DENSITY-INDUCED SPIN-NEMATIC SQUEEZING IN A SPIN-1 BOSE-EINSTEIN CONDENSATE

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Measure twice. Cut once.

My Dad

For the ones who came before me, and the ones who will come after.

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SUMMARY

Density or pressure modulation of materials is an important method for tuning and engineering interactions within materials studied in condensed matter systems. This tuning is often used to alter or modify the underlying properties of the material, leading to the crossing of a phase transition or enhanced chemical or mechanical properties. This thesis investigates the possibility of whether a similar approach might be employed in the study of ultracold atoms present within a spinor condensate. In our system we use the con ning trap potential to modulate and increase the density of the system in such a way as to push the cloud of atoms from non-interacting to interacting, and across a quantum critical point. By crossing over into this new phase, we are able to perform a constant magnetic eld quench to observe both spin mixing and spin-nematic squeezing. This allows us to achieve -8.4 0.8 dB of squeezing and shows promise for future density-driven interactions.

CHAPTER 1

INTRODUCTION AND BACKGROUND

From the macroscopic transformation of ice to water to steam, to the microscopic creation of a superconductor, phase transitions are a part and parcel of our daily lives. The same can be said for quantum phase transitions [1]. From a thermodynamic perspective, the crossing of a critical point from one phase to another occurs from the increase in thermal fluctuations. In the case of quantum phase transitions, these transitions are driven by quantum fluctuations. Just like in conventional, thermodynamic phase transitions, these transitions can be first order or second order (continuous) [2]. Continuous phase transitions often exhibit other interesting properties, such as spontaneous symmetry breaking. It stands to reason then, that with the advent of cold atomic systems and their high degrees of control-ability, that one natural avenue of research would be to try and employ these tools to explore quantum phase transitions. These endeavors have probed the many-body behavior of quantum gases in a variety of contexts.

A large aspect of experimentation in the field of condensed matter is designing desirable samples to study. One way to explore particular types of transitions, is to choose a material with the desired properties or phase transition. This however, may still present experimental challenges if the interesting aspects of the material are still outside the available experimental controls, such as a phase transition that occurs at absolute zero or a material with only weak to moderate structural coupling. However, if one could take a material that has close to the desired qualities, one might be able to devise a way to slightly alter its properties to achieve more manageable outcomes. This is where the technique of pressure-tuning comes into play. A technique often used in condensed matter, it can be utilized to change the structure and correlations in a given material, allowing for critical points and in some cases, exotic phases, to become accessible. We use this to draw parallels to our

own system where we have devised a means of traversing the quantum critical point in our system using density modulation. This will involve us brie y reviewing the background of our spinor BEC system, as well as touch on how pressure-tuning can be used in condensed matter systems, before we return to the question of how cold atoms can be used to study the behavior of quantum phase transitions.

1.1 Density and Pressure-Tuned Quantum Phase Transitions

As mentioned, many condensed matter studies revolve around the types of materials used or created. These studies center around a wide-range of topics from the practical to the purely scienti c. However in many of these areas, pressure tuning has been found to be a useful tool both for the engineering and the testing of the properties of a material.

In the realm of quantum studies, many studies have used pressure-induced QPT's in the study of quantum and fundamental scienti c studies [3, 5, 6, 7, 8, 9, 10, 11, 12, 4]. Pressure tuning, along with other types of experimental tuning such as temperature, voltage, doping, or magnetic eld, can be used to look at a wide range of phenomenon. One early example of this are studies investigating Mott transitions [5], in which pressure was used to search for the rst order phase metal to Mott insulator transition in $_1(V_{1-x}Cr_x)_2O_3$ materials. This work has been extended to other studies of similar systems [11]. Mott insulators are important in the studies of superconductivity, and the transition between metal to Mott insulator is a rst-order transition. A similar study was performed in an organic material [7], and showed a similar relationship to pressure tuning that the original metallic studies had shown. This largely has to do with the role that pressure and temperature play in the tuning of these transitions which look at the correlations of electrons. In these studies, doping has nearly the same effect as pressure-tuning [11, 5]. Additionally, some of these materials also touch on other interesting topics, as the organic material $-(BEDT\text{-}TTF_4)Hg_{289}B_8$ in [12] is additionally a quantum spin liquid candidate material.

The studies of Mott insulators using pressure and doping as experimental knobs show

us there are even more complex topics to study using pressure-tuning of transitions. Topics involving superconducting to charge-density wave phases have been explored in [8] using rare-earth materials. For the spin-1/2 ladder compound $C_9H_{18}N_2CuBr_4$, pressure tuning affected the magnetic structure and spin dynamics [9] in such a way as to exhibit rarely seen fractionalization from an organized Neel order when the continuous quantum phase transition is crossed using high pressures.

Different types of material geometry also exhibit interesting collective behavior, when tuned with pressure. The triangular lattice system found in $CsCuCl_4$ [10] shows that the application of pressure at low temperature leads to eld-induced magnetic phases; furthermore, high pressures can be used to suppress quantum uctuations for precise measurement. The hexagonal, kagome lattice found in Sn_3S_2 undergoes a magnetic-topological phase transition when tuned with pressure and appropriate temperature, opening a new avenue to investigate potential Weyl semimetal and 3D topological insulator transitions.

The effects pressure has on the creation of frustrated magnets in the spin dimer system, $TiCuCl_3$, is also a very interesting system [13, 14, 7]. It has a continuous transition which goes from disordered (spin liquid) to ordered (antiferromagnetic) for a critical pressure. This critical pressure point can be similarly shifted using other external parameters, as the point where the spin energy gap is closed is between the singlet and triplet states.

Other applications of pressure-tuning involve engineering a material's properties for more practical purposes, such as changing the band gap and absorption pro les of quantum dots to create more favorable photovoltaic structures [15], or changing the magnetic and electronic properties of nanoribbons [15, 16].

These examples underpin the importance of pressure-tuning in condensed matter systems. We draw analogy from this and compare it to the quantum phase transition in our own system. For us, the “pressure-tuning” is performed by the increase in the trap power, which increases the density of the cloud of atoms trapped in its minimum potential. The density in our system is a tuning parameter, as the relationship for the quantum critical point

(QCP) occurs when the density is equal to the quadratic magnetic field divided by two. For sufficiently high densities, we are able to cross our quantum critical point to induce spin interactions in the cloud. In the next section, we will outline the previous quantum work that has lead up to the work presented in this thesis.

1.2 Spinor BECs and Our Work

Describing the quantum phase transition of our system, requires us to first describe the creation and evolution of our spinor BEC work. Spinor BEC's are so termed because they involve atoms cooled to quantum degeneracy that still retain their spin degrees of freedom. BEC's were famously first created in 1995 [17, 18, 19] in a Nobel Prize-winning invention. Owing to their ease of controllability and relative theoretical simplicity, BECs have been an important tool for many researchers investigating many-bodied effects and phenomena. Early experiments with BECs studied the coherent nature of the condensate itself [20, 21], as well as designing atom-optics with them [22, 23, 24]. The quantum properties of the macroscopic condensates were also investigated, looking at their quantized angular momentum [25, 26, 27] and quantum tunneling [28, 29].

Many of these early condensates were created by performing evaporative cooling in magnetic traps. The first optical confinement of BEC was in 1998, but was not created in the optical trap itself [30]. All-optical evaporation first occurred in our lab in 2001 [31]. From there, many experiments developed looking at the spin domain formation [32, 33, 34], spin mixing [35, 36], and mean-field formulation of the ground states [37, 38]. The quantum formalism for these spinor collisional interactions for an optical potential were further elaborated on by the Bigelow group [39, 40, 41].

In our lab, we looked at the mean-field dynamics of the hyperfine levels through theoretical description [42, 43], and experimental observation in rubidium-87. Our lab looked at the interactions within the $F = 1$ ground state [44, 45]. The excited manifold of $F = 2$ states were studied by the Sengstock group [46, 47, 48]. Other groups also began to look at

the collisional properties of BECs in optical lattices [49, 50]. More complex studies of spin and spatial structures were looked at with larger rubidium condensates [51, 52], magnetic trap studies in the $F = 2$ manifold [53], and spin wave demonstrations [54].

By looking at higher spin in spinor condensates [51, 52], studies grew involving quantum phase transitions present in several spin-1 species. This was investigated early on [35], but the observation of polar and ferromagnetic regions in rubidium [44], and later the measurement of an antiferromagnetic transition in sodium [55], led to theoretical explorations of the effects of magnetic field quenches [56] and ferromagnetic spinor BECs [57] in the context of a quantum phase transition. The ferromagnetic quantum phase transition in rubidium is the heart of this thesis and will be described in the next section, along with a brief overview of explorations of quantum phase transitions in cold atoms.

1.2.1 Quantum Phase Transitions in Cold Atomic Systems

Early work involving the study of quantum phase transitions with ultracold atoms began with the use of the optical lattices [58]. The interactions between the atoms in the lattice are controlled by changing the trap depth by changing the laser potential, allowing the system to be driven from the Mott to superfluid phase, as described by the Bose-Hubbard model. The Bose-Hubbard model describes the occupation and tunnelling between lattice sites for bosonic atoms. These experiments [59] offered insight into the simulation power of cold atoms, and led to further studies such as the Fermi-Hubbbard model [60], two-component quantum magnetism [61, 62] and nematic spinor phases with spin-1 gases [63].

In the case of spinor BECs, the mean-field theories referenced in the previous section lead to phase diagrams for our particular quantum phase transition. These can be used to study the many-body behavior of the system, by looking at the change in the order parameter across the transition. This has been looked at in a number of different dimensions as well, including studies dealing with QPTs in zero-dimensions [64, 65, 66, 67, 68]. Zero-dimensionality is a feature by which the spatial evolution of a system is inhibited, such

that the other degrees of freedom carry the dynamics. In our case, the interactions of the system are restricted to the spin variables under a simpli ed Hamiltonian (see chapter 2 for details).

Our system exhibits a continuous transition which results in a change in the spin symmetry of the ground state when quenched into the broken-axis symmetry phase (or BA phase) [57]. The effective potential for the system, when written in terms of the collective atomic spin, exhibits a potential of the form of a Landau, or double-well, potential [69]. The choice of what well to occupy creates a simultaneous breaking of a U(1) symmetry. This has been shown experimentally for rubidium and has been investigated in detail in the Chapman lab's work, as will be touched on in the next section.

Previous Chapman Lab BEC Work

From our early, innovative work on all-optical BEC formation [31], to our description and understanding of the mean- eld dynamics [70, 71, 72, 73], our many efforts greatly enhanced our current understanding of ferromagnetic, spin-1 BECs. Other important works as a result of these ndings led to the description of the mean- eld dynamics as an inverted, unstable pendulum [73]. Meanwhile, the evolution of the state across the quantum critical point, was described to lead to “number suppression" in the magnetization, as pair-wise, spin-interacting collisions led to the formation of correlated pairs [74]. We were able to measure this phenomenon experimentally leading to an effect called “number squeezing." This has become a standard protocol on our experiment and has given us a clear way to measure our detection system to determine our overall detection limit.

Likewise, a short time after, formulism for the understanding of spin-squeezing was developed and measured in our lab as well [75]. With the ability to accurately determine our detection limits, we were able to measure a high degree of squeezing called “spin-nematic" squeezing. Using a magnetic eld quench to lower the quadratic Zeeman energy, the spin contact interaction dominates to create correlated pairs. The tomography of this

state is then performed to measure the degree of spin-nematic squeezing. Its development is crucial to the current understanding of what squeezing in higher spin systems looks like. We were able to measure a high degree of squeezing in that system, as well as develop a deeper theoretical understanding of the spin-1 space. This has also become a standard experiment to attempt to replicate for our system.

Quantum Kibble-Zureck Mechanism

A more recent and important work for the study of crossing the quantum phase transition (QPT) in our rubidium spin-1 BEC, is our work investigating the quantum version of the Kibble-Zurek mechanism. This investigates the effects the speed of a quench has on the system dynamics as it crosses the quantum critical point (QCP). The affect applies to continuous QPTs, such as our spin-1 polar to broken-axis transition, to the Kibble-Zurek Mechanism (KZM) [56, 76]. It was originally studied with cold atoms initially in the context of the Mott to super uid transition in optical lattices, but did not show good agreement with the KZM predicted behavior [77]. However, the transition to the broken-axis phase was a good candidate for observing the quantum KZM dynamics [78, 56, 57, 77], as it restricts the experiment to a zero-dimensional system. The rst evidence of the quantum KZM was measured and con rmed in our lab, showing good agreement with the predicted theory [79]. A key development out of this experiment was the ability to measure the spin interaction energy(c_2) of the system, using a critical magnetic eld scan (see Sub-Section 3.9.3). This is currently used in the lab to measure the critical eld at which the quadratic energy is equal to two times the spin interaction energy of the condensate – the critical point of our system (see Chapter 2). This is a fast and reliable tool that we employ often in our daily measurements.

Adiabatic Passage of a QPT

A particularly relevant work is the adiabatic passage and characterization of our spin-1 quantum phase transition [69]. This work used coherent oscillations to measure the energy gap and gave predictions for potential nite-numbered condensate studies in the future (see Section 5.1). Additionally, this work looked at a piece-wise optimized magnetic eld quench to cross the QCP with minimal disruptions to the ground state. Unfortunately, the atom loss due to the nite lifetime of the trap lead to too many atoms being lost for an accurate number of highly entangled pairs to be counted, but this would also be a future goal from this work. Measuring the coherent oscillations of the energy gap is an important tool in our experiments for con rming and checking the stability of the spinor dynamics.

Other Recent Chapman Lab Efforts

Other recent experiments that the lab has been involved in is the study of the measurement of geometric phases in spin-1 atoms [80], and the development of a circular polarized antenna used to selectively drive individual microwave transitions [81]. While these studies did not necessarily use condensates for their work, their theoretical understanding and experimental demonstration help illustrate the versatility of the spin-1 system for addressing and solving both theoretical and technical challenges. The study of geometric phases demonstrates the existence of topological phases in our experiments, and how we might measure and control them. Our microwave work is a very interesting technique which could be used in quantum state preparation for quantum engineering. The ability to individually drive transitions and suppress off-resonant excitations is very important to performing precise computations with microwaves which provide low fault occurrences.

Our most recent work involves using the changing of the ground state near the critical point to engineer a partially squeezed state [82]. This state can be preserved because it never actually crosses the critical point, meaning that it does not exponentially evolve away. By timing the quench sequence appropriately, we can create and preserve a squeezed state for

the duration of the lifetime of the condensate. This is an experimentally interesting result as it demonstrates the ability to create a long-lived squeezed state which could prove a useful tool for future metrological applications which may hope to utilize spin squeezed states.

1.3 Thesis Organization and Contribution

The structure of this thesis starts out with an overview of spin-1, spinor BEC theory. In Chapter 2, we will introduce the mean- eld description of the dynamics and how it is used in our experiments. A quantum description of the spin interactions is also presented, along with a discussion of the connection to our semi-classical representations.

Next, we build on this understanding in Chapter 3, as we highlight the key aspects of the experimental apparatus which creates our spinor condensates. Furthermore, we describe several procedures performed routinely on the experiment, and how they demonstrate our control and understanding of the spin interactions.

The content of Chapter 4 contains the main thrust of this thesis's work, as we present the technique of density modulation of the condensate. We are able to do this to a precise degree and use trap modulation shift the quantum phase transition of the system. Doing so, allows us to control the interactions of the system and to observe these interactions through spin mixing and spin-nematic squeezing.

The nal chapter, Chapter 5, details previous, relevant investigations performed on the experiment. The descriptions of our work involving nite-numbered condensates demonstrate the motivations behind these investigations and the progress we've made in beginning this work. A technical upgrade to our experiment with the potential addition of a 2D-MOT as a cold atom source is also discussed. We examine the main results of our previous studies and present future designs where a 2D-MOT could be readily incorporated into the experiment. The nal section of this chapter relates improvements to our imaging system and how it could impact our current studies, as well as our future studies of condensates involving nite atoms.

CHAPTER 2

SPINOR BECS: BACKGROUND AND THEORY

2.1 Gross-Pitaevski Equation and our Simpli ed Hamiltonian

In order to understand our contributions to the understanding of quantum phase transitions, it is important to contextualize the experimental system with which we work. Our laboratory specializes in working with what are now termed as “spinor Bose-Einstein condensates" or “spinor BECs." BECs were originally Nobel prize-winning inventions, rst created in 1995 [18, 19, 17] by cooling a bosonic atomic species to quantum degeneracy, as predicted by Bose-Einstein statistics [2]. Over the years, they have proven their utility in many fundamental and applied elds of physics, owing to their ease of theoretical description and range of experimental control. While BECs were rst created through evaporative cooling in a magnetic trap, the innovation of trapping and creating a BEC using all-optical means (rst pioneered in our lab [31]) allows all (hyper ne) atomic spins in the ground state to be preserved. In our lab, our rubidium BEC is called a “spin-1" BEC, as the hyper ne ground state of the atom is $F = 1$, leaving them $m_f = 1; 0; 1$ Zeeman sublevels to interact. As we will demonstrate, this creates a rich environment in which to study the behaviors of more complex topics, such as the connection between quantum and semi-classical theories, and the crossing of quantum phase transitions.

2.1.1 Our Hamiltonian

The Hamiltonian for N-identical atoms in an optical potential, or dipole trap, can be written in the form known as the Gross-Pitaevskii equation:

$$H = \sum_{i=1}^{N} \frac{-\hbar^2}{2m} r_i^2 + V_T(r_i) + U: \tag{2.1}$$

Here, the terms summed over N pertain to the kinetic energy of the atoms and the trapping potential energy of the optical dipole trap. U represents a psuedo-potential term which arises from the two-body contact interaction between the atoms. The form of this contact interaction depends greatly on the properties of the particles themselves. For example, because our system reaches such low temperature, the effective scattering of the system is dominated by s-wave scattering. Likewise, as the system consists of bosonic particles, only values of the total hyper ne spin state (the combined hyper ne values of the two atoms colliding), are allowed. This yields for our $f = 1$ bosons, a total spin F channel of 0 or 2, where $F = f_1 + f_2$ represents the sum of the atoms' maximum hyper ne spin value. This allows us to represent the psuedo-potential as,

$$U = \sum_{i<j} (\vec{r}_i - \vec{r}_j) \sum_{F=0,2} g_F \sum_{m_F=-F}^{F} |F, m_F\rangle\langle F, m_F| \tag{2.2}$$

Here, $g_F = 4\pi\hbar^2 a_F/M$, where a_F is the s-wave scattering length used to describe the two-body coupling strength of the total spin channel. Expanding the outer product summation we obtain a number of terms which, in combination with the other terms in our original Hamiltonian, can be rewritten in the form of symmetric spin-preserving terms, and asymmetric spin-dependent terms. Doing so, we find:

$$H = H_{symm} + H_{asym}; \tag{2.3}$$

where for $i = 0, 1$,

$$H_{symm} = \sum_i \int d^3r\, \hat{\psi}_i^\dagger \left[\frac{-\hbar^2}{2m} \nabla_i^2 + V_T \right] \hat{\psi}_i + \frac{c_0}{2} \sum_{ij} \int d^3r\, \hat{\psi}_i^\dagger \hat{\psi}_j^\dagger \hat{\psi}_i \hat{\psi}_j \tag{2.4}$$

and,

$$
\begin{aligned}
H_{asym} = \frac{c_2}{2}\int d^3r\,(& \hat{\psi}_1^\dagger \hat{\psi}_1^\dagger \hat{\psi}_1 \hat{\psi}_1 + \hat{\psi}_1^\dagger \hat{\psi}_1^\dagger \hat{\psi}_1 \hat{\psi}_1 \\
& 2\hat{\psi}_1^\dagger \hat{\psi}_1^\dagger \hat{\psi}_1 \hat{\psi}_1 + 2\hat{\psi}_1^\dagger \hat{\psi}_0^\dagger \hat{\psi}_1 \hat{\psi}_0 \\
& + 2\hat{\psi}_1^\dagger \hat{\psi}_0^\dagger \hat{\psi}_0 \hat{\psi}_0 + 2\hat{\psi}_0^\dagger \hat{\psi}_0^\dagger \hat{\psi}_1 \hat{\psi}_1 \\
& + 2\hat{\psi}_1^\dagger \hat{\psi}_1^\dagger \hat{\psi}_0 \hat{\psi}_0)
\end{aligned}
\tag{2.5}
$$

Note that the symmetric and asymmetric expressions of the Hamiltonian share their own coupling strengths denoted by $c_0 = (2g_2 + g_0)/3$ and $c_2 = (g_2 - g_0)/3$. Both arising from a linear combination of the coupling strengths, g_F, listed above and grouped into spin preserving (symmetric) and spin-dependent (asymmetric) channels. Thus, H_{sym} describes the spatial dynamics of the atoms, and H_{asym} describes the spin exchanges.

2.1.2 Single Mode Approximation

Using these expressions, one can now describe the spatial and spin evolution of the system of spin-1 bosons in an optical potential. However, if we would like to consider a subset of the dynamics, say solely the spin interactions, we need to compare the relative length scales of the respective Hamiltonians. In the case of the spin-dependent Hamiltonian, the relevant length scale is defined by the so-called spin-healing length, $\xi_2 = \hbar/\sqrt{2m|c_2|n}$, where n is the number density, and c_2 is as defined above. This defines the characteristic scale in which spin domains in the clouds of atoms are formed. For typical densities and our rubidium-87 atoms, this value is typically on the order of 10^{-5} m. That means for clouds smaller than this size, all atomic spins will reside within the same domain, and be allowed to interact [38]. Such is the case in our all-optical, spin-1 BEC which typically consists of 40,000 ^{87}Rb atoms in a 10.6 μm dipole trap, crossed with an additional 850 nm laser to create a roughly spherical trap. The trap volume is such that these spin domains are inhibited, and the spatial degrees of freedom can be largely ignored in the problem,

consistent with what's known as the single-mode approximation (or SMA)[38, 83, 39, 84].

The SMA allows us to write all the spin components into the a single wave function of the form

$$\hat{}_i \quad \hat{a}_i \ (\vec{r}); \tag{2.6}$$

where $\hat{a}_i = j1; i i$ is de ned as a bosonic annihilation operator for each spin component, and $(\vec{r})$ is the spatial wave function. This spatial wave function is a simultaneous solution of the symmetric Hamiltonian, H_{symm}, de ned above. This is the dominant term, as the coupling constants for the Hamiltonians have the relationship where c_0 and c_2 are determined by the scattering lengths for ^{87}Rb which has $a_0 > a_2$ by about a 2% difference [85].

We can now de ne the spin mixing Hamiltonian by taking the mean- eld form of the Gross-Pitaevskii equation written as:

$$\hat{H}_{sym} = \frac{\sim^2}{2m} r^2 + V_T + c_0 N j \ j^2 \quad = \quad ; \quad \int d^3 r j \ (\vec{r})j^2 = 1; \tag{2.7}$$

and integrate over the condensate. Neglecting the kinetic energy in our low momentum, ultracold case, we arrive at the SMA Hamiltonian in the symmetric and asymmetric forms:

$$\hat{H}_{sym} = \hat{N} \quad c\hat{N} \quad \hat{N} \quad 1 \tag{2.8}$$

$$\begin{aligned} \hat{H}_{asym} = c_2 (& \hat{a}_1^y \hat{a}_1^y \hat{a}_1 \hat{a}_1 + \hat{a}^y{}_1 \hat{a}^y{}_1 \hat{a}_{\ 1} \hat{a}_{\ 1} \quad 2\hat{a}_1^y \hat{a}^y{}_1 \hat{a}_1 \hat{a}_{\ 1} \\ & + 2\hat{a}_1^y \hat{a}_0^y \hat{a}_1 \hat{a}_0 + 2\hat{a}^y{}_1 \hat{a}_0^y \hat{a}_{\ 1} \hat{a}_0 \\ & + 2\hat{a}_0^y \hat{a}_0^y \hat{a}_1 \hat{a}_{\ 1} + 2\hat{a}_1^y \hat{a}^y{}_1 \hat{a}_0 \hat{a}_0): \end{aligned} \tag{2.9}$$

In the above case, $\hat{N} = \hat{a}_1^y \hat{a}_1 + \hat{a}_0^y \hat{a}_0 + \hat{a}^y{}_1 \hat{a}_{\ 1}$ represents the total number of atoms, and $c_i = \frac{c_i}{2} \int j \ (\vec{r})j^4 d^3 r$ is the spatially integrated interaction strength. If the total number of atoms is constant, along with the trap potential, then H_{sym} is constant, and the dynamics

Table 2.1: Spin-1 Operators and Matrices. Matrices are in the spherical polar basis of $|f, m_F\rangle$.

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{S}_x = \frac{1}{\sqrt{2}}\left(\hat{a}^\dagger_{+1}\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{+1} + \hat{a}^\dagger_0\hat{a}_{-1} + \hat{a}^\dagger_{-1}\hat{a}_0\right)$$

$$S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{S}_y = \frac{i}{\sqrt{2}}\left(\hat{a}^\dagger_{+1}\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{+1} - \hat{a}^\dagger_0\hat{a}_{-1} - \hat{a}^\dagger_{-1}\hat{a}_0\right)$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad \hat{S}_z = \hat{a}^\dagger_{+1}\hat{a}_{+1} - \hat{a}^\dagger_{-1}\hat{a}_{-1}$$

can be attributed solely to the spin-dependent Hamiltonian H_{asym}.

This simpli ed case can now be looked at through the lenses of both mean eld and quantum mechanical approaches.

2.2 Quantum Mechanical Description

The quantum representation of the asymmetric, spin-dependent Hamiltonian is made possible by expressing the states of the system in terms of second-quantized operators [75, 86, 87, 88]. This is written as,

$$\hat{H}_{asym} = c_2\left(\hat{S}^2 - 2\hat{N}\right); \tag{2.10}$$

where $\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$ and can be shown to be written as spin-1 dipole operators [86] (see Table 2.1). $\hat{N}$ is as de ned above. The nal consideration we must look at involves the addition of the magnetic eld interaction to our system, given by the linear and quadratic Zeeman effects.

This becomes easier to see when we note that the quadratic moment operator can be de ned as $\hat{Q}_{zz} = 2\left(\hat{N}_1 + \hat{N}_{-1}\right) - \frac{4}{3}\hat{N}$ (see Table 2.2) and can be used to represent the quadratic Zeeman interaction, while $\hat{S}_z = \hat{N}_1 + \hat{N}_{-1}$ represents the linear Zeeman terms. This gives,

Table 2.2: Spin-1 Nematic, Quadrapole Matrices. Matrices are in the spherical polar basis of $|f, m_F\rangle$.

$$Q_{xx} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \hat{Q}_{xx} = \frac{1}{3}\hat{a}^\dagger_{+1}\hat{a}_{+1} + \frac{2}{3}\hat{a}^\dagger_0\hat{a}_0 - \frac{1}{3}\hat{a}^\dagger_{-1}\hat{a}_{-1} + \hat{a}^\dagger_{+1}\hat{a}_{-1} + \hat{a}^\dagger_{-1}\hat{a}_{+1}$$

$$Q_{yy} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad \hat{Q}_{yy} = \frac{1}{3}\hat{a}^\dagger_{+1}\hat{a}_{+1} + \frac{2}{3}\hat{a}^\dagger_0\hat{a}_0 - \frac{1}{3}\hat{a}^\dagger_{-1}\hat{a}_{-1} - \hat{a}^\dagger_{+1}\hat{a}_{-1} - \hat{a}^\dagger_{-1}\hat{a}_{+1}$$

$$Q_{zz} = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \hat{Q}_{zz} = \frac{2}{3}\hat{a}^\dagger_{+1}\hat{a}_{+1} - \frac{4}{3}\hat{a}^\dagger_0\hat{a}_0 + \frac{2}{3}\hat{a}^\dagger_{-1}\hat{a}_{-1}$$

$$Q_{xy} = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{Q}_{xy} = i\left(\hat{a}^\dagger_{+1}\hat{a}_{-1} + \hat{a}^\dagger_{-1}\hat{a}_{+1}\right)$$

$$Q_{xz} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{Q}_{xz} = \frac{1}{\sqrt{2}}\left(\hat{a}^\dagger_{+1}\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{+1} - \hat{a}^\dagger_0\hat{a}_{-1} - \hat{a}^\dagger_{-1}\hat{a}_0\right)$$

$$Q_{yz} = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{Q}_{yz} = \frac{i}{\sqrt{2}}\left(\hat{a}^\dagger_{+1}\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{+1} + \hat{a}^\dagger_0\hat{a}_{-1} - \hat{a}^\dagger_{-1}\hat{a}_0\right)$$

$$\hat{H}_{asym} = c_2\left(\hat{S}^2 - 2\hat{N}\right) + p\hat{S}_z + \frac{q}{2}\hat{Q}_{zz} \tag{2.11}$$

with $p = \mu_B B_z g_F$ and $q = \mu_B^2 B_z^2/(\hbar^2 \Delta E_{hf}) = q_z B_z^2$, defined with constants the Bohr magneton (μ_B), the Landé g-factor (g_F), and the hyperfine energy gap (ΔE_{HF}). This Hamiltonian is nontrivial to find simultaneous eigenstates for, as not all of the operators commute with each other, namely $\hat{Q}_{zz}$ and $\hat{S}^2$ (see Table 2.3). This can be further simplified by noting that both the magnetization ($\hat{S}_z$ ($= M = 0$) and the atom number ($\hat{N}$) are conserved, and so won't affect the dynamics. The simplified version of this Hamiltonian ($\hat{H}$ becomes:

$$\hat{H}_{asym} = c_2\hat{S}^2 - \frac{q}{2}\hat{Q}_z, \tag{2.12}$$

as we define the new variable $\hat{Q}_z = \hat{N} - \hat{Q}_{zz}$. This version of the Hamiltonian is

useful in our simulations and depictions of the dynamics in the spin-1 space.

Looking back at Equation 2.11, the asymmetric Hamiltonian was constructed in the spin-1 Fock basis $|N_{-1}; N_0; N_{+1}\rangle$. This can equivalently be transformed to be written in terms of $|N; M; k\rangle$, where N is the total number of atoms, M is the magnetization, and k is the number of pairs of atoms in the $m_f = -1$ state. As we will see, this basis is convenient due to several conserved quantities.

The total atom number can be assumed to be a constant throughout our experiments. Likewise, the magnetization, de ned as $M = N_1 - N_{-1}$, is also conserved; this leaves the parameter k as the single parameter over which the Hamiltonian can be solved. The Fock states therefore have a dimensionality of $N/2 + 1$, as it spans the number of available pairs for $k \in [0, \frac{N}{2}]$.

Taking this, we can write Equation 2.11 in the Fock number basis as,

$$\begin{aligned} \hat{H}_{asym} = & -c_2 \left(\hat{N}_1 - \hat{N}_{-1}\right)^2 + \left(2\hat{N}_0 - 1\right)\left(\hat{N}_1 + \hat{N}_{-1}\right) \\ & + c_2 \left(2\hat{a}_1^\dagger \hat{a}_{-1}^\dagger \hat{a}_0 \hat{a}_0 + 2\hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_{-1}\right) \\ & + p\left(\hat{N}_1 - \hat{N}_{-1}\right) + q\left(\hat{N}_1 + \hat{N}_{-1}\right) + \hat{N} E_0 : \end{aligned} \tag{2.13}$$

From the above equation, noting again that $M = 0$ and the initial state for the atomic cloud is $|f; m_f = 0\rangle$, we gather that the elemental form of the Hamiltonian matrix follows as [75],

$$\begin{aligned} H_{k;k'} = & \left(2c_2 k\left(2(N - 2k) - 1\right) + 2qk\right)\delta_{k;k'} \\ & + 2c_2\left[(k'+1)\sqrt{(N - 2k')(N - 2k' - 1)}\,\delta_{k;k'+1}\right. \\ & \left. + k'\sqrt{(N - 2k' + 1)(N - 2k' + 2)}\,\delta_{k;k'-1}\right]. \end{aligned} \tag{2.14}$$

These elements form a symmetric, tridiagonal matrix. Using this form, we can numerically integrate the Schrödinger equation of the form $i\hbar\frac{\partial}{\partial t} = H$, to look at the dynamics of the system in the quantum formulation.

2.3 Mean-Field Approach

In the mean field description, we can describe a large number N of atoms in the form of N-independent wavefunctions which satisfy the constraints given by the solution to the Gross-Pitaevski equation [39]. For our tightly confining trap, our system satisfies the single mode approximation (or SMA) [39] and we can consider the whole condensate to be in the same spatial state. This allows us to write a spinor wavefunction of the form,

$$\psi_i = \sqrt{N}\, |\psi_i| e^{i\theta_i} \tag{2.15}$$

where $|\psi_i|^2 = \rho_i = N_i/N$ are the fractional spin populations and θ_i are the phases for the $m_F = 0, \pm 1$ modes. This can be written as a form of coupled Gross-Pitaevskii equations, which we can use to simulate the evolution of the states in time. They take the form:

$$\begin{aligned}
i\hbar\frac{\partial \psi_1}{\partial t} &= E_1 \psi_1 + c[(\rho_1 + \rho_0 - \rho_{-1})\psi_1 + \psi_0^2 \psi_{-1}^*] \\
i\hbar\frac{\partial \psi_0}{\partial t} &= E_0 \psi_0 + c[(\rho_1 + \rho_{-1})\psi_0 + 2\psi_1 \psi_{-1} \psi_0^*] \\
i\hbar\frac{\partial \psi_{-1}}{\partial t} &= E_{-1} \psi_{-1} + c[(\rho_{-1} + \rho_0 - \rho_1)\psi_{-1} + \psi_0^2 \psi_1^*]
\end{aligned} \tag{2.16}$$

with $c = 2\epsilon N$ (the spinor dynamical rate) and $E_{m_F} = q(m_F)p + q$ (the expression for the linear ($p = p_z B$) and quadratic ($q = q_z B^2$) magnetic field energy shifts felt by each hyperfine sublevel (m_F)) being defined above. This allows us to write the wavefunction:

$$\psi = (\psi_1, \psi_0, \psi_{-1})^T. \tag{2.17}$$

Additional constraints further simplify our expression, as we take into account the conservation of magnetization ($m = \rho_1 - \rho_{-1}$) and the normalization condition $\sum_{i=1}^{3} \rho_i = \sum_{i=1}^{3} |\psi_i|^2 = 1$. This leads us to rewrite the order parameter by defining $\rho = \rho_1 - \rho_0$ to

give:

$$= \left(\sqrt{\frac{(1 + m)}{2}} e^{i \phi_+}; \sqrt{n_0}; \sqrt{\frac{(1 - m)}{2}} e^{i \phi_-} \right) : \quad (2.18)$$

Defining the spinor phase, $\theta_s = \phi_{+1} + \phi_{-1} - 2\phi_0$ and the magnetization phase $\theta_m = \phi_{+1} - \phi_{-1}$, we can write the mean-field spinor energy per particle [72]:

$$E = \frac{c}{2} m^2 + c_0 n_0 (1 - n_0) + q(1 - n_0) + \sqrt{(1 - n_0)^2 - m^2} \cos\theta_s + pm + q(1 - n_0) : \quad (2.19)$$

As will be seen in the next section, the mean-field description can be linked to the quantum formulism to provide a semi-classical picture, making it easier to simulate the complex dynamics for our ensemble of atoms.

2.4 Phase Space and Spin-1 Pictures

Several useful pictures can be described using the mean-field and quantum descriptions. They help us visualize the space and dynamics in which our spin dynamics reside. The first of which is obtained by looking at the phase space created by the conjugate variables n_0 and θ_s [72]. These can be obtained by taking the derivative of the energy functional in Equation 2.19:

$$\dot{n}_0 = \frac{2}{\hbar} \frac{\partial E}{\partial \theta_s}; \quad (2.20)$$

giving,

$$\dot{n}_0 = \frac{2c_0}{\hbar} \sqrt{(1 - n_0)^2 - m^2} \sin\theta_s : \quad (2.21)$$

Likewise,

$$\dot{\theta}_s = \frac{2}{\hbar} \frac{\partial E}{\partial n_0} \quad (2.22)$$

yields,

$$\dot{\theta}_s = \frac{2q}{\hbar} + \frac{2c}{\hbar} (1 - 2n_0) + \left[\frac{(1 - n_0)(1 - 2n_0) - m^2}{\sqrt{(1 - n_0)^2 - m^2}} \right] \cos\theta_s : \quad (2.23)$$

As will be discussed below, our system undergoes a change in observable behaviors for certain values of spin collision energy c_1 (hereafter simplified to "c") and quadratic Zeeman energy q. Using the $_0$, $_s$ phase space, we can simulate the mean-field behavior of a collection of atoms for various q=c values. These dynamics follow orbits around constant energy contours, and give us insight into how the semi-classical phase space changes. We can express the mean-field variables $_0$, $_s$, , and m in terms of the expectation values of the spin-1 operators presented earlier as [86, 88]:

$$
\begin{aligned}
\tan\ _+ &= \frac{S_y + Q_{yz}}{S_x \quad Q_{xz}} \\
\tan\ \ &= \frac{S_y \quad Q_{yz}}{S_x \quad Q_{xz}} \\
_0 &= \frac{1}{2}\ 1 + \quad 1 \quad \frac{1}{2}\ \frac{S_x + Q_{xz}}{\cos\ _+}^2 + \frac{S_x \quad Q_{xz}}{\cos}^2 \\
m &= \frac{1}{8\ _0}\ \frac{S_x + Q_{xz}}{\cos\ _+}^2 \quad \frac{S_x \quad Q_{xz}}{\cos}^2 \\
_s &= \ _+ +
\end{aligned}
\qquad (2.24)
$$

Furthermore, we can use the $_0$ and $_s$ variables to create a spherical projection of the spin-1 space. We call this the “spin-nematic" (SN) sphere. Remembering the operator $\hat{Q}_z$ presented in our Hamiltonian (Equation 2.12), we note that we can define a sphere with the normalized, mean-field relation:

$$1 = S_\perp^2 + Q_\perp^2 + Q_z^2. \qquad (2.25)$$

This incorporates the definitions of the transverse spin, $S_\perp^2 = S_x^2 + S_y^2$, and the off-diagonal quadrupole moments $Q_\perp^2 = Q_{xz}^2 + Q_{yz}^2$. These equations take the form [86, 88]:

Figure 2.1: Energy contours in the spinor phase space (θ, s_z) for different values of $q/|c|$:
The plots in clockwise order, from top left to bottom left, show the contours for values of $q/|c| = 10$; $q/|c| = 2$, $q/|c| = 1$, and $q/|c| = 0.5$. Values for $q/|c| > 2$ reside in the polar phase space, while values less than two belong to the broken-axis symmetry phase, where the oscillations away from the closed orbit ground state (shown in red) are characterized by larger phase-winding oscillations.

$$
\begin{aligned}
S_x &= \langle j S_x j \rangle = 2\sqrt{{}_0(1 - {}_0)}\cos \cos {}_L = S_\perp \cos {}_L \\
Q_{yz} &= 2\sqrt{{}_0(1 - {}_0)}\sin \cos {}_L = Q_\perp \cos {}_L \\
S_y &= 2\sqrt{{}_0(1 - {}_0)}\cos \sin {}_L = S_\perp \sin {}_L \\
Q_{xz} &= 2\sqrt{{}_0(1 - {}_0)}\sin \sin {}_L = Q_\perp \sin {}_L \\
Q_z &= 2 {}_0 - 1
\end{aligned}
\tag{2.26}
$$

This creates a projection onto a Bloch-type sphere and is useful for watching the evolution of the semi-classical space constructed from dipole-quadrupole operators defined as $Q_{ij} = \hat{Q}_{ij}/N$. Q_z is constructed in a way to maintain a range $[0,1]$ for the variable ${}_0 \equiv N_0/N$. Together, the set of variables S_x, $Q_\perp$, and Q_z don't create a proper $SU(2)$ subspace of the $SU(3)$ phase space, shown in [86], but they do allow us to visualize the dynamics in the higher, spin-1 space. Moreover, it becomes an excellent way to understand the squeezing dynamics which develop in the $S_\perp$, $Q_\perp$ space, which we will demonstrate in the next section. Various values of $\{j, c\}$ are shown on the spin-nematic spheres to shown the energy contours for the various phases of the system.

Figure 2.2: Energy contours in the spin-nematic phase space ($\hat{S}_x, \hat{Q}_{xy}$, and $\hat{Q}_z$) for different values of $|q/c|$: The plots in clockwise order, from top left to bottom left, show the contours for values of $|q/c| = 10$; $|q/c| = 2$, $|q/c| = 1$, and $|q/c| = 0.5$ as projected on the spin-nematic sphere. Values for $|q/c| > 2$ reside in the polar phase space. As in the spinor phase space, values of $|q/c| < 2$ are characterized by closed oscillations (in red) near the ground state, while phase winding oscillations appear in blue. The black line depicts the separatrix between these two types of oscillations.

Table 2.3: Commutation relations for the spin-1 operators.

[#; !]	S_y	S_z	Q_{yz}	Q_{xz}	Q_{xy}	Q_{xx}	Q_{yy}	Q_{zz}
S_x	iS_z	iS_y	$i(Q_{zz} \quad Q_{yy})$	iQ_{xy}	iQ_{xz}	0	$2iQ_{yz}$	$2iQ_{yz}$
S_y	-	iS_x	iQ_{xy}	$i(Q_{xx} \quad Q_{zz})$	iQ_{yz}	$2iQ_{xz}$	0	$2iQ_{xz}$
S_z	-	-	iQ_{xz}	iQ_{yz}	$i(Q_{yy} \quad Q_{xx})$	$2iQ_{xy}$	$2iQ_{xy}$	0
Q_{yz}	-	-	-	iS_z	iS_y	0	$2iS_x$	$2iS_x$
Q_{xz}	-	-	-	-	iS_x	$2iS_y$	0	$2iS_y$
Q_{xy}	-	-	-	-	-	$2iS_z$	$2iS_z$	0
Q_{xx}	-	-	-	-	-	-	0	0
Q_{yy}	-	-	-	-	-	-	-	0

CHAPTER 3

EXPERIMENTAL APPARATUS

In order to perform our particular flavor of spinor BEC studies, our experimental techniques rely on a number of generational advances, many of which are the by-product of many Nobel-worthy advances the field of atomic physics. The apparatus and techniques written about in this section also span several generations of graduate students in the Chapman laboratory, many of who's work is describe in other works [89, 81, 90, 91, 86, 92, 93].

This chapter will present an overview of the current apparatus used to create and perform the studies described in this thesis. We will briefly discuss the general types of techniques used to cool, trap, interact, and measure our BEC. We'll then outline how these tools are made to work practically in an experimental sequence to study spinor dynamics.

3.1 Lasers

Lasers are a key part of our experiment, as they provide the cooling, trapping, and imaging of the atoms in the experiment. The sections below detail the setup and operation of these lasers, as well as their roles and layouts in the experiment.

3.1.1 Experimental Diode Lasers

Diode lasers provide an economic and effective way to interact with rubidium [94], as its atomic transitions are in the near-infrared wavelength range of many diode lasers (for us 780 nm). These lasers are easy to make a stable frequency, or single mode, through appropriate temperature and current controls. In the previous versions of our experiment, the stability of our diode lasers were further strengthened through the use of an external grating. This grating is a standard technique which reflects the first order diffractive order back into the diode's internal cavity [94], allowing one to create a coherent laser beam of a

single, narrow frequency. This laser setup is referred to as an “external cavity diode laser" of ECDL. A schematic of a typical setup is shown below Figure 3.2.

A second type of diode laser newly introduced in the experiment is the distributed Bragg re ector laser or “DBR" laser. Similar to the conventional ECDL laser, it involves a grating to provide the single mode feedback for the laser; however instead of being located external to the diode housing, the grating is microfabricated to be located within the diode housing. This allows for more stable control of the temperature of the laser, and thus stability and tunability of the laser itself.

Our experiment currently employs both types of diode lasers. The former, ECDL type was used in the experiment for many decades. Within the last two years however, we added DBRs to our arsenal and after some testing, swapped out some of the ECDLs for equivalent DBR lasers. We'll talk about how we use these diode lasers now in the experiment and some of their locking characteristics.

3.1.2 Frequency Stabilization of Diode Lasers Used in the Experiment

Our system uses a saturation absorption spectroscopy lock to electronically feedback to and lock our lasers onto a single frequency. First demonstrated in 1995 [95], it works by dithering the lasers' current with a high frequency modulation (MHz). This is then electronically mixed [86] to create an error signal using the signal from the photodiode signal and the input modulation (see Figure 3.2). Once the current and temperature settings are found to put the laser near resonance, the lines from the sub-Doppler peaks can be used to lock the lasers (see Figure 3.1).

A table for the modulation and locking settings used in the experiment is shown below in Table 3.1. Here the reported values are from Fall 2022. These modulations were later updated (circa Spring 2023) in our heterodyne studies of the DBR lasers to give smaller frequency spreads (FWHMs) of the output from the laser beams. Data presented in this thesis is taken primarily with the laser modulations listed in the table below.

Figure 3.1: DBR cycling laser saturation absorption signal (yellow) and the DBR error signal (blue) depict the main signals used to lock the experimental cooling and imaging transition. Full schematic shown in the following gure to depict how this signal is used to stabilize our laser to a particular frequency to less than a 5 MHz linewidth [89].

Table 3.1: Experimental diode lasers listed with their modulation strengths and measured with heterodyne measurements to estimate the linewidth of the laser's output when locked using the FM modulation.

Diode Laser	Est. FWHM	Modulation Freq.	Modulation Strength
Probe ECDL	3-4 MHz	0.3 MHz	+10 dBm
Cycling DBR	5+ MHz	1.8 MHz	-4.3 dBm
Repump DBR	5+ MHz	1.8 MHz	-4.1 dBm

Once this light is generated from the FM spectroscopy, it can be locked by the locking electronics using our home-built boxes [86]. Our locks use only proportional and integral feedback to the error signal to maintain the lock at its setpoint. In our older setup, the slower integral feedback went to the piezoelectric motor which controlled the grating position for the ECDL lasers, while the proportional feedback was applied to modulate the current of the diode itself. In the current con guration for the DBRs, we instead combine the proportional and integral outputs, with appropriate attenuation, to directly feedback to the DBR current (see Figure 3.3). In this con guration, the lasers can remained lock for most of the day, in the case of the ECDLs, and for many days, in the case of the DBR lasers. Additionally, while the ECDL lasers are temperature sensitive above a change of a few

Figure 3.2: FM locking electronics circuit used to lock the ECDL laser on the experiment. Currently only used for the probe laser, as it incorporates a separate integral feedback for the piezo grating.

degrees in the room (as the external grating and experiences thermal shifts outside of the correctable range of the electronic feedback), the DBR lasers stay on resonance as the diode and grating are thermally and internally isolated from the environment.

A nal point of consideration for the laser lock of the DBR lasers controlling what's known as the cycling or cooling transition of rubidium, is the frequency shifts provided by acoustic-optical modulators (AOMs). As will be discussed in the next section, the frequency shifts determine the output light's precise frequency used to cool or image the atoms. The AOMs for the DBR lasers, however lie within the laser locking path, and when the frequency of the AOM is changed, both the direction and amplitude of the outgoing

Figure 3.3: FM locking electronics circuit used to lock the DBR lasers used on the experiment. This circuit applies to both the cycling and repump DBR lasers and is used to stabilize the output frequency of the lasers.

beam is changed (see Figure 3.9 for optical layout). This inevitably changes the amount of light going to the saturation spectroscopy setup, effectively changing the error signal size (and offset to a smaller extent.) With appropriate gain settings, and a slow enough shift in the frequency, the DBR cycling laser can reliably follow the experimental cycle for the duration of the day. More details of the timing and size of the frequency shifts will be further discussed below.

3.1.3 MOT and Imaging Beams

As noted above, the diode lasers used in our experiment perform the initial stages of cooling and trapping for our rubidium atoms, and are used at the end of the cycle to image the atoms. The way we accomplish this is through a series of AOMs (mentioned above). As depicted in the level diagram shown below, the cycling transition ($F = 2 \rightarrow F = 3$) is responsible for the initial cooling stage of the room temperature rubidium-87 atoms to the millikelvin range. This occurs in a process known as “Doppler cooling," which originated in an Nobel-worthy advancement in the laser cooling of atoms [96]. Likewise, we use a simpler, though similar, locking scheme for the repump laser, which helps create a closed cooling system for our rubidium-87 system.

Rubidium is one of the easier atoms to cool because as an alkali atom, it is relatively simple to create a “closed" cooling system with two frequencies (the cycling and the repump, as mentioned previously). The cycling laser provides the

Frequency Scanning of the Laser Detuning

The so called “cycling laser" is called that as it is close in frequency to the “cycling" transition ($F = 2 \rightarrow F' = 3$, see Figure 3.4). This transition is used for both the main cooling mechanism in the first stage of the experiment (also known as the MOT phase). To cool and trap the most atoms we find a detuning of around -20 MHz to be the optimal detuning from the resonant, cycling transition. The second stage is used to further cool the lasers using a

Figure 3.4: MOT Transitions in Rubidium-87

far-detuned, “dark" MOT setting, which is typically found to be optimal around -200 MHz from the cycling resonance. The nal use of these lasers comes from In order to achieve a lockable laser capable of scanning over 200MHz, while staying locked, we use a series of AOMs and locking points to create a laser capable of reaching an on-resonance frequency.

We lock the cycling DBR laser, used for both the MOT and imaging light, to the “crossover" peak. This peak appears from the moving atoms that are resonant with either the incoming or re ected beam [95] and so produce a peak that lies between two resonant lines (here$F = 2 \rightarrow F = 3'$and$F = 2 \rightarrow F' = 1$). The crossover we us is the “3-1" crossover, which is -211.8 MHz away from resonance. This peak is large enough to lock to and when combined with the available AOMs allow us to reach the cycling frequency. The locking point combined with the AOM shifts works out to:

Table 3.2: Frequency shifts for the cycling light used to generate the MOT and imaging light for the experiment. Shifts produced by a combination of the locking point and the shifts provided by various AOMs (depicted in optical outline below).

Frequency Shifts	MHz	Type
Lock Point	- 211.8	Fixed
Lock AOM	- 160	Fixed
Double Pass	+ (2x)(111.6 241.8)	Variable
Post-TA AOM	- 110	Fixed
Net Shift	(-258.6 +1.8)	Range

The range of output frequencies allows us to empirically optimize the frequencies for the MOT, dark MOT, and imaging stages. In the past, we could only scan up to a maximum of -6 MHz. But when the new DBR lasers were installed, a 1.5x ampli er was added which allowed the control voltage from the computer (0-10 V output) to be increased. This allows a higher input to the voltage control oscillator (VCO), which changes the output of the AOM. In our case, the VCO-AOM driver circuit modulates the double-pass AOM, adding a variable frequency range to the output from this path. The plot of the net frequency shift is shown in Figure 3.5 for the new cycling DBR laser as a function of voltage input from the computer to the VCO. This however can be tricky, as the angle of de ection

Figure 3.5: Net Frequency shift for the cycling DBR detuning as a function of experimental control voltage applied from the computer to the double pass AOM in the DBR cycling locking path. Shows wide tuning range used to tune between the dark MOT stages (around -200 MHz detuned) to imaging stages (around 0 MHz detuned) of the experiment.

of the frequency-shifted beam is dependent on the frequency. Thus, the amount of light downstream from the double pass AOM changes, changes due to the changing angle and ef ciency for different frequencies.

For this reason, we try top optimize the double-pass AOM's angle in the middle of the frequency range to ensure there isn't a drastic change in the amount of light that makes its way to the saturation absorption setup used for locking. As a consequence, the saturation absorption spectra shifts by small amounts, due to the variable frequency settings, causing the error signal to have a slight DC offset and amplitude change. By using the appropriate locking settings (e.g. suf cient feedback, offset, etc.) and changing the detuning to be at a suf ciently slow rate (usually 20 - 40 ms for large detuning shifts), we can lock the cycling DBR laser for the entirety of the day without re-locking.

A similar procedure is done for the probe laser that is used in absorptive imaging of the BEC. It works by shining resonant light onto the condensate, which effectively gets totally

absorbed, creating a shadow of the cloud on the camera. In order for this to occur, the probe must be resonant with the cycling transition. To ensure this, we lock to the same transition as with the cycling DBR laser mentioned above (the (3,1) crossover, see Figure 3.6). The locking path also has a fixed -160 MHz AOM in it, as it was formerly the “master" or primary experimental cycling laser before we added the DBR lasers to the system. This gives a net detuning of around -372 MHz, before the light reaches the probe's double-pass AOM. The frequency modulation of the AOM has a range of values if can be set to, but the exact frequency and strength of the modulation is determined quasi-empirically, as we look at the absorptive image. Usually this value is very close to the nominally expected value of 186.0 MHz 0.1 MHz. See Table 3.3 below for the summary of the frequency shifts for the probe laser.

Table 3.3: Frequency shifts for the probe produced by a combination of the locking point and the shifts provided by various AOMs (depicted in optical outline below).

Frequency Shifts	MHz	Type
Lock Point	- 211.8	Fixed
Lock AOM	- 160	Fixed
Double Pass	+ (2x)(186.0)	Set
Net Shift	+ 0.1	Set

The repump laser is simplest of all of the diode lasers, in terms of frequency shifts needed to make it resonant with the “repump" transition, as it only has one frequency shift involved. The repump laser itself is on resonance with the $F = 1 \rightarrow F' = 2$ transition (see Figure 3.4). To achieve this, we lock the repump to the (2,1) crossover of the saturation absorption signal (see Figure 3.7). This proves to be a useful feature to lock to as it is one of the larger features on an already smaller signal (owing to the strength of the transition). It is also easily accessible with a fixed AOM shift of +80 MHz, as the crossover peak is located -80 MHz away from the true repump resonance.

A final function that our AOMs provide for all three of the lasers, is that the last AOMs in each of their respective paths have the ability to turn on and off with ns precision. This