

A NETWORK FLOW APPROACH TO  
COMMON USER COMMUNICATIONS CONFIGURATION

A THESIS

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Richard James Mallion

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
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## SUMMARY

The problem of controlling common user telephone communication networks during periods when requirements exceed capacities is approached using network theoretics. Since the multicommodity linear programming formulation for a large network of  $n$  nodes having up to  $\frac{1}{2}n(n-1)$  commodities is not easily solved, the network is partitioned into small subnetworks, the maxflow solutions to which can be obtained with relative ease. Allowed calls within and between subnetworks are distributed to enable maxflow in accordance with specified performance criteria while not exceeding the inter-net and intra-net capacity limitations.

A simple method for estimating changes in requirements caused by adding or deleting users, adding or deleting nodes, changing user locations, or altering network connectivity is presented. By using matrix multiplication the revised requirements for an entire network can be simultaneously generated whenever changes occur which affect the distribution of calls.

A procedure is outlined for using network theoretic techniques in a tactical military situation, and a series of examples is used to demonstrate this procedure. While

developed primarily as a means for rapidly allocating and redistributing service in potentially-overloaded and frequently-reconfigured networks, the procedure could be modified to determine optimal communication system configurations. It could also be extended to assist in the development of network performance criteria and measures of effectiveness.

## CHAPTER I

## INTRODUCTION

An historical account of communications during World War I titled "Circuits of Victory" contains the following paragraph which describes the problem facing a communication traffic engineer in a combat environment.

Every nerve in America's body was now tingling. Every long distance wire of electrical communication was alive with throbbing impulse. These wires must not be overcharged, they must not be overcrowded with the tremendous traffic of speech through space, or down would tumble the whole structure like a house of cards, delay irreparable would follow the collapse and--TOO LATE would be the epitaph on the tombstone of democracy.

That quotation, while couched in a style which is dramatic to excess, conveys an impression of the problem which this thesis addresses. Communications are indeed essential to the planning and execution of military operations, and overcrowding or collapse of vital networks must be prevented. Of course, the communication devices of World War I have been relegated to museums and replaced by the marvels of technology which are an integral part not only of the military establishment but of the American life-style. War, which was horrible enough milleniums ago, can now be waged with weaponry which multiplies the horror by almost-incomprehensible orders of magnitude. There have been astounding

changes since the quoted book was written, but the problem of communications management is no less acute now than it apparently was over fifty years ago.

### Definition Of The Problem

This thesis addresses the problem of providing communications to military organizations engaged in combat and combat-related activities. Specifically, the question which this research attempts to answer is stated:

How can available communication assets be best used to provide common user voice communication service to the U. S. Army in the field?

The following sections of this chapter will develop the problem further and explain the term "common user".

Certain professional and personal biases which prompted the initiation of this research are acknowledged. It is contended that:

1. Common user service has never been accorded proper consideration by either commanders or communicators.
2. The electronic marvels which are now (or will soon be) available are not the panacea for all present and future problems.
3. A reasonably-approximate solution arrived at quickly and easily is preferable to an elegantly-exact solution which is available only after the problem resolves itself or after the situation deteriorates to a point

beyond which problem resolution is possible.

4. Simplicity in any endeavor is a virtue.

It is further contended that while these biases are present, they should not detract from the legitimacy of the research herein reported.

#### Brief History Leading To The Problem

Communications are generally divided in two broad classes: sole user and common user. Sole user requirements are met by providing dedicated equipment and circuitry between two individuals or organizations. Figure 1 shows a simple sole user situation.

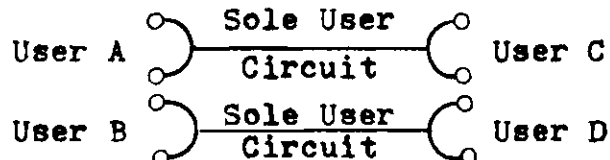


Figure 1. Sole User Situation

Common user requirements are met by having callers share equipment and/or circuitry. Figure 2 shows a simple common user situation.

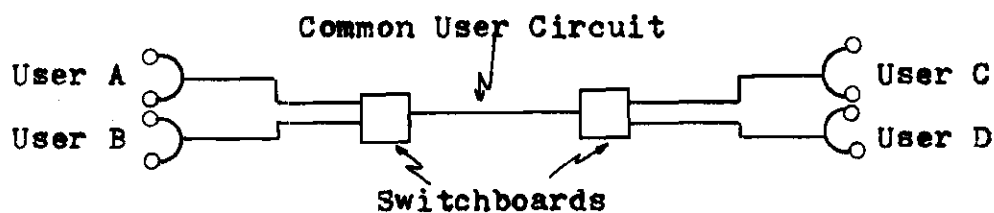


Figure 2. Common User Situation

There are some circumstances in which sole user service is the only way to meet speed and quality criteria, and sole user circuits will always consume a portion of available communication assets. However, most military organizations' communication needs are satisfied by a common user network. Rather than being "satisfied", many needs have been left unsatisfied. This condition has prompted commanders to request and, unfortunately, receive dedicated circuits. The circuits for this new sole user service were obtained from the common user network, and the other common user organizations suffered a degradation of service. This resulted in more dissatisfaction with the common user network which caused commanders to request and, unfortunately, receive sole user service . . . ad nauseum. Sole user circuits have proliferated while common user circuits have dwindled. Additional circuits have been made available and improved switching equipment has been developed, but common user service has not improved in many instances.

Commanders become used to commercial grades of service in garrison situations and to unrealistically good service provided during command post and field exercises. They expect, but will probably never receive, equivalent service during combat. Military communication networks are not like commercial networks for many reasons, among which the most significant are:

1. Commercial users are generally static, but military users have no fixed locations. As the situation dictates, units move, their communication requirements change both in location and magnitude, and the network must be altered to meet these changing demands.

2. Commercial networks are subject to normal failures, but military networks, while subject to these same "normal" failures, are also faced with massive irreparable destruction of facilities.

3. Time restrictions and asset limitations are more severe under combat conditions than they are in more peaceful situations.

For the reasons described above and prompted by the biases expounded in the previous section, it is perceived that common user service should be accorded a different perspective and that there exists a reasonable possibility of improving common user service or halting further degradation if a non-traditional approach to tactical communications traffic engineering is attempted. This feeling is shared by Jacobaeus and Elldin(60) who have stated:

Telephone traffic theory has from the start been ahead of other applied sciences. Operation research was employed long before the name operation research was coined. . . . There is no reason to expect that in the future the theory will remain in strictly rigid forms, but one may expect continued vigorous approaches to the problems from new angles. The significance of a well developed telephone traffic technique is so great that the theoreticians are quite simply forced to produce practically usable results.

### Purpose Of The Research

Military communications traffic engineering has resembled that of the commercial communication establishment. This is a natural tendency because of the amazing advances made in the commercial sector and the fact that most military traffic engineers are commercially-trained. Elegant simulation models have been built to synthesize and alter network configurations. Massive studies have been conducted to obtain, reduce and analyze vast quantities of data on traffic volumes and patterns. The precepts of queuing theory have been applied to situations and scenarios of every type. But the existence of sophisticated equipment, techniques and knowledge is of little use to the persons charged with operating and maintaining the network under adverse circumstances and without recourse to the same tools used by the network designers and analysts. Furthermore, normal traffic engineering procedures assume independent service requirements which would not be the case in many combat situations; for example, communication demand by users could not be considered independent immediately after an adversary's preemptive nuclear attack.

This research attempts to develop some useful traffic engineering techniques based on network flow theory rather than on the more-traditional queuing theory. No astonishing theoretical discoveries should be anticipated for there are none forthcoming. The intent is to review

the literature of network flow theory and to adapt the work of learned researchers to the unique tactical common user communication environment. Any techniques thus developed must be simple to understand and relatively easy to apply; eventually, a well-trained technician should be able to apply the procedure--certainly that technician would not have a graduate-level education in operations research.

In essence, it is not the purpose of this research to significantly advance the frontiers of network flow theory. It is an attempt to apply some existing theories to a significant military communication problem. The emphasis is placed on allocation of residual assets rather than on configuring the network prior to the initiation of hostilities. Particular emphasis has been placed on developing a method of estimating the changes in requirements which are brought about by the many events which impact on the network. The problem solutions thus derived may not be strictly optimal, but in a tactical environment the adage "Something is better than nothing" applies to communications. There is certainly merit to a planning sequence which will maximize the usage of the network without allowing an overload condition which can lead to the eventual collapse of the network.

#### Review Of Literature

A very extensive literature search was conducted in

the areas of telephone traffic engineering and network flow theory. This section mentions but a few of the primary references in the major areas of interest. Other sources are cited in subsequent chapters, and a lengthy bibliography is included for those who may wish to pursue the subjects further.

Telephone traffic engineering has been the subject of many books and articles. Among the most frequently cited books are those of Beckmann(3), Benes(4), Roualt(95), and Syski(99). The Bell System Technical Journal has published numerous pertinent articles since its inception in 1921. The series of ten chapters by Mina(80) in Telephony between April 1971 and May 1973 is an excellent source of information for a network manager who must work with traffic engineers but not necessarily perform the engineering functions. Other prime sources include the Dutch periodical Phillips Telecommunications Review and the Swedish periodical Ericsson Review. The U. S. Army publication TM 11-486-2(21) is hopelessly outdated in many respects but does give some of the still-applicable fundamentals of traffic engineering for small manually-operated networks. The best source of practical traffic engineering techniques is the "Bell System Practices", but these are company-proprietary documents and are generally unavailable to non-Bell System personnel. Mallion(75) attempted to adapt conventional

traffic engineering practices to the field army network; this present thesis is prompted, in part, by the realization that even such an adaptation may not provide the rapid redistributions necessary in a combat situation.

Network flow theory is a relatively recent development with the foundations being formalized during the early 1950's. The books most frequently cited are those of Ford and Fulkerson(31), Hu(56), and Frank and Frisch(36). The paper generally considered to be the foundation of network flow theory is that of Ford and Fulkerson(32). The same authors(34) provided the first solution for the multicommodity flow problem. Mayeda(78) is credited with the initial work on multiterminal flows. Chapter III contains sections on routing of flows through networks, multiterminal flows and multicommodity flows; the pertinent literature is traced in each of those sections so, to avoid unnecessary duplication, it is not reviewed here.

#### Organization Of The Research

Chapter I introduces the problem, provides some background on the motivation of the research, and reviews a portion of the vast body of network theory literature.

Chapter II is a brief propaedeutic on tactical communications. It provides sufficient background for the reader to realize the scope of the problem and to be aware

of some limits which equipment and doctrine impose on any effective solution. The inapplicability of traditional traffic engineering practices is also hypothesized.

Chapter III develops the specific problems which must be solved if available common user assets are to be properly managed. The large literature of network flow theory has been consulted to provide techniques and algorithms for solving these problems.

Chapter IV presents a method of simultaneously estimating all requirements revisions whenever some event occurs. A proof of this simple matrix multiplication technique is given, and several examples are included.

Chapter V outlines a tactical traffic engineering procedure which is derived from the problems and solutions discussed in Chapters III and IV.

Chapter VI outlines possible extensions of the procedure which was presented in Chapter V.

Chapter VII briefly discusses the obvious limitations of this research, makes some conclusions about the validity of the approach and the efficacy of the procedure, and provides some recommendations for future research.

Several appendices are provided to display examples of techniques presented in the body of the thesis.

An extensive bibliography is included.

## CHAPTER II

## TACTICAL COMMUNICATIONS

This chapter is a brief propaedeutic on tactical communications. It provides only enough background for the reader to be aware of how communications are provided in combat and to recognize constraints which must be dealt with when solving a military communication problem.

Scope Of Service

The U.S. Army publishes Tables of Organization and Equipment (TOE) which list authorizations for tactical units. As this is being written, organizational changes are being implemented to reflect a radical change in the doctrine(24), but typical strength figures from prior to the change should be relatively the same and give some indication of the population and area being serviced by the tactical common user networks.

A deployment of two corps in a theater of operations might be composed of 1,400 discrete units representing the entire spectrum of combat, combat support, and combat service support organizations. The size of these units varies from the very small to the very large. For example, a Military History Detachment has a strength of two personnel, while an Armored Division has a strength

of 16,850 personnel.

The four-division type corps used as an instructional vehicle by the U.S. Army Command and General Staff College(23) has an assigned strength of 105,501 personnel, and an estimated 117,264 personnel might be required to support a two-corps deployment. This gives an approximation of 328,266 Army personnel assigned to a two-corps deployment, and personnel of other Services will also be engaged in the operational area. This operational area might be 250 kilometers wide by 280 kilometers deep with variations depending on terrain, mission, etc.. Common user networks can be established at levels as low as individual companies of approximately 200 men, and all networks are interfaced to form what amounts to a theater common user telephone system.

These strength and size approximations indicate that the integrated common user networks serving the deployment area are providing communications to a population equivalent to that of a major city spread over an area in excess of 25,000 square miles with the added difficulties attendant to the mobility and dispersion required of a military force.

For the purpose of this thesis, no attempt is made to discuss requirements on a unit-by-unit basis. The requirements discussed throughout the research are considered to be those consolidated at the various network switching nodes.

### Trunking And Switching Equipment

A traffic engineer will necessarily have considerable knowledge of the communication equipment which is used to derive the common user network. The equipment can be divided into four categories: terminal instruments, access equipment, trunking equipment, and switching equipment. Terminal instruments and access equipment have no impact on the problems discussed in this thesis.

Trunking equipment is defined as that required to interconnect the switching centers and is usually a radio/carrier or a cable/carrier system. The Army Area Communications System Project (AACOMS) has developed a family of integrated equipment which meets military specifications for quality, durability, ease of operation, etc.. To enhance the cost effectiveness, standard-sized assemblages have been procured. The channel capacity limitations within the equipment family place a trunking restriction on the traffic engineer. These standard assemblages can derive only systems of 6, 12, 24, 48, or 96 channels. It is generally not possible to replace, for example, a 24-channel system with the next-higher-capacity 48-channel system simply because the residual circuit assets of the smaller system are insufficient to provide an adequate common user network. Therefore, a premium is placed on the efficient use of the residual assets because the traffic engineer must realistically

assume that few or, perhaps, no system overbuilds will be possible.

Switching equipments are currently manually-operated switchboards varying in capacity from 12 lines to 600 lines. Soon-to-be-issued electronic switches for higher echelons of the network have a capacity of either 344 or 620 lines. Both manual and electronic switches have distinct advantages and disadvantages. Manual switchboards provide slow service, but it is easy to exercise control in overload situations. Electronic switches provide fast service, but overload conditions can cause deterioration and collapse of the network.(8) Also, it is easier to change the routing scheme used by a manual switchboard operator than it is to rewrite the routing software for a network of electronic switches. There are many other factors that can be compared, but it is safe to assume that for a reasonable time horizon the tactical communication networks will contain both manual and automatic switching equipment.

For the purpose of this thesis, trunking equipment will be represented by the arcs and switching equipment by the nodes of the graph theoretic approach presented in the following chapters. It is sufficient to realize that there are specific branch and node capacity restrictions which are imposed by trunking and switching capacities of the equipment used to derive the network.

### Doctrine

Both strategy and tactics as well as communication doctrine impact on the job of the traffic engineer. From a strategic viewpoint, for example, the requirements for a classical anti-guerilla campaign (e.g., the British experience in Malaysia) are certainly different than the requirements in conventional warfare on a large landmass (e.g., the Allied campaigns in Europe). And yet there will be inevitable anomalies such as the recent involvement in Viet Nam which was, in many ways, an anti-guerilla campaign waged using conventional warfare methods. From a tactical viewpoint, the requirements for mobility and dispersion in conventional warfare are certainly different than the requirements in active nuclear warfare. In spite of the many possible strategic and tactical variations in which the Army might be involved, it is still necessary to provide the best possible quantity and quality of common user service.

Communication doctrine is also a changeable element. For example, the "grid communication system" was once prescribed for the Army Area Communication System of some sixteen switching centers. The basic configuration was as depicted in Figure 3.

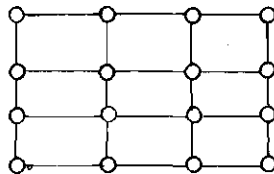


Figure 3. Grid Communication System

The latest army area doctrine prescribed a "tandem switching system" such as depicted in Figure 4; there are the same sixteen switching centers, but switching is on a hierarchial basis and eight fewer links are required to complete the basic network.

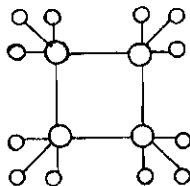


Figure 4. Tandem Switching System

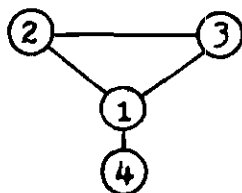
Each configuration has distinct advantages and disadvantages, but when the doctrine change was promulgated, the traffic engineer was forced to adapt all his procedures to the latter version.

At the time of this writing, the field army is no longer used by the U. S. Army, but the tandem switching philosophy will probably be retained for the network which will serve the new organizational structure.

For the purpose of this thesis, the particular strategic and tactical setting should not alter the efficacy of the procedures developed in subsequent chapters. However, a conventional warfare situation escalating into a tactical nuclear exchange is depicted in the scenario of Appendix D. This scenario was chosen only because it can provide several events which will require the communication

system to be reconfigured and the common user requirements to be redistributed.

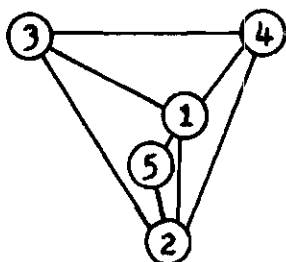
The specific configurations used as examples in the sequel are hypothetical switching systems which could realistically serve the type of organization with which each is associated. A battalion network might be of the configuration depicted in Figure 5.



1 Battalion HQ  
2,3 Engaged Companies  
4 Reserve Company

Figure 5. Hypothetical Battalion Network

A division network might be of the configuration depicted in Figure 6, but there could be different numbers of links and/or nodes.



1 Division HQ  
2 Support/Alt HQ  
3,4 Engaged Brigades  
5 Reserve Brigade

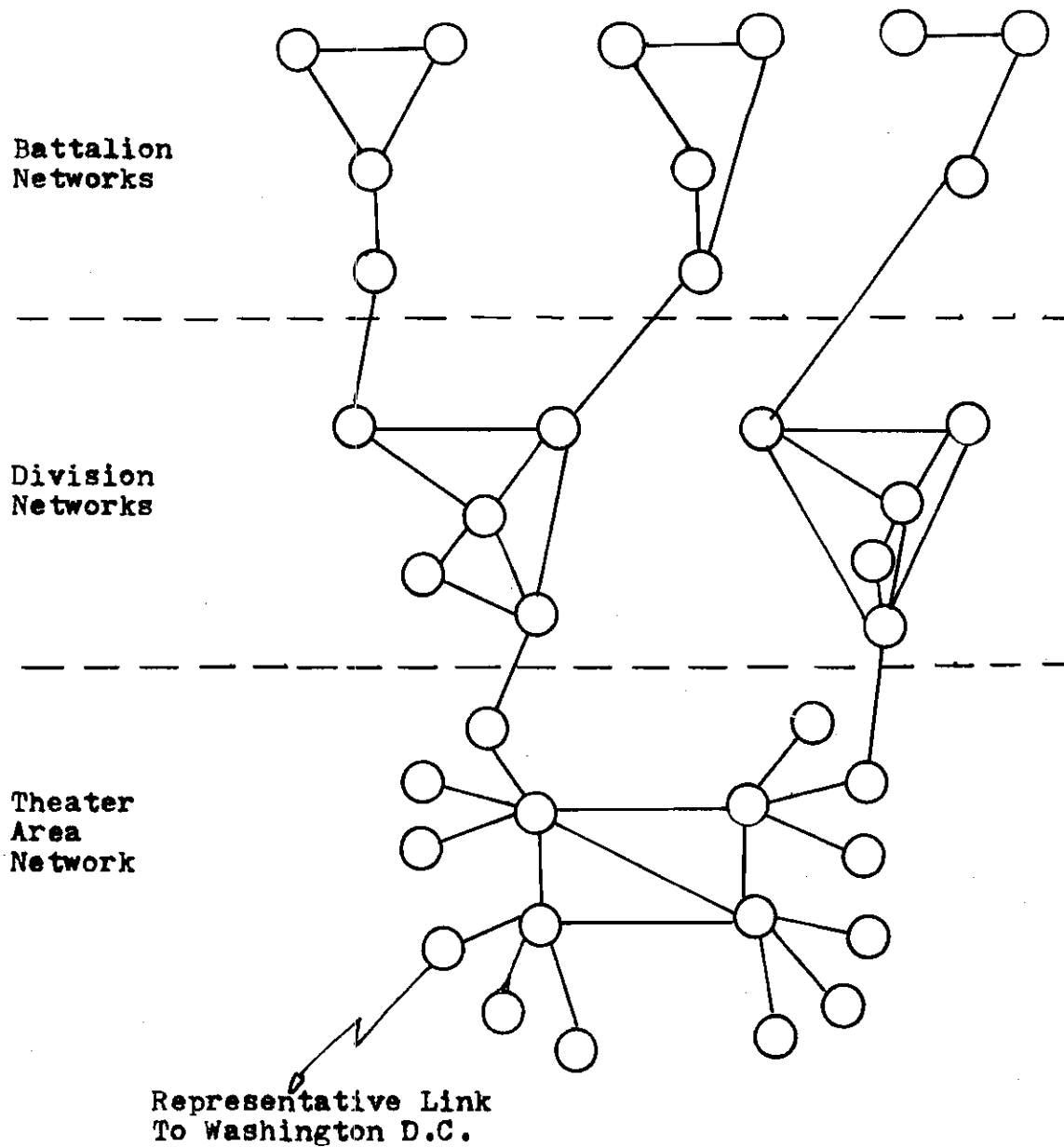
Figure 6. Hypothetical Division Network

There are other doctrinal considerations such as vulnerability and survivability which will be discussed briefly in Chapter III; these will certainly be factors in determining which configurations might be acceptable in a

given situation. However, the graph theoretic and mathematical programming approaches of the following chapters do not depend on the particular configuration to which they are applied.

The various battalion, division, and higher echelon networks are all interconnected, and it should be possible for any user to call any other user at any level if no restrictions are placed on inter-net traffic. Figure 7 shows how some of the networks in a theater of operations might be interconnected; a division might have up to fifteen battalions, a corps might have up to five divisions, and a theater could conceivably deploy several corps. A large scale deployment will involve the interconnection of many separate networks.

Communications "from the foxhole to the White House" are possible; however, it is unlikely that requirements of that nature exist. It is essential to realize that each network will have intra-net and inter-net requirements. Thus, while the requirements in any one network might be independent of the requirements in the adjacent or distant networks, the satisfaction of each networks' requirements is not a process which can be conducted independently without considering the whole scheme of interconnection and the complete set of requirements generated by the entire user population.



**Figure 7. Individual Networks Interconnected**

### Common User Requirements

Communications fall into the broad categories of sole user and common user as was explained in Chapter I, and requirements exist for service using varied media. Voice, teletypewriter, data, facsimile, and television all are potentially useful in the combat environment. Sole user requirements are given highest priority and are thus satisfied first. Even though designated as "common user", requirements in some media (e.g. teletypewriter and secure voice) require special handling within the communication system; they are generally satisfied immediately after sole user requirements. Any assets yet remaining are normally allocated for the common user voice network.

In general, it is very difficult to predict what the common user requirements will be in any given situation. Empirical data from past conflicts are available, but extrapolating this historical information to predict future requirements would not be valid.

The typical approach to generating a set of requirements has been distributing a questionnaire to interested organizations and asking them to enumerate anticipated requirements. Pooling these data led to an overall network service requirement. Initially contemplated was an exercise of this type to determine "realistic" common user voice requirements for the scenario of Appendix D. However, the construction of an appropriate

multiple-criteria model, the collection and reduction of adequate data, and the subsequent interpretation of results was self-adjudged to be a task of such magnitude as to far surpass the time and resource constraints imposed for this thesis.

The traffic engineer will generally be able to satisfy the sole user and special service common user requirements. This will leave a residual of circuits from which he can derive the common user voice network. "Normal" requirements might then be determined from studying the traffic under "normal" circumstances and applying accepted commercial traffic engineering practices. This approach has been tried both in combat and by computer simulation. The accuracy of the predictions resulting from a simulation study can be judged only after comparing the predicted and actual requirements--and there is no way to validate the result of the simulation short of engaging in combat. The capabilities and limitations of the commercial practices are discussed in the next section.

The approach which this study takes is to assume that some set of initial common user voice requirements is available. The problem is then one of using the residual circuit assets to satisfy this requirement according to some criterion or criteria which doctrine or the commander deems appropriate.

### Inapplicability Of Traffic Engineering Theory

It was mentioned in Chapter I that military and commercial networks differ because of factors such as mobility, dispersion, susceptibility to damage, and time constraints for reconfiguration. These differences make it impractical, at best, or impossible, at worst, to apply traditional traffic engineering theory and practices to a tactical network.

In a commercial network there are some times when traffic loads are quite predictable. For example, Mina(80) explains that the busiest hour of the day is usually in late morning, the busiest days of the week are Monday and Friday, the busiest season is Autumn, and the busiest days of the year are Christmas and Mother's Day. No such predictions are applicable in combat.

A basic assumption of commercial traffic engineering is that individual calls are independent except during civil emergencies. Comparable "emergencies" are more the rule than the exception in a combat situation, so the independence assumption is most inappropriate. Holding times of commercial calls are assumed to have a negative exponential distribution; this assumption has some validity for military calls, but the military can enforce shorter holding times and thus truncate the postulated distribution.

Commercial carriers try to balance customer satisfaction and plant expenditures to maximize profit. No

such objective is readily available for a military traffic engineer, so the definition of his goal is imprecise. It is often stated as a deceptively simple goal--"provide the best possible service"--with the specific objectives and constraints to be determined by the engineer.

Commercial traffic studies take place over extended periods of time and accumulate vast quantities of data; the relative stability of commercial operations makes this an efficient procedure and enables the profit maximization to be quite precise. However, lengthy traffic studies in combat would be senseless because of the ever-changing status of the network; system configuration changes would render data obsolete before they could be properly analyzed and acted on to improve service.

Overloads are not common in commercial networks; when they do occur it is for time periods measured in seconds, and such overloads can be handled effectively by practices known as "line load control". However, such was not the case when President F. D. Roosevelt died; the overload precipitated by the announcement of his demise caused the nation-wide dialing system to collapse, and it was several hours before normal service could be restored. Overloads will be common in a military network because calls are not independent during crises and capacities are severely limited. More consideration must be given to maximizing the flow of essential communications and

reducing the overloads by application of control procedures.

There is great routing flexibility in the Bell System long distance network. For example, it would not be impractical or unusual to route a group (12 circuits) or a supergroup (60 Circuits) from New York to Denver to Dallas to Atlanta in order to complete New York - Atlanta calls. Such group routing is not an admissable practice in a tactical network.

There are other reasons why commercial practices are not easily transferred to a military network, but from the differences mentioned above it should be clear why the military networks must be treated as unique cases and not merely as subcases or extensions of commercial networks. This thesis is concerned with exploiting network flow theory to fill gaps where traffic engineering theory and practice are inappropriate.

## CHAPTER III

## NETWORK THEORETICS

There are several communication situations to which graph theoretic and network flow approaches might be applied. This chapter discusses the potentials of shortest path and routing techniques, multiterminal flow techniques, and multicommodity flow algorithms for solving some problems of common user network management. Particular emphasis is placed on controlling the network during periods of overload and under circumstances requiring frequent and rapid network reconfigurations and call redistributions.

In all cases, the common user network will be derived only from circuits remaining after all sole user and special service requirements are considered to be satisfied or unsatisfiable. Initial distributions are considered assuming that some set of initial requirements is available either by dictate of the commander, by application of doctrinal principles, or from analysis of traffic data. Subsequent redistributions will be considered based on the tactical events, the effect of these events on the communication system, and some appropriate revision of the initial requirements.

### Routing Of Calls Through Networks

The common user network can be described by a graph  $N(S,L)$  where  $S = \{s_1, s_2, \dots, s_m\}$  are  $m$  switching centers and  $L = \{l_1, l_2, \dots, l_n\}$  are  $n$  undirected links connecting the switching centers in some configuration. Let  $l_{ij}$  be an undirected link connecting  $s_i$  and  $s_j$  having a circuit capacity  $c_{ij}$ .  $C = [c_{ij}]$  is an  $m \times m$  capacity matrix where  $c_{ij}$  is the capacity of  $l_{ij}$ .  $L = [l_{ij}]$  is an  $m \times m$  incidence matrix where  $l_{ij} = 1$  if  $c_{ij} > 0$  and  $l_{ij} = 0$  if  $c_{ij} = 0 \forall i, j$ .  $C$  and  $L$  are both symmetric matrices; either matrix can be used to determine the configuration of  $N$ . For example, consider the network of Figure 8.

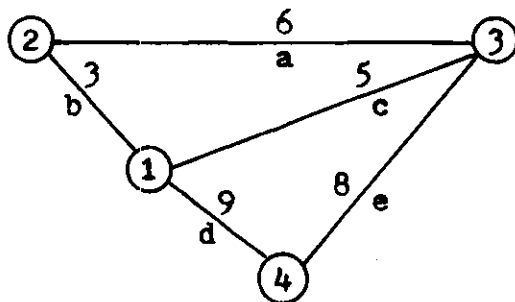


Figure 8. Reference Network

Circles represent switching centers (nodes) and the numbers therein denote the  $i^{\text{th}}$  switching center. The numbers on the links denote the link capacities and the letters are merely another way to represent respective  $l_{ij}$ 's. From a network diagram it is a simple task to construct either the capacity matrix or the incidence matrix. For the

specific network of Figure 8:

$$L = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 3 & 5 & 9 \\ 3 & 0 & 6 & 0 \\ 5 & 6 & 0 & 8 \\ 9 & 0 & 8 & 0 \end{bmatrix}$$

It is easy to determine the shortest routes in simple networks; for the anticipated tactical configurations the solution is trivial. To avoid confusion, routings will always be considered from  $s_i \rightarrow s_j$ ,  $i < j$ ; only the portions of  $L$  and  $C$  above the main diagonal constitute allowable routes regardless of whether the call originates at  $s_i$  or  $s_j$ . Although the links have some specific length (either the length of cable between  $s_i$  and  $s_j$  for a cable/carrier system or the straight-line distance between  $s_i$  and  $s_j$  for a radio/carrier system) we are concerned only with the number of circuits used in a route; therefore, the length of a link can be considered as equal to 1 if the link has a capacity greater than 0. If no link exists between a pair of switching centers, the length of the direct link between them is  $\infty$ .

The Army's largest communication simulation model, SIMCE, uses the following routing doctrine(17):

1. All traffic must be routed over the shortest path; that is, through the minimum number of switching centers.
2. All calls between a pair of switching centers

must follow the same route.

3. When more than one minimum path exists, the most direct path away from the committed forces must be selected.

Presumably, this doctrine might also be used for an actual network. As can be shown, these rules do not allow for optimization or sub-optimization of some criteria which have been adjudged reasonable measures of network efficiency. For the network of Figure 8 the SIMCE doctrine would allow only the following routings:

<u>Pair</u>	<u>Route</u>
1,2	b
1,3	c
1,4	d
2,3	a
2,4	bd
3,4	e

No use is made of capacities which might remain on non-SIMCE routes, and it is likely that some requirements might not be satisfied. Neither the optimal multiflow solutions nor the heuristic flow procedure solutions presented in this chapter are so restrictive as the SIMCE doctrine.

There are, however, distinct disadvantages to allowing calls between node pairs to use all possible routes. Every circuit used in a routing is one less circuit that could be used to form some other routing. Alternate routing, therefore, can become a major factor in causing overloads and the ensuing collapse of the network. In

essence, as the load increases the amount of alternate routing should be decreased commensurate with the amount and the nature (i.e., network-wide or localized) of the overload. This problem is much discussed in the communication literature including the articles by Lewis and Schwenzfeger(71), Laude(69), Burke(8), and Benes(6).

Alternate routing, on the other hand, has distinct advantages if one is considering the vulnerability and survivability of the network as it relates to the exercise of command and control. Considerable research has been conducted to determine what system configurations best meet specified survivability and vulnerability criteria and also what strategies are best for the attacker and defender of a communication system; c.f. Jarvis(61). However, this present thesis is not concerned with developing an optimal communication system except as discussed in Chapter VI. The tactical system configurations are determined more by tactical and geographical considerations and, possibly, by the sole user requirements. We are presently concerned only with the common user network derived from residual assets of whatever communication system is deployed to meet these other criteria.

Referring once again to Figure 8, it is easy to determine all possible routes in the network such that no switching center is used more than once. All possible

routes are enumerated below.

<u>Pair</u>	<u>All Possible Routes</u>
1,2	b, ca, dea
1,3	c, ba, de
1,4	d, ce, bae
2,3	a, bc, bde
2,4	bd, ae, acd, bce
3,4	e, cd, abd

The question of which routes should be allowed is one which must be tempered by the experience of the traffic engineer but, as will be shown later, the use of alternate routes will be necessary in many situations, and the shortest-route-only policy is not necessarily the best. Also to be considered is the fact that audibility and intelligibility are related to the number of links in a route.

For more complicated networks the shortest route and the  $n^{\text{th}}$  shortest route problems may not be so simple, but numerous algorithms exist for these computations. Price(90) describes six such algorithms attributed to other researchers; any of these algorithms and many more found throughout the literature could be used if a network is so complex as to defy route determination by inspection. Since the networks with which this thesis is concerned are simple and all existing links have length = 1, the route determination will be done by inspection; the selection of allowable routes will be discussed later.

An easy check to determine if all two-link routes

have been enumerated can be accomplished using the methods described by Seshu and Reed(97). The entries of  $L^z$  enumerate the number of routes of length  $z$  through the network, both proper and redundant (i.e., intersecting) paths being included. For the network of Figure 8:

$$L^2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

Specifically, the  $l_{ij}^2$  entries show the number of two-link routes between  $s_i$  and  $s_j$ ,  $i \neq j$ , and the  $l_{ii}^2$  entries show the number of links incident at  $s_i$ . The  $l_{ij}^3$ ,  $i \neq j$ , entries of  $L^3$  will show the number of three-link routes between  $s_i$  and  $s_j$ , but these routes may pass through some switching center more than once; this reduces the usefulness of  $L^3$ ,  $L^4$ , etc. except to note that these matrices can be used to determine the number of links in the longest of the shortest routes between all node pairs in the network. That number is the smallest number  $y$  such that

$$A(y) = L + L^2 + \dots + L^y$$

has no zeros in it. For the network of Figure 8,  $bd(2,4)$  is obviously one of the longest of the shortest routes and  $A(2) = L + L^2$  has no zeros in it. In a small network it is usually a trivial task to find the fewest number of links required to connect any pair of nodes; however in a larger

network made up of interconnected smaller networks (such as the scheme depicted in Figure 7) the solution of this problem is anything but trivial and the multiplication of the incidence matrix becomes a valuable technique.

If one examines the matrix  $P = LC$  it is noticed that  $p_{ij}$  gives the number of circuits which are incident to  $s_j$ . Since there are capacity limitations at switching centers, the number of residual circuits incident at any switching center is not necessarily the number of circuits that can be used in the common user network. For the network of Figure 8:

$$P = LC = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 3 & 5 & 9 \\ 3 & 0 & 6 & 0 \\ 5 & 6 & 0 & 8 \\ 9 & 0 & 8 & 0 \end{bmatrix} = \begin{bmatrix} 17 & & & \\ & 9 & & \\ & & 19 & \\ & & & 17 \end{bmatrix}$$

Obviously, if each switching center had a twelve-line switching capacity, the number of available circuits is in excess of the number that can be used in the common user network.

No specific common user call routing doctrine such as the one used for SIMCE is prescribed here. Rather, the policy should depend on the particular situation and the traffic engineer's assessment of all pertinent factors. Surely any policy will consider the obvious principles such as:

1. Shortest routing should be considered first

but not exclusively.

2. Excessive alternate routing can contribute to overloads.

3. No cyclical routing (referred to in commercial jargon as "ring-around-the-rosy") is allowable.

4. Switching capacities must not be exceeded.

It must be noted that routing control is far simpler in manual networks than it is in automatic networks. This situation will add great complexity to the software of the tactical electronic switches unless some technological breakthrough renders present switching concepts obsolete.

#### Multiterminal Flows

This section considers the problem of determining the maximum flow of calls between any node pair in the network, all other nodes being considered as intermediate switching centers. In a tactical situation this might be equivalent to usurping the entire common user network to maximize the calling capacity between two designated nodes and then allowing any leftover capacity to revert to the common user network. Such an event would not be unrealistic under certain emergency situations.

If some minimum multiterminal flow requirements are specified, it is possible to construct a network satisfying those flows. This problem was first solved by Mayeda(78) who dealt only with undirected networks of the

type with which this thesis is concerned. Minimum cost undirected network realizations have been presented by Gomory and Hu(44), Chien(10), and Wing and Chien(111). The synthesis problem has been extended to unsymmetric requirements, negative branch capacities, and pseudo-symmetric networks; Frisch and Sen(41) have published a general algorithm. Realization of a set of multiterminal requirements is not of interest for this thesis because of the restrictive nature of the residual network with which we must deal.

Multiterminal flow analysis is of concern. Given a network, we wish to determine what the multiterminal capacities are and, possibly, to analyze the sensitivity of these capacities to various events affecting the network. A simple multiterminal analysis algorithm was proposed by Gomory and Hu(46). Sensitivity analysis was presented by Elmaghraby(30). It is these presentations which are briefly discussed in the following paragraphs.

In any network there are  $\frac{1}{2}m(m-1)$  possible node pairs. In very simple networks the multiterminal analysis can often be done by inspection. For more complex networks it might appear that  $\frac{1}{2}m(m-1)$  maxflow problems would have to be solved, but that is not the case. Analysis can be accomplished by doing only  $m-1$  maxflow problems, and each of these is done on a network which is probably simpler than the complete network.

Elmaghraby has shown how to analyze the effect of a decrease or increase in the capacity of any link. It is evident that the terminal capacity of any node pair which has a specific arc in its minimal cut set is linearly affected by an increase or decrease in the capacity of that arc. He further shows that there is a set of "critical values" for each specific arc; when these values are reached the terminal capacities of some other pair or pairs which did not initially have that arc in their minimal cut sets are now reduced as that arc finally enters their minimal cut sets. Increases in arc capacity can be analyzed by making the arc capacity somewhat larger than the intended increase and analyzing sensitivity to reduction from this inflated capacity thus determining precisely the effect in the region of the intended link overbuild.

Multiterminal analysis techniques are demonstrated by simple examples in Appendix B.

#### Multicommodity Flows

The multicommodity maximum flow problem is stated: given a network with specified link capacities, a set of source nodes  $\{a, b, c, \dots\}$ , and a set of sink nodes  $\{a', b', c', \dots\}$ , what is the maximum total flow of commodities in the network where commodity A has source a and sink a', commodity B has source b and sink b', etc., and

all commodities flow simultaneously.

A common user telephone network is actually a multicommodity flow situation where  $C = [c_{ij}]$  is the capacity matrix defined previously and  $R = [r_{ij}]$  will be a requirements matrix where  $r_{ij}$  expresses, in some manner, the requirements between  $s_i$  and  $s_j$ . All  $r_{ij}$  must be met simultaneously to satisfy all users; the units of  $r_{ij}$  will be discussed in the next section.

Ford and Fulkerson(34) presented the seminal paper on multicommodity flows in which they formulated the arc-chain solution. Tomlin(104) showed that the node-arc formulation of the same problem is equivalent to the arc-chain formulation within a change of variable; Jarvis(62) proved that the node-arc and arc-chain formulations are completely equivalent. Other papers of interest in the general multicommodity flow area are those of Hakimi(51), Tang(100), and Hu(57). Rothschild and Whinston(94) presented a max-flow min-cut theorem for two-commodity networks. Rothfarb and Frisch(93) provide a solution for three-commodity flow problems in networks with no internal nodes. Tang(101) and Onaga, Kakuso and Kato(84) provide algorithms for multicommodity flow in a restricted class of bipath networks.

There is no general optimal multicommodity flow assignment algorithm. Problems with more than three

commodities must usually be solved by linear programming techniques. The problem can be stated as(65):

$$\begin{array}{llll}
 \text{MAX} & \sum f_{ijj} & & \\
 \text{ST} & \sum_m f_{imj} \leq r_{ij} & \text{Demand} & \\
 & \sum_k f_{ijk} \leq c_{ij} & \text{Link Capacity} & \\
 & \left. \begin{array}{l} \sum_{j,k} f_{ijk} \leq s_i \\ \sum_{j,k} f_{kij} \leq s_i \end{array} \right\} & \text{Switching Capacity} & \\
 & f_{ijk} \geq 0 & \forall i,j,k & 
 \end{array}$$

In an ideal common user network it is possible for any user to call any other user; presumably there is a known non-negative requirement between each pair of potential users. In general, a network containing  $p$  users could generate a set of  $p(p-1)$  requirements. Since it takes two users to complete a call there are, conceivably,  $\frac{1}{2}p(p-1)$  possible simultaneous requirements. We consider all calls flowing from  $s_i$  to  $s_j$ ,  $i < j$ , regardless of where the call originates. Thus  $P = [p_{ij}]$  is a  $p \times p$  symmetrical matrix where  $p_{ij}$  is the calling requirement between the  $i^{\text{th}}$  and  $j^{\text{th}}$  individual users. Since  $P$  is symmetrical only the requirements above the main diagonal need be considered.

Rather than deal with  $p$  users, it is convenient

to consolidate their individual requirements at the  $m$  switching centers. This allows us to use  $R = [r_{ij}]$  as the basis for engineering the network;  $R$  is an  $m \times m$  matrix. We are now faced with a multicommodity flow problem involving  $\frac{1}{2}m(m-1)$  commodities. Thus a two-node network has but a single commodity and its solution is trivial. The networks pictured as examples in Chapter II are very small but the four-node network has six commodities; the five-node network has ten commodities; and the sixteen-node network has 120 commodities. Clearly no heuristic flow procedure will provide an optimal solution in such situations.

The linear programming formulations are also quite formidable. Juncosa and Kalaba(64) state that there are about  $2m^2$  constraints and almost  $m^3$  variables for the usual problems. Thus a four-node network might have 32 constraints and nearly 64 variables; the five-node network might have 50 constraints and almost 125 variables; and the sixteen-node network might have 512 constraints and almost 4096 variables. Even the smallest network problems are not solved easily, and the larger problems are solvable only by large-capacity computer programs. There is no apparent technique for initially configuring a constrained common user network--at least there is no technique which is easy enough for tactical employment. There is certainly no simple technique for

making the rapid redistribution decisions which will be required in an operational environment, and there is no obvious way of using multicommodity flow techniques during the anticipated periods of extreme overloads. It is these particular problems which are addressed next.

#### Applicable Multiflow Techniques

We now analyze the common user network with the intent of determining how to maximize some measure of calling capacity given a set of requirements and a set of link capacities. The requirements could be expressed as either for simultaneous calls at any point in time or for calls during some interval of time.

When expressed as a requirement at a point in time we, in effect, develop a configuration which would result from providing high priority users with a preemption capability ensuring completion of specified calls under all conditions including overload. If so expressed, we will be able to specify which requirements will be satisfied and what routing each call will take at such time as all of the preemption capabilities are executed.

When expressed as requirements for some interval of time we will be able to specify the number of calls to be allowed between nodes during the given interval and the routings the calls will take. The minimum number of calls to be handled will depend on the enforceable holding

time; if the holding time is specified to be as long as the interval of concern we are solving the same problem as for the point-in-time requirement.

To change from a point-in-time problem to a time interval problem the link capacity matrix  $C = [c_{ij}]$  must be multiplied by the scalar  $I/H$  where  $I$  is the time interval and  $H$  is the holding time, both quantities being expressed in the same units.

These formulations are equally valid depending on the situation which is being considered. The point-in-time requirements might be applicable during the initial period of overload (e.g. immediately following the initiation of a tactical nuclear exchange) while the interval requirements might be applicable during prolonged periods of known overload (e.g. during several hours following the cessation of the same weapons exchange). Both problems can be solved in exactly the same way; it can be shown that the solutions are equivalent and can be directly converted from one situation to the other. In practice it would be necessary to specify whether the capacity and requirements matrices are for the point-in-time or time interval formulation; the units of both capacity and requirements are generally left out in this thesis. As a result of the maxflow computation we will provide a matrix showing which requirements can be distributed through the network. We now consider how the maxflow calculations might be accomplished.

### Optimal Methods

The only known optimal method for networks of any appreciable size and having several commodities is linear programming. One general formulation was presented earlier in this chapter and was shown to be of formidable size. For small networks such as those with which this thesis is concerned, optimal solutions can be calculated and used as bases for comparing the efficiencies of sub-optimal solutions. This is done to some extent in Appendix A.

### Route Restriction

It is possible to reduce the size of the linear programming formulation by specifying restrictions on the routes that can be used to complete calls. For example, the network of Figure 6 has 73 possible routes distributed by length as follows:

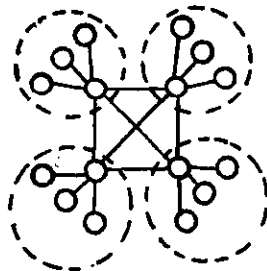
<u>Links Per Route</u>	<u>Number Of Routes</u>
1	8
2	19
3	28
4	18

Restricting the number of links used in any route to some limit (e.g. one, two or three for the network of Figure 6) should provide a sub-optimal solution with the condition of optimality being approached as longer routes are permitted. A route restriction decision will also make for easier implementation of a routing control scheme. Route restricted

solutions are demonstrated and compared to some optimal solutions in Appendix A.

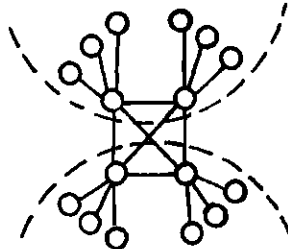
### Partitioning

It is possible to get sub-optimal solutions by partitioning a network. Using the sixteen-node network there are several partitioning schemes that deserve consideration. If traffic is generally local it might be logical to partition as shown below:



Solve the intra-area problems for each of the encircled subnetworks; then solve the inter-area maxflow problem which involves the tandem switching nodes and the links connecting them.

It might be reasonable to partition the network into forward and rearward elements as shown in the following drawing:



Another approach would be to solve the inter-area maxflow problem first and then solve the intra-area problems for each subnetwork. This would require the assumption that longer-distanced calls were of more import than local area calls; such a community-of-interest determination is unlikely in a combat situation. For a capacitated network with overloading requirements, the order in which the inter-area and intra-area problems are solved will greatly affect the maximum traffic distribution.

#### Bipath Networks

Onaga, Kakusko and Kato(85) have presented an algorithm for multicommodity flows in bipath networks; the bipath network was first introduced by Tang(101) who solved the problem of multiflow by linear programming. An undirected network is called bipath if there are no more than two properly disjoint routes between any pair of nodes in the network; a route is properly disjoint if it has at least two links and all intermediate nodes on these routes are distinct.

As an example of how this bipath property might be exploited in the sixteen-node network, consider only the problem of maximizing flow in the tandem portion of the network. There are four nodes and six links (a fully-connected graph) and there are six commodities. This portion of the network is usually drawn as in the diagram which appears at the top of the following page.



The links between source and sink pairs will have the same capacities as the links of the original network; all other constructed links will have infinite capacity. The network shown in the third diagram on the preceding page has the bipath property and the bipath algorithm can be applied to give an optimal solution.

### Heuristic Flow Procedure

A very easy heuristic procedure can be shown to provide sub-optimal but generally close-to-optimal multicommodity flows for networks with which we are concerned. This procedure has been adapted specifically for the common user network from a method applied to directed teletypewriter networks by Pollack and Wallace(89).

1. Let  $r_{ij}$  be the requirement between  $s_i$  and  $s_j$ , and let  $c_k$  be the capacity of route  $k$  between  $s_i$  and  $s_j$ . Let  $A = \text{Min}(r_{ij}, c_k)$ . When  $A$  calls are assigned to route  $c_k$  the unassigned requirement is equal to  $r_{ij} - A$  and the remaining capacity of each link  $l_{vw}$  in route  $k$  is  $c_{vw} - A$ .

2. Starting with  $a = 1$ , a requirement is never assigned to a route of length  $a + 1$  if any requirements can be assigned to an  $a$ -link route.

3. If more than one  $a$ -link route is available for assignment, the maximal capacity route is chosen.

4. Any requirement can be assigned to more than one route; a route can be of any length.

5. Once a partial requirement has been assigned

to a route of length  $a$ , the remainder of the requirement must remain unassigned until every other requirement has had an opportunity to be assigned.

6. Once a requirement has been assigned, the next requirement chosen for assignment should originate from and be destined for another pair of stations.

Results of this procedure are shown in Appendix A, and the efficiency is compared to those of the optimal and sub-optimal linear programming solutions of the same problems for a very restricted size network with capacities and requirements randomly generated over specific ranges.

## CHAPTER IV

## REVISION OF COMMON USER REQUIREMENTS

The most difficult problems encountered by a military traffic engineer will be caused by the requirements and capacities changing at the same time due, usually, to some untoward event such as a switch failure, nuclear attack, etc.. This chapter discusses the events which will cause the traffic engineer to reassess the network and, possibly, reconfigure the network or realign the distribution of calls. A technique for estimating the revised requirements is presented along with a proof of the formula used in the estimation procedure.

Changing Network Characteristics

The network characteristics which are subject to change are link capacities and the number of nodes in the network. Link capacity changes may come about for a variety of reasons, some of which are:

1. Link failure, in which case the link capacity is reduced to zero. This may be temporary or permanent.
2. Circuit failure, in which case the link capacity is reduced by the number of circuits which fail; this is more likely to be temporary than permanent.
3. Conversion of circuits to dedicated use, in

which case the link capacity is reduced by the number of circuits converted; this is more likely to be permanent than temporary.

4. Release of dedicated circuits, which is the converse of the preceding reason.

5. Link overbuilds, in which case additional circuits will be made available providing the entire capacity of the overbuild is not allocated for sole user or special services.

6. Node relocations, in which case present links will be eliminated and new links will be created. This situation usually results when switching centers displace in support of changing troop dispositions.

The number of nodes in a network may change. Reduction can occur either by the destruction of a switching center or by the removal of a switching center to another network. In either case the elimination of a node can be treated as a change in link capacities by reducing the capacity of all incident links to zero. An increase in the number of nodes might be less common because entire switching centers are not kept in reserve. However, the displacement of a switching center from another network might be contemplated to improve service in a second network. Also the combining of two networks as forces are reconstituted after suffering losses could be treated as the addition of switching centers to one or the other of the

networks being combined.

As this research was started it was expected that some easy way would be found to handle these link capacity and network reconfiguration changes. For those changes involving a small number of circuits the required redistributions might be obvious, but for changes involving a large number of circuits or multiple links the required redistributions are anything but obvious.

There is apparently no recourse other than constructing a new capacity matrix and solving anew the call distribution problem. If a linear programming formulation is being used this will mean changing the right-hand sides of the capacity constraints or adding new link capacity constraints for each new link introduced. It will also mean reenumerating the routes that calls between node pairs might use and correcting the objective function, the requirements constraints, and the capacity constraints. This is tantamount to solving a new network problem rather than doing a sensitivity analysis of the previous network solution.

The multiterminal flow solution is also sensitive to link capacity changes and the addition or deletion of switching centers. The multiterminal analysis must be redone when changes take place because, as is shown in Appendix B, any change which is such that it encounters a critical value of the affected link or links will change

the terminal capacity of all node pairs for which that link or links are in the minimal cut set.

The assessment that, in general, the call distribution problem must be completely resolved when link capacity or node additions take place or when link capacity decreases occur (noting that a node deletion can be considered as the reduction to zero of the capacity of all incident links) is somewhat disappointing. A less-involved solution was anticipated but has not yet been found.

#### Estimation Of Requirements Changes

As drastic events take place the traffic engineer needs a rapid method for estimating the changing requirements. The following method is suggested as a simple way of estimating requirements given only the original requirements set and knowing the nature of the events. The requirements matrix  $R_k = [r_{ij}]$  is defined as before but the matrix is subscripted with  $k = 0, 1, 2, \dots$  merely to denote the sequence in which the matrices are generated as the situation changes.

Let  $M = [m_{ij}]$  be a  $q \times m$  manipulation matrix where  $m$  is the number of nodes in the network prior to the event and  $q$  is the number of nodes in the network after the event. If  $m=q$  and no redistribution of traffic occurs as a result of the event,  $M=I$  (the identity matrix). If a change of requirements occurs at  $s_i$ ,  $m_{ii}$  of  $I$  is

changed to reflect the fraction of the present traffic which will be required after the reconfiguration; the  $m_{ij}$ 's are changed to reflect the portion of the requirements at  $s_i$  which will be transferred to  $s_j$  after the reconfiguration. After each  $s_i$ ,  $i = 1, \dots, m$ , has been so treated, all rows and columns which have no positive entries are deleted. The resultant  $q \times m$  matrix is called the manipulation matrix.

Further discussion in the next section proves that the requirements after some event can be estimated by the formula:

$$R_{k+1} = M R_k M^t$$

A series of examples is now used to demonstrate the method.

Let  $R_0$  be the requirements matrix for some four-node network.

$$R_0 = \begin{array}{c} \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \end{array} \begin{array}{c} \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \\ \left[ \begin{array}{cccc} 0 & 3 & 3 & 2 \\ 3 & 0 & 3 & 2 \\ 3 & 3 & 0 & 3 \\ 2 & 2 & 3 & 0 \end{array} \right] \end{array}$$

Destruction of  $s_2$  and all served units will eliminate all requirements to and from  $s_2$  which amounts to eliminating the second row and the second column of  $R_0$ . To show how the same result is obtained using the formula to generate  $R_1$  we first construct a manipulation matrix.

$$M = \begin{array}{c} s_1 \\ s_3 \\ s_4 \end{array} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \\ \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \end{array}$$

Then  $R_1 = M R_0 M^t$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 3 & 3 & 2 \\ 3 & 0 & 3 & 2 \\ 3 & 3 & 0 & 3 \\ 2 & 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{array}{c} s_1 \\ s_3 \\ s_4 \end{array} \begin{array}{ccc} s_1 & s_3 & s_4 \\ \left[ \begin{array}{ccc} 0 & 3 & 2 \\ 3 & 0 & 3 \\ 2 & 3 & 0 \end{array} \right] \end{array}$$

Suppose that instead of completely destroying all served units at  $s_2$ , half of them survive and rehome on  $s_4$  for service. Then:

$$M = \begin{array}{c} s_1 \\ s_3 \\ s_4 \end{array} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \\ \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{array} \right] \end{array}$$

Now:

$$R_1 = M R_0 M^t = \begin{array}{c} s_1 \\ s_3 \\ s_4 \end{array} \begin{array}{ccc} s_1 & s_3 & s_4 \\ \left[ \begin{array}{ccc} 0 & 3 & 3\frac{1}{2} \\ 3 & 0 & 4\frac{1}{2} \\ 3\frac{1}{2} & 4\frac{1}{2} & 0 \end{array} \right] \end{array}$$

A requirement for a fraction of a call is not meaningful, and the traffic engineer will adjust such requirements upward or downward based on his knowledge of the situation, prior experience and, possibly, intuition. The requirements shown in  $R_1$  above might be adjusted to the following:

$$R_1 = \begin{bmatrix} 0 & 3 & 4 \\ 3 & 0 & 5 \\ 4 & 5 & 0 \end{bmatrix}$$

Suppose now that a new switching center  $s_5$  is being made available and  $2/3$  of the units now being served by  $s_3$  will be moved to  $s_5$  with the other  $1/3$  remaining at  $s_3$ . Then:

$$M = \begin{array}{c} \begin{matrix} & s_1 & s_3 & s_4 \\ s_1 & \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\ s_3 & \begin{bmatrix} 0 & 1/3 & 0 \end{bmatrix} \\ s_4 & \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \\ s_5 & \begin{bmatrix} 0 & 2/3 & 0 \end{bmatrix} \end{matrix} \end{array}$$

Now:

$$R_2 = \begin{array}{c} \begin{matrix} & s_1 & s_3 & s_4 & s_5 \\ s_1 & \begin{bmatrix} 0 & 1 & 4 & 2 \end{bmatrix} \\ s_3 & \begin{bmatrix} 1 & 0 & 5/3 & 2/3 \end{bmatrix} \\ s_4 & \begin{bmatrix} 4 & 5/3 & 0 & 10/3 \end{bmatrix} \\ s_5 & \begin{bmatrix} 2 & 2/3 & 10/3 & 0 \end{bmatrix} \end{matrix} \end{array}$$

Or, to the next-higher integer value:

$$R_2 = \begin{bmatrix} 0 & 1 & 4 & 2 \\ 1 & 0 & 2 & 1 \\ 4 & 2 & 0 & 4 \\ 2 & 1 & 4 & 0 \end{bmatrix}$$

Suppose now that  $s_5$  is serving three units and that  $s_1$  is serving two units. One unit from  $s_1$  is being dispatched to  $s_5$ . If the unit being dispatched can be considered as taking its requirements  $r_{1j}$  with it, we can form the manipulation matrix shown at the top of the following page.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

This will give the new requirements matrix:

$$R_3 = M R_2 M^t = \begin{bmatrix} 0 & 1 & 4 & 3 \\ 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 5 \\ 3 & 1 & 5 & 0 \end{bmatrix}$$

If the unit being dispatched can be considered as assuming requirements equivalent to those of the units presently at  $s_5$ , we have:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4/3 \end{bmatrix}$$

To the next-higher integer values, the new requirements matrix for this variation is:

$$R_3 = M R_2 M^t = \begin{bmatrix} 0 & 1 & 4 & 3 \\ 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 6 \\ 3 & 1 & 6 & 0 \end{bmatrix}$$

Notice that the two  $R_3$  matrices differ only in  $r_{34}$  even though the requirements were shifted according to different assumptions. Depending on the assumptions and the  $R_k$  matrix values, the  $R_{k+1}$  matrix values may differ greatly, be very much the same, or fall somewhere between the two extremes.

If the actual requirements generated by the

movement of units or the occurrence of untoward events are known with certainty, the requirements matrices should be adjusted accordingly. The method demonstrated here is an approximation, albeit a reasonable one, and it involves a very simple matrix multiplication. Appendix C shows a program written in BASIC computer language and operating in an interactive mode. This program will solve the new requirements matrices by interrogating the computer terminal and having the operator input the required information as to network size and present requirements. Also included in Appendix C is the computer-generated solution to the last problem used as an example in this section.

#### Proof of Matrix Revision Technique

##### Statement of the Formula

Calls are defined between nodes  $s_i$  and  $s_j$  only for  $i < j$  regardless of which node initiates the call.

Let  $R_k = [r_{ij}]$  be a symmetric  $n \times n$  requirements matrix for the present network having  $n$  nodes where  $r_{ij}$  are the requirements between  $s_i$  and  $s_j$  for  $i=1, \dots, n-1$ ;  $j=2, \dots, n$ ;  $r_{ii} = 0$ ; and  $r_{ij} = r_{ji}$ . (Note that the total requirements between  $s_i$  and  $s_j$  are  $r_{ij} = r_{ji}$  and not the sum of  $r_{ij}$  and  $r_{ji}$ . The portion of the matrix below the main diagonal is constructed only to facilitate the calculations which follow.)

Let  $M = [m_{ij}]$  be a  $q \times n$  manipulation matrix where

q is the number of post-reconfiguration nodes and n is the number of pre-reconfiguration nodes;  $m_{ij}$  is the fraction of the pre-reconfiguration requirements generators at  $s_j$  which will be transferred to  $s_i$  as post-reconfiguration requirements generators. It is once again explained that individual user-to-user requirements are consolidated to form the node-to-node requirements matrices. It will be assumed that transferring a fraction of the requirements generators will also transfer the same fraction of the requirements. The revised requirements will then be the expected values of the transferred requirements if no information about specific generators is known.

Using the matrices defined above and accepting the assumption about the transfer of requirements generators, the formula

$$R_{k+1} = M R_k M^t$$

will provide a reasonable estimate of post-reconfiguration requirements.

#### Proof of Formula

The formula is proven by logically explaining the transfer of requirements generators and then showing that the same answer can be obtained by multiplying matrices; in fact, the entire revised requirements set is generated in the process.

The requirements between any node pair after

reconfiguration are functions of the pre-reconfiguration requirements and the redistribution of the requirements generators. Let:

$\bar{r}_{ij}$  = post-reconfiguration requirements between  $s_i$  and  $s_j$ .

$r_{ij}$  = pre-reconfiguration requirements between  $s_i$  and  $s_j$ .

$m_{ij}$  = fraction of  $s_j$  requirements generators transferred to  $s_i$  (which, by assumption, is also the fraction of requirements transferred).

Then:

$m_{ib}m_{ja}$  = fraction of  $r_{ab}$  which will be required between  $s_i$  and  $s_j$  due to the transfer of generators from  $s_b$  to  $s_i$  and the transfer of generators from  $s_a$  to  $s_j$ .

Similarly:

$m_{ia}m_{jb}$  = fraction of  $r_{ab}$  which will be required between  $s_i$  and  $s_j$  due to the transfer of generators from  $s_a$  to  $s_i$  and the transfer of generators from  $s_b$  to  $s_j$ .

Thus:

$m_{ib}m_{ja} + m_{ia}m_{jb}$  = fraction of  $r_{ab}$  which will be required between  $s_i$  and  $s_j$  due to all transfers from  $s_a$  and  $s_b$  to  $s_i$  and  $s_j$ .

Hence:  $r_{ab}(m_{ib}m_{ja} + m_{ia}m_{jb})$  is the post-reconfiguration requirement between  $s_i$  and  $s_j$  attributable only to the pre-reconfiguration requirements between  $s_a$  and  $s_b$  and the

subsequent transfer of some fraction of those generators to  $s_i$  and  $s_j$ . The post-reconfiguration requirements attributable to the pre-reconfiguration requirements of every node pair in the network and the subsequent transfer of requirements generators can be similarly calculated. The total post-reconfiguration requirement  $r_{ij}$  is:

$$\bar{r}_{ij} = \sum_{a=1}^{n-1} \sum_{b=2}^n r_{ab} (m_{ib} m_{ja} + m_{ia} m_{jb})$$

$$\forall a, b \Rightarrow a < b; i=1, \dots, n-1; j=2, \dots, n; i < j.$$

It is now shown that the entries of  $R_{k+1}$  can be obtained simultaneously by a matrix multiplication technique. From the definition of the requirements matrix we see that it can be displayed in expanded form as:

$$R_k = \begin{bmatrix} 0 & r_{12} & r_{13} & \dots & r_{1,n-1} & r_{1n} \\ r_{12} & 0 & r_{23} & \dots & r_{2,n-1} & r_{2n} \\ r_{13} & r_{23} & 0 & \dots & r_{3,n-1} & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{1,n-1} & r_{2,n-1} & \dots & \dots & 0 & r_{n-1,n} \\ r_{1n} & r_{2n} & \dots & \dots & r_{n-1,n} & 0 \end{bmatrix}$$

From the definition of the manipulation matrix we see that it can be displayed in expanded form as:

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & \cdots & m_{1n} \\ m_{21} & m_{22} & m_{23} & \cdots & m_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{q1} & m_{q2} & m_{q3} & \cdots & m_{qn} \end{bmatrix}$$

Note that  $M$  is not necessarily a square matrix. If there is a net loss of nodes,  $q < n$ ; conversely, if there is a net gain of nodes,  $q > n$ .  $M$  can be formed first as a square matrix and refined by deleting rows or columns which have no positive entries.

We now form a  $q \times q$  matrix according to the formula:

$$R_{k+1} = M R_k M^t$$

$$\begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{q1} & m_{q2} & \cdots & m_{qn} \end{bmatrix} \begin{bmatrix} 0 & r_{12} & \cdots & r_{1n} \\ r_{12} & 0 & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{1n} & r_{2n} & \cdots & 0 \end{bmatrix} \begin{bmatrix} m_{11} & \cdots & m_{q1} \\ m_{12} & \cdots & m_{q2} \\ \vdots & \ddots & \vdots \\ m_{1n} & \cdots & m_{qn} \end{bmatrix}$$

The matrix multiplications required to calculate a single element of  $R_{k+1}$  are now demonstrated.

$$MR = \begin{bmatrix} m_{i1} & m_{i2} & \cdots & m_{i,n-1} & m_{in} \end{bmatrix} \begin{bmatrix} 0 & r_{12} \cdots r_{1,n-1} & r_{1n} \\ r_{12} & 0 & \cdots & r_{2,n-1} & r_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{1n} & r_{2n} \cdots r_{n,n-1} & 0 \end{bmatrix}$$

This generates as a general row of the matrix  $MR_k$ :

$$\left[ \sum_{a=1}^n r_{1a} m_{ia} \quad \sum_{a=1}^n r_{2a} m_{ia} \quad \dots \quad \sum_{a=1}^n r_{na} m_{ia} \right]$$

Now multiplying this general row by a general column of the matrix  $M^t$  we have:

$$\left[ MR_k \right] \begin{bmatrix} m_{j1} \\ m_{j2} \\ m_{j3} \\ \vdots \\ m_{j,n-1} \\ m_{jn} \end{bmatrix}$$

This gives the general element:

$$\begin{aligned} \overline{r_{ij}} &= \sum_{a=1}^n r_{1a} m_{ia} m_{j1} + \sum_{a=1}^n r_{2a} m_{ia} m_{j2} + \dots + \sum_{a=1}^n r_{na} m_{ia} m_{jn} \\ &= \sum_{b=1}^n \sum_{a=1}^n r_{ab} m_{ia} m_{jb} \end{aligned}$$

Now the elements  $r_{ii}$  on the main diagonal are known to be zero for this formulation; see the following section for an interpretation and extension. We also know that the portion of the matrix below the diagonal was created as an

artifice to render the calculations easier. We are interested, therefore, only in the new node-pair requirements where  $i < j$  and old node-pair requirements where  $a < b$ ; this is made possible by defining all calls as being from  $s_i$  to  $s_j$  with  $i < j$ .

Looking once again at the general element

$$\sum_{a=1}^n \sum_{b=1}^n r_{ab} m_{ia} m_{jb}$$

and applying the noted restrictions on the directions of call flows where  $i < j$  and  $a < b$ , we have:

$$\bar{r}_{ij} = \sum_{a=1}^n \sum_{b=a+1}^n r_{ab} (m_{ia} m_{jb} + m_{ib} m_{ja})$$

for  $i = 1, \dots, n-1$ ;  $j = 2, \dots, n$ ;  $i < j$ .

This is precisely the same result obtained earlier by logically generating a general post-reconfiguration requirement. Hence, the matrix formula  $R_{k+1} = M R_k M^t$  has been proved to be a method of calculating all post-reconfiguration requirements simultaneously. QED.

### Extension

Non-zero entries on the main diagonal of  $R_{k+1}$  can be interpreted as a measure of the added switching capacity

required at  $s_i$  after reconfiguration due to generators from both nodes of a node-pair being transferred to the same node. For normal trunking problems the non-zero main diagonal entries can be changed to  $r_{ii} = 0$  for every switching center  $s_i$  which is in the reconfigured network.

If the intra-node requirements are inserted on the main diagonal of  $R_k$  in place of  $r_{ii} = 0$ , the main diagonal of  $R_{k+1}$  will reflect the total post-reconfiguration intra-node requirements for each node  $s_i$ . Then:

$$\bar{r}_{ij} = \sum_{a=1}^n \sum_{b=a}^n r_{ab}^{(m_{ia} m_{jb})}$$

$$\text{for } i = 1, \dots, n; \quad j = 1, \dots, n; \quad i \leq j.$$

This formulation will give the inter-node and intra-node requirements for every node in the network; this extension could be used when there is concern about the capability of the switching centers to handle the total load offered after a reconfiguration.

## CHAPTER V

## A PROCEDURE FOR DEVELOPING OVERLOAD CONTROL PLAN INPUTS

The purpose of this chapter is to describe a procedure for using network theoretic techniques to solve common user maxflow communication problems; the eventual output of the procedure is an overload control plan for each identified network. Mention is also made of network and graph theoretic techniques which can be applied to answer other questions concerning the connectivity and capacity of networks.

In Appendix D a short demonstration is presented by applying the procedure to a small segment of a hypothetical network supporting a plausible scenario. It is emphasized that the many sub-networks cannot be individually engineered without accounting for inter-net requirements and capacities.

The procedure is presented as a brief description of the events, actions, and information flows depicted in Figure 9. The procedure will provide input to the overload control plan for each sub-network; this control plan should be of assistance in preventing serious degradation of service and the eventual collapse of the network due to overload.

Communication system configuration changes are not developed because the intention is to make the best common

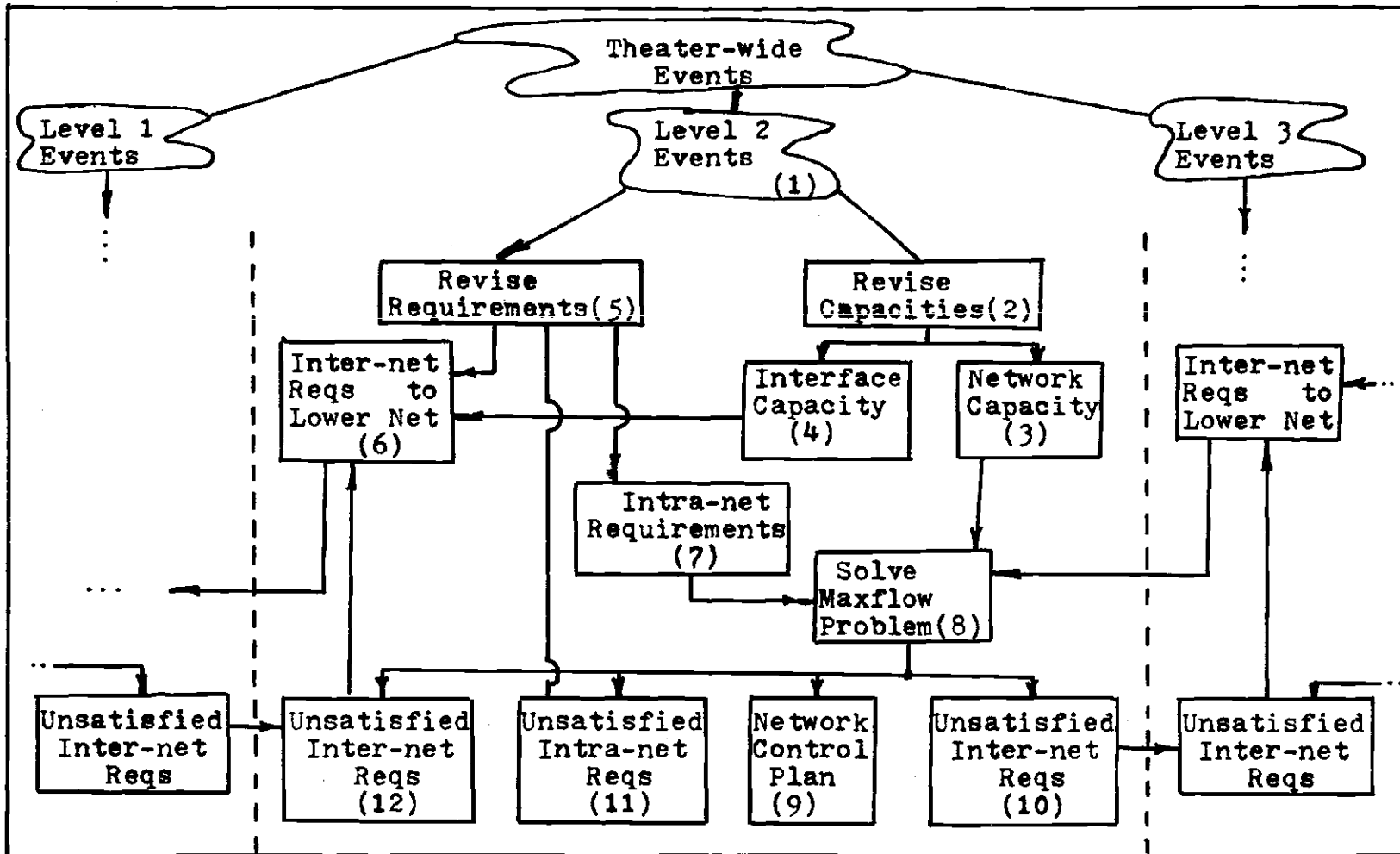


Figure 9. Call Distribution Procedure

user utilization of circuits which are not required for other purposes. However, Chapter VI hypothesizes an extension of this procedure which might determine optimal system configurations.

#### Distribution Of Calling Load

This section is but the briefest possible description of the proposed procedure. If such a procedure is ever executed, very detailed plans would be required. The procedure is depicted in Figure 9. Level 1 is simply the lowest order network (i.e., a network which has no interface with a subordinate network). Level 2 is a network which is interfaced with both higher- and lower-order networks; there might be successive Level 2 networks. Level 3 would be a network which is interfaced with no networks of higher order. Since Level 2 contains both possible interfaces, the description of the procedural steps is keyed to the numbered blocks of Level 2 in Figure 9. The numbers do not imply that the steps are accomplished in numerical order.

**1-Events:** An event may have theater-wide effects or impact only on the sub-network in question. Any occurrence which changes network capacities or user requirements must be considered to have an effect on the distribution of common user calls in the network. Events include but are not limited to: circuit failures, link failures,

node failures, node destructions, unit destructions, tactical redeployments, system overbuilds, changes in sole user requirements, and mission changes.

2-Revise Capacities: As events occur the network capacity matrix must be revised continually.

3-Network Capacity: This refers to the common user capacity on links connecting the nodes of the network.

4-Interface Capacity: This refers to common user capacities on links connecting nodes of two different networks.

5-Revise Requirements: Intra-net and inter-net requirements must be identified separately, then combined for solving the maxflow problem, and then must be re-separated for analysis.

6-Inter-net Requirements to Lower Net: These are determined by the higher-order network, revised as necessary, and communicated to the lower-order network. In no case should the requirement to the low-order network exceed the known capacity of the interface.

7-Intra-net Requirements: These are requirements between node pairs of the network in question.

The total requirements matrix is the sum of the intra-net requirements matrix and the inter-net requirements matrices; there is an inter-net requirements matrix for each lower-order and higher-order network with which the network in question is being interfaced. A method of

estimating revised requirements matrices was presented in Chapter IV.

8-Solve Maxflow Problem: The maxflow problem is solved by whatever methods are available and suitable. If the optimal solution cannot be calculated, the heuristic flow procedure can be used to generate a probably-suboptimal solution.

9-Network Control Plan: A listing of satisfiable and unsatisfiable requirements will enable the controller to develop an overload control plan.

10-Unsatisfied Inter-net Requirements: The unsatisfied requirements generated from the higher-order networks are transmitted back to that level for reassessment of their inter-net requirements and change of their inter-net requirements matrices.

11-Unsatisfied Intra-net Requirements: The unsatisfied requirements generated by intra-net users are used to reassess the intra-net requirements and change the intra-net requirements matrix.

12-Unsatisfied Inter-net Requirements: The inter-net requirements which cannot be satisfied in one or both networks are used to reassess the inter-net requirements and change the inter-net requirements matrices.

It may be necessary to generate the inter-net requirements and the several inter-net requirements matrices

iteratively before an acceptable maxflow solution is found for both higher- and lower-order networks. The problem can be further complicated if there are multiple interfaces between a pair of networks. Although complicated, multiple interface problems could be solved; however, no such solutions are presented here.

#### Other Applicable Techniques

Routing techniques were discussed in Chapter III. If necessary, such techniques could be used to determine the maximal or minimal capacity route between two nodes, the fewest or greatest number of links between two nodes, all possible routes between two nodes, the  $n^{\text{th}}$  shortest route between two nodes, etc..

The multiterminal flow techniques discussed in Chapter III and Appendix B will provide information about maximum calling capacity between all pairs of nodes when, for a given pair, all other nodes are used only for intermediary purposes. The multiterminal analysis problem can be solved when it is necessary for the common user network to revert to command and control utilization.

The slack variable values in the final tableau of a maxflow linear programming solution can be used to determine the "strong" and "weak" links of the network. Such information could impact on system overbuilds and troop disposition decisions. The dual variables, which

measure the rate of change of the maxflow with respect to unit changes in the right-hand-side of the constraints, can also be used to assist in determining where system overbuilds can best be used.

### Intuitive Assessments

Requirements can be generated either by subjective means or by measuring actual attempts to use the system. Regardless of what methods are used, the intuition of traffic engineers, commanders, and users must also be considered when determining which requirement set will be satisfied. Some aspects of communication service cannot be quantified, so even detailed procedures must provide latitude to account for the execution of decisions which are not based solely on the results provided by a mathematical model of the network.

## CHAPTER VI

EXTENSION OF PROCEDURE TO DETERMINE  
OPTIMAL SYSTEM CONFIGURATIONS

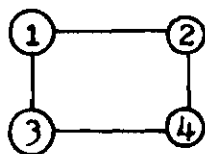
In Chapter I the problems of deriving a common user network from the residual assets of a communication system were discussed. The configuration of the common user network is determined by the system configuration and the requirements for sole user and special services. The common user network configuration is not likely to be optimal, and the "reconfiguration" procedure which has been developed herein is, more correctly, only a "redistribution" procedure which will prevent overloads and the collapse of networks during crisis periods.

Optimal Configurations

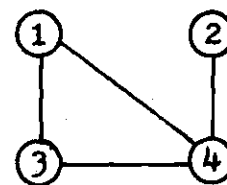
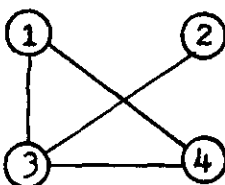
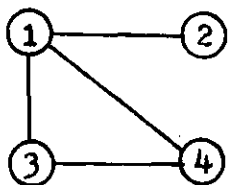
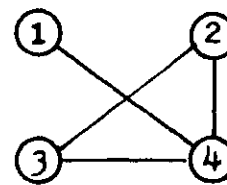
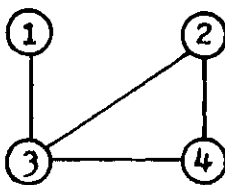
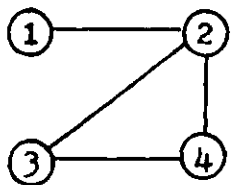
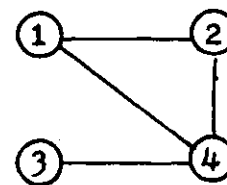
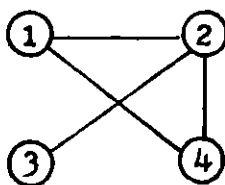
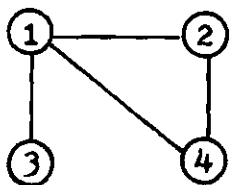
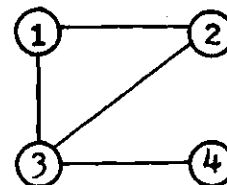
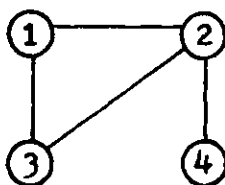
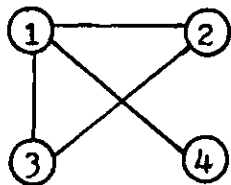
The basic communication system configurations from which all sole user, special, and common user circuits are derived may be far from optimal. The common user distribution procedure developed in this thesis might be used as the basis for developing optimal communication system configurations which would, in turn, improve the common user network configurations and allow for distributing more calls in that network. This section briefly hypothesizes how the procedure could be extended to

provide system configuration recommendations.

Suppose a network or a sub-network of a larger entity is configured as follows:



There is no assurance that this configuration is, in any sense, optimal. There are twelve other four-node, four-link configurations, any one of which might be "better".



Also, a single interface with another network could be at any one of the four nodes making a total of 52 possible interfaced network configurations.

If all tactical and practical considerations were set aside and the links could be established in any configuration, the best of the 52 configurations could be determined in the following manner:

1. Establish a network model for which each configuration to be tested will be a sub-network.
2. Select a specific sub-network configuration (i.e., one of the 52 possible) for the model.
3. Optimally allocate circuits to satisfy all inter-net and intra-net sole user and special service requirements.
4. Distribute common user requirements among the residual assets remaining after Step 3; a number of randomly-generated (but plausible) sets of common user requirements can be used in this step with each requirements set providing, probably, a different maxflow.
5. Determine the "effectiveness" of the sub-network configuration being tested. (See the following section.)
6. Return to Step 2 until the effectiveness of all potential configurations has been determined.
7. Compare the effectiveness measures of all tested configurations and adopt that configuration which best meets some specified common user criteria.

### Measures Of Effectiveness

Some common user effectiveness measure or measures must be agreed on prior to implementing a redistribution procedure. It may be that multiple criteria are needed to make effective use of the procedure and accurately reflect the needs of the users. Potential measures of effectiveness and methods for selecting criteria are briefly discussed in this section.

The objective function for the maxflow formulation used in this thesis has been:

$$\text{MAX } \sum f_{ij}$$

This simply maximizes the number of requirements which are satisfied by the call distribution. Kalaba and Juncosa(64) also propose as an objective function

$$\text{MAX } \Theta = \sum f_{ij}/r_{ij}$$

where  $\Theta$  is termed an "efficiency index". When either is used as the objective function of a linear programming formulation there are likely to be alternate optimum solutions, in which case it may be possible to use some secondary effectiveness measure for selection of the best alternative.

Secondary criteria which might be considered are:

1. Maximize excess capacity after distributing a maxflow set of requirements.

2. Select the maxflow requirements set and routings which provide either the fewest routes (for simplicity and controllability) or the most routes (for survivability and flexibility).

3. Maximize inter-net satisfaction to the detriment of intra-net satisfaction.

4. Maximize intra-net satisfaction to the detriment of inter-net satisfaction.

5. Distribute assets proportionately among competing requirements on capacitated links.

It is, of course, possible that other criteria could be considered appropriate, and it is also possible that some combination of criteria might be used to select the eventual call distribution.

The first problem is to select the criteria to be considered. This can be done only by determining the desires and needs of the commanders for whom the network will be providing service. The second problem will be weighting the multiple criteria so that the resultant call distributions will accurately reflect the commanders' perceived needs. Once again, the information needed to determine the weightings can be gained only from the commanders.

It is not the intent of this thesis to develop specific procedures for determining appropriate criteria

and weights for each criterion. There is an extensive literature dealing with the multiple criterion problem. For example, Eckenrode(29) compared six methods of weighting multiple criteria and concluded that ranking, probably the easiest weighting method, is the most efficient. Geoffrion, Dyer, and Feinberg(42) have developed an interactive approach for multi-criterion optimization. These are but two papers in the field; application of the ideas contained in those papers and others to the specific problems of common user communications might be an appropriate area for further research.

The procedure of Chapter V could be modified to test the selected criteria and weightings. By making multiple runs on a specific network configuration with fixed requirements and capacity sets while varying the criteria and weightings it might be possible to determine which criteria combination does indeed reflect the desires of the commander. Once again, it will be the commander who eventually determines whether or not the call distribution and his perceptions are in accord.

In summary, the redistribution procedure might be effectively extended to:

1. Determine optimal system configurations given some set of criteria, or
2. Determine a set of criteria and appropriate weightings (more specifically, assist the commander in

selecting the appropriate criteria and helping him to assign weightings) given a specific system configuration.

By successive iterations of the procedure between the above-mentioned purposes it might be possible to eventually obtain a common user network which will provide a maximum amount of satisfaction during crisis periods when it is impossible to meet all potential requirements.

## CHAPTER VII

## CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS

The procedure described in Chapter V implements the techniques discussed in Chapter III, and it should be an effective way of determining common user load distributions for capacitated common user networks. The problem of estimating the revised requirements is efficiently solved by using a matrix multiplication technique which determines all new requirements simultaneously.

Call distributions for a complete multi-node network can be handled by partitioning the larger entity into sub-networks, each of which then has intra-net and inter-net requirements competing for capacities within the sub-networks; the inter-net requirements also compete for capacities on the interfaces. The advantage to partitioning is that the maxflow calculations for small networks are relatively simple, whereas calculations involving multi-commodity flows in larger networks quickly become intractable as network size and the number of commodities increase.

One limitation of the procedure is that the means for obtaining optimal multicommodity flow solutions may not be readily available. In such instances the heuristic flow procedure will provide a result which, although

sub-optimal, may be sufficient for subsequent development of a load control plan.

Yet another limitation is the considerable amount of "bookkeeping" which may be required while balancing the inter-net load distributions. Several iterations of maxflow calculations and route distributions may be necessary before the accepted criteria are satisfied in interfaced networks; multiple interfaces add further complications.

The following areas are recommended as being suitable for further research:

1. Measures of common user effectiveness need to be developed and refined for situations when it is obvious that commercial criteria are inapplicable.
2. Anticipated requirements need to be developed for the many tactical situations which could arise; the need for such a study has also been recognized by the U.S. Army Material Systems Analysis Agency.
3. Assessment of present doctrinal configurations might be done as suggested in Chapter VI to support system design changes which could improve common user service.
4. Further computational effort is required to measure the comparative effectiveness of the several multiflow techniques used in tactical environments.

5. There should be a comparison of the efficiency of a network controlled by present methods and a network controlled by a plan developed from the approach used in this thesis. Such a comparison should, ideally, be done under field conditions, but it might be successfully accomplished with a simulation model.

Control of common user communications has been looked at from a different perspective and, while no excessive claims are made for the efficacy of the proposed procedure, it does seem to be a reasonable way by which to approach an interesting problem. Therefore, while acknowledging the obvious limitations of this work, it is concluded that a network theoretic approach to optimal distribution of common user calls has certain merit.

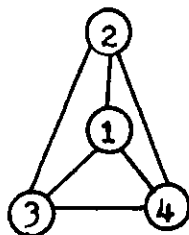
**APPENDICES**

## APPENDIX A

SOME COMPARISONS OF OPTIMAL, ROUTE-RESTRICTED, AND  
ASSIGNMENT-TYPE MULTIFLOW TECHNIQUES

This appendix analyzes the results of a series of maxflow solutions for one specific network configuration using capacities and requirements which were randomly generated over specific ranges. Only broad generalizations can be made from such an analysis.

The network used is the fully-connected four node configuration:



In runs 1-5, requirements were held at 6 for each node pair; capacities were randomly generated for each link according to a uniform distribution on the range from 4 to 9. In runs 6-10, capacities were held at 6 for each link; requirements were randomly generated for each node pair according to a uniform distribution on the range from 3 to 8. In runs 11-15, both capacities and requirements

were randomly generated over the previously-mentioned ranges. The first of the accompanying tables shows the capacities and requirements for each of the runs; the second table summarizes the results of these runs.

It is noted that the route-restricted solutions tend to allow less flow than the heuristic solutions and that heuristic solutions provided all or nearly all of the flow provided by the optimal solution for the specific examples used here. It can be said that the heuristic flow procedure provided reasonable results for these examples; no further generalizations should be made without analyses of an additional large number of runs on networks with different configurations, capacities, and requirements.

Table 1. Capacities And Requirements Used In Examples

Run	Link Capacities						Node-Pair Requirements					
	12	13	14	23	24	34	12	13	14	23	24	34
1	4	9	4	8	8	6						
2	9	5	8	6	7	6						
3	9	7	6	7	8	7			6			
4	7	7	8	9	7	6						
5	4	8	4	9	8	5						
6							4	7	4	4	6	4
7							6	8	5	5	4	6
8			6				7	7	7	8	6	8
9							6	8	4	8	6	6
10							6	7	6	4	7	4
11	5	6	5	5	9	6	8	6	6	6	7	4
12	4	8	8	9	6	5	7	7	4	4	8	5
13	4	4	5	7	8	5	7	4	8	8	3	8
14	5	8	7	5	9	5	4	7	8	4	4	3
15	6	9	6	8	9	4	3	8	5	8	5	7

Table 2. Summary Of Test Results: Four-Node Network

Run Number	Total Capacity	Total Requirements	Optimal Solution		2-Link Restricted Solution			Heuristic Algorithm Solution		
			Satisfied	% Satisfied	Satisfied	% Satisfied	% of Optimal	Satisfied	% Satisfied	% of Optimal
1	39	36	34	94.4	33	91.7	97.1	34	94.4	100.0
2	41	36	36	100.0	35	97.8	97.8	35	97.8	97.8
3	44	36	36	100.0	36	100.0	100.0	36	100.0	100.0
4	44	36	36	100.0	36	100.0	100.0	36	100.0	100.0
5	38	36	34	94.4	36	94.4	100.0	33	91.7	97.1
6	36	29	29	100.0	29	100.0	100.0	29	100.0	100.0
7	36	34	33	97.1	33	97.1	100.0	33	97.1	100.0
8	36	43	36	83.7	36	83.7	100.0	36	83.7	100.0
9	36	38	34	89.5	34	89.5	100.0	34	89.5	100.0
10	36	34	33	97.1	32½	95.6	98.5	33	97.1	100.0
11	36	37	33	89.2	33	89.2	100.0	33	89.2	100.0
12	40	35	31	88.6	29	82.9	93.5	31	88.6	100.0
13	33	38	28	73.7	28	73.7	100.0	28	73.7	100.0
14	39	30	30	100.0	30	100.0	100.0	30	100.0	100.0
15	42	36	35½	98.6	34	94.4	95.8	34	94.4	95.6

## APPENDIX B

## MULTITERMINAL ANALYSIS EXAMPLES

The algorithm of Gomory and Hu(44) will be applied to the network of Figure 8 as a means of describing the analysis procedure. The algorithm is outlined by Hu(56) as:

STEP 1. Do a maximal flow computation for two terminal nodes on a network which is usually smaller than the original network, since subsets are condensed into single nodes. Based on the flow computation, we get a minimum cut. Go to Step 2.

STEP 2. Use the minimum cut just obtained in Step 1 and construct a link of the tree network  $N'$ . The algorithm ends when  $m-1$  links are constructed. Select a pair of nodes which will serve as the source and the sink in Step 1, and condense certain subsets of the original network into single nodes. This is the network that will be used to do the maximal flow computation in Step 1. Go to Step 1.

To begin the analysis of the network of Figure 8 we redraw the network as Diagram A, arbitrarily select two nodes, solve a maximal flow problem between these nodes, and thereby locate the minimal cut. Selecting nodes 3 and 4 for the initial computation we see that the minimal cut, as shown in Diagram A, contains links 1-2, 1-3, and 3-4.

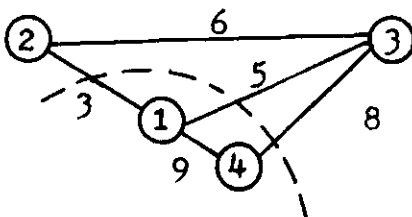


Diagram A

This cut has a capacity of 16 and separates the network into two parts: (  $\{1,4\}$  ,  $\{2,3\}$  ). The network can now be represented by the tree of Diagram B where the nodes of the sets which make up the cut have been consolidated.

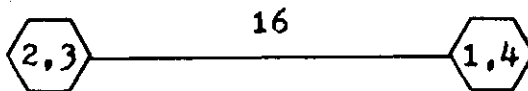


Diagram B

In the set  $\{2,3\}$  we see that the minimal cut is as shown in Diagram C. It contains links 2-3 and 2- $\{1,4\}$  and has a capacity of 9 circuits.

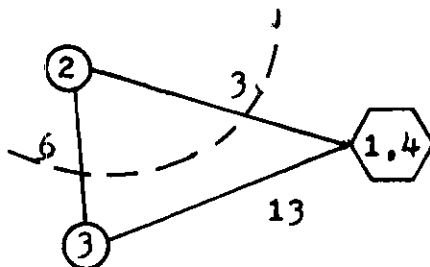


Diagram C

The tree of Diagram B can now be represented by the tree of Diagram D.

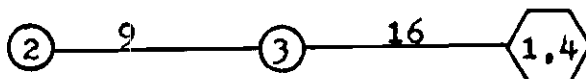


Diagram D

Considering nodes 1 and 4 for the final computation we see that the minimal cut, as shown in Diagram E, contains links 1- $\{2,3\}$  and 1-4 with a capacity of 17 circuits

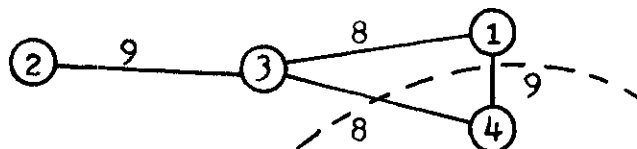


Diagram E

The tree of Diagram D can now be represented by the tree of Diagram F.

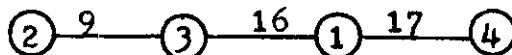


Diagram F

We have now developed a network equivalent to that of Figure 8 with respect to the terminal capacities of all node pairs. However the realized network is a tree or, to be more specific, a cut tree. The cut tree thus formed is not unique but this fact is of no consequence since, regardless of the result, the terminal capacity between two nodes is simply the capacity of the minimal link on the unique route between those nodes. It is now a simple matter to determine the multiterminal flow capacities and to write down the terminal capacity matrix.

Let  $T = [t_{ij}]$  be an  $n \times n$  symmetric matrix where  $t_{ij}$  is the terminal capacity between  $s_i$  and  $s_j$ . Then for this network in Diagram F which is equivalent to the network of Figure 8 we can construct the terminal capacity matrix shown at the top of the following page.

$$T = [t_{ij}] = \begin{bmatrix} - & 9 & 16 & 17 \\ 9 & - & 9 & 9 \\ 16 & 9 & - & 16 \\ 17 & 9 & 16 & - \end{bmatrix}$$

This is, admittedly, a very simple example, but more complicated networks can be analyzed almost as easily. A very clear exposition of the technique is given by Ford and Fulkerson(31).

Note that the sixteen-node network depicted in Diagram G can be analyzed very easily because of its structure. One need only analyze the portion of the network encircled by the dotted line and determine the terminal capacity matrix of those four nodes.

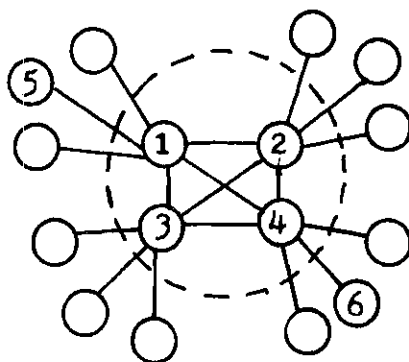


Diagram G

Then the sixteen node  $16 \times 16$  terminal capacity matrix can be constructed as is done for the following example. To determine the terminal capacity of the node pair 5,6 (numbered nodes in Diagram G) we:

1. Construct the terminal capacity matrix of the

sub-network (1,2,3,4) using the previously-stated algorithm.

$$2. \quad t_{56} = \text{Min} \{t_{14}, c_{15}, c_{46}\}$$

To show a simple application of Elmaghraby's procedure consider again the network of Figure 8 with interest being centered on decreases of capacity in the link 2-3. Let  $\delta$  = the amount by which the link capacity is reduced. See Diagram H.

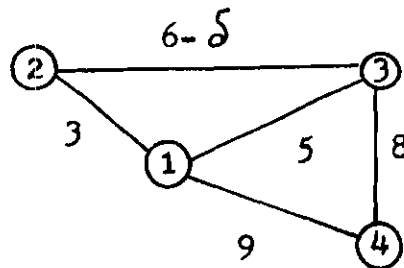


Diagram H

The multiterminal analysis is applied as it was for the network before the arc capacity changes were to be considered. The cut tree for  $\delta = 0, 1, 2, 3$  can be shown to be as in Diagram I.

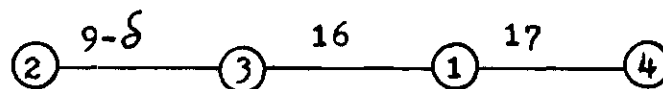


Diagram I

For  $\delta = 3, 4, 5, 6$  the link 2-3 is (or could be) in the minimal cut set of all node pairs. The cut tree can be shown to be as in Diagram J.

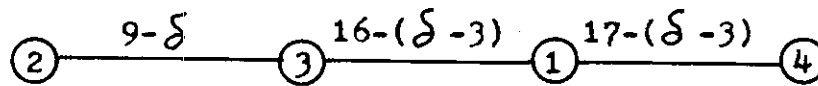


Diagram J

When the capacity of link 2-3 is reduced to zero (i.e.,  $\delta = 6$ ), the cut tree is as shown in Diagram K.

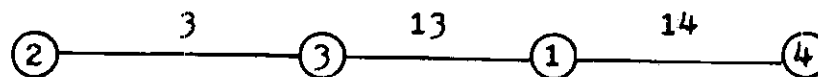


Diagram K

The critical values for link 2-3 are thus  $\{3, 0\}$ .

For more complex networks the procedure is more lengthy, but Elmaghraby shows exactly how to perform the analysis on any undirected network. This technique could be useful for demonstrating some of the effects that taking common user circuits for other uses might have not only on the links from which the circuits are taken but on the network-wide capability to handle common user calls.

## APPENDIX C

## BASIC PROGRAM FOR SOLVING REQUIREMENTS ESTIMATION PROBLEMS

This program was written in BASIC computer language to solve simple requirements matrix manipulations for networks having up to five nodes. The program interrogates the user who supplies the asked-for information.

Following the program is an example of the computer-generated solution to one of the problems which was demonstrated in Chapter IV.

```
10 PRINT 'THIS PROGRAM WILL FURNISH REVISED REQUIREMENTS'
11 PRINT 'MATRICES FOR NETWORKS HAVING UP TO FIVE NODES.'
12 PRINT 'WHEN ASKED TO "ENTER DATA", TYPE IN ALL ELEMENTS'
13 PRINT 'OF THE PERTINENT MATRIX WITH ELEMENTS'
14 PRINT 'SEPARATED BY COMMAS.'
20 PRINT 'IF YOU WISH TO WORK A SITUATION, TYPE "1".'
21 PRINT 'IF YOU HAVE NO NEED FOR "FIVENODES", TYPE "0".'
30 INPUT X
40 IF X = 0 THEN 999
50 PRINT 'ENTER THE NUMBER OF NODES IN THE PRESENT NETWORK.'
51 PRINT 'THIS NUMBER CAN BE 1,2,3,4, OR 5.'
60 INPUT Y
70 PRINT 'ENTER REQUIREMENTS DATA FOR THE PRESENT NETWORK.'
80 ON Y THEN 90,100,110,120,130
90 DIM A(1,1)
91 MAT INPUT A
92 MAT M = A
93 GO TO 190
100 DIM B(2,2)
101 MAT INPUT B
102 MAT M = B
103 GO TO 190
110 DIM C(3,3)
111 MAT INPUT C
112 MAT M = C
113 GO TO 190
120 DIM D(4,4)
121 MAT INPUT D
122 MAT M = D
123 GO TO 190
130 DIM E(5,5)
131 MAT INPUT E
132 MAT M = E
133 GO TO 190
140 DIM F(1,2)
141 MAT INPUT F
142 MAT T = F
143 GO TO 310
145 DIM G(2,3)
146 MAT INPUT G
147 MAT T = G
148 GO TO 310
150 DIM H(3,4)
151 MAT INPUT H
152 MAT T = H
153 GO TO 310
155 DIM I(4,5)
156 MAT INPUT I
157 MAT T = I
158 GO TO 310
```

```
160 DIM J(2,1)
161 MAT INPUT J
162 MAT T = J
163 GO TO 310
165 DIM K(3,2)
166 MAT INPUT K
167 MAT T = K
168 GO TO 310
170 DIM L(4,3)
171 MAT INPUT L
172 MAT T = L
173 GO TO 310
175 DIM V(5,4)
176 MAT INPUT V
177 MAT T = V
178 GO TO 310
190 IF X = 0 THEN 300
191 MAT R = M
192 PRINT 'THIS IS THE PRESENT REQUIREMENTS MATRIX:'
193 MAT PRINT R
194 X = 0
200 PRINT 'ENTER THE SITUATION NUMBER TO BE SOLVED'
201 PRINT '      1      ADD A NODE'
202 PRINT '      2      LOSE A NODE'
203 PRINT '      3      LOSE A NODE, BUT UNITS RELOCATE'
204 PRINT '      4      SAME NODES, BUT UNITS RELOCATE'
210 INPUT Z
220 PRINT 'ENTER THE MANIPULATION DATA FOR THE NEW NETWORK:'
230 ON Z THEN 240,260,260,250
240 ON Y THEN 160,165,170,175,997
250 ON Y THEN 997,100,110,120,130
260 ON Y THEN 997,140,145,150,155
300 MAT T = M
310 PRINT 'THE MANIPULATION MATRIX IS:'
320 MAT PRINT T
330 MAT O = TRN(T)
340 MAT P = T*R
350 MAT Q = P*O
360 PRINT 'THE REVISED REQUIREMENTS MATRIX IS:'
370 MAT PRINT Q
380 GO TO 20
997 PRINT 'SITUATION IS UNSOLVABLE WITH THIS PROGRAM.'
998 GO TO 20
999 END
```

Example Of Interactive Computer Input And Output

NOTE: Entries on lines starting with "?" are the inputs made by the program user; all other lines are produced by the program.

THIS PROGRAM WILL FURNISH REVISED REQUIREMENTS MATRICES FOR NETWORKS HAVING UP TO FIVE NODES. WHEN ASKED TO "ENTER DATA", TYPE IN ALL ELEMENTS OF THE PERTINENT MATRIX WITH ELEMENTS SEPARATED BY COMMAS. IF YOU WISH TO WORK A SITUATION, TYPE "1". IF YOU HAVE NO NEED FOR "FIVENODES", TYPE "0".

?1

ENTER THE NUMBER OF NODES IN THE PRESENT NETWORK. THIS NUMBER CAN BE 1,2,3,4, OR 5.

?4

ENTER THE REQUIREMENTS DATA FOR THE PRESENT NETWORK.

?0,1,4,2,1,0,2,1,4,2,0,4,2,1,4,0

THIS IS THE PRESENT REQUIREMENTS MATRIX:

0	1	4	2
1	0	2	1
4	2	0	4
2	1	4	0

ENTER THE SITUATION NUMBER TO BE SOLVED

1	ADD A NODE
2	LOSE A NODE
3	LOSE A NODE, BUT UNITS RELOCATE
4	SAME NODES, BUT UNITS RELOCATE

?4

ENTER THE MANIPULATION DATA FOR THE NEW NETWORK:

?1,0,0,0,0,.5,0,0,0,0,1,0,0,0,0,1.33

THE REVISED REQUIREMENTS MATRIX IS

0	.5	4	2.67
.5	0	1	.67
4	1	0	5.33
2.67	.67	5.33	0

IF YOU WISH TO WORK A SITUATION, TYPE "1".  
IF YOU HAVE NO NEED FOR "FIVENODES", TYPE "0".

?0

## APPENDIX D

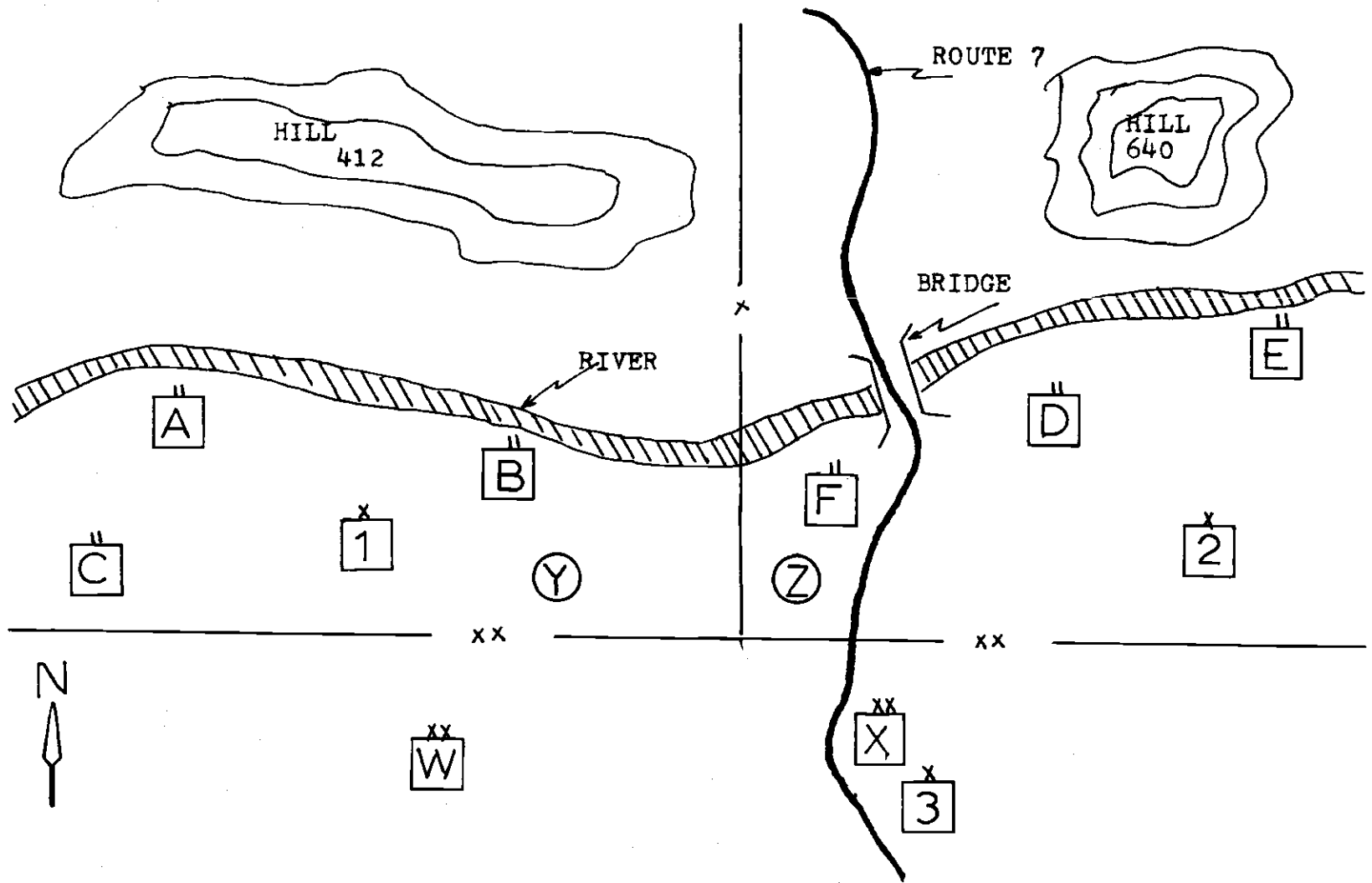
APPLICATION OF PROCEDURES AND  
TECHNIQUES TO A HYPOTHETICAL SCENARIO

An infantry division (Headquarters W) is deployed as shown on Map 1. Its mission is to seize crossings over the river, gain control of Highway 7 north of the river, and prepare to continue the attack northward.

The 1-Brigade on the left will cross the river by boat while the 2-Brigade on the right will seize the bridge over the river. The 3-Brigade is attached to Division Alternate (Headquarters X) for use as a reserve force or to exploit success in either of the committed brigades' sectors. Y and Z represent Division Area Signal Centers.

Prior to the attack communication requirements have been refined. The present network is shown in Diagram A with the numbers on the links indicating common user capacities. Due to the stability of the pre-attack situation and realignment of sole user requirements there is sufficient capacity to satisfy all common user requirements. Requirements matrices are shown in Charts 1-4. The routing tables are shown in Chart 5.

A computer capability exists at each location where traffic engineering is done; the necessary programs for



Map 1. Initial Disposition Of Divisional Units

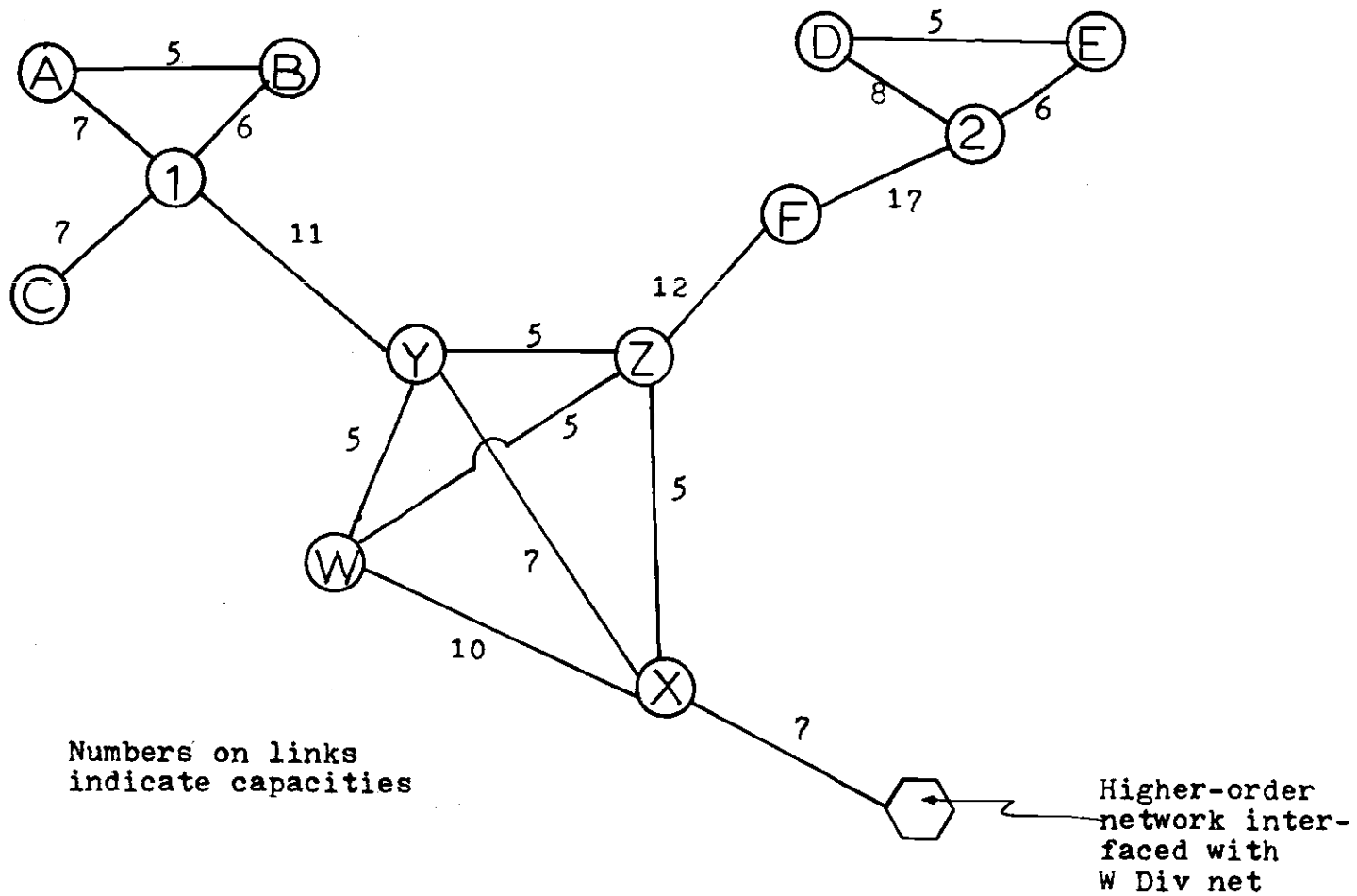


Diagram A. Initial Configurations Of Common User Networks



Chart 2. 1-Brigade Requirements Matrices

$$R_1(\text{intra}) = \begin{array}{c} 1 \\ A \\ B \\ C \end{array} \begin{bmatrix} 0 & 3 & 5 & 4 \\ & 0 & 4 & 1 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$$

$$R_1(\text{inter}) = \begin{array}{c} 1 \\ A \\ B \\ C \end{array} \begin{bmatrix} 0 & 2 & 2 & 2 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$$

+5 from Y to 1

$$R_1(\text{total}) = \begin{array}{c} 1 \\ A \\ B \\ C \end{array} \begin{bmatrix} 0 & 5 & 7 & 6 \\ & 0 & 4 & 1 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$$

with 11 interfaced to  
the W-Div network

Chart 3. 2-Brigade Requirements Matrices

$$R_2(\text{intra}) = \begin{array}{c} 2 \\ D \\ E \\ F \end{array} \begin{bmatrix} 0 & 2 & 4 & 4 \\ & 0 & 3 & 2 \\ & & 0 & 2 \\ & & & 0 \end{bmatrix}$$

$$R_2(\text{inter}) = \begin{array}{c} 2 \\ D \\ E \\ F \end{array} \begin{bmatrix} 0 & 0 & 0 & 5 \\ & 0 & 0 & 2 \\ & & 0 & 2 \\ & & & 0 \end{bmatrix}$$

+3 from Z to 2

$$R_2(\text{total}) = \begin{array}{c} 2 \\ D \\ E \\ F \end{array} \begin{bmatrix} 0 & 2 & 4 & 9 \\ & 0 & 3 & 4 \\ & & 0 & 4 \\ & & & 0 \end{bmatrix}$$

with 12 interfaced to  
the W-Div network

Chart 4. W-Division Requirements Matrices

$$R_W(\text{intra}) = \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 4 & 2 & 2 \\ & 0 & 2 & 2 \\ & & 0 & 3 \\ & & & 0 \end{bmatrix}$$

$$R_W(\text{inter-1}) = \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 0 & 4 & 0 \\ & 0 & 2 & 0 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$$

+5 from Y to 1

$$R_W(\text{inter-2}) = \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 0 & 0 & 5 \\ & 0 & 0 & 1 \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$$

+5 from Z to F

$$R_W(\text{inter-higher}) = \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 3 & 0 & 0 \\ & 0 & 1 & 1 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix}$$

+2 from higher to X

$$R_W(\text{total}) = \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \begin{bmatrix} W & X & Y & Z \\ 0 & 7 & 6 & 7 \\ & 0 & 5 & 4 \\ & & 0 & 4 \\ & & & 0 \end{bmatrix}$$

With 11 interfaced to 1  
 12 interfaced to F  
 7 interfaced to higher

Chart 5. Routing of Initial Requirements

NETWORK	NUMBER OF REQUIREMENTS	FROM, TO	ROUTE
1-Brigade	5	1,A	1-A
	6	1,B	1-B
	1	1,B	1-A-B
	6	1,C	1-C
	4	A,B	A-B
	1	A,C	A-1-C
2-Brigade	2	2,D	2-D
	4	2,E	2-E
	9	2,F	2-F
	3	D,E	D-E
	4	D,F	D-F
	2	E,F	E-2-F
	2	E,F	E-D-2-F
W-Division	5	W,Y	W-Y
	1	W,Y	W-X-Y
	5	W,Z	W-Z
	1	W,Z	W-X-Z
	1	W,Z	W-X-Y-Z
	4	Y,Z	Y-Z
	7	W,X	W-X
	5	X,Y	X-Y
	4	X,Z	X-Z

NOTE: Interfaces are on direct links indicated on Diagram A.

### Application Of Procedure To Continuation Of Scenario

From the pre-attack positions already described, three sequential events are now introduced; the several techniques are applied to answer specific questions which are posed after each event is described. Each question is designated as "Q-", and the answer is later denoted by "A-".

References to the units deployed in the scenario will be abbreviated. For example, the 2-Brigade will be noted as 2 Bde; the F-Battalion as F Bn; and the infantry division as W Div. Links and routings will be hyphenated strings of letters and numbers; for example, a routing from F Bn through the 2 Bde to the Z area center would be noted as F-2-Z.

#### Event 1

The D Bn and the E Bn successfully seize the bridge, and while the E Bn holds the bridge and road the D Bn moves onto Hill 640. It is not possible to re-establish the D-E link, so it is directed that two sole user circuits be derived from common user assets to provide direct communications between the two battalions.

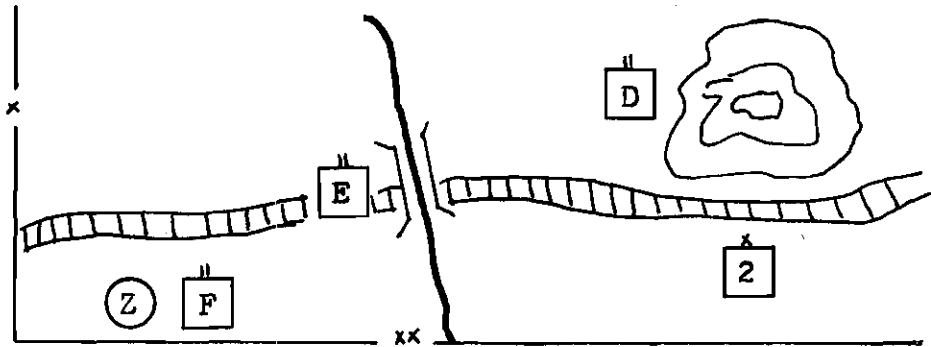
Q1: What is the maximum common user flow now possible in the 2 Bde network?

Q2: Using as a secondary criterion the dictum that partial allocations should be equally distributed among competing requirements if alternate optima exist, what is

the recommended flow?

Q3: How does this event impact on the W Div network?

The 2 Bde sector map is redrawn as Map 2.



Map 2. 2 Bde Sector After Event 1

The 2 Bde portion of the network is now configured as shown in Diagram B.

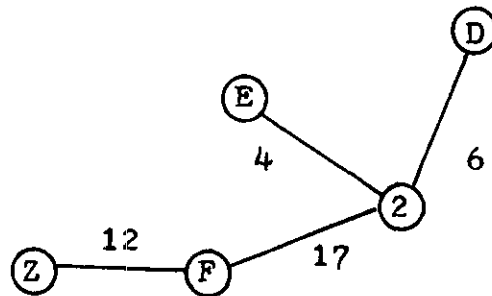


Diagram B. 2 Bde Network After Event 1

There is no longer a D-E link, so in the capacity matrix the entries  $C_{ED} = C_{EF} = C_{DF} = 0$ . The two circuits between D and E must be routed through 2 reducing the capacities on both the D-2 and E-2 links. The revised capacity matrix for the 2 Bde network is shown at the top of the following page.

$$\begin{array}{c}
 2 \quad E \quad D \quad F \\
 2 \left[ \begin{array}{cccc}
 0 & 4 & 6 & 17 \\
 & 0 & 0 & 0 \\
 & & 0 & 0 \\
 & & & 0
 \end{array} \right] \\
 E \\
 D \\
 F
 \end{array}$$

The total requirements matrix for the 2 Bde network is unchanged.

A1: A maxflow problem can be set up for this network in the following manner:

$$\begin{array}{l}
 \text{MAX} \quad f_{2D} + f_{2E} + f_{2F} + f_{DE} + f_{DF} + f_{EF} \\
 \text{ST} \quad f_{2D} \leq 2 \\
 \quad \quad f_{2E} \leq 4 \\
 \quad \quad f_{2F} \leq 9 \\
 \quad \quad f_{DE} \leq 3 \\
 \quad \quad f_{DF} \leq 4 \\
 \quad \quad f_{EF} \leq 4 \\
 \quad \quad f_{2D} + f_{DE} + f_{DF} \leq c_{2D} = 6 \\
 \quad \quad f_{2E} + f_{DE} + f_{EF} \leq c_{2E} = 4 \\
 \quad \quad f_{2F} + f_{DF} + f_{EF} \leq c_{2F} = 17 \\
 \quad \quad f_{ij} \geq 0
 \end{array}$$

The maximum common user flow (optimal solution) satisfies 19 requirements; there are alternate optimum solutions so it is necessary to select the alternative which will satisfy the established secondary criteria.

A2: Using the specified secondary criterion to select among alternatives, the following requirements will be satisfied:

	2	E	D	F
2	0	2	2	9
E		0	0	2
D			0	4
F				0

Left unsatisfied are:

3 requirements between E and D  
 2 requirements between 2 and E  
 2 requirements between E and F

Of the unsatisfied requirements between E and F, one is intra-net and one is inter-net. Using the secondary criterion once again we will satisfy half of each of the E,W and E,Z requirements and allow half of each of these requirements to remain unsatisfied.

A3: The impact on the W Div network is that one less capacity unit is used on the F-Z interface and  $\frac{1}{2}$  of a capacity unit is released on some W-Z routing. Maximum release of capacity would be realized by making the change on the W-X-Y-Z routing.

#### Event 2

The F Bn and the E Bn are consolidated into a task force, designated T, to hold the bridge; each battalion retains its original requirements to facilitate

future deployments of the units separately. A new link is established between T and D, and the interface between the 2 Bde network and Z is rehomed from T (formerly F) to 2. After reallocating sole user circuits the capacity matrix is:

$$\begin{array}{c} \text{2} \quad \text{T} \quad \text{D} \\ \text{2} \quad \left[ \begin{array}{ccc} 0 & 8 & 8 \\ \text{T} & 0 & 8 \\ \text{D} & & 0 \end{array} \right] \end{array}$$

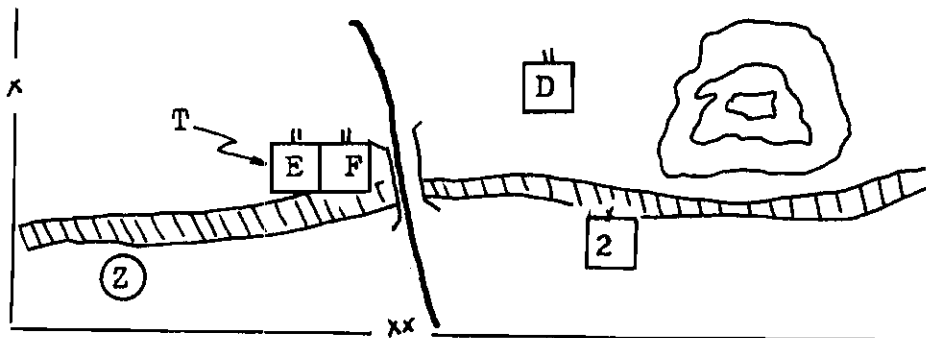
The interface capacity is now also 8. (All capacities used here are arbitrarily chosen for this event.)

Q4-6: What are the intra-net(Q4), inter-net(Q5), and total(Q6) requirements matrices for the reconfigured 2 Bde network?

Q7: What is the maximum common user flow distribution in the 2 Bde network?

Q8: How does this event impact on the W Div network?

Map 3 shows the disposition of the 2 Bde units after Event 2.



Map 3. 2 Bde Sector After Event 2

The 2 Bde portion of the network is now configured as shown in Diagram C.

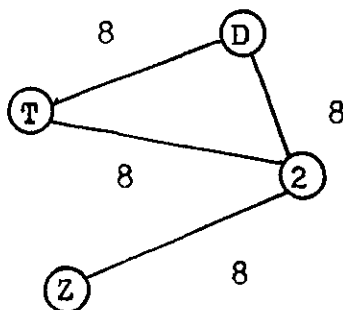


Diagram C. 2 Bde Network After Event 2

The manipulation matrix for the 2 Bde network is:

$$M_{ij} = \begin{matrix} & 2 & E & D & F \\ \begin{matrix} 2 \\ T \\ D \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

The revised requirements matrix for the intra-net requirements is computed using the formula of Chapter IV.

$$R_{k+1} = M R_k M^t$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 4 & 4 \\ 2 & 0 & 3 & 2 \\ 4 & 3 & 0 & 2 \\ 4 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 2 \\ 8 & 0 & 5 \\ 2 & 5 & 0 \end{bmatrix}$$

The revised inter-net requirements matrix if the interface was to be at T would be computed similarly using the same

manipulation matrix. It would be:

$$\begin{array}{c} 2 \quad T \quad D \\ 2 \quad \begin{bmatrix} 0 & 5 & 0 \end{bmatrix} \\ T \quad \begin{bmatrix} 5 & 0 & 2 \end{bmatrix} \\ D \quad \begin{bmatrix} 0 & 2 & 0 \end{bmatrix} \end{array}$$

plus 5 requirements from T to the W Div network nodes.

A5: However, the rehoming of the interface means that the inter-net requirements matrix actually is:

$$\begin{array}{c} 2 \quad T \quad D \\ 2 \quad \begin{bmatrix} 0 & 5 & 2 \end{bmatrix} \\ T \quad \begin{bmatrix} 5 & 0 & 0 \end{bmatrix} \\ D \quad \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \end{array}$$

plus 5 requirements from 2 to the W Div network.

A6: The total requirements matrix for the 2 Bde network is now the sum of the revised intra-net and inter-net requirements matrices:

$$\begin{array}{c} 2 \quad T \quad D \\ 2 \quad \begin{bmatrix} 0 & 13 & 4 \end{bmatrix} \\ T \quad \begin{bmatrix} 13 & 0 & 5 \end{bmatrix} \\ D \quad \begin{bmatrix} 4 & 5 & 0 \end{bmatrix} \end{array}$$

plus 12 requirements on the 2-Z interface.

Obviously, only eight of the inter-net requirements can be satisfied since the interface capacity is now reduced to 8. Equally distributing the inter-net assets will give a revised inter-net requirements matrix as shown at the top of the following page.

$$\begin{array}{c} 2 \quad T \quad D \\ \left[ \begin{array}{ccc} 0 & 10/3 & 4/3 \\ 10/3 & 0 & 0 \\ 4/3 & 0 & 0 \end{array} \right] \end{array}$$

plus  $10/3$  requirements from 2 to the W Div nodes.

Now the revised total requirements matrix is:

$$\begin{array}{c} 2 \quad T \quad D \\ \left[ \begin{array}{ccc} 0 & 34/3 & 10/3 \\ 34/3 & 0 & 5 \\ 10/3 & 5 & 0 \end{array} \right] \end{array}$$

of which 8 requirements are interfaced with the W Div network.

A7: The maxflow solution allows satisfaction of the requirements on the following routings:

<u>Requirement</u>	<u>Number</u>	<u>Routing</u>
2,T	8	2-T
2,T	3	2-D-T
2,D	$10/3$	2-D
T,D	5	T-D

One way of distributing the capacity (and probably the easiest way) is to reduce the intra-net satisfaction of the 2,T requirement from 8 to  $23/3$  and satisfy all of the remaining revised requirements. Other redistributions could be accomplished but only by sacrificing ease of solution. For example, the satisfaction of the 2,T requirement could be reduced from 8 to  $47/6$  and the

satisfaction of the T,D requirement could be reduced from 5 to  $29/6$ . Either redistribution would allow the fullest satisfaction of the already-reduced inter-net requirements.

A8: The W Div inter-net requirements are reduced because of the limited interface capacity. The satisfied inter-net requirements to the 2 Bde network are:

	W	X	Y	Z
W	0	0	0	$10/3$
X	0	0	0	$2/3$
Y	0	0	0	$2/3$
Z	$10/3$	$2/3$	$2/3$	0

plus  $10/3$  of the requirements between 2 Bde and Z.

The revised total W Div requirements matrix is:

	W	X	Y	Z
W	0	7	7	$16/3$
X	7	0	5	$11/3$
Y	6	5	0	$11/3$
Z	$16/3$	$11/3$	$11/3$	0

This will allow elimination of the W-X-Y-Z routing and reductions on the W-X-Z, X-Z, and Y-Z routings.

### Event 3

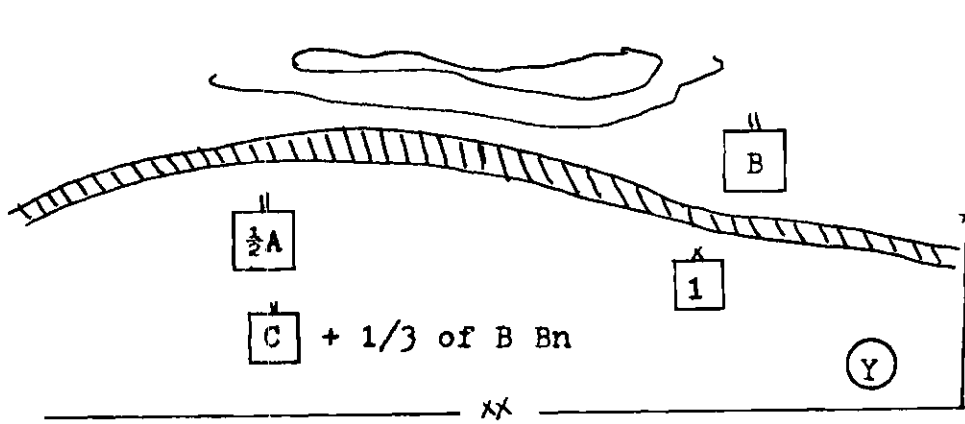
While attempting to cross the river, a nuclear strike destroyed half of the A Bn;  $2/3$  of the B Bn, including all communication elements, crossed the river successfully and the remaining  $1/3$  of the B Bn was reassigned to the C Bn.

Q9-11: What are the 1 Bde's revised intra-net(Q9), inter-net(Q10), and total(Q11) requirements matrices?

Q12: What is the maxflow distribution in the 1 Bde network?

Q13: What impact does this event have on the W Div network?

Map 4 shows the disposition of units in the 1 Bde sector at the completion of Event 3.



Map 4. 1 Bde Sector After Event 3

The 1 Bde network configuration is unchanged; links and capacities are the same as those originally established. The 1 Bde portion of the network is redrawn in Diagram D as it appeared in Diagram A.

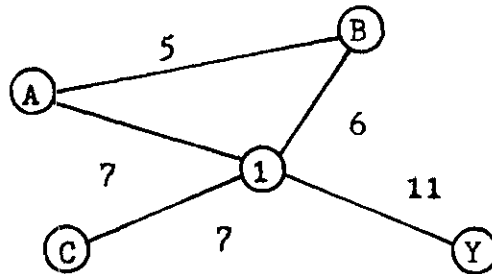


Diagram D. 1 Bde Network After Event 3



It is apparent that all requirements can be satisfied except those which transit the 1-C link. The 1-C requirement plus the A-C requirement is  $25/3 + 7/6 = 9\frac{1}{2}$  requirements which are competing for a capacity of only 7 on the 1-C link. Satisfying equal percentages of the competing requirements, that is  $14/19$ ths of each user-user requirement which must transit the link, will allow the following flows:

<u>Requirement</u>	<u>Number</u>	<u>Routing</u>
1,A	5/2	1-A
1,B	14/3	1-B
1,C	43/7	1-C
A,B	4/3	A-B
A,C	6/7	A-1-C

The interface requirement becomes:

<u>From</u>	<u>Number</u>
1	5
A	1
B	4/3
C	2
Total	28/3

A13: About  $5/3$  units are no longer needed on the 1-Y interface; by equitably distributing inter-net requirements it is also possible to release  $7/6$  of a unit on the W-Y routings which were needed initially (see Chart 5). This will eliminate the need for the W-X-Y routing and

fractionally decrease the need for the W-Y routing.

The cumulative revised total W Div requirements matrix after Events 1, 2, and 3 is:

	W	X	Y	Z
W	0	7	35/6	16/3
X	7	0	5	11/3
Y	35/6	5	0	11/3
Z	16/3	11/3	11/3	0

The three events depicted in this Appendix do not exhaust the possibilities that might occur in an operational situation; they do, however, demonstrate a variety of incidents which could arise. The techniques developed in this thesis have provided simple and rapid responses to the questions which exist in such situations, and the maxflow solutions and routings can serve as inputs to an overload control plan which uses network theoretic control measures.

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