

SUMMARY OF THE DEVELOPMENT OF GAPPS7  
COMPUTER PROGRAM

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FINITE ELEMENT PROGRAM FOR THE ANALYSIS  
OF GEOTECHNICAL PROBLEMS

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## INTRODUCTION

This report briefly describes selected examples used to debug the GAPPS7 version of the FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF GEOTECHNICAL PROBLEMS. Many of the examples used in developing this program are not, however, presented in this report. The GAPPS7 finite element computer program is based on the earlier GAPIN program [1]. The new features of the GAPPS7 program not present in the GAPIN program are summarized as follows:

1. Plane Strain Loading Option
2. Selected Output Capability
3. Multiple Load Application Capability

A complete description of the data input for this program is given in the Users Manual for the GAPPS7 Program [2]. GAPPS7 is the last version (September 1981) of the GAPIN program. The backup copy is GAPPS6; two additional copies of the program with the stiffness matrix (A) reduced to (3000) are given as GAPPS8 and GAPPS9.

In changing the program from GAPIN to GAPPS7 the (1) plane strain and (2) cyclic load options added to the program were carefully verified.

The cyclic load option was first checked using relatively simple examples. For the initial fabric tests the model of a fixed end beam with fabric at the bottom fiber was used (Data file: DAMFAB1). Several cycles of load applications were imposed to the model and the stress-strain response of the fabric was analyzed. Then a small, twelve element model with all types of elements (fabric, interface, gravel) was tested first under a small load and then with increasing load to obtain a nonlinear response of the model. The examples were run with the data files: DAMGEX1, DAMAEX2, DAMAEX3, DAMAEX4, DAMAEX5. The geometry of the

models is the same as used in the GAPPS7 User's Manual, Example No. 1 except the material properties and loads were varied.

After these test problems were successfully run and GAPPS7 corrected, to test the body weight application during the cycles and the unbalanced forces during unloading, a large mesh problem with 56 elements was run using data file: DA4CY56. This model was subjected to 4 cycles of load applications and the results are presented subsequently in this report. Stress levels of both 20 psi and 70 psi were used in debugging the program.

#### LIST OF TEST EXAMPLES

The following is a general list of test examples used for testing the multiple load application, fabric unload, printing capabilities, plane strain tests and load option capabilities developed in the GAPPS7 program:

- DAMAEX 1\* - From User's Manual, Sept. 4, 1981, Example No. 1
- DAMEX14 - Two cycles with different modulus - 4 element
- DAMEX12 - Multiple load application, 4 cycles - 4 elements
- DAMEX 15 - Variable E; 1 cycle - 4 elements
- DAMFAB1\* - Beam with fabric; Plane Strain
- DAMFAB2\* - Beam without fabric; Plane Strain
- DAMEX42 - Homogeneous layer; 42 elements; uniform load - Plane Strain
- DAMEX44 - Homogeneous layer; 44 elements; rigid footing - Plane Strain
- DAMEX56 - 2 layer model; Fabric; Rigid Footing - 2 cycles, axisymmetric
- DA4CY56\* - 4 cycles, Large mesh example
- DAMGEX1\* - User's Manual example with 2 layers

DAMAEX2,3,4,5\* - Variations of the User's Manual example for different loads and load weight options

Data Files for Plot:

DGCUED - for CUIIPT

DGEXPLT - for PTMESH (Plots a mesh)

\* Revised and run in September 1981 using the GAPPS7 version of the GAPPS program.

#### PLANE STRAIN OPTION

One check used to verify the plane strain option was an elastic half-space linear elastic solid subjected to an infinitely long, uniform strip loading. For this example (DAMEX42), the vertical stresses calculated using GAPPS7 are compared in Table 1 with those obtained from Poulos and Davis [3]. This comparison which is plotted in Figure 1 shows good agreement for vertical stress between the two methods. Likewise good agreement was also obtained for radial stress and vertical surface deflection. The vertical surface deflection computed by the program results in values of 0.0847 inch for the center of a flexible loaded area and 0.0559 inch for the edge. Using the formula proposed by Poulos & Davis, p. 103, the edge deflection is:

$$\rho_z = \frac{10 \times 18}{\pi \times 765} \times 0.76 = 0.0569 \text{ inch.}$$

The computed computer value is 1.7% less than the theoretical solution as given in graphical form.

#### CYCLIC LOAD APPLICATION

Infinitely Long Plate with Fabric. Several tests were run for the infinitely long plate shown in Figure 2 having each side fixed. The

Table 1. Vertical Stress for Plane Strain Option, GAPPS7 Program Compared with Poulos and Davis Solution for Elastic Half-Space Solution.

Poulos & Davis		GAPPS7 DAMEX42	
z inch	$\sigma_2$ psi	z inch	$\sigma_2$ psi
0	10	0.0	10
1.5	9.59	0.75	9.614
3	8.18	2.25	8.689
4.5	6.68	3.75	7.306
6.0	5.51	5.25	6.08
7.5	4.62	7.5	4.817
9.0	3.95		
10.5	3.46	10.5	3.802
12.0	3.05	15	2.99

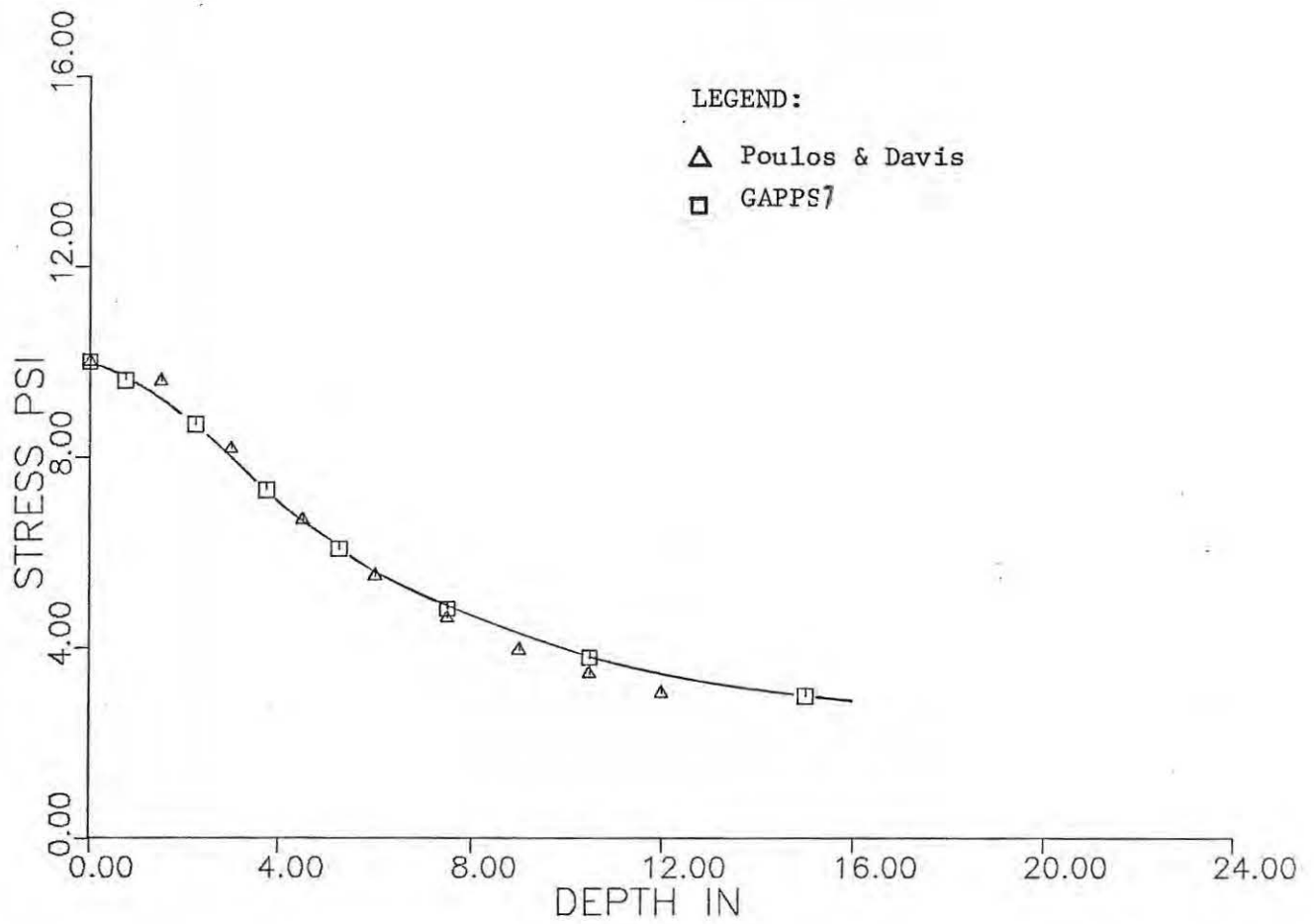


FIGURE 1. COMPARISON OF FINITE ELEMENT AND THEORETICAL VERTICAL STRESSES FOR PLANE STRAIN STRIP LOAD EXAMPLE.

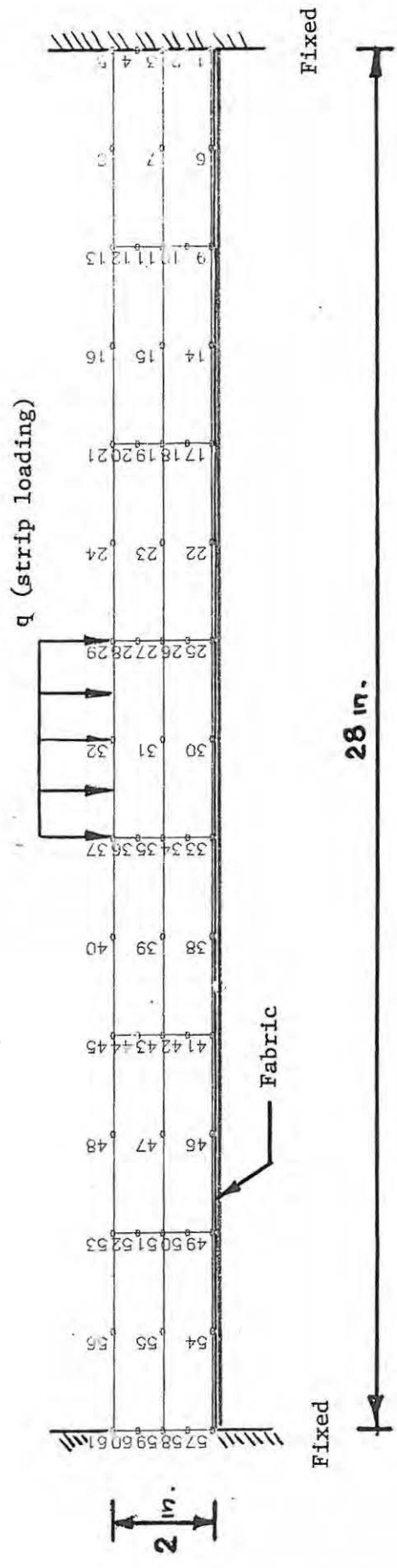


FIGURE 2. PLANE STRAIN BEAM WITH FABRIC ON THE BOTTOM (DATAFILE DAMFABI).

plate model is one of plane strain and the corresponding data files are DAMFAB1 (with fabric) and DAMFAB2 (without fabric). Both examples showed residual stresses to exist after unloading, even if the material was in the elastic range. These residual stresses were due to the deformed geometry used when the unloading "load" is applied to the system (for the large displacement option).

To verify the unloading routine both examples were run without updating the coordinates after each load application. The results show that after the stress is removed the stresses are essentially zero (less than  $10^{-4}$ ) for the model without fabric. These results indicate the program is unloading correctly. The change in geometry produces a change in stiffness and some residual stresses are left in the system when the coordinates are updated using the large displacement option. The examples are EXI and EXII run without updating coordinates. For the same models with updating of coordinates the results are given in examples EXIII, EXIV and EXV (Data files: DAMFAB1 & DAMFAB2).

The verification of the program and the changes from GAPPS4 to GAPPS7 are shown in yellow in the LISTING GAPPS7. The general flow diagram is shown in Figure 3. The fabric modulus interpolation is shown in Figure 4 which is a test of the interpolation of the fabric nonlinear properties.

The output of the DAM<sup>F</sup>AB1 run shows the fabric not to have stress upon unloading for elastic fabric response. The stress-strain curve for the fabric is shown in Figure 5. The unloading of the fabric is computed by the program considering the elastic modulus for unloading<sup>(1)</sup>.

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1. The unload modulus for all material is taken equal to the initial loading modulus as shown in Figure 5. Unload is done in a single increment.

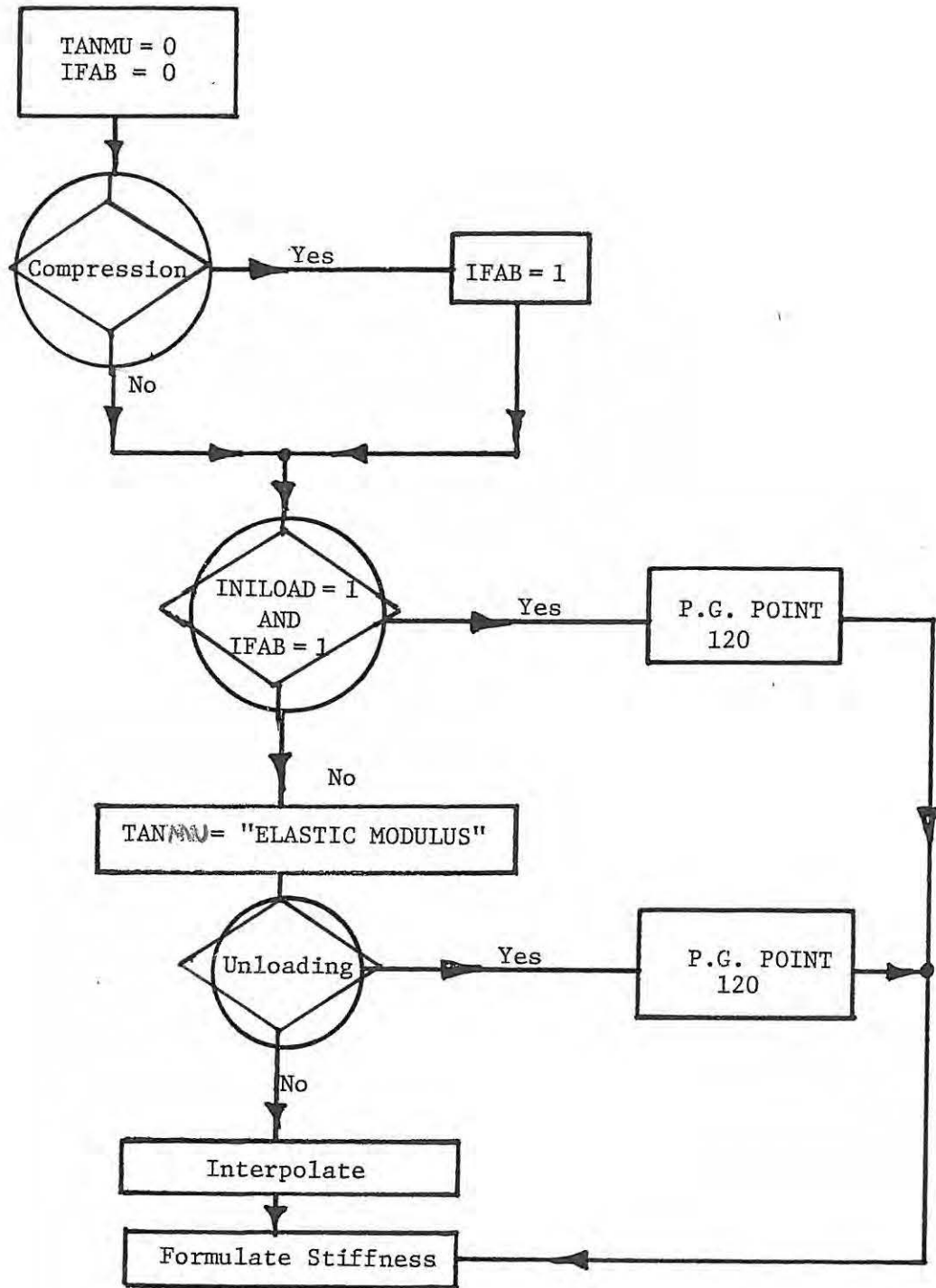


FIGURE 3. GENERALIZED FABSTIF SUBROUTINE FLOW DIAGRAM.

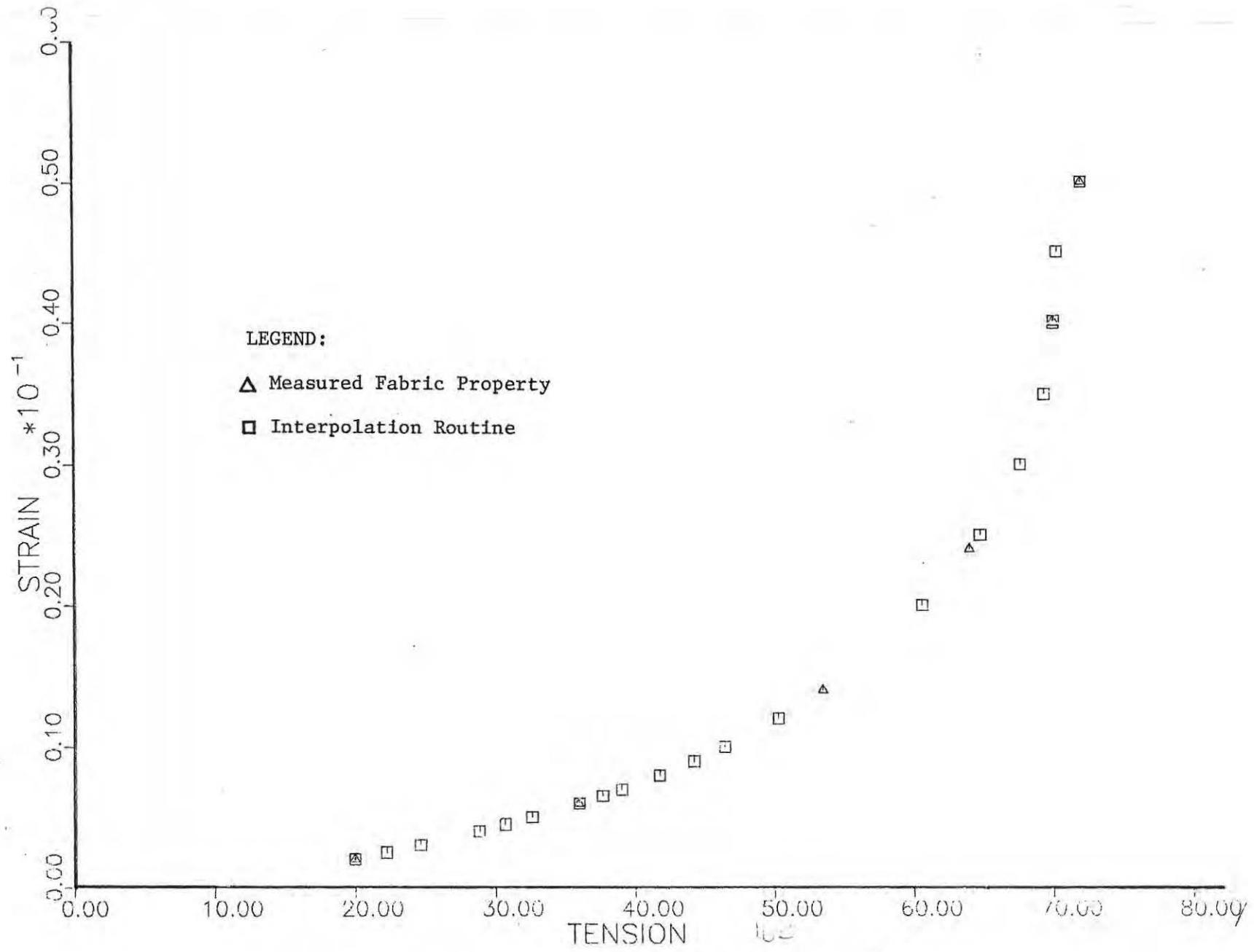


FIGURE 4. FABRIC INTERPOLATION.

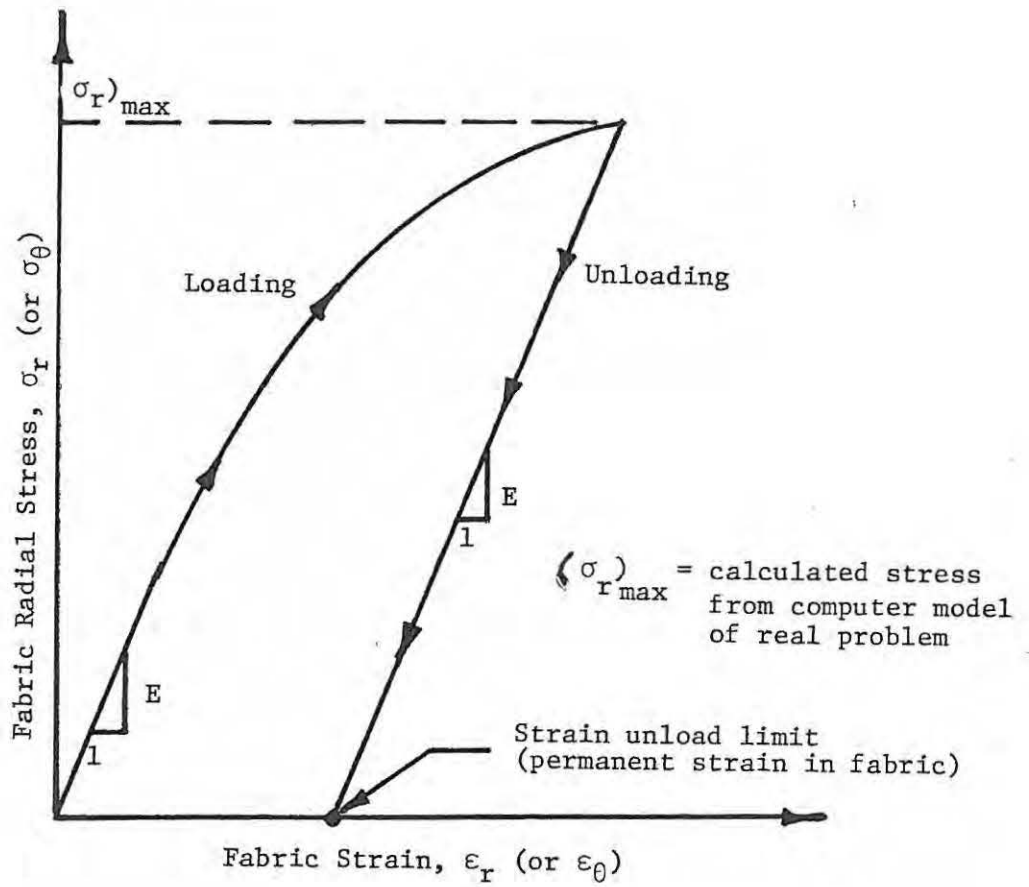


FIGURE 5. LOAD-UNLOAD PROPERTIES OF FABRIC FROM LABORATORY TESTS.

If the fabric in the model has a permanent strain upon unloading smaller than the strain unload limit defined by the laboratory test (Figure 4), the fabric will not have any residual stress in the finite element model.

Computations of the deflection at the center of the plate agree well with the results computed by the GAPPS7 program. The total centerline deflection values are as follows:

<u>Case</u>	<u>GAPPS7</u>	<u>Theoretical Formulas</u>
With Fabric	0.554 inch	0.529 inch
No Fabric	0.619 inch	0.612 inch

The theoretical computation was performed using the beam deflection formula<sup>(1)</sup> for the loaded conditions and resulted in a 0.673 inch centerline deflection. Since the computer model is a plane strain solution of an infinitely long beam, a beam solution could not be directly used to calculate the deflection. However the deflection of the plane strain plate is equal to [4]:

$$\delta = \delta^* (1 - \nu^2)$$

where  $\delta^*$  is the corresponding beam deflection. For the plate considered in this example the deflection is then

$$\delta = 0.673 \times (1 - 0.3^2) = 0.612 \text{ inch}$$

which agrees very well with the computer solution.

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1. The deflections of the center of the plate were calculated using an equivalent transformed section to handle the fabric stiffness.

Twelve Element Soil-Fabric Example. The multiple load application capability was tested for models with the geometry of Example No. 1 of the User's Manual. The unloading and the load weight options were revised and corrected in GAPPS7. Of importance is the finding that when the load weight option LWFLAG = 0 is used, separation between the fabric and soil is likely to occur because very small stresses (sometime tension) will remain in the system upon loading (often these stresses are very close to zero but still in tension). To reduce the chances of separation from occurring, the option LWFLAG = 1 should be used to maintain body weight all the time in the model for cyclic load applications.

The cyclic load examples run using the 12 element soil-fabric model are as follows for different loading conditions and materials:

DAMGEX1 - EX VI (two runs)  
 DAMAEX2 - EX VII  
 DAMAEX3 - EX VIII  
 DAMAEX4 - EX IX  
 DAMAEX5 - EX X

All of these models except DAMGEX1 consist of a fabric sandwiched between two layers of clay having the same properties. The DAMGEX1 example has gravel overlying the fabric with clay below.

DAMAEX5 was used to initially test the unloading portion of the program and the cyclic load capability. DAMAEX5 was used before the other examples in the above list.

Example DAMGEX1 was run with the gravel cohesion  $c = 0$  and also with  $c = 20$  psi to keep from having no tension corrections in the model.

Example EX IV, DAMAEX2 shows that LWFLAG = 0 will not go to completion since separation produces a non-positive stiffness matrix. Example DAMAEX3,

which uses a small 2 psi load, shows correct unloading with only small stresses remaining. For a 20 psi loading, example DAMEX4 EXIX shows the system is highly nonlinear. For this system the residual stresses upon unloading go essentially to the applied body stresses indicating correct load-unload characteristics.

Before running the above examples, the computation of the unbalanced forces for the unloading case was corrected. Since the total applied load to the system during unloading consists only of the body weight so the unbalanced force  $P_{un}$  is computed with

$$P_{un} = PWT - PRES$$

where: PWT = body weight forces

PRES = internal forces

The application of PWT with the multiple load application option was verified in examples DAMAEX3,4&5.

General Fabric Model. As a final verification of the program, a large 56 element model was run several times. This model consists of gravel, fabric and clay with the finite element idealization being shown in Figure 6 (Data File DA4CY56). The material properties are the same as those given by Zeevaert [5].

Important results are shown in Figures 7 and 8 where the behavior of the system under a moderate cyclic load of 20 psi is presented. Four cycles of load-unload were applied to the system. The data file used is DA4CY56 - Example XI. The deformation at the center of the loaded area at the surface as a function of the total applied stress is shown in Figure 7, and the variation of the fabric radial stress with the cyclic

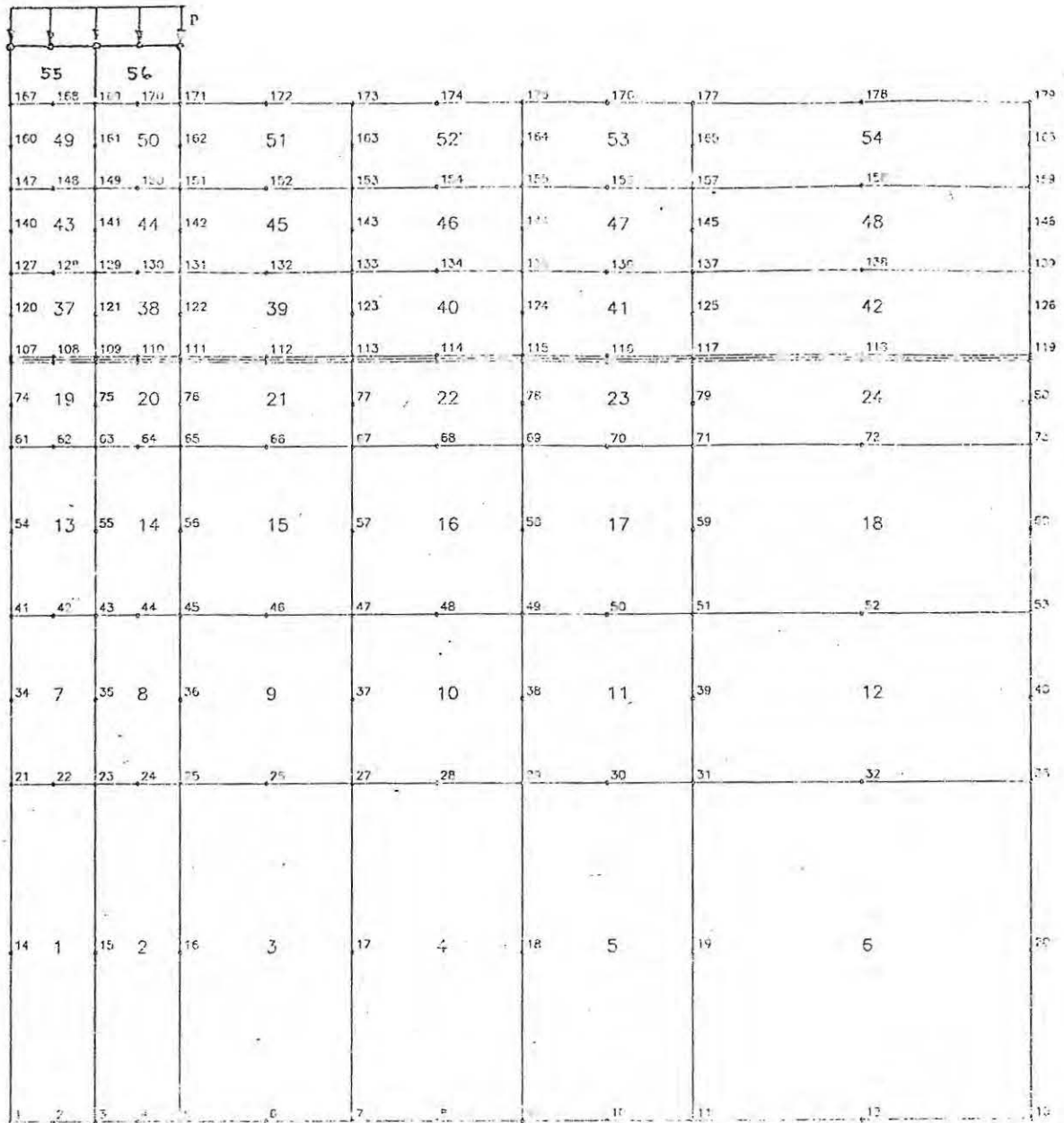


FIGURE 6. LARGE 56 ELEMENT SOIL-FABRIC FINITE MODEL.

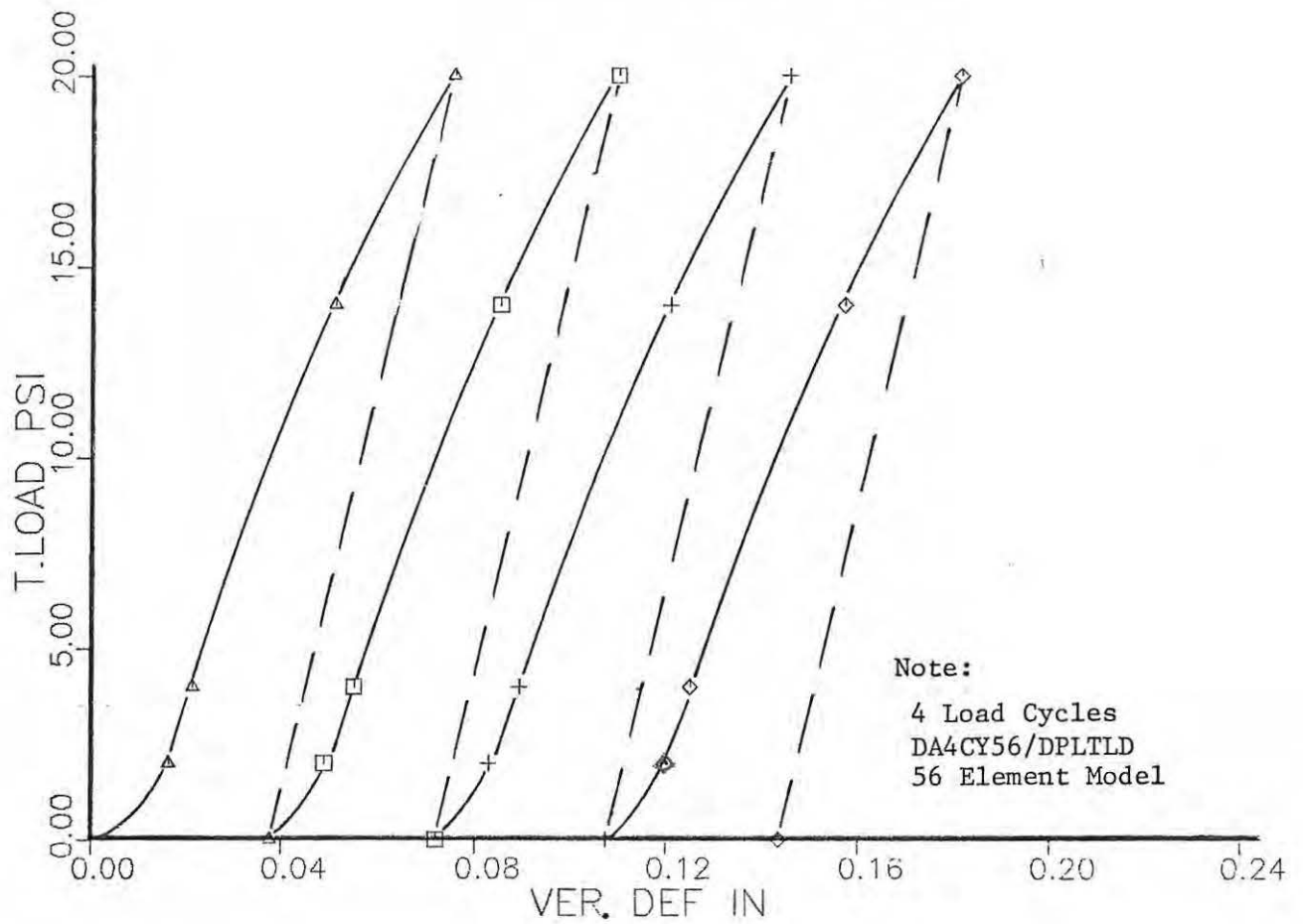


FIGURE 7. TOTAL VERTICAL DEFORMATION UNDER THE CENTER OF LOAD AS A FUNCTION OF LOAD CYCLES AND APPLIED STRESS - 20 psi MAXIMUM PRESSURE.

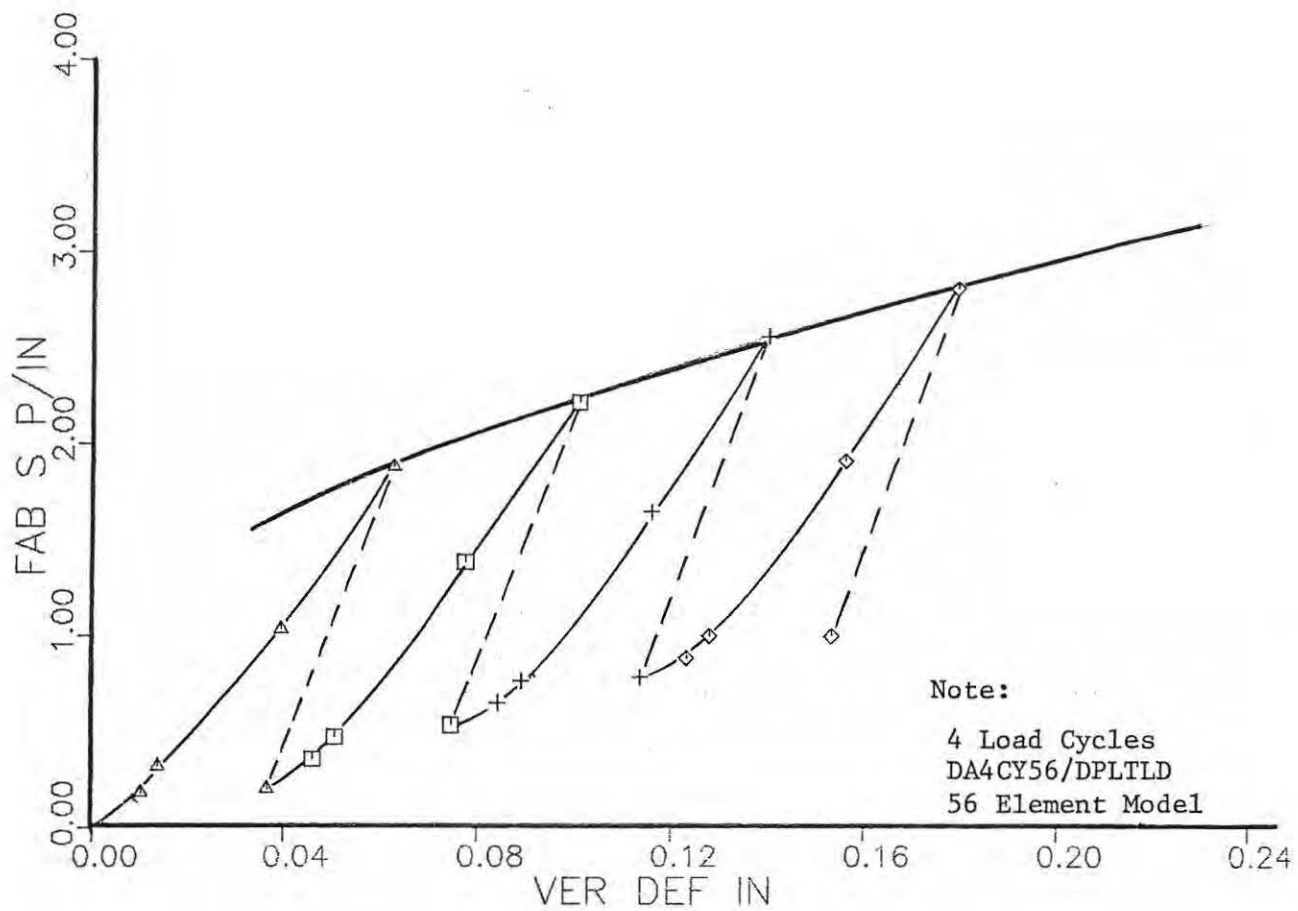


FIGURE 8. FABRIC STRESS AS A FUNCTION OF LOAD REPETITION AND VERTICAL CENTERLINE SYSTEM DEFLECTION - 20 psi MAXIMUM PRESSURE.

load application is given in Figure 8. An important increase in radial stress in the fabric occurs for each application of load due to the permanent deformation of the system. The elastic unloading of the fabric is shown in Figure 8.

The same large mesh model was loaded to 70 psi in 10 load applications (Example XII - Data File DA4CY56). The soil-fabric model after one load application of 70 psi produced an unstable structure, due primarily to the large tensile stresses in the bottom layer of the gravel, that produced failure conditions that could not be equilibrated. This is not surprising since the model uses only 4.5 inches of gravel.

Also, for the thin layer of gravel used, at a 70 psi surface loading the fabric started slipping during the first load increment. Slip initially occurred in element 8 and 9 between the stone and the fabric. These results indicate the interface strength properties are important properties influencing the behavior of the composite structure for the properties and geometry used in this example.

The results for the first cycle of load application are shown in Figures 9 and 10. In Figure 9 the total applied load as a function of the centerline vertical deformation is shown. In Figure 10 the fabric radial stress versus the vertical deformation is given. The nonlinear behavior of the soil-fabric system is illustrated in this example. The large residual stresses in the fabric remain in the fabric due to permanent deformations of the system.

Both examples were run with the large displacement option flag `LDFLAG = -1`. The data file used is `D4CLD56` and the output files are given in Examples XIII and XIV.

To complete the verification of the program, several additional

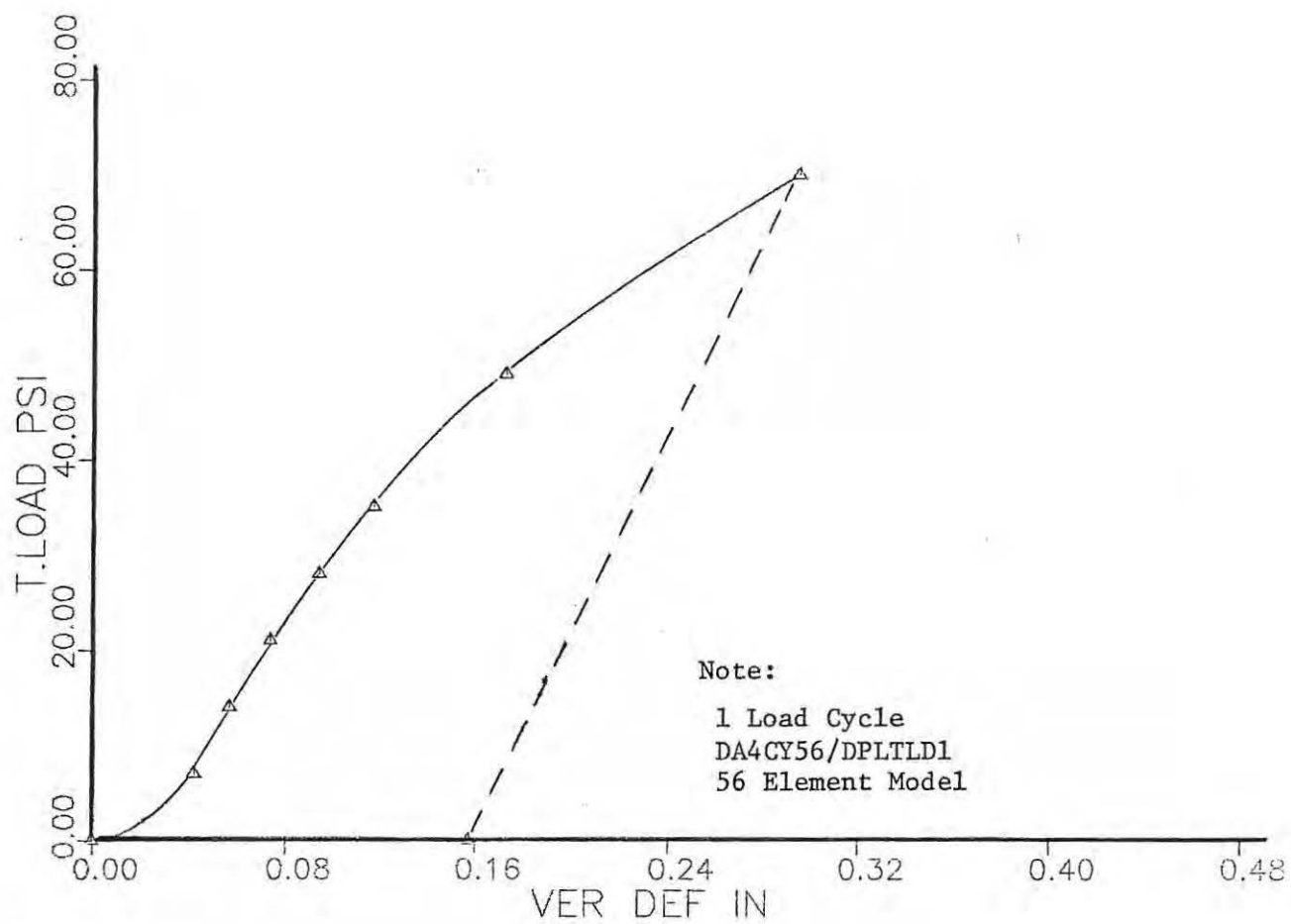


FIGURE 9. TOTAL LOAD AS A FUNCTION OF CENTERLINE DEFLECTION - 70 psi MAXIMUM PRESSURE.

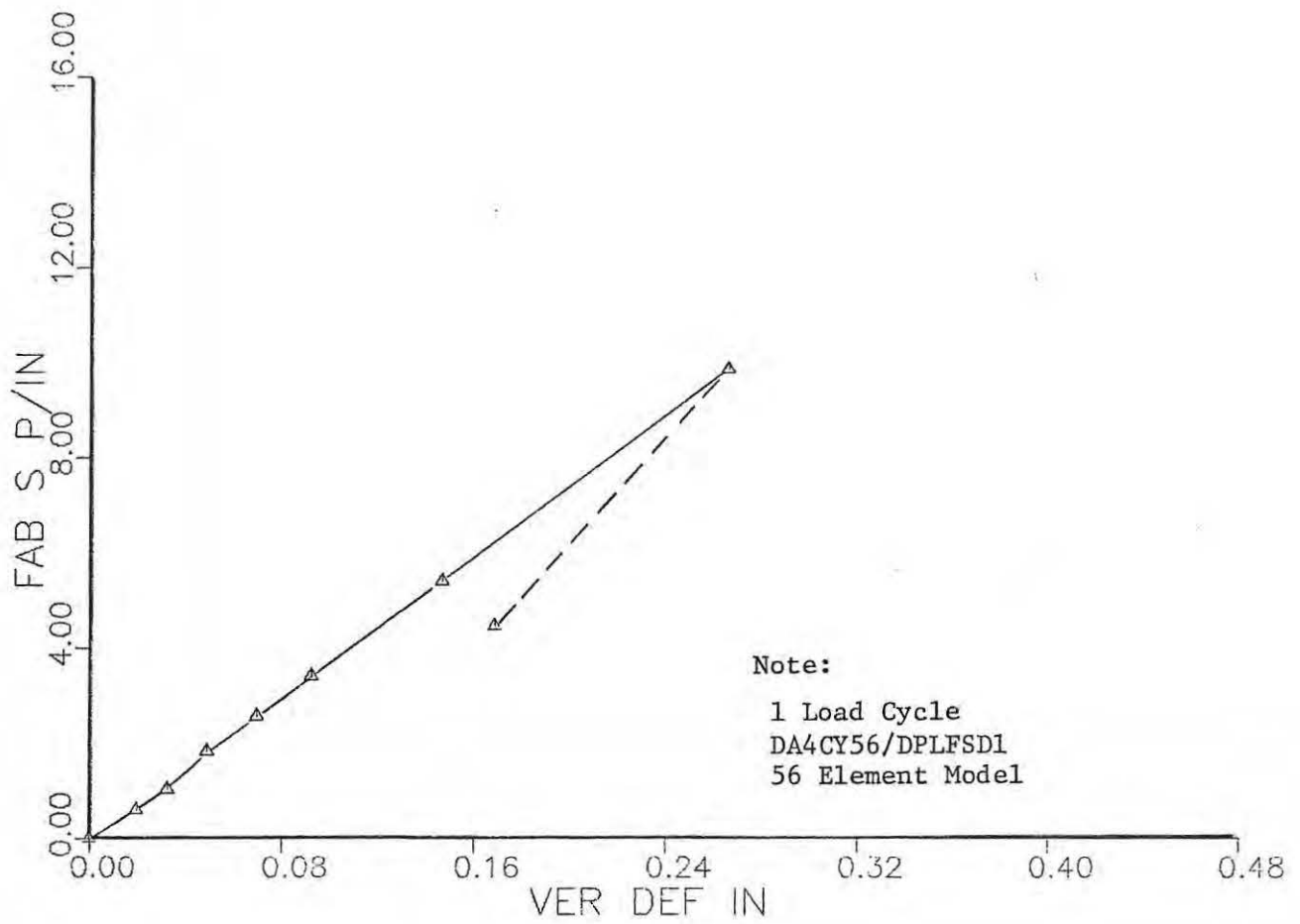


FIGURE 10. FABRIC STRESS AS A FUNCTION OF VERTICAL CENTERLINE SYSTEM DEFLECTION.

examples were run using the 56 element finite element mesh. All of these examples consisted of a gravel-fabric-soil model and are summarized as follows:

EX-XIV (Data File: DA4CY56). This example included the large displacement option with 10 load increments and 4 load cycles. The execution time was 3359 sec. The applied load was 20 psi.

EX XV (Data File: D4C7056). This model introduced a large cohesion in the gravel ( $c = 15$  psi) and loaded the model to 70 psi using 5 load increments; 4 load cycles were used requiring an execution time of 2294 sec.

EX XVI (Data File: D2CYB56). In this model the gravel was divided into three layers having a total thickness of 9 inches. To reduce the effects of tension in the gravel (refer to the next section), an apparent cohesion was introduced in this model for the bottom gravel layer. The model was loaded with two applications of 70 psi load; 5 load increments per application was used. To compute the apparent cohesion introduced in the model a cross-anisotropic elastic analysis was first run with the bottom gravel layer having a horizontal modulus of elasticity  $E_h$  of 200 psi and a vertical  $E = 2000$  psi. Using the vertical stress calculated in the centerline element, the apparent cohesion was calculated by first computing the maximum tension<sup>(1)</sup>

$$\sigma_{r_{\max}} = \sigma_1 \tan (0.75\phi)$$

where  $\sigma_1$  is the major principal stress and  $\phi$  the angle of internal friction in the gravel. For this example  $\sigma_{r_{\max}} = 3.169 \tan (.75 \times 48^\circ)$  giving  $\sigma_{r_{\max}} = 2.3$  psi. The apparent cohesion  $c_a$  is then

$$\begin{aligned} c_a &= \sigma_{r_{\max}} \tan 48^\circ = 2.3 \times 1.11 \\ c_a &= 2.55 \text{ psi} \end{aligned}$$

This value of apparent cohesion was introduced in the model together with the normally used modulus of elasticity variation with deviator stress for the cohesive material.

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1. Refer to the next section for a discussion of this technique.

EX XVII (Data File: D2CYR56). This example is identical to EX XVI except an apparent cohesion was not used. The program was run using the usual input variables.

The results for EX XVI and EX XVII are summarized in Tables 2 and 3. Both examples did not present slip or instability problems and ran to completion. Both analyses gave similar results, even though the modulus properties were not the same for the bottom gravel layer. For both cases the stress in the fabric increased upon the second application of load. Further, the stress in the fabric was about 8% greater when apparent cohesion was not used.

Apparent Cohesion in Base. Analysis of the tensile state of stress which exists in the bottom of the base is quite complicated. This analysis is usually made considering the material to be infinitesimally small particles comprising a continuum. However, consideration of the base as a series of discrete particles such as exist in a haul road constructed using coarse stone is more realistic.

Consider two large rock particles touching each other as shown in Figure 11. Let the normal stress between the two particles be equal to  $\sigma_z$ . Then the maximum horizontal tensile stress,  $\sigma_r$  that can be applied is equal to

$$\sigma_r = \sigma_z (\tan \bar{\phi})$$

where  $\bar{\phi}$  is the effective friction angle between the two stones for a horizontal tensile state of stress. This reasoning indicates that a granular base stone subjected to vertical stress can take some tension before failure.

Considering the failure state in the granular stone to be as shown in Figure 12, the definition of apparent cohesion is introduced.

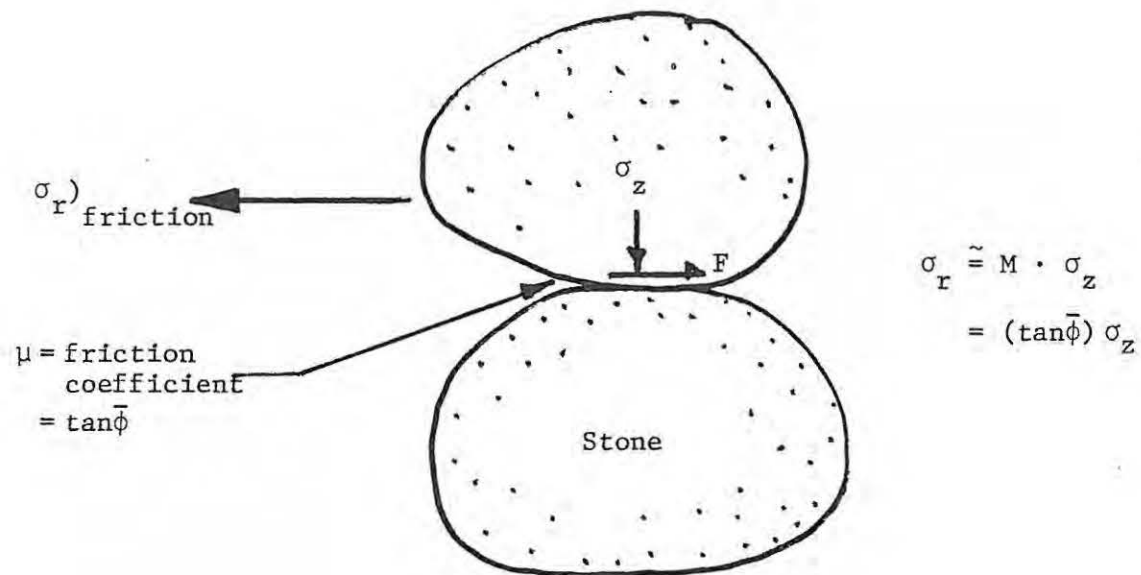


FIGURE 11. DEVELOPMENT OF RADIAL TENSILE STRESS IN GRANULAR BASE DUE TO FRICTION.

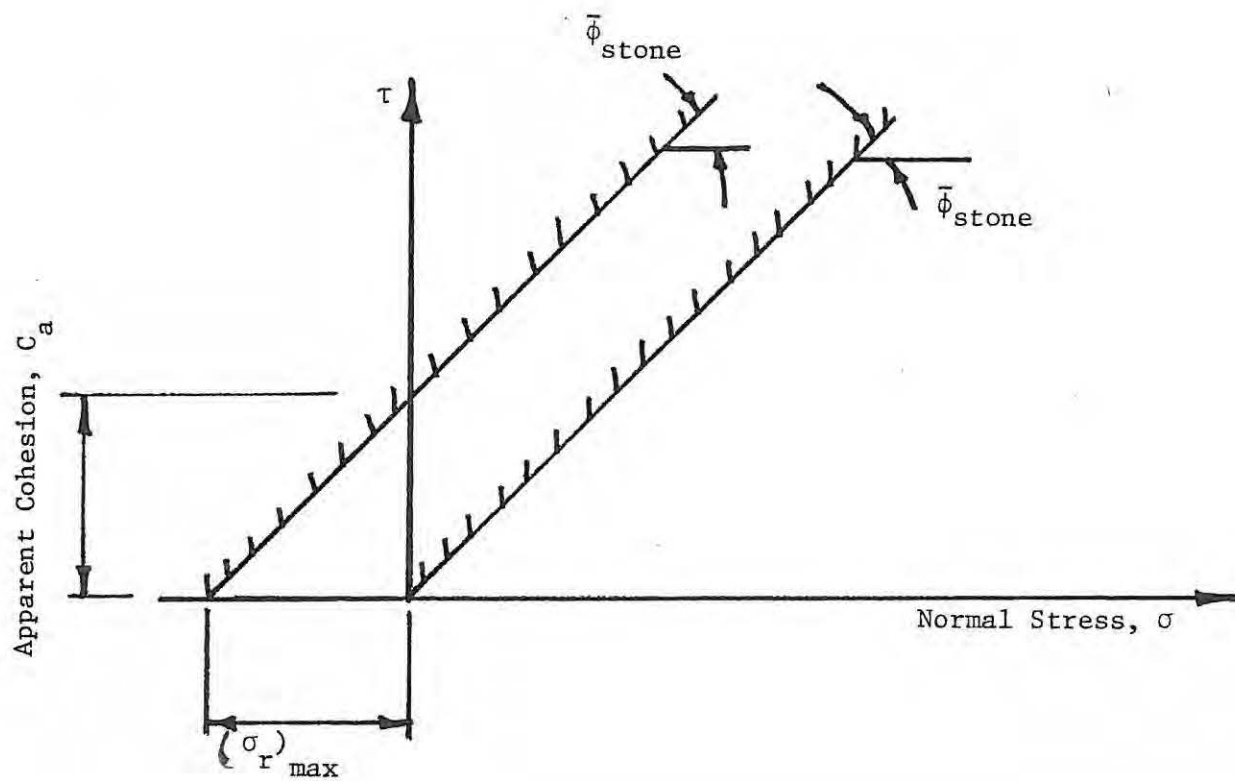


FIGURE 12. DEFINITION OF APPARENT COHESION IN TENSILE ZONE OF GRANULAR PAVEMENT STONE.

Table 2. Example XVI - GAPP57 with Apparent Cohesion.

CYCLE	P (psi)	$\delta_{G_L}$ (inches)	Fabric Stress $G_L$ (lbs/inch)
1	14	0.0415	0.157
	28	0.0487	0.292
	42	0.0551	0.435
	56	0.0621	0.596
	70	0.0695	0.789
	U.L.	0.0389	0.327
2	14	0.0727	0.497
	28	0.0805	0.666
	42	0.0878	0.835
	56	0.0951	1.08
	70	0.103	1.305
	U.L.	0.0729	0.897

- Notes: 1. Gravel thickness 13 inch.  
 2. Data: D2CYB56 (Apparent Cohesion)  
 3. U.L. = Unload

Table 3. Example XVII - GAPPS7 Regular Run.

CYCLE	(psi)	(inches)	Fabric Stress $G_L$ (lbs/inch)
1	14	0.0416	0.154
	28	0.049	0.289
	42	0.0567	0.429
	56	0.0648	0.668
	70	0.0736	0.852
	U.L. (1)	0.0452	0.297
2	14	0.0789	0.358
	28	0.0868	0.518
	42	0.0946	0.628
	56	0.1027	0.870
	70	0.1106	1.044
	U.L. (1)	0.0829	0.474

- Notes: 1. U.L. = Unload  
 2. Gravel Thickness 13 inch.  
 3. Data: D2CYR56 (No Apparent Cohesion)

In figuring the apparent cohesion, to be conservative the tentative recommendation is made that  $\bar{\phi} = 0.75\phi_{\text{stone}}$  be used at least until more experience is gained applying this concept in practice. Use of an apparent cohesion may, in some applications, reduce or perhaps eliminate problems with instability of the system. *Note that tension effects and instability problems in the bottom of the granular base can be eliminated entirely by using a large value of apparent cohesion in the analysis.* Use of excessively large values of fictitious cohesion in the bottom of the base may give erroneous results.

The vertical stress used to calculate the apparent cohesion can be estimated by using the results from an initial, cross-anisotropic elastic run. In this and subsequent runs the granular layer can be divided into say three sublayers, and  $\sigma_z$  calculated in the center of each layer beneath the load. To obtain a reasonable estimate of  $\sigma_z$  in the lowest layer the horizontal elastic modulus  $E_h$  in that layer can be taken equal to 1/10 to 1/20 of the vertical modulus  $E_v$  used in the same layer; this approach will partially account for the tension effects in the lower layer, at least for the preliminary run used to estimate apparent cohesion. In the computer program rather than using the  $\sigma_z$  and  $\sigma_r$  stresses for calculating apparent cohesion, the  $\sigma_1$  and  $\sigma_3$  stresses are used since shear stresses are not present on the principal planes.

#### PROGRAM LIMITATIONS

The GAPPS7 program has a number of limitations and must be used with extreme caution; only someone familiar with both the program and the associated nonlinear pavement material properties used in it should

attempt to use the program. Particular care should always be used in selecting a sufficient number of load increments to give convergence of the iterations. Also the results should be carefully reviewed for any data input errors or answers that do not appear to be correct. Before a large nonlinear run is made with or without cyclic loading, an elastic run should be first made to be sure the grid and program are correct.

A summary of some of the more important general limitations are as follows:

1. For nonlinear problems a sufficient number of load increments must be used to insure convergence. Although the number of increments used depends on the degree of nonlinearity, 5 to 10 increments should be considered a minimum.
2. Care should be taken to look out for instability and non-convergence of the results.
3. The program was developed mainly for horizontal layered systems. For other geotechnical problems like retaining walls, sheet pile walls and other passive pressure problems, the load increment capability, unload and no tension analysis should be verified and results compared with actual measurements. Caution should also be exercised in applying the program for fabric reinforced overlays.
4. The program unloads elastically even at failure; upon unloading the stresses are not corrected for no tension.
5. The program unloads elastically the fabric material using the initial elastic modulus of the fabric.

6. After a cycle of load is applied if slip or separation conditions are encountered, upon unloading the interface is modeled with the initial spring stiffness.
7. The plot program was developed for the interface models for horizontal numbering. For vertically numbered meshes without interfaces, the PTMESH4 program can be used for plotting.
8. Maximum Tension  $\sigma_t$ . The variable TENMAX is used in the program for the case of limiting small vertical tensile stresses. This variable is for limiting the allowable vertical tension that a material can handle. The variable does not limit the maximum horizontal tensile stress. In other words, the  $\sigma_t - \text{TENMAX}$  variable is not designed to skip the force elimination routine for lateral tensile stresses. In the case of lateral tensile stresses the elimination routine is used regardless of the value given to TENMAX.
10. The no-tension analysis used in the program is approximate. It is considered, however, as a good approximation for the horizontally layered pavement problem.

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