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AN ANALYSIS OF CARPET INDUSTRY
PRODUCTION REQUIREMENTS

A THESIS

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PRODUCTION REQUIREMENTS

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SUMMARY

The objectives of this study were to (1) predict long-term carpet shipments quarterly, four years in advance, and (2) predict short-term carpet shipments, that is, carpet shipments one quarter in advance.

The long-term prediction was limited to a systematic technique comprising trend projection, seasonal analysis, and cyclical analysis. The short-term prediction was restricted to a statistical technique of multiple regression analysis. The available data limited the prediction time period to one quarter or longer. A time series from 1952 to 1959 was analyzed to reduce the influence of the Korean War on carpet shipments.

A multiplicative model which contained trend, seasonal, cyclical, and error elements was utilized for the long-term prediction. This model projects seasonal and cyclical elements of increased magnitude as the trend element increases.

In order to project the trend component, equations of polynomials of degrees one through four were calculated by the method of least squares. The linear equation was found to be statistically significant with the parameter $\alpha = 0.10$ in the "F" test. This significance level was assumed to be the error which the carpet industry would accept.

The seasonal component was determined by adjusting the quarterly averages of the seasonal element times the cyclical element times the error element. The cyclical element was determined from a five point

moving average of the cyclical element times the error element. The error term was tested for randomness by the theory of runs and was found to be random at the 0.05 significance level.

The total United States carpet shipments are composed primarily of imported carpet, tufted carpet, and woven carpet. Imported carpet deliveries were predicted with the aid of a linear equation calculated by the method of averages. Projections of tufted carpet shipments were made with a cubic equation selected by the method of differences.

A multiple regression model was constructed which contained independent variables with a relationship to the total United States carpet shipments. The statistical significance of the variables was tested with the t test. Two variables were significant at the 0.10 level. The error terms in the predictions resulting from the multiple regression equations were tested for randomness by the theory of runs. The error term of the full regression equation was found to be random, while the error term of the reduced regression equation was found not to be random at the 0.05 significance level.

The systematic technique provides a better prediction for both the long-term and short-term projection of carpet shipments. The prediction made with this technique has a smaller range and average error than the prediction resulting from the multiple regression model.

In view of the coefficient of multiple correlation of 0.957, obtained with the available basic data, additional study should be devoted to multiple regression analysis of carpet shipments data of an individual company.

The total United States carpet shipments in 1963 are projected to be 211,620,000 square yards as compared to 161,620,000 square yards in 1959. Tufted carpet shipments are projected to increase from 96,040,000 square yards in 1959 to 156,510,000 square yards in 1963. Woven carpet shipments are projected to decline from 58,508,000 square yards in 1959 to 46,940,000 square yards in 1963.

CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Introduction.--A carpet is a durable consumer good. It is a style item, the demand for which is influenced by the changes in consumer preferences for patterns and colors. In recent years carpet has ceased to be regarded as a luxury item.

The two common methods of manufacturing carpet are woven and tufted. Woven carpet is manufactured on a loom. Warp yarn is fed from a creel or a beam into the loom; a shuttle, which passes back and forth across the loom, inserts filling yarn perpendicularly to the warp yarn. The loom weaves a carpet by interlacing the warp and filling yarns.

Tufted carpet is manufactured on a wide sewing machine. Jute fabric is fed into the machine from a roll, and as the jute passes through the machine, needles, threaded with yarn, sew tufts which form the carpet.

In 1951, carpet production totaled 64,766,000 square yards valued at $449,188,000. In 1959, carpet production had increased to 154,535,000 square yards valued at $670,062,000 (1).

Carpet is manufactured by three major producers and a large number of small plants. The 1959 sales, as a per cent of carpet industry total sales, are listed in Table 1 for the three major carpet companies (2).

Table 1

1959 Sales - Per Cent of Industry Total

Manufacturer	Per Cent
Mohasco Industries Incorporated	15.1
James Lees and Sons	12.5
Bigelow-Sanford Carpet Company	10.7

The introduction of tufted carpet has changed the industry's requirements for production facilities. The number of carpet looms in production has decreased from 5,000 in 1954 to 3,200 in 1958 (3). Table 2 illustrates the shift from woven to tufted carpet (4, 5).

Table 2

Broadloom Carpet Shipments by Method of Manufacture

	Per Cent of Total Yardage		
Method of Manufacture	1939	1952	1959
Woven	100.0	85.7	40.5
Tufted	0.0	14.3	59.5
Total	100.0	100.0	100.0

The carpet industry requires an adequate forecast of future production requirements in order to (1) provide optimum long term production facilities, (2) satisfactorily control inventories, and (3) stabilize employment and production. These objectives are not easily attained in the carpet industry due to such factors as the lengthy manufacturing cycle from fiber to finished carpet, short production runs for colors and patterns, and the production of carpet for stock to satisfy customer demands for quick delivery.

A case study of carpet forecasting, prepared by Sprague of the Bigelow-Sanford Carpet Company, is included in the American Management Association's Special Report Number 27 (6). This forecast, based on historical data, reveals a correlation between carpet shipments and disposable income. Sprague estimated the carpet industry's average share of disposable income at 0.214 per cent.

Data prepared by the American Carpet Institute indicates that carpet sales averaged 0.185 per cent of disposable income during the period, 1951 to 1959 (7). See Table 3.

<u>Statement of the Problem</u>.--The objectives of the present study are to:

(1) Predict long-term carpet shipments quarterly, four years in advance.

(2) Predict short-term carpet shipments, that is, carpet shipments one quarter in advance.

It is proposed to achieve objectives by the following method of procedure:

(a) Isolation of the trend, seasonal, cyclical, and random components from the total of quarterly domestic carpet shipments and import carpet deliveries for the period from 1952 through 1959.

(b) Projection of total carpet shipments quarterly through 1963.

(c) Separation of import carpet deliveries and the domestic carpet shipments from the total carpet shipments.

Table 3

Estimated Value of Manufacturers' Carpet Shipments

Year	Disposable Income (Billions)	Dollar Shipments (Millions)	Shipments (Per Cent of Income)
1951	$227.5	$449.2	0.19
1952	238.7	421.8	0.18
1953	252.5	446.1	0.18
1954	256.9	434.0	0.17
1955	274.4	520.9	0.19
1956	292.9	569.3	0.19
1957	307.9	567.6	0.18
1958	316.5	559.4	0.18
1959	334.6	670.1	0.20
Average	227.9	515.4	0.185

(d) Separation of domestic carpet shipments by method of manufacture, namely, woven and tufted.

(e) Specification of the variables which explain carpet shipments.

(f) Construction of a multiple regression model of the type

$$Y_i = B_0 + B_1 X_{1i} + B_2 X_{2i} + \ldots\ldots + B_p X_{pi} + \epsilon_i \; .$$

(g) Solution of this equation for estimates of B_0, B_1, B_2, ... B_p.

(h) Determination of the validity of the prediction models by a comparison of the range of error, average error, randomness of the error, and coefficient of multiple correlation.

CHAPTER II

SURVEY OF PREDICTING TECHNIQUES

For the purpose of this study, predicting techniques are classified into five groups, namely:

(1) Systematic Techniques.

(2) Index Techniques.

(3) Ratio Techniques.

(4) Survey Techniques.

(5) Statistical Techniques.

Systematic Techniques.--This classification of techniques includes the fitting and extrapolation of mathematical trends and the analysis of cyclical and seasonal variations about the trend line.

The analysis of time series usually results in two conventional models with four components, the additive model and the multiplicative model (8).

Additive Model: $Y_i = T_i + S_i + C_i + R_i$

Multiplicative Model: $Y_i = T_i \times S_i \times C_i \times R_i$

Y_i = the data at the ith time period

T_i = the trend component at the ith time period

S_i = the seasonal component at the ith time period

C_i = the cyclical component at the ith time period

R_i = the random error component at the ith time period

The multiplicative model is utilized when the seasonal component increases in magnitude as the trend component increases.

The method of auto-correlation is similar to the methods of trend projection and cyclical analysis. Auto-correlation is the projection of a series by means of a correlation of the series with itself at different points in time.

Index Techniques.--Forecasters have sought indexes which would change consistently with the index they wish to forecast. In 1938, Roos examined 238 indexes to determine their lead characteristics (9). Those that had lead characteristics in 1938 had the same lead characteristics in 1954. G. H. Moore, in 1950, studied 801 monthly and quarterly time series for the United States (10, 11).

Moore found that the following eight series, regularly available in current publications, had lead characteristics in relation to general business activity.

(1) residential building contracts

(2) commercial and industrial building contracts

(3) new orders for durable goods

(4) prices of industrial common stocks

(5) wholesale prices of basic commodities

(6) average work week in manufacturing

(7) number of new incorporations

(8) business failures

Moore devised the diffusion index (12) to predict a reversal in the general business trend. He first smoothed a group of individual series using a moving average. He then counted the number of series that were rising at a given time and calculated the percentage of rising series. If less than fifty per cent were rising, these data indicated a business contraction, and conversely, an expansion. However, this method indicated contractions in mid-1946, early 1947, late 1947, and early 1951 when no generally recognized business recession followed. Therefore, the validity of this method is questionable.

Wright has developed a statistical indicator method (13) of determining business cycle turns. His method is based on the assumptions that: (a) cyclical peaks in individual series are spread over several months before and after turns in general business, and (b) the individual peaks show a clustering tendency in the vicinity of general business peaks and troughs. By use of the normal distribution, Wright attempts to estimate the turning points in general business by discovering clustering tendencies in the peaks of individual time series included in a sample. Wright concluded that this technique would predict a business cycle turn three times out of four.

Ratio Techniques.--Ratios are relative indexes. For example, the ratio of raw material inventories to finished goods production is a guide to raw material prices.

Roos (14) found that relative indexes are guides to the satisfactory explanation of the demand for capital goods. He found that the ratio of seasonally corrected carloadings to cars in a serviceable condition gives a reasonably satisfactory forecast of cars that will be ordered by railroads from six to twelve months in the future.

Survey Techniques.--Several agencies conduct surveys, for example, Fortune Magazine, McGraw-Hill Book Company, and the United States Department of Commerce. These surveys are based on a wide variety of experimental designs.

The Survey Research Center (15) conducted original interviews and reinterviews of 1,245 subjects in 1949 to determine their intentions for purchasing automobiles. The data shows that the intentions of consumers to purchase automobiles are not valid.

The Survey of Consumer Finance (16) surveyed the intentions of consumers to purchase refrigerators, furniture, television sets, and radios. Consumer intentions on these items have less validity than their intentions to purchase automobiles.

The staff of the Board of Governors of the Federal Reserve Board (17) has been testing the hypotheses that (a) consumers can furnish useful clues to their future economic behavior, and (b) that consumer buying plans represent the single most powerful predictor of changes in specific expenditures. The Federal Reserve System analyst organize and analyze the data contained in the Consumer Survey of Finance to predict demand for durable goods. From 1948 to 1954, out of 46 predictions of changes in specific expenditures, 39 were correct. Limitations of their work are (a) the analysis of data requires judgment, (b) consumers change their plans, (c) the basic data was originally obtained from consumers, and (d) there is a time lag to obtain data and publish the survey. No conclusion has been reached as to the validity of this method of prediction.

Ferber (18) found that the predictions of durable goods purchases made from information obtained by surveys of consumer intentions were valid for no more than six months.

Schweiger states that,

> "The evidence to date indicates that, in general, individuals are not sensitive to small changes in price or income, that their response to economic happenings are frequently unsophisticated and emotional and are limited by lack of knowledge and uncertainty. Forecasts that assume the rational 'economic man,' aware of the full range of choice available to him, may easily go astray. . . . . . Threats of a shortage will get a strong reaction, but prospects of a small price change appear to have little effect on durable goods purchases."[1]

Therefore, data obtained by surveys of consumer intentions to buy should be used with considerable caution. A through knowledge of the design of the survey is necessary to reasonably interpret the data.

Statistical Techniques.--These techniques involve the derivation of equations, utilizing statistical regression and correlation analysis, to predict future requirements. Regression models should contain independent variables which have an explainable relationship to the dependent variable.

Using multiple regression analysis, the Timken Roller Bearing Company (19) developed the following model for predicting company sales.

$$\text{Forecast} = B_1(1Z) + B_2(3D) + B_3(2G) + B_4(8H) + B_5(6F) + B_6(7D) + B_7(1G)$$

The seven indicators are:

1Z = failure liabilities

3D = manufacturers' new orders

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[1]Schweiger, Irvin, "The Contribution of Consumer Anticipations in Forecasting Consumer Demands," Short Term Economic Forecasting, National Bureau of Economic Research, 1955.

2G = machine tool index

8H = fabricated structural steel

6F = manufacturers' unfilled orders

7D = Federal Reserve Index of Production

1G = manufacturers sales

B_j = constant multipliers or parameters

Wagenhals of the Timken Roller Bearing Company states that,

> "The results of this model have been from good to poor. The problem is to locate leading indicators which are consistent over the years. It is also necessary to find variables which will explain the forecasted sales at the 0.05 or 0.01 significance level."[1]

A system of cross-section analysis is used to determine the sensitivity of an item to the change in a dependent variable. Two periods in time are selected and detailed data are obtained. Brandeen (20) illustrates this method of analysis for automobile consumption using a cross-section of 1940 and 1950. He concluded that the income sensitivity of automobile consumption was 0.9.

Statistical techniques have been criticized for the following reasons (21):

(1) Most of the mathematical models are static and inflexible to short-run changes.

(2) Mathematical precision is often applied to rough data.

(3) Mathematical assumptions necessary to derive regression equations do not apply in practice.

---

[1]Wagenhals, R. E., unpublished letter, Timken Roller Bearing Company, Canton, Ohio, July 6, 1960.

(4) Statisticians do not consider the element of objective judgment.

(5) Independent variables often cannot be predicted with any more accuracy than the dependent variable.

Multiple regression models have been questioned as a result of the two mathematical assumptions necessary to derive the mathematical theory (23). They are:

(1) The dependent variable is, except for random deviations, a linear function of the independent variables.

(2) The random deviations are normally and independently distributed with the same standard deviation, regardless of the value of the independent variable.

Application of Techniques.--The American Management Association (23) conducted a survey in 1956 to determine the techniques actually being used in industrial forecasting practice. The results of this survey, which included a cross-section of all industry in the United States, are listed in Table 4.

Table 4

Survey of Industry Forecasting Techniques

Technique	Number of Companies Using Technique
Survey	1,308
Systematic	278
Index	213
Ratio	151
Miscellaneous	34
Total Companies Reporting	1,984

CHAPTER III

DATA COLLECTION

The data pertaining to domestic carpet shipments and import carpet deliveries are limited to total data for the entire United States.

Domestic woven carpet shipments in the ith time period, denoted by WC_i, were obtained from the American Carpet Institute. This is the only easily available data of domestic woven carpet shipments on a monthly basis.

Domestic tufted carpet shipments in the ith time period, denoted by TC_i, were obtained from the Tufted Textile Manufacturers Association. This data was prepared from the United States Department of Commerce Series M22L-129, "Tufted Textile Products." The domestic carpet shipments are rugs, carpets, and roll goods four feet by six feet or larger.

Machine-made import carpet deliveries were obtained from the American Carpet Institute. Import carpet deliveries in the ith time period are denoted by IC_i. These data are based on the United States Department of Commerce Report FT 110, "United States Imports of Merchandise for Consumption in Commodity by Country of Origin Arrangement."

The total carpet shipments in the ith time period, denoted by Y_i, are calculated by equation 1.

$$Y_i = WC_i + TC_i + IC_i \qquad (1)$$

The total United States carpet shipments on a quarterly basis were obtained by adding the monthly woven carpet shipments, and prorating the semi-annual tufted carpet shipments and the annual machine-made import carpet deliveries. This proration was based on the quarterly per cent of annual woven carpet shipments. As a result, the seasonal component of Y_i is weighted toward the seasonal fluctuation of woven carpet.

The data used to obtain the B_0 values in the multiple regression equation of Chapter V, equation 17, were obtained from Business Statistics which is published by the United States Department of Commerce. One item of data, the number of household in the United States, was obtained from the "Current Population Reports," Series P-20. These reports are published by the United States Department of Commerce. This annual series of data was interpolated quarterly on a straight line basis.

All data for the independent variables in the regression equation of Chapter V, equation 17,were "unadjusted" except for the disposable personal income, which is seasonally adjusted. The National Income Division of the United States Department of Commerce does not publish this data on an "unadjusted" basis.[1]

A diagram of the flow of carpet from shipments to consumption is shown in Figure 1.

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[1]Graham, Robert E., unpublished letter, Reference BE-54. U. S. Department of Commerce, Washington 25, D. C., October 10, 1960.

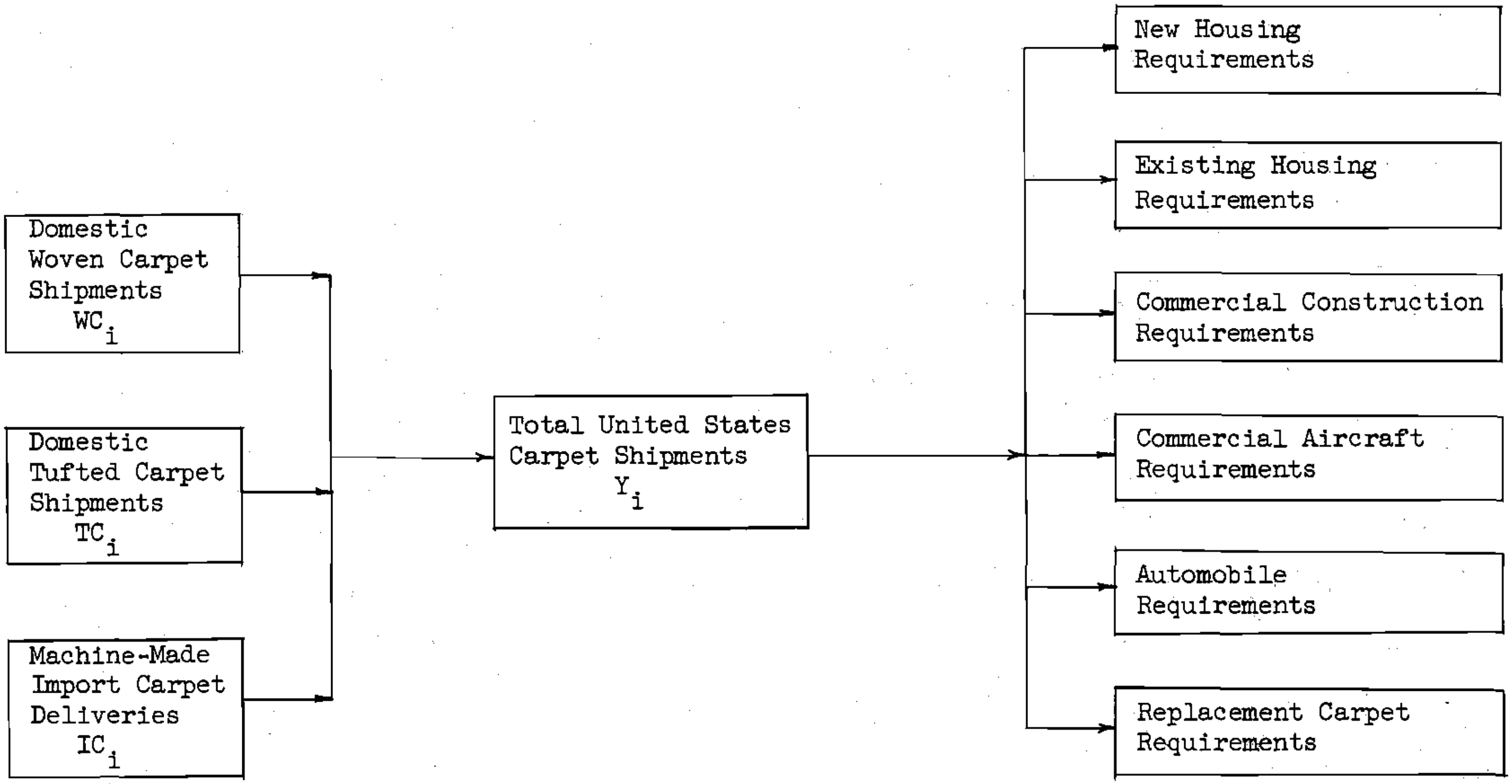

Figure 1. Diagram of Carpet Shipments

CHAPTER IV

A SYSTEMATIC ANALYSIS OF LONG TERM CARPET SHIPMENTS

The objective of this analysis is to predict carpet shipments quarterly, four years in advance, by method of manufacture. This prediction is limited to the systematic technique of trend projection, seasonal analysis, and cyclical analysis. A time series from 1952 to 1959 is analyzed in order to reduce the influence of the Korean War on carpet shipments.

The quarterly carpet shipments as a per cent of the annual total carpet shipments are a reasonably constant percentage as shown in Table 5. The second and third quarters are less variable than the first and fourth quarters.

This type of data, where the quantity of the quarterly variation from the trend increases as the magnitude of the trend increases, are projected with the following multiplicative model:

$$Y_i = T_i \times S_i \times C_i \times \epsilon_i \qquad (2)$$

Taking the logarithm of equation 2, we get

$$\log Y_i = \log T_i + \log S_i + \log C_i + \log \epsilon_i \qquad (3)$$

where Y_i = the data at the ith time period,

T_i = the trend component at the ith time period,

S_i = the seasonal component at the ith time period,

Table 5

Carpet Shipments - Per Cent of Annual Total

Year	Quarter 1	2	3	4	Total
1952	23.85	22.83	23.30	30.02	100.00
1953	30.68	23.32	22.56	23.44	100.00
1954	26.75	23.92	23.79	25.54	100.00
1955	25.51	24.51	22.34	27.64	100.00
1956	27.22	23.78	23.05	25.95	100.00
1957	28.85	23.65	23.19	24.31	100.00
1958	24.25	22.42	23.35	30.00	100.00
1959	26.27	24.95	22.48	25.80	100.00
Average	26.73	23.67	23.01	26.59	100.00
Range From	-2.88	-1.25	-0.53	-3.25	
Range To	+3.95	+1.28	+0.79	+3.23	
Range	6.83	2.53	1.32	6.58	

C_i = the cyclical component at the ith time period,

and ϵ_i = the random error component at the ith time period.

The quarterly data utilized in this analysis is listed in Table 6. The woven carpet shipments, tufted carpet shipments, and import carpet deliveries which make-up the total United States carpet shipments are tabulated in Appendix I.

In order to establish the trend component, T_i, equations of polynomials of degrees one through four were calculated by the method of least squares. The equations of polynomials and the statistical data in Table 7 were calculated using the IBM 650 computer program PL 05.

If all effects other than trend are assumed non-existent, then $\hat{Y}_i = \hat{T}_i$ and $Y_i = T_i$.

The four equations are:

Linear $$\hat{T}_i = +1622.36 \quad +74.23i \tag{4}$$

Quadratic $$\hat{T}_i = +1708.51 \quad +59.03i \quad +0.46i^2 \tag{5}$$

Cubic $$\hat{T}_i = +1635.60 \quad +83.67i \quad -1.38i^2 \quad +0.04i^3 \tag{6}$$

Quartic $$\hat{T}_i = +1936.67 \quad -76.70i \quad +19.72i^2 \quad -0.95i^3 \quad +0.01i^4 \tag{7}$$

where $\hat{T}_i$ = estimate of the true T_i.

The statistical significance of these equations was tested using the "F" test. The analysis shown in Table 7 indicates that the linear equation is significant at the 0.001 level. The other three equations

Table 6

Quarterly United States Carpet Shipments

Year	Quarter	Period i	Shipments, Y_i (10,000 Square Yards)
1952	1	1	1,772
	2	2	1,696
	3	3	1,792
	4	4	2,310
1953	1	5	2,442
	2	6	1,856
	3	7	1,960
	4	8	2,037
1954	1	9	2,201
	2	10	1,968
	3	11	2,256
	4	12	2,422
1955	1	13	2,852
	2	14	2,740
	3	15	2,615
	4	16	3,325
1956	1	17	3,181
	2	18	2,780
	3	19	2,976
	4	20	3,351
1957	1	21	3,558
	2	22	2,917
	3	23	3,158
	4	24	3,311
1958	1	25	3,150
	2	26	2,914
	3	27	3,282
	4	28	4,218
1959	1	29	4,192
	2	30	3,908
	3	31	3,753
	4	32	4,308

are not significant at the 0.10 level. The 0.10 level was selected because the basic data is not precise. In addition, this significance level was believed to be acceptable to the carpet industry. The "F" statistic for $\alpha = 0.10$ with one and twenty seven degrees of freedom is 2.90.

Table 7

Analysis of Variance Table

Source	$SS\times10^{12}$	df	$MS\times10^{12}$	F	
Linear	1503.32	1	1503.32	168.53	Significant
Quadratic	3.93	1	3.93	0.44	Not Significant
Cubic	1.67	1	1.67	0.19	Not Significant
Quartic	17.22	1	17.22	1.93	Not Significant
Residual	240.95	27	8.92		
Total	1767.09	31			

The trend equation calculated by the method of least squares is $\hat{T}_i = +1622.36 + 74.23i$. This estimate determined by the method of averages is:

$$\hat{T}_i = +1635.74 + 73.44i \tag{8}$$

The method of averages is reasonably satisfactory for $\hat{T}_i$ calculations of linear trends in industrial situations where computers are not available. Bryant (24) obtained a similar small variation in trend equations which are calculated by the two methods.

The linear trend equation and the actual Y_i values are shown in Figure 2.

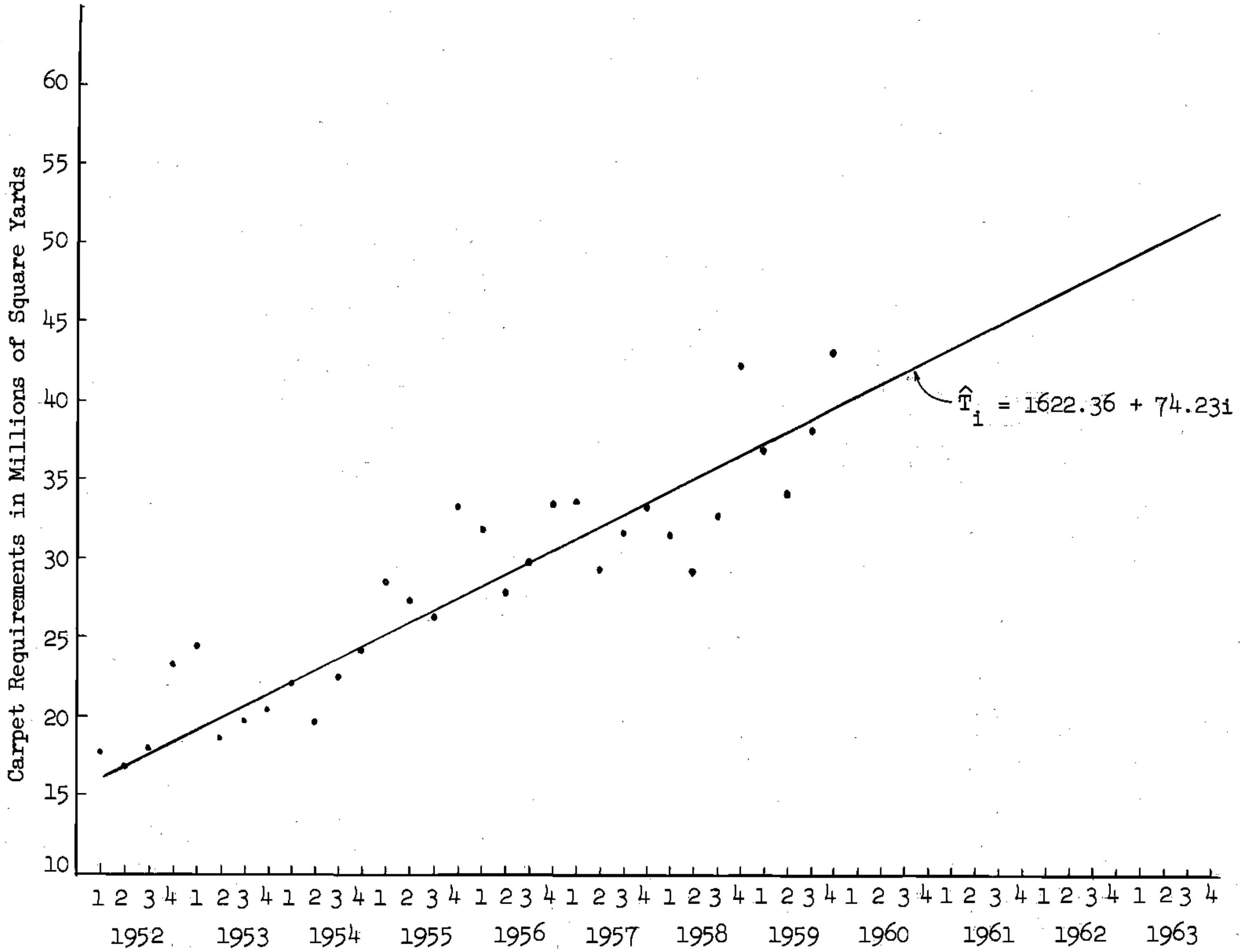

Figure 2. Linear Trend Equation and Actual Y_i's

Assuming the trend component, as given by equation 4, is correct, $\hat{T}_i$ error equals zero, the log Y_i - log $\hat{T}_i$ = log $(S_i \times C_i \times \epsilon_i)$ in the multiplicative model. The seasonal component, S_i, was calculated using quarterly averages of log $(S_i \times C_i \times \epsilon_i)$. The quarterly seasonal components, Table 8, are calculated in Appendix I.

Table 8

Quarterly Seasonal Components

Quarter	Logarithm of Seasonal Component
1	+0.0301
2	-0.0340
3	-0.0229
4	+0.0268

With the assumption that the $\hat{S}_i$ error is zero, the log Y_i -log $\hat{T}_i$ -log $\hat{S}_i$ = log $(C_i \times \epsilon_i)$ in the multiplicative model. The cyclical component, C_i, was calculated using a five point moving average to smooth the data, log $(C_i \times \epsilon_i)$. The results of the smoothing indicates a fifteen quarter cycle which is shown in Figure 3.

From the relationship

$$\log \hat{Y}_i = \log (1622.36 + 74.23i) + \log \hat{S}_i + \log \hat{C}_i, \qquad (9)$$

the total United States carpet shipments were predicted quarterly for the period from 1952 through 1959. A comparison of this prediction with the actual Y_i values is listed in Table 9. The average error of this systematic

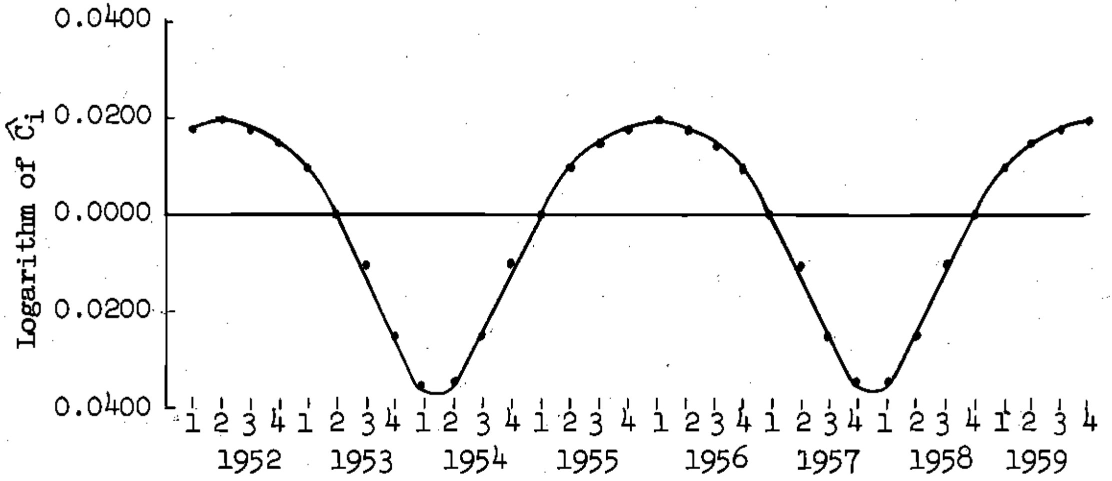

Figure 3. Cyclical Component of United States Carpet Shipments

Fifteen Quarter Cycle
Origin 1953-2

Quarter	Logarithm
0	0.0000
1	-0.0100
2	-0.0250
3	-0.0350
4	-0.0350
5	-0.0250
6	-0.0100
7	0.0000
8	+0.0100
9	+0.0150
10	+0.0175
11	+0.0200
12	+0.0175
13	+0.0150
14	+0.0100
15	0.0000

Table 9

Comparison of $\hat{Y}_i$ from Equation 9 with Y_i

Period i	Prediction $\hat{Y}_i$	Actual Y_i	Residual e_i
1	1,894	1,772	122
2	1,717	1,696	21
3	1,822	1,792	30
4	2,113	2,310	-197
5	2,187	2,442	-255
6	1,913	1,856	57
7	1,986	1,960	26
8	2,225	2,037	188
9	2,264	2,201	63
10	2,017	1,968	49
11	2,184	2,256	-72
12	2,612	2,422	190
13	2,773	2,852	-79
14	2,519	2,740	-221
15	2,687	2,615	72
16	3,112	3,235	-123
17	3,237	3,181	56
18	2,848	2,780	68
19	2,979	2,976	3
20	3,326	3,351	-25
21	3,410	3,558	-148
22	2,942	2,917	25
23	2,982	3,158	-176
24	3,340	3,311	29
25	3,439	3,150	289
26	3,101	2,914	187
27	3,362	3,282	80
28	3,936	4,218	-282
29	4,140	4,192	-52
30	3,684	3,908	-224
31	3,875	3,753	122
32	4,452	4,308	144

projection is 4.0 per cent ranging from -255 to +289.

The randomness of the residual error, e_i, was tested by the theory of runs (25). With twelve minus signs and twenty plus signs, the critical number of runs is eleven at the 0.05 significance level (26). The residual error, Table 9, has fifteen runs. Therefore, the residual error is random at the tested significance level. The 0.05 level was selected as the maximum error the carpet industry would accept.

The projected quarterly carpet shipments, $\hat{Y}_i$, for the period from 1960 through 1963 are listed in Table 13 and are shown in Figure 4. The actual carpet shipments for the period from 1952 through 1959 are plotted in Figure 4.

An equation to estimate annual import carpet deliveries was calculated by the method of averages. The data in Table 10 is related by the linear equation 11.

$$\widehat{IC}_i = 197.46 + 51.62i \quad (11)$$

Table 10

Annual Import Carpet Deliveries

Year	Import Carpet, IC_i (10,000 Square Yards)
1952	260
1953	332
1954	299
1955	415
1956	470
1957	486
1958	469
1959	707

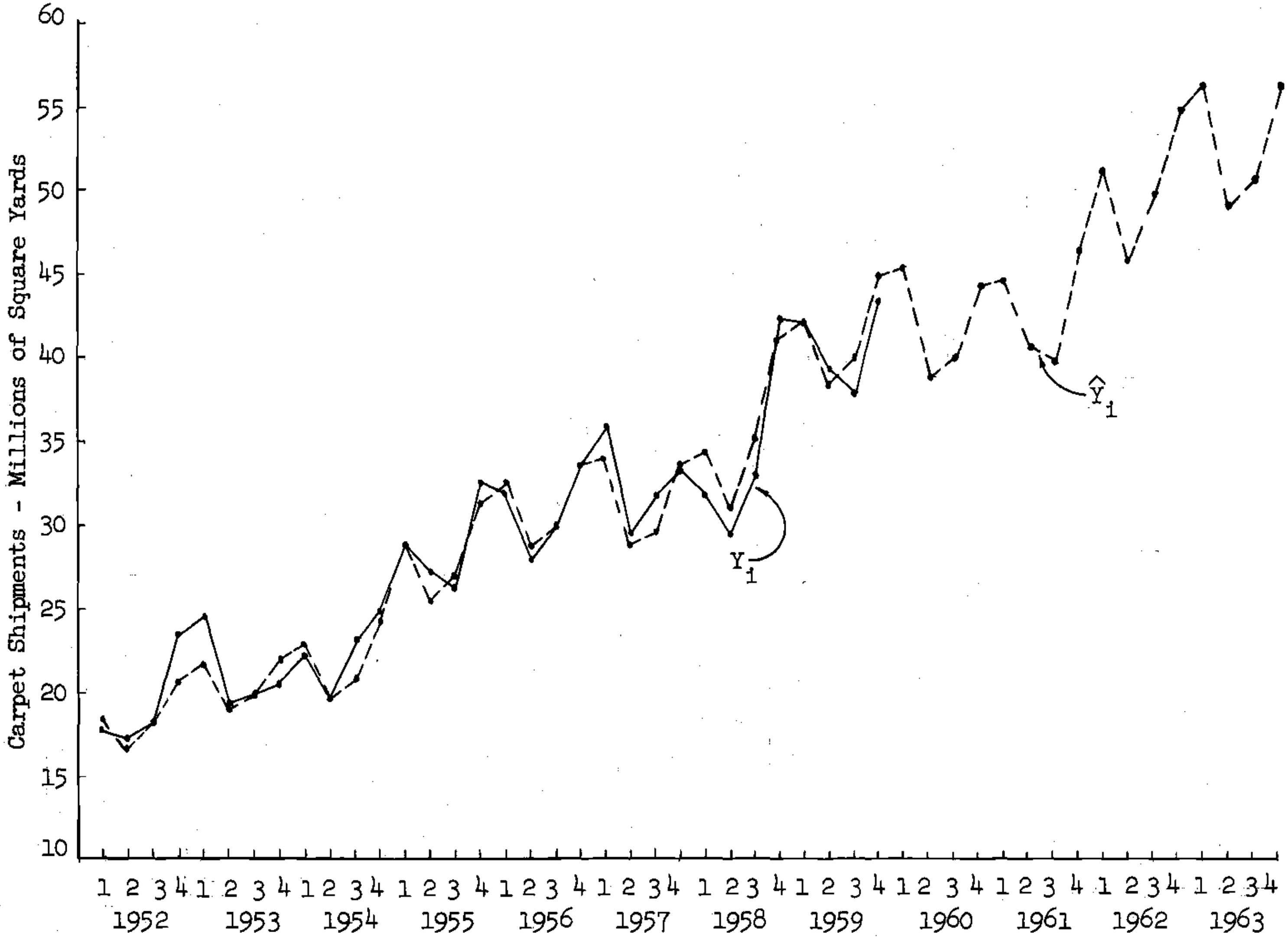

Figure 4. Total United States Carpet Shipments

Import carpet deliveries projected from 1960 through 1963 are listed in Table 11. The annual projection was prorated quarterly based on the projected quarterly per cent of annual carpet shipments. The domestic United States carpet shipments are obtained by subtracting $\widehat{IC}_i$ from $\hat{Y}_i$.

Table 11

Import Carpet Deliveries Projection

Year	Quarter	Import Carpet, $\widehat{IC}_i$ (10,000 Square Yards)
1960	1	175
	2	153
	3	158
	4	176
1961	1	193
	2	164
	3	167
	4	190
1962	1	191
	2	174
	3	185
	4	215
1963	1	213
	2	187
	3	196
	4	221

An equation for the tufted carpet per cent of domestic carpet shipments in the ith time period, denoted by $(\text{TC Per Cent})_i$, was calculated in order to separate the domestic carpet shipments by method of manufacture. The basic data is listed in Table 12.

Table 12

Tufted Carpet Per Cent of Domestic Carpet Shipments

Year	Period (i)	$(\text{TC Per Cent})_i$
1952	1	14.26
1953	2	20.45
1954	3	34.54
1955	4	42.93
1956	5	45.89
1957	6	55.33
1958	7	60.77
1959	8	62.14

By application of the method of differences, see Appendix I, a third degree polynomial equation was selected to fit to the data. Four points of annual data, 1953, 1955, 1957, and 1959, were selected for this equation. The Gauss elimination technique was used to solve the four linear equations. These calculations are listed in Appendix I. The resulting solution is given in equation 12. The actual and projected tufted carpet per cent of domestic carpet shipments is plotted in Figure 5.

$$(\widehat{\text{TC}}\ \text{Per Cent})_i = -16.60 + 22.91i - 2.38i^2 + 0.094i^3 \qquad (12)$$

Based on equation 12, the $(\widehat{\text{TC}}\ \text{Per Cent})_i$ was projected for the period from 1960 through 1963. The annual domestic tufted carpet shipments in square yards are calculated by

$$\widehat{\text{TC}}_i = (\widehat{\text{TC}}\ \text{Per Cent})_i \times (\hat{Y}_i - \widehat{\text{IC}}_i). \qquad (13)$$

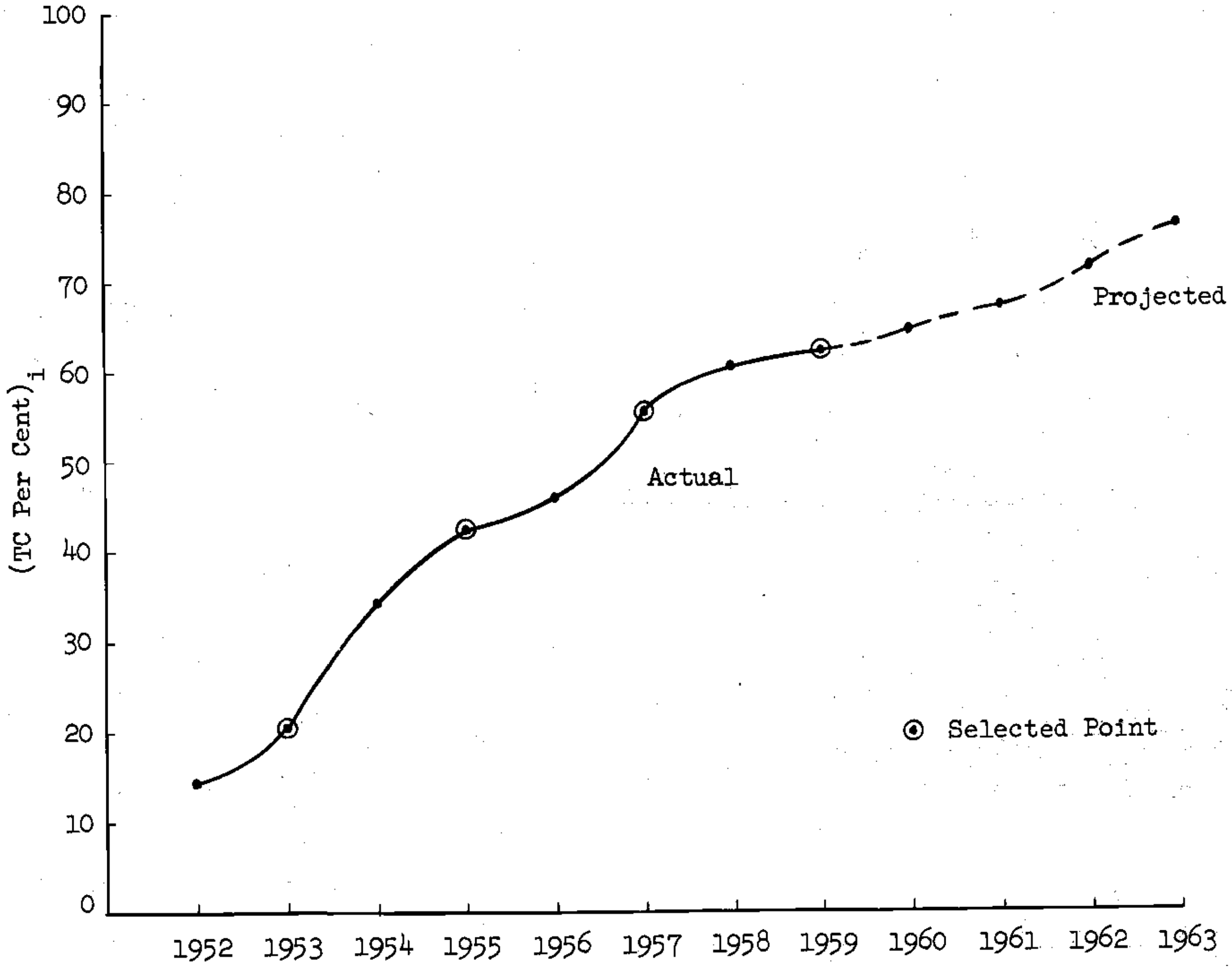

Figure 5. Actual and Projected $(\text{TC Per Cent})_i$

The annual data is prorated quarterly based on the quarterly per cent of annual United States carpet shipments.

Woven carpet shipments, WC_i, are calculated by

$$\widehat{WC}_i = \hat{Y}_i - \widehat{IC}_i - \widehat{TC}_i \qquad (14)$$

The projected United States carpet shipments, import carpet deliveries, and domestic carpet shipments by method of manufacture are listed in Table 13 and are plotted in Figure 6.

The systematic projection for the period from 1960 through 1963 assumes that the same conditions which existed from 1952 through 1959 will exist in the projected period. This projection indicates that carpet shipments will decline in 1961 as the cyclical component reaches a trough. An upturn is indicated in the fourth quarter of 1961. This upturn is projected to continue to a peak in 1963. Total United States carpet shipments in 1963 are projected to be 211,162,000 square yards as compared to 161,620,000 square yards in 1959.

The major increase is expected in tufted carpet. This type carpet is predicted to increase from 96,040,000 square yards in 1959 to 156,510,000 square yards in 1963. During the same period, woven carpet shipments are projected to decline from 58,508,000 square yards in 1959 to 46,940,000 square yards in 1963. Import carpet deliveries are expected to total 8,170,000 square yards in 1963 as compared to 7,070,000 square yards in 1959.

The cost of carpet production varies substantially between woven and tufted manufacturing methods. A typical carpet loom cost about three

Table 13

Quarterly Carpet Shipments Projection
1960 through 1963

Year	Quarter	Shipments Projection - 10,000 Square Yards			
		Y_i	IC_i	TC_i	WC_i
1960	1	4,544	175	2,833	1,536
	2	3,968	153	2,474	1,341
	3	4,096	158	2,554	1,384
	4	4,569	176	2,849	1,544
Total		17,177	662	10,710	5,805
1961	1	4,575	193	2,972	1,410
	2	3,879	164	2,521	1,194
	3	3,944	167	2,562	1,215
	4	4,506	190	2,928	1,388
Total		16,904	714	10,983	5,207
1962	1	4,725	191	3,249	1,285
	2	4,284	174	2,947	1,163
	3	4,567	185	3,141	1,241
	4	5,314	215	3,655	1,444
Total		18,890	765	12,992	5,133
1963	1	5,506	213	4,072	1,221
	2	4,850	187	3,587	1,076
	3	5,065	196	3,745	1,124
	4	5,741	221	4,247	1,273
Total		21,162	817	15,651	4,694

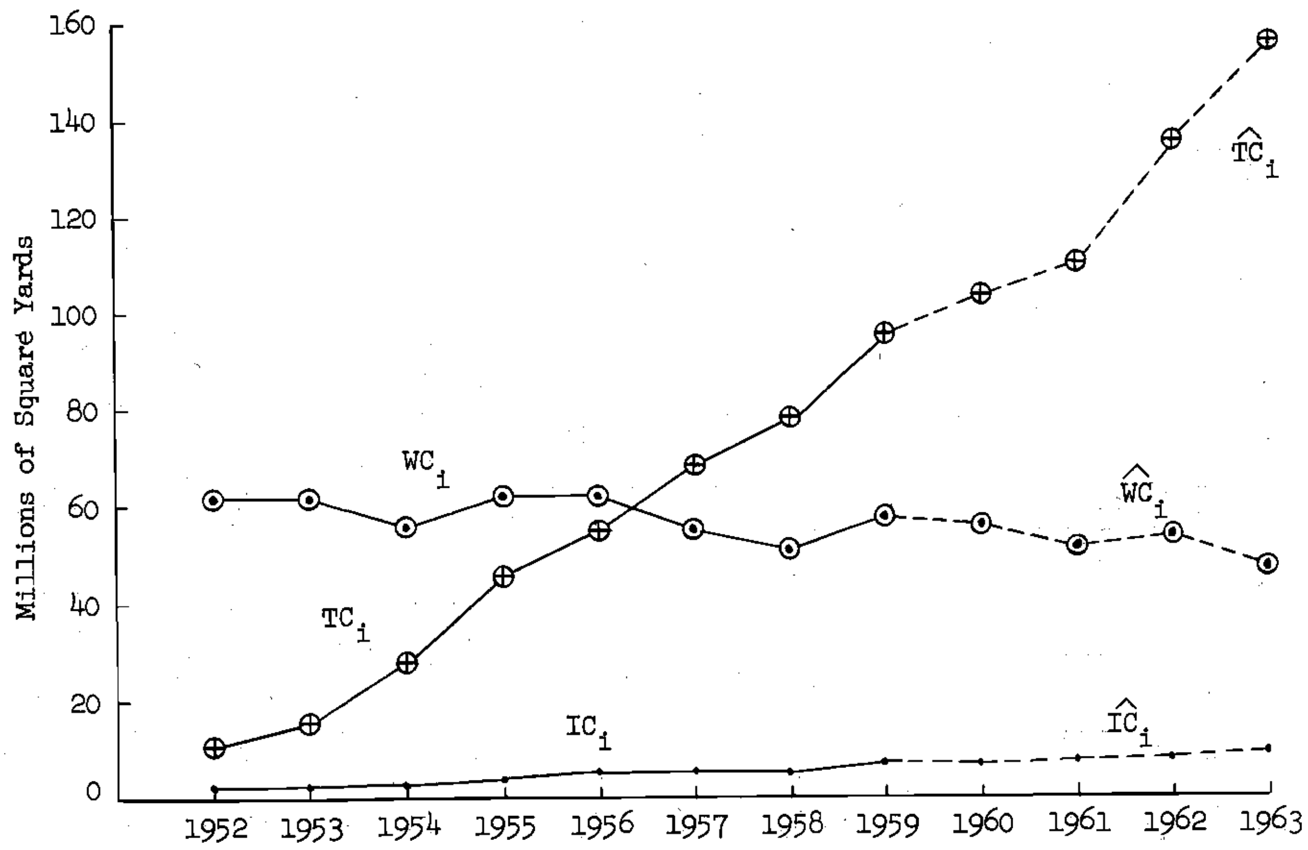

Figure 6. Annual Carpet Shipments - 1952-1963

times the cost of a tufting machine. In addition, the production rate of a loom is approximately one-eighth that of a tufting machine.

CHAPTER V

MULTIPLE REGRESSION ANALYSIS

The objective of this analysis is to develop a method of predicting short-term carpet shipments, that is, shipments one quarter in advance. This analysis is limited to the statistical technique of multiple regression analysis. The available data limits the prediction time period to one quarter or longer. The independent variables selected for the multiple regression model were based on an explainable relationship to carpet shipments.

The variables were classified in the following manner:

<u>Dependent Variable</u>

Y_i = carpet shipments in ten thousands of square yards during the ith time period in quarters.

<u>Independent Variables</u>

$X_{1(i-1)}$ = new housing starts in millions of dollars during the (i-1)th time period.

$X_{2(i-2)}$ = disposable personal income per household in dollars during the (i-2)th time period.

$X_{3(i-1)}$ = commercial construction in millions of dollars during the (i-1)th time period.

$X_{4(i+2)}$ = commercial aircraft shipments in number of aircraft during the (i+2)th time period.

$X_{5(i+1)}$ = factory sales in thousands of cars sold during the (i+1)th time period.

$X_{6(i-36)}$ = carpet shipments in thousands of square yards during the (i-36)th time period.

New housing starts were utilized as a variable because carpet is installed in new homes. After a study of the data in Figure 7, the time period i-1 was selected on the basis that a house must be started before carpet could be shipped from the carpet plant.

The second variable, disposable personal income per household, is a measure of the consumer's ability to buy carpet. After studying the actual data in Figure 7, the time period i-2 appeared to be the most feasible. The fact that the disposable personal income is seasonally adjusted is more important than the specific time period selected. Unadjusted data were not available from any publication. Neither the Research Department of the Federal Reserve Bank of Atlanta nor the National Income Division of the United States Department of Commerce could supply this information.

Commercial construction is an indicator of the quantity of carpet which will be installed in commercial buildings such as hotels and motels. The (i-1)th time period was selected by analysis of the data in Figure 8. This time lag compensates for the delay between starting construction and shipment of the carpet.

The fourth variable, commercial aircraft construction, was selected in order to include the carpet installed in aircraft. After a study of the data in Figure 8, the period i+2 was correlated because carpet must be shipped from the carpet plant in advance of delivery of the aircraft.

Factory sales of automobiles were correlated to measure the carpet installed in automobiles. After a study of the data in Figure 9, the time period i+1 was selected. This leading relationship is necessary to allow for shipment and processing of the carpet before the sale of the car.

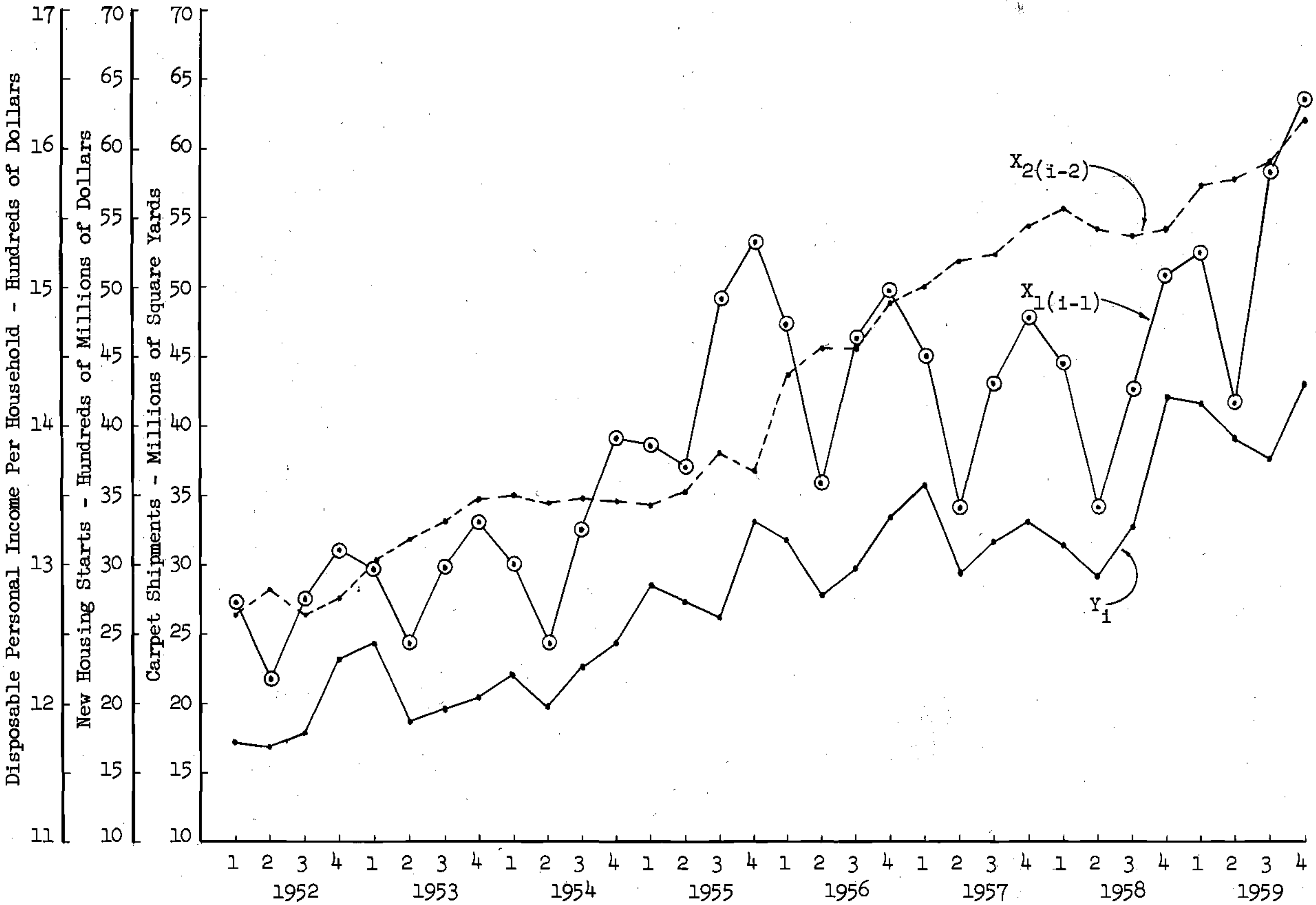

Figure 7. Selection of Time Periods - X_1 and X_2

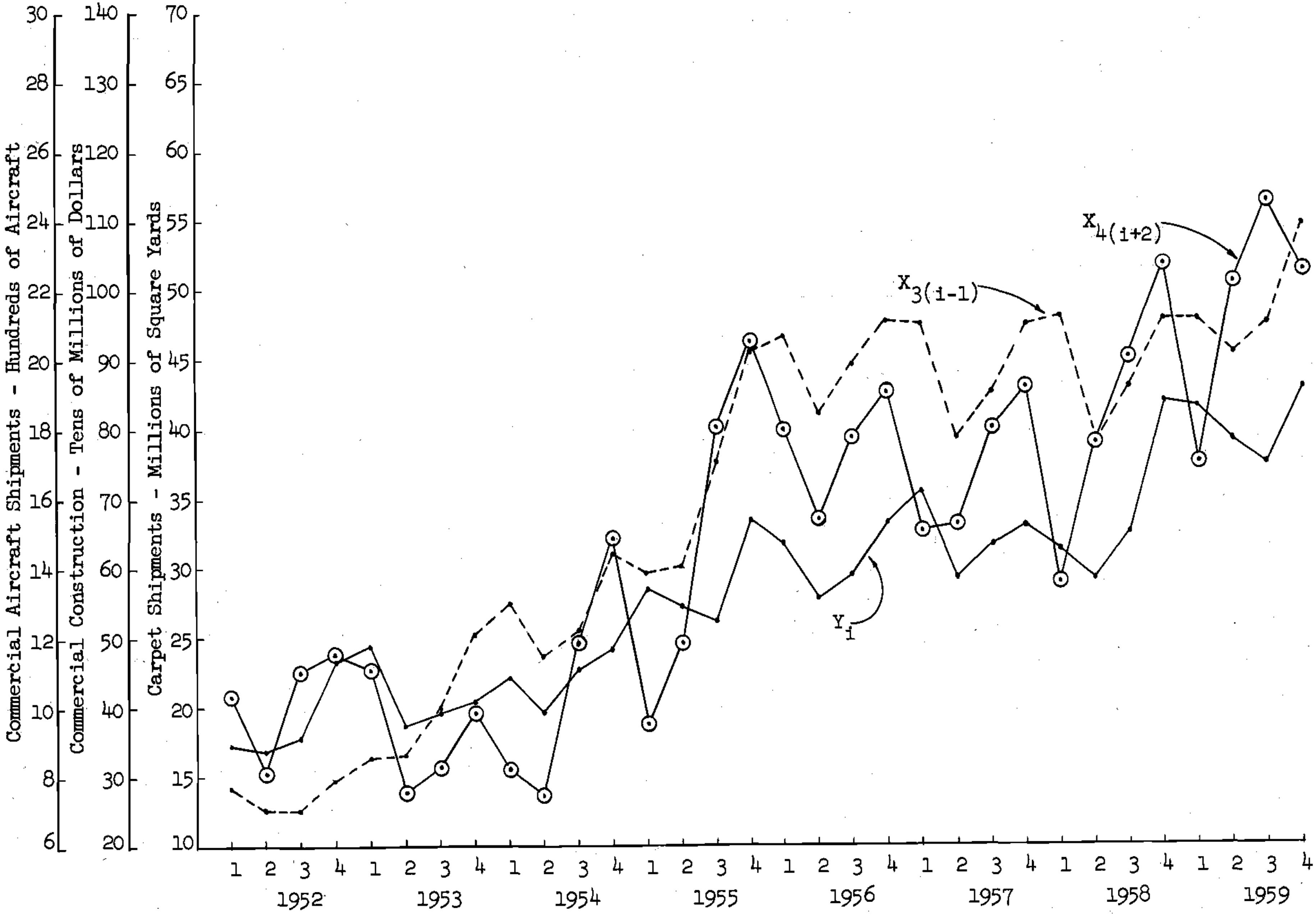

Figure 8. Selection of Time Periods - X_3 and X_4

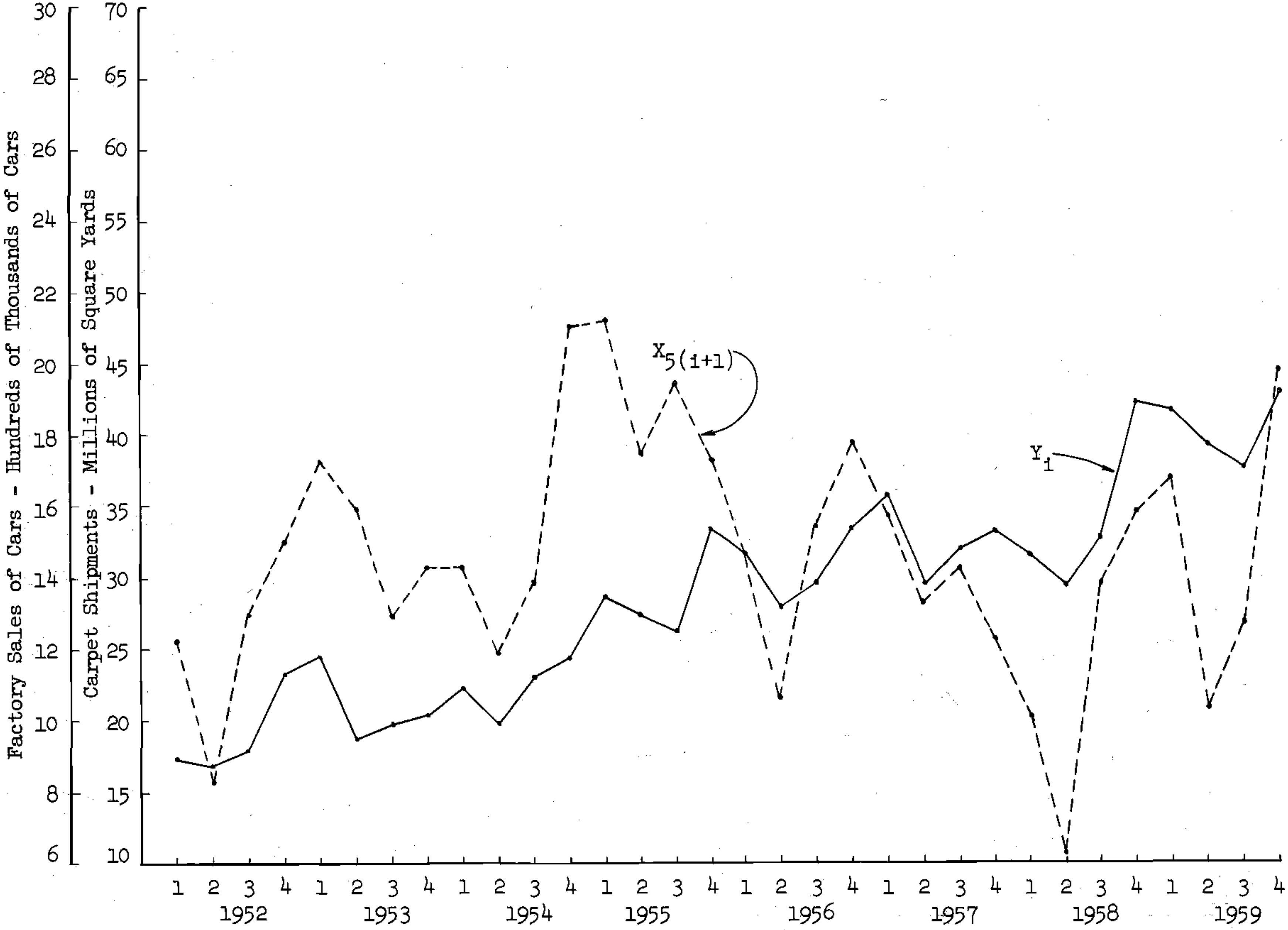

Figure 9. Selection of Time Period - X_5

The sixth variable, carpet shipments during the (1-36)th time period, is an auto-correlation variable to measure the replacement requirements for carpet. This period was based on a study of the data in Figure 10, and on discussions with representatives of the carpet industry.

The proposed multiple regression model, given in equation 15 below, assumes that carpet shipments are a linear function of each independent variable.

$$Y_i = B_0X_0 + B_1X_{1(i-1)} + B_2X_{2(i-2)} + B_3X_{3(i-1)} + B_4X_{4(i+2)} + B_5X_{5(i+1)} + B_6X_{6(i-36)} + \epsilon_i \tag{15}$$

where B_j = true regression coefficient for the jth independent variable

and ϵ_i = factors not otherwise accounted for in the equation.

The IBM 650 computer routine STO-1 was used in computing the correlation and regression statistics given in Table 15. The basic data for the analysis are listed in Table 14.

The multiple regression equation for the model equation 15 is given as follows:

$$Y_{i(\text{predicted})} = \hat{Y}_i = b_0X_0 + b_1X_{1(i-1)} + b_2X_{2(i-2)} + b_3X_{3(i-1)} + b_4X_{4(i+2)} + b_5X_{5(i+1)} + b_6X_{6(i-36)} \tag{16}$$

where $\hat{Y}_i - Y_i = e_i$ = estimate of the residual error for the ith time period.

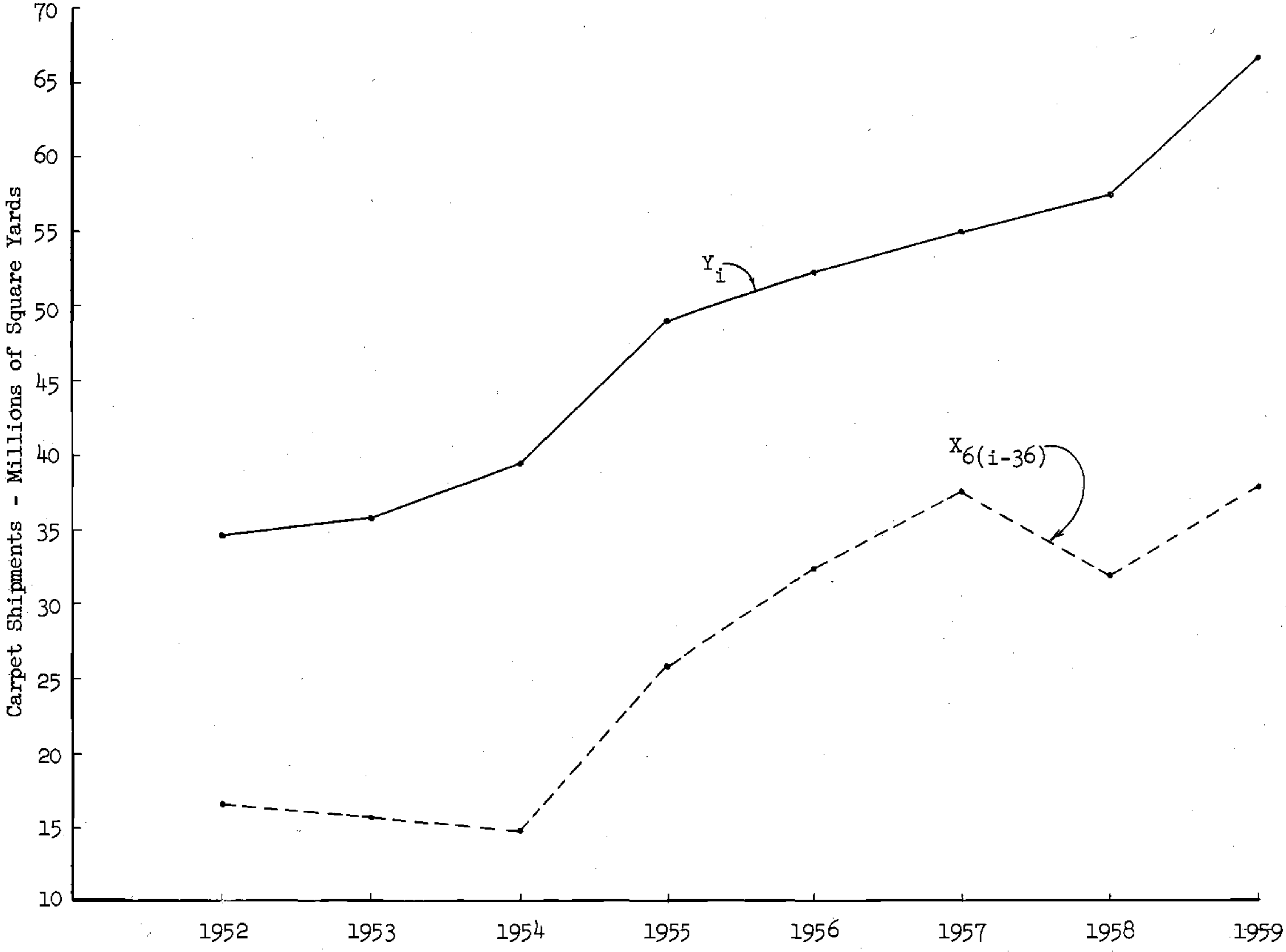

Figure 10. Selection of Time Period - X_6

Table 14

Basic Data for Regression Equation

Period	i	Y_i	$X_{1(i-1)}$	$X_{2(i-2)}$	$X_{3(i-1)}$	$X_{4(i+2)}$	$X_{5(i+1)}$	$X_{6(i-36)}$
1952-1	1	1,722	2,733	1,264	283	1,039	1,221	8,387
2	2	1,696	2,194	1,284	253	815	825	7,434
3	3	1,792	2,769	1,265	256	1,105	1,296	5,794
4	4	2,310	3,120	1,277	298	1,158	1,504	5,716
1953-1	5	2,442	2,999	1,304	330	1,111	1,730	7,293
2	6	1,856	2,437	1,318	334	760	1,586	6,403
3	7	1,960	2,999	1,333	395	830	1,297	6,046
4	8	2,037	3,333	1,349	509	984	1,433	5,538
1954-1	9	2,201	3,061	1,351	553	823	1,439	5,127
2	10	1,968	2,437	1,345	475	752	1,198	4,722
3	11	2,256	3,280	1,349	514	1,187	1,389	4,924
4	12	2,422	3,907	1,347	626	1,488	2,104	8,032
1955-1	13	2,852	3,872	1,343	597	959	2,122	11,034
2	14	2,740	3,719	1,353	603	1,186	1,747	13,079
3	15	2,615	4,915	1,381	757	1,807	1,946	12,520
4	16	3,325	5,346	1,373	919	2,054	1,735	15,443
1956-1	17	3,181	4,725	1,437	939	1,801	1,473	15,147
2	18	2,780	3,576	1,458	821	1,543	1,062	17,622
3	19	2,976	4,607	1,459	895	1,782	1,546	16,720
4	20	3,351	4,986	1,491	961	1,919	1,784	19,711
1957-1	21	3,558	4,508	1,501	954	1,510	1,575	21,348
2	22	2,917	3,429	1,521	789	1,534	1,324	22,562
3	23	3,158	4,307	1,525	856	1,808	1,430	19,281
4	24	3,311	4,808	1,547	951	1,930	1,234	20,789
1958-1	25	3,150	4,475	1,558	968	1,373	1,020	18,821
2	26	2,914	3,420	1,544	791	1,771	614	14,989
3	27	3,282	4,268	1,539	869	2,020	1,393	15,006
4	28	4,218	5,104	1,544	966	2,283	1,591	19,999
1959-1	29	4,192	5,255	1,575	963	1,709	1,686	20,993
2	30	3,908	4,167	1,579	811	2,230	1,033	20,834
3	31	3,753	5,867	1,594	960	2,469	1,280	22,670
4	32	4,308	6,390	1,623	1,100	2,264	1,993	21,223

Table 15

Results of Correlation Analysis

		Y_i	$X_{1(i-1)}$	$X_{2(i-2)}$	$X_{3(i-1)}$	$X_{4(i+2)}$	$X_{5(i+1)}$	$X_{6(i-36)}$
Averages		2,847.2	3,969.2	1,429.1	696.7	1,500.1	1,456.6	13,600.2
Standard Deviation		743.1	1,040.5	110.8	258.6	499.3	342.9	6,323.7
Simple Correlation Coefficients	r_y	1.00	0.88	0.88	0.89	0.86	0.20	0.88
	r_{X_1}	0.88	1.00	0.75	0.89	0.87	0.39	0.78
	r_{X_2}	0.88	0.75	1.00	0.89	0.81	-0.12	0.90
	r_{X_3}	0.89	0.89	0.89	1.00	0.84	0.14	0.89
	r_{X_4}	0.86	0.87	0.81	0.84	1.00	0.09	0.82
	r_{X_5}	0.20	0.39	-0.12	0.14	0.09	1.00	-0.01
	r_{X_6}	0.88	0.78	0.90	0.89	0.82	-0.01	1.00
Partial Correlation Coefficients		-1.00	0.24	0.53	-0.14	0.17	0.37	0.28
Regression Coefficients $b_{j's}$		-1.000	0.178	3.681	-0.421	0.187	0.390	0.026
Standard Error $S_{b_{j's}}$			0.148	1.191	0.592	0.219	0.198	0.018

The results of the correlation analysis were used to obtain the multiple regression equation:

$$\hat{Y}_i = -4{,}028.68 + 0.178X_{1(i-1)} + 3{,}681X_{2(i-2)} - 0.421X_{3(i-1)} + 0.187X_{4(i+2)} + 0.390X_{5(i+1)} + 0.026X_{6(i-36)}. \quad (17)$$

The constant b_0 is calculated in Appendix II and $X_0 = 1.00$.

The t test with (N-n) degrees of freedom was used to test the hypothesis that $b_j = B_j$, $(j = 2, . ., 7)$.

$$t_j = \frac{b_j - B_j}{S_{b_j}}, \quad B_j = 0 \quad (18)$$

N = number of observations

n = total number of variables = 6

S_{b_j} = standard error of b_j

The test statistic, t, with 32 - 6 degrees of freedom at the ninety per cent confidence level is 1.706. This significance level was chosen because the basic data is not precise. In addition, this significance level is believed to be acceptable to the carpet industry. The results of the t test are listed in Table 16.

Table 16

Results of t Test

Variable	t	Significance
$X_{1(i-1)}$	1.203	Not Significant at $\alpha = 0.10$
$X_{2(i-2)}$	3.091	Significant at $\alpha = 0.001$
$X_{3(i-1)}$	0.711	Not Significant at $\alpha = 0.10$
$X_{4(i+2)}$	0.854	Not Significant at $\alpha = 0.10$
$X_{5(i+1)}$	1.970	Significant at $\alpha = 0.10$
$X_{6(i-36)}$	1.444	Not Significant at $\alpha = 0.10$

The coefficient of multiple correlation, R, is calculated in Appendix II. This value for the complete regression equation is 0.957. This coefficient implies that 91.63 per cent, $R^2 \times 100$, of the variation of Y_i can be explained by the complete regression equation 17. A comparison of the prediction, $\hat{Y}_i$, with the actual Y_i values is shown in Table 17.

The average residual error of the prediction is 177.5 or 6.2 per cent. The range of error is 911, from -493 to +418.

When the variables which do not add significantly to the equation are deleted, the reduced regression equation becomes

$$\hat{Y}_i = b_0X_0 + b_2X_{2(i-2)} + b_5X_{5(i+1)} \qquad (19)$$

This reduced equation is further justified by the minor inter-correlation, r = -0.12, between variables $X_{2(i-2)}$ and $X_{5(i+1)}$.

Table 17

Comparison of $\hat{Y}_i$ from Equation 17 with Y_i

Period i	Prediction $\hat{Y}_i$	Actual Y_i	Residual e_i
1	1,878	1,772	+106
2	1,647	1,696	- 49
3	1,873	1,792	+ 81
4	2,051	2,310	-259
5	2,236	2,442	-206
6	2,041	1,856	+185
7	2,061	1,960	+101
8	2,200	2,037	+163
9	2,102	2,201	- 99
10	1,884	1,968	- 84
11	2,194	2,256	- 62
12	2,667	2,422	+245
13	2,645	2,852	-207
14	2,602	2,740	-138
15	3,033	2,615	+418
16	3,053	3,235	-182
17	3,013	3,181	-168
18	2,790	2,780	+ 10
19	3,156	2,976	+180
20	3,511	3,351	- 40
21	3,351	3,558	-207
22	3,240	2,917	+323
23	3,389	3,158	+231
24	3,506	3,311	+195
25	3,241	3,150	+ 91
26	2,891	2,914	- 23
27	3,341	3,282	+ 59
28	3,725	4,218	-493
29	3,824	4,192	-368
30	3,547	3,908	-361
31	4,032	3,753	+279
32	4,372	4,308	+ 66

The results of the multiple correlation analysis with the reduced equation are listed in Table 18.

Table 18

Results of Correlation Analysis

	Y_i	$X_{2(i-2)}$	$X_{5(i+1)}$
Averages	2,847.2	1,429.1	1,456.6
Standard Deviation	743.1	110.8	342.9
Simple Correlation Coefficients r_y	1.00	0.88	0.20
r_{X_2}	0.88	1.00	-0.12
r_{X_5}	0.20	-0.12	1.00
Partial Correlation Coefficients	-1.00	0.93	0.66
Regression Coefficients, $b_{j's}$	-1.000	6.172	0.674
Standard Error, $S_{b_{j's}}$		0.436	0.141

The results of the t test for the reduced regression equation 20 are listed in Table 19. The constant, b_0, is calculated in Appendix II.

$$\hat{Y}_i = -6,954.87 + 6.172X_{2(i-2)} + 0.674X_{5(i+1)} \tag{20}$$

Table 19

Results of t Test

Variable	t	Significance
$X_{2(i-2)}$	14.15	Significant at $\alpha = 0.001$
$X_{5(i+1)}$	4.77	Significant at $\alpha = 0.001$

Bryant (27) illustrates a method of testing the hypothesis that no regression is present, that is: H_o: $B_2 = B_5 = 0$. The results of this analysis for the reduced equation 20 are listed in Table 20.

Table 20

Analysis of Variance Table
Regression Data

Source	$SS \times 10^3$	df	$MS \times 10^3$	F
Linear Regression	9,222	2	4,611	16.6
Residuals from Regression	8,864	29	277	
Total	18,086	31		

The F value of 16.6 in Table 20 is significant at the 0.001 level.

The multiple correlation coefficient for the reduced equation 20 is 0.937. The reduced equation explains 87.8 per cent of the variation of Y_i.

The ninety per cent confidence interval for the variance of an individual $\hat{Y}_i$ is given by Hader and Grandage (28) as:

$$V(\hat{Y}_i) = \pm t_{\epsilon} \; s \sqrt{1 + \underline{X}'A^{-1}\underline{X}} \qquad (21)$$

where t = "Students t" test statistic,

s = square root of the residual error variance,

$\underline{X}'$ = vector $\left[X_0, X_{2(i-1)}, X_{5(i+1)}\right]$,

A^{-1} = inverse of the A matrix, Appendix II,

and $\underline{X}$ = vector $\begin{bmatrix} X_o \\ X_{2(i-1)} \\ X_{5(i+1)} \end{bmatrix}$.

The confidence intervals for several magnitudes of $X_{j's}$ at $t_{(0.90,\ 30)}$ are calculated in Appendix II and are listed in Table 21.

Table 21

Confidence Intervals of the Reduced Regression Equation 20

i	$\hat{Y}_i$	$X_{2(i-1)}$	$X_{5(i+1)}$	Interval
8	2,037	1,349	1,433	±615
25	3,150	1,558	1,020	±648
32	4,308	1,623	1,993	±667

A comparison of the prediction, $\hat{Y}_i$, with the actual Y_i values is shown in Table 22. The average error of the prediction from the reduced regression equation is 225.4 or 7.9 per cent. The range of error is 1,136 from -571 to +565.

Table 22

Comparison of $\hat{Y}_i$ from Equation (20) with Y_i

Period i	Prediction $\hat{Y}_i$	Actual Y_i	Residual e_i
1	1,675	1,772	- 97
2	1,526	1,696	-170
3	1,726	1,792	- 66
4	1,940	2,310	-370
5	2,259	2,442	-183
6	2,249	1,856	393
7	2,147	1,960	187
8	2,330	2,037	293
9	2,353	2,201	152
10	2,154	1,968	186
11	2,307	2,256	51
12	2,777	2,422	355
13	2,764	2,852	- 88
14	2,573	2,740	-167
15	3,180	2,615	565
16	2,689	3,235	-546
17	2,907	3,181	-374
18	2,760	2,780	- 20
19	3,092	2,976	116
20	3,450	3,351	99
21	3,371	3,558	-187
22	3,325	2,917	408
23	3,421	3,158	263
24	3,425	3,311	114
25	3,349	3,150	199
26	2,989	2,914	75
27	3,483	3,282	201
28	3,647	4,218	-571
29	3,902	4,192	-290
30	3,487	3,908	-421
31	3,746	3,753	- 9
32	4,406	4,308	- 98

A comparison of the reduced regression equation 20 with the full regression equation 17 is listed in Table 23.

Table 23

Comparison of Equations 17 and 20

Statistic	Equation 17	Equation 20
R	0.957	0.937
R^2	0.916	0.878
Average Error	177.5	225.4
Range of Error	911	1,136

Hader and Grandage (29) state that the rationale for discarding of variables in multiple regression equations is not yet sufficiently well developed. With this statement and the reduced precision, Table 23, of the prediction resulting from the reduced equation, the full regression equation could be utilized.

The theory of runs (30) lends additional support to the use of the full regression equation. The residual error of the prediction resulting from the full regression equation 17 has seventeen minus signs and seventeen plus signs. The critical number of runs at the 0.05 level of significance is eleven (31). The residual error, Table 17 has seventeen runs. Therefore, the residual error is considered random at the tested significance level. The residual error of the prediction resulting from the reduced regression equation 20 has fifteen minus signs and seventeen plus signs. The critical number of runs at the 0.05 significance level is eleven (32). The number of runs in the residual error, Table 22, is ten. Therefore, the residual error is not random at the 0.05 level of significance.

The results indicate that the full regression equation 17 should be utilized for prediction purposes.

CHAPTER VI

SUMMARY OF CONCLUSIONS AND DISCUSSION

Conclusions.--The findings of this study are based on a time series of total United States carpet shipments for eight years which were analyzed quarterly. With this limitation, the conclusions of this investigation are as follows:

(1) The systematic projection provides a better prediction of carpet shipments than the multiple regression equations.

(2) The systematic projection should be used for both the long-term and short-term prediction of carpet shipments.

Discussion.--The systematic technique provides a better prediction of carpet shipments than the multiple regression equations because:

(1) The average error of the systematic projection is 4.0 per cent as compared to an average error of 6.9 per cent for the prediction made from the full multiple regression equation.

(2) The range of error of the prediction made by the systematic technique is 544 as compared to a range of 911 in the error of the prediction made by the full multiple regression equation. (Note: The unit of error is 10,000 square yards.)

(3) The multiple regression equation contains two variables, commercial aircraft shipments and factory sales of cars, which lead the time period i by one and two quarters. These variables would usually be predicted by a systematic technique which would reduce the regression analysis to partial systematic analysis.

The full regression equation provides a prediction with a coefficient of multiple correlation of 0.957. This coefficient was obtained with a variable, disposable personal income, which was seasonally adjusted. Since this variable is the more significant of the six variables, additional study should be directed toward obtaining unadjusted disposable income data.

The multiple regression analysis was based on industry-wide data of questionable precision. This technique lends itself better to the study of shipments data of an individual company.

The systematic projection for the period from 1960 through 1963 assumes that the same conditions which existed from 1952 to 1959 will exist in the projected period. Variation in economic conditions and manufacturing technology will affect the validity of this projection.

The systematic projection indicates that carpet shipments will decline in 1961 as the cyclical component reaches a trough. An upturn is indicated in the fourth quarter of 1961. This upturn is projected to continue to a peak in 1963. The total United States carpet shipments in 1963 are projected to be 211,162,000 square yards as compared to 161,620,000 square yards in 1959.

The major increase is expected in tufted carpet. This type carpet is predicted to increase from 96,040,000 square yards in 1959 to 156,510,000 square yards in 1963. During the same period, woven carpet shipments are projected to decline from 58,508,000 square yards in 1959 to 46,940,000 square yards in 1963. A small increase in import carpet deliveries is indicated. Import carpet deliveries are projected to increase to

8,170,000 square yards in 1963 as compared to 7,070,000 square yards in 1959.

A comparison of the actual and predicted carpet shipments for the first two quarters of 1960 is listed in Table 24.

Table 24

Comparison of Actual and Predicted Carpet Shipments - January Through June 1960

	1,000 Square Yards		
	Predicted	Actual	Error
Woven Carpet	28,770	26,216	2,554
Tufted Carpet	53,070	56,140	-3,070
Imported Carpet	3,280	4,159	- 879
Total	85,120	86,515	-1,395

The cost of carpet production varies substantially between woven and tufted manufacturing methods. A typical carpet loom cost about three times the cost of a tufting machine. In addition, the production rate of a loom is approximately one-eighth that of a tufting machine.

Based on the data and limitations of this study, the production requirements of the carpet industry will continue to change as the trend from woven to tufted carpet continues into the period from 1960 to 1963. Industrial engineers should study these predicted changes in order to plan optimum manufacturing facilities for minimum cost operation of carpet plants.

APPENDIX I

Basic Data in 1,000 Square Yard Units for Table 6

Year	Quarter	Domestic Woven Carpet Shipments	Domestic Tufted Carpet Shipments	Import Carpet Deliveries Machine-Made	Total
1952	1	14,949	2,151	621	17,721
	2	14,307	2,059	594	16,960
	3	14,602	2,714	607	17,923
	4	18,819	3,497	781	23,097
1953	1	19,435	3,964	1,019	24,418
	2	14,771	3,012	774	18,557
	3	14,289	4,565	750	19,604
	4	14,851	4,744	778	20,373
1954	1	14,967	6,240	801	22,008
	2	13,382	5,578	716	19,676
	3	13,313	8,536	712	22,561
	4	14,294	9,166	764	24,224
1955	1	16,054	11,410	1,060	28,524
	2	15,423	10,963	1,018	27,404
	3	14,063	11,162	928	26,153
	4	17,395	13,810	1,148	32,353
1956	1	17,406	13,128	1,280	31,814
	2	15,206	11,471	1,118	27,795
	3	14,741	13,937	1,084	29,762
	4	16,593	15,692	1,220	33,505
1957	1	16,057	18,124	1,402	35,583
	2	13,162	14,859	1,150	29,171
	3	12,906	17,547	1,127	31,580
	4	13,528	18,396	1,182	33,106
1958	1	12,444	17,916	1,137	31,497
	2	11,515	16,577	1,053	29,145
	3	11,992	19,731	1,096	32,819
	4	15,414	25,359	1,408	42,181
1959	1	15,662	24,366	1,893	41,921
	2	14,600	22,718	1,764	39,082
	3	13,150	22,794	1,590	37,534
	4	15,096	26,162	1,825	43,083

Trend Prediction - Equation 4

Period	i	10,000 Square Yards Y_i	$\hat{T}_i$
1952-1	1	1,772	1,697
2	2	1,696	1,771
3	3	1,792	1,845
4	4	2,310	1,919
1953-1	5	2,442	1,994
2	6	1,856	2,068
3	7	1,960	2,142
4	8	2,037	2,216
1954-1	9	2,201	2,290
2	10	1,968	2,365
3	11	2,256	2,439
4	12	2,422	2,513
1955-1	13	2,852	2,587
2	14	2,740	2,662
3	15	2,615	2,736
4	16	3,235	2,810
1956-1	17	3,181	2,884
2	18	2,780	2,959
3	19	2,976	3,033
4	20	3,351	3,107
1957-1	21	3,558	3,181
2	22	2,917	3,256
3	23	3,158	3,330
4	24	3,311	3,404
1958-1	25	3,150	3,478
2	26	2,914	3,552
3	27	3,282	3,627
4	28	4,218	3,701
1959-1	29	4,192	3,775
2	30	3,908	3,849
3	31	3,753	3,924
4	32	4,308	3,998

Cycle Calculation - Figure 3

Period	i	log Y_i	log $\hat{T}_i$	log(C_ixS_ix ϵ_i)	log $\hat{S}_i$	log T_i+log S_i	log(C_ix ϵ_i)	5 Point Moving Average	log $\hat{C}_i$
1952-1	1	3.2485	3.2297	+0.0188	+0.0301	3.2598	-0.0113		+0.0175
2	2	3.2294	3.2482	-0.0188	-0.0340	3.2142	+0.0152		+0.0200
3	3	3.2533	3.2660	-0.0127	-0.0229	3.2431	+0.0102	+0.0136	+0.0175
4	4	3.3636	3.2831	+0.0806	+0.0268	3.3098	+0.0538	+0.0248	+0.0150
1953-1	5	3.3877	3.2997	+0.0880	+0.0301	3.3298	+0.0579	+0.0187	+0.0100
2	6	3.2686	3.3156	-0.0470	-0.0340	3.2816	-0.0130	+0.0039	0.0000
3	7	3.2923	3.3308	-0.0385	-0.0229	3.3079	-0.0156	-0.0163	-0.0100
4	8	3.3090	3.3456	-0.0366	+0.0268	3.3724	-0.0634	-0.0370	-0.0250
1954-1	9	3.3426	3.3598	-0.0172	+0.0301	3.3899	-0.0473	-0.0366	-0.0350
2	10	3.2940	3.3738	-0.0798	-0.0340	3.3398	-0.0458	-0.0421	-0.0350
3	11	3.3533	3.3872	-0.0339	-0.0229	3.3643	-0.0110	-0.0269	-0.0250
4	12	3.3842	3.4002	-0.0160	+0.0268	3.4270	-0.0428	-0.0081	-0.0100
1955-1	13	3.4552	3.4128	+0.0424	+0.0301	3.4429	+0.0123	+0.0017	0.0000
2	14	3.4378	3.4252	+0.0126	-0.0340	3.3912	+0.0466	+0.0108	+0.0100
3	15	3.4175	3.4371	-0.0196	-0.0229	3.4142	+0.0033	+0.0218	+0.0150
4	16	3.5099	3.4487	+0.0612	+0.0268	3.4755	+0.0344	+0.0207	+0.0175
1956-1	17	3.5026	3.4600	+0.0426	+0.0301	3.4901	+0.0125	+0.0143	+0.0200
2	18	3.4440	3.4711	-0.0271	-0.0340	3.4371	+0.0069	+0.0149	+0.0175
3	19	3.4736	3.4819	-0.0083	-0.0229	3.4590	+0.0146	+0.0117	+0.0150
4	20	3.5252	3.4923	+0.0329	+0.0268	3.5191	+0.0061	+0.0065	+0.0100
1957-1	21	3.5512	3.5026	+0.0486	+0.0301	3.5327	+0.0185	+0.0051	+0.0000
2	22	3.4649	3.5127	-0.0478	-0.0340	3.4787	-0.0138	-0.0056	-0.0100
3	23	3.4994	3.5224	-0.0230	-0.0229	3.4995	-0.0001	-0.0215	-0.0250
4	24	3.5200	3.5320	-0.0120	+0.0268	3.5588	-0.0388	-0.0356	-0.0350
1958-1	25	3.4983	3.5413	-0.0430	+0.0301	3.5714	-0.0731	-0.0369	-0.0350
2	26	3.4645	3.5505	-0.0860	-0.0340	3.5165	-0.0520	-0.0309	-0.0250
3	27	3.5161	3.5595	-0.0434	-0.0229	3.5366	-0.0205	-0.0200	-0.0100
4	28	3.6251	3.5683	+0.0568	+0.0268	3.5951	+0.0300	-0.0027	0.0000
1959-1	29	3.6264	3.5769	+0.0455	+0.0301	3.6070	+0.0154	+0.0138	+0.0100
2	30	3.5920	3.5853	+0.0067	-0.0340	3.5513	+0.0407	+0.0191	+0.0150
3	31	3.5744	3.5937	-0.0193	-0.0229	3.5707	+0.0036		+0.0175
4	32	3.6343	3.6018	+0.0325	+0.0268	3.6286	+0.0057		+0.0200

Seasonal Calculation - Table 8

Year	Quarter 1	2	3	4
1952	+0.0188	-0.0188	-0.0127	+0.0806
1953	+0.0880	-0.0470	-0.0385	-0.0366
1954	-0.0172	-0.0798	-0.0339	-0.0160
1955	+0.0424	+0.0126	-0.0196	+0.0612
1956	+0.0426	-0.0271	-0.0083	+0.0329
1957	+0.0486	-0.0478	-0.0230	-0.0120
1958	-0.0430	-0.0860	-0.0434	+0.0568
1959	+0.0455	+0.0067	-0.0193	+0.0325
Total	+0.2257	-0.2872	-0.1987	+0.1994
Average	+0.0282	-0.0359	-0.0248	+0.0249
Adjustment	+0.0019	+0.0019	-0.0019	+0.0019
Adjusted Average	+0.0301	-0.0340	-0.0229	+0.0268

Calculation of Y_i and e_i - Table 9

Period	i	$\hat{T}_i$	log $\hat{T}_i$	log $\hat{S}_i$	log $\hat{C}_i$	log $\hat{Y}_i$	$\hat{Y}_i$	Y_i	e_i
1952-1	1	1,697	3.2297	+0.0301	+0.0175	3.2773	1,894	1,772	+122
2	2	1,771	3.2482	-0.0340	+0.0200	3.2342	1,717	1,696	+ 21
3	3	1,845	3.2660	-0.0229	+0.0175	3.2606	1,822	1,792	+ 30
4	4	1,919	3.2831	+0.0268	+0.0150	3.3249	2,113	2,310	-197
1953-1	5	1,994	3.2997	+0.0301	+0.0100	3.3398	2,187	2,442	-255
2	6	2,068	3.3156	-0.0340	0.0000	3.2816	1,913	1,856	+ 57
3	7	2,141	3.3308	-0.0229	-0.0100	3.2979	1,986	1,960	+ 26
4	8	2,216	3.3456	+0.0268	-0.0250	3.3474	2,225	2,037	+188
1954-1	9	2,290	3.3598	+0.0301	-0.0350	3.3549	2,264	2,201	+ 63
2	10	2,365	3.3738	-0.0340	-0.0350	3.3048	2,017	1,968	+ 49
3	11	2,439	3.3872	-0.0229	-0.0250	3.3393	2,184	2,256	- 72
4	12	2,513	3.4002	+0.0268	-0.0100	3.4170	2,612	2,422	+190
1955-1	13	2,587	3.4128	+0.0301	0.0000	3.4429	2,773	2,852	- 79
2	14	2,662	3.4252	-0.0340	+0.0100	3.4012	2,519	2,740	-221
3	15	2,736	3.4371	-0.0229	+0.0150	3.4292	2,687	2,615	+ 72
4	16	2,810	3.4487	+0.0268	+0.0175	3.4930	3,112	3,235	-123
1956-1	17	2,884	3.4600	+0.0301	+0.0200	3.5101	3,237	3,181	+ 56
2	18	2,959	3.4711	-0.0340	+0.0175	3.4546	2,848	2,780	+ 68
3	19	3,033	3.4819	-0.0229	+0.0150	3.4740	2,979	2,976	+ 3
4	20	3,107	3.4923	+0.0268	+0.0100	3.5219	3,326	3,351	- 25
1957-1	21	3,181	3.5026	+0.0301	0.0000	3.5327	3,410	3,558	-148
2	22	3,256	3.5127	-0.0340	-0.0100	3.4687	2,942	2,917	+ 25
3	23	3,330	3.5224	-0.0229	-0.0250	3.4745	2,982	3,158	-176
4	24	3,404	3.5320	+0.0268	-0.0350	3.5238	3,340	3,311	+ 29
1958-1	25	3,478	3.5413	+0.0301	-0.0350	3.5364	3,439	3,150	+289
2	26	3,552	3.5505	-0.0340	-0.0250	3.4915	3,101	2,914	+187
3	27	3,627	3.5595	-0.0229	-0.0100	3.5266	3,362	3,282	+ 80
4	28	3,701	3.5683	+0.0268	0.0000	3.5951	3,936	4,218	-282
1959-1	29	3,775	3.5769	+0.0301	+0.0100	3.6170	4,140	4,192	- 52
2	30	3,849	3.5853	-0.0340	+0.0150	3.5663	3,684	3,908	-224
3	31	3,924	3.5937	-0.0229	+0.0175	3.5883	3,875	3,753	+122
4	32	3,998	3.6018	+0.0268	+0.0200	3.6486	4,452	4,308	+144

Projection of $\hat{Y}_i$ - Table 13

Period	i	$\hat{T}_i$	log $\hat{T}_i$	log $\hat{S}_i$	log $\hat{C}_i$	log $\hat{Y}_i$	$\hat{Y}_i$
1960-1	33	4,072	3.6098	+0.0301	+0.0175	3.6574	4,544
2	34	4,146	3.6176	-0.0340	+0.0150	3.5986	3,968
3	35	4,220	3.6253	-0.0229	+0.0100	3.6124	4,096
4	36	4,295	3.6330	+0.0268	0.0000	3.6598	4,569
1961-1	37	4,368	3.6403	+0.0301	-0.0100	3.6604	4,575
2	38	4,443	3.6477	-0.0340	-0.0250	3.5887	3,879
3	39	4,517	3.6548	-0.0229	-0.0350	3.5969	3,944
4	40	4,592	3.6620	+0.0268	-0.0350	3.6538	4,506
1962-1	41	4,666	3.6689	+0.0301	-0.0250	3.6740	4,725
2	42	4,470	3.6758	-0.0340	-0.0100	3.6318	4,284
3	43	4,814	3.6825	-0.0229	0.0000	3.6596	4,567
4	44	4,888	3.6886	+0.0268	+0.0100	3.7254	5,314
1963-1	45	4,963	3.6957	+0.0301	+0.0150	3.7408	5,506
2	46	5,037	3.7022	-0.0340	+0.0175	3.6857	4,850
3	47	5,111	3.7085	-0.0229	+0.0200	3.7056	5,065
4	48	5,185	3.7147	+0.0268	+0.0175	3.7590	5,741

Calculation of Equation 11

$$IC_i = a + bi$$

Year	i	IC_i (10,000 Square Yards)
1952	1	260
1953	2	332
1954	3	299
1955	4	415
Average	$2.5 = \bar{i}_1$	$326.5 = \overline{IC}_{i1}$
1956	5	470
1957	6	486
1958	7	469
1959	8	707
Average	$6.5 = \bar{i}_2$	$533.0 = \overline{IC}_{i2}$
Adjusted Average	$4.5 = \bar{i}$	$429.75 = \overline{IC}_i$

$$a = \overline{IC}_i - bi = 429.75 - 51.62(4.5) = +197.46$$

$$b = \frac{\overline{IC}_{i2} - \overline{IC}_{i1}}{\bar{i}_2 - \bar{i}_1} = \frac{533.00 - 326.50}{6.5 - 2.5} = +51.62$$

Projection of Equation 11

Year	i	$\hat{IC}_i$ (10,000 Square Yards)
1960	9	662
1961	10	714
1962	11	765
1963	12	817

Method of Differences

Selection of Polynomial Equation 12

$$(\widehat{TC}\ \text{Per Cent})_i = a + bi + ci^2 + di^3$$

Year	i	(TC Per Cent)$_i$	ΔY	$\Delta^2 Y$	$\Delta^3 Y$
1952	1	14.3			
1953	2	20.4	6.1		
1954	3	34.5	14.1	8.0	
1955	4	42.9	8.4	5.7	2.3
1956	5	45.9	3.0	5.4	2.7
1957	6	55.3	9.4	6.4	1.0
1958	7	60.8	5.5	3.9	1.5
1959	8	62.1	1.3	4.2	0.3

Selected Points

$$i \quad (\widehat{TC}\ \text{Per Cent})_i = a + bi + ci^2 + di^3$$

$$\begin{matrix} 2 \\ 4 \\ 6 \\ 8 \end{matrix} \quad \begin{bmatrix} 20.45 \\ 42.93 \\ 55.33 \\ 62.14 \end{bmatrix} \begin{matrix} = \\ = \\ = \\ = \end{matrix} \begin{bmatrix} a + b(2) + c(2)^2 + d(2)^3 \\ a + b(4) + c(4)^2 + d(4)^3 \\ a + b(6) + c(6)^2 + d(6)^3 \\ a + b(8) + c(8)^2 + d(8)^3 \end{bmatrix}$$

Solution of Selected Points Equations

Initial Tableau

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 4 & 16 & 64 \\ 1 & 6 & 36 & 216 \\ 1 & 8 & 64 & 512 \end{bmatrix} \qquad \begin{bmatrix} 20.45 \\ 42.93 \\ 55.33 \\ 62.14 \end{bmatrix}$$

Solution Tableau

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -16.5980 \\ +22.9140 \\ -\ 2.3820 \\ +\ 0.0935 \end{bmatrix}$$

Projection of Equation 12

Year	i	$(\widehat{TC}\ \text{Per Cent})_i$
1960	9	64.85
1961	10	67.84
1962	11	71.68
1963	12	76.93

APPENDIX II

Formulas for Computing Statistics

Mean: $\bar{X}_i = \frac{\Sigma X_i}{N}$

Where N = Total Number of Observations

Standard Deviation: $\sigma_i = \sqrt{(V_{ij})}$

Where $V_{ij} = \frac{1}{N} \left[\Sigma X_i X_j - \bar{X} \Sigma X_j\right]$

Simple Correlation Coefficient $r_{ij} = \frac{V_{ij}}{\sigma_i \sigma_j}$

Sample Calculations Equation 17

The matrix of simple correlation coefficients for the regression model is defined as A.

$$A = \begin{bmatrix} r_{yy} & r_{yx_1} & r_{yx_2} & r_{yx_3} & r_{yx_4} & r_{yx_5} & r_{yx_6} \\ r_{x_1y} & r_{x_1x_1} & r_{x_1x_2} & r_{x_1x_3} & r_{x_1x_4} & r_{x_1x_5} & r_{x_1x_6} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ r_{x_jy} & r_{x_jx_i} & r_{x_jx_2} & r_{x_jx_3} & r_{x_jx_4} & r_{x_jx_5} & r_{x_jx_6} \end{bmatrix}$$

For the sample data analyzed in this study, the A matrix is listed in Table 15. The inverse of this matrix is as follows:

$$\begin{bmatrix} 11.95 & -2.98 & -6.56 & 1.75 & -1.50 & -2.15 & -2.67 \\ -2.98 & 13.05 & 1.90 & -6.96 & -5.07 & -2.84 & 1.09 \\ -6.56 & 1.90 & 12.65 & -5.94 & -0.04 & 2.89 & -1.75 \\ 1.75 & -6.96 & -5.94 & 12.41 & 1.22 & -0.17 & -2.80 \\ -1.50 & -5.07 & -0.04 & 1.22 & 6.39 & 1.52 & -1.00 \\ -2.15 & -2.84 & 2.89 & -0.17 & 1.52 & 2.77 & 0.44 \\ -2.67 & 1.09 & -1.75 & -2.80 & -1.00 & 0.44 & 7.41 \end{bmatrix}$$

Where $(a_{ij}) = (r_{ij})^{-1}$

Where $(r_{ij})^{-1} = A^{-1}$ which is the inverse of the simple correlation coefficients. The elements a_{ij}, of this inverse matrix, are used in computing the following statistics.

Multiple Regression Coefficients

$$b_j = - \frac{a_{1j} \sigma_1}{a_{11} \sigma_j} \; (j = 2,3,4,5,6,7) \text{ where } \sigma j =$$

standard deviation of the jth variable, and σ_1 = standard deviation of the dependent variable, Y_i.

$$b_2 = - \left[\frac{-2.982 \times 0.743}{11.947 \times 1.041} \right] = 0.178$$

$$b_3 = - \left[\frac{-6.558 \times 0.743}{11.947 \times 0.1108} \right] = 3.681$$

$$b_4 = - \left[\frac{1.750 \times 0.743}{11.947 \times 0.2586} \right] = 0.421$$

$$b_5 = - \left[\frac{-1.502 \times 0.743}{11.947 \times 0.4993} \right] = 0.187$$

$$b_6 = - \left[\frac{-2.1473 \times 0.743}{11.947 \times 0.3429} \right] = 0.390$$

$$b_7 = - \left[\frac{-2.666 \times 0.743}{11.947 \times 6.324} \right] = 0.026$$

Partial Correlation Coefficients

$$r_{ij} = r_{yj} = \frac{-a_{ij}}{\sqrt{a_{11}\ a_{jj}}}$$

$$r_{12} = -\left[\frac{-2.982}{\sqrt{11.947 \times 13.048}}\right] = 0.239$$

$$r_{13} = -\left[\frac{-6.558}{\sqrt{11.947 \times 12.649}}\right] = 0.533$$

$$r_{14} = -\left[\frac{1.750}{\sqrt{11.947 \times 12.414}}\right] = 0.144$$

$$r_{15} = -\left[\frac{-1.502}{\sqrt{11.947 \times 12.414}}\right] = 0.172$$

$$r_{16} = -\left[\frac{-2.147}{\sqrt{11.947 \times 2.772}}\right] = 0.373$$

$$r_{17} = -\left[\frac{-2.666}{\sqrt{11.947 \times 7.405}}\right] = 0.283$$

Biased Standard Error of the Estimate

$$S_e' = \frac{\sigma\ 1}{\sqrt{a_{11}}} = \frac{0.743}{\sqrt{11.947}} = 0.215$$

Unbiased Standard Error of the Estimate

$$S_e = \sqrt{\frac{N}{N - n}}\ S_e' = \sqrt{\frac{32}{32-7}}\ (0.215) = 0.243$$

Multiple Correlation Coefficient

$$R = \sqrt{1 - \frac{1}{a_{11}}} = \sqrt{1 - \frac{1}{11.947}} = 0.957$$

Unbiased Standard Error of Multiple Regression Coefficients

$$S_{bj} = \frac{b_j}{r_{1j}} \sqrt{\frac{1 - r_{1j}^2}{N - n}}$$ for (j = 2, 3, 4, 5, 6, 7) where

r_{12} = partial correlation coefficient between the dependent variable and the 2th variable and b_j = regression coefficient.

N = Total number of observations.

n = Total number of variables.

$$S_2 = \frac{0.178}{0.239} \sqrt{\frac{1 - (0.239)^2}{(32 - 7)}} = 0.147$$

$$S_3 = \frac{3.681}{0.533} \sqrt{\frac{1 - (0.533)^2}{32 - 7}} = 1.191$$

$$S_4 = \frac{-0.421}{-0.144} \sqrt{\frac{1 - (-0.144)^2}{32 - 7}} = 0.592$$

$$S_5 = \frac{0.187}{0.172} \sqrt{\frac{1 - (0.172)^2}{32 - 7}} = 0.219$$

$$S_6 = \frac{0.390}{0.373} \sqrt{\frac{1 - (0.373)^2}{32 - 7}} = 0.198$$

$$S_7 = \frac{0.026}{0.283} \sqrt{\frac{1 - (0.283)^2}{32 - 7}} = 0.018$$

Calculation of b_o, Equation 17

$$\overline{Y} = b_o X_o + b_1 \overline{X}_{1(i-1)} + b_2 \overline{X}_{2(i-2)} + b_3 \overline{X}_{3(i-1)}$$

$$+ b_4 \overline{X}_{4(i+2)} + b_5 \overline{X}_{5(i+1)} + b_6 \overline{X}_{6(i-36)}$$

where $\overline{X}_o = 1.00$

$$b_o = \overline{Y} - b_1\,\overline{X}_{1(i-1)} - b_2\,\overline{X}_{2(i-2)} - b_3\,\overline{X}_{3(i-1)}$$

$$- b_4\,\overline{X}_{4(i+2)} - b_5\,\overline{X}_{5(i+1)} - b_6\,\overline{X}_{6(i-36)}$$

$\overline{Y} =$			$+2,847.22$
$-b_1\,\overline{X}_{1(i-1)}$	$= -(0.178)(3969.156)$	$=$	$-\ \ 706.51$
$-b_2\,\overline{X}_{2(i-2)}$	$= -(3.681)(1429.094)$	$=$	$-5,260.50$
$-b_3\,\overline{X}_{3(i-1)}$	$= -(-0.421)(696.656)$	$=$	$+\ \ 293.29$
$-b_4\,\overline{X}_{4(i+2)}$	$= -(0.187)(1500.125)$	$=$	$-\ \ 280.52$
$-b_5\,\overline{X}_{5(i+1)}$	$= -(0.390)(1456.563)$	$=$	$-\ \ 568.06$
$-b_6\,\overline{X}_{6(i-36)}$	$= -(0.026)(13,600.22)$	$=$	$-\ \ 353.60$
b_o		$=$	$-4,028.68$

Sample Calculations Equation 20

The matrix of simple correlation coefficients for the regression model is defined as A.

$$A = \begin{bmatrix} r_{yy} & r_{yx_2} & r_{yx_5} \\ r_{x_2y} & r_{x_2x_2} & r_{x_2x_5} \\ r_{x_5y} & r_{x_5x_2} & r_{x_5x_5} \end{bmatrix}$$

For the sample data analyzed in this study, the A matrix is listed in Table 18. The inverse of this matrix is as follows:

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{15} \\ a_{21} & a_{22} & a_{25} \\ a_{51} & a_{52} & a_{55} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 8.228 & -7.587 & -2.561 \\ -7.587 & 8.010 & 2.483 \\ -2.561 & 2.483 & 1.812 \end{bmatrix}$$

where $(a_{ij}) = (r_{ij})^{-1}$

where $(r_{ij})^{-1} = A^{-1}$ which is the inverse of the simple correlation coefficients. The elements a_{ij}, of this inverse matrix, are used in computing the following statistics.

<u>Multiple Regression Coefficients</u>

$$b_j = -\frac{a_{1j}\,\sigma_1}{a_{11}\,\sigma_j} \quad (j = 2,5) \text{ where } \sigma_j =$$

standard deviation of the jth variable, and σ_1 = standard deviation of the dependent variable, Y_i.

$$b_2 = -\left[\frac{-7.587 \times 0.743}{8.228 \times 0.111}\right] = 6.172$$

$$b_5 = -\left[\frac{-2.561 \times 0.734}{8.228 \times 0.343}\right] = 0.674$$

Partial Correlation Coefficients

$$r_{ij} = r_{yj} = \frac{-a_{ij}}{\sqrt{a_{11}a_{jj}}}$$

$$r_{12} = -\left[\frac{-7.587}{\sqrt{8.228 \times 8.010}}\right] = 0.934$$

$$r_{15} = -\left[\frac{-2.561}{\sqrt{8.228 \times 1.812}}\right] = 0.663$$

Biased Standard Error of the Estimate

$$S_e' = \frac{\sigma_1}{\sqrt{a_{11}}} = \frac{0.743}{\sqrt{8.228}} = 0.259$$

Unbiased Standard Error of the Estimate

$$S_e = \sqrt{\frac{N}{N - n}}\ S_e' = \sqrt{\frac{32}{32 - 3}}\ (0.259) = 0.272$$

Multiple Regression Coefficient

$$R = \sqrt{1 - \frac{1}{a_{11}}} = \sqrt{1 - \frac{1}{8.228}} = 0.937$$

Unbiased Standard Error of Multiple Regression Coefficients

$$S_{b_j} = \frac{b_j}{r_{1j}} \sqrt{\frac{1 - (r_{1j})^2}{N - n}}$$

$$S_2 = \frac{6.172}{0.934} \sqrt{\frac{1 - (0.934)^2}{32 - 3}} = 0.436$$

$$S_5 = \frac{0.674}{0.663} \sqrt{\frac{1 - (0.663)^2}{32 - 3}} = 0.141$$

Calculation of b_o, Equation 20

$$\bar{Y} = b_o X_o + b_2 \bar{X}_{2(i-2)} + b_{5(i+1)}$$

where $\bar{X}_o = 1.00$

$$b_o = \bar{Y} - b_2 X_{2(i-2)} - b_5 \bar{X}_{5(i+1)}$$

$\bar{Y}$		= +2,847.22
$-b_2 \bar{X}_{2(i-2)}$	= (6.172)(1429.094)	= -8,820.37
$-b_5 \bar{X}_{5(i+1)}$	= (0.674)(1456.563)	= - 981.72
		-6,954.87

Analysis of Variance Calculation - Table 20

$$G_{2y} = n \Sigma(X_{2(i-2)}Y) - \Sigma X_{2(i-2)} \Sigma Y = 74,617,387$$

$$G_{5y} = n \Sigma(X_{5(i+1)}Y) - \Sigma X_{5(i+1)} \Sigma Y = 52,435,010$$

$$G_{yy} = n \Sigma Y^2 - (\Sigma Y)^2 = 578,771,375$$

Linear Regression $= (\frac{1}{n})(b_2 G_{2y} + b_5 G_{5y}) = 9.222 \times 10^3$

Residual From Regression $= (\frac{G_{yy}}{n}) - (\frac{1}{n})(b_2 G_{2y} + b_5 G_{5y}) = 8,864 \times 10^3$

Mean Square of Linear Regression $= \frac{\text{Linear Regression}}{\text{df}} = 4,611 \times 10^3$

Mean Square of Residual From Regression $= \frac{\text{Residual From Regression}}{\text{df}} = 277 \times 10^3$

$$\text{"F" Statistic} = \frac{4.611 \times 10^3}{277 \times 10^3} = 16.6$$

90 Per Cent Confidence Interval, Multiple Regression

$$\text{Variance } (\hat{Y}_i) = + T_{0.10}\, S \sqrt{1 + \underline{X}'\, A^{-1}\, \underline{X}}$$

From "T " Tables, $_{0.10} = 1.699$ (d.f. = 29)

$$S = \sqrt{\frac{(\hat{Y}_i - Y_i)^2}{n - 3}} = 287.237$$

$$\underline{X}' = \begin{bmatrix} X_o & X_{2(i-2)} & X_{5(i+1)} \end{bmatrix}$$

$$A = \begin{bmatrix} \Sigma X_o X_o & \Sigma X_o X_{2(i-2)} & \Sigma X_{5(i+1)} \\ \Sigma X_{2(i-2)} & \Sigma (X_{2(i-2)})^2 & \Sigma X_{2(i-2)} X_{5(i+1)} \\ \Sigma X_{5(i+1)} & \Sigma X_{2(i-2)} X_{5(i+1)} & \Sigma (X_{5(i+1)})^2 \end{bmatrix}$$

where $X_o = 1$

$$A = \begin{bmatrix} 32 & 45{,}730 & 46{,}610 \\ 45{,}730 & 65{,}750 \times 10^3 & 66{,}460 \times 10^3 \\ 46{,}610 & 66{,}460 \times 10^3 & 71{,}650 \times 10^3 \end{bmatrix}$$

$$A^{-1} \begin{bmatrix} 6.218 & -3.582 \times 10^{-3} & -0.537 \times 10^{-3} \\ -3.582 \times 10^{-3} & 2.544 \times 10^{-6} & 0.101 \times 10^{-6} \\ -0.537 \times 10^{-3} & 0.101 \times 10^{-6} & 0.270 \times 10^{-6} \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} X_o \\ X_{2(i-2)} \\ X_{5(i+1)} \end{bmatrix}$$

Calculation of 90 Per Cent Confidence Interval - Equation 20, Table 21

$i = 8$

$$\underline{X}' = \begin{bmatrix} 1 & 1349 & 1433 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 1 \\ 1349 \\ 1433 \end{bmatrix}$$

$$\text{Variance } (\hat{Y}_i) = \pm 1.699(287.237)\sqrt{1 + 0.5879} = \pm 615$$

$i = 25$

$$X' = \begin{bmatrix} 1 & 1558 & 1020 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 1 \\ 1558 \\ 1020 \end{bmatrix}$$

$$\text{Variance } (\hat{Y}_i) = +1.699(287.237)\sqrt{1 + 0.7661} = \pm 648$$

$i = 32$

$$\underline{X}' = \begin{bmatrix} 1 & 1623 & 1993 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} 1 \\ 1623 \\ 1993 \end{bmatrix}$$

$$\text{Variance } (\hat{Y}_i) = \pm 1.699(287.237)\sqrt{1 + 0.8669} = +667$$

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