and C_2 were assigned different values with no appreciable change in results.

CHAPTER III

SIMULATION EXPERIMENT

This chapter provides a description of the simulation model used in this research along with a discussion of the test data and the methods used to verify the simulation process. Lastly, this chapter delineates the sequence followed in conducting the simulation testing and resulting statistical analysis. All of the computer programs used in this research were executed using the UNIVAC 1108 computer located at Georgia Institute of Technology.

The Simulation Model

This research is centered around a comparative analysis of an uncontrolled inventory system using independently derived optimal ordering policies for each item to a system which uses the control procedure discussed in Chapter II. To accomplish this, a computer simulation program was written in GASP II (9), a programming language which inherently has the necessary housekeeping routines for conducting a simulation while the user writes subroutines pertinent to those events being simulated. This program models a multi-item inventory system which operates under a continuous review (Q, r) policy with backorders permitted. It consists of a main program and 30 subroutines of which 6 are user written. Appendices C and D contain these latter routines and certain selected output listings relevant to this program. This program is capable of simulating demands, orders, receipts, and performing accounting functions and pertinent calculations to derive cost and statistical data for each item in the system. Figure 3 is a general system flow chart depicting the functions of this model.

To accommodate the unique ordering approach used by the newly developed control procedure, the basic model was altered slightly. Since COPEAK is in essence a periodic review system in that all items are reviewed _daily_ for possible ordering, a modification to the method of review and ordering had to be made to the simulation model in order to test this procedure. A separate subroutine, ORDER, was written to accomplish this. Subroutine ORDER, Appendix C, functions around the use of the prediction matrix and uses an ordering procedure and other processing steps as outlined in Figure 2, General Flow Chart of the Control Procedure, COPEAK.

The particular inventory system under study allows for ordering costs, holding costs, and time variable backorder costs. All demands which are incurred by the system when out of stock are thus met through backordering. Demands are assumed to follow a stochastic pattern with a normally distributed time between demand occurrences.

In the uncontrolled procedure, each item is reordered in an amount Q_i^* exactly when its reorder point is reached regardless of the inventory position of other items in the system. For this research the reorder quantity and reorder position for each item used in both the uncontrolled and controlled systems were determined through approximate methods with adjustments after preliminary simulation trials.

The simulation model provides total operating costs by computing the ordering costs, holding costs, and backordering costs incurred by the

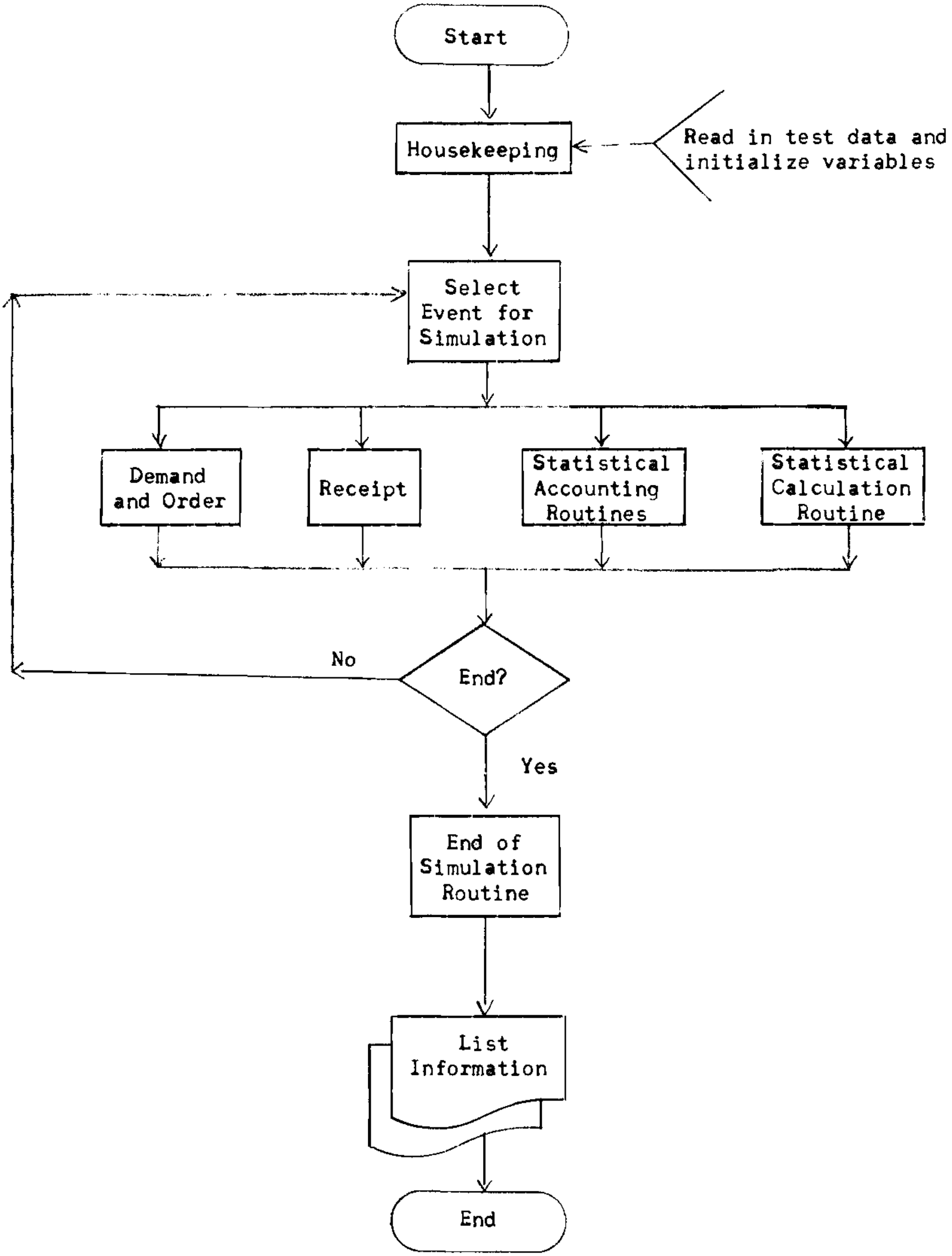

Figure 3. General System Chart of the Simulation Model.

system. It also computes the measure of dispersion, $(MD)^2$, as described in Chapter I.

System Parameters

The parameters composing the test data were not acquired from any real world situation. Certain rules used to establish parameters for use in recent research by Byrne (10) were used as guidelines for creating the test data in this research. Byrne found that past and current literature on the setting of system parameters seemed to indicate that any value for one would be acceptable so long as it reasonably lent itself to the particular research and provided it did not weight the use of the other parameters creating possible correlation between results and selected items of test data. The parameters pertinent to the test data used in this thesis are the following:

Ordering Cost: The value selected was chosen arbitrarily and assumed constant for each item.

Unit Cost: Unit costs were randomly determined from a range of 0 to 110 dollars per item.

Inventory Holding Rate: Research conducted by Byrne (10) and his findings in the literature yielded a minimum value of .15 dollars per unit of stock held per year to a maximum value of .35. All inventory holding rates were randomly chosen with uniform probability using these limits.

Backorder Cost: A time-variable backorder cost was randomly selected from a uniform distribution using values between 0 and 100 dollars. No fixed backorder costs were assigned.

System Operating Cost: The simulation model did not consider the real cost of implementing and executing either a continuous review system or a periodic review imposed by the control procedure, COPEAK. Although not explicitly used in the statistical evaluations, this cost can be important and will be considered in Chapter IV.

Lead Time: Procurement lead times were assumed to be constant for each item and were assigned using intuitive logic to insure a complete and representative mix of parameter data.

Mean Demand Size: This parameter was assumed to be unity for each item.

Time Between Demands: This parameter was considered to be normally distributed. The mean for each item was randomly selected from an uniform distribution with limits .01 to 4.0 days between demands. Research has shown for items having a reasonably large demand rate per fixed time period with inter-arrival times normally distributed that the demand rate itself is approximately normal (11). Appendix A lists two sets of test data, data set 1 and set 2, constructed from these parameters and which were used in this research.

Program Verification and Preliminary Experimentation

Great care was taken to insure that the simulation program was operating properly. Results of preliminary runs were compared with approximate solutions via analytical techniques. Simulation solutions obtained from using data set 1, for example, were directly compared with approximate analytical solutions. Results are shown in Appendix C.

An analysis was performed on the independence of the daily AOHI observations used to compute the measure of dispersion, $(MD)^2$. Data set 1 was used in five-year simulations with and without COPEAK in which the last 200 daily AOHI amounts were subjected to a length of run test and a test for runs above and below the median as described in Duncan (12). Results showed that the daily observations of the aggregate inventory level could not be considered perfectly independent. Because of this, the term "Variance" was not used to describe the measure of dispersion of observations about the average aggregate inventory level.

The computer simulation program used in this research was designed to operate for a maximum simulation period of five years. Since the computer time required for either a 3, 4, or 5 year simulation was about the same, it seemed desirable to select the maximum run time possible to insure a representative mixture of simulated events. To insure that the length of simulation in no way affected the experimental results, data set 1 was used in a 5, 4, and 3 year simulation. The initial random number seed was changed five times with each seed being used in a new 5, 4, 3 year sequence of computer runs. These seeds were subjected to five tests to insure randomness of the resulting stream of numbers. The tests included (1) Chi-Square Goodness of Fit Test, (2) Kolmogorov-Smirnoff Goodness of Fit Test, (3) Chi-Square Serial Test, (4) Chi-Square test for length of run, and (5) a test for total number of runs. The results of the five replications using data set 1 provided information to construct a one-way Analysis of Variance (ANOVA) Table (13), (14). An F test was used to detect any significant difference in the resulting average AOHI for each simulation run length. Appendix B, Table 6, displays the observations obtained and the resulting ANOVA table. With the significance level of the test (α error) set as high as .75 a $F_{.75,2,12}$

statistic yielded a value of .295. Thus, the null hypothesis that the average AOHI obtained from either a 5, 4, or 3 year simulation came from the same population could not be rejected. A similar analysis was conducted to test the measure of dispersion about the average AOHI for each of the three time simulations. Appendix B, Table 7, displays the observations and the resulting ANOVA table. With an alpha error of .75, and an $F_{.75,2,12}$ statistic of .295 the null hypothesis that the means for the measure of dispersions from any of the three different time simulations were the same could not be rejected. All further experimentation was conducted using a length of five years.

Experimental Procedure

With the measure of effectiveness for use in evaluating the merits of the control procedure, COPEAK, already determined, the first step in the experimental procedure was to acquire information using data set 1 without COPEAK. Five computer runs, each with a different random number seed, were conducted using a system with independently derived optimal ordering policies (Q, r) for each item in data set 1. From these runs an average value of approximately 133 was derived for the average AOHI and 35 for the measure of dispersion about this average. The resulting annual operating cost averaged $1034.0.

To effectively compare COPEAK with its uncontrolled counterpart, it was highly desirable to keep the average aggregate inventory at the same level for both procedures. Thus, the measure of dispersion, $(MD)^2$, would be about the same mean inventory level in both cases. The major vehicle in controlling the average inventory level under COPEAK was the use of the control parameter MGLVL. This parameter was set to maintain the same average inventory level as under an uncontrolled system. It was found that an unique value for MGLVL existed for each COPEAK ordering priority rule used. The determination of these various values was not independent of the other parameters inherent in COPEAK and its four rules. In the process of acquiring the MGLVL's, reasonable values of these other parameters were determined. No attempts were made to reach their optimum values. The random number seeds were changed five times and computer runs made for each priority rule at its appropriate MGLVL. The results were all then subjected to an ANOVA and accompanying F test to evaluate the effectiveness of COPEAK in reducing the measure of dispersion of the AOHI.

CHAPTER IV

ANALYSIS OF RESULTS

This chapter describes the statistical results derived from comparing computer simulations of data set 1 under an uncontrolled multi-item inventory system and under a system using COPEAK. A brief discussion of some sensitivity analysis performed on the parameters used within COPEAK and its four ordering priority rules is presented. Lastly, a comparison of the Lagrangian Multiplier technique used in a constrained inventory system is made to a similar use of COPEAK.

Simulation Results Using COPEAK

As described in Chapter III after testing the random number seeds computer runs were made of date set 1 operating under COPEAK using each of the four ordering priority rules and also operating under a similar system without COPEAK. Five runs each with a different seed were conducted for each case. The resulting cost data, average aggregate inventory level, and measure of dispersion observations are listed in Table 1. A F test was conducted to determine if any significant differences existed between the average AOHI observations. ANOVA results (Figure 4) yielded a F test statistic of 3.58. A value of 4.43 from Ostel (15) for a $F_{.01,4,20}$ was obtained. Consequently, at a significance level of .01 no differences were determined to exist between the average AOHI observations under any of the systems tested.

Table 1. Results for Data Set 1

		Independently Derived Policies	COPEAK Rule 1	COPEAK Rule 2	COPEAK Rule 3	COPEAK Rule 4
Run # 1	$	1035.1	1212.1	1188.0	1219.9	1185.6
	$\overline{\text{AOHI}}$	133.08	132.82	132.49	131.34	131.36
	$(\text{MD})^2$	33.66	26.75	27.01	27.31	26.18
Run # 2	$	1034.3	1214.6	1228.1	1206.7	1198.5
	$\overline{\text{AOHI}}$	133.09	133.44	133.53	132.91	133.26
	$(\text{MD})^2$	35.23	26.11	26.58	27.19	26.95
Run # 3	$	1032.4	1174.6	1195.4	1241.0	1189.2
	$\overline{\text{AOHI}}$	132.63	133.15	132.23	132.30	131.44
	$(\text{MD})^2$	34.62	27.07	25.96	27.34	26.43
Run # 4	$	1034.2	1191.0	1182.7	1230.2	1227.9
	$\overline{\text{AOHI}}$	132.68	132.31	132.24	130.43	130.79
	$(\text{MD})^2$	35.15	26.56	25.49	26.83	25.56
Run # 5	$	1034.0	1179.1	1185.9	1232.0	1195.7
	$\overline{\text{AOHI}}$	133.26	133.12	132.08	131.93	132.15
	$(\text{MD})^2$	34.83	26.87	26.29	27.07	25.96
AVERAGE	$	1034.0	1194.3	1194.4	1226.0	1199.4
	$\overline{\text{AOHI}}$	132.95	132.97	132.51	131.78	131.80
	$(\text{MD})^2$	34.70	26.67	26.27	27.15	26.22

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	6.869	4	1.71725	3.58
Error	9.590	20	.47952	
Total	16.459	24		

Figure 4. ANOVA (Average AOHI Observations - Data Set 1).

Another ANOVA was performed on the test data, this time to detect any significant differences between the measure of dispersions resulting from simulations of the uncontrolled system and one using COPEAK. Figure 5 summarizes the ANOVA results. The resulting F statistic of 283 when compared to an $F_{.01,4,20}$ value of 4.43 resulted in the rejection of the null hypothesis that all five treatments did in fact have a common mean value for the measure of dispersion criterion. To further pursue this, a Duncan's Multiple Range Test (14) used to analyze the treatment means was conducted. A set of least significant ranges was derived, and when the treatment means were arranged in ascending order, all four COPEAK methods were determined to possess significantly different mean values for the measure of dispersion criterion when compared to the results from the uncontrolled system, (Table 20, Appendix E). It should be noted that rules two and four also exhibited a significant difference from rule 3.

A histogram was prepared, Figure 6, comparing the AOHI amounts which resulted from a simulation of data set 1 under both the uncontrolled

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	266.711	4	66.678	283
Error	4.723	20	.236	
Total	271.434	24		

Figure 5. ANOVA (Measure of Dispersion Observations - Data Set 1).

and controlled system. Inventory fluctuations under COPEAK appear to be reduced about the average AOHI of 133.

To determine if any significant differences existed in terms of COPEAK's annual operating costs, the cost data relevant to COPEAK's four priority rules was also subjected to a F test (Figure 7). A F test statistic of 3.89 was computed and compared to a $F_{.01,3,16}$ value of 4.89 to arrive at a conclusion that no differences did in fact exist at a significance level of .01.

At this point in the research a comparison was drawn between the cost of using COPEAK to an uncontrolled system in relation to the reduction in the measure of dispersion about the average AOHI. From Table 1, the following average information for the uncontrolled system using independently derived optimal policies was obtained:

Average Annual Operating Cost: $1034.0

Average Measure of Dispersion: 34.70

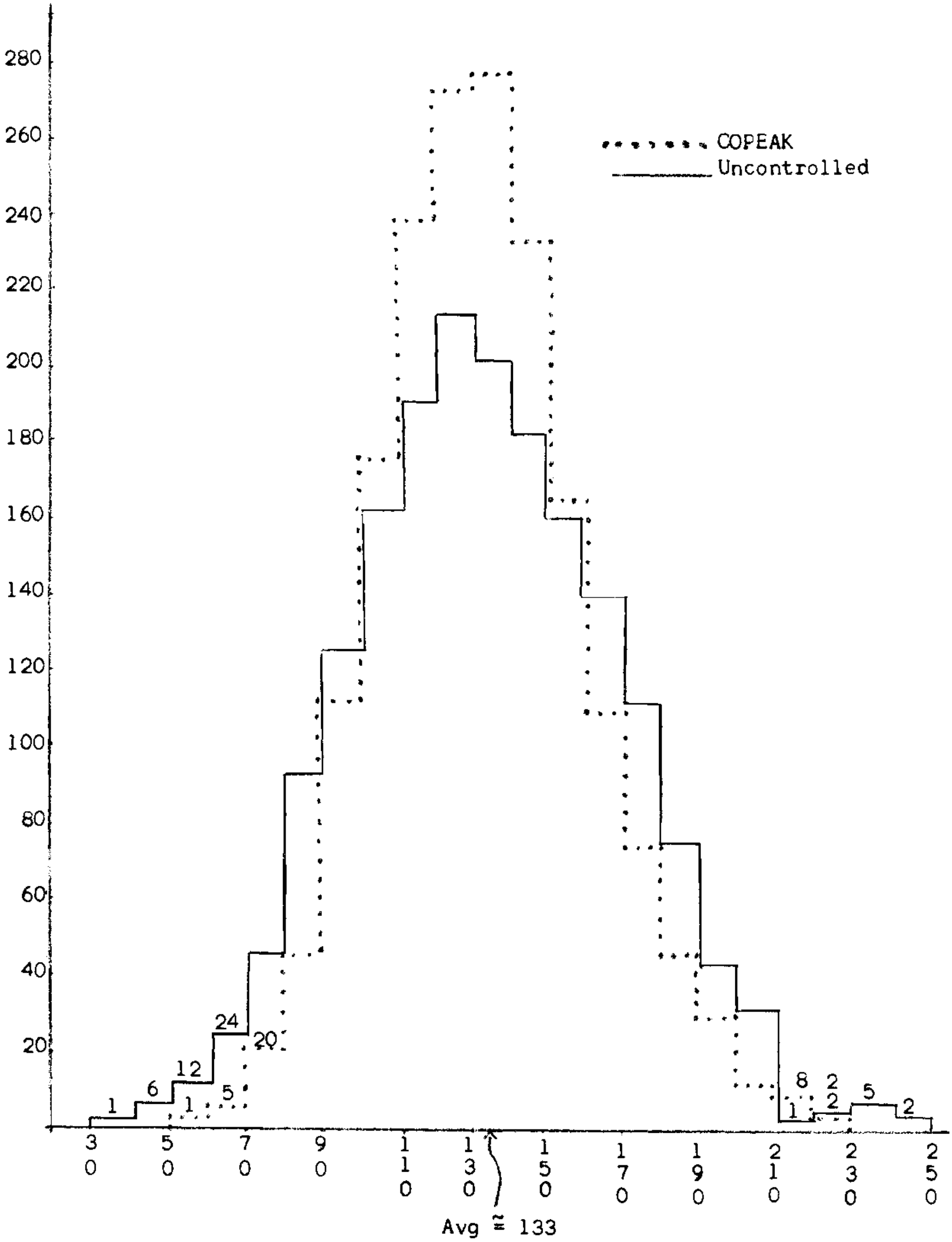

Figure 6. Histogram of AOHI Amounts.

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	3306.70	3	1102.23	3.89
Error	4536.78	16	283.55	
Total	7843.48	19		

Figure 7. ANOVA (Cost Observations - Data Set 1)

From a similar analysis using COPEAK (Rule 1) the following was obtained:

Average Annual Operating Cost: $1194.3

Average Measure of Dispersion: 26.67

The latter represented an average reduction of 23.1% on the measure of dispersion with an accompanying rise of 15.5% in operating costs. As was discussed earlier, operating cost is composed of ordering, holding, and backorder costs, whereas the cost of operating the management information system is not included. Depending on one's subjective evaluation of the financial savings to his system incurred by implementation of a controlled inventory policy, this rise in cost may very well be acceptable. No further investigation of cost information was pursued.

A second set of test data, Appendix A, was evaluated. With appropriate MGLVL values acquired for this data, similar analyses as with data set 1 were conducted. A list of resulting values for the average AOHI and measure of dispersion observations (Rule 1 and 2 only) are shown in Appendix F. An ANOVA was performed on the average AOHI observations resulting in a F test statistic of 3.71. From Ostel (15), an $F_{.01,2,12}$ has a value of 6.93; therefore, as with the original set of test data, the null hypothesis that the means for each treatment are the same could not be rejected at a significance level of .01. Following this, a second F test was conducted on the measure of dispersion observations. The results are at Table 22, Appendix F. An F test statistic of 458 was derived and when compared to an $F_{.01,2,12}$ value of 6.93, the null hypothesis that the treatments yielded the same means could not be accepted.

As with the first set of test data, a comparison of percentage change in measure of dispersion and operating costs between the controlled system (using rule 2) and the uncontrolled system was performed. From the information listed in Table 21 the following was obtained:

Average Annual Operating Cost, (Uncontrolled): $1586.2
Average Annual Operating Cost, (Controlled): $1773.6
Percentage rise in cost: 12.0

Average Measure of Dispersion, (Uncontrolled): 68.9
Average Measure of Dispersion, (Controlled): 48.1
Percentage reduction in Measure of Dispersion: 30.0

No further analysis was conducted using data set 2.

Sensitivity Analysis

A brief sensitivity analysis was performed on different input parameters belonging to COPEAK's four ordering priority rules. Several simulations were made in which the values of these items were changed.

Appendix G summarizes the results obtained. Of interest here are the following points:

1. When AMTPIN (the percentage level of the "projected" due-in Row 2 of the prediction matrix used within the ordering subroutine) was changed from 0% to a value of 75%, the average aggregate inventory level fell drastically as would be expected.
2. When "Days-to-Order" parameter was increased, (reference runs 1 - 4) the average AOHI also increased. This fact made any comparisons of the measures of dispersions extremely difficult.
3. Concerning the constants C_1 and C_2 used in COPEAK (rule 4). As these constants were varied (reference runs 17 - 25) through a series of combinations, no extreme deviations were detected within their average AOHI and measure of dispersion observations.
4. With all other parameters zero, when AMTQ was changed from 75% to a value of 50% (reference runs 2 and 9) total operating cost and the measure of dispersion decreased while the average AOHI increased slightly.
5. When ILOOK was changed from 0 to 4 days (reference runs 2,14,15,16) the average aggregate inventory level fell with an accompanying increase in total operating costs.

It should be emphasized here that oftentimes changing the values for these parameters resulted in new average AOHI amounts which made any comparisons of their measure of dispersions almost impossible. However, further experimentation to acquire the appropriate MGLVL's, which when interacting with given mixes of parameters yielded the same average AOHI amounts, would make comparisons of measures of dispersion possible. Other than what has been mentioned earlier in this chapter, no attempts were made to pursue this fact during the remainder of this research.

A Contrast Between COPEAK and Lagrangian Multiplier Techniques

A three item constrained inventory system with parameters as listed in Table 2 was designed for the purpose of contrasting results obtained from using COPEAK and those acquired by employing the standard Lagrangian Multiplier techniques. A warehouse capacity constraint was placed on the maximum cubic storage space allowable. Constrained optimal Q_i^* were derived analytically using Lagrangian Multipliers. These Q_i^*'s along with their calculated reorder points were subjected to a five year simulation without the use of any control procedure resulting in an average AOHI of 193, a measure of dispersion of 72, and an average annual operating cost of \$3883. The same data using the unconstrained Wilson Q_i^*'s for this data was subjected to a five year simulation using COPEAK (rule 1) with the following results: an average AOHI of 193 with a measure of dispersion of 65, and an average annual operating cost of about \$3908. Use of the control procedure represented a 10% reduction in the measure of dispersion over the Lagrangian method with only an accompanying increase in costs of approximately 1%. (It should be noted, however, that although the Lagrangian method resulted in no back-orders, by the very nature of the design of COPEAK and its ordering

Table 2. Data for Lagrangian Multiplier Example

---

1. Data:

Item	Ordering Cost	Unit Cost	Inventory Holding Rate	Lead Time (days)	Cubic Space per Item
1	$50	$420	.2	30	1 foot
2	$75	$100	.2	30	1 foot
3	$100	$50	.2	30	1 foot

2. Warehouse Capacity Constraint: 391 cubic feet of storage space

3. Constrained Optimal Q_i^*'s derived analytically:

Q_1 : 141

Q_2 : 60

Q_3 : 190

4. COPEAK Parameters:

a. AMTQ: 75%

b. MGLVL: 268

c. AMPIN: 0%

d. ILOOK: 0%

e. Days-to-Order: 7

---

procedures, item 1 was on backorder status 3% of the five year simulation time; item 2 for 2%; and item 3 for 2%. No backorder costs were assessed in this example.)

Table 2. Data for Lagrangian Multiplier Example

---

1. Data:

Item	Ordering Cost	Unit Cost	Inventory Holding Rate	Lead Time (days)	Cubic Space per Item
1	\$50	\$420	.2	30	1 foot
2	\$75	\$100	.2	30	1 foot
3	\$100	\$50	.2	30	1 foot

2. Warehouse Capacity Constraint: 391 cubic feet of storage space

3. Constrained Optimal Q_i^*'s were derived analytically using the following equation.

$$Q_i^* = \sqrt{\frac{2\lambda_i A_i}{I_i C_i + 2\theta^* f_i}}$$

(λ_i: yearly demand rate

I_i: inventory carrying rate

C_i: unit cost

A_i: ordering cost

f_i: cubic storage space

θ^*: Lagrangian Multiplier. θ^* is the value which satisfies the equation $\sum_{i=1}^{n} f_i Q_i^*$ = the total capacity constraint.)

Q_1 : 141 Q_2 : 60 Q_3 : 190

4. COPEAK Parameters:

   a. AMTQ: 75%

   b. MGLVL: 268

   c. AMPIN: 0%

   d. ILOOK: 0%

   e. Days-to-Order: 7

procedures, item 1 was on backorder status 3% of the five year simulation time; item 2 for 2%; and item 3 for 2%. No backorder costs were assessed in this example.)

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

As a result of the research conducted in the preparation of this thesis, the following conclusions have been made:

1. With the sets of test data used in this research, COPEAK provided a reasonable approach to the problem of reducing the level of inventory fluctuations about the average AOHI in a multi-item inventory system.

2. Each of the four ordering priority rules appeared to work effectively with rules 2 and 4 showing a significant difference from rule 3 (rules 2 and 4 having a smaller Measure of Dispersion value). These conclusions as stated are not necessarily related to the administrative cost of implementing and conducting a system such as COPEAK. These costs, although often difficult to quantify, would generally relate to the unique characteristics of the system under study and should be considered when selecting it for implementation.

Recommendations for Further Study

1. A study to attempt to determine the exact relationship between the parameter MGLVL and the average aggregate inventory resulting from simulations using COPEAK should be performed. As has been demonstrated with data set 1, a different MGLVL value for each COPEAK priority rule had to be used to insure resulting averages of the AOHI

close to the performance criterion of 133 obtained from the uncontrolled system. The relationship that exists between this parameter and the uniqueness of the other parameters in the given sets of test data appears to have a direct bearing on the resulting average AOHI.

2. As a recommendation similar to no. 1, further analysis concerning the relationship of the other COPEAK parameters to the average AOHI and measure of dispersion is warranted. In the Sensitivity Analysis section of Chapter IV it was seen that, for example, when the AMTPIN parameter was given a non-zero value, the average AOHI reduced drastically. Additionally, when a value of 50% for AMTQ was used with all other parameters at a zero level, a reduction in the measure of dispersion with an accompanying minor rise in cost yielded promising results.

3. As alluded to in Chapter IV, the control procedure COPEAK might offer potential merit as an alternative method over the Lagrangian procedure for use in solving a constrained inventory warehousing problem. It could conceivably reduce the fluctuations of the AOHI sufficient enough to warrant increasing the average AOHI higher than that held under the Lagrangian solution. Depending on the nature of the inventory problem under study, such action could be desirable in offsetting any resulting backorder or system implementation costs. Further study of the feasibility of COPEAK in this role is appropriate.

4. A time series plot of the aggregate inventory levels acquired during this research displayed a tendency for the observations to be cyclic. It would be appropriate to analyze and define the parameters comprising this time series. The interaction of the variables in the

inventory system and their effect upon this cyclic behavior should be analyzed. It might be possible in this way to analytically investigate the problem of controlling aggregate inventory levels.

APPENDICES

APPENDIX A

PARAMETER DATA FOR DATA SET 1 and DATA SET 2

Table 3. Data Set 1 Parameters

Item	A	C	I	$\hat{\pi}$	Q^*	r^*	Time Between Demands	Lead Time	Average Cycle Time	Average Daily Demand	Average Yearly Demand
1	3.73	3.13	.29	7	84	19	.50	14	40.0	1.9964	759
2	3.73	4.87	.16	47	48	8	1.60	14	75.8	.6247	231
3	3.73	15.01	.28	69	15	7	2.90	21	43.6	.3462	126
4	3.73	10.00	.34	38	35	26	.72	21	25.1	1.3895	503
5	3.73	39.91	.23	19	35	14	.37	10	12.7	2.6947	1000
6	3.73	20.00	.15	73	31	14	.98	15	30.4	1.0213	372
7	3.73	50.00	.27	16	13	7	2.25	28	28.8	.4421	164
8	3.73	70.00	.21	88	8	6	3.69	25	28.2	.2716	99
9	3.73	21.00	.18	8	48	6	.45	10	22.0	2.2270	800
10	3.73	13.50	.31	29	35	30	.61	21	21.3	1.6410	600

Table 4. Data Set 2 Parameters

Item	A	C	I	$\hat{\pi}$	Q^*	r^*	Time Between Demands	Lead Time	Average Cycle Time	Average Daily Demand	Average Yearly Demand
1	8.00	2.00	.30	12	117	13	.75	14	87.50	.7481	486
2	8.00	2.49	.19	39	108	11	1.10	14	118.00	1.0960	332
3	8.00	4.90	.32	21	65	7	.96	14	61.78	.9603	380
4	8.00	5.99	.19	48	95	24	.57	14	53.92	.5705	640
5	8.00	6.90	.21	24	72	12	.83	14	59.40	.8307	439
6	8.00	9.98	.22	9	66	6	.77	14	50.53	.7697	474
7	8.00	30.80	.21	54	30	14	1.20	21	35.15	1.1960	304
8	8.00	53.16	.22	51	30	20	.70	18	20.70	.7015	420
9	8.00	74.99	.30	42	14	7	2.15	25	29.20	2.1510	169
10	8.00	103.49	.20	60	25	15	.61	13	15.16	.6146	600

APPENDIX B

PROGRAM VERIFICATION DATA

Table 5. Cost Comparisons for Data Set 1 Between the Simulation Model and Analytically Derived Results

Item	Ordering	Holding	Backorders	Total
1				
Model	32.80	30.80	3.00	66.60
Analytical	33.70	29.60	4.17	67.47
2				
Model	17.90	18.40	.10	36.40
Analytical	17.95	17.93	.49	36.37
3				
Model	32.10	32.50	.10	64.70
Analytical	31.33	27.44	2.30	61.07
4				
Model	54.50	50.30	5.30	110.00
Analytical	53.61	49.74	4.89	108.24
5				
Model	105.90	65.90	43.60	215.40
Analytical	106.57	69.37	39.09	215.03
6				
Model	45.50	44.10	.80	90.40
Analytical	44.76	40.69	4.71	90.16
7				
Model	46.30	36.60	13.50	96.30
Analytical	47.06	25.44	22.15	94.65
8				
Model	47.00	54.00	1.90	102.90
Analytical	46.16	45.02	5.50	96.68
9				
Model	63.40	40.40	21.10	124.90
Analytical	62.17	40.32	21.33	123.82
10				
Model	64.20	56.90	8.40	129.40
Analytical	63.94	53.74	10.36	128.04

Table 6. Program Verification - Effects of the Length of the Simulation on the Average AOHI Observations

---

1. Resulting Observations:

seed	5 years	4 years	3 years
1	132.8	132.8	133.3
2	133.4	133.9	134.2
3	133.2	133.1	131.1
4	132.3	131.9	131.8
5	133.1	132.8	132.9

2. ANOVA:

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	.252	2	.126	.1717
Error	8.804	12	.734	
Total	9.056	14		

Table 7. Program Verification - Effects of the Length of the Simulation on the Measure of Dispersion Observations

---

1. Resulting Observations:

seed	5 years	4 years	3 years
1	26.8	26.6	27.4
2	26.1	26.4	26.5
3	27.1	27.2	26.5
4	26.6	26.2	26.1
5	26.9	26.9	27.2

2. ANOVA:

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	.016	2	.008	.04
Error	2.384	12	.199	
Total	2.40	14		

APPENDIX C

SUBROUTINES FOR COMPUTER SIMULATION MODEL

The GASP II simulation technique uses several standard subroutines to accomplish the normal procedures involved in any simulation effort. User written subroutines are added to accommodate the particular problem at hand. The program had an EVENTS subroutine, Table 9, which triggers the simulation of such events as demands, receipts, and the calculation of annual operating costs. It has a RECEIPT subroutine, Table 13, and a DEMAND subroutine, Table 10, which not only simulated demands based on a stochastic inter-arrival time but also performed the necessary actions to generate orders for Q_i^*. The unique development of COPEAK necessitated a different DEMAND subroutine for its use employing no ordering section. Thus, subroutine ORDER, Table 12, was specifically written to accommodate this research.

Table 8. Main Program

```
      DIMENSION NSET(7,120)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30),MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
     1ISTOP
      DIMENSION K(25,30)
      NCRDR=5
      NPRNT=6
      NRUNS=1
   10 FORMAT(6A2,A6,3I5)
   15 FORMAT( )
      DO 20 I=1,20
      DO 25J=1,40
   25 D(I,J)=0
   20 CONTINUE
      READ(NCRDR,10)NAME,NPROJ,ISTOCK,ISTOP,IBUG
      WRITE(NPRNT,55)
      WRITE(6,50)
   50 FORMAT(T48,'INVENTORY SIMULATION PROGRAM',2(/))
      WRITE(NPRNT,110) NAME,      ISTOCK,ISTOP
  110 FORMAT(T5,'PROGRAMER = ',6A2,T35,', NUMBER OF ITEMS = ',I5,T68,' N
     1UMBER OF YEARS TO SIMULATE = ',I5,2(/))
      WRITE(NPRNT,115)
  115 FORMAT(T45,'INITIAL CONDITION OF D(I,J)',2(/))
      WRITE(NPRNT,120)
  120 FORMAT(T45,'ATRIBUTE NUMBER',2(/))
      DO 30 (I=1,ISTOCK
   30 READ(NCRDR,15)(D(I,J),J=1,30)
      IF (IDP .NE. 0) GO TO 299
```

Table 8. (Continued)

```
      WRITE(NPRNT,125)
  125 FORMAT(T5,'1',T14,'2',T23,'3',T32,'4',T41,'5',T50,'6',T59,'7',T68
     1'8',T77,'9',T86,'10',T95,'11',T104,'12',T111,'13',2(/))
      DO 34 I=1,ISTOCK
      DO 33 J=1,30
   33 K(I,J)=D(I,J)
   34 CONTINUE
      DO 35 I=1,ISTOCK
      D(I,40)=0.0
      IF (D(I,13) .LE. 0.01) D(I,40)=100
   35 WRITE(NPRNT,130) K(I,1),K(I,2),(D(I,J),J=3,7),(K(I,J),J=8,11),D(I,
     112),K(I,13)
  130 FORMAT(I6,3X,I6,4F9.3,F9.1,3X,I6,3X,I6,3X,I6,3X,I6,2X,F6.2,2X,I6)
      WRITE(NPRNT,135)
  135 FORMAT(//,5X,24HREFER TO ATTRIBUTE SHEET)
      WRITE(6,55)
      WRITE(6,120)
      WRITE(NPRNT,140)
  140 FORMAT(T5,'1',T11,'14',T17,'15',T23,'16',T29,'17',T35,'18',T41,'19
     1',T47,'20',T53,'21',T59,'22',T65,'23',T71,'24',T77,'25',T83,'26',T
     289,'27',T95,'28',T101,'29',T107,'30',2(/))
      DO 40 I=1,ISTOCK
   40 WRITE(NPRNT,145) K(I,1),K(I,14),(D(I,J),J=15,23),(K(I,J),J=24,30)
  145 FORMAT(2I6,9F6.2,7I6)
      WRITE(6,150)
  150 FORMAT(//,5X,24HREFER TO ATTRIBUTE SHEET)
      WRITE(NPRNT,55)
   55 FORMAT(1H1)
  299 DO 300 I=1,ISTOCK
  300 HOLD(I)=0.0
      CALL GASP(NSET)
      END
```

Table 9. Events Subroutine

```
      SUBROUTINE EVNTS(I,NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30),MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      DIMENSION NSET(7,1)
      GO TO (1,2,3,4,5,6),I
    1 CALL DMAND(NSET)
      RETURN
    2 CALL RECPT(NSET)
      RETURN
    3 CALL ENDSM(NSET)
      RETURN
    4 CALL ACCT(NSET)
      RETURN
    5 CALL CALC(NSET)
      RETURN
    6 CALL REVIEW(NSET)
      RETURN
      END
```

Table 10. Demand Subroutine

```
      SUBROUTINE DMAND(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,22),KRANK(30),MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      DIMENSION NSET(7,1)
      EXTERNAL RN1
      I= ATRIB(3) + 0.001
      DEM = ATRIB(4)
      D(I,24)= D(I,24)-DEM
      D(I,25) = D(I,25) + DEM
      BAC = D(I,28)
      IF (D(I,13) .GE" DEM) GO TO 30
      IF (D(I,13)"GT" 0.000001) GO TO 40
   11 CALL TMST (BAC,TNOW,I,NSET)
      D(I,27)= D(I,27) + DEM
      D(I,28)= D(I,28) + DEM
      GO TO 55
   30 D(I,13)= D(I,13)- DEM
      GO TO 55
   40 CALL TMST (BAC,TNOW,I,NSET)
      IF (BAC .GT. .01) CALL ERROR(269,NSET)
      D(I,28)=DEM-D(I,13)
      D(I,27)= D(I,27) +D(I,28)
      D(I,13)= 0.0
   55 ATRIB(2)= 1.0 + 0.00001
      BACK = D(I,40)
      I20 = I + 20
      CALL TMST(BACK,TNOW,I20,NSET)
      D(I,40)=0.0
      IF (D(I,13) .LE. 0.01) D(I,40)=100.
      IDT= D(I,8)
      GO TO (101,102,103,104,105),IDT
  105 RETURN
  101 AT= D(I,16)
      GO TO 106
  102 A= D(I,16)
      B= D(I,17)
      AT= UNFRM(A,B)
      GO TO 106
  103 AT= RNORM(I)
      GO TO 106
  104 AT=-D(I,16)*ALOG(RN1(K))
```

Table 10. (Continued)

```
      IF (AT .GT. D(I,18)) AT=D(I,18)
      IF (AT .LT. D(I,17)) AT= D(I,17)
106   IF ( AT .LT. 0.0) CALL ERROR(222,NSET)
      ATRIB(1)=TNOW + AT
      I20= I + 20
      CALL COLCT(AT,I20,NSET)
      CALL FILEM(1,NSET)
      ID2=D(I,2) + 0.001
      IF(ID2 "EQ. 2) RETURN
      IF (D(I,24) .GE. D(I,11) + 0.00001) RETURN
      ATRIB(4) = D(I,10)
      D(I,29)=D(I,29)+D(I,10)
      D(I,24) = D(I,10) + D(I,24)
      D(I,26)=D(I,26) + 1.0
      D(I,30)= D(I,30) + D(I,10)
      IDT=D(I,9) + 0.001
      IND=0
      GO TO (107,108,109,110,111),IDT
107   DS= D(I,20)
      GO TO 112
108   A=D(I,20)
      B= D(I,21)
      DS= UNFRM(A,B)
      GO TO 112
109   JEVNT=2
      DS=RNORM(I)
      JEVNT=1
      GO TO 112
110   DS=-D(I,20)*ALOG(RN1(K))
      IF (DS .GT. D(I,22)) DS=D(I,22)
      IF (DS .LT. D(I,21)) DS=D(I,21)
      GO TO 112
111   DS=D(I,22)
      IF (D(I,23) .GT. 0.0) IND=1
      D(I,23)= D(I,10) + D(I,23)
      ATRIB(4)= D(I,20)
      IF (D(I,20) .GT. D(I,10))ATRIB(4)=D(I,10)
112   ATRIB(1)= TNOW +DS
      ATRIB(2)= 2.00001
      I40= I+ 40
      CALL COLCT(DS,I40,NSET)
      IF (IND .EQ.1) RETURN
      CALL FILEM(1,NSET)
      RETURN
      END
```

Table 11. Demand - COPEAK Subroutine

```
      SUBROUTINE DMAND(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30)MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
     1ISTOP,PM(5,100),OM(20,5),TEMP(5),DAYORD,AMTQ,AMTPIN,INDKEY,RMGLVL,
     2S,ILOOK
      DIMENSION NSET(7,1)
      EXTERNAL RN1
      I= ATRIB(3) + 0.001
      DEM = ATRIB(4)
      D(I,24)= D(I,24)-DEM
      D(I,25) = D(I,25) + DEM
      BAC = D(I,28)
      IF (D(I,13) .GE. DEM) GO TO 30
      IF (D(I,13) .GT. 0.000001) GO TO 40
   11 CALL TMST (BAC,TNOW,I,NSET)
      D(I,27)= D(I,27) + DEM
      D(I,28)= D(I,28)+ DEM
      GO TO 55
   30 D(I,13)= D(I,13)- DEM
      GO TO 55
   40 CALL TMST (BAC,TNOW,I,NSET)
      IF (BAC .GT. .01) CALL ERROR(269,NSET)
      D(I,28)=DEM-D(I,13)
      D(I,27)= D(I,27) +D(I,28)
      D(I,13)= 0.0
   55 ATRIB(2)= 1.0 + 0.00001
      BACK = D(I,40)
      I20 = I + 20
      CALL TMST(BACK,TNOW,I20,NSET)
      D(I,40)=0.0
      IF (D(I,13) .LE. 0.01) D(I,40)=100.
      IDT= D(I,8)
      GO TO (101,102,103,104,105),IDT
  105 RETURN
  101 AT=D(I,16)
      GO TO 106
  102 A= D(I,16)
      B= D(I,17)
      AT= UNFRM(A,B)
      GO TO 106
```

Table 11. (Continued)

```
 103 AT= RNORM(I)
     GO TO 106
 104 AT=-D(I,16)*ALOG(RN1(K))
     IF (AT .GT. D(I,18)) AT=D(I,18)
     IF (AT .LT. D(I,17)) AT= D(I,17)
 106 IF ( AT .LT. 0.0) CALL ERROR(222,NSET)
     ATRIB(1)=TNOW + AT
     I20= I + 20
     CALL COLCT(AT,I20,NSET)
     CALL FILEM(1,NSET)
     ID2=D(I,2) + 0.001
     IF (ID2 .EQ. 2) RETURN
     RETURN
     END
```

Table 12. Order - COPEAK Subroutine

```
       SUBROUTINE ORDER(NSET)
       COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
      1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
      2TBDG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
       COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30),MAXNQ(30),
      1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
      2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
       COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      1ISTOP,PM(5,100),OM(20,5),TEMP(5),DAYORD,AMTQ,AMTPIN,INDKEY,RMGLVL,
      2S,ILOOK,DDAY,C1,C2
       DIMENSION NSET(7,1)
C   FIRST SHIFT IN TIME ROUTINE FOR THE PM
C   SHIFT ALL 5 ROWS IN THE PM
       DO 16 I=1,5
       DO 16 J=1,99
    16 PM(I,J)=PM(I,J+1)
C    NOW, INITIALIZE THE DAY 100 IN THE PM
       DO 19 K=1,5
    19 PM(K,100)=0.
C   NEXT, LOAD 100 WITH PROJECTED OUTPUT (DEMAND)
       PM(1,100)=S
C   NEXT, UPDATE THE PROJECTED INPUT ROW 3 OF THE PM
C   FIRST, DETERMINE THE DAY FOR YOUR FIRST ARRIVAL(BASED ON THE CURRENT
C   INVENTORY POSITION AT THE END OF THE DAY)
       DO 7 I=1,100
    7  PM(3,I)=0.0
       SSOHQ=0.0
       DO 3 I=1,ISTOCK
       IDAY=(D(I,24)-D(I,11))/D(I,32)+D(I,33)+.5
       SSOHQ=SSOHQ+D(I,13)
       IF(IDAY.LT.0)GO TO 334
       IF(IDAY.GT.100)GO TO 3
       PM(3,IDAY)=PM(3,IDAY)+D(I,10)
C   NOW LOAD IN THE PROJECTED DUE-IN BASED ON Q AMOUNTS STARTING AT IDAY
```

Table 12. (Continued)

```
 334   CNTR=1.0
   2   B1=CNTR*D(I,31)
       IB1=B1+.00001
       MDAY=IDAY+IB1
       IF(MDAY.LT.0)GO TO 333
       IF(MDAY.GT.100)GO TO 3
       PM(3,MDAY)=PM(3,MDAY)+D(I,10)
 333   CNTR=CNTR+1.
       GO TO 2
   3   CONTINUE
C  COMPUTE PM(4,K) SUCH THAT IT IS A RUNNING FIGURE EQUAL TO THE SUM OF THE
C  STARTING AMOUNTS + THE SUM OF THE CERTAIN DUE-IN,S + (AMTPIN)*(SUM OF THE
C   PROJECTED DUE-IN,S) - THE SUM OF THE PROJECTED DAILY DEMANS UP TO AND
C   INCLUDING DAY K
       PM(4,1)=SSOHQ+PM(2,1)+AMPTIN*PM(3,1)-PM(1,1)
       DO 8 K=2,100
    8  PM(4,K)=PM(4,K-1)+PM(2,K)-PM(1,K)+AMTPIN*PM(3,K)
C    NEXT, COMPUTE THE ORDER MATRIX
C   COMPUTE DAYS TILL NEXT ANTICIPATED ORDER DATE FOR EACH ITEM.
C   USE THE CURRENT INVENTORY POSITION, D(I,24)
       DO 6 I=1,ISTOCK
       OM(I,1)=I
       IF(D(I,34).EQ.0)GO TO 81
       RNEW=D(I,34)
       GO TO 82
81     RNEW=D(I,11)
82     OM(I,2)=((D(I,24)-RNEW)/D(I,32))+.5
6      CONTINUE
C  NEXT, SORT OM ON COLUMN 2,  DAYS TO ORDER  AND GET THE OM IN  DAYS TO ORDER
C    SEQUENCE
       IISTOC=ISTOCK-1
       DO 4 I=1,IISTOC
       IP1=I+1
       M=I
```

Table 12. (Continued)

```
      DO 5 J=IP1,ISTOCK
      IF(OM(J,2)-OM(M,2))9,5,5
    9 M=J
    5 CONTINUE
      DO 70 L=1,5
   70 TEMP(L)=OM(I,L)
      DO 11 K=1,5
   11 OM(I,K)=OM(M,K)
      DO 12 L=1,5
   12 OM(M,L)=TEMP(L)
    4 CONTINUE
C NEXT USING THE INDEX KEY,INDKEY, COMPUTE AN INDEX FOR EACH ITEM WHOSE  DAYS
C  TO ORDER  IS LE.  DAYORD
      GO TO (60,61,61,61),INDKEY
C NEXT PICK OFF THOSE ITEMS WHOSE DAYS TO ORDER,OM(I,2), IS LE DAYORD
   61 LN=0
      DO 38 JN=1,ISTOCK
      IF(OM(JN,2).GT"DAYORD)GO TO 39
      LN=LN+1
C   LN IS THE NUMBER OF ITEMS WITH A DAYS TO ORDER  LE  TO DAYORD
   38 CONTINUE
C    IF THERE ARE NO ITEMS WITH DAYS TO ORDER  LE  DAYORD GO TO 485
   39 IF(LN.EQ.0)GO TO 485
      DO 980 J=1,LN
      I=OM(J,1)+.00001
      GO TO (42,42,43,44),INDKEY
C  KEY 2 IS TO SELECT ITEMS BASED ON THE NEXT FORMULA
   42 OM(J,4)=(DDAY+OM(J,2))*((D(I,3)*D(I,5)/365.)*(SUMA(I,1)/(SUMA(I,3)
     1+.001)))/((D(I,7)/365.)*((SSUMA(I,2)/(SSUMA(I,1)-TBEG))+.001))
      GO TO 980
C  KEY 3 IS TO USE THE FORMULA WITHIN THIS ROUTINE
   43 IF(OM(J,2).GT.0)GO TO 51
      IF(OM(J,2).EQ.0)GO TO 52
```

Table 12. (Continued)

```
      OM(J,4)=(D(I,5)*D(I,3))/(D(I,7)*(OM(J,2)*(-1.)))
      GO TO 980
 51   OM(J,4)=((D(I,5)*D(I,3))*OM(J,2))/D(I,7)
      GO TO 980
 52   OM(J,4)=(D(I,5)*(I,3))/D(I,7)
      GO TO 980
C  KEY 4 IS TO USE THE NEXT FORMULA
 44   OM(J,4)=OM(J,2)+(C1*D(I,7))+(C2*D(I,5)*D(I,3))
 980  CONTINUE
C  NEXT SORT FIRST LN ROWS OF THE OM ON COLUMN 4, THE PRIORITY INDEX
      IISTOC=LN-1
      DO 31 I=1,IISTOC
      IP1=I+1
      M=I
      DO 32 J=IP1,LN
      IF (OM(J,4)-OM(M,4))33,32,32
 33   M=J
 32   CONTINUE
      DO 35 L=1,5
 35   TEMP(L)=OM(I,L)
      DO 36 K=1,5
 36   OM(I,K)=OM(M,K)
      DO 37 L=1,5
 37   OM(M,L)=TEMP(L)
 31   CONTINUE
C  NEXT, TRY TO ORDER THE FIRST LN ITEMS
C KEY 1 IS TO INDEX THE SELECTED ITEMS SOLEY ON THEIR  DAYS TO ORDER  RANKING.
  60  K=1
 473  IF(OM(K,2).LE.DAYORD)GO TO 479
C  IF NO MORE ITEMS ARE IN THE INDEX QUEUE, TERMINATE THIS DAILY ORDER ROUTINE.
      GO TO 485
 479  I=OM(K,1)+.00001
C  COMPUTE THE PREDICTED DATE OF ARRIVAL,IDD,FOR THE SHIPMENT FOR ITEM I.
      W1=D(I,24)/D(I,32)
```

Table 12. (Continued)

```
      IF(W1.GE.D(I,33))GO TO 68
      IDD=D(I,33)
      GO TO 69
  68  IDD=W1
C  COMPUTE THE GRAND SUM,GSUM, FOR THE DUE DATE,IDD, IN THE PM.
  69  GSUM=PM(4,IDD)+D(I,10)
      A1=GSUM-D(I,10)
      IF(A1.GE.RMGLVL)GO TO 482
      A2=RMGLVL-A1
      IF(D(I,10).LE.A2)GO TO 480
      IF(A2.LT.(AMTQ*D(I.10)))GO TO 482
      B63=A2
C  THIS PORTION IS ADDED TO THE ORDERING ROUTINE TO CHECK FOR RLOOK DAYS
C AHEAD. IT TRYS TO ENSURE THAT THE ON HAND INVENTORY FOR THE NEXT
C  RLOOK DAYS AHEAD STAYS BELOW THE RMGLVL
  490 HE=0.
      IF(ILOOK.EQ.0)GO TO 481
      DO 559 L=1,ILOOK
      B99=(B63-(L*D(I,32)))+PM(4,IDD+L)
      IF(B99.GT.RMGLVL)HE=HE+1.
  559 CONTINUE
      IF(HE.GT.0.0)GO TO 482
      GO TO 481
C   OR ORDER THE ITEM I IN SIZE OF D(I,10)
  480 B63=D(I,10)
      GO TO 490
C  NOW, ORDER THE ITEM IN SIZE B63
  481 ATRIB(4)=B63
      D(I,29)=D(I,29)+B63
      D(I,24)=B63+D(I,24)
      D(I,26)=D(I,26)+1.0
      D(I,30)=D(I,30)+B63
C  NOW RECORD THE FUTURE RECEIPT OF THIS ITEM
```

Table 12. (Continued)

```
C  FIRST GENERATE THE RANDOM VARIABLE  DISTRIBUTION LEAD TIME
C  COMPUTE DISTRIBUTION LEAD TIME
      IDT=D(I,9)+.00001
      IND=0
      GO TO (107,108,109,110,111),IDT
  107 DS=D(I,20)
      GO TO 112
  108 A=D(I,20)
      B=D(I,21)
      DS=UNFRM(A,B)
      GO TO 112
  109 JEVNT=2
      DS=RNORM(I)
      JEVNT=1
      GO TO 112
  110 DS=-D(I,20)*ALOG(RN1(K))
      IF(DS.GT.D(I,22))DS=D(I,22)
      IF(DS.LT.D(I,22))DS=D(I,21)
      GO TO 112
  111 DS=D(I,22)
      IF(D(I,23).GT.0.0)IND=1
      D(I,23)=D(I,10)+D(I,22)
      ATRIB(4)=D(I,20)
      IF(D(I,20).GT.D(I,10))ATRIB(4)=D(I,10)
  112 CONTINUE
C  NOW RECORD THE RECEIPT
      IF(W1.GE.D(I,33))GO TO 62
      ATRIB(1)=TNOW+DS
      GO TO 63
  62  W2=W1-D(I,33)
      ATRIB(1)=TNOW+W2+DS
  63  ATRIB(2)=2.00001
      ATRIB(3)=I
```

Table 12. (Continued)

```
      I40=I+40
      CALL COLCT(DS,I40,NSET)
      CALL FILEM(1,NSET)
      GO TO 483
C  NOW, UPDATE THE CERTAIN DUE-IN ROW 2 OF THE PM AND PROJECTED DUE-IN ROW 3.
C  FIRST, ERASE THE PRESENT PROJECTED DUE-IN FOR ITEM I.
  483 IDAY=(D(I,24)-B63-D(I,11))/D(I,32)+D(I,33)+.5
C  IDAY IS THE DAY THAT WE USED TO LOAD IN THE FIRST PROJECTED INPUT FOR
C  ITEM I WHEN WE DID OUR DAILY UPDATE ON THE PROJECTED DUE-IN ROW 3 OF THE
C  PM.
C  NOW, USING IDAY SUBTRACT OUT D(I,10) FROM PM(3,IDAY) AND THEN
C  INCREMENT BY PM(3,IDAY+CNTR*D(I,31)).
   20 IF(IDAY.GT.100)GO TO 71
      PM(3,IDAY)=PM(3,IDAY)-D(I,10)
      CNTR=1.0
   98 B1=CNTR*D(I,31)
      IB1=B1+.00001
      MDAY=IDAY+IB1
      IF(MDAY.GT.100)GO TO 71
      PM(3,MDAY)=PM(3,MDAY)-D(I,10)
      CNTR=CNTR+1.0
      GO TO 98
C  NOW, UPDATE THE CERTAIN DUE-IN COLUMN 2 OF THE PM
   71 PM(2,IDD)=PM(2,IDD)+B63
C  UPDATE THE PROJECTED DUE_IN COLUMN 3, STARTING AT IDD
   10 RCNTR=1.0
C  NOTE, IF B63 WAS A  PART  PRDER, THE FIRST PROJECTED DUE-IN AMOUNT WILL
C  NOT ARRIVE IN ONE CYCLE BUT IN LESS TIME.
      IF(B63.LT.D(I.10))GO TO 22
C  IF NOT, THEN WE HAVE ORDERED A FULL Q AMOUNT AND OUR FIRST PROJECTED
C  DUE-IN WILL BE BASED ON THE CYCLE DATE.
   23 B5=IDD+D(I,31)*RCNTR
      IF(B5.GT.100)GO TO 15
      IB5=B5+.00001
```

Table 12. (Continued)

```
      PM(3,IB5)=PM(3,IB5)+D(I,10)
      RCNTR=RCNTR+1.0
      GO TO 23
C DIVIDE AMOUNT ORDERED,B63, BY AVGDM FOR ITEM I TO GET THE DATE FOR THE
C FIRST PROJECTED DUE-IN.
   22 B68=B63/D(I,32)
      B5=IDD+1.+B68
C B5 IS THE DATE FOR THE FIRST PROJECTED Q AMOUNT DUE-IN
      IF(B5.GT.100)GO TO 15
      IB5=B5+.00001
      PM(3,IB5)=PM(3,IB5)+D(I,10)
C NEXT, DETERMINE WHEN THE REST OF THE PROJECTED Q:S WILL COME IN.
      ACNTR=1.0
   14 AB1=B5+D(I,31)*ACNTR
      IF(AB1.GT.100)GO TO 15
      IAB1=AB1+.00001
      PM(3,IAB1)=PM(3,IAB1)+D(I,10)
      ACNTR=ACNTR+1.0
      GO TO 14
C NEXT, RECOMPUTE THE RUNNING TOTAL ROW 4 OF THE PM.
  15 PM(4,1)=SSOHQ+PM(2,1)+AMTPIN*PM(3,1)-PM(1,1)
      DO 24 K=2,100
  24  PM(4,k)=PM(4,K-1)+PM(2,K)-PM(1,K)+AMTPIN*PM(3,K)
C  NOW SEE IF THERE ARE ANY MORE ITEMS IN THE INDEX QUEUE.
  482 K=K+1
      IF(K.LE"ISTOCK)GO TO 473
  485 CONTINUE
      RETURN
      END
```

Table 13. Receipt Subroutine

```
      SUBROUTINE RECPT(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MSC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30),MAXNQ(30), MAIN005
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      DIMENSION NSET(7,1)
      I= ATRIB(3) + 0.0001
      BAC= D(I,28)
      CALL TMST(BAC,TNOW,I,NSET)
      REC= ATRIB(4)
      D(I,29)=D(I,29)-REC
      D(I,13)= D(I,13) + REC
      IF (D(I,28) .LE. 0.00001) GO TO 28
      IF (D(I,13) .GE. D(I,28)) GO TO 30
      D(I,28)= D(I,28)-D(I,13)
      D(I,13)= 0.0
      GO TO 28
   30 D(I,13)=D(I,13)-D(I,28)
      D(I,28)= 0.0
   28 BACK= D(I,40)
      I20= I + 20
      CALL TMST(BACK,TNOW,I20,NSET)
      D(I,40)=0.0
      IF (D(I,13) .LE. 0.01) D(I,40)=100.
      IF (D,9) .LE. 4.5 .OR. D(I,9) .GT. 5.5) RETURN
      D(I,21)=D(I,21) + REC
      D(I,23)=D(I,23)-D(I,20)
      IF (D(I,23) .LE. 0.01) D(I,23)=0.0
      IF (D(I,23) .LE. .001) RETURN
```

TABLE 13. (Continued)

```
      ATRIB(1)=TNOW+ 1.0
      ATRIB(4)=D(I,20)
      IF(D(I,20) .GT. D(I,23)) ATRIB(4)=D(I,23)
      CALL FILEM (1,NSET)
      RETURN
      END
```

Table 14. Account Subroutine

```
      SUBROUTINE ACCT(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,22),KRANK(30),MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      DATA NDAY/0/
      DIMENSION NSET(7,1)
      AAHI1=0.0
      AAHI2=0.0
      NDAY=NDAY+1
      DO 1 I=1,10
      XOH=D(I,13)
      AAHI1=AAHI1+XOH
    1 CALL COLCT(XOH,I,NSET)
      CALL COLCT(AAHI1,61,NSET)
      DO 2 I=11,20
      XOH=D(I,13)
      AAHI2=AAHI2+XOH
    2 CALL COLCT(XOH,I,NSET)
      CALL COLCT(AAHI2,62,NSET)
      AAHI=AAHI1+AAHI2
      CALL COLCT(AAHI,63,NSET)
      AAA(NDAY)=AAHI
      ATRIB(1)=TNOW+1.0
      CALL FILEM(1,NSET)
      RETURN
      END
```

Table 15. Calculation Subroutine

```
      SUBROUTINE CALC(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2TBEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,35),KRANK(30),MAXNG(30),MAIN005
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
     1ISTOP
      DIMENSION NSET(7,1)
      DIMENSION A(20),COS(20,8)
      DIMENSION IA(20)
      ATRIB(1)=TNOW+365.0
      CALL FILEM(1,NSET)
      IYEAR=IYEAR + 1
      DATA((COS(I,J),I=1,20),J=1,8)/160*0.0/
      DO 1 I=1,ISTOCK
      XX = D(I,28)
      I20=I+20
      XY=0.0
      IF (D(I,13) .LT. 0.001) XY=100.0
      CALL TMST(XY,TNOW,I20,NSET)
    1 CALL TMST(XX,TNOW,I,NSET)
      WRITE(6,50) IYEAR
   50 FORMAT(T35,'RESULTS FOR YEAR =',I5,2(/))
      WRITE(6,30)
   30 FORMAT(T2,'NUMBER  NUMBER   ORDERS',T27,'AVERAGE AV.INV"   AVE. ON'
     1,T57,'NUMBER   ORDERING  HOLDING',T84,'BACKORDER  TOTAL')
      WRITE(6,35)
   35 FORMAT(T1,'DEMANDED ORDERED  PLACED',T26,'INVENTORY  VALUE',T44,'B
     1ACKORDER   BACKORDED',T68,'COST     COST',T86,'COST   OPERATING CO
     2ST',2(/))
```

Table 15. (Continued)

```
      DO 20 I=1,ISTOCK
      A(1)=D(I,25)-COS(I,1)
      A(2)=D(I,30)-COS(I,2)
      A(3)=D(I,26)-COS(I,3)
      A(4)=(SUMA(I,1)-COS(I,4))/365.0
      A(5)=(SSUMA(I,2)-COS(I,5))/365.0
      A(6)=D(I,27) - COS(I,6)
      A(7) = A(2) * D(I,3)
      A(8) = A(3) * D(I,4)
      A(9) = A(4) * D(I,3) * D(I,5)
      A(10)= (A5) * D(I,7) + A(6)  * D(I,6)
      A(11)= A(8) + A(9) + A(10)
      A4= A(4) * D(I,3)
      DO 25 K=1,11
 25   IA(K)=A(K)
      WRITE(6,21) (IA(K),K=1,4),A4, A(5),IA(6),(A(K),K=8,11)
 21   FORMAT(I6,3X,I6,3X,I3,5X,I6,2X,F7.1,2X,F7.1,2X,F6.1'5X,I6,5X,3F8.1,F9.1)
      COS(I,1) = D(I,25)
      COS(I,2) = D(I,30)
      COS(I,3) = D(I,26)
      COS(I,4) = SUMA(I,1)
      COS(I,5) = SSUMA(I,2)
      COS(I,6) = D(I,27)
 20   CONTINUE
      ISTOP=TSTOP + 0.001
      IF(IYEAR .GE. ISTOP) CALL ENDSM(NSET)
      RETURN
      END
```

Table 16. End of Simulation Subroutine

```
      SUBROUTINE ENDSM(NSET)
      COMMON ID,IM,INIT,JEVNT,JMNIT,MFA,MSTOP,MX,MXC,NCLCT,NHIST,
     1NOQ,NORPT,NOT,NPRMS,NRUN,NRUNS,NSTAT,OUT,SCALE,ISEED,TNOW,
     2BEG,TFIN,MXX,NPRNT,NCRDR,NEP,VNQ(30)
      COMMON ATRIB(15),ENQ(30),INN(30),JCELS(20,22),KRANK(30),MAXNQ(30),
     1MFE(30),MLC(30),MLE(30),NCELS(20),NQ(30),PARAM(40,4),QTIME(30),
     2SSUMA(60,5),SUMA(65,5),NAME(6),NPROJ,MON,NDAY,NYR,JCLR
      COMMON   D(20,40),IR,IYEAR,ISTOCK,TSTOP,HOLD(20),IBUG,AAA(2000),
      DIMENSION NSET(7,1)
      DIMENSION K(25,30)
      DIMENSION A(20),IA(20)
      DO 2777 I=1,20
      A(I)=0.0
 2777 IA(I)=0
      MSTOP=-1
      SUMOR=0.0
      SUMHO=0.0
      SUMBO=0.0
      SUMTOT=0.0
      NORPT=0
      WRITE(6,115)
  115 FORMAT(1H1,T50,'FINAL CONDITION OF D(I,4)',2(/))
      WRITE(6,120)
  120 FORMAT(T50,'ATRIBUTE NUMBER',2(/))
      WRITE(NPRNT,125)
  125 FORMAT(T5,'1',T14,'2',T23,'3',T32,'4',T41,'5',T50,'6',T59,'7',T68,
     1'8',T77,'9',T86,'10',T95,'11',T104,'12',T111,'13',2(/))
      DO 34 I=1,ISTOCK
      DO 33 J=1,30
   33 K(I,J)=D(I,J)
   34 CONTINUE
      DO 35 I=1,ISTOCK
   35 WRITE(NPRNT,130) K(I,1),K(I,2),(D(I,J),J=3,7),(K(I,J),J=8,11),D(I,
```

Table 16. (Continued)

```
     112),K(I,13)
 130  FORMAT(I6,3X,I6,4F9,=,F9.1,3X,I6,3X,I6,3X,I6,3X,I6,2X,F6.2,2X,I6)
      WRITE(NPRNT,135)
 135  FORMAT(//,5X,24HREFER TO ATTRIBUTE SHEET)
      WRITE(6,55)
  55  FORMAT(1H1)
      WRITE(6,120)
      WRITE(NPRNT,140)
 140  FORMAT(T5,'1',T11,'14',T17,'15',T23,'16',T29,'17',T35,'18',T41,'19
     1',T47,'20',T53,'21',T59,'22',T65,'23',T71,'24',T77,'25',T83,'26',T
     289,'27',T95,'28',T101,'29',T107,'30',2(/))
      DO 40 I=1,ISTOCK
  40  WRITE(NPRNT,145) K(I,1),K(I,14),(D(I,J),J=15,23),(K(I,J),J=24,30)
 145  FORMAT(2I6,9F6.2,7I6)
      WRITE(6,150)
 150  FORMAT(//,5X,24HREFER TO ATTRIBUTE SHEET)
      XD=IYEAR*365.0
      WRITE(6,10)
  10  FORMAT(1H1,T21,'FINAL COST SUMMARY',2(/),
     1T5,'ITEM  TYPE   Q  OR   R  OR   ORDER    HOLD    BACKORDER    TOTA
     2L',1(/),T5,'NO.  POLICY  1ST S   2ND S   COST     COST         COST
     3    COST',2(/))
      DO 221 I=1,ISTOCK
      IA(1)=K(I,1)
      IA(2)=K(I,2)
      IA(3)=K(I,10)
      IA(4)=K(I,11)
      A(5)=D(I,26)*D(I,4)/TSTOP
      A(6)=SUMA(I,1)/XD * D(I,3)*D(I,5)
      A(7)=SSUMA(I,2)/XD*D(I,7) +D(I,27)/TSTOP*D(I,6)
      A(8)=A(5) + A(6) + A(7)
      SUMOR=A(5)+SUMOR
      SUMHO=A(6)+SUMHO
```

Table 16 (Continued)

```
      SUMBO=A(7)+SUMBO
      SUMTOT=A(8)+SUMTOT
      A(10)=A(5) + A(10)
      A(11)= A(6)+A(11)
      A(12)=A(7)+A(12)
      A(13)=A(8)+A(13)
  221 WRITE(6,230) (IA(K),K=1,4),(A(K),K=5,8)
  230 FORMAT(4X,I2,4X,I2,6X,I4,2X,I4,3X,F8.1,1X,F8.1,1X,F8.1,3X,F8.1)
      WRITE(6,971)SUMOR,SUMHO,SUMBO,SUMTOT
  971 FORMAT(3/,5X,30HAVG AGGREGATE ORDERING COST   ,F8.1,2/,5X,30HAVG A
     1GGREGATE HOLDING COST    ,F8.1,2/,5X,30HAVG AGGREGATE BACKORDER CO
     2ST  ,F8.1,2/,5X,30HAVG AGGREGATE TOTAL COST      ,F8.1)
      RETURN
      END
```

APPENDIX D

SAMPLE OUTPUT LISTINGS

Table 17. Initial Condition of Input

```
                                        INVENTORY SIMULATION PROGRAM

PROGRAMER =   BEHNE-J-R      , NUMBER OF ITEMS =    10        NUMBER OF YEARS TO SIMULATE =    5

 MGLVL= 120.0  DAYORD= 7.0  INDKEY= 4  AMTQ= .75   AMTPIN= .00  ILOOK= 0  DDAY= 25.0 C1=-.10 C2= .10
                                        INITIAL CONDITION OF D(I,J)

                                          ATTRIBUTE NUMBER

 1      2        3         4        5       6        7       8      9      10     11     12     13

 1      1       3.14      3.73     .29     .0       7.0     3      1      84     19     .0     42
 2      1       4.87      3.73     .16     .0      47.0     3      1      48      8     .0     24
 3      1      15.01      3.73     .28     .0      69.0     3      1      15      7     .0      7
 4      1      10.00      3.73     .34     .0      38.0     3      1      35     26     .0     17
 5      1      39.91      3.73     .23     .0      19.0     3      1      35     14     .0     17
 6      1      20.00      3.73     .15     .0      73.0     3      1      31     14     .0     15
 7      1      50.00      3.73     .27     .0      16.0     3      1      13      7     .0      6
 8      1      70.00      3.73     .21     .0      88.0     3      1       8      6     .0      4
 9      1      21.00      3.73     .18     .0       8.0     3      1      48      6     .0     24
10      1      13.50      3.73     .31     .0      29.0     3      1      35     30     .0     17

1=ITEM NUMBER                                 8=DISTRIBUTION TYPE, TIME BETWEEN DEMANDS
2=TYPE OF INVENTORY POLICY(1=Q,R;2=S,S)       9=DISTRIBUTION TYPE, LEAD TIME
3=ITEM COST, C                                10=REORDER QUANTITY (IF Q,R POLICY)OR 1ST S IN (S,S) POLICY
4=ORDERING COST, A                            11=REORDER POSITION (IF Q,R POLICY) OR 2ND S IN (S,S)POLICY
5=INVENTORY HOLDING RATE, I                   12=TIME BETWEEN REVIEWS FOR PERIODIC REVIEW
6=BACKORDER COST, FIXED PER UNIT,             13=CURRENT ON HAND QUANTITY
7=BACKORDER COST PER UNIT PER YEAR,
```

Table 17. (Continued)

ATTRIBUTE NUMBER																				
1	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
1	1	.01	.50	.10	.20	.80	14.0	.0	.0	.0	42	0	0	0	0	0	0	40.0	2.0	14.0
2	1	.01	1.60	.25	1.00	2.20	14.0	.0	.0	.0	24	0	0	0	0	0	0	75.8	.6	14.0
3	1	.01	2.90	.50	1.50	4.00	21.0	.0	.0	.0	7	0	0	0	0	0	0	43.6	.3	21.0
4	1	.01	.72	.10	.40	1.00	21.0	.0	.0	.0	17	0	0	0	0	0	0	25.1	1.4	21.0
5	1	.01	.37	.10	.15	.70	10.0	.0	.0	.0	17	0	0	0	0	0	0	12.7	2.7	10.0
6	1	.01	.98	.10	.50	1.30	15.0	.0	.0	.0	15	0	0	0	0	0	0	30.4	1.0	15.0
7	1	.01	2.25	.50	1.00	3.50	28.0	.0	.0	.0	6	0	0	0	0	0	0	28.8	.4	28.0
8	1	.01	3.69	.50	2.00	5.00	25.0	.0	.0	.0	4	0	0	0	0	0	0	28.2	.3	25.0
9	1	.01	.45	.10	.15	.75	10.0	.0	.0	.0	24	0	0	0	0	0	0	22.0	2.2	10.0
10	1	.01	.61	.10	.25	.95	21.0	.0	.0	.0	17	0	0	0	0	0	0	21.3	1.7	21.0

1 = ITEM NUMBER
14 = NUMBER OF ITEMS PER DEMAND
15 = TIME FOR 1ST DEMAND TO OCCUR
16 =PARAMETER 1=TIME BETWEEN DEMAND DIST.(DAYS)
17 = PARAMETER 2= TIME BETWEEN DEMANDS
18 = PARAMETER 3= TIME BETWEEN DEMANDS
19 = PARAMETER 4= TIME BETWEEN DEMANDS
20 = PARAMETER 1= LEAD TIME DIST.(DAYS)
21 = PARAMETER 2= LEAD TIME DISTRIBUTION
22 = PARAMETER 3= LEAD TIME DISTRIBUTION
23 =PARAMETER 4= LEAD TIME DISTRIBUTION
24 = CURRENT INVENTORY POSITION
25 = CUMULATIVE N ITEMS DEMANDED
26 = CUMULATIVE NUMBER OF ORDERS PLACED
27 = CUMULATIVE BACKORDERS
28 = CURRENT BACKORDERS
29 = DUE IN QUANTITY
30 = CUMULATIVE ITEMS ORDERED
31 = AVERAGE CYCLE TIME (DAYS)
32 = AVERAGE DAILY DEMAND RATE
33 = AVERAGE PROCUREMENT LEAD TIME

Table 18. Sample List of Yearly Cost Summaries

```
                                        **INTERMEDIATE RESULTS**

                                        RESULTS FOR YEAR  =   1

 NUMBER    NUMBER ORDERS  AVERAGE  AVG. INV. AVG. ON    NUMBER   ORDERING HOLDING BACKORDER   TOTAL
DEMANDED  ORDERED PLACED INVENTORY   VALUE  BACKORDER BACKORDED    COST     COST     COST    OPERATING
                                                                                               COST

   733       699     9       25       80.3       .8        82      33.6     23.3      5.9       62.7
   229       236     5       20       99.3       .0         2      18.6     15.9       .5       35.1
   124       135     9        6      104.0       .0         0      33.6     29.1       .0       62.7
   507       550    16       15      154.5       .2        14      59.7     52.5      6.7      118.9
   988       992    30        7      303.7      4.7       385     111.9     69.9     89.2      270.9
   372       397    13       12      252.5       .1        12      48.5     37.9      4.0       90.4
   160       169    13        3      179.6       .5        39      48.5     48.5      7.2      104.2
   101       112    14        3      260.1       .0         4      52.2     54.6      3.9      110.7
   824       826    19        2       50.1     14.0       559      70.9      9.0    111.8      191.7
   598       623    18       15      209.6       .3        23      67.1     65.0      9.0      141.1
```

Table 19. Sample List of Final Cost Analysis

ITEM NO.	TYPE POLICY	Q OR 1ST S	R OR 2ND S	ORDER COST	HOLD COST	BACKORDER COST	TOTAL COST
1	1	84	19	35.4	21.3	18.6	75.3
2	1	48	8	18.6	17.1	.3	36.0
3	1	15	7	31.7	31.7	.0	63.4
4	1	35	26	55.9	57.1	3.4	116.4
5	1	35	14	111.9	75.1	88.6	275.6
6	1	31	14	46.6	42.5	2.1	91.2
7	1	13	7	46.6	51.0	6.6	104.2
8	1	8	6	48.5	59.8	1.9	110.2
9	1	48	6	67.1	17.5	87.7	172.3
10	1	35	30	67.1	69.5	4.6	141.2

AVG AGGREGATE ORDERING COST	529.7
AVG AGGREGATE HOLDING COST	442.5
AVG AGGREGATE BACKORDER COST	213.8
AVG AGGREGATE TOTAL COST	1185.9

APPENDIX E

DUNCAN'S MULTIPLE RANGE TEST - DATA SET 1

Table 20. Duncan's Multiple Range Test on Measure of Dispersion Observations for Data Set 1

1. Treatment Means in Ascending Order:

$\bar{X}_{C4} = 26.22$

$\bar{X}_{C2} = 26.27$

$\bar{X}_{C1} = 26.67$

$\bar{X}_{C3} = 27.15$

$\bar{X}_{IDP} = 34.70$

2. Set of Least Significant Ranges:

2	3	4	5
.641	.674	.691	.706

3. Comparisons:

C4 vs IDP = 8.482 > .706

C4 vs C3 = .932 > .691

C4 vs C1 = .456 < .674

C4 vs C2 = .05 < .641

C2 vs IDP = 8.432 > .691

C2 vs C3 = .882 > .674

C2 vs C1 = .406 < .641

C1 vs IDP = 8.026 > .674

C1 vs 3 = .476 < .641

C3 vs IDP = 7.550 > .641

APPENDIX F

ANALYSIS OF RESULTS USING DATA SET 2

Table 21. Analysis of Results - Data Set 2 for Average AOHI Observations and Average Annual Operating Costs

1. Resulting Observations:

Indept. Derived Policies	COPEAK RULE 1	COPEAK RULE 2
267.1 ($1586.0)	265.6 ($1765.3)	263.6 ($1762.5)
267.0 ($1587.5)	264.0 ($1759.6)	265.3 ($1775.7)
266.8 ($1589.3)	262.8 ($1762.0)	264.8 ($1782.9)
266.6 ($1589.3)	266.4 ($1759.4)	267.2 ($1767.1)
266.8 ($1583.7)	266.0 ($1765.2)	262.1 ($1779.6)

2. ANOVA:

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	14.75	2	7.375	3.712
Error	23.84	12	1.987	
Total	38.59	14		

Table 22. Analysis of Results - Data Set 2 for Measure of Dispersion Observations

1. Resulting Observations:

Indept Derived Policies	COPEAK RULE 1	COPEAK RULE 2
68.6	48.7	47.6
70.3	46.2	49.6
69.5	50.1	48.4
67.9	46.2	47.0
68.1	47.5	47.9

2. ANOVA:

SOURCE	SS	Degrees Freedom	MS	F_o
Between Treatments	1464.73	2	732.365	458.30
Error	19.18	12	1.598	
Total	1483.91	14		

APPENDIX G

SENSITIVITY ANALYSIS

Table 23. Results from Sensitivity Analysis

MGLVL	RUN	RULE	"Days-to Order	AMTQ	AMTPIN	ILOOK	C_1	C_2	COST	AVG AOHI/M-of-D
122	1	1	4	.75	0	0			1196.1	124.83/25.16
122	2	1	7	.75	0	0			1212.1	132.82/26.75
122	3	1	10	.75	0	0			1217.7	138.50/28.28
122	4	1	14	.75	0	0			1300.4	142.59/28.24
122	5	1	7	.75	.75	0			1303.8	108.88/25.32
122	6	1	7	.75	.75	1			1536.5	95.32/25.87
122	7	1	7	.75	.75	2			1717.0	87.88/24.97
122	8	1	7	.75	.75	4			5112.3	70.20/22.17
122	9	1	7	.50	0	0			1165.4	133.91/25.67
122	10	1	7	.50	.50	0			1191.7	124.80/24.50
122	11	1	7	.50	.50	1			1250.5	115.54/23.16
122	12	1	7	.50	.50	2			1437.5	106.14/22.17
122	13	1	7	.50	.50	4			2264.9	93.68/23.56
122	14	1	7	.75	0	1			1281.4	127.08/25.92
122	15	1	7	.75	0	2			1339.1	123.12/26.64
122	16	1	7	.75	0	4			1630.5	119.09/26.20
123	17	4	7	.75	0	0	-.1	.1	1185.8	131.38/26.18
123	18	4	7	.75	0	0	-.1	.5	1219.3	132.83/27.33
123	19	4	7	.75	0	0	-.1	.9	1237.9	134.21/28.40
123	20	4	7	.75	0	0	-.5	.1	1176.9	131.71/26.18
123	21	4	7	.75	0	0	-.5	.5	1201.3	130.62/27.10
123	22	4	7	.75	0	0	-.5	.9	1184.5	129.76/26.76
123	23	4	7	.75	0	0	-.9	.1	1179.4	128.42/25.26
123	24	4	7	.75	0	0	-.9	.5	1210.3	127.55/25.83
123	25	4	7	.75	0	0	-.9	.9	1197.2	127.89/26.34

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