

EFFECT OF YARN LENGTH ON TENSILE  
STRENGTH AND ITS DISTRIBUTION

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EFFECT OF YARN LENGTH ON TENSILE  
STRENGTH AND ITS DISTRIBUTION

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## SUMMARY

The tensile stress-strain characteristics of yarns are very useful physical properties. However, the stress-strain properties depend greatly not only on the inherent properties of the materials but also on the measuring conditions and methods, such as extension speed, sample length and so on. In addition to these variables, yarns themselves are not uniform, and the tensile strengths along the length of the yarn are variable.

In 1926, Peirce proposed the weakest link theory to explain the distribution of strength variance, and to estimate the strength at multiple lengths of the original length. It has been found, however, that Peirce's theory deviates from the experimental data.

As the preliminary experiment of this thesis, the effect of sample length, rate of testing, and breaking time on the breaking strength and elongation was determined by using nylon and polyester filament yarns and acrylic and cotton spun yarns. The relationships between parameters of tensile properties were made, explained, and compared with results of Meredith and others.

Several thousand tensile tests were made on nylon filament and acrylic spun yarns in various lengths, and the distributions of their breaking strengths and elongations were calculated. Evaluation of the test results indicated that the decrease in strength with increasing sample length agreed generally with Peirce's results.

It was found that the longer the sample length, the lower became

## CHAPTER I

### INTRODUCTION

#### Statement of the Problem

The stress-strain characteristics of yarns, comparatively speaking, can be measured easily; therefore, these characteristics have been regarded as the most useful physical properties for controlling the quality and appraising the merits of yarns.

But the stress-strain properties depend greatly not only on inherent properties of the materials but also on the measuring methods and conditions; for example, types of testing machines, the specimen length, the humidity and temperature, and so on.

On the other hand, some test methods to determine yarn strength and elongation are prescribed in ASTM,<sup>1,2,3,4</sup> BS,<sup>5,6</sup> JIS,<sup>7</sup> etc., and the measurement values by those methods are prescribed as inherent ones whose results are calculated as the average of a number of specimens on the nominal gage length.

However, supposing these observed values do not arise from a symmetrical distribution, there should be some differences between the arithmetic mean, median and mode. Further, this variability is considered to be affected by the specimen length of yarns, too. The increase of the specimen length, in general, is considered to decrease its average tensile strength, that is, the average strength of a long specimen is lower than that of a short specimen. But it is observed, in reality, that some long specimens are often stronger than short ones, since the specimens have

On the other hand, George (2) presented, in 1951, stress-strain curves for a nylon monofilament, extended at constant "true" rates of extension,  $e$ , which are given by

$$e = (L_0/L) (dL/dt) \quad [2]$$

where  $L_0$  is the initial length and  $L$  is the length at time  $t$ .

By using this term, he was concerned with the macro-and microscopic evidence for the existence of structural domains within filaments.

Peirce (3) proposed the weakest link theory in 1926: that the strength of a test specimen is that of its weakest element of length, and the tensile strength decreases with the length of specimen in a way which is definitely calculated from the distribution of strength of a short specimen  $l$ .

He concluded successfully that the decrease in mean strength and in irregularity is directly proportional to the irregularity of the short specimens and to a factor, depending only on the multiple by which the length is increased and very simply calculated therefrom.

As shown in Figure 1, the distribution of the original short specimen strength ( $n=1$ ) is assumed to be Gaussian. The mean of the distribution for various-sized specimens ( $n=2,3,10,100$ ) is indicated by a vertical line in each case, which is calculated from the normal distribution at  $n=1$ .

Morton and Hearle (4) described Peirce's theory in their book Physical Properties of Textile Fibers.

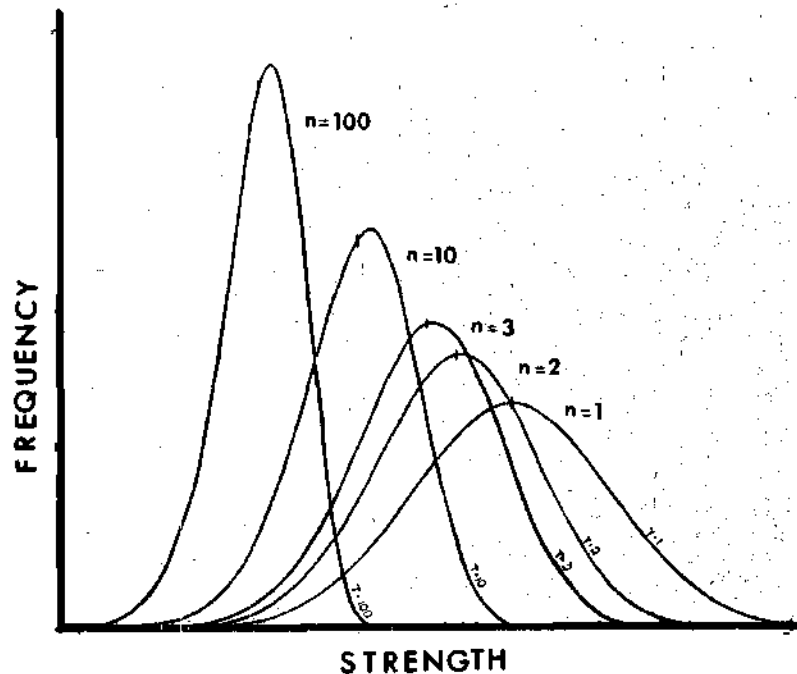


Figure 1. Frequency Curves of Strength for Multiple Length (after Peirce)

Let  $f_p(x) \cdot dx$  be the probability that the strength of a specimen of length  $l$  should lie between  $x$  and  $(x+dx)$ . The function  $f_p(x)$  gives the distributions of breaking loads. It is assumed that this function does not vary significantly from one part of the batch of specimens to another. From the values of the distribution  $f_p(x)$ , one can work out the mean value,  $\mu_p$ , and the standard deviation,  $\sigma_p$ , by the usual methods.

We now wish to find the distribution of breaking loads for specimens of length  $nl$ , that is, the probability,  $f_{nl}(x) \cdot dx$ , that the strength of a specimen of length  $nl$  lies between  $x$  and  $(x + dx)$ . The condition for this to occur is that the weakest of the  $n$  portions of length  $l$  of which the complete specimen of length  $nl$  is made up should have a strength lying between  $x$  and  $(x + dx)$ . In other words, any one of the  $n$  portions must have a strength greater than  $x$ .

But the probability that any one of  $n$  lengths  $l$  has a strength between  $x$  and  $(x + dx)$  is  $nf_p(x)dx$ .

The probability that the strength of a length  $l$  shall be greater than  $x$  is  $\int_x^{\infty} f_p(x)dx$ ;

and so the probability that all  $(n-1)$  lengths shall have a strength greater than  $x$  is

$$\left[ \int_x^{\infty} f_l(x) dx \right]^{(n-1)}$$

The probability that the strength of specimen of length  $nl$  lies between  $x$  and  $(x + dx)$  will therefore be given by the product of these two terms. That is:

$$f_{nl}(x) = n f_l(x) \left[ \int_x^{\infty} f_l(x) dx \right]^{(n-1)} \quad [3]$$

Using this relation, the frequency distribution can be worked out for any length of specimen. The relation is valid whether  $n$  is less than or greater than unity."

Figure 2 shows an example of the application of this formula to cotton yarns.

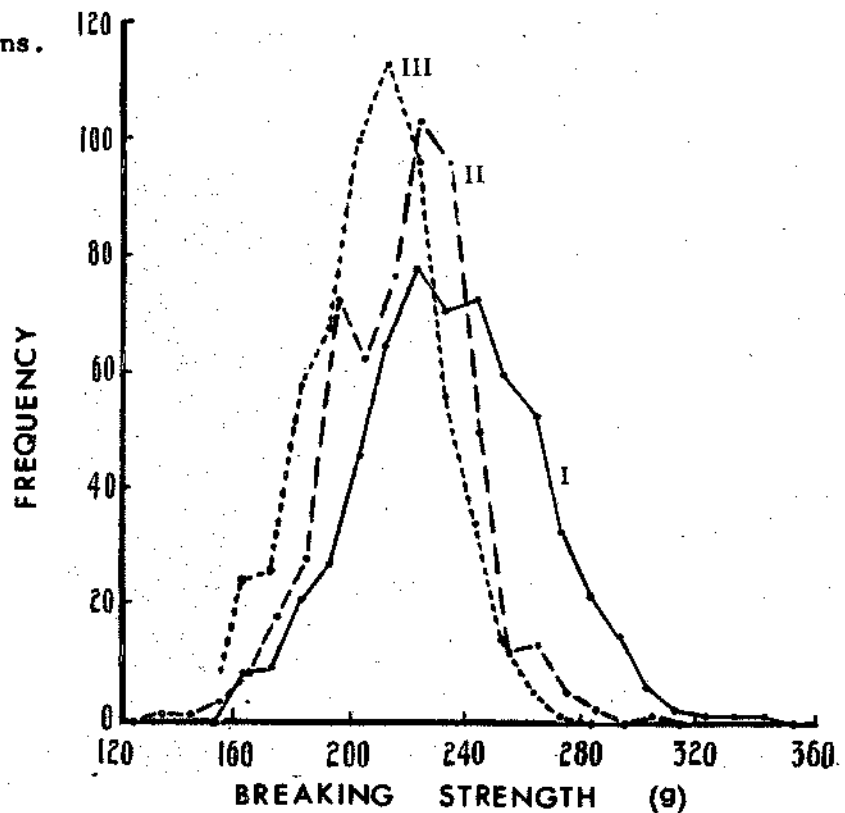


Figure 2. Application of Peirce's Theory to Cotton Yarn. (Curves 1 and 2 are experimental curves for test-lengths of 9 in. and 27 in. respectively; curve 3 is the calculated curve for 27 in. test-lengths from the data in curve 1.)

Table 1. The Strength of Multiple Lengths (after Peirce)

Specimen Length n	$\frac{w\ell - \mu n\ell}{\sigma}$	$3\sqrt{2}(1-n-\frac{1}{5})$	$\frac{\sigma n\ell}{\sigma\ell}$	$n - \frac{1}{5}$
$\ell$	0	0	1	1
2 $\ell$	.564	.549	.82	.87
3 $\ell$	.846	.836	.74	.80
4 $\ell$	1.027	1.028	.70	.76
5 $\ell$	1.160	1.169	.66	.72
10 $\ell$	1.541	1.570	.58	.63
30 $\ell$	2.051	2.096	.49	.51
40 $\ell$	2.164	2.220	.48	.48
80 $\ell$	2.432	2.478	.44	.42
100 $\ell$	2.503	2.560	.42	.40
160 $\ell$	2.673	2.701	.41	.36
275 $\ell$	2.857	2.859	.39	.33
1000 $\ell$	3.253	3.182	.35	.25

For further mathematical development, it is necessary to assume a form for the function  $f_{\rho}(x)$ . It is simplest to assume a normal distribution.

$$f_{\rho}(x) = \frac{1}{\sqrt{2\pi} \sigma_{\rho}} e^{-\frac{(x-\mu_{\rho})^2}{2\sigma_{\rho}^2}} \quad \dots\dots [4]$$

This relation can be substituted in equation [3], and the new distribution is then defined. Figure 1 shows an example of this. It will be noticed that, even though we start with a symmetrical normal distribution, the derived distribution at other lengths is skew.

The distribution  $f_{n\rho}(x)$  is thus known in terms of  $\mu_{\rho}$ ,  $\sigma_{\rho}$  and  $n$ . Analysing this expression, and making some mathematical approximations, Peirce obtained equations giving the mean strength,  $\mu_{n\rho}$ , and standard deviation,  $\sigma_{n\rho}$ , for specimens of length  $n\rho$ . The relations are:

$$\mu_{\rho} - \mu_{n\rho} = 3\sqrt{2} (1-n^{-\frac{1}{5}}) \sigma_{\rho} \quad \dots\dots [5]$$

$$\sigma_{n\rho}/\sigma_{\rho} = n^{-\frac{1}{5}} \quad \dots\dots [6]$$

Table 1 shows the values obtained using these approximations with his theoretical results.

Thus, Peirce's theory is a useful approximation. But this theory makes the assumptions that the distribution of strength is independent of the part of the sample considered, and that the frequency distribution of strength of the original short specimen is normal. Usually Peirce's relation gives deviations between approximate and experimental results. These approximate values also depend on the

size of the original short specimen.

In 1965, Nakashima and Ota (5) wrote their article, "Dependence of Strength of Specimen on its Length", to explain the length effect on strength, with the theory based on a return probability and the distribution of specimen length zero.

The following equation was given to estimate the strength distribution for a given specimen length  $l$ .

$$1-F_l(x) = [1-F_0(x)] \exp \left[ - \frac{f_0(x) / f_0(M_0)}{1-F_0(x)} \frac{l}{2\lambda} \right] \quad \dots [7]$$

where  $f_l(x)$  and  $F_0(x)$  are the distribution functions for length  $l$  and zero, respectively.  $f_l(x)$  is the probability density function for specimen length  $l$ .  $f_0(x)$  and  $f_0(M_0)$  are the probability density functions at strengths  $x$  and  $M_0$  (median of strength), respectively, for specimen length zero.  $\lambda$  is the mean return interval of  $M_0$ .

## CHAPTER II

## PRELIMINARY EXPERIMENT

Definition of Terms and Their Relation

V: Extension speed (inch  $\cdot$  min<sup>-1</sup>)

L: Sample Length (inch)

F: Breaking Strength (lb)

$l$ : Breaking Extension (inch)

e: Breaking Elongation (%)

A: Elongation Speed (min<sup>-1</sup>)

t: Breaking Time (sec)

$$A = V/L \quad \dots [8]$$

$$e = 100 l/L \quad \dots [9]$$

$$t = 60(l/V) = 60(eL/100/AL) = (60/100)(e/A) \quad \dots [10]$$

Effect of Sample LengthEffect of Sample Length on Breaking Strength

In general, it has been shown that the mean measured strength of a specimen decreases as the sample length is increased. This fact is well known as the "weak-link" effect. Suppose that the yarn strength at every point along the length is like Figure 3. If the sample lengths are  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{DE}$ , the measured strengths should be 2, 3, 1, and 4, respectively, which are the weakest places in each sample length, and the

mean strength is the average of 2,3,1, and 4. But if the sample is tested at twice the length,  $\overline{AC}$  and  $\overline{CE}$ , each will break at its own weakest place, 2 and 1, and the mean strength is the average of 2 and 1. Further if the sample is tested at length  $\overline{AE}$ , the strength must be 1.

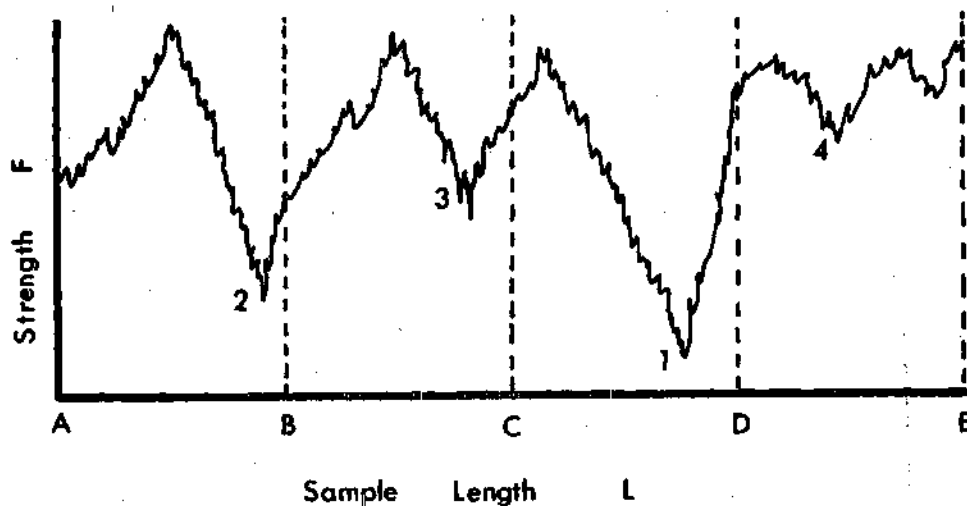


Figure 3. Weak-link Effect

In this way, the mean strength decreases with the increase of sample length.

Tables 3 to 6 and Figure 5 show the practical data obtained in this preliminary experiment. As shown in Figure 5, the breaking strength  $F$  of every sample, nylon and polyester filament yarns, and acrylic and cotton spun yarns, decreased with the increase of sample length at every extension speed, 5, 10, 12, and 20 inches per minute. The rates of strength decrease, for example, are 0.06 lb. and 0.16 lb. with an increase in sample length from five inches to 20 inches for nylon and

acrylic, respectively.

It can be seen that each sample has a different rate of strength decrease with an increase in sample length.

#### Effect of Sample Length on Breaking Elongation

Breaking elongation  $e$ , in general, decreases with an increase in sample length  $L$ . Suppose  $l$  is the measured breaking extension for a tensile test of sample length  $L$ .

In Figure 4, suppose  $\overline{P_1P} + \overline{Q_1Q} = \Delta l$

$$L + \Delta L = L_1$$

where  $L_1$  is the true sample length.

Thus, the true breaking elongation  $e_1$  is

$$\begin{aligned} e_1 &= \frac{100 l}{L_1} \\ &= \frac{100 l}{L + \Delta L} \\ &= \frac{100 l}{L} \cdot \frac{1}{1 + (\Delta L/L)} \\ &= e \cdot \frac{1}{1 + (\Delta L/L)} \end{aligned}$$

That is,  $e = e_1(1 + \Delta L/L)$  ..... [11]

Therefore, the value of the measured breaking elongation  $e$  becomes greater than that of the true breaking elongation  $e_1$  with a decrease in sample length  $L$ , that is, the breaking elongation  $e$  decreases with an increase in sample length  $L$ .

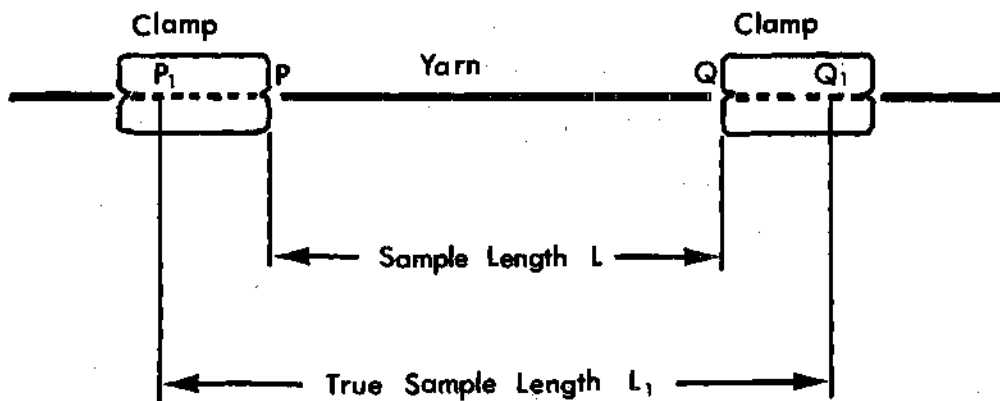


Figure 4. The True Sample Length

Tables 3 to 6 and Figure 6 show the practical data obtained in this preliminary experiment. As shown in Figure 6, the breaking elongation,  $e$ , of every sample tended to decrease with increase in sample length  $L$  at every extension speed, following a hyperbolic curve. The rates of decrease of polyester and nylon filament yarns are greater than those of cotton and acrylic spun yarns. These two points can be explained by using equation [11]. In equation [11],  $e$  varies inversely as  $L$  against the constant  $e_1$  and  $\Delta L$ . Therefore, the relation between  $e$  and  $L$  should be a hyperbola, and the different rates of decrease occurred probably because  $\Delta L$  of the filament yarns was greater than that of the spun yarns.

#### Effect of Extension Speed

Experimental data are given in Tables 3 to 6 and Figures 7 and 8. Breaking Strength  $F$  is shown in Figure 7, with increase of the Extension Speed  $V$ . An increase in the extension speed  $V$  causes, shown in equation [10], decrease in the Breaking Time  $t$ . The effect of extension speed,

therefore, is mentioned later as the effect of breaking time.

Regarding the effect of Extension Speed  $V$  on the Breaking Elongation  $e$ , the breaking elongation may be, as shown in Figure 8, independent of the extension speed, as an approximation.

#### Effect of Breaking Time

As shown in the equation [10], the Breaking Time,  $t$ , is concerned with the Breaking Extension  $l$  and the Extension Speed,  $V$ . The Breaking Time,  $t$ , therefore, can be one of the main parameters affecting the mechanical properties of yarns. Experimental data of the Breaking Strength,  $F$ , shown in Figure 9, were plotted against a log scale of the Breaking Time,  $t$ . The Breaking Strength,  $F$ , decreased linearly with the increase of Breaking Time,  $t$ , on a log scale.

To compare these experimental data with the results by Meredith, only nylon filament data were calculated by using the formula [1] obtained by Meredith, as shown in Table 7 and Figure 10. The straight line in Figure 10 is the Meredith result, which was obtained by using the value  $-0.080$  for the strength-time coefficient  $k$  for a nylon filament. The experimental data fitted well with Meredith's results, although his formula was obtained by the rate of loading and these experimental data were obtained by the rate of extension.

Thus, the formula of Meredith can be applied to different types of tests, and these results can be easily compared by calculating an equivalent breaking time.

## CHAPTER III

### PROCEDURE

After the preliminary experiment, it was decided that nylon filament and acrylic spun yarns would be used in the main experiment as follows: (1) nylon continuous filament yarn (140 denier) (2) acrylic spun yarn (Orlon fiber, 20 count).

The tensile testing machine used was the Instron Tester in which the rate of increase of specimen length was uniform with time.

As shown in Table 2, 2,200 stress-strain curves of tensile tests were made with the two different materials and nine different sample lengths at a constant rate of extension of 10 inches/min. ASTM Standard D2256-69 was followed for all tests except for the sample length, number of tests, and rate of extension. A H-P 9100A desk type computer was used to compute the data and to calculate the distribution of strength and elongation, including the mean and the standard deviation.

"Uster" evenness and spectrograph testers were used to determine yarn thickness variation.

Table 2. Experimental Procedure

Material	Sample Length (in.)	Number of test specimens	Rate of Extension (in./min)
Nylon	5	100	10
Continuous	10	100	10
Filament	15	100	10
(140 denier)	20	100	10
Acrylic	3	200	10
Spun	5	200	10
(Orlon, 20's)	7.5	200	10
	10	200	10
	12.5	200	10
	15	200	10
	17.5	200	10
	20	200	10
	22.5	200	10

## CHAPTER IV

## DISCUSSION OF RESULTS

Strength DistributionExperimental Strength Distribution

The frequencies from the tests on each group of 200 specimens at nine different sample lengths, 3 in. to 22.5 in., of acrylic spun yarn are shown in Table 8 with the class interval of 0.01 pound, and also the frequencies from the test on each group of 100 specimens at four different sample lengths, 5 in. to 20 in., of nylon filament yarn are shown in Table 9. The frequency curves of only three Sample Lengths L, 5, 10, and 20 in., are shown in Figure 11 as examples.

Tables 10 and 11 show the observed cumulative frequencies of acrylic spun and nylon filament yarns, respectively. These frequency curves are shown in Figure 12.

Mean Strength  $\mu$  and Standard Deviation  $\sigma$ 

Experimental values of Mean Strength  $\mu$  and Standard Deviation  $\sigma$  for strength distribution at each sample length are listed in Table 12.

The mean breaking strengths obtained in the experiment, shown in Figure 13, decreased with an increase of sample length. The mean breaking strength of sample length 5 inches for nylon filament yarn, for example, was 1.82 pounds and that of 20 inches was 1.72 pounds. That is, the decrease in strength was nearly 6 percent with a four times increase in sample length. The rate of decrease was not linear. The

same mean breaking strength values were plotted on a log scale of sample length in Figure 14, and now the rate of decrease gave a straight line plot. This reason is discussed later.

The standard deviations obtained in the experiment, shown in Figure 15, decreased non-linearly with increase of sample length, although the standard deviations for nylon filament yarn were remarkably smaller than those for acrylic spun yarn.

#### Comparison of Experimental Data with Peirce's Theory

The data for 5 in. sample length were taken as the original length to compare the obtained experimental data, listed in Table 12, with Peirce's theory, listed in Table 1. The results are shown in Figure 16. Both results for nylon filament and acrylic spun yarns were observed to be quite different from Peirce's theory, that is, one of them was higher and another was lower than Peirce's theory.

The rate of strength decrease for nylon filament yarn, whose standard deviations were observed to be remarkably smaller than those for acrylic spun yarn, was greater than that for Peirce's theory and acrylic spun, whose rate of decrease was smaller than that for Peirce's theory.

#### Depending upon the Original Length

However, it seemed that these results were due to the fact that the 5 in. sample length was taken as the original length.

Table 13 and Figure 17 were made to confirm whether or not the strength of multiple lengths depended upon the original length.

As shown in Figure 17, the curve for the shorter original length of nylon filament yarn approached more closely to Peirce's theory. A curve for an original length shorter than 5 in. can be expected to agree

with Peirce's theory.

Regarding acrylic spun yarn, the curve approached closer to Peirce's theory as the original length changed from 5 in. to 10 in. The curve for 15 in. original length was exactly on the curve for Peirce's theory, and then the curve separated from Peirce's theory as the original length changed from 15 in. to 20 in.

Therefore, it was made clear that the strength of multiple lengths by Peirce's theory depended upon the original length. It is said that Peirce's theory deviates from the experimental data. However, if the proper original length can be estimated easily, such as 15 in. length in acrylic spun yarn, Peirce's theory becomes an adequate theory to explain the relationship between the sample length and its tensile breaking strength. Further, once the proper original length can be determined, a tensile breaking strength of any sample length can be estimated by using the values of mean strength and standard deviation for the strength distribution on this original length as a test specimen whether it be a yarn, fiber, or a metal rod.

Table 14 shows a comparison of values obtained using Peirce's approximation relations, equation [5] and [6], and the results of 15 in. adequate original length determined in Figure 17, with experimental results for acrylic spun yarn.

#### Correlation of Breaking Elongation with Breaking Strength

##### Mean Elongation $\mu$ and Standard Deviation $\sigma$

Experimental values of Mean Elongation  $\mu$  and Standard Deviation  $\sigma$  for elongation distribution at each sample length are listed in Table 15.

The mean breaking elongation obtained in the experiment, shown in

Figure 18, decreased with an increase of sample length, the same as for the mean breaking strength.

The standard deviations for elongation distribution obtained in the experiment, shown in Figure 19, decreased with an increase of sample length. The standard deviations of elongation for nylon filament yarn were larger than those for acrylic spun yarn, although the standard deviation of strength for nylon was remarkably smaller than that for acrylic. Therefore, the distribution curves of elongation for nylon filament yarn showed a larger variability than the sharp shape curves for acrylic spun yarn, as shown in Figure 20, although opposite phenomena for strength distribution have been shown in Figure 11.

#### Relation between Breaking Strength and Elongation

As shown in Figure 9, the relation between Breaking Strength  $F$  and Breaking Time  $t$  is as follows:

$$F = a \log t + b \quad \dots [12]$$

where  $a$  and  $b$  are constant.

However, from equation [9] and [10]

$$t = \frac{60 eL}{100 V} \quad \dots [13]$$

and, from equation [11]

$$L = \frac{e_1 \Delta L}{e - e_1} \quad \dots [14]$$

Substituting equation [13] and [14] in [12],

$$F = a \log \left( \frac{60}{100} \cdot \frac{e_1 \Delta L}{V} \cdot \frac{e}{e - e_1} \right) + b \quad \dots [15]$$

The equation [15], now, shows the relationship between Breaking Strength  $F$  and Breaking Elongation  $e$ . Further, the term  $(60/100) (e_1 \Delta L/V)$  is constant because each  $e_1$  and  $\Delta L$  is constant as mentioned in Figure 4, and the extension speed  $V$  was fixed at a constant 10 in./min. in this experiment. The relation between Breaking Strength  $F$  and Breaking Elongation  $e$ , therefore, must be approximately linear on a log scale of Breaking Elongation  $e$ .

By the way, the relation between Breaking Strength  $F$  and Sample Length  $L$  can be obtained if equations [11] and [13] are substituted for equation [12]

$$F = a \log \frac{60e_1}{100V} (L + \Delta L) + b \quad \dots [16]$$

Equation [16] shows that Breaking Strength  $F$  is linear in a log scale of Sample Length  $L$ . Figure 14 is an experimental data plot showing a linear relation of breaking strength to sample length on a log scale.

#### Correlation of Breaking Elongation with Breaking Strength

There must be an approximately linear relation between Breaking Strength  $F$  and Breaking Elongation  $e$  on a log scale, as shown in equation [15].

An empirical relationship for this was calculated from experimental data by using the least squares method, as shown in Table 16. According to this table, equation [17] was obtained for acrylic spun yarn, and equation [18] for nylon filament yarn.

$$F = 1.084 \log e + 0.091 \quad \dots [17]$$

$$F = 0.725 \log e + 0.773 \quad \dots [18]$$

Figure 21 shows these relations, which were plotted as Breaking Strength  $F$  against Breaking Elongation  $e$  on a log scale. The breaking strength of this nylon filament yarn (140 denier), for example, can be estimated at 1.787 lb. at the breaking elongation 25 percent from equation [18]

Estimating for the Strength Distribution of Sample Length Zero

Equations for Estimating

Nakashima and Ota's theory as shown in equation [7], which is based on the distribution of sample length zero and a return probability, is useful to estimate the strength distribution of sample length zero.

$$1-F_{\ell}(x) = [1-F_0(x)] \exp\left[ \frac{f_0(x)/f_0(M_0)}{1-F_0(x)} \cdot \frac{\ell}{2\lambda} \right] \dots [7]$$

This equation [7] should be changed to equation [19] when sample length changes from  $\ell$  to  $n\ell$ .

$$1-F_{n\ell}(x) = [1-F_0(x)] \exp\left[ \frac{f_0(x)/f_0(M_0)}{1-F_0(x)} \cdot \frac{n\ell}{2\lambda} \right] \dots [19]$$

Raising both sides in equation [7] to  $n$ -th power,

$$[1-F_{\ell}(x)]^n = [1-F_0(x)]^n \exp\left[ \frac{f_0(x)/f_0(M_0)}{1-F_0(x)} \cdot \frac{n\ell}{2\lambda} \right] \dots [20]$$

From equation [19] and [20],

$$\exp\left[ \frac{f_0(x)/f_0(M_0)}{1-F_0(x)} \cdot \frac{n\ell}{2\lambda} \right] = \frac{1-F_{n\ell}(x)}{1-F_0(x)} = \frac{[1-F_{\ell}(x)]^n}{[1-F_0(x)]^n}$$

That is,

$$[1-F_0(x)]^{n-1} = \frac{[1-F_l(x)]^n}{1-F_{nl}(x)} \quad \dots [21]$$

Therefore, if the strength distribution of sample length  $l$ ,  $F_l(x)$ , and the strength distribution of sample length  $nl$ ,  $F_{nl}(x)$ , are known actually, the strength distribution of sample length zero  $F_0(x)$ , can be calculated by using equation [21].

#### Procedures to Estimate

The distributions of sample length 5 in., 10 in., and 20 in., shown in Figure 11, were used, that is, ( $l=5$ ,  $n=2$ ) and ( $l=5$ ,  $n=4$ ) according to equation [21] to obtain the distributions of sample length zero.

Actual procedures are shown in Tables 17 to 20 in the cases of ( $l=5$ ,  $n=2$ ) and ( $l=5$ ,  $n=4$ ) for acrylic spun and nylon filament yarns.

#### Calculated Frequencies

Comparing Table 18 with 17, and Table 20 with 19, there was little difference in the results for the cases of ( $l=5$ ,  $n=2$ ) and ( $l=5$ ,  $n=4$ ). To obtain the frequency  $f_0(x)$ , therefore, the cumulative frequencies  $F_0(x)$  obtained in the case of ( $l=5$ ,  $n=4$ ) were used in Tables 21 and 22 for acrylic spun and nylon filament yarns, respectively.

The calculated distributions of sample length zero are shown in Figure 22 by bar charts.

#### Estimated Normal Distributions

Assuming the normal distribution for the strength distribution of sample length zero, this distribution also can be calculated from actual data, although Peirce assumed a normal distribution curve for a finite

original sample length.

Tables 23 and 24 were made to obtain the normal distribution curves by using the values of standard deviations 0.1136 and 0.030, and means 0.9922 and 1.829 which were calculated in Tables 21 and 22 for acrylic spun and nylon filament yarns.

The results obtained are shown by two curves in Figure 22.

## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The effect of sample length, rate of testing, and breaking time on the breaking strength and elongation was determined in the preliminary experiment by using nylon and polyester filament yarns and acrylic and cotton spun yarns.

The relations between parameters of yarn tensile properties were made, and the results were compared with the results by Meredith and others.

Several thousand tensile tests were made of nylon filament and acrylic spun yarns in various lengths, and the distributions of their breaking strengths and elongations were calculated.

For the original frequency curve, a normal distribution was assumed using Peirce's "the weakest link theory", and it was observed, generally, that Peirce's theory deviates from the experimental data.

The decrease in breaking strength with increasing sample length in this experiment agreed generally with Peirce's theory. It was found, however, that the longer the sample length, the lower became the value of standard deviation of its distribution, and the values of standard deviation for filament yarn were much smaller than those for spun yarns. It was suspected, therefore, that estimation of the strength of multiple lengths depended upon the original length.

The curve for the shorter original length of nylon filament yarn

approached more closely to Peirce's theory, and a curve for an original length shorter than 5 in. could be expected to agree with Peirce's theory. Regarding acrylic spun yarn, the curve approached closer to Peirce's theory as the original length changed from 5 in. to 10 in. The curve for 15 in. original length was exactly on the curve for Peirce's theory, and then the curve separated from Peirce's theory as the original length changed from 15 in. to 20 in. Thus, it was made experimentally clear that the strength of multiple lengths by Peirce's theory depended upon the original length.

Once the proper original length such as 15 in. length in acrylic spun yarn can be determined, Peirce's theory becomes an adequate theory to explain the relationship between the sample length and its tensile breaking strength, and a tensile breaking strength of any sample length can be estimated by using the values of mean strength and standard deviation for the strength distribution based on this original length as a test specimen, whether it be a yarn, fiber, or a metal rod. The strength of sample length 500 in. of acrylic spun yarn used in this experiment, for example, can be estimated to be 0.747 lb. although it is difficult to measure actually the strength of such a long specimen length.

Furthermore, the correlation of breaking elongation with breaking strength was considered, and also the strength distributions for sample length zero were estimated by using data for a finite sample length.

#### Recommendations

A parameter of breaking time  $t$  includes some factors such as

breaking elongation  $e$ , sample length  $L$ , and extension speed  $V$ , as shown in the following equation:

$$t = (60/100)(eL/V)$$

In addition to this fact, the breaking strength decreases linearly with the increase of breaking time on a log scale regardless of different types of testing machines, such as constant rate of extension, constant rate of traverse, and constant rate of load. The parameter of breaking time, therefore, is very useful for comparing results in any kind of tensile test.

To estimate the breaking strength of multiple sample lengths, Peirce's theory is very useful, provided that the values of mean strength and standard deviation for the strength distribution on the proper original length are used.

A rough proper original length of any kind of yarn, fiber, or a metal rod can be estimated if tensile tests are made on a few sample lengths as in this experiment. It is not easy, however, to determine actually the proper original length such as 15 in. of acrylic spun yarn and less than 3 in. of nylon filament yarn, as in this experiment.

The variance length curve of thickness and its spectrogram can be obtained easily with an evenness tester. The spectrograms of acrylic spun and nylon filament yarns used in this experiment show 4 in. and 3 in. periodic waves of thickness variance, respectively. It seems that there is little relation between thickness and strength variances because the weakest place is not always the thinnest place in twisted spun yarn and multiple filament yarn.

An easy way to estimate the proper original length, such as using an evenness tester for thickness evenness still must be developed.

**APPENDIX**

Table 3. F, e, C.V., and t at Various Sample Lengths and Extension Speeds.

Nylon Filament (140 denier)

Sample Length L		5	10	12	20	Mean
		inch				
Extension Speed V						
	5 (in/min)	F	1.77	1.71	1.70	1.66
C.V.		2.15	2.11	1.41	1.63	1.83
e		25.52	21.13	20.40	19.89	21.74
C.V.		12.35	10.13	13.44	7.24	10.79
t		15.35	25.36	29.38	47.74	29.46
10	F	1.78	1.78	1.77	1.73	1.77
	C.V.	2.87	2.65	1.13	1.33	2.00
	e	23.12	22.03	21.42	20.43	21.75
	C.V.	16.52	12.67	9.30	7.17	11.42
	t	6.94	13.22	15.42	24.51	15.02
12	F	1.81	1.76	1.75	1.75	1.77
	C.V.	2.66	1.99	2.46	0.97	2.02
	e	24.02	20.16	18.43	20.76	20.84
	C.V.	14.65	16.17	6.88	6.12	10.96
	t	6.01	10.08	11.06	19.26	11.60
20	F	1.83	1.80	1.79	1.77	1.80
	C.V.	3.05	2.55	1.90	1.19	2.17
	e	27.82	20.99	21.51	19.73	22.51
	C.V.	14.45	14.05	11.97	7.53	12.00
	t	4.17	6.30	7.74	11.84	7.51
Mean	F	1.79	1.76	1.75	1.73	
	C.V.	2.68	2.33	1.73	1.28	
	e	25.12	21.08	20.44	20.20	
	C.V.	14.49	13.26	10.40	7.02	
	t	8.12	13.74	15.90	25.84	

F: Breaking Strength (lb.)  
e: Breaking Elongation (%)  
C.V: Coefficient of Variation (%)  
t: Breaking Time (sec)

Table 4. F, e, C.V., and t at Various Sample Lengths and Extension Speeds.

Polyester Filament (150 denier)

Extension Speed V	Sample Length L	5 (inch)	10	12	20	Mean
	5 (in/min)	F	1.48	1.43	1.39	1.42
C.V.		2.77	2.31	2.51	3.30	2.72
e		29.84	25.39	24.48	24.97	26.17
C.V.		9.99	13.00	10.52	10.41	10.98
10	t	17.90	30.47	35.26	59.93	35.89
	F	1.50	1.42	1.42	1.43	1.43
	C.V.	5.45	5.98	4.02	4.18	4.91
	e	31.06	23.49	22.94	23.87	25.34
12	C.V.	13.20	24.82	14.86	14.18	16.77
	t	9.32	14.09	16.52	28.64	17.14
	F	1.49	1.47	1.45	1.46	1.47
	C.V.	5.03	2.17	4.70	5.08	4.25
20	e	29.66	25.81	24.81	23.07	25.84
	C.V.	15.78	7.25	16.90	12.05	13.00
	t	7.42	12.90	14.82	23.07	14.55
	F	1.52	1.52	1.46	1.43	1.48
Mean	C.V.	6.31	7.91	3.71	4.75	5.67
	e	28.76	26.10	23.68	23.40	25.49
	C.V.	27.68	23.87	10.70	15.86	19.53
	t	4.31	7.83	8.53	14.04	8.68
Mean	F	1.50	1.46	1.43	1.44	
	C.V.	4.89	4.59	3.74	4.33	
	e	29.83	25.20	23.98	23.83	
	C.V.	16.66	17.24	13.25	13.13	
	t	9.74	16.32	18.78	28.67	

Table 6. F, e, C.V., and t at Various Sample Lengths and Extension Speeds.

Cotton Spun (20 count)

Extension Speed V	Sample Length L	5 (inch)	10	12	20	Mean
	5 (in/min)	F	0.88	0.80	0.82	0.78
C.V.		10.87	9.40	6.50	8.08	8.71
e		9.98	7.89	7.79	6.75	8.10
C.V.		9.02	7.99	6.10	4.22	6.83
10	t	5.99	9.47	11.22	16.21	10.72
	F	0.78	0.79	0.78	0.82	0.79
	C.V.	7.57	8.62	7.30	8.16	7.91
	e	8.86	7.69	7.38	7.01	7.74
12	C.V.	6.10	6.89	7.78	6.99	6.94
	t	2.66	4.61	5.32	8.41	5.25
	F	0.83	0.82	0.80	0.81	0.82
	C.V.	11.79	10.43	4.64	9.62	9.12
20	e	8.94	7.64	7.55	6.97	7.71
	C.V.	10.96	8.51	5.19	7.68	8.09
	t	2.24	3.82	4.53	6.97	4.39
	F	0.79	0.79	0.77	0.79	0.78
Mean	C.V.	9.71	10.35	9.53	7.70	9.32
	e	8.60	7.45	7.15	6.65	7.46
	C.V.	6.28	7.52	7.34	6.02	6.79
	t	1.29	2.24	2.57	3.99	2.52
Mean	F	0.82	0.80	0.79	0.80	
	C.V.	9.99	9.70	6.99	8.39	
	e	9.10	7.67	7.47	6.85	
	C.V.	8.09	7.73	6.61	6.23	
	t	3.05	5.04	5.91	8.90	

Table 7. The Relation Between Breaking Strength  $F$  and Breaking Time  $t$

Nylon Filament (140 denier)

Sample Length L		5 (inches)	10	12	20	Mean
Extension Speed V						
5 (in/min)	$F$	1.77	1.71	1.70	1.66	1.71
	$(F_1 - F)/F_1$	0.033	0.066	0.071	0.093	0.066
	$t$	15.35	25.36	29.38	47.74	29.46
	$t/t_1$	3.681	6.082	7.046	11.448	7.064
	$\log(t/t_1)$	0.566	0.784	0.848	1.059	0.849
10	$F$	1.78	1.78	1.77	1.73	1.77
	$(F_1 - F)/F_1$	0.027	0.027	0.033	0.055	0.036
	$t$	6.94	13.22	15.42	24.51	15.02
	$t/t_1$	1.664	3.170	3.698	5.878	3.603
	$\log(t/t_1)$	0.221	0.501	0.568	0.769	0.557
12	$F$	1.81	1.76	1.75	1.75	1.77
	$(F_1 - F)/F_1$	0.011	0.038	0.044	0.044	0.034
	$t$	6.01	10.08	11.06	19.26	11.60
	$t/t_1$	1.441	2.417	2.652	4.619	2.782
	$\log(t/t_1)$	0.159	0.383	0.424	0.665	0.444
20	$F$	$F_1 = 1.83$	1.80	1.79	1.77	1.80
	$(F_1 - F)/F_1$	0	0.016	0.022	0.033	0.018
	$t$	$t_1 = 4.17$	6.30	7.74	11.84	7.51
	$t/t_1$	1	1.511	1.856	2.839	1.802
	$\log(t/t_1)$	0	0.179	0.269	0.453	0.256
Mean	$F$	1.79	1.76	1.75	1.73	
	$(F_1 - F)/F_1$	0.018	0.037	0.043	0.056	
	$t$	8.12	13.74	15.90	25.84	
	$t/t_1$	1.947	3.295	3.813	6.196	
	$\log(t/t_1)$	0.289	0.518	0.581	0.792	

Table 8. Breaking Strength Frequency (Acrylic Spun)

inches lbs.	3	5	7.5	10	12.5	15	17.5	20	22.5
0.63 - 0.64	0	0	0	0	1	0	0	0	
0.65 - 0.66	1	0	0	0	1	0	0	0	
0.67 - 0.68	1	0	1	1	0	0	1	0	
0.69 - 0.70	1	1	1	2	1	0	0	0	1
0.71 - 0.72	2	2	0	1	0	0	0	1	1
0.73 - 0.74	2	3	1	3	1	2	1	1	2
0.75 - 0.76	2	1	2	2	2	0	2	3	3
0.77 - 0.78	3	3	1	5	3	1	0	3	6
0.79 - 0.80	1	4	5	4	5	4	4	3	11
0.81 - 0.82	1	4	2	5	5	3	3	5	10
0.83 - 0.84	5	6	2	9	9	10	7	13	12
0.85 - 0.86	3	12	2	6	5	14	9	14	16
0.87 - 0.88	7	8	7	10	8	10	14	14	19
0.89 - 0.90	8	13	12	16	16	17	16	27	18
0.91 - 0.92	5	10	10	7	21	22	20	16	23
0.93 - 0.94	13	9	13	18	13	13	17	22	21
0.95 - 0.96	11	11	23	12	18	20	25	26	25
0.97 - 0.98	14	13	25	16	14	11	17	13	9
0.99 - 1.00	13	17	17	13	18	22	15	11	15
1.01 - 1.02	5	11	19	13	17	17	18	7	2
1.03 - 1.04	20	11	14	19	17	8	9	10	5
1.05 - 1.06	16	12	11	10	12	6	6	9	1



Table 9. Breaking Strength Frequency (Nylon Filament)

Sample Length Class Interval (0.01 lb.)	5	10	15	20
1.65 - 1.66	1	1	1	1
1.67 - 1.68	0	0	2	3
1.69 - 1.70	0	1	1	17
1.71 - 1.72	0	6	14	39
1.73 - 1.74	1	9	20	34
1.75 - 1.76	3	22	46	6
1.77 - 1.78	3	36	16	0
1.79 - 1.80	14	20	0	0
1.81 - 1.82	28	4	0	0
1.83 - 1.84	28	1	0	0
1.85 - 1.86	17	0	0	0
1.87 - 1.88	5	0	0	0

Table 10. Breaking Strength Cumulative Frequency (Acrylic Spun)

inches lbs.	3	5	7.55	10	12.5	15	17.5	20	22.5
0.63 - 0.64					1				
0.65 - 0.66	1				2				
0.67 - 0.68	2		1	1	2		1		
0.69 - 0.70	3	1	2	3	3		1		1
0.71 - 0.72	5	3	2	4	3		1	1	2
0.73 - 0.74	7	6	3	7	4	2	2	2	4
0.75 - 0.76	9	7	5	9	6	2	4	5	7
0.77 - 0.78	12	10	6	14	9	3	4	8	13
0.79 - 0.80	13	14	11	18	14	7	8	11	24
0.81 - 0.82	14	18	13	23	19	10	11	16	34
0.83 - 0.84	19	24	15	32	28	20	18	29	46
0.85 - 0.86	22	36	17	38	33	34	27	43	62
0.87 - 0.88	29	44	24	48	41	44	41	57	81
0.89 - 0.90	37	57	36	64	57	61	57	84	99
0.91 - 0.92	42	67	46	71	78	83	77	100	122
0.93 - 0.94	55	76	59	89	91	96	94	122	143
0.95 - 0.96	66	87	82	101	109	116	119	148	168
0.97 - 0.98	80	100	107	117	123	127	136	161	177
0.99 - 1.00	93	117	124	130	141	149	151	172	192
1.01 - 1.02	98	128	143	143	158	166	169	179	194
1.03 - 1.04	118	139	157	162	175	174	178	189	199
1.05 - 1.06	134	151	168	172	187	180	184	198	200



Table 11. Breaking Strength Cumulative Frequency (Nylon Filament)

Class Interval (0.01 lb.) \ Sample Length (inches)	5	10	15	20
1.65 - 1.66	1	1	1	1
1.67 - 1.68	1	1	3	4
1.69 - 1.70	1	2	4	21
1.71 - 1.72	1	8	18	60
1.73 - 1.74	2	17	38	94
1.75 - 1.76	5	39	84	100
1.77 - 1.78	8	75	100	
1.79 - 1.80	22	95		
1.81 - 1.82	50	99		
1.83 - 1.84	78	100		
1.85 - 1.86	95			
1.87 - 1.88	100			

Table 12. The Strength of Multiple Lengths  
(Experimental)

		Sample Length (ins.)								
		3	5	7.5	10	12.5	15	17.5	20	22.5
Acrylic (Spun)	Mean $\mu$ (lbs.)	1.008	0.977	0.973	0.954	0.947	0.931	0.918	0.912	0.898
	Standard Deviation $\sigma$	0.121	0.112	0.109	0.104	0.090	0.086	0.078	0.073	0.071
Nylon (Filament)	$\mu$ (lbs.)	--	1.82	--	1.77	--	1.74	--	1.72	--
	$\sigma$	--	0.032	--	0.028	--	0.024	--	0.018	--

Table 13. The Strength of Multiple Lengths depending upon the Original Length.

Acrylic Spun	Sample Length (in.)								
	3	5	7.5	10	12.5	15	17.5	20	22.5
$\frac{\mu_5 - \mu}{\sigma_5}$	-0.277	0	0.036	0.205	0.218	0.411	0.527	0.580	0.705
$\frac{\mu_{10} - \mu}{\sigma_5}$	-0.519	-0.221	-0.183	0	0.067	0.221	0.346	0.404	0.538
$\frac{\mu_{15} - \mu}{\sigma_{15}}$	-0.895	-0.535	-0.488	-0.267	-0.186	0	0.151	0.221	0.384
$\frac{\mu_{20} - \mu}{\sigma_{20}}$	-1.315	-0.890	-0.836	-0.575	-0.479	-0.260	-0.082	0	0.192

Nylon Filament									
$\frac{\mu_5 - \mu}{\sigma_5}$	--	0	--	1.563	--	2.500	--	3.125	--
$\frac{\mu_{10} - \mu}{\sigma_{10}}$	--	-1.786	--	0	--	1.071	--	1.786	--
$\frac{\mu_{15} - \mu}{\sigma_{15}}$	--	-3.333	--	-1.250	--	0	--	0.833	--
$\frac{\mu_{20} - \mu}{\sigma_{20}}$	--	-5.556	--	-2.778	--	-1.111	--	0	--

Table 15. The Elongation of Multiple Lengths  
(Experimental)

		Sample Length (in.)								
		3	5	7.5	10	12.5	15	17.5	20	22.5
Acrylic Spun)	Mean $\mu$ (%)	9.13	5.14	6.56	5.89	5.86	5.79	5.46	5.30	5.08
	Standard Deviation $\sigma$	1.17	0.92	0.65	0.71	0.64	0.51	0.53	0.46	0.47
Nylon Filament	$\mu$	--	28.14	--	23.00	--	21.37	--	20.89	--
	$\sigma$	--	2.34	--	2.12	--	1.95	--	1.69	--

Table 16. Least Squares Estimates of the Slope

Acrylic Spun					
Sample Length (in)	Strength y (lbs)	Elongation x (%)	log x	y · log x	(log x) <sup>2</sup>
3	1.008	9.13	0.96047	0.96815376	0.9225026209
5	0.977	7.14	0.85370	0.83406490	0.7288036900
7.5	0.873	6.56	0.81690	0.79484370	0.6673256100
10	0.954	5.89	0.77012	0.73469448	0.5930848144
12.5	0.947	5.86 q	0.76790	0.72720130	0.5896704100
15	0.931	5.79	0.76268	0.71005508	0.5816807824
17.5	0.918	5.46	0.73719	0.67674042	0.5434490961
20	0.912	5.30	0.72428	0.66054336	0.5245815184
222.5	0.898	5.08	0.70586	0.63386228	0.4982383396
$\Sigma$	8.518	--	0.70991	6.74015928	5.6493368818
				b = 1.0840	
				a = 0.0914	
Nylon Filament					
5	1.82	28.14	1.44932	2.6377624	2.1005284624
10	1.77	23.00	1.36173	2.4102621	1.8543085929
15	1.74	21.37	1.32980	2.3138520	1.7683680400
20	1.72	20.89	1.31994	2.2702968	1.7422416036
$\Sigma$	7.05	--	5.46079	9.6321733	7.4654466989
				b = 0.7248	
				a = 0.7730	

Table 17. Estimation for the Strength Distribution of Sample  
Length zero (Acrylic spun,  $l = 5$ ,  $n = 2$ )

$x$	$F_l(x)$	$[1-F_l(x)]^n$	$F_{nl}(x)$	$1-F_{nl}(x)$	$[1-F_o(x)]^{n-1}$	$1-F_o(x)$	$F_o(x)$
0.67-0.68			0.5	0.5	0	0	1
0.69-0.70	0.5	0.25	1.5	-0.5	-0.5	-0.5	1.5
0.71-0.72	1.5	0.25	2	-1	-0.25	-0.25	1.25
0.73-0.74	3	4	3.5	-2.5	-1.6	-1.6	2.6
0.75-0.76	3.5	6.25	4.5	-3.5	-1.8	-1.8	2.8
0.77-0.78	5	16	7	-6	-2.7	-2.7	3.7
0.79-0.80	7	36	9	-8	-4.5	-4.5	5.5
0.81-0.82	9	64	11.5	-10.5	-6.1	-6.1	7.1
0.83-0.84	12	121	16	-15	-8.1	-8.1	9.1
0.85-0.86	18	289	19	-18	-16.1	-16.1	17.1
0.87-0.88	22	441	24	-23	-19.2	-19.2	20.2
0.89-0.90	28.5	756.25	32	-31	-24.4	-24.4	25.4
0.91-0.92	33.5	1056.25	35.5	-34.5	-30.6	-30.6	31.6
0.93-0.94	38	1369	44.5	-43.5	-31.5	-31.5	32.5
0.95-0.96	43.5	1806.25	50.5	-49.5	-36.5	-36.5	37.5
0.97-0.98	50	2401	58.5	-57.5	-41.8	-41.8	442.8
0.99-1.00	58.5	3306.25	65	-64	-51.7	-51.7	52.7
1.01-1.02	64	3969	71.5	-70.5	-56.3	-56.3	57.3
1.03-1.04	69.5	4692.25	81	-80	-58.7	-58.7	58.7
1.05-1.06	75.5	5550.25	86	-85	-65.3	-65.3	66.3
1.07-1.08	80.5	1320.25	91	-90	-70.2	-70.2	71.2
1.09-1.10	88	7569	95	-94	-80.5	-80.5	81.5
1.11-1.12	93	8464	96.5	-95.5	-88.6	-88.6	89.6
1.13-1.14	94.5	8742.25	97.5	-96.5	-90.6	-90.6	91.6
1.15-1.16	96	9025	99	-98	-92.1	-92.1	93.1
1.17-1.18	99.5	9702.25	99.5	-98.5	-98.5	-98.5	99.5
1.19-1.20	99.5	9702.25	99.5	-98.5	-98.5	-98.5	99.5
1.21-1.22	99.5	99702.25	99.5	-98.5	-98.5	-98.5	99.5
1.23-1.24	100	9801	100	-99	-99	-99	100
1.25-1.26							

Table 18. Estimation for the Strength Distribution of Sample Length Zero (Acrylic spun,  $\rho = 5$ ,  $n = 4$ ).

$x$	$F_{\rho}(x)$	$[1-F_{\rho}(x)]^n$	$F_{n\rho}(x)$	$1-F_{n\rho}(x)$	$[1-F_{\rho}(x)]^{n-1}$	$1-F_{\rho}(x)$	$F_{\rho}(x)$
0.67-0.68							
0.69-0.70	0.5		0.06				
0.71-0.72	1.5		0.06	0.5	0.5	0.1	
0.73-0.74	3		16	1	0		
0.75-0.76	3.5		39.06	2.5	-1.5	-26.0	-3 4
0.77-0.78	5		256	4	-3	-85.3	-4.4 5.4
0.79-0.80	7		1296	5.5	-4.5	-288	-6.6 7.6
0.81-0.82	9		4096	8	-7	-585.1	- 8.4 9.4
0.83-0.84	12		14641	14.5	-13.5	-1084.5	-10.2 11.2
0.85-0.86	18		83521	21.5	-20.5	-4074.2	-16.0 17.0
0.87-0.88	22		194481	28.5	-27.5	-7072.0	-19.2 20.2
0.89-0.90	28.5		571914.06	42.	-41	-13949.1	-24.1 25.1
0.91-0.92	33.5		1115664.06	50.	-49	-22768.7	-28.3 29.3
0.93-0.94	38		1874161.00	61	-60	-31236.0	-31.4 32.4
0.95-0.96	43.5		3262539.06	74	-73	-44692.3	-35.4 36.4
0.97-0.98	50		5764801	80.5	-79.5	-72513.2	-41.7 42.7
0.99-1.00	58.5		10931289.06	86	-85	-128603.4	-50.4 51.4
1.01-1.02	64		15752961	89.5	-88.5	-177999.6	-56.2 57.2
1.03-1.04	69.5		22017210.06	94.5	-93.5	-235478.2	-61.7 2.7
1.05-1.06	75.5		30805275.06	99	-98	-314339.5	-68.0 69.0
1.07-1.08	80.5		3994556.06	99.5	-98.5	-405538.7	-74.0 75.0
1.09-1.10	88		57289761	99.5	-98.5	-581621.9	-83.4 84.4
1.11-1.12	93		71639296	100	-99	-723629.3	-89.8 90.8
1.13-1.14	94.5		76426935.06	100	-99	-771989.2	-91.7 92.7
1.15-1.16	96		81450625	100	-99	-822733.6	-93.7 94.7
1.17-1.18	99.5		94133655.06	100	-99	-950845.0	-98.3 99.3
1.19-1.20	99.5		94133655.06	100	-99	-950845.0	-98.3 99.3
1.21-1.22	99.5		94133655.06	100	-99	-950845.0	-98.3 99.3
1.23-1.24	100		96059601	100	-99	-970299.0	-99 100
1.25-1.26							

Table 19. Estimation for the Strength Distribution of Sample Length Zero (Nylon filament,  $l = 5$ ,  $n = 2$ )

$x$	$F_g(x)$	$[1-F_g(x)]^n$	$F_{ng}(x)$	$1-F_{ng}(x)$	$[1-F_o(x)]^{n-1}$	$1-F_o(x)$	$F_o(x)$
1.65-1.66	1	0	1	0	0	0	1
1.67-1.68	1	0	1	0	0	0	1
1.69-1.70	1	0	2	-1	0	0	1
1.71-1.72	1	0	8	-7	0	0	1
1.73-1.74	2	1	17	-16	-0.06	-0.06	1.06
1.75-1.76	5	16	39	-38	-0.42	-0.42	1.42
1.77-1.78	8	49	75	-74	-0.66	-0.66	1.66
1.79-1.80	22	461	95	-94	-4.90	-4.90	5.90
1.81-1.82	50	2401	99	-98	-24.50	-24.50	25.50
1.83-1.84	78	5929	100	-99	-59.89	-59.89	60.89
1.85-1.86	95	8836	100	-99	-89.25	-89.25	90.25
1.87-1.88	100	9801	100	-99	-99.00	-99.00	100.00

Table 20. Estimation for the Strength Distribution of Sample Length Zero (Nylon Filament,  $l = 5$ ,  $n = 4$ )

$x$	$F(x)$	$[1-F_l(x)]^n$	$F_{nl}(x)$	$1-F_{nl}(x)$	$[1-F_0(x)]^{n-1}$	$1-F_0(x)$	$F_0(x)$
1.65-1.66	1	0	1	0	0	0	1
1.67-1.68	1	0	4	-3	0	0	1
1.69-1.70	1	0	21	-20	0	0	1
1.71-1.72	1	0	60	-59	0	0	1
1.73-1.74	2	1	94	-93	-0.01	0	1
1.75-1.76	5	256	100	-99	-2.59	-1.4	2.4
1.77-1.78	8	2401	100	-99	-24.25	-2.9	3.9
1.79-1.80	22	194481	100	-99	-1964.45	-12.5	13.5
1.81-1.82	50	5764801	100	-99	-58230.31	-38.8	39.8
1.83-1.84	78	35153041	100	-99	-355081.22	-70.8	71.8
1.85-1.86	95	78074896	100	-99	-788635.31	-92.4	93.4
1.87-1.88	100	96059601	100	-99	-970299.00	-99.0	100.0

Table 21. Calculated Frequency of Strength Distribution of Sample Length Zero (Acrylic Spun)

x	k=5, n=4		
	Calculated $F_0(x)$	Estimated $F_0(x)$	$F_0(x)$
0.67-0.68	4	4	4
0.69-0.70	5.4	5	1
0.71-0.72	7.6	8	3
0.73-0.74	9.4	9	1
0.75-0.76	11.2	11	2
0.77-0.78	17.0	17	6
0.79-0.80	20.2	20	3
0.81-0.82	25.1	25	5
0.83-0.84	29.3	29	4
0.85-0.86	32.4	32	3
0.87-0.88	36.4	36	4
0.89-0.90	42.7	43	7
0.91-0.92	51.4	51	8
0.93-0.94	57.2	57	6
0.95-0.96	62.7	63	6
0.97-0.98	69.0	69	6
0.99-1.00	75.0	75	6
1.01-1.02	84.4	84	9
1.03-1.04	90.8	91	7
1.05-1.06	92.7	93	2
1.07-1.08	94.7	95	2
1.09-1.10	99.3	99	4
1.11-1.12			
1.13-1.14			
1.15-1.16			
1.17-1.18			

Table 22. Calculated Frequency of Strength Distribution of Sample Length Zero (Nylon Filament)

x	Calculated	Estimated	
	$F_0(x)$ $l=5, n=4$	$F_0(x)$ F	$f_0(x)$
1.65-1.66	1	1	1
1.67-1.68	1	1	0
1.69-1.70	1	1	0
1.71-1.72	1	1	0
1.73-1.74	1	1	0
1.75-1.76	2.4	2	1
1.77-1.78	3.9	4	2
1.79-1.80	13.5	14	10
1.81-1.82	39.8	40	26
1.83-1.84	71.8	72	32
1.85-1.86	93.4	93	21
1.87-1.88	100	100	7

$$\mu_0 = 1.829$$

$$\sigma_0 = 0.030$$

Table 23. Estimated Normal Distribution for Strength of Sample Length Zero (Acrylic Spun)  $\mu = 0.9922$ ,  $\sigma = 0.1136$

x	$f(x) = (1/\sqrt{2\pi} \sigma) \exp[-(x-\mu)^2/2\sigma^2]$	
	f(x)	%
0.70	0.132	0.2
0.72	0.203	0.4
0.74	0.304	0.6
0.76	0.441	0.8
0.78	0.621	1.2
0.80	0.8479	1.6
0.82	1.121	2.1
0.84	1.439	2.7
0.86	1.790	3.4
0.88	2.160	4.1
0.90	2.527	4.8
0.92	2.867	5.4
0.94	3.154	6.0
0.96	3.364	6.4
0.98	3.480	6.6
0.992	3.499	6.6
1.00	3.491	6.6
1.02	3.396	6.4
1.04	3.203	6.1
1.06	2.929	5.5
1.08	2.598	4.9
1.10	2.234	4.2
1.12	1.863	3.5
1.14	1.507	2.8
1.16	1.181	2.2
1.18	0.898	1.7
1.20	0.662	1.3

Table 23. (cont'd)

x	$f(x) = (1/\sqrt{2\pi}\sigma)\exp[-(x-\mu)^2/2\sigma^2]$	
	f(x)	%
1.22	0.474	0.9
1.24	0.328	0.6
1.26	0.221	0.4

Table 24. Estimated Normal Distribution for Strength of Sample Length Zero (Nylon Filament)  $\mu = 1.829$ ,  $\sigma = 0.030$

x	$f(x) = (1 / \sqrt{2\pi} \sigma) \exp[-(x - \mu)^2 / 2 \sigma^2]$	
	f(x)	%
1.54	0.000	0.0
1.56	0.000	0.0
1.58	0.000	0.0
1.60	0.000	0.0
1.62	0.000	0.0
1.64	0.000	0.0
1.66	0.000	0.0
1.68	0.000	0.0
1.70	0.001	0.0
1.72	0.018	0.0
1.74	0.163	0.3
1.76	0.944	1.5
1.78	3.503	5.5
1.80	8.334	13.2
1.82	12.713	20.1
1.829	13.298	21.0
1.84	12.434	19.6
1.86	7.797	12.3
1.88	3.135	5.0
1.90	0.808	1.3
1.92	0.134	0.2
1.94	0.014	0.0
1.96	0.001	0.0
1.98	0.000	0.0
2.00	0.000	0.0
2.02	0.000	0.0
2.04	0.000	0.0
2.06	0.000	0.0

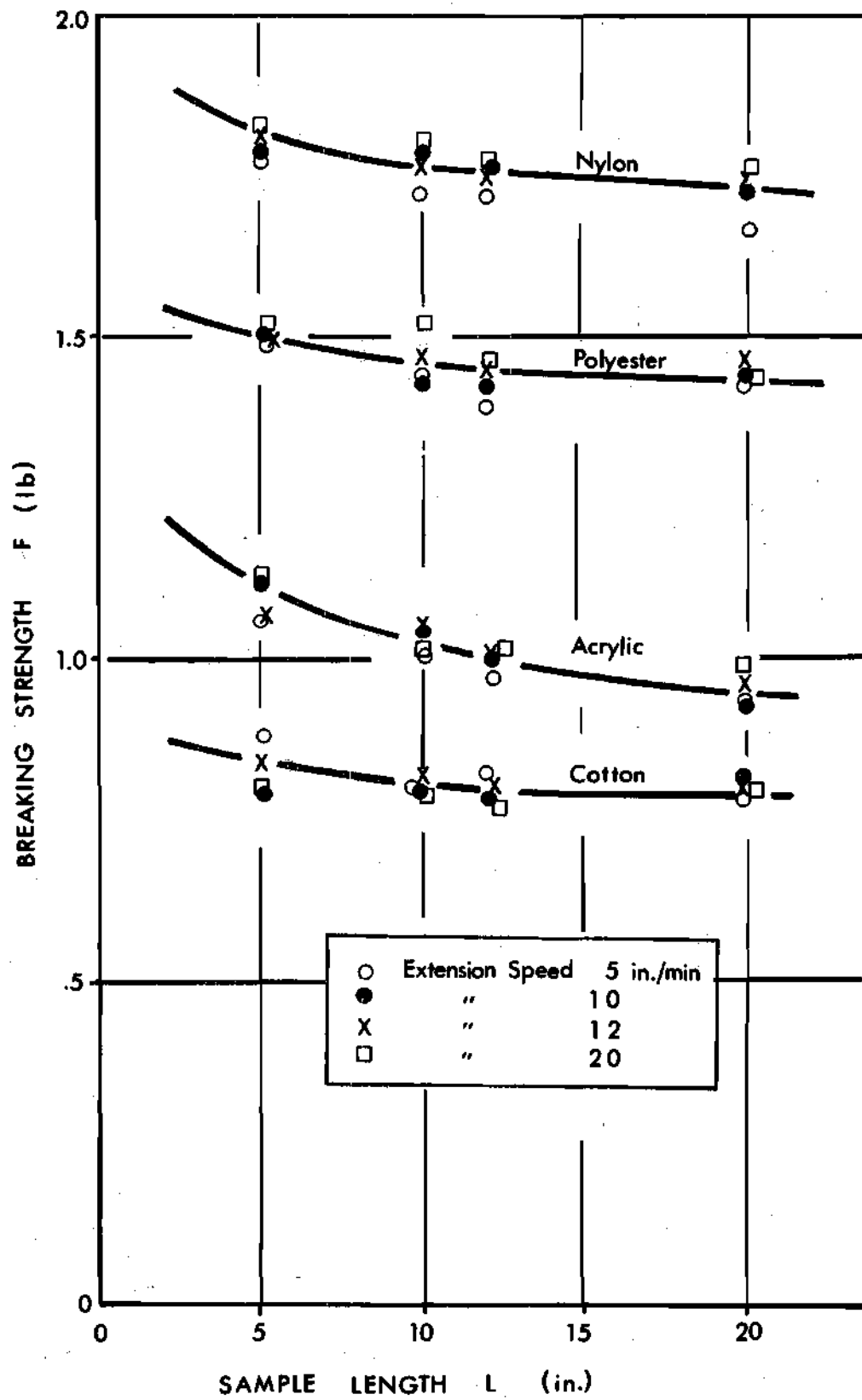


Figure 5. Breaking Strength and Sample Length.

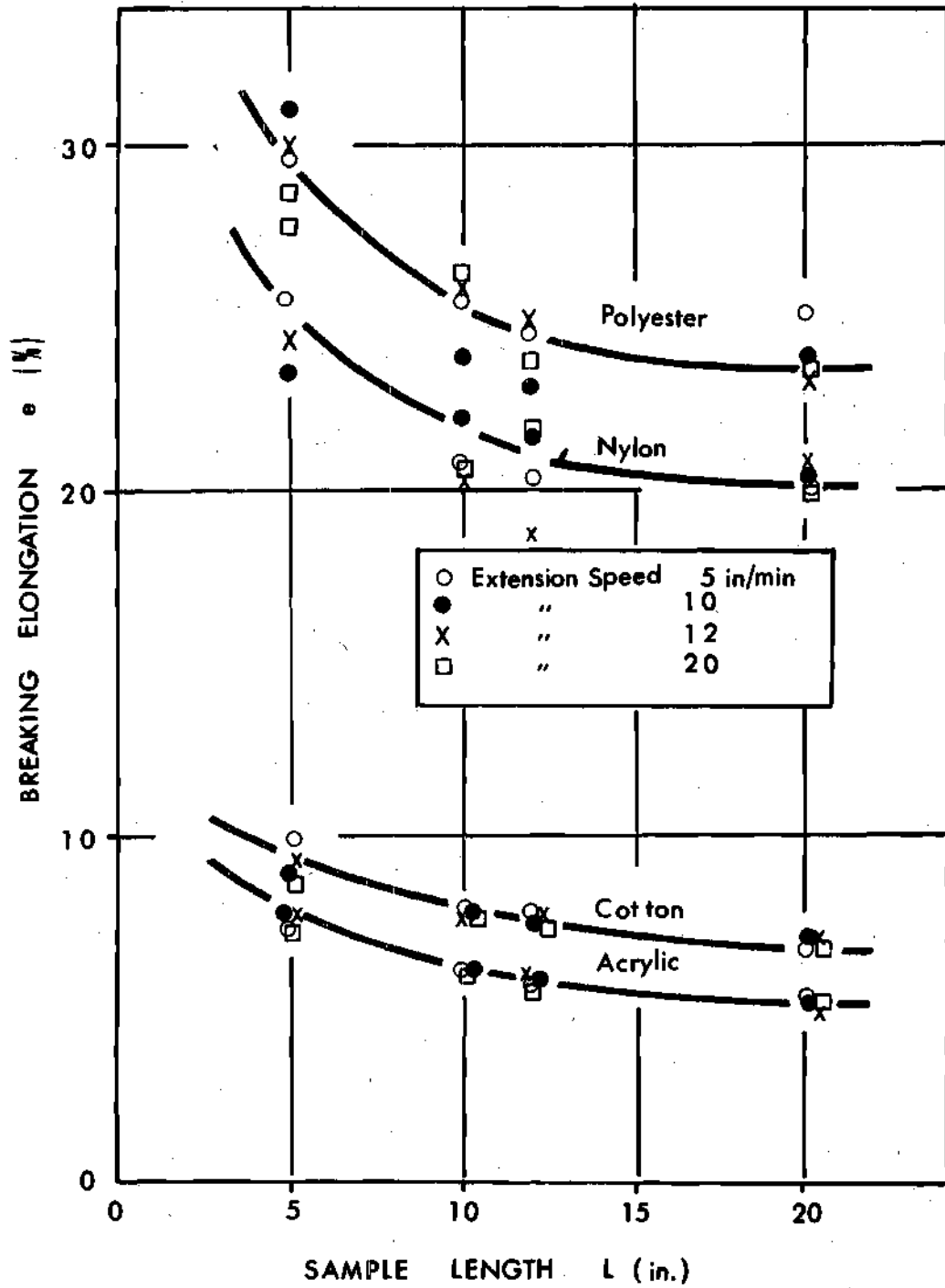


Figure 6. Breaking Elongation and Sample Length.

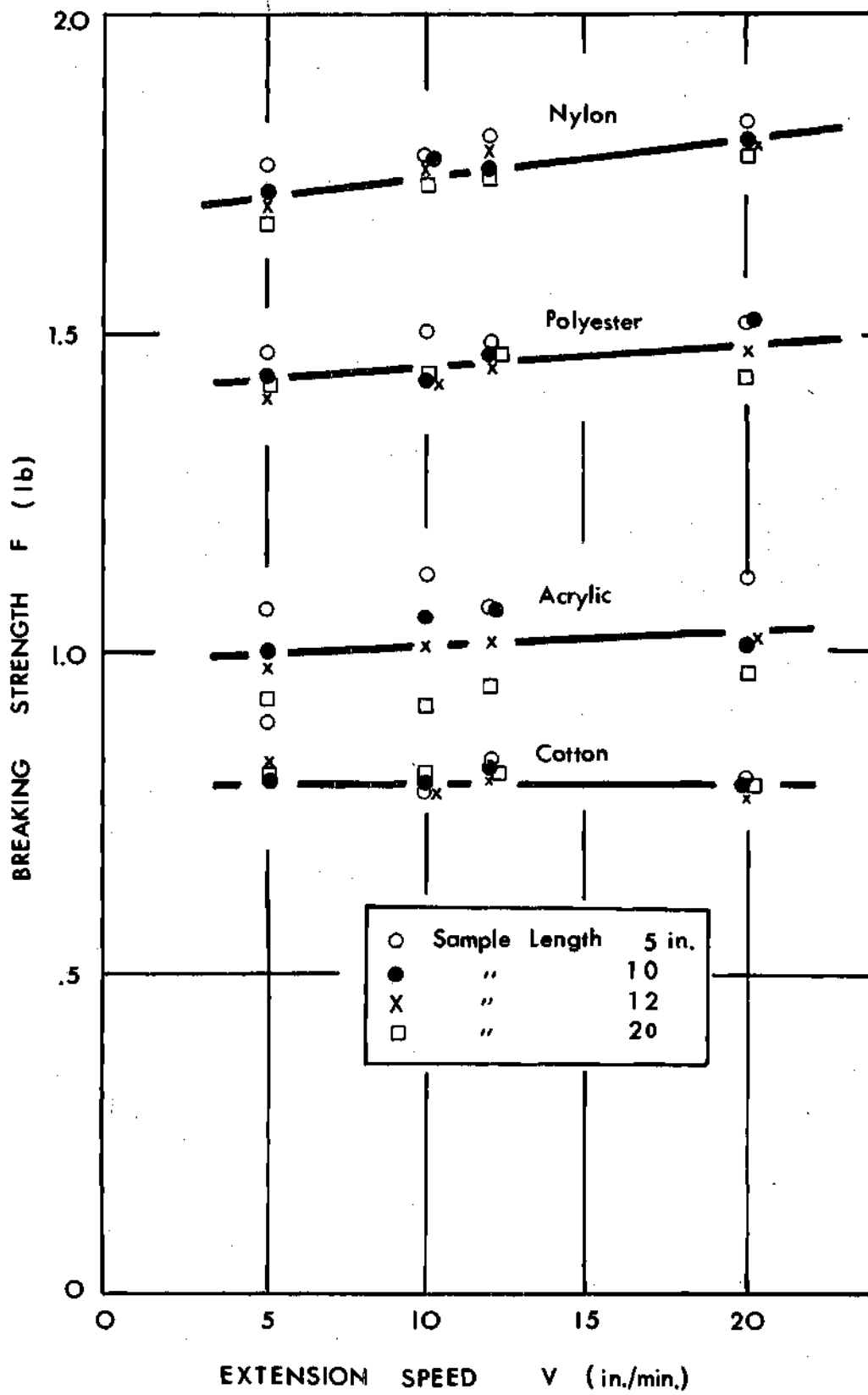


Figure 7. Breaking Strength and Extension Speed.

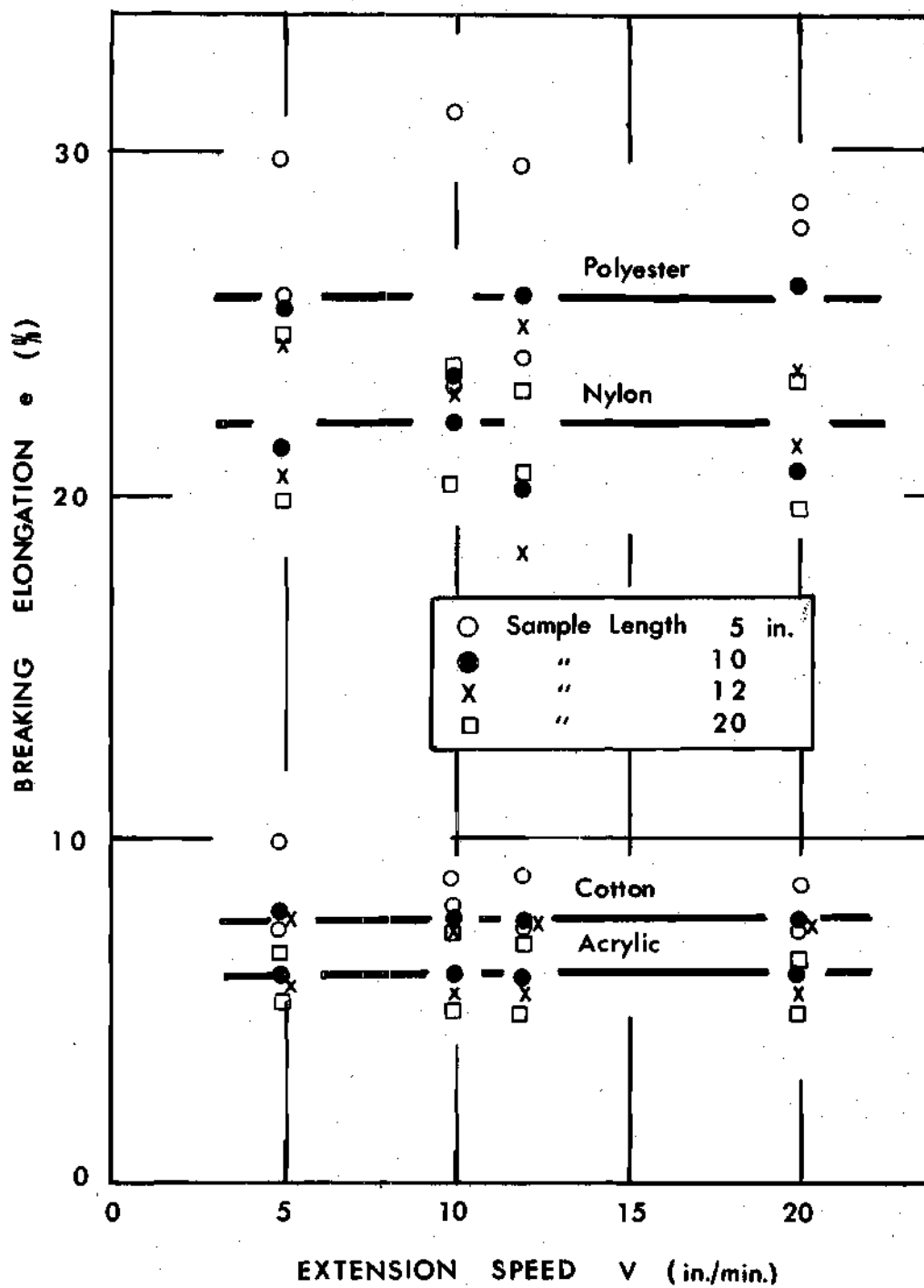


Figure 8. Breaking Elongation and Extension Speed.

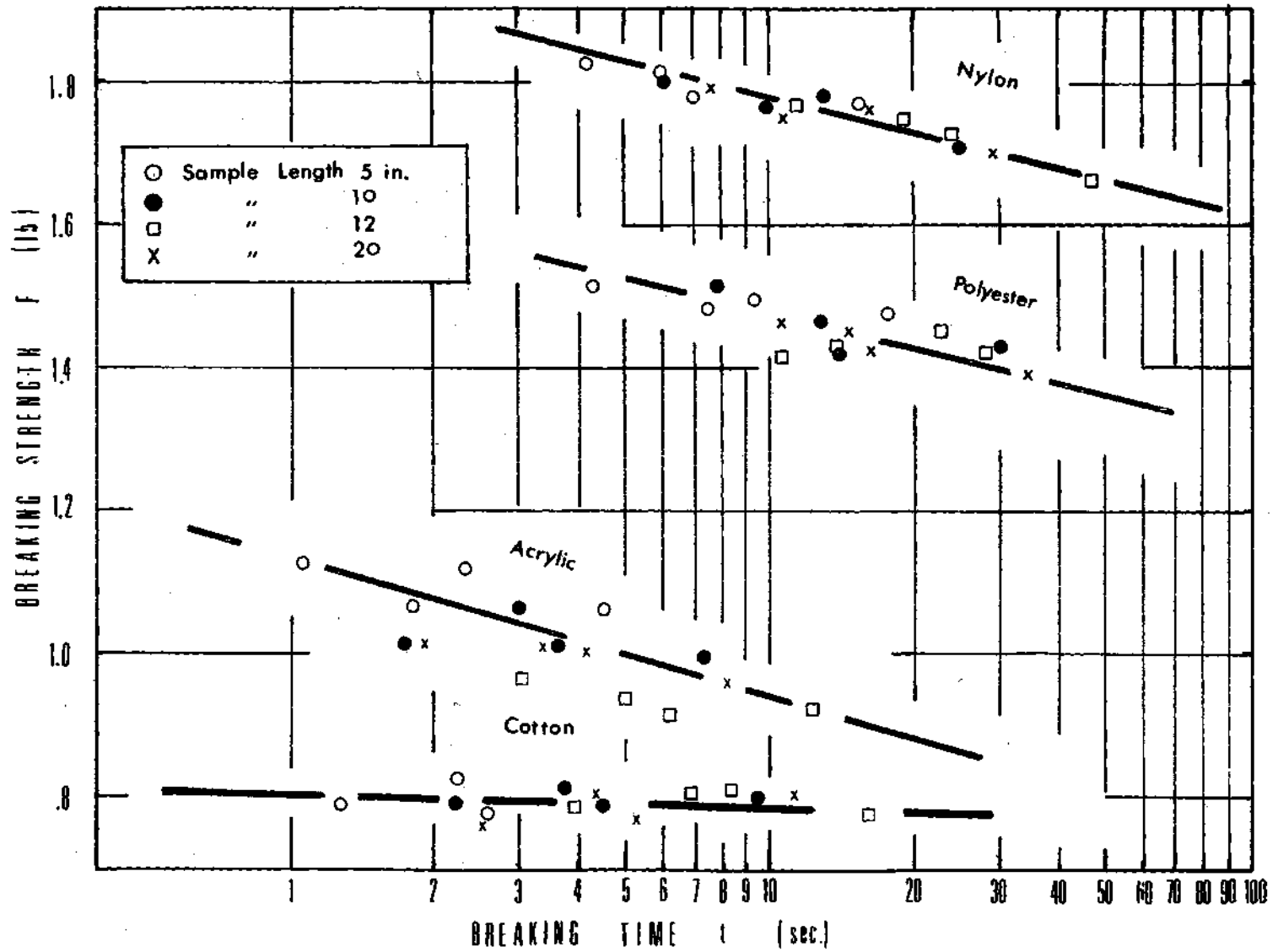


Figure 9. Breaking Strength and Breaking Time.

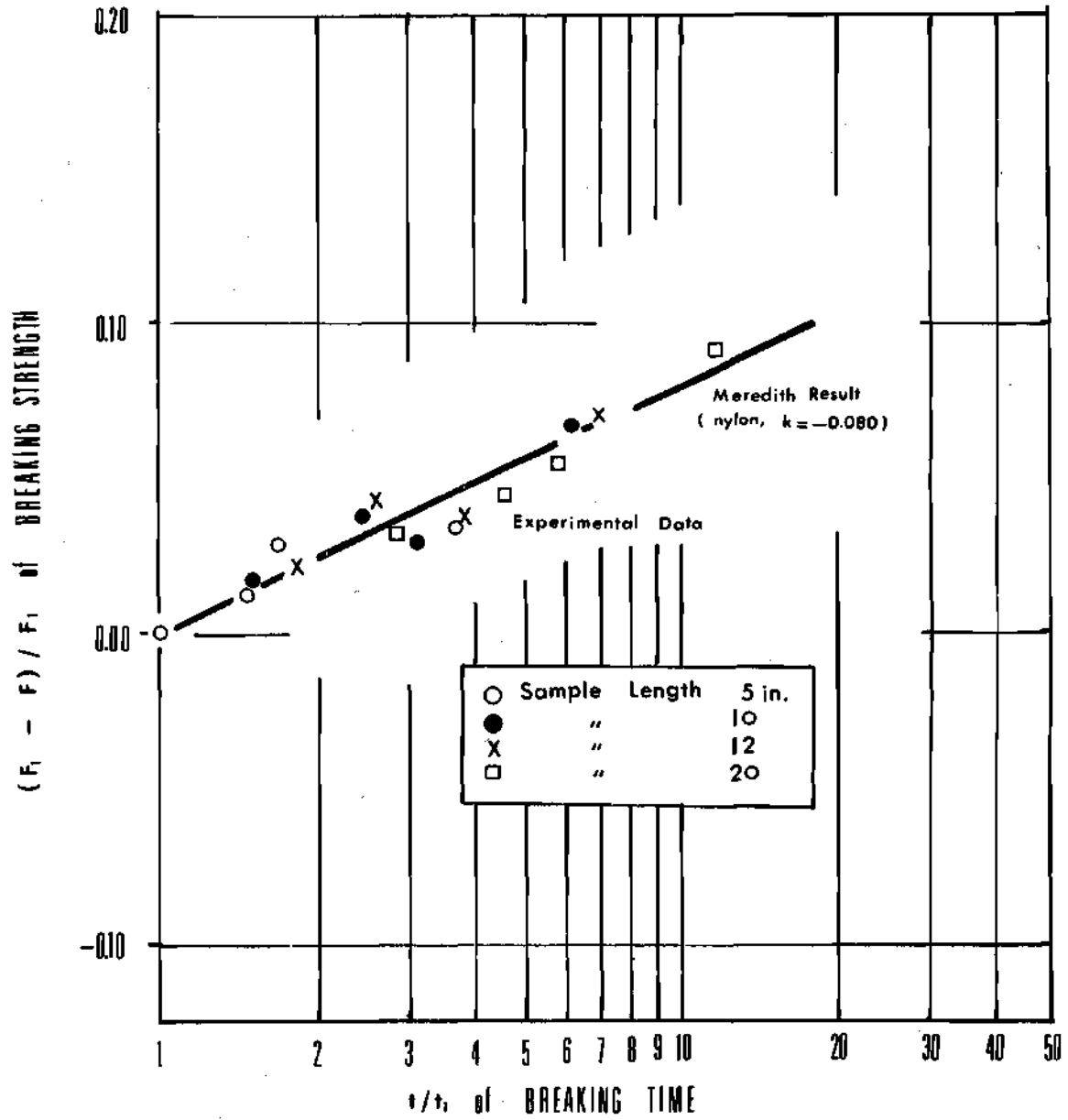


Figure 10. Comparison with Meredith's Results.

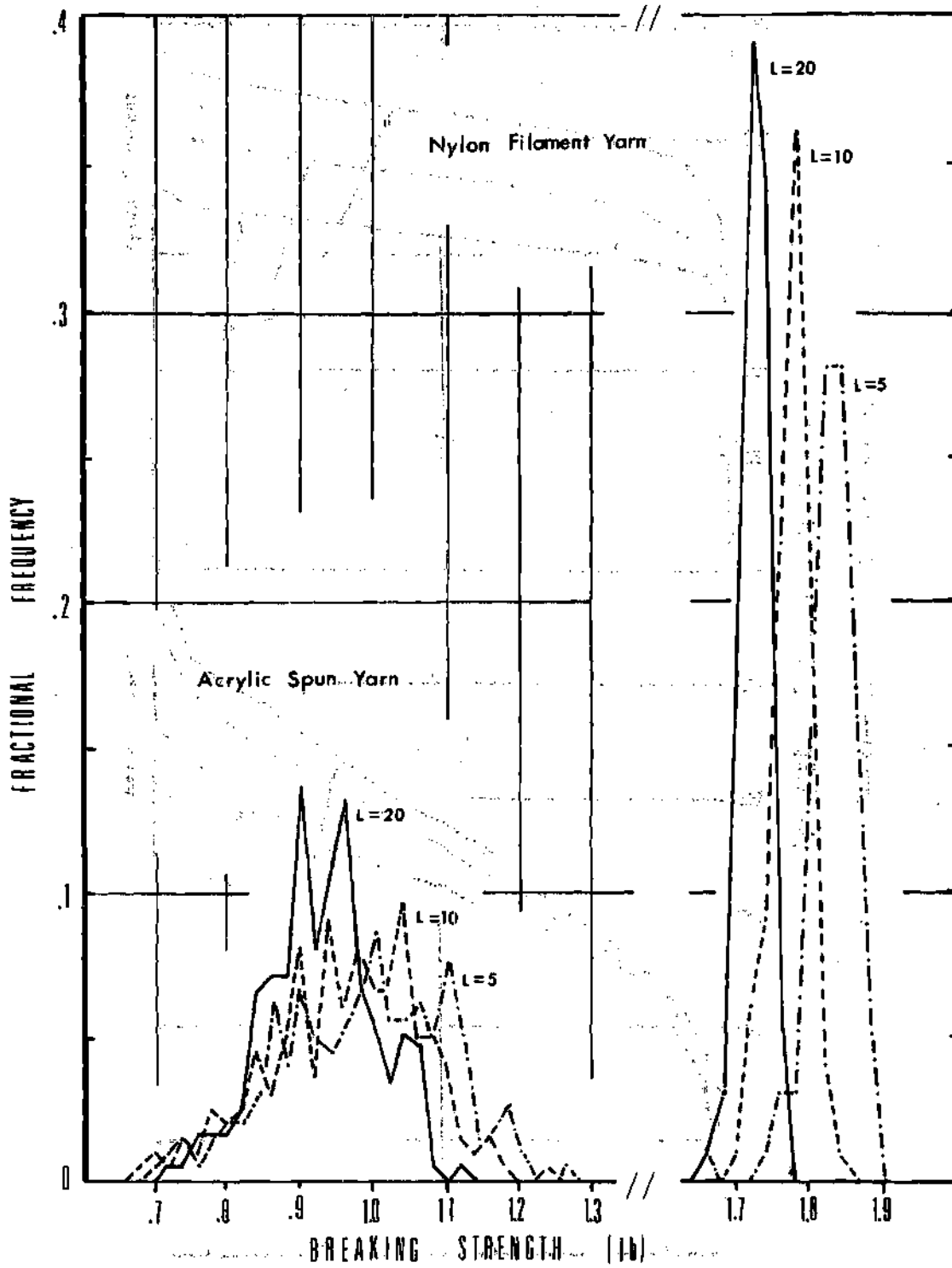


Figure 11. Experimental Strength Distribution.

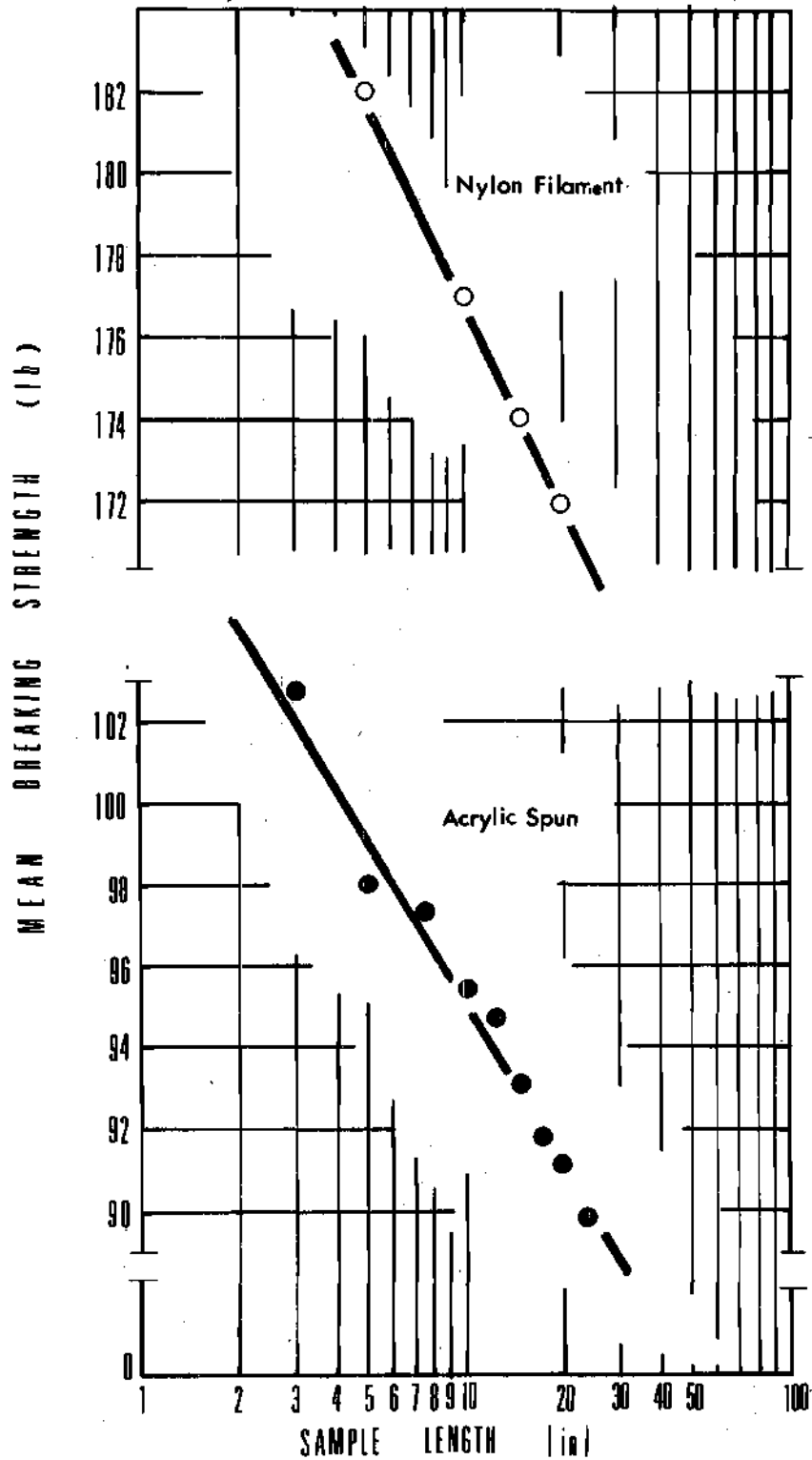


Figure 14. Mean Breaking Strength and Sample Length on log scale.

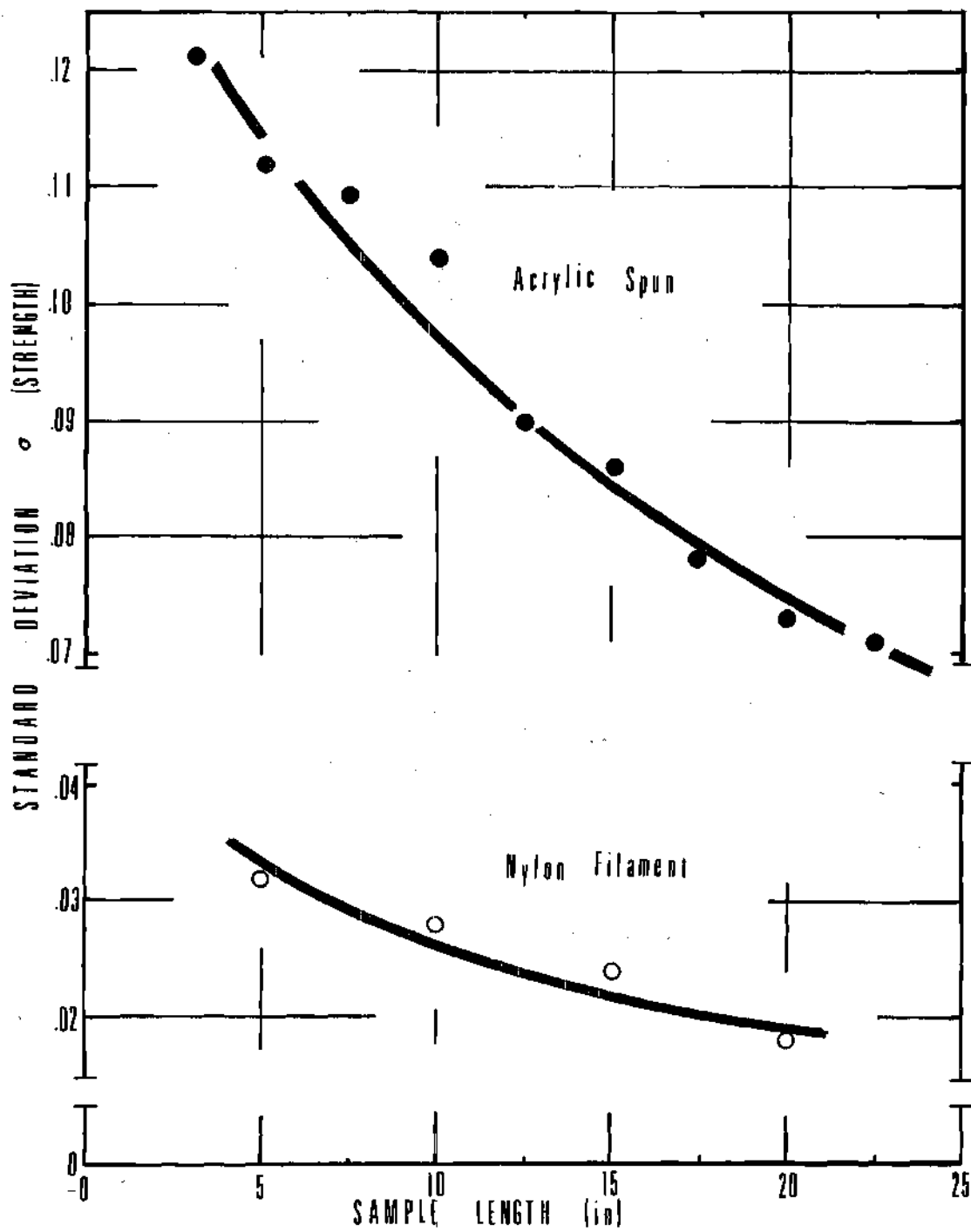


Figure 15. Standard Deviation and Sample Length (strength).

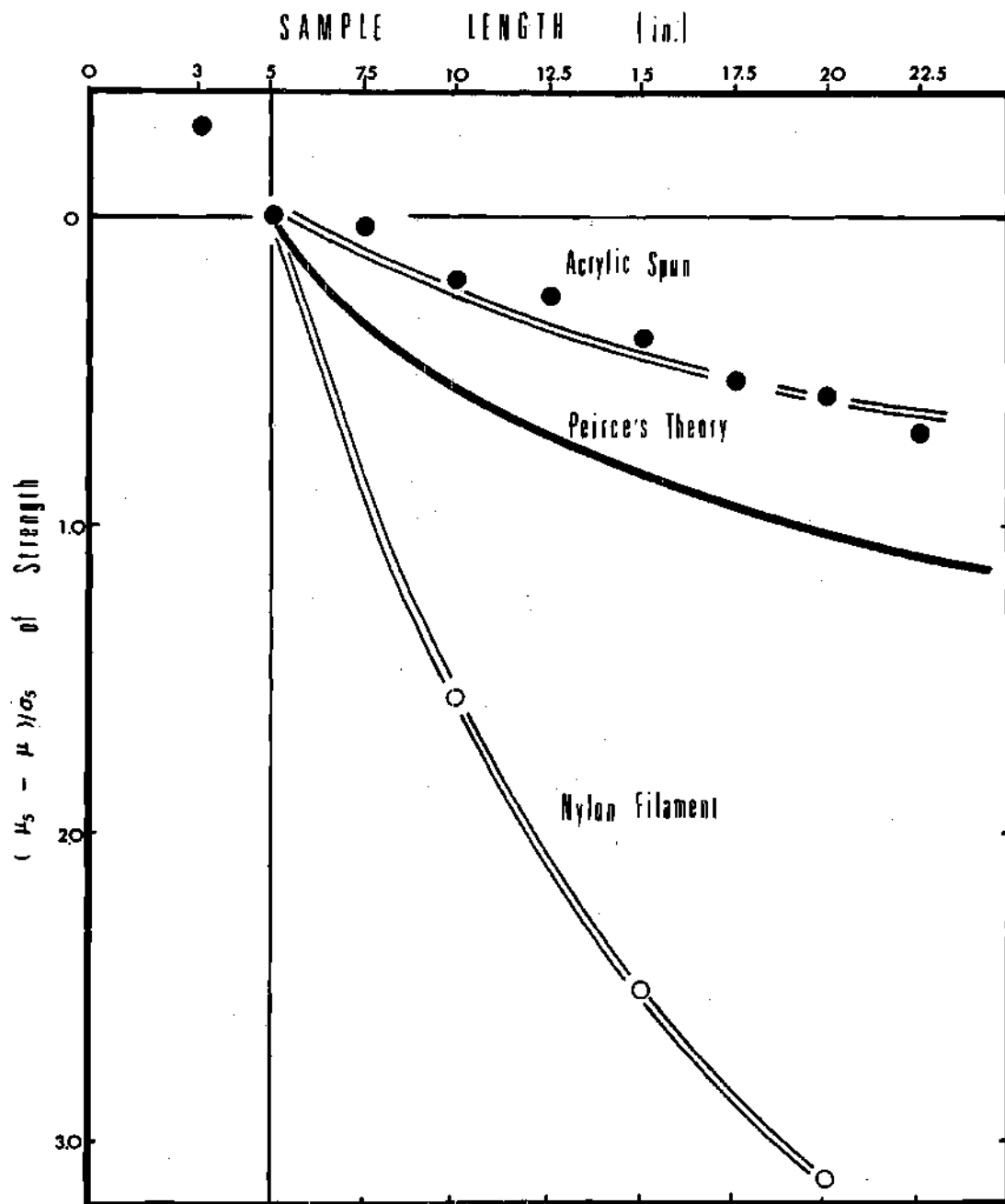


Figure 16. Comparison Experimental Data with Peirce's Theory.

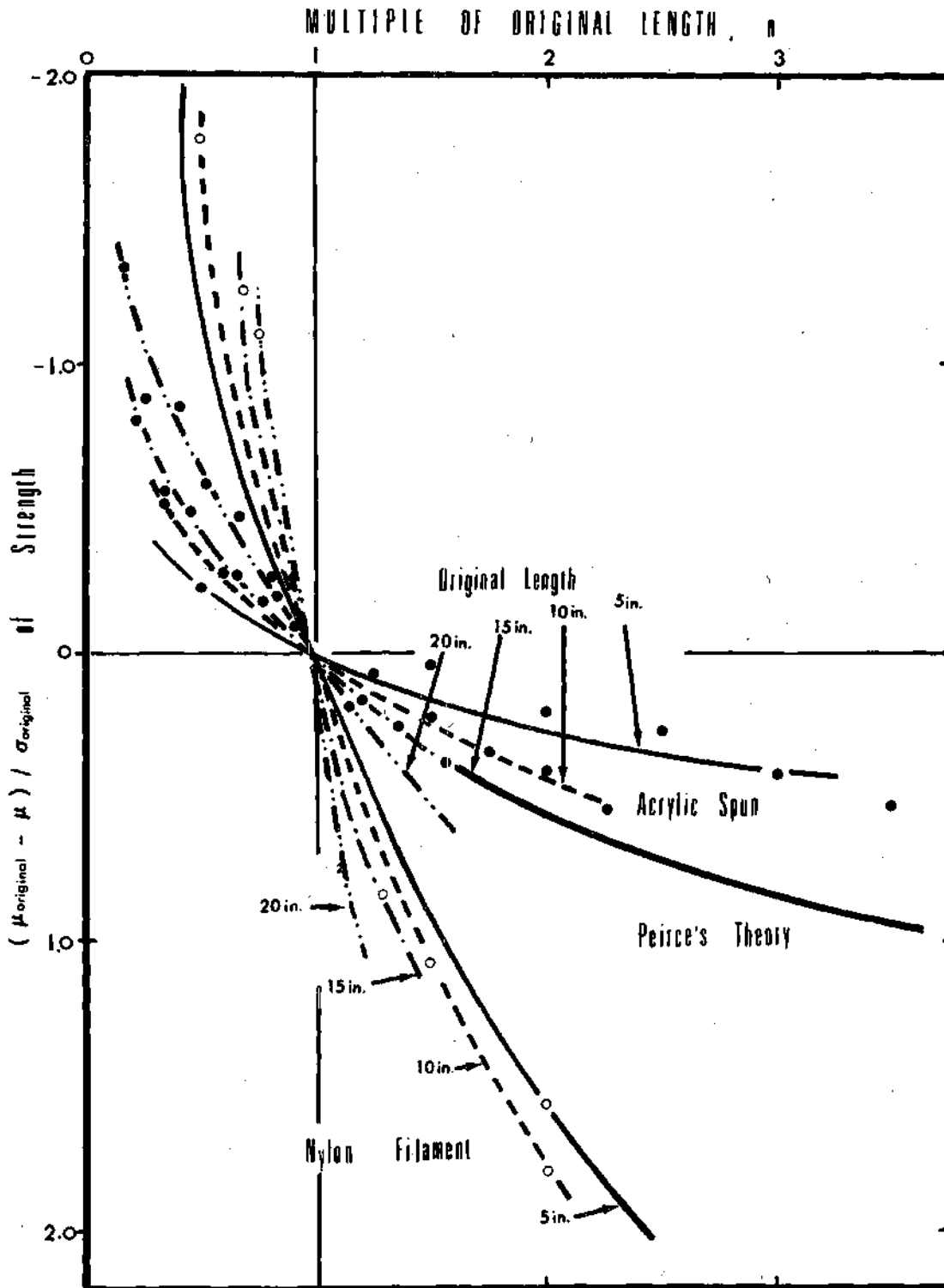


Figure 17. The Strength of Multiple Lengths depending upon the Original Length.

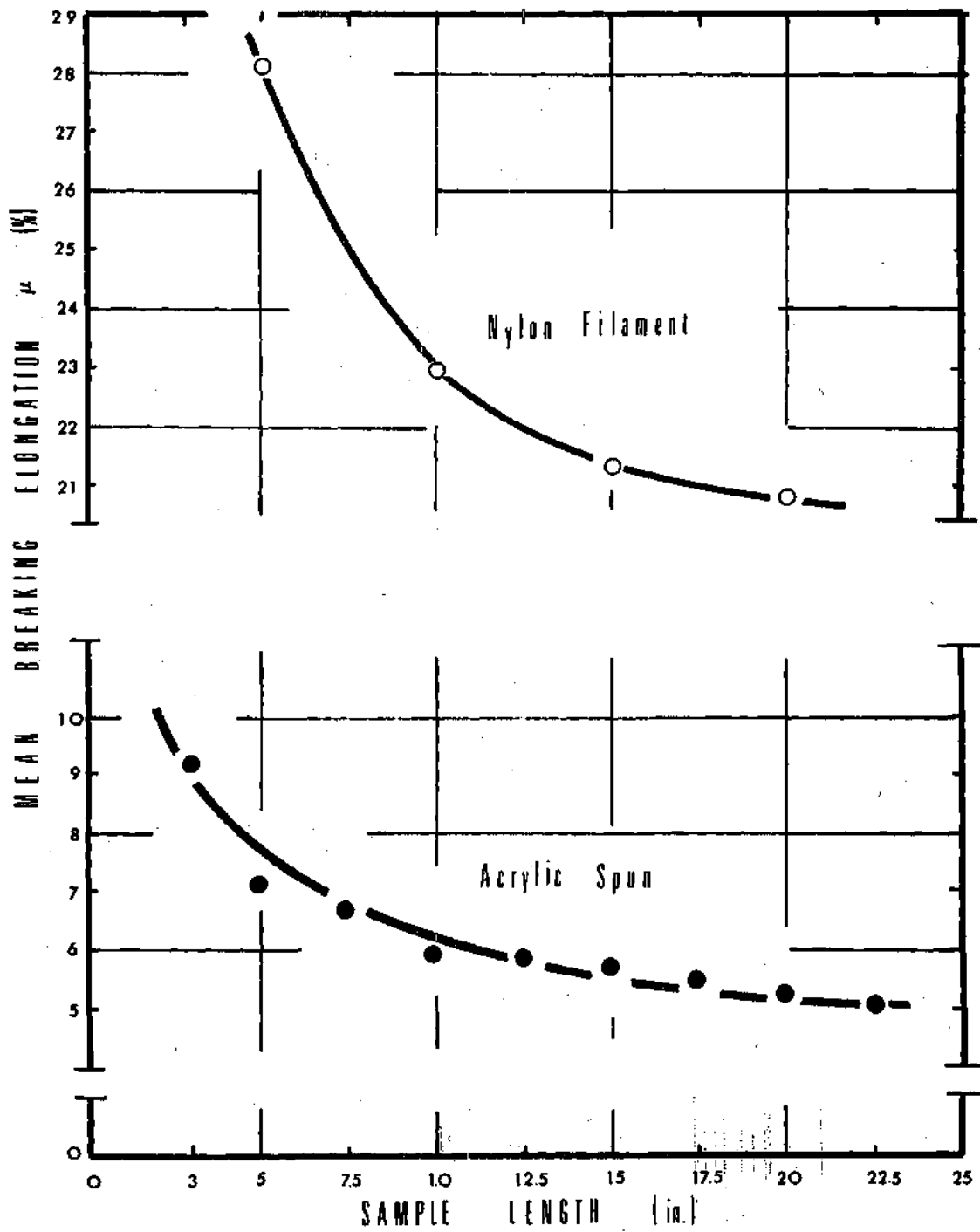


Figure 18. Mean Breaking Strength and Sample Length.

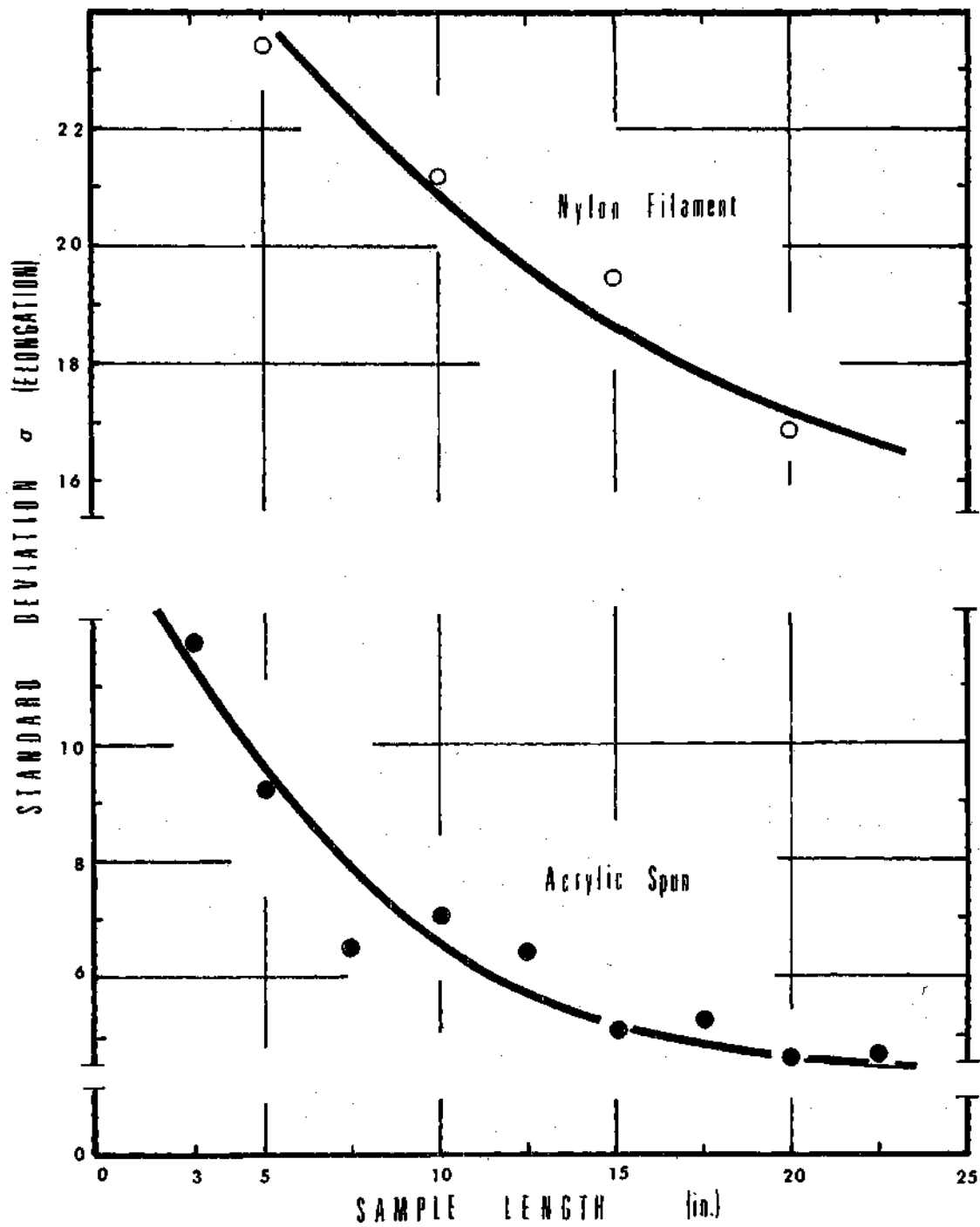


Figure 19. Standard Deviation and Sample Length (Elongation).

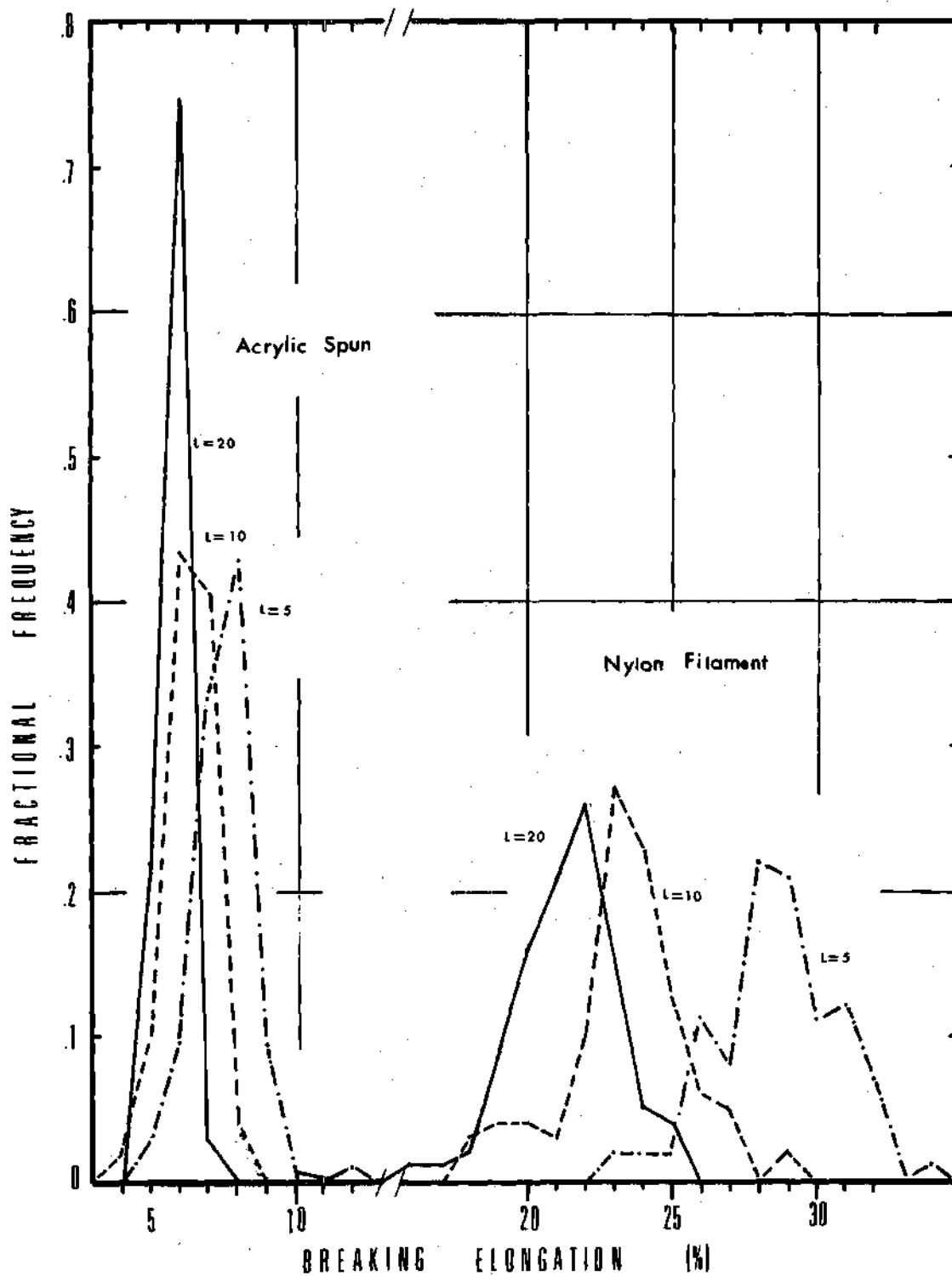


Figure 20. Experimental Elongation Distribution.

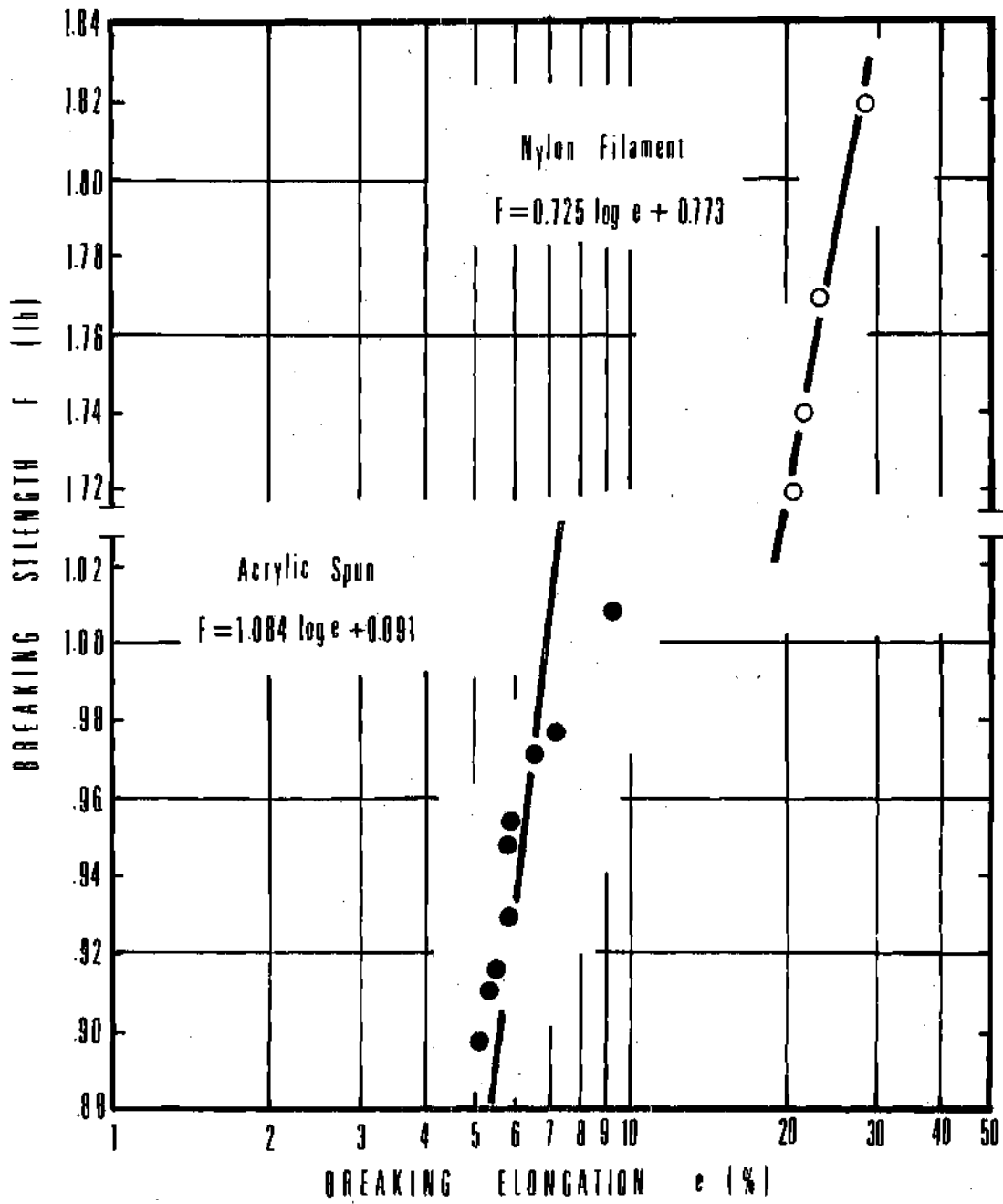


Figure 21. Correlation of Breaking Elongation with Breaking Strength.

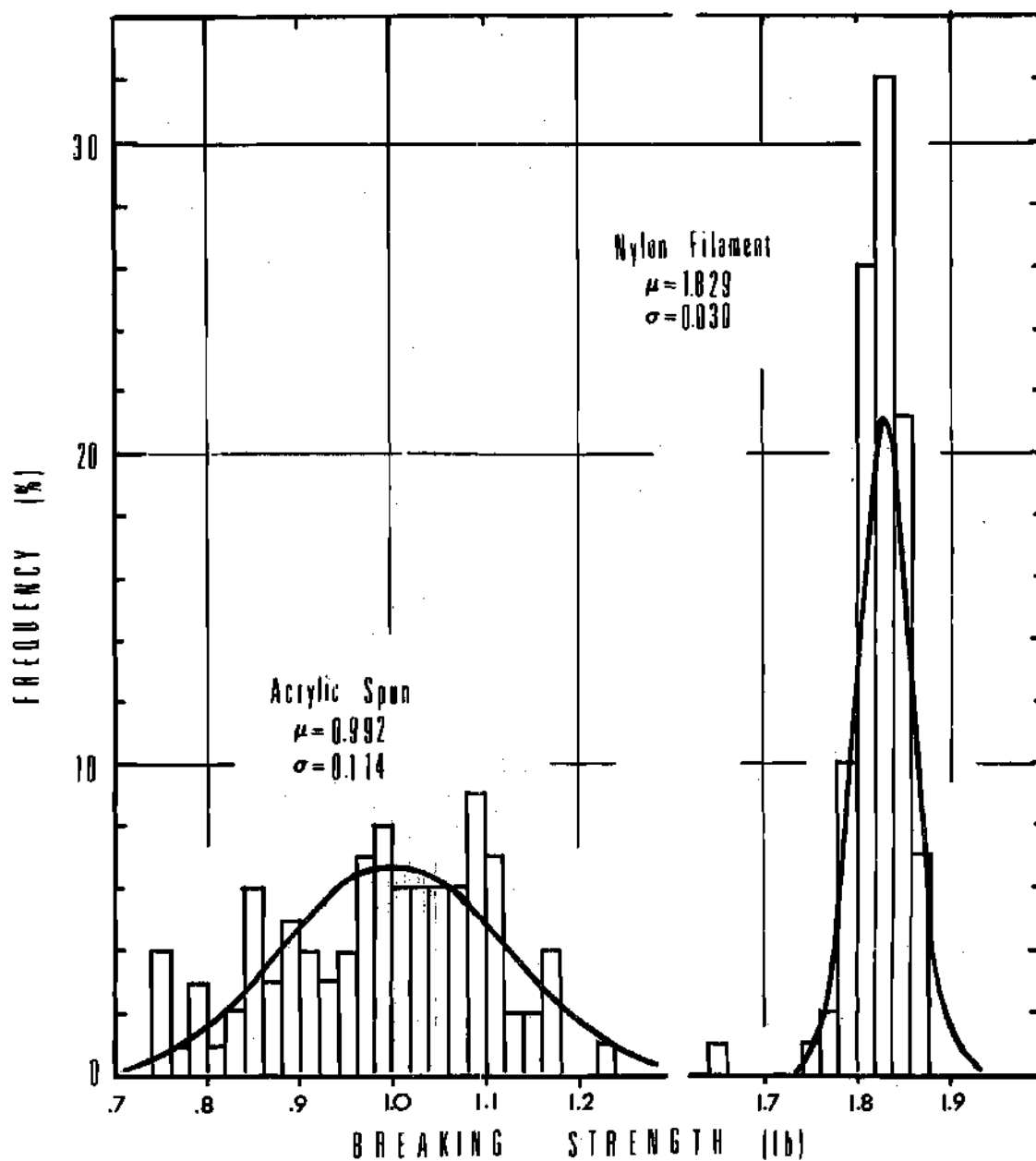
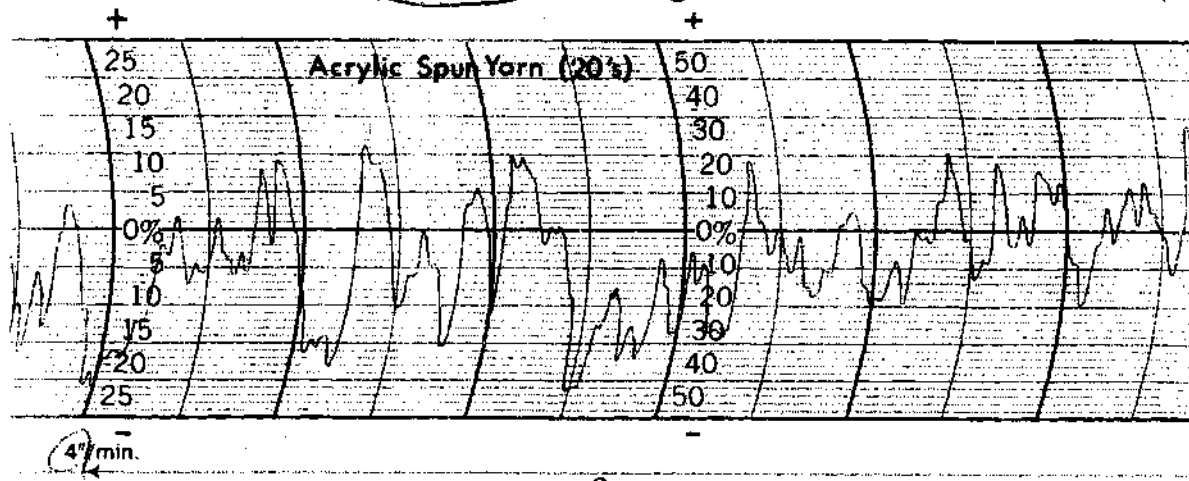


Figure 22. Estimated Distribution for Strength of Sample Length Zero.

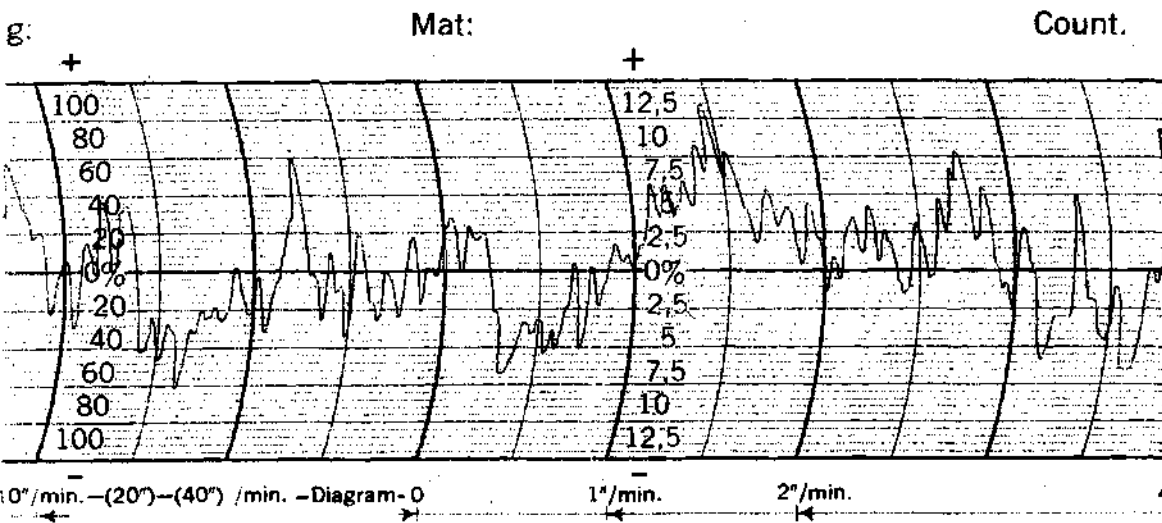
it. Normal Inert Test Irreg: 12.5 % Date: S:



Mat: 1, 2, 4, 8, 25, 50, 100 yards/min.

CHART No. 120451-4

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Figure 23. Yarn Thickness Variation of Test Speciman (Acrylic Spun).

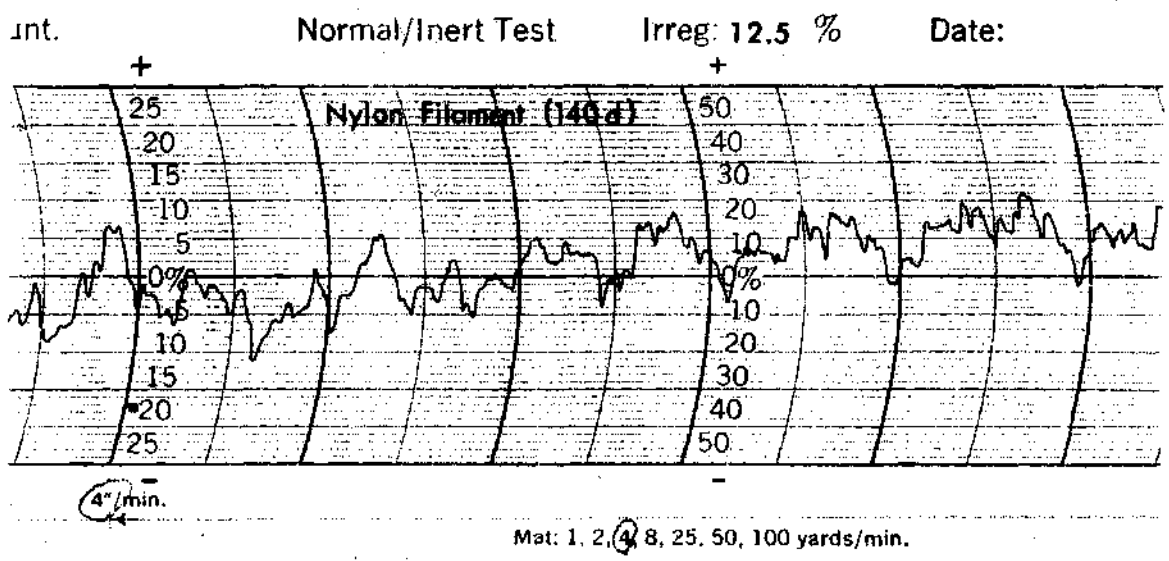
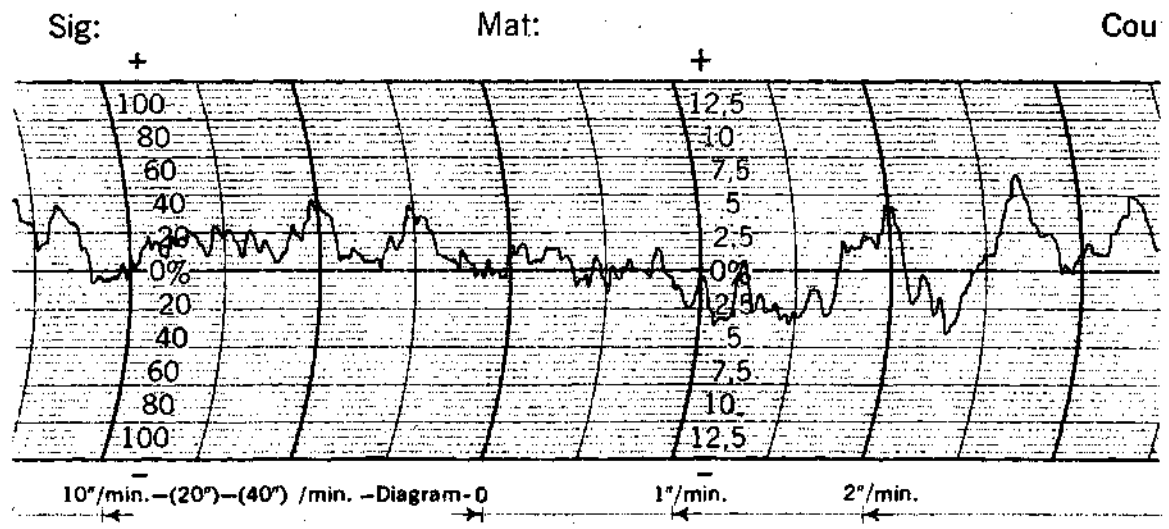


CHART No. 120451-4

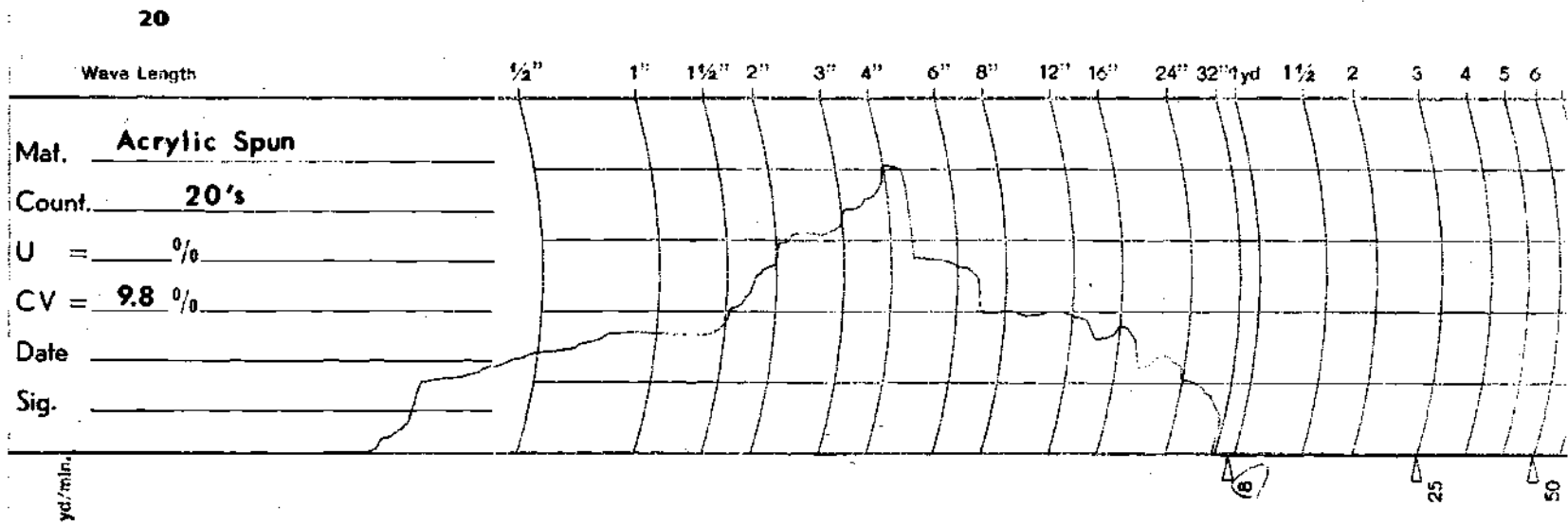


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Figure 24. Yarn Thickness Variation of Test Specimen (Nylon Filament).

Figure 25. Spectrograph (Acrylic Spun).



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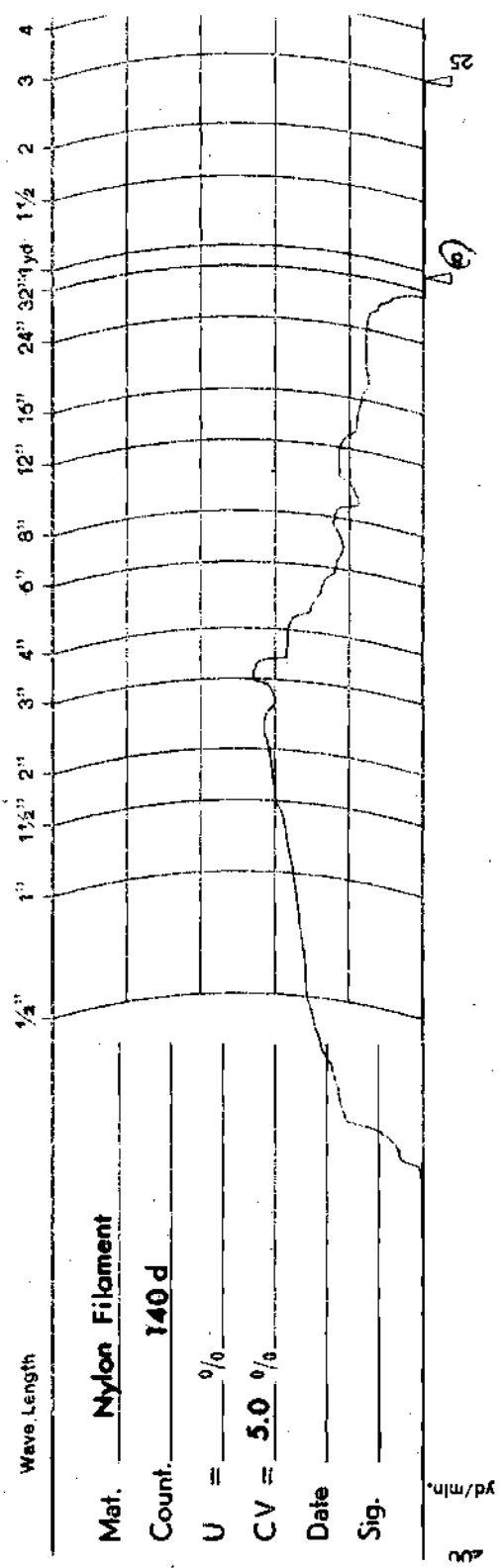


Figure 26. Spectrograph (Nylon Filament).

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## STANDARD FOR TENSILE TESTING

1. ASTM D76-67, Tensile Testing Machines for Textile Materials.
2. ASTM D123-69a, Terms relating to Textile Materials.
3. ASTM D1906-62T, Tentative Method of Test for Estimation of effective gage length by evaluation of clamp error in single fiber testing.
4. ASTM D2256-69, Breaking Load (Strength) and Elongation of Yarn by the single-strand method.
5. BS 1932, Methods of yarns from Packages Part I, Determination of Breaking Load and Elongation.
6. BS 3411, Method for determination of the tensile properties of individual textile fibers and filaments.
7. JIS L 1067.