AN APPROXIMATE SOLUTION FOR THE STREAMLINES ABOUT A LIFTING ROTOR HAVING A UNIFORM LOADING AND OPERATING IN THE LOW SPEED VERTICAL DESCENT RANGE

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AN APPROXIMATE SOLUTION FOR THE STREAMLINES ABOUT A LIFTING

ROTOR HAVING A UNIFORM LOADING AND OPERATING

IN THE LOW SPEED VERTICAL DESCENT RANGE

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TABLE OF CONTENTS

		Page
ACKNOWLEDGEMENTS		ii
LIST OF TABLES		iv
LIST OF FIGURES		v
NOTATION		vi
SUMMARY		viii
Chapter		
I.	INTRODUCTION	1
II.	PROCEDURE	3
III.	DISCUSSION OF RESULTS	10
IV.	CONCLUSIONS	12
APPENDIX		13
BIBLIOGRAPHY		32

LIST OF TABLES

Table		Page
1a.	Values of the Stream Function Above the Plane of the Rotor for $V/\gamma = .05$	14
1b.	Values of the Stream Function Below the Plane of the Rotor for $V/\gamma = .05$	15
2a.	Values of the Stream Function Above the Plane of the Rotor for $V/\gamma = .10$	16
2b.	Values of the Stream Function Below the Plane of the Rotor for $V/\gamma = .10$	17
3a.	Values of the Stream Function Above the Plane of the Rotor for $V/\gamma = .15$	18
3b.	Values of the Stream Function Below the Plane of the Rotor for $V/\gamma = .15$	19

LIST OF FIGURES

Figure		Page
1.	Schematic flow pattern for low rates of vertical descent .	20
2.	Streamlines for $V/\sigma = .1$ ($\lambda_z \approx .215$) for solution using a uniform vortex cylinder	21
3.	Streamlines for the flow induced by a uniform distribution of sinks over the rotor disk	22
4.	Streamlines for the displacement velocity of a disk . . .	23
5.	Streamlines for the flow induced by a ring source at the periphery of the disk	24
6.	Schematic diagram for the geometry of present analysis .	25
7.	Values of initial wake radius r_o	26
8.	Values of ultimate wake radius r_∞	26
9.	Streamlines for $V/\sigma = .05$	27
10.	Streamlines for $V/\sigma = .10$	28
11.	Streamlines for $V/\sigma = .15$	29
12.	Smoke flow diagram for a 4 foot rotor with constant chord untwisted blades operating at $V/\sigma = .15$ . . .	30
13.	Relation between V/σ and nondimensional freestream velocity, λ_z	31

NOTATION

r	radius at point P (r, z) from rotor axis (figure 6)
r_o	radius of wake vortex sheet at rotor disk
r_w	wake radius
r_∞	radius of wake vortex sheet at a large distance upstream (below the rotor)
v_o	normal component of velocity at the rotor disk due to the stream function ψ_o for the actuator disk displacement velocity
v_s	normal component of velocity at the rotor due to the stream function ψ_s for the uniform distribution of sinks over the rotor disk
R	rotor radius
T	rotor thrust
V	freestream velocity
V_r	radial component of velocity at point P (r, z)
V_z	normal component of velocity at point P (r, z)
γ	wake vortex sheet strength at a large distance upstream (below the rotor)
λ_z	nondimensional freestream velocity
ρ	mass density of air
ψ	stream function for the rotor flow as computed from the present analysis

ψ_o stream function for the displacement velocity of a disk

ψ_o^* nondimensional value of the stream function for the displacement velocity of a disk

ψ_r stream function for a ring source at the periphery of the rotor

ψ_r^* nondimensional value of the stream function for a ring source at the periphery of the rotor

ψ_s stream function for a uniform distribution of sinks over the rotor disk (i.e. for the induced flow outside a uniform vortex cylinder)

ψ_s^* nondimensional value of the stream function for a uniform distribution of sinks over the rotor disk

$\psi_{tot.}$ the value of the stream function on the wake boundary

ψ_v stream function for the freestream velocity

SUMMARY

It is shown that a useful approximation can be obtained for the flow pattern of a uniformly loaded lifting rotor operating at very low rates of vertical descent, of the order of 450 feet per minute or less, by adding several elementary potential flow solutions.

The assumption that a developed wake extends a large distance below the rotor is made, and the flow is represented by combining the stream functions for a uniform distribution of sinks over the rotor disk, the displacement velocity of a disk, a ring source at the periphery of the rotor, and the freestream velocity.

It appears that for very low rates of vertical descent, the velocity distribution in the vicinity of the rotor as given by the present analysis may furnish a basis for computing the induced velocity at a helicopter tailplane or the interference velocities on multi-rotor helicopters.

CHAPTER I

INTRODUCTION

In order to calculate the forces on the rotors of a multi-rotor helicopter, or the horizontal tailplane of a helicopter, it is essential to have a means for computing the velocity distribution in the vicinity of a rotor. However, there have been no theoretical solutions for the flow induced by a rotor operating at small rates of vertical descent.

It appears from the smoke flow studies by Castles and Gray (1) and Drees and Hendal (2) that for small rates of vertical descent a developed wake extends to an appreciable distance upstream (below the rotor) and that this segment of wake ends in a stagnation region in which the fluid that constituted the wake undergoes a turbulent mixing with the freestream fluid. As a result of the mixing, the wake boundaries are spread, the strength of the vortex sheet enclosing the wake is degraded, and together with the fluid inside the wake, the vortex sheet is folded back in the form of a secondary wake which is carried downstream (above the rotor) by the freestream flow. The schematic flow pattern is shown in Fig. 1.

As a first approximation, the wake vortex distribution could be represented by a uniform vortex cylinder extending from the rim of the rotor down to infinity. As it is shown by Küchemann and Weber (3) that the flow induced outside the wake by a uniform vortex cylinder is identical to that of a uniform distribution of sinks over the end of the cylinder, the stream function of the uniform distribution of sinks could be added to the stream function for the freestream flow to compute the composite flow pattern.

The resulting flow pattern would satisfy conditions at infinity in that the velocity outside the wake at infinity would approach that of the freestream, and the wake boundary streamline would converge to the diameter satisfying momentum considerations. However, the uniform distribution of sinks over the rotor disk results in an infinite radial velocity component at the rim of the disk and, in addition, for very low values of the freestream velocity, the flow in the lower quadrants does not satisfy continuity. If for very low rates of vertical descent, it is assumed that the primary wake extends to infinity (i.e. the turbulent mixing occurs at a very large distance below the rotor and can be neglected); then the flow pattern in the vicinity of the rotor can be approximated by a potential flow solution. An approximate solution which satisfies continuity in the vicinity of the rotor can be obtained by combining the flow induced by a uniform vortex cylinder with other simple singularity distributions.

The equivalent problem for vertical ascent has been considered by Castles (4).

CHAPTER II

PROCEDURE

Analysis.--It appears that a semi-infinite vortex cylinder ending at the rotor does not alone afford a good approximation for the actual flow. This can be seen from Fig. 2 drawn for the small rate of vertical descent for which

$$\lambda_z = \frac{V}{\sqrt{\frac{T}{2\rho\pi R^2}}} \approx .215$$

where

- V is the freestream velocity
- T is the rotor thrust
- ρ is the mass density of air

and

- R is the rotor radius

Following the procedure outlined by Castles (4), Fig. 2 was computed using values of the stream function ψ_s, for a uniform distribution of sinks as given by Küchemann and Weber (3). Equidistant values of ψ_s are shown in Fig. 3.

A better approximation can be obtained if the singularity distribution for the displacement velocity of a disk is added to that for the vortex cylinder. Equidistant values of ψ_o, the stream function for the displacement velocity of the disk, are shown in Fig. 4. The values of ψ_o for Fig. 4 were taken from Castles (4). It is to be noted that the normal components of velocity due to ψ_o and ψ_s are uniform over the rotor, which appears to be a required condition for uniform loading. In addition, it is necessary to add the stream function for a ring source, ψ_r , at the periphery of the rotor in order to satisfy continuity over the outer portions of the rotor. Values of ψ_r are given in Küchemann and Weber (3), and the streamlines are shown in Fig. 5.

Before using the aforementioned singularity distributions together with the stream function, ψ_v , for the freestream velocity to obtain the stream function of the resultant flow, it should be pointed out that the equations which will be derived using the above singularity distributions do not furnish an exact solution to the actual flow, but appear to yield results which are in fair agreement with experiment for the flow in the vicinity of the rotor.

Let the rotor thrust, T , be uniformly distributed over the rotor disk of radius R , and let the freestream velocity be V. Let the radius of the wake boundary streamline be r_o at the rotor and r_∞ at a large distance below the rotor where the wake boundaries have approached parallelism and the wake vortex sheet has a uniform strength of γ as denoted in Fig. 6. Following the procedure outlined by Castles (4), the thrust, T , is equal to the rate of transport of excess momentum across a section of the ultimate wake or

$$T = \rho \pi r_\infty^2 \gamma (\gamma - V) \tag{1}$$

Let v_o be the normal component of velocity at the rotor attributed to the stream function ψ_o , and v_s the normal component of velocity due to

ψ_s. Then equating total values of the stream function, ψ, on the opposite faces of the wake boundary at a large distance below the rotor where the radius is r_∞, the velocity is the freestream value V, and the value of ψ_o is zero, yields the following equation

$$\pi r_\infty^2 (\gamma - V) = 2 \pi R^2 v_s - \pi r_\infty^2 V - \psi_{r_{tot.}} \qquad (2)$$

For continuity of the flow within the closed streamline around the rotor rim, the relative magnitude of the ring source must be such that

$$\psi_{r_{tot.}} = 2 \pi (R^2 - r_o^2) v_s \qquad (3)$$

Solving equations (2) and (3) for v_s yields

$$v_s = \frac{1}{2} \gamma \left(\frac{r_\infty}{r_o} \right)^2 \qquad (4)$$

When the value of ψ on the wake boundary streamline at r_o is equated to the value of ψ at radius r_∞ on the wake boundary streamline in the ultimate wake, the following is obtained

$$\pi r_o^2 (v_s + v_o - V) = \pi r_\infty^2 (\gamma - V) \qquad (5)$$

Substituting the value of v_s from equation (4) gives

$$v_o = \left(\frac{r_\infty}{r_o} \right)^2 \left[\frac{1}{2} \gamma - V \right] + V \qquad (6)$$

The normal components of velocity due to the uniform distribution of sinks and the displacement of the actuator disk, as given by equations (4) and (6), are dependent on r_o and r_∞ for any given thrust and freestream velocity. An equation relating the value of r_o to r_∞, v_s, and v_o was derived by Castles (4) for a uniformly loaded rotor in vertical ascent. For the descent flight range under consideration, a similar derivation yields the following identical result

$$\left[\frac{r_\infty}{r_o}\right]^4 \left[1 + \frac{4\, v_o^{\,2}\; r_o^{\,2}}{\pi^2\, v_s^{\,2}\,(1 - r_o^{\,2})}\right] = 1 \qquad (7)$$

Equations (4), (6) and (7), together with the condition that r_o be single valued, define the values of v_s/σ, v_o/σ, r_o/R and r_∞/R for any given value of V and T.

Plots of r_o/R and $(r_\infty/R)^2$ are shown in Figs. 7 and 8, where the parameter V/σ relates thrust to freestream velocity as shown by equation (1).

Computation of the Value of the Stream Function.--Values of the stream function for three small rates of vertical descent, at V/σ = .05, .10, .15, are given in Tables 1, 2, and 3 in the Appendix. The streamlines for these three flight conditions are shown in Figs. 9, 10, and 11.

The following procedure was used to calculate the values of ψ given in Tables 1, 2, and 3.

(1) For any given value of V/σ, equation (7) together with

the condition that r_o be single valued were used in an iterative solution for r_o/R and r_∞/R.

(2) The values v_o/σ , v_s/σ , and $\psi_{r_{tot.}}$, were computed for the given values of V/σ , r_o/R, and r_∞/R.

(3) Then for the region outside the wake

$$\psi = \left| \psi_o + \psi_s - \psi_v - \psi_r \right| \qquad (8)$$

where

$$\psi_o = (2\, R^2\, v_o/\sigma\,)\, \psi_o^* \qquad (8a)$$

$$\psi_s = (2\, R^2\, v_s/\sigma\,)\, \psi_s^* \text{ for upper quadrants} \qquad (8b)$$

$$\psi_s = (2\, R^2\, v_s/\sigma\,)\, (\pi - \psi_s^*) \text{ for lower quadrants} \qquad (8c)$$

$$\psi_r = \frac{\psi_{r_{tot.}}\, \psi_r^*}{2\,\pi} \qquad (8d)$$

The starred values of ψ are the nondimensional values of the individual singularity distributions used.

In computing the streamlines inside the wake, it was necessary to make the assumption that the axial component of velocity was uniform across all sections of the wake. This assumption appears to be reasonable since the axial component of velocity is uniform at the rotor and almost uniform over the region where the wake boundaries have approached parallelism. Tabulated values of ψ for the region inside the wake, are thus based on the approximation that

$$\psi = \left[\frac{r}{r_w}\right]^2 \psi_{tot.} \qquad (9)$$

where r_w is the radius of the wake at any station z as determined from the solution for the external flow.

Application of Results.--If it is desired to obtain the axial and radial velocity components at some point in the field of a rotor operating at any intermediate flight condition for which values of ψ are not tabulated, it is necessary to perform a graphical or numerical interpolation and differentiation. For example, if the coordinates of the point in question are z' and r' measured from the rotor hub, the procedure could be as follows:

(1) For any given thrust loading, $T/\pi R^2$, and freestream velocity V, solve for $(r_\infty/R)^2$ as a function of γ from equation (1).

(2) Use trial values of γ in the equation obtained from step (1) until the resulting value for V/γ is the same as that obtained from Fig. 7.

(3) Graphically interpolate between Tables 1, 2, and 3 for the values of $\psi/\psi_{tot.}$ at the required V/γ for a box of about nine points enclosing the values of z'/R and r'/R.

(4) Multiply the value of $\psi/\psi_{tot.} = \pi r_\infty^2 (\gamma - V)$ to obtain the values of ψ for the nine points.

(5) Interpolate in the box of values of ψ for the three values at $z = z'$ and $r = r_1$, r_2, and r_3, and for the three values at $r = r'$ and $z = z_1$, z_2, and z_3.

(6) Plot ψ versus r at $z = z'$ and measure the slope $\partial\psi/\partial r$ at $r = r'$. Then the axial velocity component at (z', r') is

$$V_z \Big|_{(z',r')} = \frac{1}{2\pi r'} \frac{\partial\psi}{\partial r}\Big|_{(z',r')} \tag{10}$$

(7) Plot ψ versus z at $r = r'$ and measure the slope $\partial \psi / \partial z$ at $z = z'$. Then the radial velocity component is

$$V_r \Big|_{(z',r')} = - \frac{1}{2 \pi r'} \frac{\partial \psi}{\partial z} \Big|_{(z',r')} \qquad (11)$$

CHAPTER III

DISCUSSION OF RESULTS

A smoke flow diagram for a 4 foot rotor with constant chord, untwisted blades, operating at $V/\sigma = .15$ or ($\lambda_z \approx .3$), as obtained from Castles and Gray (1), is shown in Fig. 12. The non-dimensional free-stream velocity, λ_z , is plotted versus V/σ in Fig. 13. The theoretical flow pattern for this value of V/σ , as shown in Fig. 11, appears to be in fair agreement for the streamlines in the vicinity of the rotor. It should be noted that the initial slopes of the smoke filaments leaving the nozzles, as shown in Fig. 12, are not representative because the smoke was introduced into the flow with a certain initial velocity directed against the freestream. As a result, the smoke flow indicated the correct flow pattern only after the initial velocity of the smoke jets was cancelled out. Consequently the theoretical and experimental flow patterns are in better agreement than Figs. 11 and 12 indicate.

Fig. 7 shows the spread between the theoretical and experimental values for r_o/R at the higher rates of vertical descent. (Experimental values for r_o/R were obtained from Castles and Gray (1).)

A reasonable upper limit for the values of V/σ within which the present analysis should be useful appears to be about $V/\sigma = .15$ or $\lambda_z = .3$. For higher rates of descent, the present analysis will considerably overestimate the velocities in the vicinity of the rotor because the stagnation region is now fairly close to the rotor and the assumption that the wake extends a large distance below the rotor is no longer valid.

CHAPTER IV

CONCLUSIONS

For very low rates of vertical descent, $\lambda_z \leq .3$, or rates of descent less than values of the order of 450 feet per minute, for a rotor with conventional disk loading, it appears that the flow pattern in the vicinity of a uniformly loaded rotor can be approximated by the superposition of the following simple flow patterns:

(1) The flow for a uniform distribution of sinks over the rotor disk (i.e. the flow outside a uniform vortex cylinder);

(2) The flow for the displacement velocity of a disk of radius equal to that of the rotor;

(3) The flow due to a ring source at the periphery of the rotor;

(4) The freestream flow.

A P P E N D I X

Table 1a

Values of the Stream Function Above the Plane of the Rotor for $V/\sigma = .05$

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.058	.226	.510	.908	1.418 1.198	.746	.603	.488	.382	.276
.2	0	.042	.165	.353	.563	.667	.607	.508	.411	.314	.219
.4	0	.030	.114	.234	.354	.430	.435	.390	.323	.241	.154
.6	0	.020	.077	.154	.229	.282	.296	.249	.204	.142	.068
.8	0	.014	.048	.099	.148	.183	.193	.181	.148	.094	.026
1.2	0	.005	.018	.040	.061	.074	.073	.057	.027	-.023	-.085
1.6	0	.002	.006	.010	.013	.010	.002	-.021	-.057	-.102	-.163
2.0	0	-.002	-.003	-.006	-.016	-.029	-.051	-.081	-.118	-.168	-.228

Table 1b

Values of the Stream Function Below the Plane of

the Rotor for $V/\sigma = .05$

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.057	.226	.510	.906	.979 1.198	.746	.603	.488	.382	.276
.2	0	.082	.326	.735	.979	.905	.784	.659	.548	.437	.327
.4	0	.095	.379	.853	.966	.897	.800	.695	.587	.481	.369
.6	0	.106	.423	.952	.960	.896	.818	.722	.618	.514	.407
.8	0	.113	.452	1.000	.956	.897	.827	.742	.646	.544	.436
1.2	0	.113	.452	.999	.953	.898	.833	.758	.672	.581	.482
1.6	0	.111	.444	1.000	.960	.907	.842	.771	.690	.602	.507
2.0	0	.104	.417	.937	.966	.917	.858	.791	.713	.627	.531

Table 2a

Values of the Stream Function Above the Plane of

the Rotor for $V/\sigma = .10$

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.059	.228	.516	.917	1.434 1.193	.640	.422	.228	.032	-.170
.2	0	.041	.162	.343	.540	.610	.491	.321	.145	-.040	-.231
.4	0	.028	.106	.213	.311	.351	.305	.194	.051	-.116	-.300
.6	0	.018	.065	.125	.175	.190	.153	.071	-.047	-.197	-.369
.8	0	.011	.035	.066	.087	.082	.043	-.031	-.136	-.275	-.437
1.0	0	.006	.015	.027	.032	.013	-.032	-.108	-.213	-.345	-.513
1.4	0	-.002	-.007	-.018	-.039	-.074	-.130	-.212	-.315	-.445	-.603
1.8	0	-.003	-.015	-.042	-.077	-.128	-.195	-.284	-.392	-.524	-.677
2.0	0	-.007	-.022	-.049	-.091	-.148	-.221	-.314	-.424	-.557	-.709

Table 2b

Values of the Stream Function Below the Plane of

the Rotor for V/σ = .10

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.057	.230	.517	.918	.951 1.193	.640	.422	.228	.032	-.170
.2	0	.074	.296	.957	.974	.855	.676	.479	.288	.090	-.118
.4	0	.086	.346	.779	.954	.841	.688	.514	.329	.137	-.074
.6	0	.090	.362	.814	.945	.837	.702	.540	.360	.169	-.035
.8	0	.097	.390	.878	.939	.835	.710	.560	.388	.199	-.004
1.0	0	.101	.403	.907	.935	.835	.712	.569	.404	.219	.022
1.4	0	.097	.390	.814	.936	.835	.717	.578	.424	.248	.055
1.8	0	.096	.385	.830	.942	.847	.731	.595	.441	.269	.079
2.0	0	.095	.379	.852	.945	.851	.738	.606	.452	.282	.091

Table 3a

Values of the Stream Function Above the Plane of

the Rotor for $V/\sigma = .15$

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.060	.235	.528	.941	1.470 1.194	.517	.210	-.079	-.379	-.695
.2	0	.041	.160	.336	.503	.550	.357	.104	-.166	-.455	-.760
.4	0	.026	.097	.191	.265	.287	.155	-.034	-.267	-.537	-.833
.6	0	.014	.051	.094	.114	.085	-.011	-.168	-.374	-.624	-.907
.8	0	.006	.019	.029	.018	-.034	-.132	-.280	-.473	-.708	-.980
1.2	0	-.004	-.018	-.043	-.087	-.164	-.276	-.426	-.614	-.889	-1.158
1.6	0	-.009	-.033	-.080	-.147	-.239	-.359	-.518	-.712	-.938	-1.201
2.0	0	-.013	-.044	-.099	-.179	-.286	-.421	-.590	-.784	-1.015	-1.276

Table 3b

Values of the Stream Function Below the Plane of

the Rotor for $V/\sigma = .15$

r/R =	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
z/R						$\psi/\psi_{tot.}$					
0	0	.059	.235	.529	.940	.918 1.194	.517	.210	-.079	-.379	-.695
.2	0	.074	.296	.666	.965	.799	.549	.268	-.016	-.318	-.640
.4	0	.082	.326	.735	.939	.775	.554	.302	.024	-.270	-.595
.6	0	.086	.346	.778	.924	.765	.566	.326	.054	-.237	-.555
.8	0	.092	.367	.826	.916	.760	.572	.344	.082	-.207	-.524
1.2	0	.094	.379	.852	.907	.755	.571	.353	.106	-.171	-.478
1.6	0	.096	.385	.865	.911	.761	.576	.365	.120	-.152	-.452
2.0	0	.092	.367	.826	.917	.771	.594	.385	.143	-.127	-.428

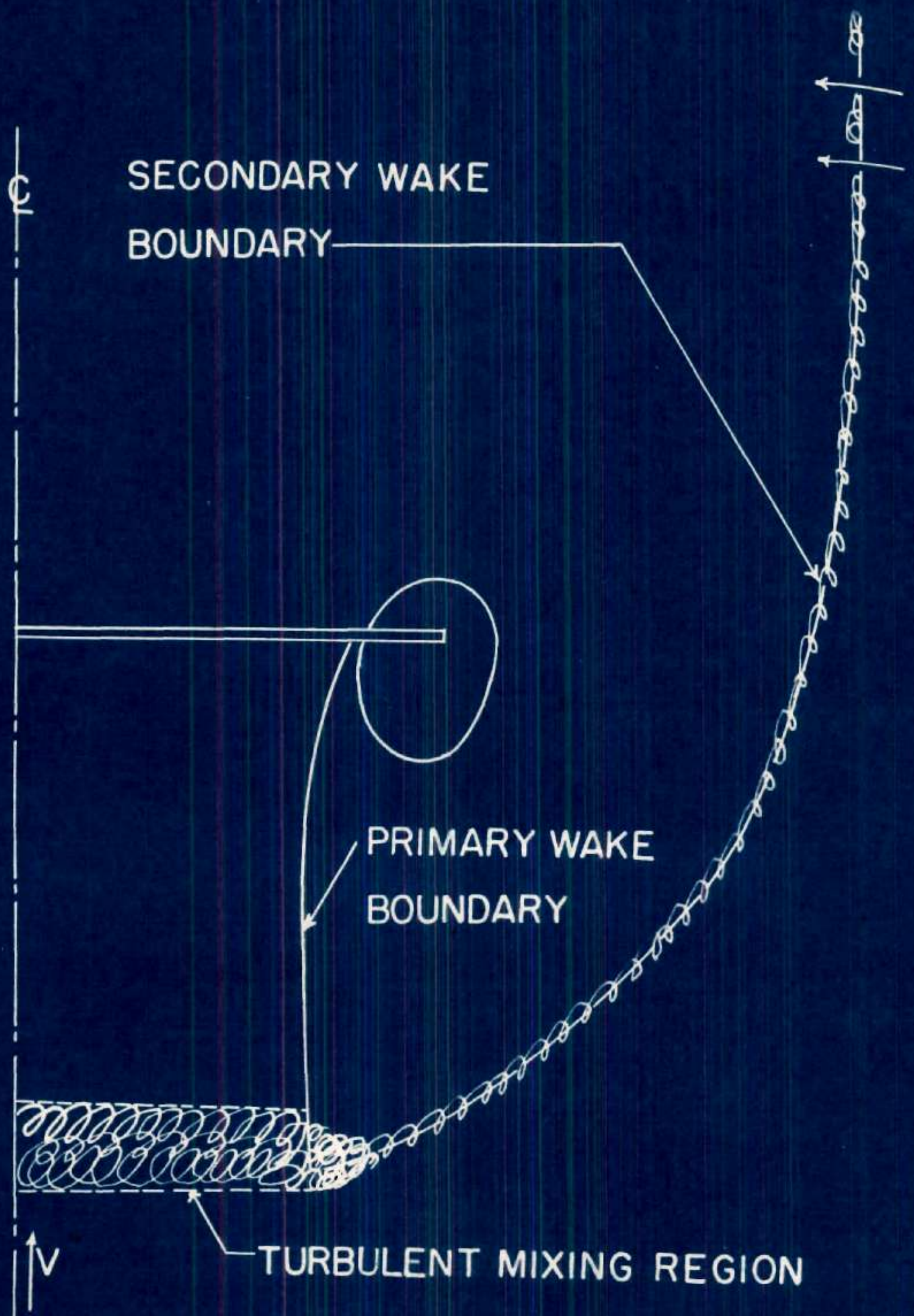

Figure 1

Schematic Flow Pattern for Low Rates of Vertical Descent

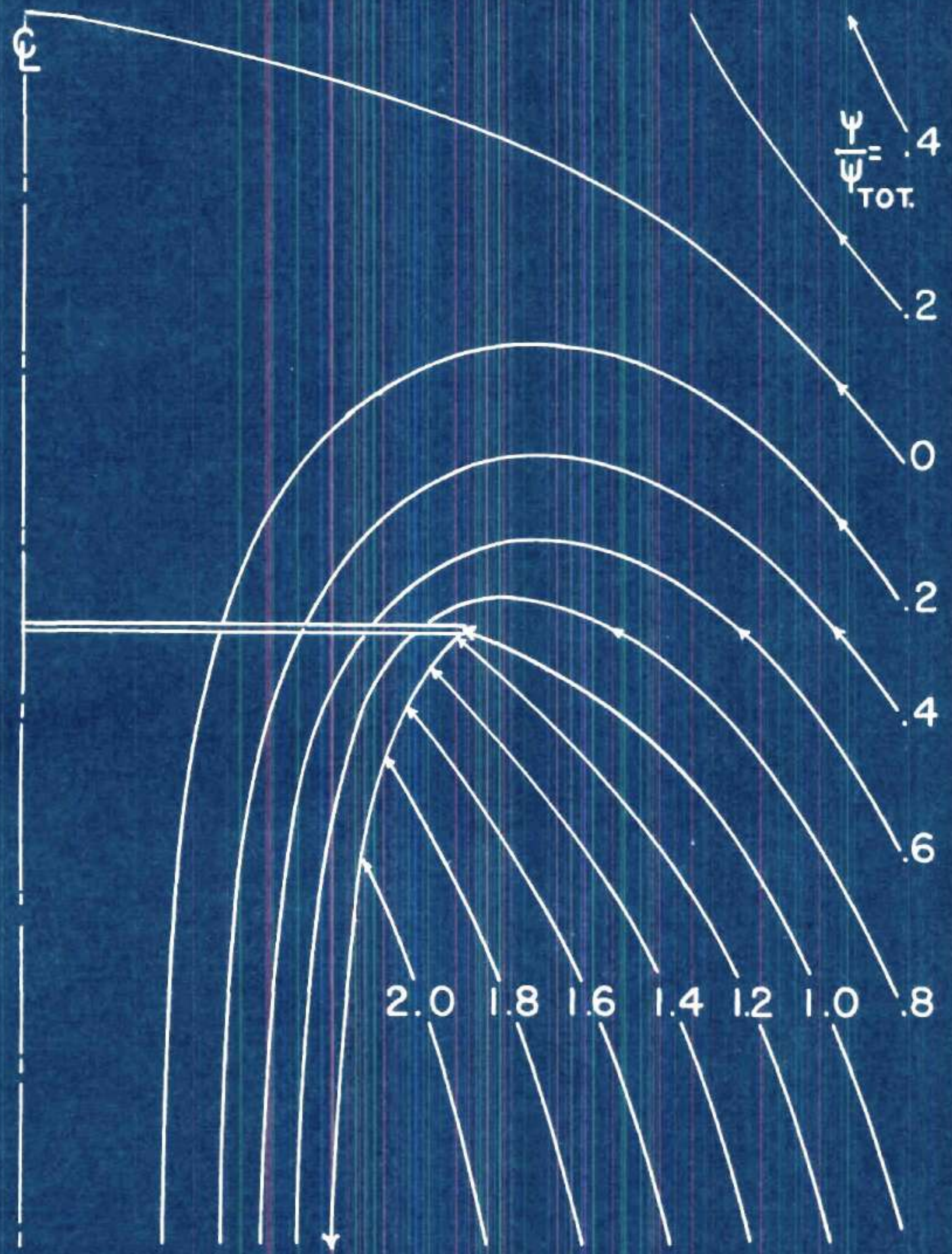

Figure 2

Streamlines for $\frac{V}{\gamma} = .1$ ($\lambda_z = .215$) for Solution Using a Uniform Vortex Cylinder.

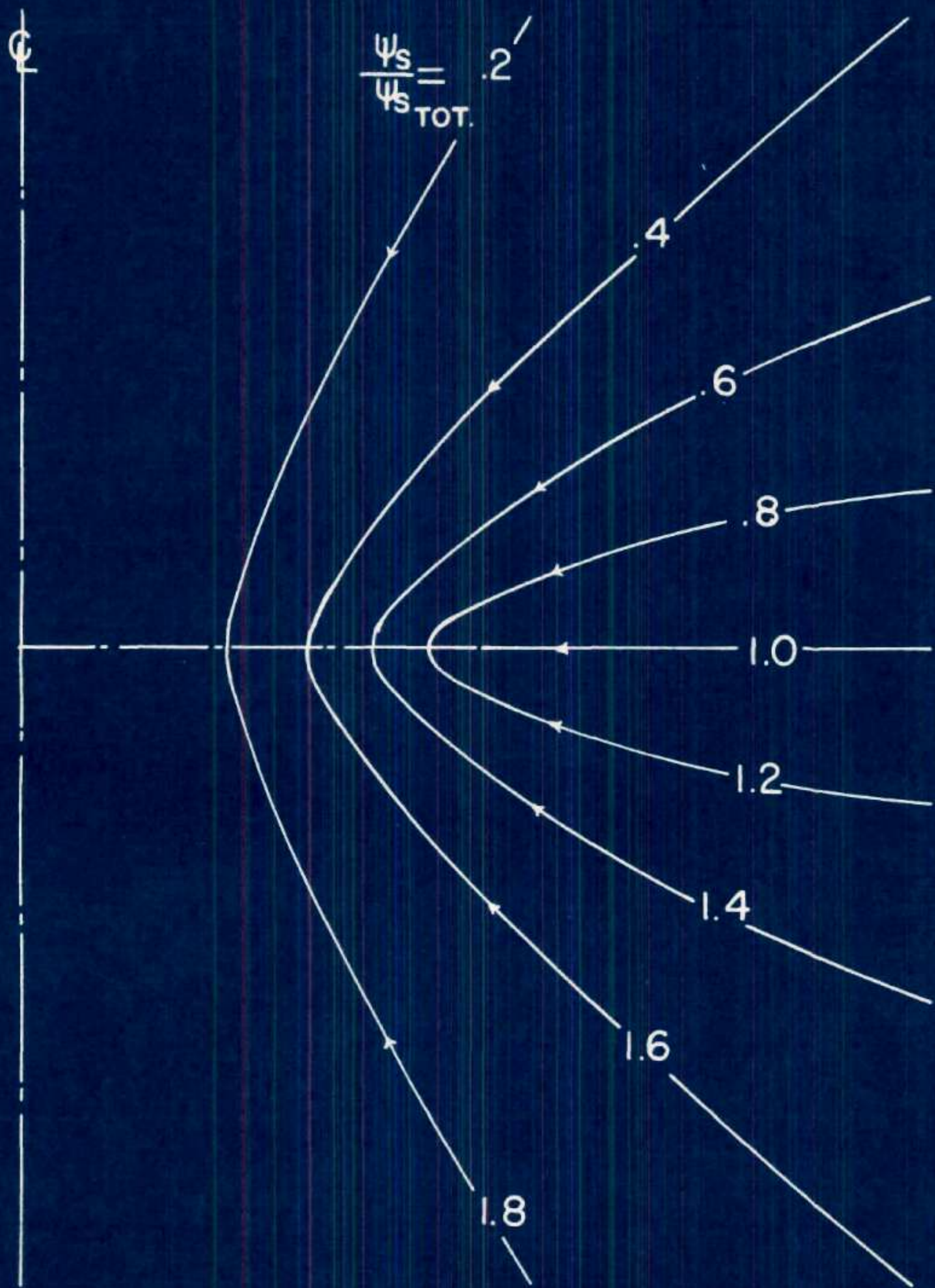

Figure 3

Streamlines for the Flow Induced by a Uniform Distribution of Sinks over the Rotor Disk.

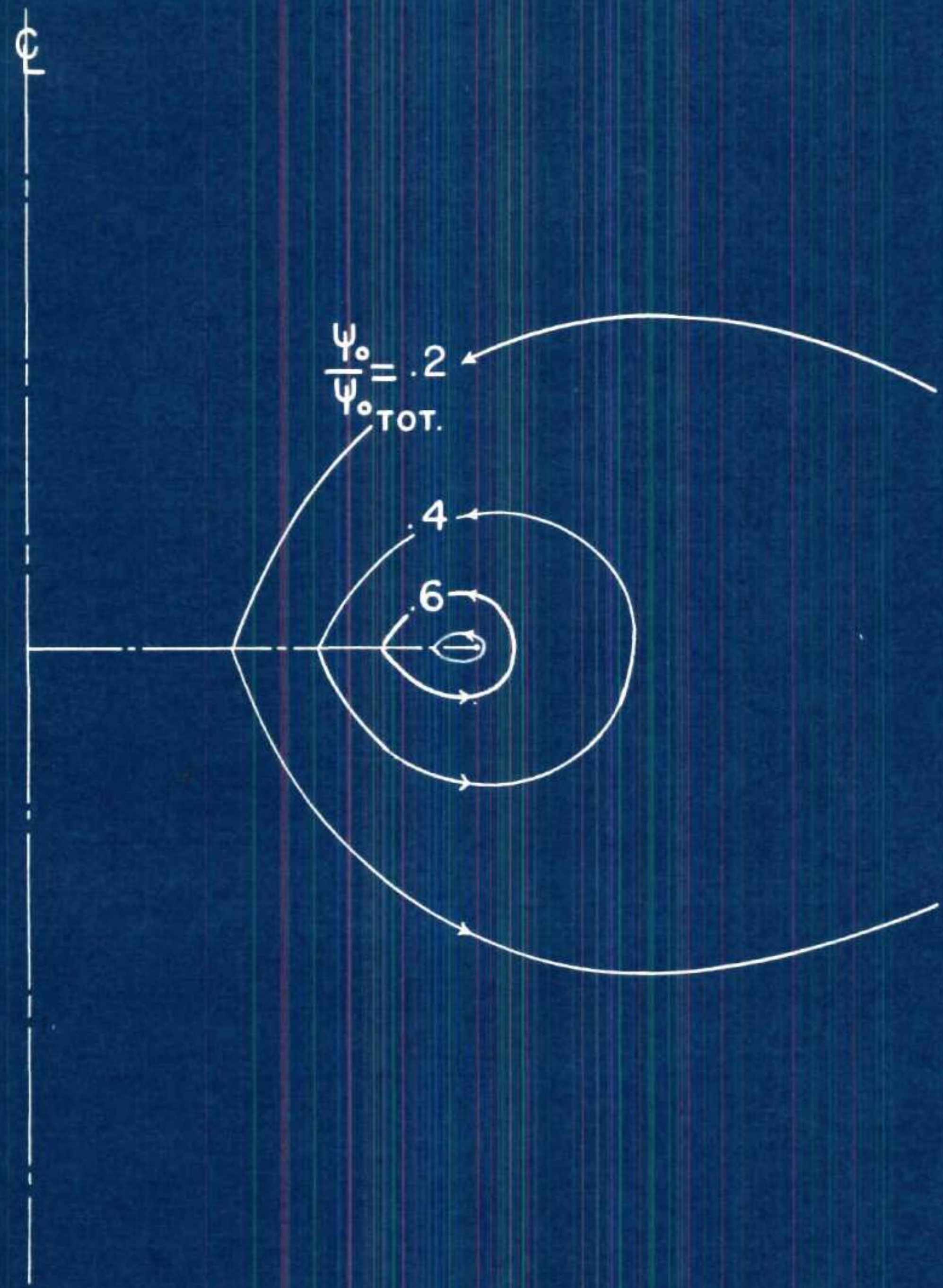

Figure 4

Streamlines for the Displacement Velocity of a Disk.

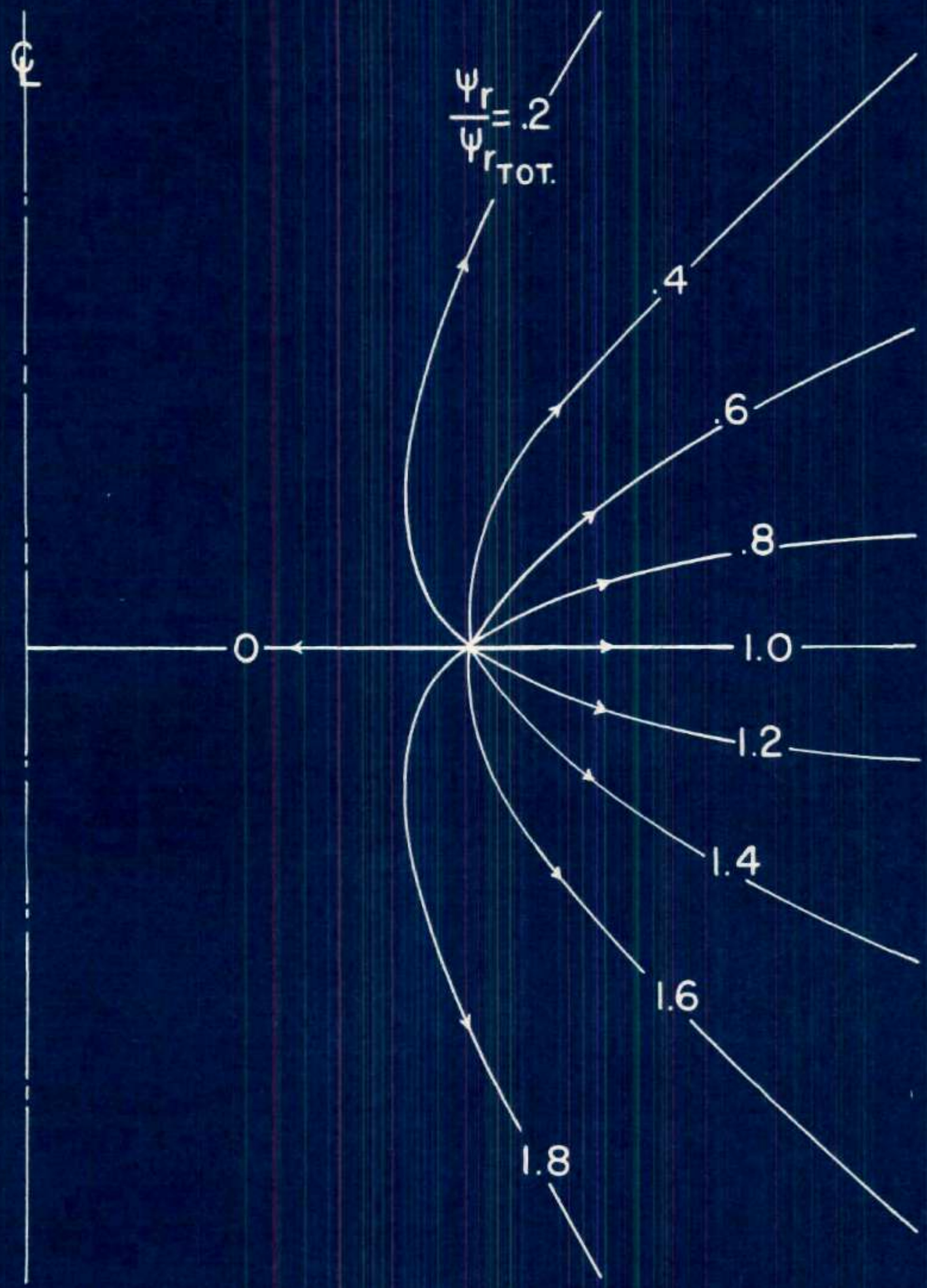

Figure 5

Streamlines for the Flow Induced by a Ring Source at the Periphery of the Rotor Disk.

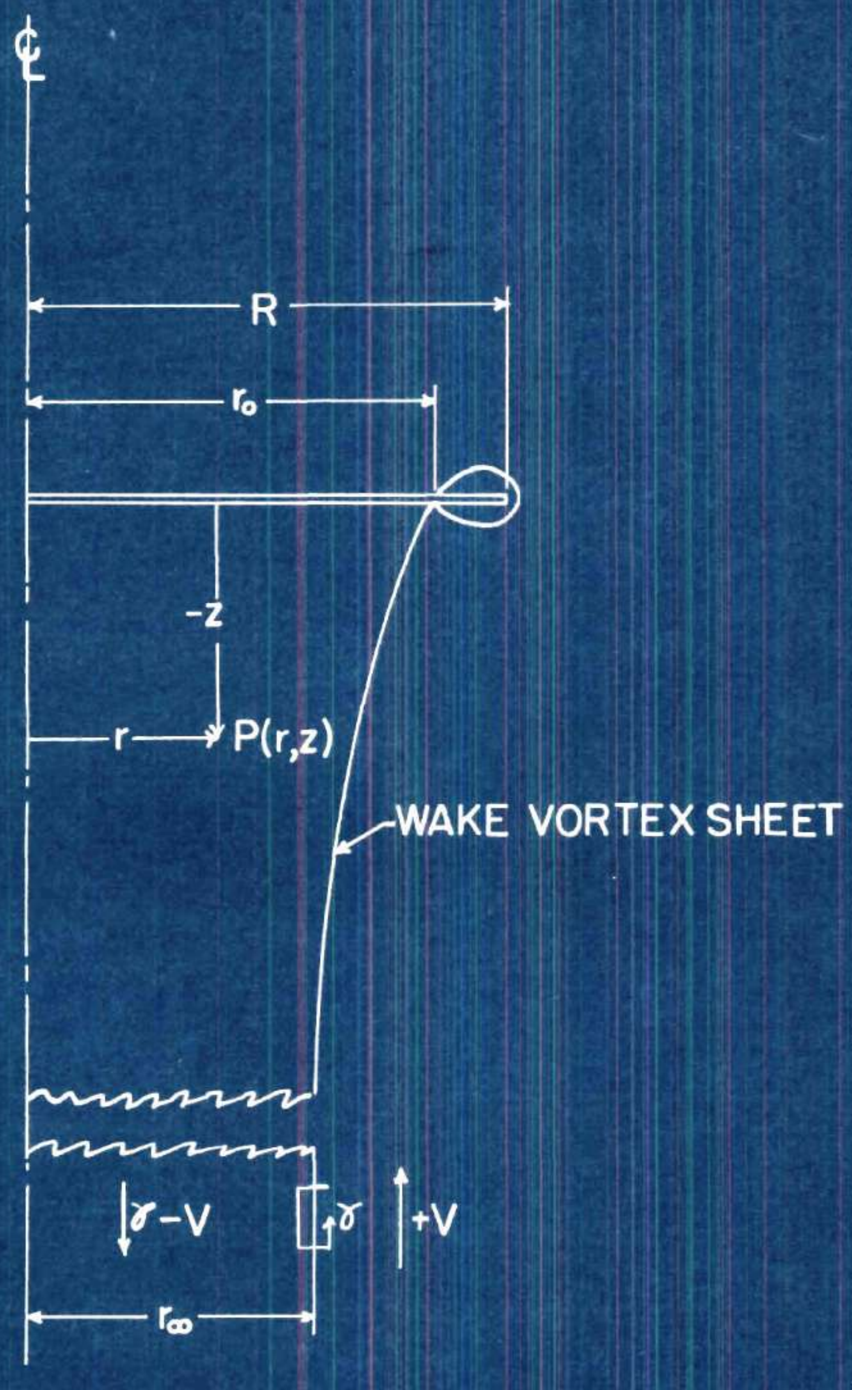

Figure 6

Schematic Diagram for the Geometry of Present Analysis.

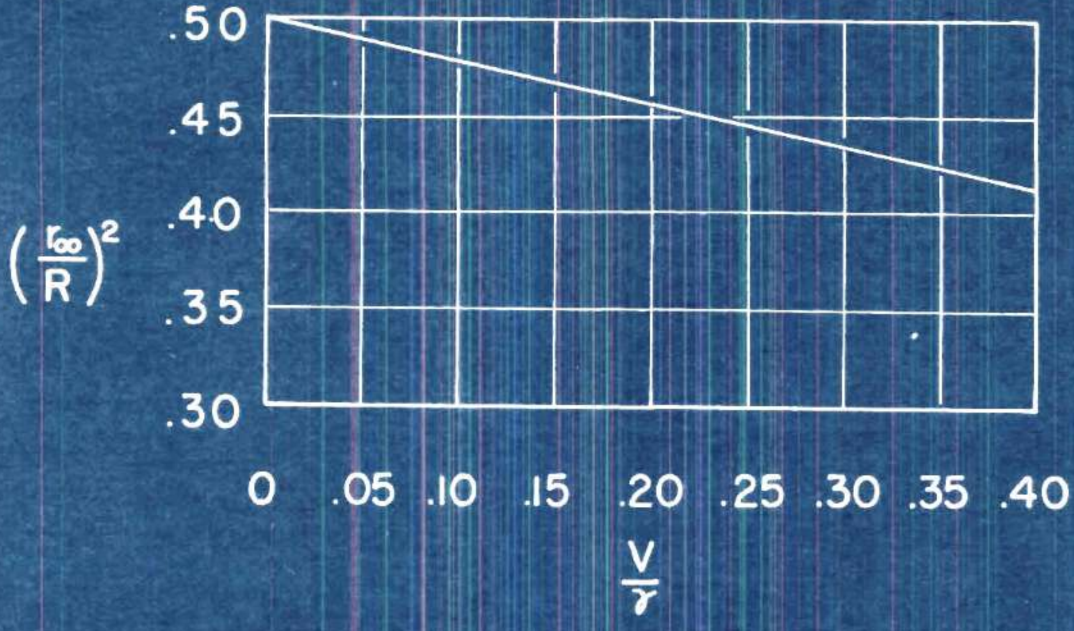

Figure 7

Values of Initial Wake Radius $\frac{r_o}{R}$

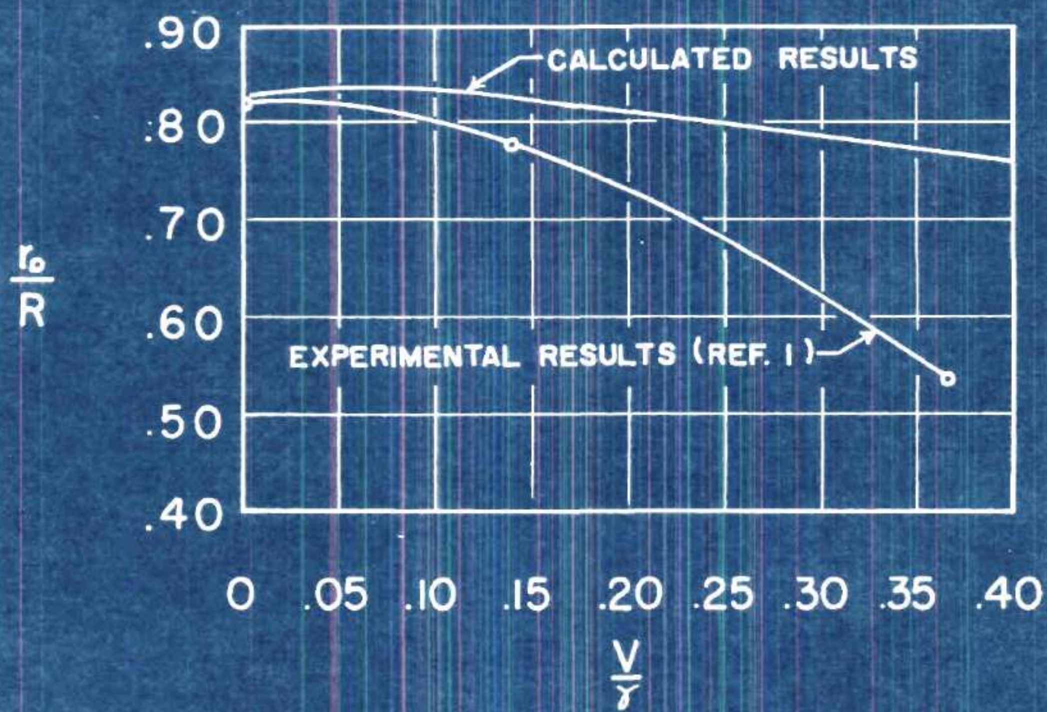

Figure 8

Values of Ultimate Wake Radius $\left(\frac{r_\infty}{R}\right)^2$

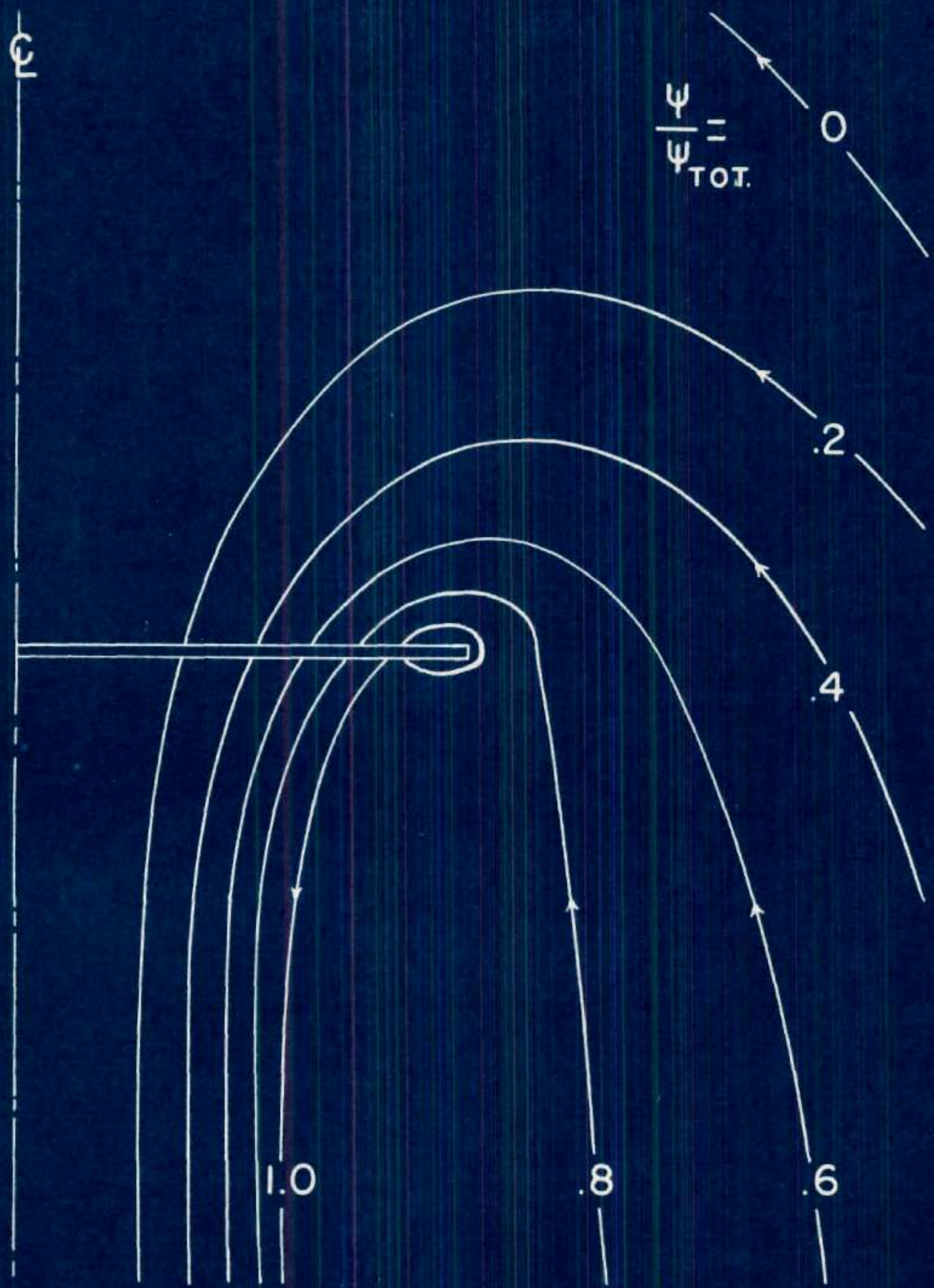

Figure 9

Streamlines for $\frac{V}{\gamma}$ = .05

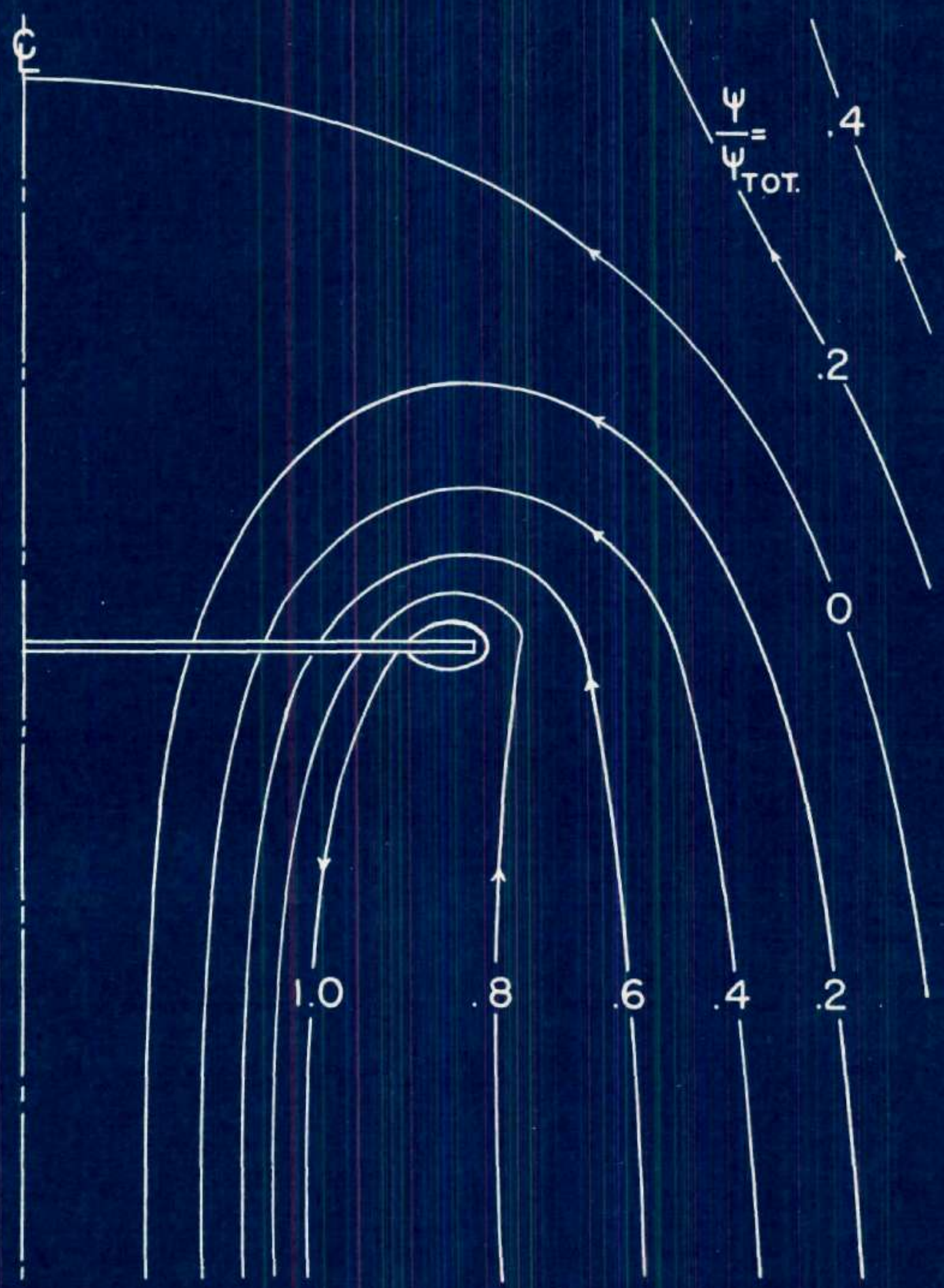

Figure 10

Streamlines for $\frac{V}{\gamma} = .10$

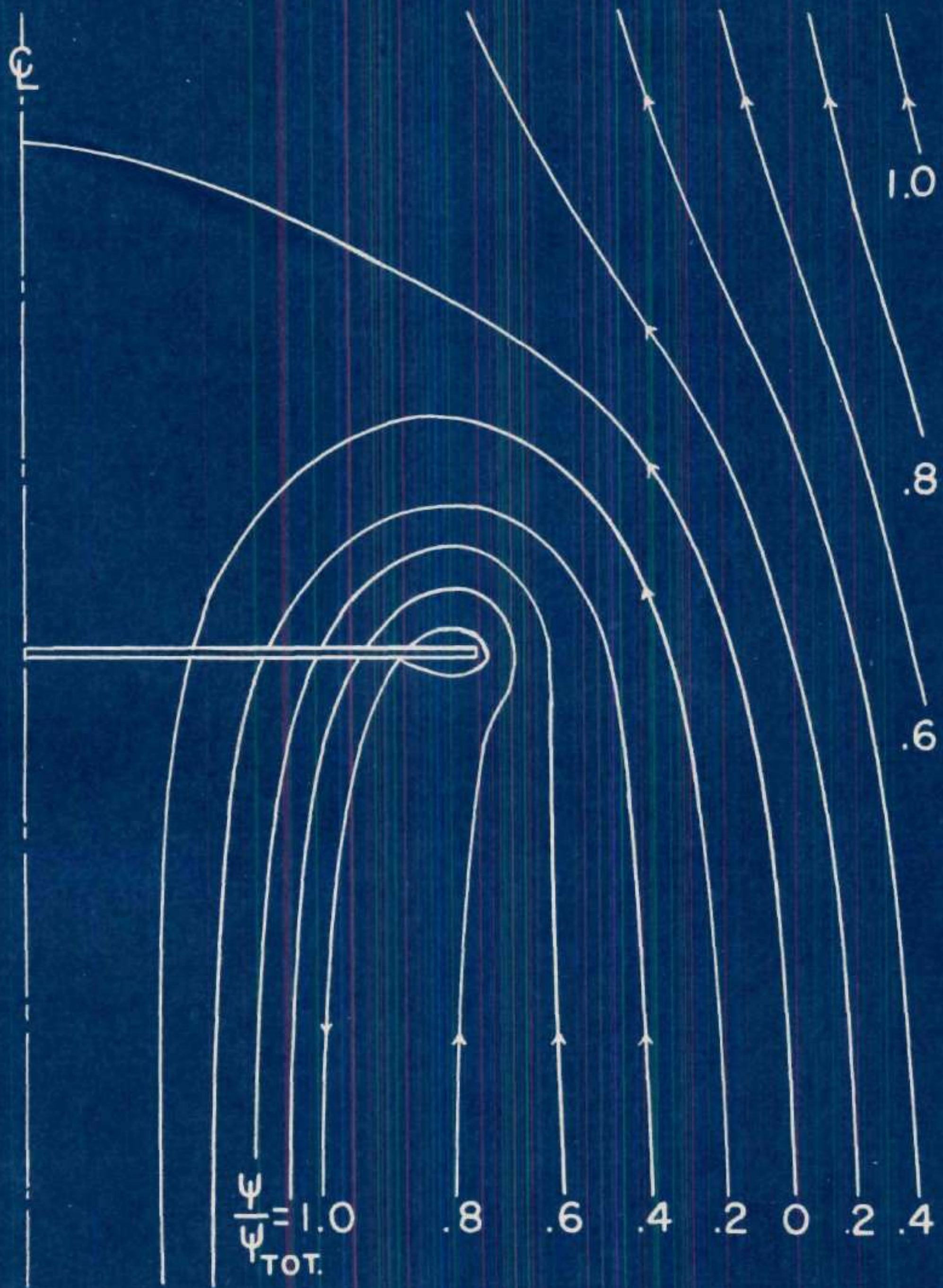

Figure 11

Streamlines for $\frac{V}{\gamma}$ = .15

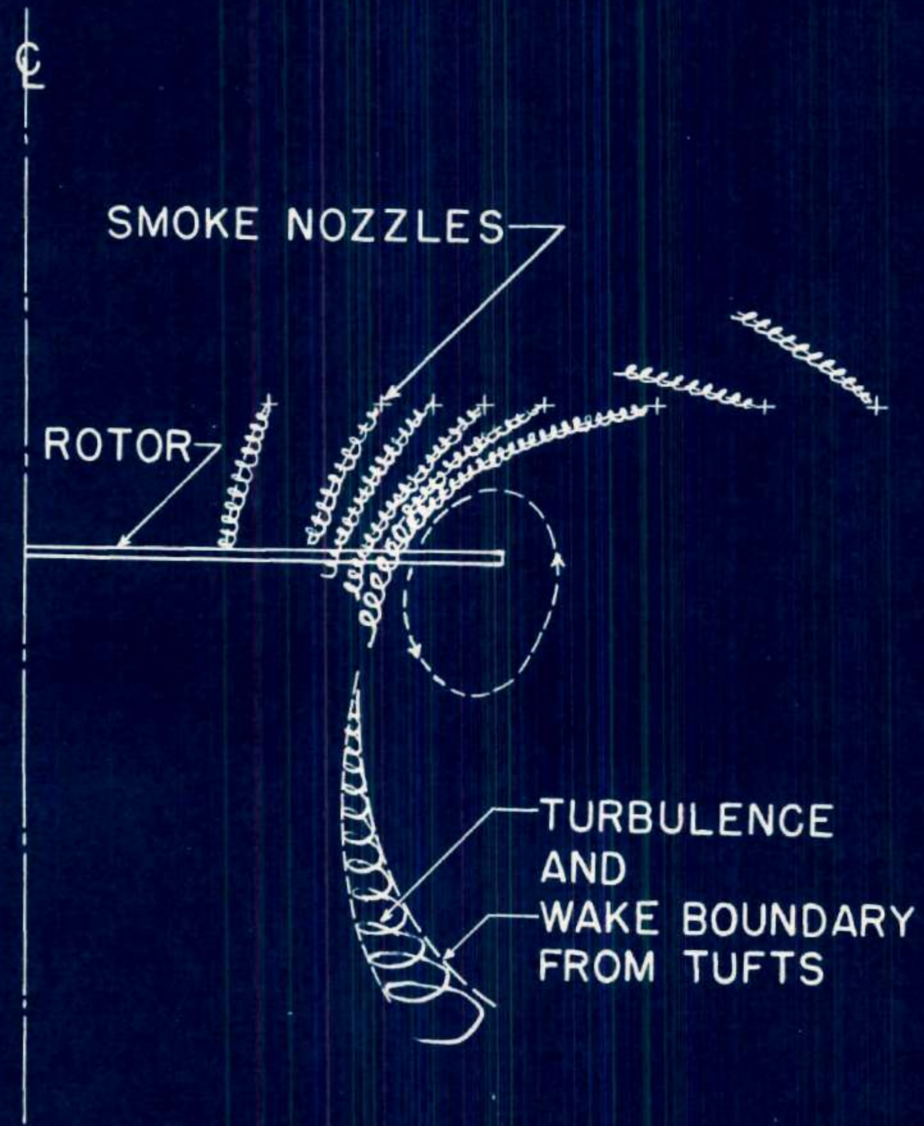

Figure 12

Smoke Flow Diagram for a 4 ft. Rotor

with Constant Chord Untwisted Blades Operating at $\frac{V}{\gamma} \approx .15$

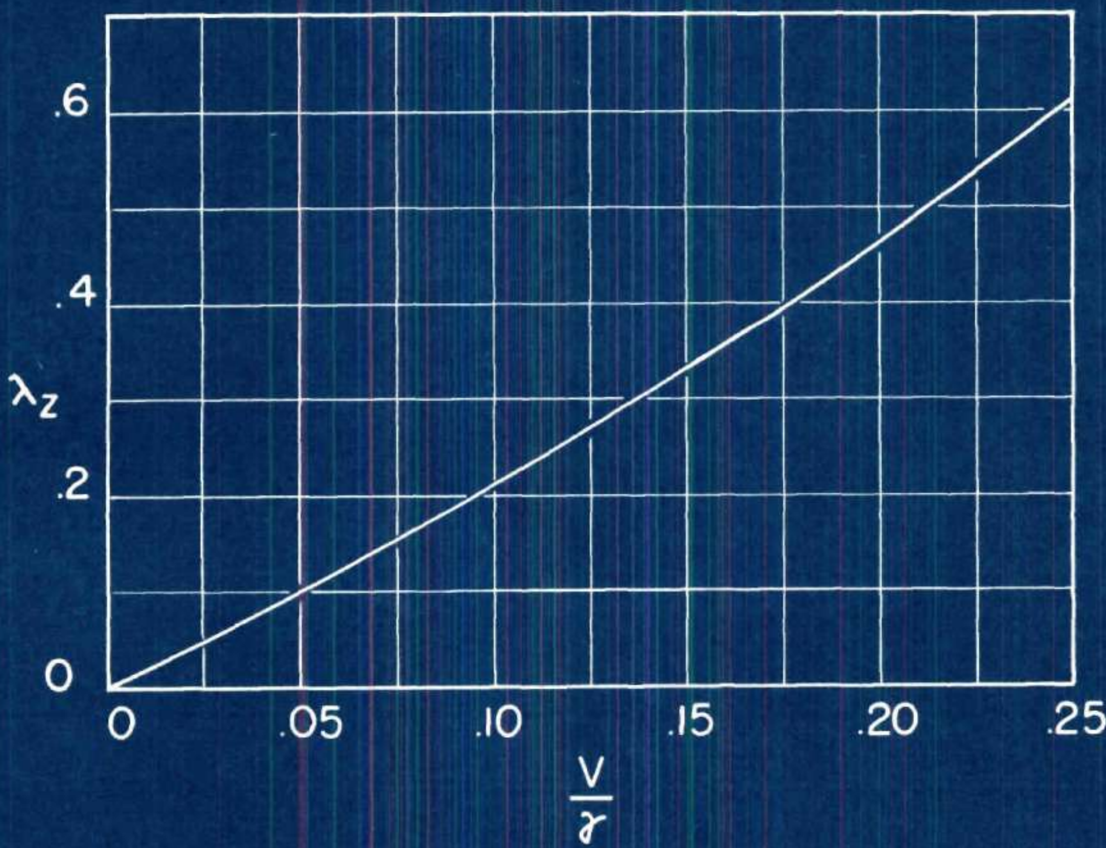

Figure 13

Relation between $\frac{V}{\gamma}$ and Nondimensional Freestream Velocity.

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