NON-REPETITIVE COLOURINGS

Graph theory

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NON-REPETITIVE SEQUENCES

$X = x_1 \, \boxed{x_2 \, x_3} \, \boxed{x_4 \, x_5} \, \ldots \, x_n$ is **non-repetitive**

if no 2 consecutive blocks are the same

NON-REPETITIVE SEQUENCES

$X = x_1 \, x_2 \, x_3 \, x_4 \, x_5 \ldots x_m$ is **non-repetitive**

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example:

1 2 3 4 2 3 4 2 1 — repetitive

1 2 3 1 3 2 1 2 3 2 1 3 — non-repetitive

1 2 3 1 3 2 1 2 3 2 1 3

THUE's THEOREM

1903

3 symbols are enough.

(2 are not)

12313212321213

THUE's THEOREM

1903

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(2 are not)

Construction via SUBSTITUTION:

1 → 12312

2 → 131232

3 → 1323132

GENERALIZATION TO GRAPHS

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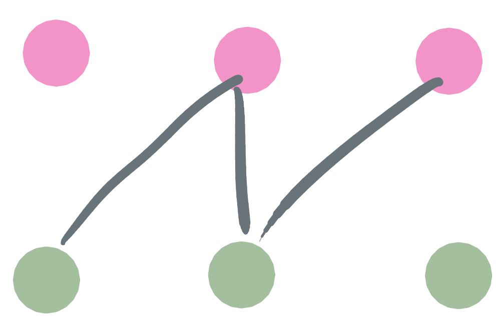

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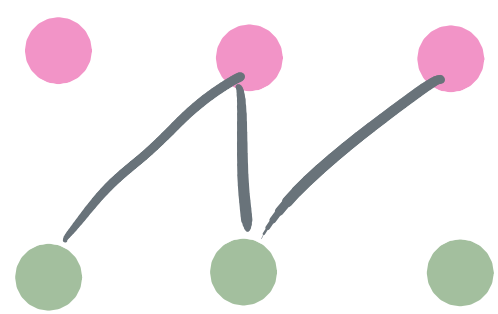


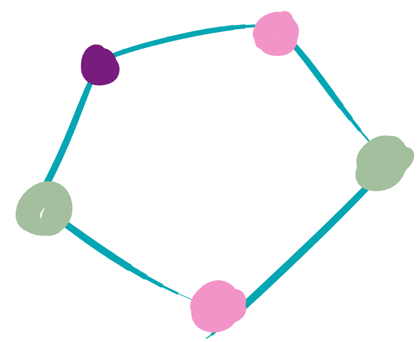
testing co-NP

Marx & Schaefer (2009)

π of GRAPH CLASSES

thue's th : $\pi(P_n) = 3$

$\pi(C_n) \leqslant 4$



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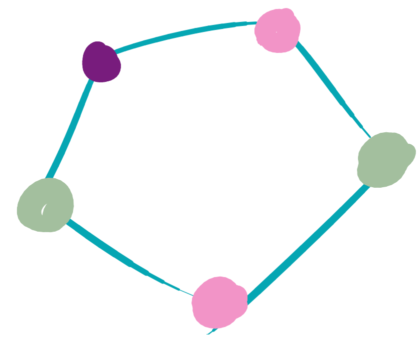

Currie (2005):

$\pi(C_n) = 3$ unless $n = 5, 7, 9, 10, 14, 17$

$\Rightarrow$ bounded when $\Delta(G) \leq 2$

BOUNDED DEGREE

Alon, Grytczuk, Hałuszczak, Riordan (2002):

$$c' \frac{\Delta^2}{\log \Delta} \leq \pi(G) \leq c \cdot \Delta^2$$

$G(n,p)$

LLL

Dujmović, Joret, Kozik, Wood (2012): $c \to 1$

via entropy compression of Moser & Tardos

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PLANAR GRAPHS

Alon et al (2002):

Is π(planar) bounded?

- true for X_{st}
- even for π_K

$\pi(\text{planar}) \le \sqrt{n}$

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$$\pi(G) \le \log n$$

$\to \forall$ planar G

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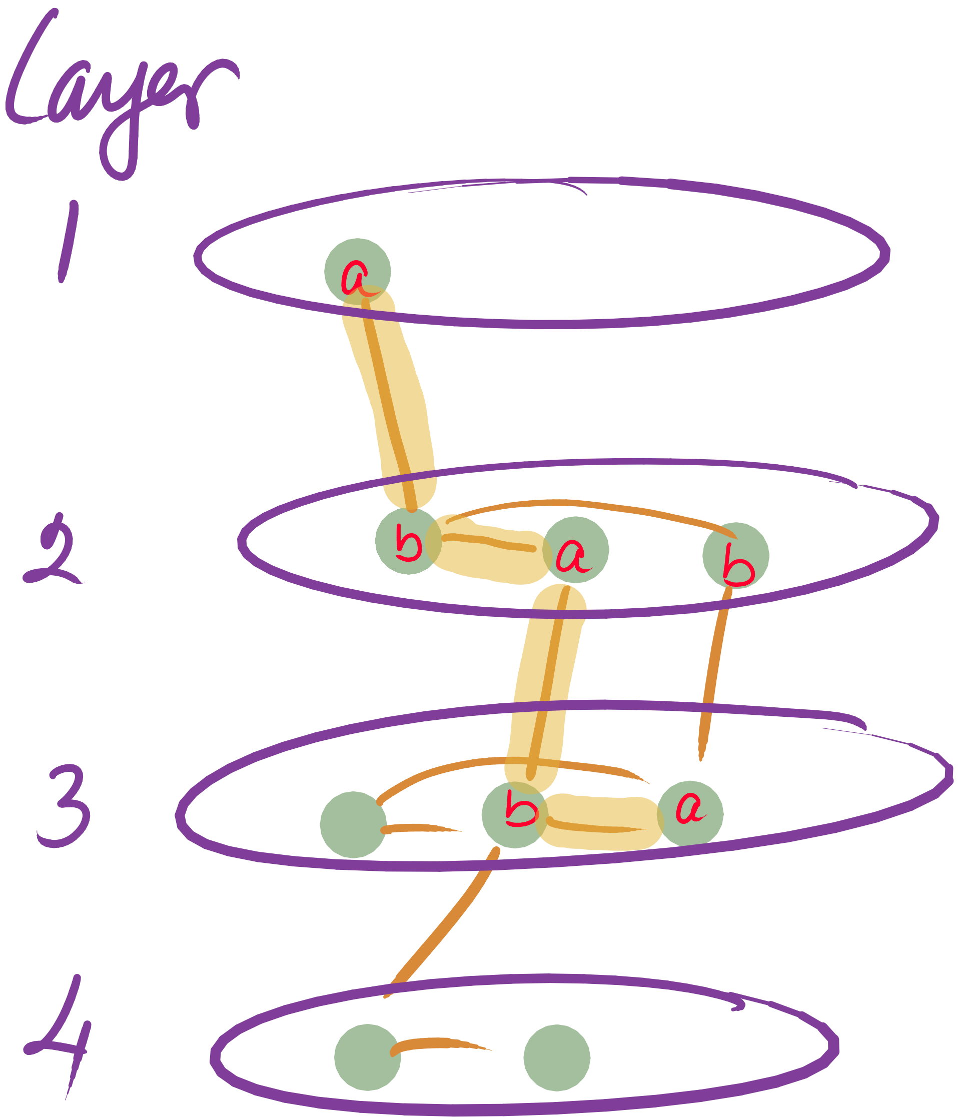
OTHER CLASSES

- trees $\pi(T) \leq 4$
- treewidth $\pi(G) \leq 4^{tw}$ Kunden, Pelsmajer (2003)
- pathwidth $\pi(G) \leq pw^2$ D., Joret, Kozik, Wood (2012)

Hellper lemma



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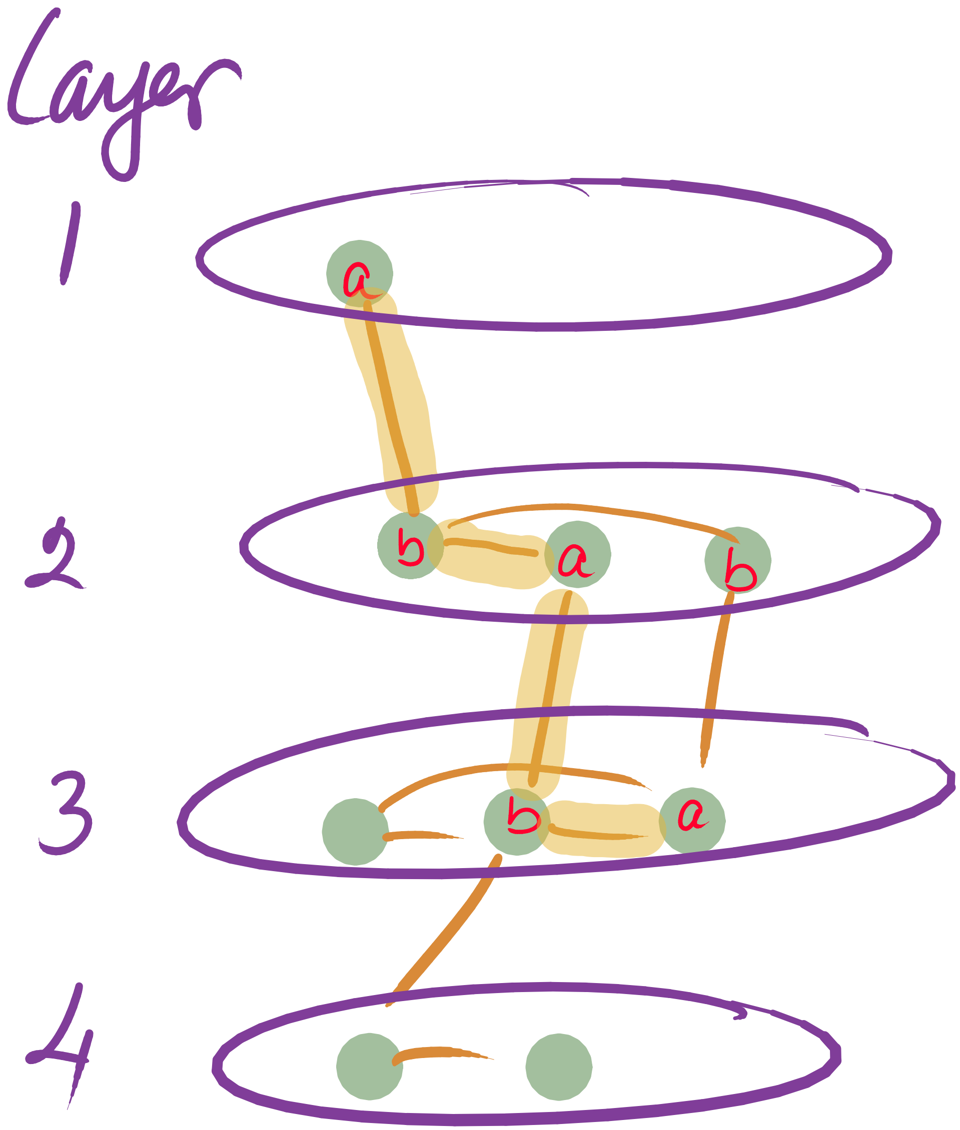
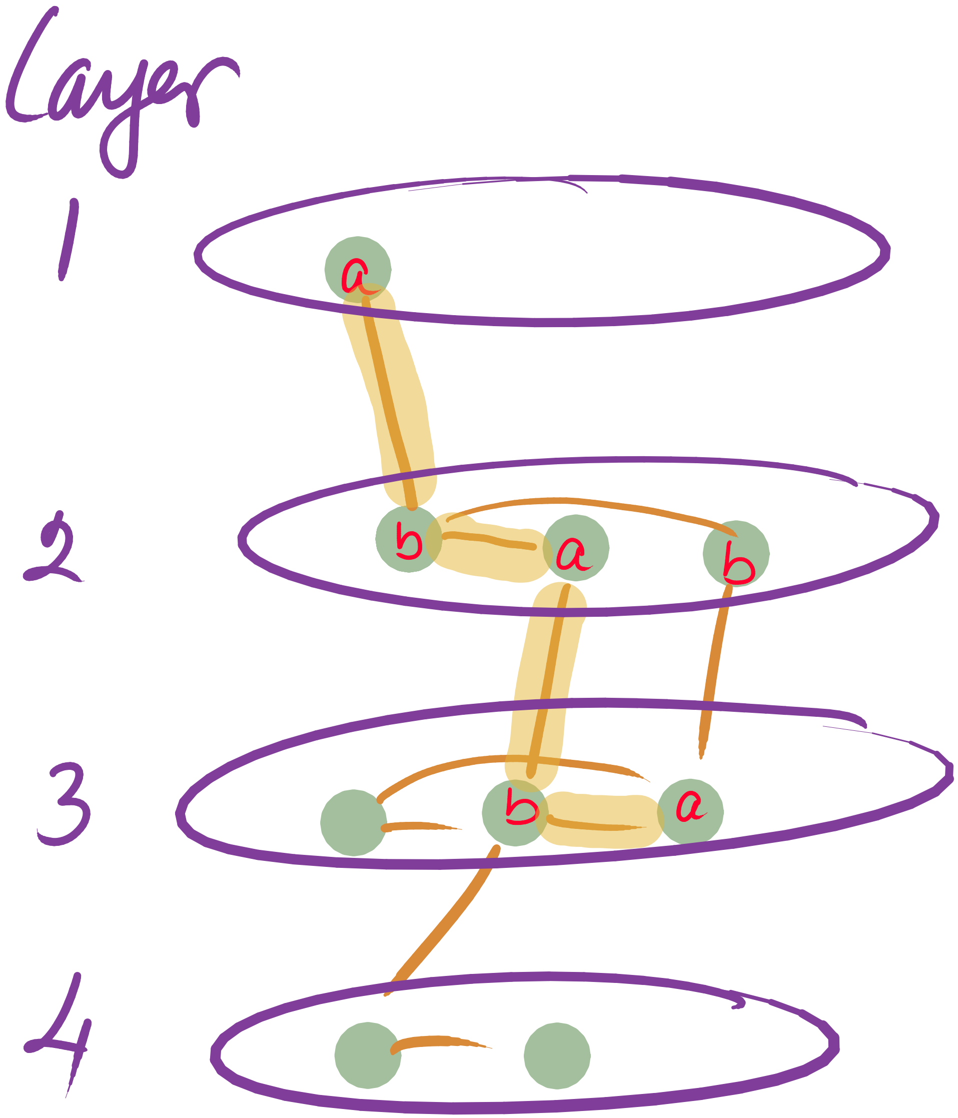


$\exists$ 4 col of $V(G)$ st. every repetitive path has the same layer pattern

Hellper lemma



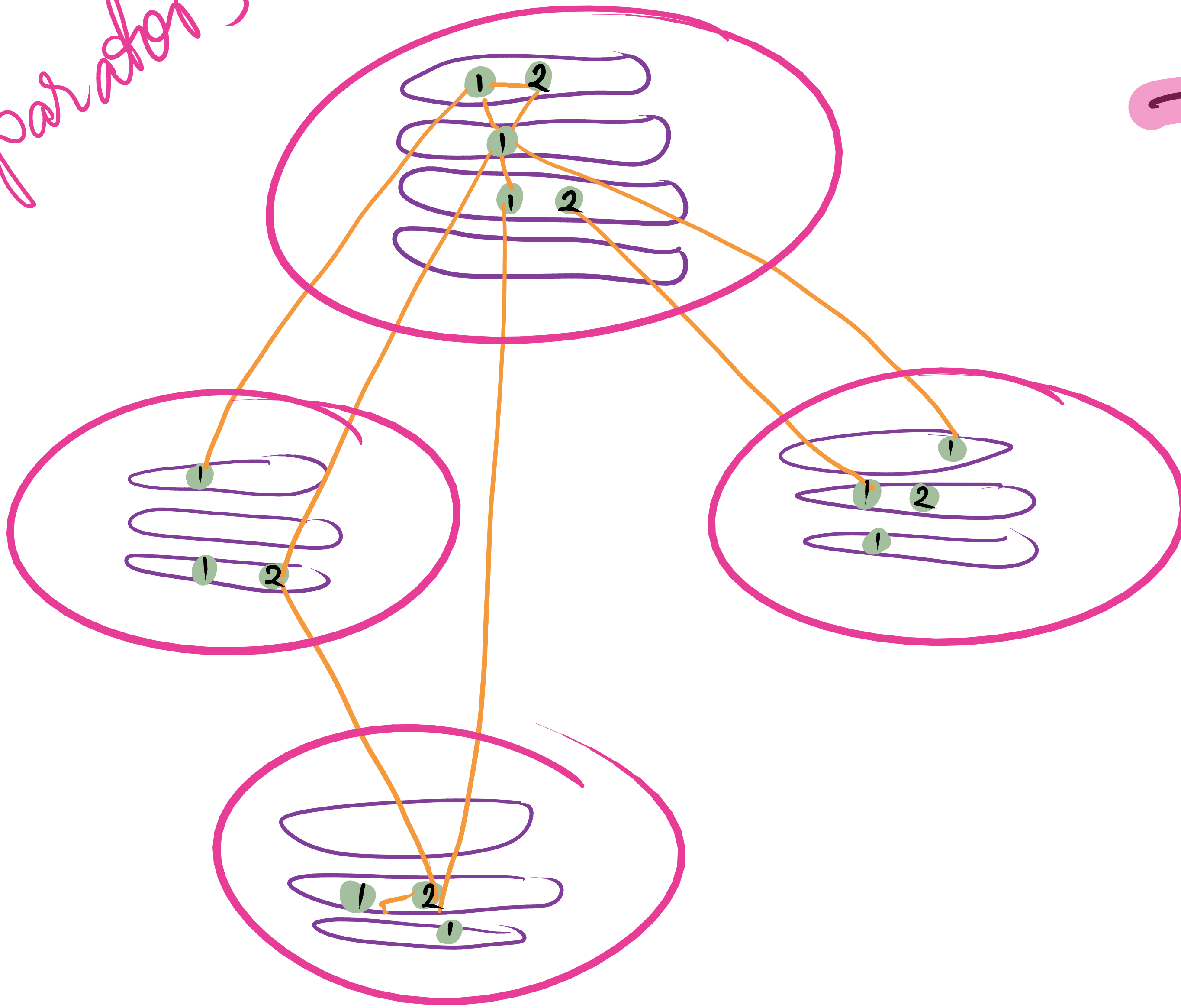
$\exists$ 4 col of $V(G)$
st. every repetitive path has the same layer pattern

a b a b
1 2 3 2
cannot happen

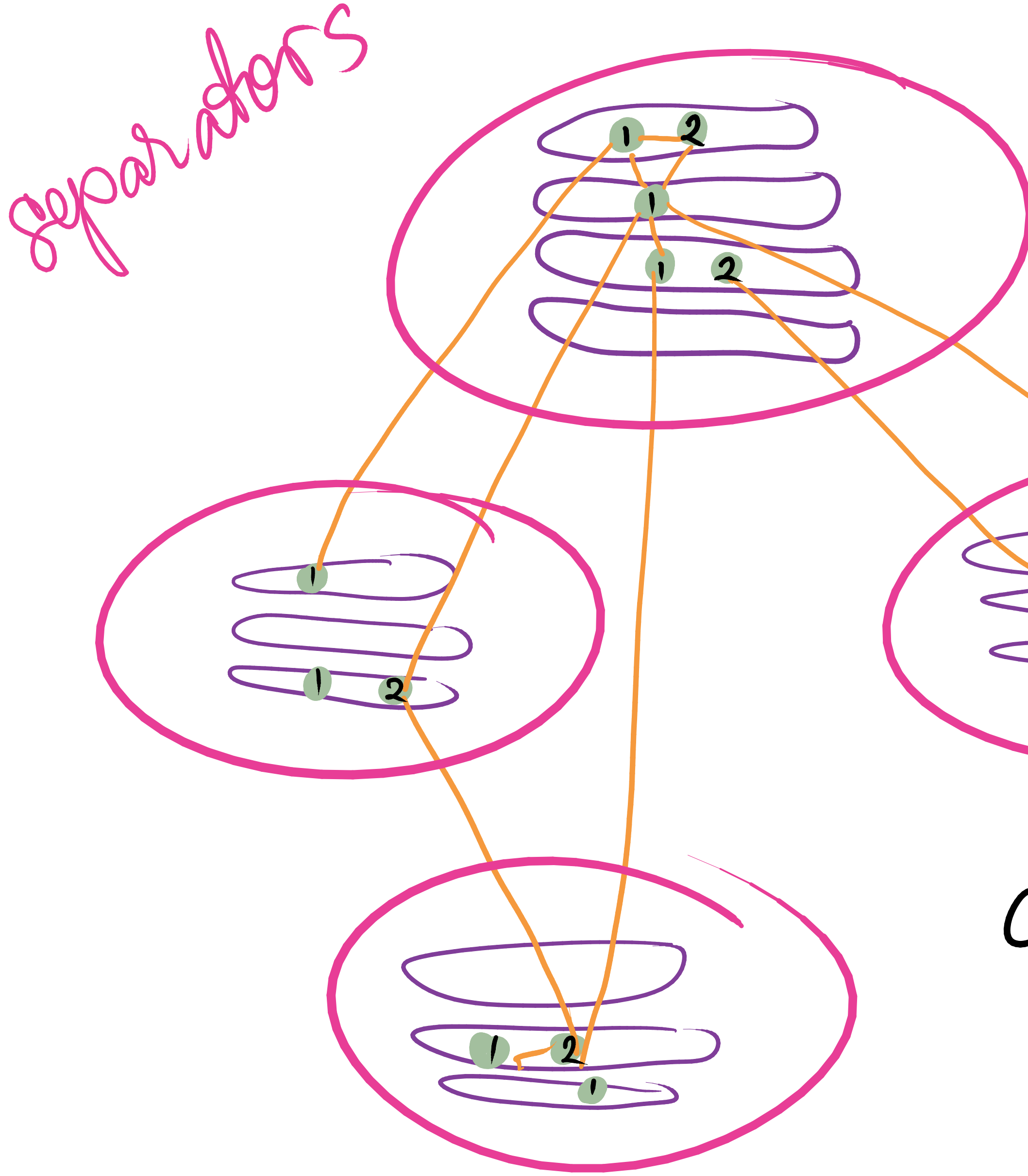
a b a b
1 2 1 2
can happen

Colouring Alg

separators



Colouring Alg



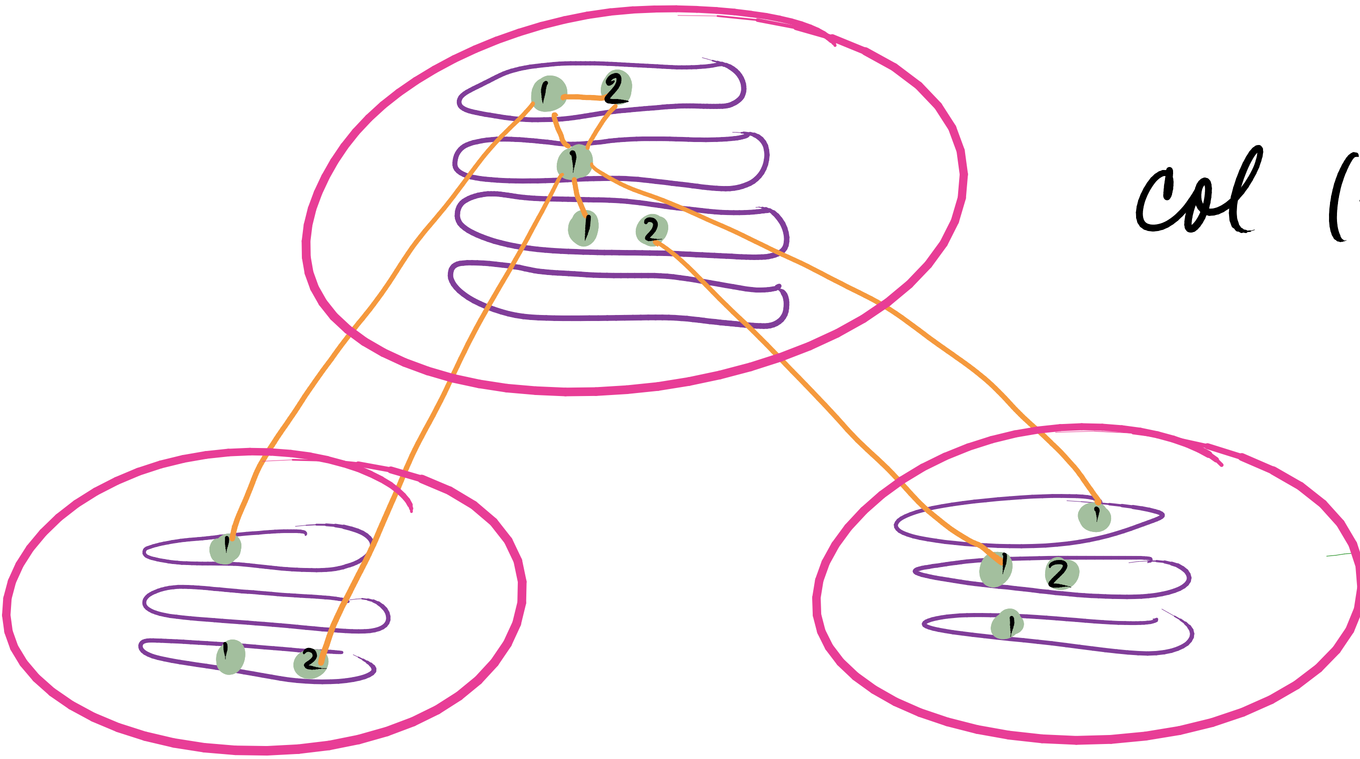
$$col(v) = \{ depth(v), label(v), help(v) \}$$

in separator tree

inside of separator

vertices of the same layer

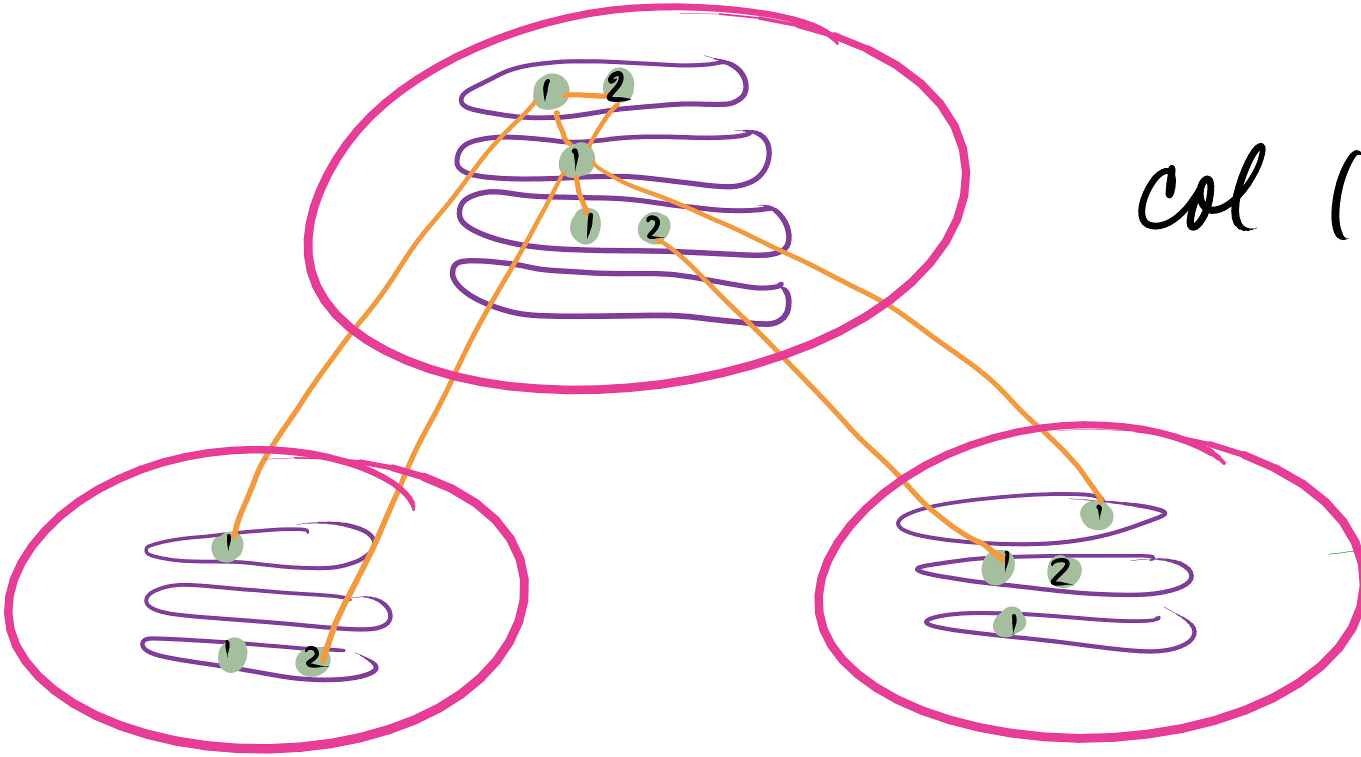
get distinct colours



$$col(v) = \begin{cases} depth(v), \\ label(v), \\ help(v) \end{cases}$$

num col:
$4 \cdot c \cdot \log n$

$a_1 a_2 \ldots a_r \; a_{r+1} \quad a_{r+2} \ldots a_{2r}$



$$col(v) = \begin{cases} depth(v), \\ label(v), \\ help(v) \end{cases}$$

$a_1 a_2 \ldots a_r \quad a_{r+1} \quad a_{r+2} \ldots a_{2r}$

smallest depth v_i:

$a_i \quad \ldots \quad a_{i+r}$

$(d, 1, h) \quad (d \;\; 1, h)$

same layer by helper lemma

Which graph families have LAYERED SEPARATORS?

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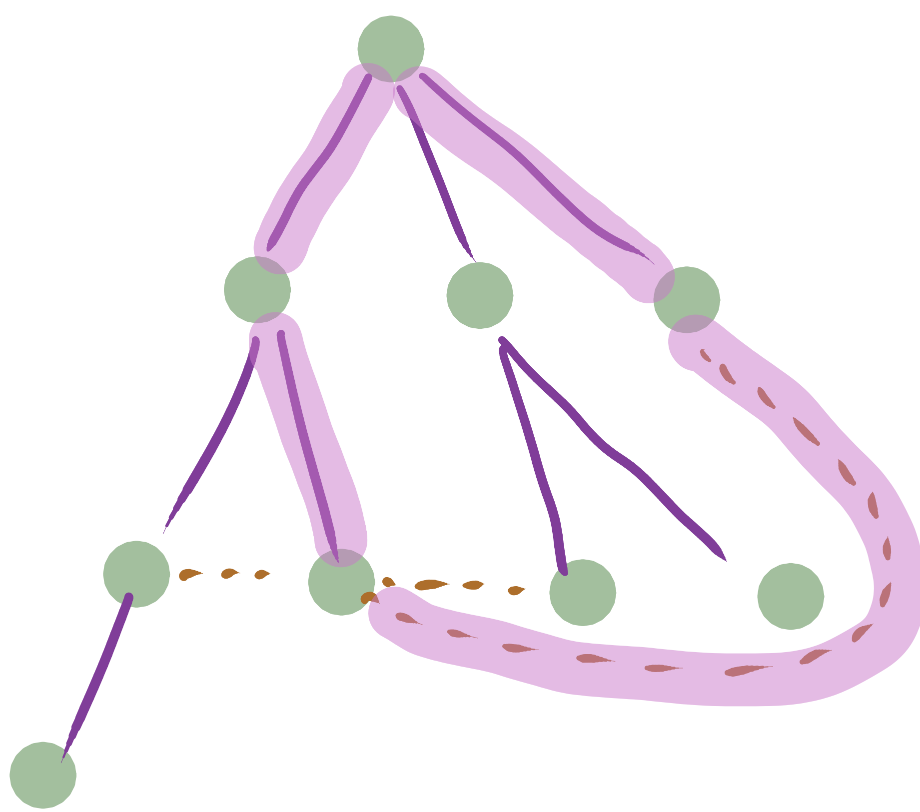
PLANAR GRAPHS : Lipton and Tarjan (1979)

have t-cycle separators



t-cycles hit each BST layer in ≤ 2 vertices

Do planar graphs have bounded non-rep chromatic # ?

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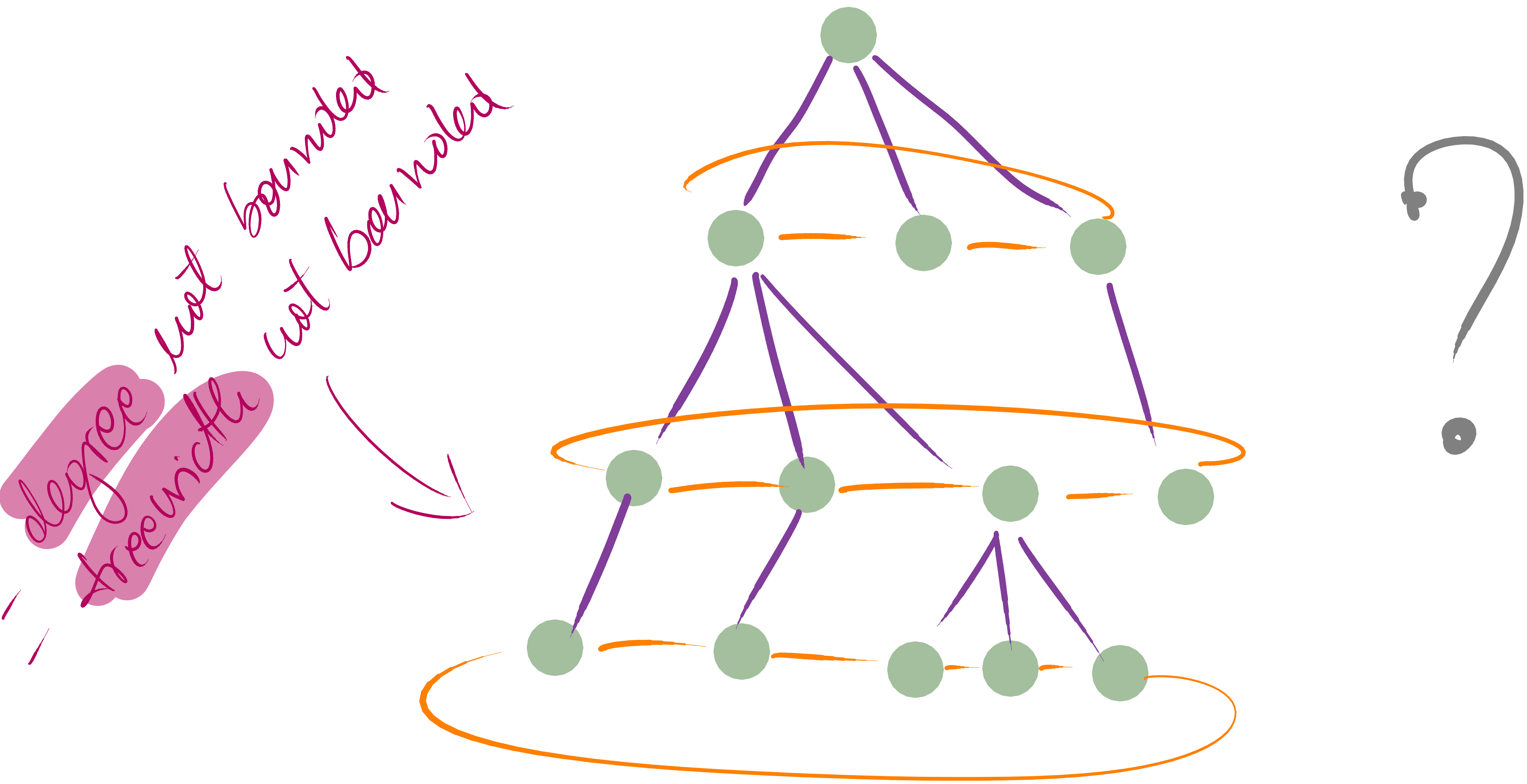
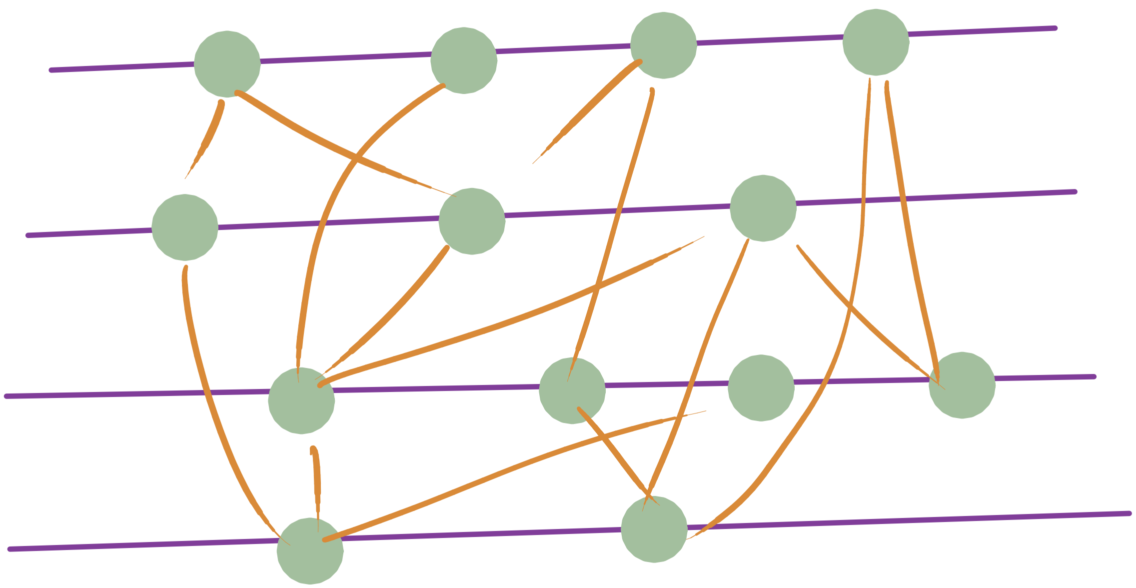
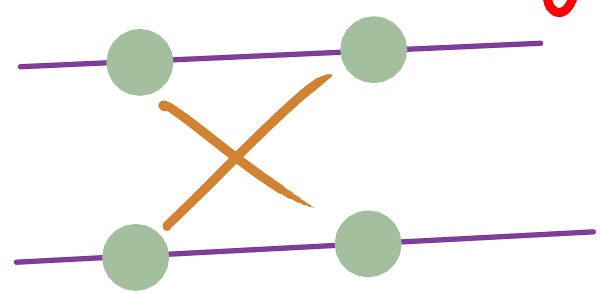


Track Layouts



independent sets
+
vertex ordering
s.t.
no X-crossings



Is track # of planar graphs bounded?

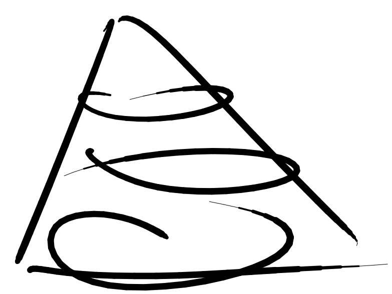
Happy birthday Robin!

10. Lipton Tarjan → t separators known

NOTE (size of separator does not matter)

surfaces

HW: color this



work in progress +

- it is not apriori clear that any finite # of symbols would do

- If it true that non planar is ~~po~~ [illegible] forced

↳ Alon

$G(n,p)$

Subdivisions

$c \frac{\Delta^2}{\log \Delta}$ > lower bound

↳ probabilistic p

- def pattern avoidance in words and study of repet
  ↳ combinat of words
- Co-NP hard to test if the coloring is non-rep

const?

- path
- cycle → mostly 3
- Δ Alon
- probabilistic method usually gives choosability here that is not possible

- subdivision - repetitively $\underline{3 \text{ col}}$

- $tw = c^{tw}$

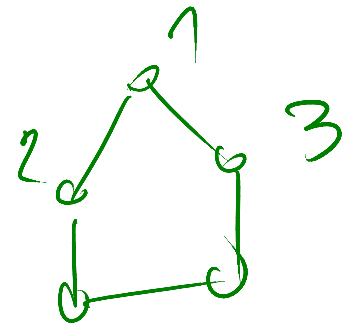
- $pw = pw^2$

- planar & proper

plug

$C_n = 3$

$n \geq 18$

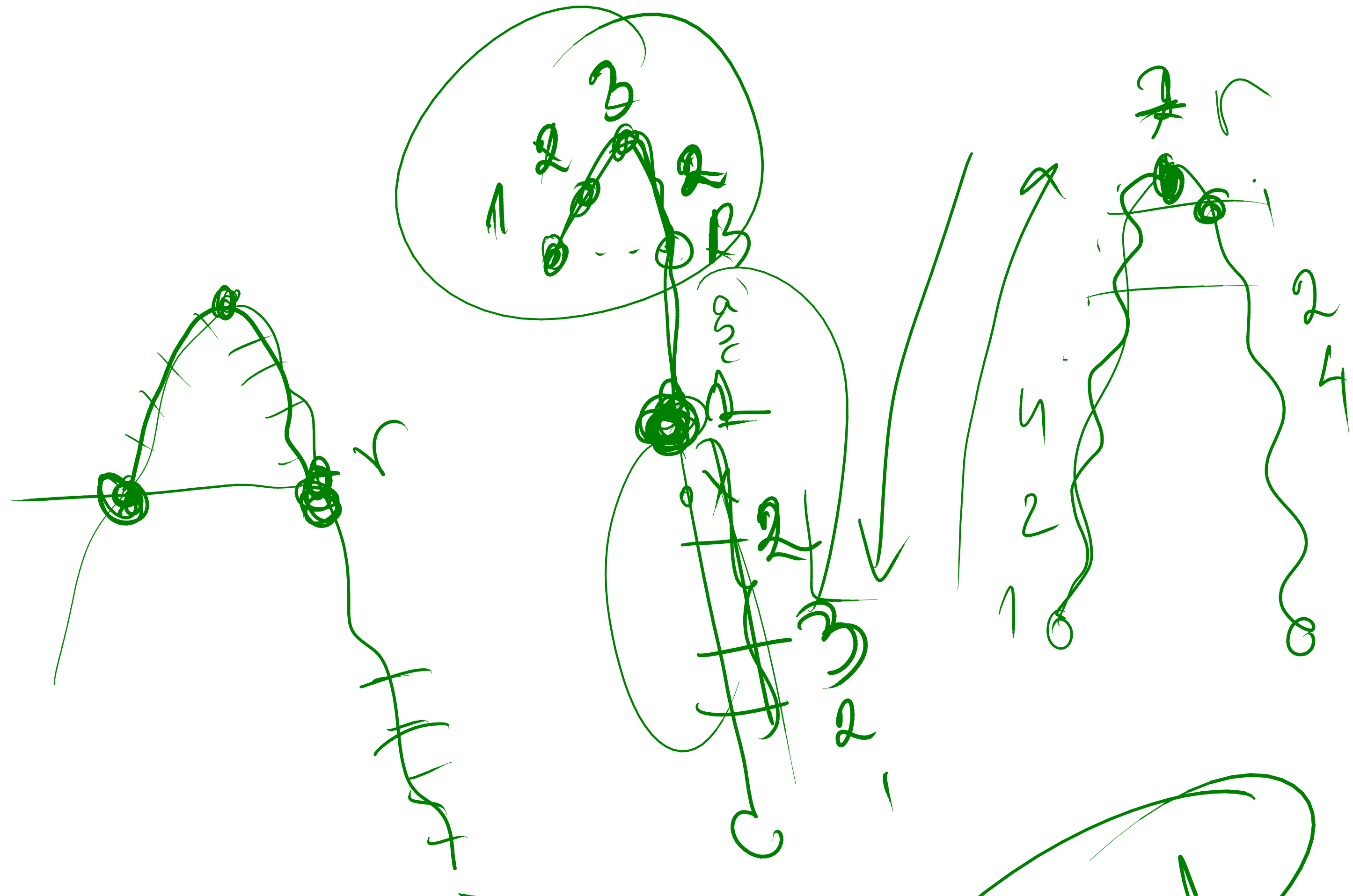


$\chi(G) \leq \chi_a(G) \leq \chi_{st} \leq 11$

Trees $\pi(T) \leq 4$

co-NP even

Dániel Marx & Marcus Schaefer



1 2 3 2 1

Happy birthday Robin!