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RESEARCH PROJECT INITIATION

Date: 23 April 1974

Project Title: Probability Analysis of Crack Initiation

Project No: E-16-646

Principal Investigator: Dr. S. Hanagud

Sponsor: NASA- Marshall Space Flight Center; MSFC, Ala.

Agreement Period: From 3-22-74 Until 3-21-75

Type Agreement: NASA Contract No. NAS8-30617

Amount:	$ 9,455	NASA funds (E-16-646)
	1,731	GIT Contrib (E-16-341)
	$11,186	Total

Reports Required: Monthly Progress Reports; Final Technical Report

Sponsor Contact Person (s):

Technical Matters	Contractual Matters
Mr. J. M. Thomas, Principal	(thru ORA)
S&E-ASTN-AS	Mr. R. J. Whitcomb (ACO)
Mr. J. H. George, Alternate	ONR Resident Rep.
S&E-ASTN-RRC	Campus
NASA - MSFC	
Marshall Space Flight Ctr., Ala 35812	

Defense Priority Rating: DO-A2 under DMS Reg. 1.

Assigned to: School of Aerospace Engineering

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SPONSORED PROJECT TERMINATION

Date: June 20, 1977

Project Title: Probability Analysis of Crack Initiation

Project No: E-16-646

Project Director: Dr. S. V. Hanagud

Sponsor: NASA, Marshall Space Flight Center, AL

Effective Termination Date: 9/30/76

Clearance of Accounting Charges: 9/30/76

Grant/Contract Closeout Actions Remaining:

- ☐ Final Invoice and Closing Documents
- ☐ Final Fiscal Report
- ☒ Final Report of Inventions
- ☐ Govt. Property Inventory & Related Certificate — Submitted 30 Mar 77
- ☐ Classified Material Certificate 9 NASA 1018 ✓
- ☐ Other ____________

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Progress Report No. 1

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor S.H/
School of Aerospace Engineering

Period Ending: May 1974

This is the first progress report on the NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation." The derived differential equations for $P(t;k\Delta L)$ were modified to include varying rates of crack growth and arbitrary distributions for crack initiation. The solutions of these equations for specific hypothesized crack initiation distributions were attempted.

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Progress Report No. 2

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor S 4/
School of Aerospace Engineering

Period Ending: June 1974

This is the second progress report on NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitles "Probability Analysis of Crack Initiation." A solution to the differential equations for $P(t;k\Delta L)$ and a hypothesized exponential distribution for crack initiation has been completed. The formulation of the problem is being extended to consider multiple critical locations in the structures of the fleet that are prone to fatigue damage.

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Progress Report No. 3

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor S.H.
School of Aerospace Engineering

Period Ending: July 1974

This is the third progress report on NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation." The formulation of the problem has been extended to include multiple critical locations by using order statistics approach and renewal theory. Computer programs are being written to use the solution (to the formulated differential equations) in estimating the parameters of a hypothesized distribution for crack initiation.

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Progress Report No. 4

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: August 1974

This is the fourth progress report on NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation." The work performed during this month included the continued development of computer programs for estimation of the parameters of a hypothesized distribution for crack initiation. The method used in the program has incorporated the stochastic model developed for crack initiation in the prescence of multiple critical locations. The work to date was discussed with Dr. Jerrel M. Thomas of Marshall Space Flight Center, Huntsville, Alabama. The program was reviewed and goals were set for further development.

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Progress Report No. 5

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor SH

School of Aerospace Engineering

Period Ending: September 1974

This is the fifth progress report on NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation". The work performed during this month included the continued development of computer programs for estimation of parameters of a hypothesized distribution for crack initiation. The computer program will be capable of estimating the parameters from the available crack data from multiple fatigue critical regions. The goals of the computer program were changed following the discussion with Dr. Jerrel M. Thomas of Marshall Space Flight Center, Huntsville, Alabama.

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Progress Report No. 6

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: October 1974

This is the sixth progress report on NASA Contract NAS 8-30617 (Georgia Tech project no. E-16-646) entitled "Probability Analysis of Crack Initiation". The work performed during the month included specific checks on the developed computer program for estimation of parameters of a hypothesized distribution crack initiation. A check was done by hypothesizing an exponential distribution for crack initiation and by considering a single fatigue critical region. All the crack growth rates between the discretized crack lengths are assumed to be constant.

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Progress Report No. 7

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor S.H.
School of Aerospace Engineering

Period Ending: November 1974

This is the seventh progress report on NASA Contract NAS 8-30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation". The work performed during the month included further specific checks on the developed computer program for estimation of a hypothesized distribution for crack initiation. In this case, the crack growth rates and the associated distributions were chosen such that the growth rates can be different and can vary as the crack length increases. The single fatigue critical region is still preserved and exponential distribution for crack initiation has been hypothesized. Weibull distribution for crack initiation will be next considered.

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Progress Report No. 8

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: December 1974

This is the eighth progress report on NASA Contract NASB - 30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation." The computer program was modified to include weibull distribution for crack initiation. A weibull distribution with arbitrary shape parameter and scale parameter was hypothesized. By using the maximum likelihood method, these parameters can be estimated from data that include varying lengths of observed crack length at different times.

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Progress Report No. 9

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

S.H

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: January 1975

This is the ninth progress report on NASA Contract NASB - 30617 (Georgia Tech Project No. E-16-646) entitled "Probability Analysis of Crack Initiation." The computer program that has been written to include weibull distribution was checked by using the available data. Basic probability model that was derived for a single critical region is being modified to include multiple critical regions.

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Progress Report . 13

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor SH/
School of Aerospace Engineering

Period Ending: 10 Feb 1975

During this period the developed computer programs were applied to fatigue data that were available. The available fatigue data contained information on the observed crack lengths and their time of observation. These data were to be used to obtain the time of crack initiation and rate of crack propagation. As a first step it was decided to provide to the computer program the rate of crack propagation as an input. This was done to minimize the variables that were estimated. The rate of crack propagation was estimated by regression techniques. The regression program is developed.

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Progress Report 11

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 March 1975

During this period the regression program was completed and the results were input into the Weibull Program. The crack initiation times were estimated.

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Progress Report 12

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 April 1975

During this period, the computer program was tested with several available data. A source of inaccuracy was detected in the method of multiple integration technique. This is being improved by a variable size of the assumed polynomials.

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Progress Report 13

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor SH/
School of Aerospace Engineering

Period Ending: 10 May 1975

During this period efforts were to develop the new numerical technique for multiple integrals and to use the results in the computer program for estimation of probability of crack initiation.

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Progress Report

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 June 1975

During this period the multiple critical regions for crack initiation were considered. Speed on the new integration technique the computer program is being modified. Also application of the validity of single crack growth parameter are being investigated. Attempts are being done to obtain analytical expressions instead of multiple integrals.

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Progress Report 17

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 July 1975

During this period efforts to develop analytical expressions (with continuous variation of crack growth rates are being continued. This will eliminate the multiple integrals when the stress is constant.

Also the fatigue test options are being considered to apply the developed techniques for decisions on fatigue testing.

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PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 August 1975

During this period a specific analytical fatigue design procedure or procedures were investigated. The objective of this investigation is to select some specific procedures so that example problems can be constructed to investigate the methodology for selecting test options.

Also the significance of one or two full scale fatigue test in estimating the reliability of the structure are being investigated on the basis of a statistical definition of "estimated life".

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PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

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Period Ending: 10 September 1975

During this contract period special fail-safe design procedure for fatigue were considered for application. Periodic proofing and periodic or non-periodic inspection procedures were investigated in detail. Application of the developed stochastic model for fatigue failure to these fail safe design procedures were investigated. The objective of the investigation was to select a specific procedure for further development.

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PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor S.H.
School of Aerospace Engineering

Period Ending: 10 October 1975

During this contract period, a procedure for reliability-based fail safe design on the basis of periodic proofing has been further developed. The developments to date were discussed with the technical monitor at the Marshall Space Flight Center in Huntsville, Alabama. The research program for the next few months was outlined. It was decided to develop the fail-safe design procedure based on the periodic proof concept. Work on these lines are in progress at present.

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Progress Report 9

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 November 1975

After discussion with Dr. Thomas at Marshall Space Flight Center, Alabama, the objectives of the proposal was redefined. During the contract period the problem of estimating the probability of failure of SRM was specified. Procedures for the estimation of probability of failure of SRM are being formulated.

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Progress Report 10

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 December 1975

During the period investigations were continued in estimating the probability of failure of SRM. Specifically the material supplied to us by the Marshall Space Flight Center was studied. The uncertainties and the variables that should be considered were defined.

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PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor SH/
School of Aerospace Engineering

Period Ending: 10 January 1976

During the period the complete problem of optimization and cost effective design of SRM was defined on the basis of periodic proofing. A general procedure was formulated for estimation of certain design parameters on the basis of minimum expected cost. The procedure for estimation depends on the expected cost of failure which in turn depends on the probability of failure and other specific parameters.

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Progress Report. 2 P.

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor 5/
School of Aerospace Engineering

Period Ending: 10 February 1976

During the period the established procedure for cost optimization and the goals of the work to be done during the remainder of the contract period was discussed with Dr. Thomas and Mr. Bianca of Marshall Space Flight Center. It was agreed that only a methodology with guidelines to use the methodology will be provided. Work is proceeding in the development of the methodology.

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Progress Report 23

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 March 1976

During this contract period, basic equations for considering the probabilistic growth of cracks in SRM were formulated. The next step was to hypothesize and estimate the initial flaw distribution. Johnson S_u distribution was considered to be suitable for accurate representation of the initial flaw distribution.

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Progress Report 24

PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 April 1976

During this contract period, fracture mechanics equations (Paris, Foreman and Collipriest) were used to estimate the probability of the presence of a crack of a certain length after n service uses and m proof tests. The crack lengths were assumed to grow in discrete units. Attempts are being made to obtain a closed form solution for the deterministic counterpart of Foreman's equation.

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PROBABILITY ANALYSIS OF CRACK INITIATION

NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 May 1976

During this contract period, the following investigations were carried out. Instead of Johnson S_u distribution, S_b distribution was found more suitable for initial flaw distribution. The stress intensity factor is a function of crack length and the crack depth. For illustration purposes only crack depth has been retained as the random variable. A similar procedure can, however, be used to include crack length also as a random variable. This would include consideration of a two-dimensional S_b distribution. In order to develop a procedure that is easily understood, only crack depth has been examined as a random variable.

Equations are now being formulated for the probability of crack depth after each cycle.

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Progress Report 26

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 June 1976

During this contract period, a complete program was written to estimate the change of probability distribution of crack depth after each flight and each proof test. The computer program has been checked out. The analysis needed a quadratic or a cubic approximation for ϕ^2 and an integer approximation of n.

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Progress Report 17

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 July 1976

During the contract period, the complete reliability analysis of the membrane of the cylindrical segments has been formulated. The developments of the reliability analysis based on probabilistic fracture mechanics is being illustrated by a simple problem. A computer program has been written to estimate the reliability and to optimize the proof factor and number of cycles. In a similar way, initial wall thickness and material removal can also be optimized.

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Progress Report 28

PROBABILITY ANALYSIS OF CRACK INITIATION
NASA Contract No. NAS 8-30617 (E-16-646)

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Period Ending: 10 August 1976

During this contract period, the computer program for the reliability analysis including the fracture control plan is being checked and an example problem is being worked. The final report on the contract is being written.

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Final Report Draft
NAS 8-30017
E-16-646

PROBABILISTIC FRACTURE MECHANICS AND OPTIMUM FRACTURE CONTROL OF THE SOLID ROCKET MOTOR CASE OF THE SHUTTLE

Principal Investigator: S. Hanagud, Professor
School of Aerospace Engineering

Associate Investigator: B. Uppaluri, Research Assistant
School of Aerospace Engineering

October 1976

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APPENDIX I

APPENDIX II

ABSTRACT

Development of a procedure for the reliability analysis of the solid rocket motor case of the space shuttle has been described in this report. The analysis is based on probabilistic fracture mechanics and consideration of a probability distribution for the initial flaw sizes. The reliability analysis can be used to select design variables, such as the thickness of the SRM case, projected design life and proof factor, on the basis of minimum expected cost and specified reliability bounds. Effects of fracture control plans such as the material erosion between the procedure can be used for other similar structures.

Introduction

All structural components of the solid rocket motor case of the space shuttle are considered to be fracture critical. It is also the present plan to reuse the solid rocket motor case for a designated number of missions. The expected number of missions and operations such as the tests on the case between the missions are accounted in the projected design life of the structure. A fracture control plan is necessary because fracture critical components are being reused.

In particular, this report is concerned with the fracture control of the membrane of the six cylindrical segments that are considered to be the most critical of all structural components of the case. The developed procedure can, however, be used for all similar structures. During each mission, significant loads are applied to these six cylindrical segments during the flight and "slap down" operations. The applied stresses from all other events during the mission are considered not significant enough to result in cyclic or time dependent crack growth. If the test or analysis indicate the possibility of other critical loading events they can be included in the fracture control plan by extending the reported analysis. Before each mission, the cylindrical segments are also subjected to a proof test. The loads applied during the proof tests can result in significant amount of crack growth. As a preventive measure to reduce the effective depth of cracks, the thickness of the membrane is reduced by a selected amount between two missions. While the effective depth of crack is reduced, the operation has the effect of increasing the applied stresses. This necessitates a larger initial thickness of the membranes than that would be designed without this particular plan for fracture control.

Therefore, any design of the membrane of the six cylindrical segments of the solid rocket motor case must arrive at an initial wallthickness "t", the thickness 'Δt' that will be decreased between each mission and the proof load factor 'K_p'. For example, a large value of initial wall thickness results in increased reliability, but results in the need for increased propellant, increased cost of operation and reduced pay load capability. On the other hand, a small initial wall thickness increases the probability of failure and the resulting loss of the shuttle vehicle and the pay load. Therefore, there is a need for optimizing the initial wall thickness. Similar arguments can be presented to explain the need for selecting the other design variables such as 'ΔL' and 'K_p' by optimizing the desired objective function of cost and weight.

In general, these design variables depend on the probability distribution for the initial flaw sizes present in the membrane, applied stresses during the use of the vehicle, crack growth characteristics of the material, fracture control plans, specified reliability bounds, weight and cost considerations. The report describes a reliability-based procedure that can be used to select the design variables of SRM by using probabilistic fracture mechanics and cost or weight considerations.

Method of Approach

As discussed in reference 1, careful NDI techniques can detect initial cracks greater than the surface length of $c_0 = 0.1$ inch and surface depth of $a_o = 0.5\ C_o$ with 100% success. It has been claimed that cracks corresponding to surface length $c_o = 0.1$ inch can be identified 100% of the time. If the corresponding maximum depth is 0.05 inch there is no possibility of any initial cracks of depth larger than 0.05 inch. Such an initial crack depth distribution can be analytically represented[2] by Johnson S_b distribution. The density function for the probabilistic model is written as follows

$$f_{a_o}(a_o) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(a_o - \epsilon)(\lambda - a_o + \epsilon)} \exp\left[-\frac{1}{2}\left\{\gamma + \eta \ln\left(\frac{a_o - \epsilon}{\lambda - a_o + \epsilon}\right)\right\}^2\right],$$

$$\epsilon \leq a_o \leq \epsilon + \lambda, \quad \eta > 0, \quad -\infty < \gamma < \infty, \quad \lambda > 0, \quad -\infty < \epsilon < \infty; \qquad (1)$$

The four parameters of the distribution are η, λ, ϵ and γ.

This probability distribution for initial crack depth changes after each mission, proof test and the material removal from the wall thickness. The change in distribution after each mission and proof test is due to the crack growth resulting from the applied stresses. This crack growth also depends on the lengths of the crack that are already present and the material properties that are responsible for the crack growth. In this analysis, the applied stresses and material properties are assumed to be known deterministically. If the initial crack length were also known deterministically the crack length after each use can be determined from equations such as Paris' equation[3], Foreman's equation[4] or Collipriests[5] equations. Because initial crack lengths are not known deterministically, crack length after each use of the vehicle is again another probabilistic distribution that has to be determined.

The cumulative density function for crack length after 'n' uses is denoted by $F(a_n)$. This represents the probability that $a_n \leq A$ after n uses. Each use is defined as one flight, slap down, proof test and material removal. In this analysis "slap down" effects have not been considered. The crack growth due to "slap down" effects can be considered in a similar way. Also, crack growth due to time related effects such as stress corrosion have also been neglected.

If $F(a_n)$ is known the probability distribution for the stress intensity factor (K) can be obtained from the knowledge of the applied stresses. The probability distribution $F(K_n)$ for stress intensity factor can be used to estimate the probability failure (P_f) which is the probability of stress intensity factor K_n greater than or equal to the critical stress intensoty factor during the projected design life of the structure. The critical stress intensity factor is denoted by K^c. In this analysis, stresses and the material properties are assumed to be known deterministically. However, the applied stress changes after each use due to material removal. Therefore, the probability of failure can be expressed as the probability of $a_n \geq a^c$. In this expression a^c is the critical crack depth that can be obtained from the critical stress intensity factor and the applied stress. This relationship between the stress intensity and the applied stress is discussed in the next section.

Stress Intensity Factor

For the analysis of the stress intensity factor in the membrane, an infinite plate model with elliptical surface flaws that are oriented perpendicular to the applied stress has been assumed. The relationship between the stress intensity, the applied tensile stress and crack depth is given by

$$K = \sqrt{\frac{1.2\,\pi\,\sigma^2 a}{Q\{a/c}} \qquad (2)$$

where

$$Q\{a/c\} = \phi^2 - 0.212\,\frac{\sigma}{\sigma_y} \qquad (3)$$

In this equation, σY is the yield stress and ϕ is a function of the ratio of crack depth to crack length (a/c). Variation ϕ^2 with (a/c) is given in reference 1.

Because the crack depth (a) is a random variable the stress intensity factor K is also a random variable. In general, both crack depth a and crack length c are random variables and there is a need for a joint distribution for a and c. In this analysis, only the crack depth is considered as the random variable. It is also assumed that the probability distribution for crack depth 'a' is known initially and is given by a Johnson S_b distribution. The density function for the distribution is given in equation (1). This probability distribution for crack depth changes with use. The next step will be to determine the change and the new probability distribution after each flight and proof test.

Probability Distributions for Crack Depth After Use

The following symbols are used to properly account for the changes in probability distributions.

$f(a_o)$: Probability density function for the initial crack depth

$F(a_o)$: Cumulative distribution function for initial crack depth

$F(a_{op})$: Cumulative distribution function for initial crack depth after the first proof test

$F(a_n)$: Cumulative distribution function after N flights and (N+1) tests

$F(a_{np})$: Cumulative distribution function after N flights and N proof proof tests.

$F(a_n)$: Cumulative distribution function after material removal from the wall thickness.

Similarly, density functions are denoted by lower case 'f'. As discussed before, 'slap down' effects are not considered in the analysis but can be included by following a similar procedure.

The rate at which crack depth increases is assumed to be given by Paris' equation. Then

$$\frac{da}{dn} = C \quad (\Delta K)^n$$

where C and n are empirical constants. Alternately, the rate of crack growth can be assumed to be given by Foreman's equation of Collipriest's equation if they are found to represent the situation more accurately. For example, Collipriest's equation can be written as follows:

$$\frac{da}{dn} = \exp\left[n \frac{\ln K_c - \ln \Delta K_0}{2} \operatorname{arctanh} \left\{ \frac{\ln \Delta K - \frac{\ln K_0(1-K) + \ln \Delta K_0}{2}}{\frac{\ln K_c(1-K) - \ln K_0}{2}} \right\} + \ln \left\{ C \exp\left(\frac{\ln K_c + \ln K_0}{2} n \right) \right\} \right. \quad (4)$$

where n is an empirical constant. These equations can be used to obtain crack depth after N+1 uses if the crack depth after N uses and N proof tests are known deterministically, i.e.,

$$a_{N+1} = a_{N+1}\{a_{Np}\} \qquad (5)$$

Similarly, crack depth after the proof test can be determined from equation (3) or (4) if the crack depth before the proof test is known deterministically, i.e.,

$$a_{Np} = a_{Np}\{a_N\} \qquad (6)$$

These functions represented by equations (5) or (6) can be determined analytically or in the form of quadratures from equation (3) or (4). From equation (5), a_{N+1} can be obtained for every known value of a_{NP}. Similarly, a_{Np} can be obtained for every known value of a_N from equation (6). However, both a_{NP} and a_N are random variables in the present analysis. In this case equation (5) can be used to obtain the probability distribution for a_{n+1} if the probability distribution for a_{np} is known by using the principle of transformation of random variables. It should be noted that all equations similar to (5) or (6) involving crack depths are increasing functions. This property is useful in transforming the random variables.

For example, the probability density function for a_{n+1} can be written as follows

$$f(a_{n+1}) = f\left[a_{n+1}\{a_{np}\}\right] \left| \frac{da_{np}}{da_{n+1}} \right| \qquad (7)$$

similarly

$$f(a_{np}) = f\left[a_{np}\{a_n\}\right] \left|\frac{da_n}{da_{np}}\right| \qquad (8)$$

Equations (7) and (8) can be written for every value of n from zero to the projected number of uses.

Details of obtaining these equations for the membrane of the SRM with the expression for stress intensity given by equation (2) and Paris' equation for crack growth is discussed in Appendix I.

The next step is to obtain a tool for change of probability distribution due to the material removal from the wall thickness.

Material Removal and the Change of Probability Distribution

Due to material removal after each use the effective crack depth is reduced by 'Δt'. Thus new crack depth is

$$\bar{a}_n = a_n - \Delta t \qquad (9)$$

It is assumed that Δt is a constant. Thus, by using the principles of transformation of random variables (2), the probability density function for a_n can be written as follows.

$$p(\bar{a}_n) = f(\bar{a}_n + \Delta t) \qquad (10)$$

In this equation, $p(\bar{a}_n)$ represents the density function for $\bar{a}_n$ and f represents the functional form of the probability density function for a_n.

Probability of Failure

By following the method discussed in the preceeding two sections probability density function for crack depth can be obtained after every flight, proof test and material removal. From the density function, cumulative probabilities can be obtained by integration. Integration after the transformation of variables as discussed in equations (7), (8) and (10) needs the determination of appropriate limits of integration consistent with the transformation of variables. This is also discussed in the Appendix I. If $F(a_n)$ represents the CDF after n flights and n proof tests the probability of failure is given by the probability of $a_n \geq a_c$.

It is to be noted that the probability of failure changes with different selections of the initial wall thickness t, increased loading due to proof test, the material removed Δt and the number of designated missions. The increased loading due to proof tests is denoted by a factor K_p. A cost function or a weight function can be formulated from this knowledge of probability of failure and other related unit-cost or weight. Such a cost or weight function depends on t, K_p, Δt and number of missions N. It is possible to select these design variables by minimizing the cost or weight function subject to appropriate reliability bounds. The effect of NDI is indirectly related to initial flaw distribution. Additional NDI effects such as the rejection of structures are not considered in the analysis. However, they can be included as cost units related to the probability of failure. A numerical example is illustrated in the next section to illustrate the developments of the report.

Numerical Example and the Computer Program

For the numerical example, it is assumed that the 'Johnson S_b' distribution for the initial crack depth is such that the minimum crack depth is zero and the maximum crack depth is 0.1 inch. Different possible ratios (a/c) are considered. Paris' equation for crack growth is assumed with $c = 0.847 \times 10^{-18}$ and exponent equal to 3.0. The variation of ϕ^2 with (a/c) is approximated by a quadratic relation.

The primary objective of reusing the SRM case is to reduce the cost of operation of the shuttle. However, as the number of uses (or cycles) is increased probability of failure increases because of large crack depths associated with more use. The probability of failure also increases with higher proof factors because of higher stresses. Thus, smaller number of cycles and small proof factors, result in higher reliability. However, small number of cycles increase the cost of the SRM case because it has to be replaced after relatively smaller number of uses. Then the total cost function consists of (a) the cost due to number of uses and proof factor and (b) the expected cost of failure, i.e.,

$$C_{total} = C(N,K^P) + C_3 P_f$$

In the equation$C(N,K^P)$ is the cost due to number of uses N, and proof factor K_p. The cost of failure of SRM case is denoted by C_3 and the probability of failure by P_f. The cost $C(N,K^P)$ can be expressed as

$$C(N,K^P) = C_1N^a + c_2(K^P)^b$$

It is to be noted that the expression is only for the purpose of illustration in this report and can be changed to reflect the figures more accurately.

The power 'a' is negative to reflect the fact that the effective investment cost is lower if more number of uses can be obtained from the same vehicle. Similarly the power 'b' is also negative. This is to reflect the fact that the capability of vehicle to withstand higher proof load usually indicates larger available margin of safety and increased confidence in the success of the next mission. This also includes intangible cost due to confidence. It is to be noted that t and 'Δt' are not varried in the numerical example. Therefore, there is no cost associated directly with t or Δt.

Initial thickness of the case is assumed to be 0.686 inch and it is assumed that 1% of the thickness is reduced after each use. The flight loading is assumed to be 936 psi. For the purposes of the illustrative example, the problem posed is to select the number of use cycles and proof factor for minimum expected cost. A reliability restraint can be imposed. However, the numerical example has not been considered such a restraint. Arbitrarily, the following values have been used for C_1, C_2 and C_3, $c_3 = c_1 = 1000.00$ units, $c_2 = 180$ units, $a = -0.3$ and $b = 4.0$ have been used.

The general procedure can be summarized in the following steps. A computer program has been written to carry out the needed computations.

1. Obtain the parameters of the Johnson S_b distribution for the initial flaw size.
2. Obtain the stress in the membrane from the known geometry of the case and wall thickness

$$\sigma = K^p \frac{PR}{t}$$

In the equation K_p is the proof stress factor. During flight K^p is equal to one. Pressure P is the MEOP pressure and R is the radius of the SRM case equal to 72.5 inches.

3. Obtain the new CDF and density function for the crack depth after the proof test. A value of K^p close 1.0 is assumed to start the calculation.
4. Obtain the new CDF for the crack depth during the flight following the proof test.
5. Estimate the probability of failure.
6. Compute the cost function parameters.
7. Obtain the new CDF after the material removal.
8. Repeat steps 2 to 7 for the new thickness and the next mission until the total number of missions are complete.
9. Change 'Δt', t, , N and repeat the calculations as necessary.
10. Select the design variables for the minimum value of the objective function subject to reliability constraints.

A computer program has been written to carry out these steps. Only N is varied in step number 9. The program is listed in Appendix II.

Figure 1 illustrates the variation of cost with number of cycles and proof factor in the range 1.02 to 1.20. From the assumed arbitrary cost figures minimum expected cost occurs for 16 cycles and proof factor of 1.12. The corresponding reliability is only 0.9. Lower proof factor need to be used for higher reliability. In the numerical example presented in this report, t and Δt have not been varied.

Conclusions and Recommendations

This report has demonstrated that the reliability analysis based on probabilistic fracture mechanics can be used to optimize the selection of the design variables of the SRM case. In particular, basic design variables such as the thickness and projected design life as well as the fracture control variables such as the proof factor and material erosion can be included. Accuracy in estimation of the initial flaw size distribution is reflected in the assessment of the risks involved in the design. By knowing the risks involved in the design, weight and cost can be reduced from those obtained by deterministic analysis and use of arbitrary safety margins.

This report is only a first step in the development of procedures based probabilistic fracture mechanics. Additional work that is necessary can be listed as follows:

1. A more accurate analysis can be obtained by considering the joint distribution for the crack depth and crack length along the surface.
2. Accurate methods of estimatimation of the probability distribution for the initial flaw size distribution should be developed.
3. In particular, effects of slap down and time dependent crack growth including stress corrosion should be considered in the SRM analysis.
4. Uncertainties in external loads and material properties should be considered.

5. Accuracy of the different models for crack growth (in the point of view of probabilistic fracture mechanics) should be evaluated.
6. Alternate fracture control plans and more accurate stress intensity measures based on cylindrical geometry can be considered.
7. Cost of NDI efforts in relation to the cost that will be incurred by additional safety factor should be evaluated in the point of view of improved reliability.

APPENDIX I

Estimation of the new CDF of crack depth after use from a knowledge of the old CDF and probability density before use.

Crack Growth Rate

The rate at which the crack depth increases is given by Paris equation as follows.

$$\frac{da}{dN} = C(\Delta K)^n = 0.847(\Delta K)^n \times 10^{-16}$$

For subsequent convenience in algebra, the value of 'n' is taken to be 3.0. The suggested value from current state of art is 2.48 ($C = 0.847 \times 10^{-18}$). Now substituting for ΔK

$$\frac{da}{dN} = 0.847 \left[C_4 \left\{ \frac{a}{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2} \right\}^{\frac{1}{2}} \right]^3$$

Simplifying this further,

$$\frac{da}{dN} = C_6 \left\{ \frac{a}{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2} \right\}$$

where $$C_6 = 0.847 \times (C_4)^3 \times 10^{-18}$$

Integration of $(\frac{da}{dN})$

Separating the variables a and N in $\frac{da}{dN}$, it follows that

$$dN = \frac{1}{C_6} \left\{ \frac{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2}{a} \right\}^{1.5} da$$

Integrating both sides between state (1) and state (2)

$$[N]_{N_1}^{N_2} = \frac{1}{C_6} \int_{a_1}^{a_2} \left\{ \frac{C_5 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2}{(a)^{1.5}} \right\}^{1.5} da$$

In order to evaluate the integral on the right hand side, it is found necessary to expand the numerator of the integrant binomially .

Now consider the numerator of the integrand with $C_5 - 1$. Neglecting terms of higher order than $(\frac{a}{c})^3$, it follows that

$$\left\{ 1 + C_2(\frac{a}{c}) + C_3(\frac{a}{c})^2 \right\}^{1.5}$$

$$= 1.0 + 1.5\ C_2(\frac{a}{c}) + \left\{ 1.5\ C_3 + 1.5(0.25) \right\} (\frac{a}{c})^2$$

$$+ \left\{ 1.5(0.5)\ C_2C_3 - 0.25(0.5)^2\ C_2^{\ 3} \right\} (\frac{a}{c})^3$$

Letting

$$P_1 = \frac{1}{c}\ 1.5\ C_2$$

$$P_2 = \frac{1}{c^2} \left\{ 1.5\ C_3 + 1.5(0.25)\ C_2^{\ 2} \right\}$$

and

$$P_3 = \frac{1}{c^3} \left\{ 1.5(0.5)\ C_2C_3 - (0.25)^2\ C_2^{\ 3} \right\}$$

it follows that

$$\left\{ 1 + C_2\left(\frac{a}{c}\right) + C_3 \left(\frac{a}{c}\right)^2 \right\} = 1.0 + P_1 a + P_2 a^2 + P_3 a^3.$$

Substituting in the integral

$$[N]_{N_1}^{N_2} = \frac{1}{C_6} \left[- \frac{1}{0.5} (a)^{-0.5} + \frac{P_1}{0.5} (a)^{0.5} + \frac{P_2}{1.5} (a) + \frac{P_3}{2.5} (a)^{2.5} \right]_{a_1}^{a_2}$$

Solution of a_1 as a function of a_2

Substituting the limits

$$C_6(N_2 - N_1) = -2(a_2)^{-0.5} + 2 P_1(a_2)^{0.5} + \frac{2}{3} P_2(a_2)^{1.5} + \frac{2}{5} P_3(a_2)^2$$

$$+ 2(a_1)^{-0.5} - 2 P_1(a_1)^{0.5} - \frac{2}{3} P_2(a_1)^{1.5} - \frac{2}{5} P_3(a_1)^{2.5}$$

Rearranging and neglecting terms of order higher than three, it reduces to the following

$$(a_1)^3 + P(a_1)^2 + q\, a_1 + r = 0$$

where

$$P = \frac{1.0}{\left[\frac{8}{3} P_1P_2 - \frac{8}{5} P_3 \right]} \cdot \left[4 P_1^2 - \frac{8}{3} P_2\right]$$

$$q = \frac{-1.0}{\left[\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3\right]} \cdot [8\ P_1 + (C_1)^2]$$

and

$$r = \frac{4}{\left[\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3\right]}$$

Now, the three roots of this cubic equation, $(a_1)^i$ are given by the following [CRC tables 17th edition P. 105]

$$a_1^{(1)} = A + B - \frac{P}{3}$$

$$a_1^{(2)} = -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3} - \frac{P}{3}$$

$$a_1^{(3)} = -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3} - \frac{P}{3}$$

where

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}}$$

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}}$$

$$\bar{a} = \frac{1}{3}(3q - P^2)$$

$$b = \frac{1}{27}(2P^3 - 9Pq + 27r)$$

Transformation

Probability density of a_2 is given by

$$f_{a_2}(a_2) = \frac{da_1}{da_2} f_{a_1}(a_1)$$

CDF of a_2 is then

$$\int_{\bar{a}_1}^{\bar{a}_2} f_{a_2}(a_2)\, da_2 = \int_0^{a_1(a_2)} f_{a_1}(a_1)\, da_1$$

$$= \Big[F_{a_1}(a_1)\Big]_0^{a_1(a_2)}$$

where $F_{a_1}(a_1)$ is the CDF of Johnson S_B distribution.

Now, it is needed to obtain a_1 as a function of a_2, No. of cycles etc. This can be done by solving the polynomial equation obtained previously in terms of a_1 and treating a_2, N_1 and N_2 as constants. The infinite degree polynomial equation is truncated at the 3rd degree for convenience.

Of the three roots only one will be the real root because of the physical nature of the problem, say $\hat{a}_1(a_2)$

Then, substituting in the expression for the CDF of a_2

$$F_{a_2}(a_2) = \int_0^{\hat{a}_1(a_2)} f_{a_1}(a_1)\, d(a_1)$$

or if the CDF of a_1 is known,

$$F_{a_2}(a_2) = \Big[F_a(a_1)\Big]_0^{\hat{a}_1(a_2)}$$

Thus $F_{a_2}(a_2)$ is a function of the parameters of the initial flaw distribution i.e. λ, ϵ, γ and η, the proof test factor K_p and the number of uses $(N_2 - N_1)$.

The effect of each of these parameters can be studied by calculating $F_{a_2}(a_2)$ for various cases, by means of a computer.

Parabolic Fit to $\phi^2(\frac{a}{c})$

Consider the range $0 \le (\frac{a}{c}) \le 1.0$. In this range it is attempted to fit a parabolic curve for $\phi^2(\frac{a}{c})$ such as follows.

$$\phi^2(\frac{a}{c}) = C_1 + C_2 \cdot (\frac{a}{c}) + C_3 (\frac{a}{c})^2$$

In order to determine the three constants C_1, C_2 and C_3 three points are considered on the given curve.

(i) $\frac{a}{c} = 0$ $\qquad \phi^2(\frac{a}{c}) = 1.0$

(ii) $(\frac{a}{c}) = 0.5$ $\qquad \phi^2(\frac{a}{c}) = 1.5$

(iii) $(\frac{a}{c}) = 1.0$ $\qquad \phi^2(\frac{a}{c}) = 2.5$

Substituting the values for point (i),

$$C_1 = 1.0$$

Substituting the values for point (ii)

$$1.0 + C_2(0.5) + C_3(0.25) = 1.5$$

or

$$2C_2 + C_3 = 2.0$$

Substituting the values for point (iii)

$$1.0 + C_2 + C_3 = 2.5$$

or

$$C_2 + C_3 = 1.5$$

Solving equations (2) and (3) simultaneously

$$C_2 = 0.5$$

and

$$C_3 = 1.0$$

Thus the chosen parabolic fit is as follows

$$\phi^2(\frac{a}{c}) = 1.0 + 0.5(\frac{a}{c}) + (\frac{a}{c})^2$$

Limits of Integration for the CDF of 'a_2"

By hypothesis, the initial flas 'a_1' has a Johnson - S_B distribution. Also, there is a functional relationship between the initial flaw size 'a_1' and the subsequent flaw size 'a_2' after N cycles. This relationship renders 'a_2' a random variable because 'a_1' is a random variable by hypothesis. Having known the range space of 'a_1' the range space of 'a_2' can be derived from the functional relationship between 'a_1' and 'a_2'. Thus, if the lower limit of 'a_1' is zero, it follows from the functional relationship between 'a_1 and 'a_2' that the lower limit of a_2 is also zero. Next, if the upper limit of 'a_1' is a_1 the corresponding upper limit for 'a_2' can be obtained by solving the cubic relation between a_1 and a_2, as a function of the number of cycles $N = N_2 - N_1$.

APPENDIX II

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     **********************************************************
C     THIS PROGRAM CALCULATES  THE RELIABILITY OF THE SRM CASING AT THE END OF
C     EACH PROOF TEST AND USE CYCLE GIVEN THE INITIAL FLAW DISTRIBUTION FOR
C     VARIOUS PROOF-TEST FACTORS.
      DIMENSION F(30,1),Y(30,1)
      COMPLEX AA,BB,A1HAT
      READ(5,*)ALAMDA,EATA,GAMA
      EPS=0.0
      WRITE(6,100) ALAMDA,EATA,GAMA,EPS
      ENDFILE6
100   FORMAT(1H1,/,5X,"PARAMETERS OF THE JOHNSON SB DISTRIBUTION ARE
     1!LAMBDA =",F4.1,2X,"EATA =",F5.2,2X,"GAMA =",F5.2,2X,"EPSILON =",
     1F5.2)
      DO 5000 I=1,10
      AKP=1.0+(0.02*I)
C     "AKP"  IS THE PROOF-TEST FACTOR
      WRITE(6,102)AKP
102   FORMAT(/////,5X," PROOF - TEST FACTOR =",F15.3)
      DO 4000 N=1,25
C     "N" IS THE NUMBER OF THE CYCLE CONSISTING OF ONE PROOF-TEST AND ONE USE.
      DO 3010 L=1,2
      IF(L.EQ.1) WRITE(6,104) N
      IF(L.EQ.2) WRITE(6,105) N
104   FORMAT(///,5X,"PROOF TEST NUMBER =",I13)
105   FORMAT(5X,"LAUNCH NUMBER OF SRM CASE =",I7)
C     **********************************************************
C     THIS SECTION CALCULATES THE THICKNESS AT THE END OF EACH CYCLE.
      THIK=0.486-((0.0048*(N-1)))
      WRITE(6,106) THIK
106   FORMAT(5X,"THICKNESS OF SRM CASE=",F15.4)
C     **********************************************************
C     THIS SECTION CALCULATES THE APPLIED STRESS  (SIGMP)
      SIGMP=AKP*950.7*72.5/THIK
      ACR=((93500/SIGMP)**2.0)/(1.2*3.147)
      IF(L.EQ.2) SIGMP=SIGMP/AKP
      WRITE(6,108)SIGMP
108   FORMAT(5X,"PROOF-STRESS =",F26.1)
C     **********************************************************
C     THIS SECTION  CONSIDERS THE CUBIC APPROXIMATION.
      C=0.4
      C1=1.0
      C2=0.5
      C3=1.0
      C4=(SQRT(1.2*3.147))*SIGMP
      C6=0.847*(C4**3.0)
      C6=1.0E-18*C6
      P1=1.5*C2/C
      P2=((1.5*C3)+(1.5*0.25*C2*C2))/(C*C)
      P3=((1.5*0.5*C2*C3)-(0.0625*(C2**3.0)))/(C**3.0)
      A2=ACR
      Z1=2.0*P3*(A2**2.5)/5.0
      Z2=2.0*P2*(A2**1.5)/3.0
```

```
      Z3=2.0*P1*(SQRT(A2))
      Z4=-(2.0/SQRT(A2))-(C6*N)
      C7=Z1+Z2+Z3+Z4
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=(8.0*P1*P2/3.0)-(8.0*P3/5.0)
      P=Z1/Z2
      Z1=-((8.0*P1)+(C7*C7))
      Q=Z1/Z2
      R=4.0/Z2
      ABAR=((3.0*Q)-(P*P))/3.0
      Z1=2.0*(P**3.0)
      Z2=-9*P*Q
      Z3=27*R
      B=(Z1+Z2+Z3)/27.0
      Z1=(B*B/4.0)+((ABAR/3.0)**3.0)
 112  FORMAT(/," DISCREMINENT SQUARE =",E15.5)
      AA=((-B/2.0)+CSQRT(CMPLX(Z1,0.0)))
      BB=((-B/2.0)-CSQRT(CMPLX(Z1,0.0)))
      Q1=REAL(AA)
      Q2=AIMAG(AA)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      AR1=RR*COS(THET/3.0)
      AI1=RR*SIN(THET/3.0)
      AR2=RR*COS((THET+(2.*3.147))/3.0)
      AI2=RR*SIN((THET+(2.*3.147))/3.0)
      AR3=RR*COS((THET+(4.*3.147))/3.0)
      AI3=RR*SIN((THET+(4.*3.147))/3.0)
      Q1=REAL(BB)
      Q2=AIMAG(BB)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      BR1=RR*COS(THET/3.0)
      BI1=RR*SIN(THET/3.0)
      BR2=RR*COS((THET+(2.*3.147))/3.0)
      BI2=RR*SIN((THET+(2.*3.147))/3.0)
      BR3=RR*COS((THET+(4.*3.147))/3,0)
      BI3=RR*SIN((THET+(4.*3.147))/3,0)
      A1HAT=AR1+BR1-(P/3.0)
      AA=REAL(A1HAT)
C     **************************************************************************
C     THIS SECTION CONSIDERS THE QUADRATIC APPROXIMATION.
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=-((8.0*P1)+(C7*C7))
      P=Z2/Z1
      Q=4.0/Z1
      A1=(-P+SQRT((P*P)-(4.0*Q)))/2.0
      WRITE(6,114)A1
 114  FORMAT(5X,"UPPER LIMIT OF A1 =",E24.6)
      A1HA1=A1
      IF  (A1HA1.GE.ALAMDA)  Y(N1,1)=1.0
      IF  (A1HA1.GE.ALAMDA)  GO TO 3005
C     **************************************************************************
```

```
C         THIS SECTION CALCULATES THE RELIABILITY
          N1=15
          DO 3000 K=2,N1
          A1=A1HA1*(K-1)/FLOAT(N1-1)
          Z1=EATA*ALAMDA/SQRT(2.0*3.147)
          Z2=1.0/(A1*(ALAMDA-A1))
          Z4=EATA*ALOG(A1/(ALAMDA-A1))
          Z3=EXP(-((GAMA+Z4)**2.0)/2.0)
          F(K,1)=Z1*Z2*Z3
 3000     CONTINUE
          F(1,1)=0.0
          CALL INTGRL(1,A1HA1,F,Y,N1)
 3005     WRITE(6,116)Y(N1,1)
 116      FORMAT(5X,"PROBABILITY OF NO FAILURE =",E14.5)
 C        ************************************************************
 C        THIS SECTION CALCULATES THE TOTAL COST FUNCTION .
          C8=1000.0
          C10=0.18/(AKP**4.0)
          C11=(1.0/FLOAT(N))*0.30
          ZZ1=Y(N1,1)
          CTOT=C8*(C10+C11+1.0-ZZ1)
          WRITE(6,118)CTOT
 118      FORMAT(5X,"********************** TOTAL COST FUNCTION =",F15.5)
 3010     CONTINUE
 4000     CONTINUE
 5000     CONTINUE
 999      STOP
          END
```

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PROBABILISTIC FRACTURE MECHANICS AND OPTIMUM FRACTURE CONTROL ANALYTICAL PROCEDURES FOR A REUSABLE SOLID ROCKET MOTOR CASE

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NOTATIONS

A	a specified crack depth
C	constant in Paris's equation for crack growth
H	height of casing
K	stress intensity factor
K_C	critical stress intensity factor
K_N	stress intensity factor after N uses
N	number of uses of the motor case
p	Pressure
R_o	outer radius of the casing
a	surface crack depth
a_c	critical crack depth
a_N	crack depth after N uses
c_o	half the length of a surface crack
c_1	payload cost per pound
c_2	cost of total payload
c_3	cost of articles and accessories at proof test
c_i	component cost
c_{ii}	component cost
c_{iii}	component cost
c_{iv}	component cost
c_T	total cost
f	probability density function
n	constant in Paris's equation
n	constant in Collipriest's equation
p	proof load factor
t	thickness of the case
t_N	thickness of the case after N uses
x	random variable representing crack depth
z	standard normal variable
γ,η	shape parameter
Δt	thickness decreased during grit blasting
ΔK	stress intensity range
ϵ,λ	maximum initial crack depth, scale parameter

a_o	minimum crack depth
ϕ	shape parameter
$\overline{\gamma}$	density of the material of the casing
y	yield stress

Abstract

A methodology for the reliability analysis of a reusable solid rocket motor case has been discussed in this paper. The analysis is based on probabilistic fracture mechanics and probability distribution for initial flaw sizes. The developed reliability analysis can be used select the structural design variables of the solid rocket motor case on the basis of minimum expected cost and specified reliability bounds during the projected design life of the case. Effects of failure prevention plans such as non-destructive inspection and the material erosion between missions can also be considered in the developed procedure for selection of design variables. The reliability-based procedure that has been discussed in this paper can easily be modified to consider other similar structures of reusable space vehicle systems with different failure prevention plans.

INTRODUCTION

Structural components of a solid rocket motor case are considered to be fracture critical whenever the game plan is to recover and reuse the motor case for a designated number of missions. Proof tests, conducted on the case between missions, are also significant to rendering the structural components fracture critical. Proof load levels may significantly affect the design life of the structure. A fracture control plan is, therefore, necessary and are considered in the design of the case.

In particular, this paper is concerned with the fracture control of the most critical membrane areas of the case. All discussions and methodologies presented in this paper can, however, be used whenever similar fracture critical structures of a reusable space vehicle system are designed. Some modification might be necessary in particular structures. Significant loads are applied to the motor case during flight and water recovery operation of each mission. The applied stresses from all other events during the mission are assumed in this analysis to be not significant enough to result in cyclic or time dependent crack growth. If the test or analysis indicate the possibility of other critical loading events they can be included in the fracture control plan by extending the reported analysis. Before each mission, the case is also subjected a proof test. The loads applied during the proof tests can result in significant amount of crack growth. Grit blasting is assumed to be used between each mission. This reduces the effective depth of cracks and the thickness of the membrane by a selected amount. While the effective depth of crack is reduced, the refurbishment grit blasting operation has the effect of increasing the applied stresses. This necessitates a larger initial thickness of the membranes than that would be required otherwise. Therefore, any design of the membrane of the case must arrive at an initial wall thickness t, the thickness t that will be decreased between each mission and the proof load factor p. For example, a large value of initial wall thickness results in increased reliability, but results in the need for increased propellant, increased cost of operation and reduced pay load capability. On the other hand, a small initial wall thickness increases the probability of failure and the resulting loss of the reusable space vehicle system and the pay load. Therefore, there is a need for optimizing the initial wall thickness. Similar arguments can be presented to explain the need for selecting the other design variables such as Δt and p by optimizing the desired objective function of cost and weight.

In general, these design variables depend on the probability distribution for the initial flaw sizes present in the membrane, applied stresses during the use of the vehicle, crack growth characteristics of the material, fracture control plans, specified reliability bounds, weight and cost considerations. The paper describes a reliability-based procedure that can be used to select the design variables of a solid rocket motor case in a reusable space vehicle system by using probabilistic fracture mechanics and cost or weight considerations.

Method of Approach

It is assumed that careful Non Destructive Inspection (NDI) techniques can detect initial cracks greater than the surface length of $2c_o$ and depth of a_o with 100% success. Sometimes, it is assumed that cracks corresponding to surface length 2c = 0.1 inch can be identified 100% of the time.[1] If the corresponding maximum depth is 0.05 inch there is no possibility of existence any initial cracks of depth larger than 0.05 inch. Such an initial crack depth distribution is assumed to be analytically represented by Johnson S_b distribution.[2] Reasons for this assumption can be explained as follows. One of the requirements of any assumed distribution is that the minimum and maximum crack depths be bounded within finite limits. Depending on the thickness and the available techniques of non destructive inspection techniques, there is a finite maximum depth of possible crack. It is not infinity as is provided by distributions such as normal distribution, gamma or log-normal distributions. The minimum value of depth of crack can be assumed to be zero or a small number . Such a distribution can be obtained as the transformation of the usual normal variate. One such transformation is the following.

$$z = \gamma + \eta \ \ln \frac{x - \varepsilon}{\lambda + \varepsilon - x} \qquad \varepsilon \leq x \leq \varepsilon + \lambda \qquad (1)$$

In this equation, z is the standard normal variable and x is the variable of interest i.e., the crack depth. The four available parameters are γ, η, ε, & λ. The minimum and maximum crack depths fix ε & λ respectively. The parameters can be called shape parameters and can be determined from percentiles of the observed data.

The density function for the probabilistic model is written as follows

$$f_{ao}(a_o) = \frac{\eta}{2\pi} \ \frac{\lambda}{(a_o - \varepsilon)\ (\lambda - a_o + \varepsilon)} \ \exp \left\{ -\tfrac{1}{2} \left[\gamma + \eta \ \ln \left(\frac{a_o - \varepsilon}{\lambda - a_o + \varepsilon} \right) \right]^2 \right\} \qquad (2)$$

$$\varepsilon \leq a_o \leq \varepsilon + \lambda \ , \qquad \eta > 0$$

$$-\infty \leq \gamma \leq \infty \qquad , \qquad \lambda > 0$$

$$-\infty \leq \varepsilon \leq \infty$$

This emperical distribution is called Johnson S_b distribution. It should be noted that it is possible to obtain other emperical distributions to represent the crack depths.

This probability distribution for initial crack depth changes after each mission, each proof test and each time the material as removed from the wall thickness. The change in distribution after each mission and each proof test is due to the crack growth resulting from the applied stresses. This crack growth also depends on the present length of the crack, applied stress and the material that are responsible for the crack growth. In this analysis, the applied stresses and material properties are assumed to be known deterministically. If the initial crack length were also known deterministically the crack length after each use can be determined from equations such as Paris' equation[3], Foreman's equation[4] or Collipriests equations[5]. Because initial crack lengths are not known deterministically, crack length after each use of the vehicle is again another probablistic distribution that has to be estimated.

The cumulative density function (CDF) for crack length after N uses is denoted by $F(a_N)$. This represents the probability that $a_N \leq A$ after N uses. Each use is defined as one flight, one proof test and a material removal. Crack growth due to time related effects such as stress corrosion have been neglected.

If $F(a_N)$ is known, the probability distribution for the stress intensity factor K_N can be obtained from the knowledge of the applied stresses. The probability distribution $F(K_N)$ for stress intensity factor can be used to estimate the probability failure P_f which is the probability of stress intensity factor K_N greater than or equal to the critical stress intensity factor during the projected design life of the structure. The critical stress intensity factor is denoted by K_c. In this analysis, stresses and the material properties are assumed to be known deterministically. However, the applied stress changes after each use due to material removal. Therefore, the probability of failure can be expressed as the probability of $a_N \geq a_c$. In this expression a_c is the critical crack depth that can be obtained from the critical stress intensity factor and the applied stress corresponding to that particular mission. This relationship between the stress intensity and the applied stress is discussed in the next section.

Stress Intensity Factor

For the analysis of the stress intensity factor in the membrane, an infinite plate model with elliptical surface flaws that are oriented perpendicular to the applied stress has been assumed. The relationship between the stress intensity factor, the applied tensile stress and crack depth is given by[1]

$$K = \sqrt{\frac{1.2\ \pi\ \sigma^2 a}{Q\left(\frac{a}{c}\right)}} \qquad (3)$$

where

$$Q\ (\frac{a}{c}) = \phi^2 - 0.212\left(\frac{\sigma}{\sigma_y}\right)^2 \qquad (4)$$

In this equation, is the yield stress and ϕ is a function of the ratio of crack depth to crack length (a/c). Variation ϕ^2 with (a/c) is given in reference 1.

Because the crack depth a is a random variable the stress intensity factor K is also a random variable. In general, both crack depth a and crack length 2c are random variables and there is a need for a joint distribution for a and c. In this analysis, only the crack depth is considered as the random variable. It is also assumed that the probability distribution for crack depth a is known initially and is given by a Johnson S_b distribution[2]. The density function for the distribution is given in equation (1). This probability distribution for crack depth changes with use. The next step will be to determine the change and the new probability distribution after each flight and proof test.

Probability Distributions for Crack Depth After Use

The following symbols are used to properly account for the changes in probability distributions.

$f(a_o)$: Probability density function for the initial crack depth
$F(a_o)$: Cumulative distribution function for initial crack depth
$F(a_{op})$: Cumulative distribution function for initial crack depth after the first proof test
$F(a_N)$: Cumulative distribution function after N flights and (N+1) tests
$F(a_{Np})$: Cumulative distribution function after N flights and N proof tests.
$F(a_N)$: Cumulative distribution function after material removal from the wall thickness.

Then,

$$\frac{da}{dN} = c \; (\Delta K)^n \tag{5}$$

where C and n are empirical constants. Alternately, the rate of crack growth can be assumed to be given by Foreman's equation[4] or Collipriest's equation[5], if they are found to represent the situation more accurately. For example, Collipriest's equation can be written as follows:

$$\frac{da}{dN} = D \exp \left[\bar{n} \; \frac{\ln K_c - \ln \Delta K}{z} \; \tanh^{-1} \left\{ \frac{\ln \Delta K - \frac{1}{2} (\ln K_c \; (1-R) + \ln \Delta K)}{\frac{1}{2} (\ln K_c (1-R) - \ln K)} \right\} + \ln \left\{ c \exp \left(\bar{n} \left(\frac{\ln K_c + \ln K_o}{z} \right) \right) \right] \tag{6}$$

where n is an empirical constant. By integrating either of the selected equations (5) or (6) crack depth after N+1 uses can be determined if the crack depth after N uses and N proof tests are known deterministically, i.e.,

$$a_{N+1} = a_{N+1} \left\{a_{NP}\right\} \tag{7}$$

Similarly, crack depth after the proof test can be determined from equation (5) or(6) if the crack depth before proof test is known deterministically, i.e.,

$$a_{NP} = a_{NP} \left\{a_{N}\right\} \tag{8}$$

These functions represented by equations (7) or (8) can be determined analytically or in the form of quadratures from equation (5) or (6). From equation (7), a_{N+1} can be obtained for every known value of a_{NP}. Similarly, a_{NP} can be obtained for every known value of a_N from equation (8). However, both a_{NP} and a_N are random variables in the present analysis. In this case, equation (7) can be used to obtain the probability distribution for a_{N+1} if the probability distribution for a_{NP} is known by using the principle of transformation of random variables. It should be noted that all equations similar to (7) or (8) involving crack depths are increasing functions. This property is useful in transforming the random variables.

For example, the probability density function for a_{N+1} can be written as follows

$$f(a_{N+1}) = f\left[a_{N+1}\left\{a_{Np}\right\}\right] \left|\frac{da_{Np}}{da_{N+1}}\right| \qquad (9a)$$

similarly

$$f(a_{Np}) = f\left[a_{Np}\ \left\{a_{N}\right\}\right] \left|\frac{da_{N}}{da_{Np}}\right| \qquad (9b)$$

Equations (7) and (8) can be written for every value of N from zero to the projected number of uses.

Details of obtaining these equations for the membrane of the solid rocket motor case, with the expression for stress intensity given by equation (2) and Paris' equations for crack growth, is discussed in the Appendix I.

The next step is to obtain a tool for change of probability distribution due to the material removal from the wall thickness.

Material Removal and the Change of Probability Distribution

Due to material removal after each use, the effective crack depth is reduced by Δt . Thus, new crack depth is

$$\tilde{a}_N = a_N - \Delta t \qquad (10)$$

It is assumed that Δt is a constant. Thus, by using the principles of transformation of random variables[2], the probability density function for a_N can be written as follows.

$$p(\bar{a}_N) = f\ (\bar{a}_N + \Delta t) \tag{11}$$

In this equation, $p\ (\bar{a}_n)$ represents the density function for a_N and f represents the functional form of the probability density function for a_N.

Probability of Failure

By following the method discussed in the preceeding two sections probability density function for crack depth can be obtained after every flight, proof test and material removal. From the density function, cumulative probabilities can be obtained by integration. Integration after the transformation of variables as discussed in equations (9), (10), and (11) needs the determination of appropriate limits of integration consistent with the transformation of variables. This is also discussed in the Appendix I. If $F(a_N)$ represents the cumulative density function after N flights & N proof tests the probability of failure is given by the probability of $a \geq a_{cN}$. The quanity of a_{cN} corresponds to K_c and the applied stress at the N^{th} use.

It is to be noted that the probability of failure changes with different selections of the initial wall thickness t, increased loading due to proof test, the material removed Δt and the number of designated number of missions. The increased loading due to proof tests is denoted by a factor p. A cost function or a weight function can be formulated from this knowledge of probability of failure and other related unit-cost or weight. Such a cost or weight function depends on t, p, and number of missions N. It is possible to select these design variables by minimizing the cost or weight function subject to appropriate realiability bounds. The effect of non destructive inspection (NDI) is indirectly related to initial flaw distribution. Additional NDI effects such as the rejection of structures are not considered in the analysis. However, they can be included as units related to the probability of failure. A numerical example is illustrated in the next section to illustrate the developments of the paper.

Numerical Example

For the numerical example, it is assumed that the Johnson S_b distribution for the initial crack depth is such that the minimum crack depth is zero and the maximum crack depth is 0.1 inch. Paris's equation for crack growth is assumed with

$$c = 0.847 \times 10^{-18}$$
$$n = 3.0$$

The variation of ϕ^2 with (a/c) as shown in figure 1 is approximated by a quadratic relation.

The primary objective of reusing the solid rocket motor case is to reduce the cost of operation of the reusable space vehicle system in which it is used. However, as the number of uses is increased, the probability of failure increases because of the propagation of the crack depth. On the otherhand, smaller number of uses increases reliability and also the cost is distributed over a smaller number of uses. This means the casing has to be replaced after a fewer number of uses.

A larger initial thickness would increase the weight of the casing and costs more in terms of payload. But the probability failure is less if the thickness is more. The proof test factor p and the material erosion Δt are kept constant in this example. However, they also can be varied and their effect on total cost can be considered in the most general case. The total cost function c_T, therefore, comprises the following component costs.

i) Initial cost of the casing, c_i,
ii) Expected cost of flight failure c_{ii},
iii) Expected cost of proof test failure c_{iii} and

iv) cost due to multiple usage, c_{iv}.

The initial cost c_i is given by the product of the weight of the casing and the cost per pound of the system, i.e.,

$$c_i = \pi (2R_o t_N - t_N^2) H \gamma c_1 \tag{12}$$

where R_o = outer radius of the casing
t_N = thickness of the casing at the N^{th} cycle
H = height of the casing
γ = density of the material
c_1 = payload cost per pound

The expected cost of flight failure is the product of the probability of flight failure and the entire payload cost, i.e.

$$c_{ii} = P_N \cdot c_2 \tag{13}$$

where P_N is the probability of failure at the N^{th} flight c_2 is the total cost of the payload. Similarly the cost of proof test failure is

$$c_{iii} = P_{Np}\, c_3 \tag{14}$$

where p_{np} is the probability of failure at the N^{th} proof test and c_3 is cost of articles and accessories of proof test. Finally, the cost due to multiple usage is given as follows:

$$c_{iv} = c_3 \Big/ (N)^{0.3} \tag{15}$$

Thus, substituting all the components, the total cost function c_T is given by the following equation

$$c_T = c_i + c_{ii} + c_{iii} + c_{iv} \tag{16}$$

The following numerical values are used[1,6] in evaluating equation (16).

$\bar{\gamma}$ = 0.3 lbs/cubic inch
H = 816 inches
R_o = 72.5 inches
C_1 = \$1624 per lbs.
C_2 = \$250 x 10^6
C_3 = \$2x10^6

Results

The initial thickness to is varied from 0.535" to 0.435" in steps of 0.005". Also 1% of the initial thickness is eroded after each flight. The total cost function is calculated for various initial thicknesses and use cycles by means of a digital computer. Figure 2 illustrates the variation of the cost function with t_o and N. If it is obvious that as the number of uses increases, the minimum occurs at a higher initial thickness. For example, for 18 missions the minimum cost occurs at an initial thickness of 0.48 inch. The initial thickness to give minimum cost for 20 mission cycles increases to 0.497 inch, for 22 missions the thickness required is 0.512".

Figure 3 delineates the variation of reliability with initial thickness, after 20 missions cycles. The reliability corresponding to the minimum cost for 20 uses is 99.3%. If this reliability is not adequate enough, then a higher initial thickness should be used even though the total cost will be higher than the minimum.

General Procedure

Based on the preceding example, a general procedure can be delienated in the following steps.

1. Obtain the parameters of the Johnson S_b distribution[2] for the initial flaw size.

2. Obtain the stress in the membrane from the known geometry of the case and wall thickness.

$$\sigma = p \frac{PR_o}{t}$$

In the equation p is the proof stress factor. During flight, p is replaced by a value of 1. Pressure P is the MEOP pressure on the case and R_o is the radius of the case.

3. Obtain the new CDF and density function for the crack depth after the proof test.

4. Obtain the new CDF for the crack depth during the flight following the proof test.

5. Estimate the probability of failure.

6. Compute the cost function parameters.

7. Obtain the new CDF after the material removal.

8. Repeat steps 2 to 7 for the new thickness and the next mission until the total number of missions are complete.

9. Change t and N and repeat the calculations as necessary.

10. Select the design variables for the minimum value of the objective function subject to reliability constraints.

A computer program has been written to carry out these steps (see Appendix II).

Conclusions and Recommendations

This paper has demonstrated that the reliability analysis based on probabilistic fracture mechanics can be used to optimize the selection of the design variables of a solid rocket motor case. In particular, basic design variables such as the thickness and projected design life as well as the fracture control variables such as the proof factor and material erosion can be included in the analysis. Accuracy in estimation of the initial flaw size distribution is reflected in the assessment of the risks involved in the design. By knowing the risks involved in the design, weight and cost can be reduced from those obtained by the conventional deterministic analysis and use of arbitrary safety margins.

This report is only a first step in the development of procedures based probabilistic fracture mechanics. Additional work that is necessary can be listed as follows:

1. A more accurate analysis can be obtained by considering the joint distribution for the crack dpeth and crack length along the surface.

2. Accurate methods of estimation of the probability distribution for the initial flaw size distribution should be developed.

3. In particular, effects of water impact and time dependent crack growth, stress corrosion, should be considered. This is particularly important if the missions are spaced over years.

4. Uncertainties in external loads and material properties should be considered.

5. Accuracy of the different models for crack growth (in the point of view of probabilistic fracture mechanics) should be evaluated.

6. Alternate fracture control plans and more accurate stress intensity measures based on cylindrical geometry can be considered.

7. Cost of NDI efforts in relation to the cost that will be incurred by additional safety factor should be evaluated in the point of view of improved reliability.

8. Thermal effects should be considered.

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APPENDIX I

This appendix describes estimation of the new CDF of crack depth after use from a knowledge of the old CDF and probability density before use.

Crack Growth Rate

The rate at which the crack depth increases is given by Paris' equation as follows.

$$\frac{da}{dN} = c(\Delta K)^n = 0.847\text{x}10^{-16}(\Delta K)^n$$

For subsequent convenience in algebra, the value of n is taken to be 3.0. The suggested value from current state of art is 2.48 and c is equal to 0.867 x 10^{18}. By substituting for

$$\frac{da}{dN} = 0.847 \left[c_4 \left\{ \frac{a}{c_5 + c_2 \frac{a}{c} + c_3 (\frac{a}{c})^2} \right\}^{\frac{1}{2}} \right]^3 \qquad \text{(A1)}$$

Simplifying this further,

$$\frac{da}{dN} = c_6 \left\{ \frac{a}{c_5 + c_2 (\frac{a}{c}) + c_3 (\frac{a}{c})^2} \right\}^{1.5} \qquad \text{(A2)}$$

where

$$c_6 = 0.847 \times c_4^{\,3} \times 10^{-18} \tag{A3}$$

Separating the variables a and N in $\frac{da}{dN}$, it follows that

$$dN = \frac{1}{c_6} \left\{ \frac{c_5 + c_2(\frac{a}{c}) + c_3(\frac{a}{c})^2}{a} \right\}^{1.5} da \tag{A4}$$

Integrating both sides between state (1) and state (2) the following equation is obtained

$$[N]^2 = \frac{1}{c_6}\int_{a_1}^{a_2} \left\{ \frac{c_5 + c_2(\frac{a}{c}) + c_3(\frac{a}{c})^2}{a} \right\}^{1.5} da \tag{A5}$$

In order to evaluate the integral on the right hand side, it is found necessary to expand the numerator of the integrant binomially.

Now consider the numerator of the integrand with $C_5 = 1$. Neglecting terms of higher order than $(a/_c)^3$, it follows that

$$\left\{1 + c_2 \left(\frac{a}{c}\right) + c_3 \left(\frac{a}{c}\right)^2\right\}^{1.5}$$

$$= 1.0 + 1.5\, c_2 \left(\frac{a}{c}\right)$$

$$+\left[1.5c_3 + 1.5\ (0.25)\right] \left(\frac{a}{c}\right)^2$$

$$+ \left[0.75c_2c_3 - 0.25(0.5)^2 c_2{}^3\right] \left(\frac{a}{c}\right)^3 \qquad \text{(A6)}$$

Letting

$$P_1 = \frac{1}{c}\ 1.5\ c_2 \qquad \text{(A7)}$$

$$P_2 = \frac{1}{c^2} \left\{1.5\ c_3 + 0.375\ c_2{}^2\right\} \qquad \text{(A8)}$$

and

$$P_3 = \frac{1}{c^3} \left\{0.75c_2c_3 - (0.25)^2\ c_2{}^3\right\} \qquad \text{(A9)}$$

Then, it follows that

$$\left\{1 + c_2 \left(\frac{a}{c}\right) + c_3 \left(\frac{a}{c}\right)^2\right\} = 1.0 + P_1 a + P_2 a^2 + P_3 a^3 \qquad \text{(A10)}$$

Substituting in the integral the following result is obtained

$$[N]_{N_1}^{N_2} = \frac{1}{c_6} \left[- \frac{1}{0.5} (a)^{-0.5} + \frac{P_1}{0.5} (a)^{0.5} + \frac{P_2}{1.5} (a)^{1.5} + \frac{P_2}{1.5} a^{2.5} \right]_{a_1}^{a_2} \qquad (A11)$$

Solution of a_1 as a function of a_2

Substituting the limits of integration in A(11)

$$c_6 (N_2 - N_1) = -2a_2^{-0.5} + 2P_1 (a_2)^{0.5} + \frac{2}{3} P_2 a_2^{1.5}$$

$$+ \frac{2}{5} P_3 a_2^{2.5} + 2a_1^{-0.5} - 2P_1 a_1^{0.5} - \frac{2}{3} P_2 a_2^{1.5} - \frac{2P_3}{5} a_1^{2.5} \qquad (A12)$$

Rearranging and neglecting terms of order higher than three, it reduces to the following equation

$$(a_1)^3 + p (a_1)^2 + q(a_1) + \gamma = 0 \qquad (A13)$$

where

$$p = \frac{1.0}{\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3} \left(4P_1^2 - \frac{8P_2}{3}\right) \qquad (A14)$$

$$q = \frac{-1.0}{\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3} (8P_1 + c_1^{2}) \tag{A15}$$

and

$$\gamma = \frac{4}{\frac{8}{3} P_1 P_2 - \frac{8}{5} P_3} \tag{A16}$$

Now, the three roots of this cubic equation, $(a_1)^i$ are given as follows

$$\begin{aligned} a_1^{(1)} &= A + B - \frac{P}{3} \\ a_1^{(2)} &= \frac{A + B}{z} + \frac{A - B}{z} \sqrt{-3} - \frac{P}{3} \\ a_1^{(3)} &= \frac{A + B}{z} - \frac{A - B}{z} \sqrt{-3} - \frac{P}{3} \end{aligned} \tag{A17}$$

where

$$\begin{aligned} A &= \sqrt[3]{- \frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}} \\ B &= \sqrt[3]{- \frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{\bar{a}^3}{27}}} \end{aligned} \tag{A18}$$

$$\bar{a} = \frac{1}{3} (3q - p^2) \ , \ b = \frac{1}{27} (2p^3 - qpq + 27\gamma)$$

Transformation

Probability density of a_2 is given by

$$f_{a_2}(a_2) = \frac{da_1}{da_2} f_{a_1}(a_1) \tag{A19}$$

CDF of a_2 is then

$$\int_{\bar{a}1}^{a_2} f_{a_2}(a_2)\, da_2 = \int_{o}^{a_1(a_2)} f_{a_1}(a_1)\, da_1 \tag{A20}$$

$$\int_{o}^{a_1(a_2)} f_{a_1}(a_1)\, da_1 = \Big[F_{a_1}(a_1)\Big]^{a_1(a_2)} \tag{A21}$$

where $F_{a_1}(a_1)$ is the CDF of Johnson S_B distribution^2.

Now, it is needed to obtain a_1 as a function of a_2, No. of cycles etc. This can be done by solving the polynomial equation obtained previously in terms of a_1 and treating a_2, N_1 and N_2 as constants. The infinite degree polynomial equation is truncated at the 3rd degree for convenience.

Of the three roots only one will be the real root because of the physical nature of the problem, say $a_1(a_2)$

Then by substituting in the expression for the CDF of a_2

$$F_{a_2}(a_2) = \int_{o}^{\hat{a}_1(a_2)} f_{a_1}(a_1)\, da_1 \tag{A22}$$

or if the CDF of a_1 is known,

$$F_{a_2}(a_2) = \left[F_a(a_1)\right]_o^{a_1(a_2)} \tag{A23}$$

Thus, $F_{a_2}(a_2)$ is a function of the parameters of flaw distribution i.e., ϵ, λ, γ, η , the proof test factor p and the number of uses $(N_2 - N_1)$.

The effect of each of these parameters can be studied by calculating $F_{a_2}(a_2)$ for various cases, by means of a computer.

Parabolic Fit to $\phi^2(\frac{a}{c})$

Consider the range $0 \leq \phi^2 \leq 1$. In this range

it is attempted to fit a parabolic curve for

such as follows.

$$\phi^2 (\frac{a}{c}) = \bar{c}_1, + \bar{c}_2 (\frac{a}{c}) + \bar{c}_3 (\frac{a}{c})^2 \tag{A24}$$

In order to determine the three constants C_1, C_2 and C_3 three points are considered on the given curve.

$$\frac{a}{c} = 0 \qquad \phi^2 (\frac{a}{c}) = 1.0$$

$$\frac{a}{c} = 0.5 \qquad \phi^2 (\frac{a}{c}) = 1.5 \tag{A25}$$

$$\frac{a}{c} = 1.0 \qquad \phi^2 (\frac{a}{c}) = 2.5$$

Substituting the values for point (i),

Substituting the values for point (ii)

$$1.0 + \bar{c}_2\ (0.5) + \bar{c}_3\ (0.25) = 1.5 \tag{A27}$$

or

$$2\ \bar{c}_2 + \bar{c}_3 = 2.0 \tag{A28}$$

Substituting the values for point (iii)

$$1.0 + \bar{c}_2 + \bar{c}_3 = 2.5 \tag{A29}$$

or

$$\bar{c}_2 + \bar{c}_3 = 1.5 \tag{A30}$$

Solving equations (2) and (3) simultaneously

$$c_2 = 0.5 \tag{A31}$$

and

$$c_3\ 1.0 \tag{A32}$$

Thus the chosen parabolic fit is as follows

$$\phi^2 = 1.0 + 0.5\,\frac{a}{c} + \frac{a^2}{c^2} \tag{A33}$$

Limits of Integration for the CDF of a_2

By hypothesis, the initial flaw a_1 has a Johnson S_b distribution[2]. Also, there is a functional relationship between the initial flaw size a_1 and the subsequent flaw size a_2 after N cycles. This relationship renders a_2 a random variable because a_1 is a random variable by hypothesis. Having known the range space of a_1 the range space of a_2 can be derived from the functional relationship between a_1 and a_2. Thus, if the lower limit of a_1 is zero, it follows from the functional relationship between a_1 and a_2 that the lower limit of a_2 is also zero. Next, if the upper limit of a_1 is a_1, the corresponding upper limit for a_2 can be obtained by solving the cubic relation between a_1 and a_2, as a function of the number of cycles $N^2 = N_2 - N_1$.

APPENDIX II

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C     ***********************************************************************
C     THIS PROGRAM CALCULATES  THE RELIABILITY OF THE SRM CASING AT THE END OF
C     EACH PROOF TEST AND USE CYCLE GIVEN THE INITIAL FLAW DISTRIBUTION FOR
C     VARIOUS INITIAL THICKNESSES AND LAUNCH NUMBERS.
      DIMENSION F(30,1),Y(30,1),ZZ1(5)
      COMPLEX AA,BE,A1HAT
      READ(5,*)ALAMDA,EATA,GAMA
      EPS=0.0
      WRITE(6,100) ALAMDA,EATA,GAMA,EPS
      ENDFILE6
  100 FORMAT(1H1,/,5X,"PARAMETERS OF THE JOHNSON SB DISTRIBUTION ARE
     1 LAMBDA =",F4.1,2X,"EATA =",F5.2,2X,"GAMA =",F5.2,2X,"EPSILON =",
     1F5.2)
      DO 5000 I=1,21
      AKP=1.05
C     "AKP"  IS THE PROOF-TEST FACTOR
      WRITE(6,102)AKP
  102 FORMAT(1H1,/,5X," PROOF - TEST FACTOR =",F10.3)
      STHIK=0.486*(1.-(I-11)/100.)
      WRITE(6,103)STHIK
  103 FORMAT(/,5X," INITIAL THICKNESS OF SRM CASE =",F10.3)
      DO 4000 N=1,25
C     "N" IS THE NUMBER OF THE CYCLE CONSISTING OF ONE PROOF-TEST AND ONE USE.
      DO 3010 L=1,2
      IF(L.EQ.1) WRITE(6,104) N
      IF(L.EQ.2) WRITE(6,105) N
  104 FORMAT(///,5X,"PROOF TEST NUMBER =",I13)
  105 FORMAT(5X,"LAUNCH NUMBER OF SRM CASE =",I7)
C     ***********************************************************************
C     THIS SECTION CALCULATES THE THICKNESS AT THE END OF EACH CYCLE.
      THIK=STHIK-(((STHIK/100.)*(N-1)))
      WRITE(6,106) THIK
  106 FORMAT(5X,"THICKNESS OF SRM CASE=",F15.4)
C     ***********************************************************************
C     THIS SECTION CALCULATES THE APPLIED STRESS  (SIGMP)
      SIGMP=AKP*950.7*72.5/THIK
      ACR=((93500/SIGMP)**2.0)/(1.2*3.143)
      IF(L.EQ.2) SIGMP=SIGMP/AKP
      WRITE(6,108)SIGMP
  108 FORMAT(5X,"PROOF-STRESS =",F26.1)
C     ***********************************************************************
C     THIS SECTION  CONSIDERS THE CUBIC APPROXIMATION.
      C=0.4
      C1=1.0
      C2=0.5
      C3=1.0
      C4=(SQRT(1.2*3.147))*SIGMP
      C6=0.847*(C4**3.0)
      C6=1.0E-18*C6
      P1=1.5*C2/C
      P2=((1.5*C3)+(1.5*0.25*C2*C2))/(C*C)
      P3=((1.5*0.5*C2*C3)-(0.0625*(C2**3.0)))/(C**3.0)
      A2=ACR
      Z1=2.0*P3*(A2**2.5)/5.0
      Z2=2.0*P2*(A2**1.5)/3.0
      Z3=2.0*P1*(SQRT(A2))
```

```
      Z4=-(2.0/SQRT(A2))-(C6*N)
      C7=Z1+Z2+Z3+Z4
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=(8.0*P1*P2/3.0)-(8.0*P3/5.0)
      P=Z1/Z2
      Z1=-((8.0*P1)+(C7*C7))
      Q=Z1/Z2
      R=4.0/Z2
      ABAR=((3.0*Q)-(P*P))/3.0
      Z1=2.0*(P**3.0)
      Z2=-9*P*Q
      Z3=27*R
      B=(Z1+Z2+Z3)/27.0
      Z1=(B*B/4.0)+((ABAR/3.0)**3.0)
112   FORMAT(/," DISCREMINANT SQUARE =",E15.5)
      AA=((-B/2.0)+CSQRT(CMPLX(Z1,0.0)))
      BB=((-B/2.0)-CSQRT(CMPLX(Z1,0.0)))
      Q1=REAL(AA)
      Q2=AIMAG(AA)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      AR1=RR*COS(THET/3.0)
      AI1=RR*SIN(THET/3.0)
      AR2=RR*COS((THET+(2.*3.147))/3.0)
      AI2=RR*SIN((THET+(2.*3.147))/3.0)
      AR3=RR*COS((THET+(4.*3.147))/3.0)
      AI3=RR*SIN((THET+(4.*3.147))/3.0)
      Q1=REAL(BB)
      Q2=AIMAG(BB)
      RR=(SQRT((Q1**2.0)+(Q2**2.0)))**(1.0/3.0)
      THET=ATAN(Q2/Q1)
      BR1=RR*COS(THET/3.0)
      BI1=RR*SIN(THET/3.0)
      BR2=RR*COS((THET+(2.*3.147))/3.0)
      BI2=RR*SIN((THET+(2.*3.147))/3.0)
      BR3=RR*COS((THET+(4.*3.147))/3.0)
      BI3=RR*SIN((THET+(4.*3.147))/3.0)
      A1HAT=AR1+BR1-(P/3.0)
      AA=REAL(A1HAT)
C     ******************************************************
C     THIS SECTION CONSIDERS THE QUADRATIC APPROXIMATION.
      Z1=(4.0*P1*P1)-(8.0*P2/3.0)
      Z2=-((8.0*P1)+(C7*C7))
      P=Z2/Z1
      Q=4.0/Z1
      A1=(-P+SQRT((P*P)-(4.0*Q)))/2.0
      WRITE(6,114)A1
114   FORMAT(5X,"UPPER LIMIT OF A1 =",E24.6)
      A1HA1=A1
      IF (A1HA1.GE.ALAMDA) Y(N1,1)=1.0
      IF (A1HA1.GE.ALAMDA) GO TO 3005
C     ******************************************************
C     THIS SECTION CALCULATES THE RELIABILITY
      N1=15
      DO 3000 K=2,N1
      A1=A1HA1*(K-1)/FLOAT(N1-1)
      Z1=EATA*ALAMDA/SQRT(2.0*3.147)
```

```
      Z2=1.0/(A1*(ALAMDA-A1))
      Z4=EATA*ALOG(A1/(ALAMDA-A1))
      Z3=EXP(-((GAMA+Z4)**2.0)/2.0)
      F(K,1)=Z1*Z2*Z3
 3000 CONTINUE
      F(1,1)=0.0
      CALL INTGRL(1,A1HA1,F,Y,N1)
 3005 WRITE(6,116)Y(N1,1)
  116 FORMAT(5X,"PROBABILITY OF NO FAILURE =",E14.5)
      ZZ1(L)=1.-Y(N1.1)
 3010 CONTINUE
C     ****************************************************************************
C     THIS SECTION CALCULATES THE TOTAL COST FUNCTION .
      C11=(1.0/FLOAT(N))*0.30
      C1=1624.
      C2=250.E+6
      C3=2.0E+6
      WT=3.143*816.*0.3*((145.*THIK)-(THIK*THIK))
      CTOT=(WT*C1)+(ZZ1(2)*C2)+(ZZ1(1)*C3)+(C3*C11)
      WRITE(6,118)CTOT
  118 FORMAT(5X,"********************** TOTAL COST FUNCTION =",F15.5)
 4000 CONTINUE
 5000 CONTINUE
  999 STOP
      END
```

```
      SUBROUTINE INTGRL (M,T,F,Y,N)
      DIMENSION A(30.30),B(30,30),C(30,30),F(30,1),Y(30,1)
      H=T/(720.*(N-1))
      DO 20 I=1,N
      DO 20 J=1,N
      B(I,J)=0.0
      A(I,J)=0.0
 20   CONTINUE
      DO 24 K=1,N
      DO 24 J=1,K
      B(K,J)=1.0
 24   CONTINUE
      A(2,1)=251.
      A(2,2)=646.
      A(2,3)=-264.
      A(2,4)=106.
      A(2,5)=-19.
      A(N,N-4)=-19.
      A(N,N-3)=106.
      A(N,N-2)=-264.
      A(N,N-1)=646.
      A(N,N)=251.
      A(N-1,N-4)=11.
      A(N-1,N-3)=-74.
      A(N-1,N-2)=456.
      A(N-1,N-1)=346.
      A(N-1,N)=-19.
      J=N-2
      DO 25 I=3,J
      A(I,I-2)=-19.
      A(I,I-1)=346.
      A(I,I)=456.
      A(I,I+1)=-74.
      A(I,I+2)=11.
 25   CONTINUE
      DO 30 I=1,N
      DO 30 J=1,N
      C(I,J)=0.0
      DO 30 K=1,N
      C(I,J)=C(I,J)+(B(I,K))*(A(K,J)*H)
 30   CONTINUE
      DO 35 I=1,N
      Y(I,1)=0.0
      DO 40 J=1,N
 40   Y(I,1)=Y(I,1)+((C(I,J))*(F(J,1)))
 35   CONTINUE
      IF (M.EQ.1) GO TO 45
      M=M-1
      DO 65 K=1,M
      DO 50 L=1,N
 50   F(L,1)=Y(L,1)
      DO 55 I=1,N
      Y(I,1)=0.0
      DO 60 J=1,N
 60   Y(I,1)=Y(I,1)+(C(I,J)*F(J,1))
 55   CONTINUE
 65   CONTINUE
```

```
 45   DO 80 I=1,N
 80   Y(I,1)=ABS(Y(I,1))
      RETURN
      END
```

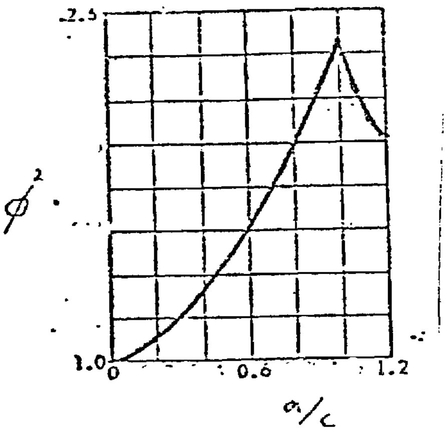

Figure No. 1. Variation of Shape Factor with a/c

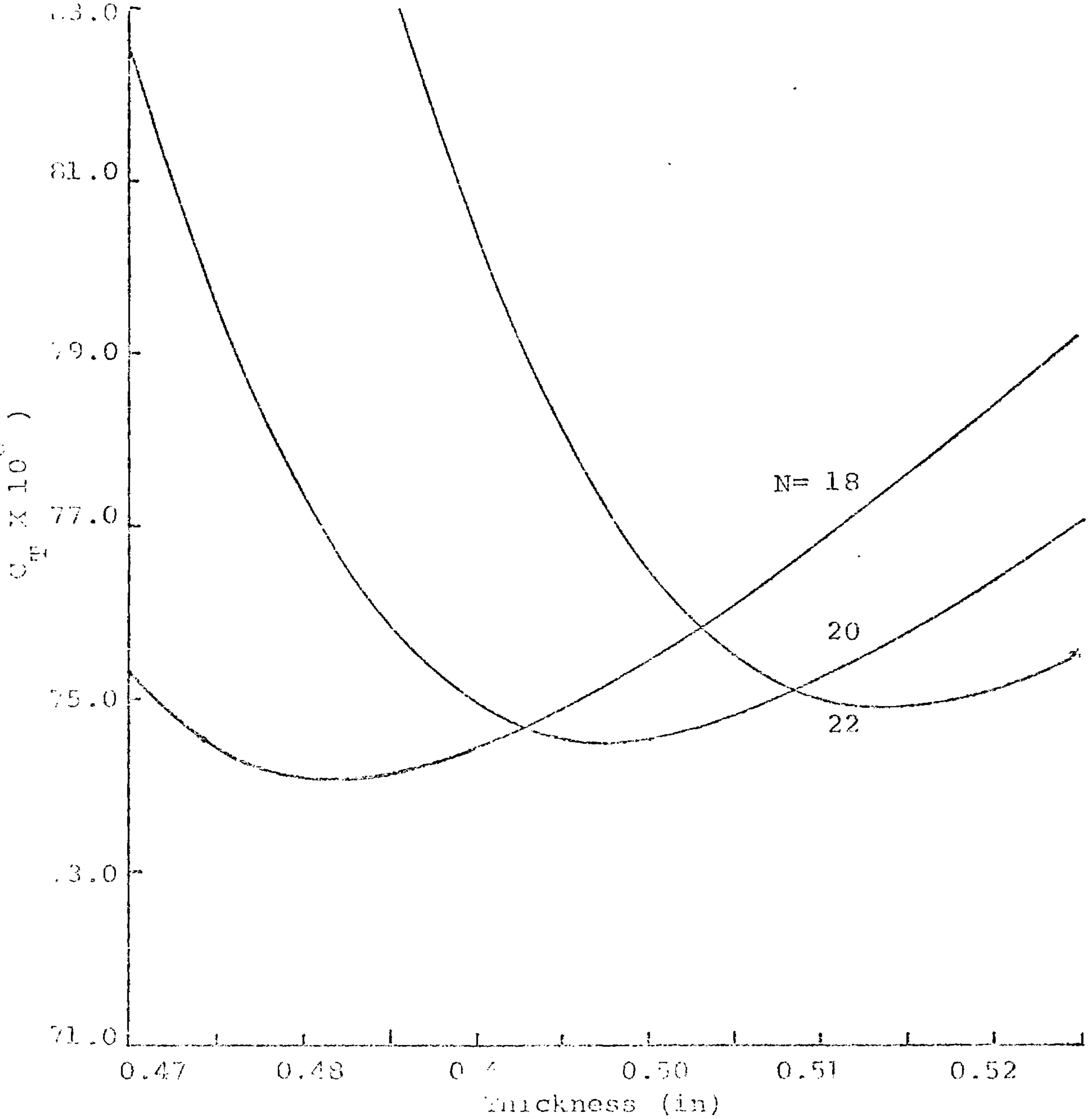

Figure 2. Optimization Curves

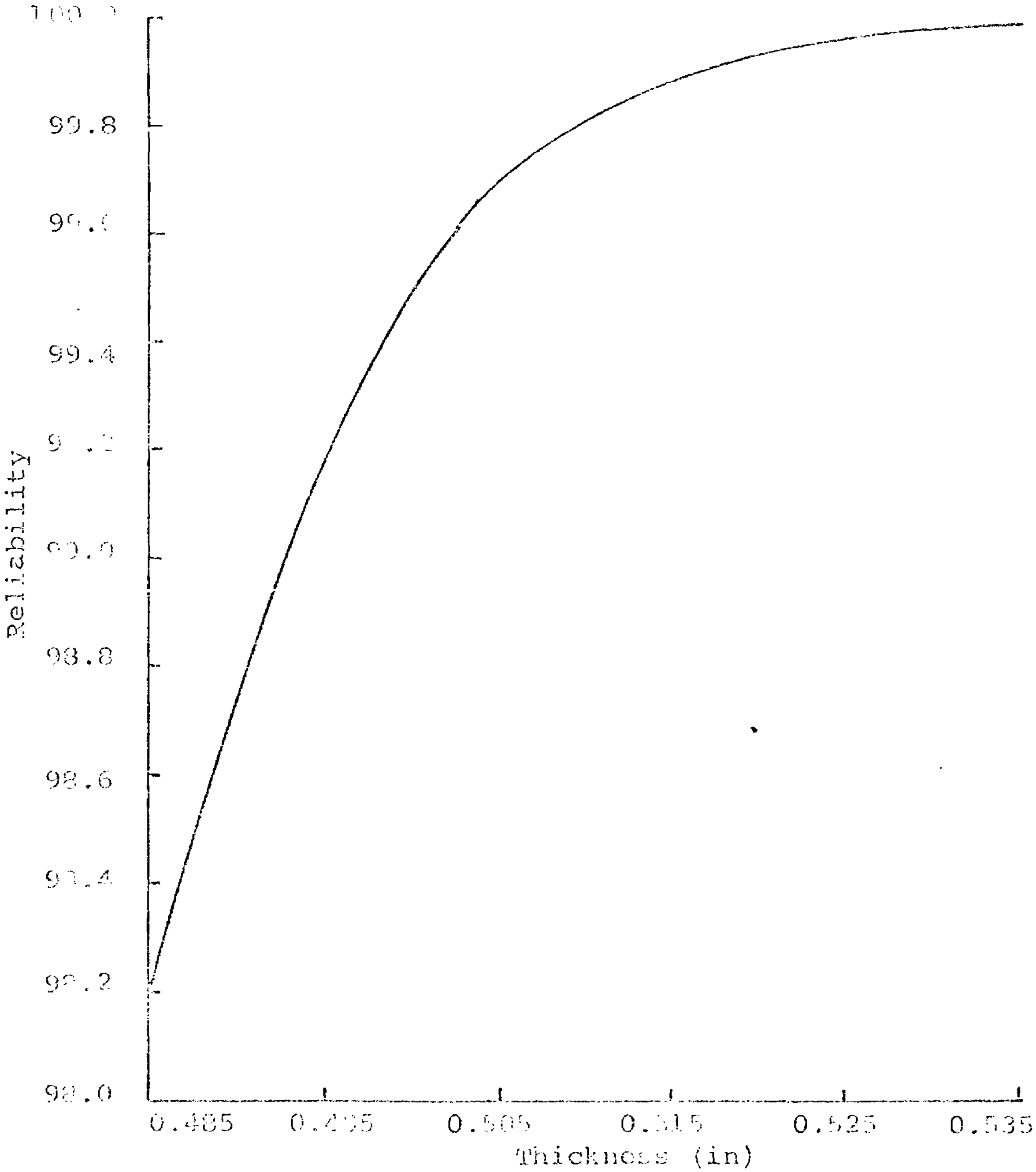

Figure **3.** Reliability After 20 Launch Cycles