CORRELATION OF HINDERED SETTLING DATA

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SETTLING DATA

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ABSTRACT

The object of this paper is to present an equation that suggests that the mechanism of "hindered" settling might be similar to that of the flow of a fluid through a fixed bed of particles, together with presenting the results of the correlation in such a fashion that one would easily be able to estimate the error incurred in using the resulting equation.

Many of the previous investigators have based their results either on very few measurements or on a limited range of particle size. It was therefore thought worthwhile to combine the data of several of these investigators in the correlation presented in this paper.

Starting with the Kozeny - Fair - Hatch equation, the results were arranged into two dimensionaless groups similar to Froude and Reynolds numbers. Next from measurements of the particle diameter and density, liquid density and viscosity and data relating the observed settling velocity of the particles to the porosity of the suspension, a very good correlation was obtained for a large number of particles, (d = 10 to d = 1740 microns) in various fluids.

The final equation was found to be a modified Stokes equation, and to predict the settling velocity of essentially spherical particles with a maximum error of ± 20 per cent for porosities up to 0.80. For porosities less than 0.725 the

per cent error is only ± 12 per cent. This equation is limited to streamline flow, but could be extended into the turbulent region by means of a correction factor. A suggestion for correlating this correction factor is presented in the conclusions of this paper.

The results also suggest that the equation might be valuable in predicting the settling velocity of non-spherical particles, a problem that has not been successfully solved to date.

CHAPTER I

INTRODUCTION

Sir George Stokes is credited with the first statement for the viscous force acting on a single spherical particle in an infinite media. The equation bearing his name results from equating the viscous force to the effective gravitational force acting on a particle, i.e.

$$3\pi\mu U_o d = \frac{\pi d^3}{6} (\rho_p - \rho_L)$$

$$U_o = \frac{d^2 (\rho_p - \rho_L) g}{18\mu} \qquad (1)$$

U_o = velocity of the particle (Stokes velocity)

ρ_p = density of particle

ρ_L = density of liquid

d = diameter of particle

μ = viscosity of fluid

g = acceleration of gravity

Equation I is limited to a single spherical particle falling in an infinite expanse of fluid there being no slipping of fluid at the surface of the particle and that the motion is streamline. Many investigators have attempted to modify the Stokes equation in order to remove these restrictions, such as concentration and shape of particles.

If the concentration of a suspension is sufficiently high so that the particles are crowded together and interfere with the settling of neighboring particles, the entire suspension will settle at a constant rate and a definite liquid interface will exist. This form of sedimentation is known as "hindered" or "line" settling. The theoretical treatments of hindered settling have met with limited success due to the complexity of the variables involved. The most recent theoretical approach is that of Brinkman (1) but it is definitely limited to low concentration.

This survey deals mainly with the data of investigators who have tried to determine the effect of concentration on the settling velocity of uniformly sized particles, by the use of émperical or semi-émperical approaches. An equation is presented which suggests that the mechanism of hindered settling might be similar to that of the flow of a fluid through a fixed bed of particles.

CHAPTER II

LITERATURE SURVEY

Robinson (2) suggested the following equation for predicting the settling velocity of very fine particles:

$$\frac{\partial H}{\partial \Theta} = \frac{kd^2(\rho_p - \rho_s)}{\mu_s} \qquad (2)$$

H = height of interface

Θ = time

k = a proportionality constant

d = average dimension of particle

ρ_p = density of particle

ρ_s = density of suspension

μ_s = viscosity of suspension

For the experimental verification of this equation, Robinson used data obtained by Adams and Glasson (3). The viscosity of the suspension was obtained as a function of the height of the suspension by experiment. The equation was then written in the following form:

$$\frac{\partial H}{\frac{\rho_p - \rho_s}{\mu_s}} = kd^2 \partial \Theta \qquad (3)$$

The left hand side of this equation was evaluated graphically. An intergal curve was then constructed and plotted against Θ. This curve gave a straight line indicating that the modified Stokes equation fits the data well.

The equation was next used to predict a settling curve. The calculated velocity compared favorable with the experimental data, but only two different experimental runs were used in the entire presentation.

Steinour (4) studied the settling rates of fine pearl tapioca and microscopic glass spheres. He reasoned that the effect of concentration could be taken into account by the following equation:

$$U_s = \frac{d^2(\rho_p - \rho_s)g\phi e}{18\mu} \quad (4)$$

U_s = velocity of particle relative to fluid

ρ_s = density of fluid

ϕe = a function of the porosity

e = volume fraction of voids in bed or suspension

The relative velocity, U_s, is related to the observed velocity, U_c, by a continuity equation, and the results are:

$$U_s = \frac{U_c}{e} \quad (5)$$

$$\rho_p - \rho_s = e(\rho_p - \rho_L) \quad (6)$$

From experimental data ϕe was found to be equal to $10^{-1.82(1-e)}$. The results of substituting these values into equation 4 is the following equation:

$$U_c = \frac{e^2 d^2 (\rho_p - \rho_L) g \; 10^{-1.82(1-e)}}{18 \mu} \qquad (7)$$

For high concentrations, e less than 0.73, the data was found to fit the equation

$$U_c = 0.123 \; U_o \frac{e^3}{1-e} \qquad (8)$$

U_o = Stokes velocity

Actually very little experimental data were used in the evaluation of these equations.

In a second article Steinour (5) concerned himself with suspensions of uniformly sized angular particles. He obtained data on two different emery particles suspended in water containing a wetting agent to prevent flocculation. It was found that equation 8 did not apply. Steinour used a correction term "wi" which was to represent "immobile" water which was supposed to be dragged along with the settling particles. The equation governing the flow of irregular shaped particles was reported to be:

$$U_c = \frac{0.123}{(1-wi)^2} \; U_o \; \frac{(e-wi)^3}{1-e} \qquad (9)$$

In the specific case of the emeries wi was found to be 0.168.

An A.E.C. report on work done by Loeffler (6) seems to verify the theory proposed by Ruth (7) that hindered settling is governed by a combination of forces that govern flow through a packed bed and the flow of a single sphere through a fluid. The resulting equation is derived by equating the sum of the Stokes and Kozeny forces corrected for turbulence to the buoyant weight of the bed.

$$
\begin{aligned}
ku(1 + Z) &= Ku \\
ku &= \frac{d\,U_c(1 + F_s)}{(1-\rho_p)\sqrt{\phi}\,\mu} \\
Ku &= \frac{g d^3 (\rho_p - \rho_L)\rho_L}{18\mu^2} \\
Z &= \frac{2K_e(1 + Fk_e)}{\phi^1 \psi^{3/2}\,(1 + F_s)}
\end{aligned}
\qquad (10)
$$

F_s = experimentally determined turbulence factor for flow past a single sphere

Fk_e = experimentally determined turbulence factor for flow through an expanded bed

ψ = (sphericity), $\frac{\text{surface area of sphere}}{\text{surface area of particle}}$, where both sphere and particle have the same volume

$\phi = \frac{e^3}{1-e}$

$\phi^1 = \frac{e^2}{1-e}$

K_e = the Kozeny constant for an expanded bed

Equation 10 should apply over the entire range of porosity and Reynolds number, since it reduces to the Stokes equation at a porosity of unity and it also includes correction terms for turbulence.

The authors state that a definite wall effect was noted in the small diameter tubes, but with tubes having a diameter greater than five centimeters no wall effect was noticed. The resulting equations are quite complicated and it would be very difficult to estimate the degree of accuracy that they will exhibit in predicting settling velocities, solely from the fact that the data exhibited a fair degree of correlation.

The very recent work by Richardson and Zaki (8) treats the problem quite differently than the previously mentioned investigators. From dimensional analysis the authors have shown that for a uniform suspension the ratio of the actual settling velocity to the Stokes velocity is as follows:

For either viscous or turbulent conditions

$$U_c/U_o = f\left(e, \frac{d}{D}\right) \qquad (11)$$

For the transition zone

$$\frac{U_c}{U_o} = f_1\left(\frac{U_o \rho_L d}{\mu}, e, d/D\right) \qquad (12)$$

D = diameter of tube

d = particle diameter

U_o = Stokes velocity

μ = viscosity of fluid

ρ_L = density of fluid

The equations were found to be of the form:

$$\frac{U_c}{U_o} = e^n$$

where $n = f_2\left(\frac{d}{D}, \frac{dU_o \rho_L}{\mu}\right)$ (13)

a) For R_e less than 0.2 $n = 4.65 + 19.65 \frac{d}{D}$

b) For R_e greater than 0.2 and less than 1 $n =$

$(4.35 + 17.5 \frac{d}{D}) R_e^{-0.01}$

c) For R_e greater than 1 and less than 200 $n \doteq$

$(4.45 + 18 \frac{d}{D}) R_e^{-0.01}$

d) For R_e greater than 200 and less than 500 $n =$

$4.45\ R_e^{-0.01}$

e) For R_e greater than 500 $n = 2.39$

f) For non-spherical particles in the turbulent region only $n = 2.7K^{0.16}$

$$K = \frac{\pi}{6} \frac{d_s^3}{d_p^3}$$

d_s = diameter of sphere with same volume of particle

d_p = the diameter of a circle of the same area as the particle when lying in its most stable position

This work should be very valuable since no special mechanism was assumed in the derivations, and also the wall effect $\frac{d}{D}$ has been correlated for the first time. The data seem to correlate well, but no effort was made to use the resulting equations to predict settling velocities, and compare the experimental and calculated velocity. This work was limited to particles greater than one hundred microns.

Robinson (2) and Steinour (4,5) both assumed that the effective buoyancy force acting on the particles was that of the suspension. For uniform particles settling at the same rate this cannot be true, since each particle must displace its own volume of fluid. Therefore if the suspended material is closely sized, the density of the fluid is the correct buoyant force.

On the other hand, if a relatively large particle is settling in a suspension composed of particles small enough to act as part of the fluid, the larger particle displaces its volume of suspension and therefore the density of the suspension should be used.

For a range of particles the correct buoyant force is quite complex, consisting of the density of the fluid for the smallest particles, up to the density of the suspension for the largest. Similarly the viscosity of the suspension is not correct for a closely sized suspension. The effect of concentration on the resistance for a given relative velocity is due to the increased velocity gradients. That is the velocity of the fluid is zero at the surface of the particle and a maximum at some distance from the particle.

CHAPTER III

EXPERIMENTAL PROCEDURE AND PRELIMINARY CALCULATIONS

The equipment used for obtaining sedimentation data is quite simple. Graduated cylinders are usually used for settling chambers along with a timer to obtain the subsidence of the liquid interface as a function of time.

A known weight of sample is placed in the container and then a measured quantity of liquid is added. The mixture is gently rocked back and forth until the suspension is thoroughly mixed. Next the cylinder is placed in an upright position and the timer started. Data is thus obtained relating the height of the interface with the corresponding time interval. See fig. I (9).

In a more elaborate system a transparent constant temperature bath can be provided for the cylinder to prevent any possible thermal effects due to the ambient room temperature.

To obtain the velocity of the interface, which is equivalent to the particle velocity since the entire suspension settles at the same rate, it is only necessary to subtract the initial height of the suspension from the height of the suspension at any time, and to plot this distance against the corresponding time interval. The results should be a straight line whose slope is equal to the observed particle velocity.

The porosity of the suspension is obtained by dividing the weight of sample by its density, then dividing this quantity by the total volume of the suspension and subtracting the resulting number from one. In this investigation the data of Steinour (4,5), Lewis et al (10), and Loeffler (11) were used which made these preliminary calculations unnecessary in this paper.

The diameters of the non-spherical emeries used by Steinour (5) were obtained by the "Wagner Turbidimeter Method"(12). This method consists essentially of a source of light of constant intensity which passes through a settling suspension and into a photoelectric cell. The current generated in the cell is a measure of the turbidity of the suspension. The turbidity is related to the surface area of the suspended particles which allows the diameter of the particle to be reported as equivalent to the diameter of a sphere having the same density of the irregular particle and obeying Stokes law, equation 1.

For a very complete treatment of particle size measurement the reader is referred to a book by DallaValle (13).

CHAPTER IV

MECHANISM OF HINDERED SETTLING AND CORRELATION OF DATA

Hindered settling can be thought of as analogous to the flow of a fluid through a packed bed. The Kozeny (14) - Fair - Hatch (15) equation deals with this type flow.

$$\frac{h}{L} = \frac{k\mu(1-e)^2 U_L\ S_v^2}{e^3\ g\rho_L} \tag{15}$$

or

$$\frac{h}{L} = \frac{k\mu(1-e)^2 U_L\ (\frac{S}{d})^2}{e^3\ g\rho_L} \tag{16}$$

h = loss of head (ft. of liquid)

L = length of bed

e = porosity

U_L = fluid velocity

k = a constant characteristic of bed

S_v = specific surface area of particle $\left(\frac{\text{surface area of particle}}{\text{volume of particle}}\right)$

S = shape factor (S = 6 for spheres)

d = size factor (d = diameter for sphere)

As the equations stand they are only limited by the restrictions that the flow must be streamline and that the pores of the suspension must be reasonably alike.

The pressure drop per unit length of bed, $\frac{\Delta P}{L}$, has been found for fluidization to be equivalent to an upward flow of fluid just sufficient to maintain the particles in a state of equilibrium. Since it is an established fact that hindered settling velocities and fluidization velocities are equivalent the term $\frac{\Delta P}{L}$ can be expressed by the following relation:

$$\frac{\Delta P}{L} = (1-e)(\rho_p - \rho_L)\frac{g}{g_c} \tag{17}$$

$$\text{Since } \frac{\Delta P}{L} = h\rho_L\frac{g}{g_c} \tag{18}$$

Therefore

$$\frac{h}{L} = \left(\frac{1-e}{e}\right)\left(\frac{\rho_p - \rho_L}{\rho_L}\right) \tag{19}$$

Now by using this value of $\frac{h}{L}$ in equation 16 and arranging the terms we obtain the following equation:

$$\left(\frac{1-e}{e^3}\right)\frac{U_c^2}{gd} = K \cdot \left[\frac{dU_c(\rho_p - \rho_L)}{\mu}\right] \tag{20}$$

where $K = \frac{1}{kS^2}$

These are two dimensionaless groups very similar to the Froude and Reynolds group. Figure 2 shows a plot of Reynolds (R_e) numbers versus Froude numbers (Fr) for several particles ranging in size from 10 to 1740 microns. The slope of the line for spherical particles is one and the constant $K = 8.0 \times 10^{-3}$. K was obtained by reading the value of Fr at Re = 1.

For non-spherical particles equation 20 is of the same form as for spherical particles with the exception that the value of $K = 5.0 \times 10^{-3}$ is different due to the different shape factor. Included in fig. 2 are data on two different angular particles. The diameter of these particles were determined by the "Wagner Turbidimeter Method".

The shape factor S can easily be calculated if it can be assumed that the constant k is the same for all suspension in hindered settling. This assumption might be justified by the fact that many investigators have found that during sedimentation the particles align themselves into an equilibrium arrangement presenting the greatest possible area to the flowing fluid. Now since this has been found to be characteristic of hindered settling suspensions, it would be possible that k, which is a constant characteristic of the bed, would be the same for all hindered settling suspensions.

For spherical particles S = 6 and K = 8.0×10^{-3}.

$$k = \frac{1}{KS^2} = 3.46$$

For non-spherical emeries K = 5.0×10^{-3} and S = 7.6 if k = 3.46.

If equation 20 is solved for U_c, the resulting equation will show directly how the predicted settling velocity compares with the observed velocity.

$$U_c = K \left[\frac{gd^2}{\mu} (\rho_p - \rho_L) \frac{e^3}{1-e}\right] \qquad (21)$$

Since U_c is directly proportional to K, it is only necessary to observe how K deviates from the predicted values of 8.0×10^{-3} for spheres and 5.0×10^{-3} for angular particles. Figure 3 shows a plot of $\frac{K(\text{observed})}{K(\text{predicted})}$ versus porosity (e), K(observed) = $\frac{Fr}{Re}$. Only data where the Reynolds numbers were less than 2 are presented in this figure.

CHAPTER V

DISCUSSION OF RESULTS

The final form of equation 21 is a modified Stokes equation even though the Kozeny (14) - Fair - Hatch (15) equation was the starting point for its derivation.

$$U_c = 18\ KU_o \frac{e^3}{1-e}$$

U_o = Stokes velocity

$$K = \frac{1}{3.46S^2}$$

S = shape factor (S = 6 for spheres)

e = porosity (void fraction)

Steinour (4) reported a very similar equation, equation 8, for spherical particles, when the porosity is less than 0.73. It is not evident from his paper just how he arrived at this form of the equation.

It is obvious from the form of equation 22 that the predicted settling velocity, U_c, will become indeterminate at a porosity of unity instead of reducing to the Stokes velocity, U_o. Figure 3 shows that for most of the particles, equation 22 will predict the settling velocity up to porosities of 0.725 with a maximum error of $\pm$ twelve per cent and $\pm$ twenty per cent for porosities between 0.725 and 0.80. The error increases very rapidly for porosities greater than 0.80 with the exception of the angular particles whose maximum error is only eleven per cent at a porosity of 0.85.

It is not evident why equation 22 should work so well for the angular particles at high porosities. A definite possibility would be the effect that angular particles have on the viscosity of the suspension. Since these effects are quite complex and not well established for concentrated suspensions this investigation has not tried to take them into account.

The value of the shape factor for angular emeries, S = 7.6, as calculated from equation 20, is very nearly equal to S = 7.7 reported by Fair and Hatch (15) for angular sand particles which closely resembled the emeries used by Steinour.

In using Steinour's (4,5) data it was necessary to assume that the viscosity of a twelve per cent sodium-hexametaphospate solution used as dispersing agent for small particles, was equal to the viscosity of pure water at the same temperature as the mixture.

Loeffler (6,11) et al reported that their data showed no evidence of wall effect when the tube diameters were greater than five centimeters. Both Steinour (4,5) and Lewis et al (10) used tube diameters between ten and twelve centimeters, therefore the data used in this report should be free from any wall effects.

Figure 2 indicates that equation 20 is definitely limited to low Reynolds numbers. For Reynolds numbers greater than two a family of curves is obtained showing that the Froude number is still a function of the Reynolds number, but that it is no longer a linear function. It would be possible to correlate a turbulence factor similar to Loeffler's (6) F_{K_e} to extend the range of equation 20 into the turbulent region.

CHAPTER VI

CONCLUSIONS

Equation 22 was found to predict the settling velocity of spherical particles under viscous flow conditions ranging in size from 13.5 to 1740 microns in various liquids, water, ethylene glycol, and oil up to porosities of 0.8 with a maximum error of $\pm$ twenty per cent. For porosities less than 0.725 the error is only $\pm$ twelve percent. Although equation 22 is not as rigorous as Zaki and Richardson's (8) equation 13 or Loeffler's (6) equation 10 it has been tested for a wider range of particle size and the results presented in such a fashion that the error incurred in using this equation is obvious.

In the case of non-spherical particles, equation 22 has not been tested sufficiently to draw any definite conclusions. The fact that the shape factor calculated from equation 20 for the emeries, S = 7.6, agreed with the shape factor for angular sand particles, S = 7.7, which closely resembled the emeries, might possibly justify further investigation of equation 22 for predicting the settling velocity of non-spherical particles.

Figure 2 shows that a definite linear relation exists between the Froude and Reynolds numbers in the viscous region and also suggests that a correlation may exist to extend equation 20 into the turbulent region. Loeffler's (6) paper should be very helpful in making such a correlation.

None of the equations found in the literature seem to have solved the problem of predicting the settling velocity of non-spherical particles. Equation 13f as reported by Zaki and Richardson (8) is limited to the turbulent region only, and Loeffler's (6) equation 10, although derived for the general case, was tested with spherical particles only.

If further investigation of equation 22 is made on non-spherical particles, the author suggests that the specific surface area of the particle be determined by methods developed by Carman and described by DallaValle (13). The apparatus used in these methods is quite simple and the actual flow conditions would more closely resemble those existing in a hindered settling suspension than would the turbidimeter method where the particles are obeying Stokes law.

APPENDIX

Appendix A

Tables 1-17. Data Relating the Hindered Settling Velocity (U_c) to the Porosity of the Suspension, (e), together with Corresponding Values of Reynolds and Froude Numbers.

d = diameter of particles in microns

ρ_p = density of particle in gm/cc

ρ_L = density of fluid in gm/cc

μ = viscosity of fluid in poise

e = porosity (void fraction)

U_c = observed settling velocity of particle in cm/sec

$$Fr = \frac{U_c^2}{gd}\,\frac{(1-e)}{e^3}$$

$$Re = \frac{dU_c(\rho_p - \rho_L)}{\mu}$$

K(observed) = Fr/Re

K(predicted) = 8.0×10^{-3} for spherical particles and 5.0×10^{-3} for angular particles

Table 1. Tapioca in Oil (4)*

e	$V_c \times 10^3$	$Fr \times 10^6$	$Re \times 10^4$	$K(observed) \times 10^3$	$\frac{K(observed)}{K(predicted)}$
0.502	4.27	0.418	0.513	8.16	1.02
0.592	7.75	0.69	0.93	7.43	0.93
0.691	15.9	1.36	1.91	7.13	0.89
0.665	11.9	0.925	1.43	6.44	0.80
0.727	22.6	2.12	2.71	7.83	0.98
0.894	61.2	3.12	7.34	4.25	0.53

*Number in parentheses refers to bibliography.

$d = 1740$

$\rho_p = 1.38$

$\rho_L = 0.89$

$\mu = 7.13$

Table 2. Glass Spheres in Water (4)

e	$V_c \times 10^3$	$Fr \times 10^6$	$Re \times 10^4$	$K(observed) \times 10^3$	$\frac{K(observed)}{K(predicted)}$
0.60	0.997	1.38	1.94	7.1	0.89
0.70	2.175	3.05	4.22	7.2	0.90
0.80	5.31	8.18	10.3	7.9	0.99

$d = 13.6$

$\rho_p = 2.32$

$\rho_L = 1$

$\mu = 9.3 \times 10^{-3}$

Table 3. Emery A in Water (5)

e	$V_c \times 10^3$	$Fr \times 10^6$	$Re \times 10^4$	$K(\text{observed}) \times 10^3$	$\frac{K(\text{observed})}{K(\text{predicted})}$
0.65	1.38	2.02	4.7	4.3	0.86
0.70	2.18	3.41	7.4	4.6	0.92
0.75	3.32	5.31	11.3	4.7	0.94
0.80	5.27	8.7	18.0	4.84	0.97
0.85	7.55	11.5	25.7	4.47	0.89

$d = 12.2$

$\rho_p = 3.79$

$\rho_L = 1$

$\mu = 10^{-2}$

Table 4. Emery B in Water (5)

e	$V_c \times 10^3$	$Fr \times 10^6$	$Re \times 10^4$	K(observed) $\times 10^3$	$\frac{K(observed)}{K(Predicted)}$
0.65	0.847	0.99	2.25	4.4	0.88
0.70	1.35	1.67	3.59	4.65	0.94
0.75	2.06	2.51	5.33	4.7	0.94
0.80	3.14	3.98	8.35	4.76	0.95
0.85	4.70	5.72	12.5	4.56	0.91

$d = 9.6$

$\rho_p = 3.77$

$\rho_L = 1$

$\mu = 10^{-2}$

Table 5. Glass Spheres in Water (10)

e	$V_c \times 10^1$	Fr x 10^1	Re x 10^{-1}	K(observed) x 10^3	$\frac{K(observed)}{K(predicted)}$
0.90	5.38	3.94	9.2	4.3	0.54
0.80	3.38	4.5	5.75	7.84	0.98
0.75	2.51	3.82	4.27	8.95	1.12
0.70	1.84	2.96	3.12	9.5	1.19
0.65	1.32	2.15	2.24	9.6	1.20

$d = 100$

$\rho_p = 2.48$

$\rho_L = 1$

$\mu = 8.7 \times 10^{-3}$

Table 6. Glass Spheres in Water (10)

e	$V_c \times 10^1$	$Fr \times 10^1$	$Re \times 10^{-1}$	K(observed) $\times 10^3$	$\frac{K(observed)}{K(predicted)}$
0.78	5.07	7.6	11.2	6.8	0.85
0.60	1.62	3.18	3.56	8.95	1.12
0.57	1.19	2.08	2.62	7.95	0.99

$d = 155$

$\rho_p = 2.32$

$\rho_L = 1$

$\mu = 9.3 \times 10^{-3}$

Table 7. Data on Glass Spheres in Ethylene Glycol (II)

e	$U_c \times 10^2$	$Fr \times 10^4$	$Re \times 10^2$	$K(observed) \times 10^3$	$\frac{K(observed)}{K(predicted)}$
0.521	2.16	0.60	0.624	9.6	1.2
0.566	2.86	0.695	0.827	8.4	1.05
0.597	3.58	0.865	1.04	8.4	1.05
0.656	5.33	1.21	1.53	7.9	0.99
0.674	5.93	1.28	1.71	7.5	0.94
0.742	8.93	1.80	2.60	6.9	0.86
0.767	10.47	1.90	3.01	6.3	0.79
0.810	13.64	2.43	3.94	6.2	0.78
0.851	17.60	2.70	5.08	5.4	0.67

$d = 287$

$\rho_p = 2.67$

$\rho_L = 1.11$

$\mu = 153 \times 10^{-3}$

Table 8. Data on Glass Spheres in Ethylene Glycol (II)

e	$U_c \times 10^1$	Fr $\times 10^3$	Re $\times 10^1$	K(observed) $\times 10^3$	$\frac{K(observed)}{K(predicted)}$
0.52	1.36	0.806	1.14	7.06	0.88
0.55	1.85	1.11	1.55	7.13	0.89
0.60	2.66	1.58	2.23	7.10	0.89
0.67	4.69	2.85	3.93	7.25	0.91
0.76	8.09	4.17	6.77	6.16	0.77
0.83	11.36	4.53	9.52	4.75	0.59
0.89	14.90	4.01	12.50	3.2	0.40

$d = 850$

$\rho_p = 2.63$

$\rho_L = 1.11$

$\mu \doteq 155 \times 10^{-3}$

Table 9. Glass Spheres in Water (II)

e	$U_c \times 10^0$	$Fr \times 10^1$	$Re \times 10^{-1}$	K(observed)*	K(observed)* / K(predicted)
0.61	2.20	1.26	2.6		
0.70	3.41	1.55	4.04		
0.78	4.42	1.36	5.24		
0.85	5.84	1.26	6.90		

*Only K's corresponding to Re less than 2 are presented.

$$d = 659$$

$$\rho_p = 2.67$$

$$\rho_L = 1.0$$

$$\mu = 9.3 \times 10^{-3}$$

Table 10. Glass Spheres in Water (II)

e	$U_c \times 10^0$	$Fr \times 10^1$	$Re \times 10^{-1}$	K(observed)*	K(observed)* / K(predicted)
0.485	0.289	1.03	1.77		
0.563	0.594	2.59	3.63		
0.603	0.770	3.22	4.70		
0.705	1.306	4.07	7.98		
0.808	2.08	4.58	12.70		
0.848	2.395	4.21	14.65		

*Only K's corresponding to Re less than 2 are presented.

$d = 340$

$\rho_p = 2.67$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table II. Glass Spheres in Water (II)

e	$U_c \times 10^0$	$Fr \times 10^2$	$Re \times 10^0$	K(observed)*	$\frac{K(observed)*}{K(predicted)*}$
0.59	2.28	11.7	36.0		
0.68	3.91	17.0	61.8		
0.76	5.0	15.3	79.0		
0.84	6.9	14.1	109.0		
0.89	7.28	9.1	115.0		

*Only K's corresponding to Re less than 2 are presented.

$d = 903$

$\rho_p = 2.63$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 12. Glass Spheres in Water (II)

e	$U_c \times 10^0$	$Fr \times 10^2$	$Re \times 10^0$	K(observed)*	$\frac{K(observed)*}{K(predicted)}$
0.505	0.716	4.27	5.68		
0.559	1.069	7.56	8.47		
0.650	1.650	7.70	13.10		
0.750	2.68	9.03	21.20		
0.850	3.8	7.85	30.10		
0.885	4.21	6.33	33.40		

*Only K's corresponding to Re less than 2 are presented.

$d = 459$

$\rho_p = 2.67$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 13. Polystyrene Spheres in Water (II)

e	$U_c \times 10^1$	$Fr \times 10^4$	$Re \times 10^2$	$K(observed) \times 10^{-3}$	$\frac{K(observed)}{K(predicted)}$
0.50	0.73	2.79	3.26	8.55	1.07
0.563	1.2	4.75	5.38	8.8	1.1
0.62	1.8	6.85	8.06	8.5	1.06
0.750	3.6	10.40	16.15	6.4	0.80
0.825	4.9	9.97	22.0	4.5	0.56
0.87	6.0	9.5	26.9	3.5	0.44

$d = 758$

$\rho_p = 1.055$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 14. Glass Spheres in Water (II)

e	$U_c \times 10^1$	$Fr \times 10^3$	$Re \times 10^1$	$K(observed) \times 10^{-3}$	$\frac{K(observed)}{K(predicted)}$
0.529	0.75	1.26	1.78	7.1	0.89
0.604	1.40	2.52	3.34	7.55	0.94
0.684	2.50	4.37	5.97	7.32	0.92
0.747	3.75	6.10	8.95	6.8	0.85
0.794	4.88	6.96	11.7	5.95	0.74
0.856	6.75	7.46	16.1	4.64	0.58

$d = 140$

$\rho_p = 2.59$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 15. Glass Spheres in Water (II)

e	$U_c \times 10^1$	$Fr \times 10^3$	$Re \times 10^1$	$K(observed) \times 10^{-3}$	$\frac{K(observed)}{K(predicted)}$
0.513	0.97	2.08	2.72	7.65	0.96
0.603	2.06	4.54	5.77	7.85	0.98
0.641	2.76	6.35	7.73	8.23	1.03
0.716	4.48	9.50	12.60	7.54	0.94
0.791	6.94	12.20	19.20	6.36	0.80
0.858	8.94	10.6	25.0	4.24	0.53

$d = 164.5$

$\rho_p = 2.58$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 16. Polystyrene Spheres in Water (II)

e	$U_c \times 10^1$	$Fr \times 10^4$	$Re \times 10^2$	K(observed) $\times 10^{-3}$	$\frac{K(observed)}{K(predicted)}$
0.51	0.63	2.34	2.5	9.35	1.17
0.58	0.93	3.47	3.87	8.96	1.12
0.64	1.53	5.05	4.6	11.0	1.38
0.71	2.20	6.01	8.75	6.86	0.86
0.76	2.90	7.11	11.5	6.2	0.78
0.82	3.90	7.72	15.5	4.98	0.62
0.91	5.40	5.30	21.4	2.48	0.31

$d = 650$

$\rho_p = 1.057$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Table 17. Polystyrene Spheres in Water (II)

e	$U_c \times 10^1$	$Fr \times 10^4$	$Re \times 10^2$	K(observed) $\times 10^{-3}$	$\frac{K(observed)}{K(predicted)}$
0.51	0.5	1.77	1.69	10.5	1.31
0.57	0.7	2.14	2.36	9.06	1.13
0.62	1.1	2.64	3.70	7.14	0.89
0.70	1.8	5.18	6.03	8.6	1.08
0.75	2.3	5.78	7.75	7.5	0.94
0.80	3.0	6.45	10.10	6.45	0.81
0.85	3.7	6.25	12.50	5.0	0.62

$d = 550$

$\rho_p = 1.057$

$\rho_L = 1.0$

$\mu = 9.3 \times 10^{-3}$

Appendix B

4 Seconds

250 Seconds

500 Seconds

750 Seconds

Figure 1. The Hindered Setting of Silicon Carbide (400 Grit) in Methyl Alcohol (858 g./1.)

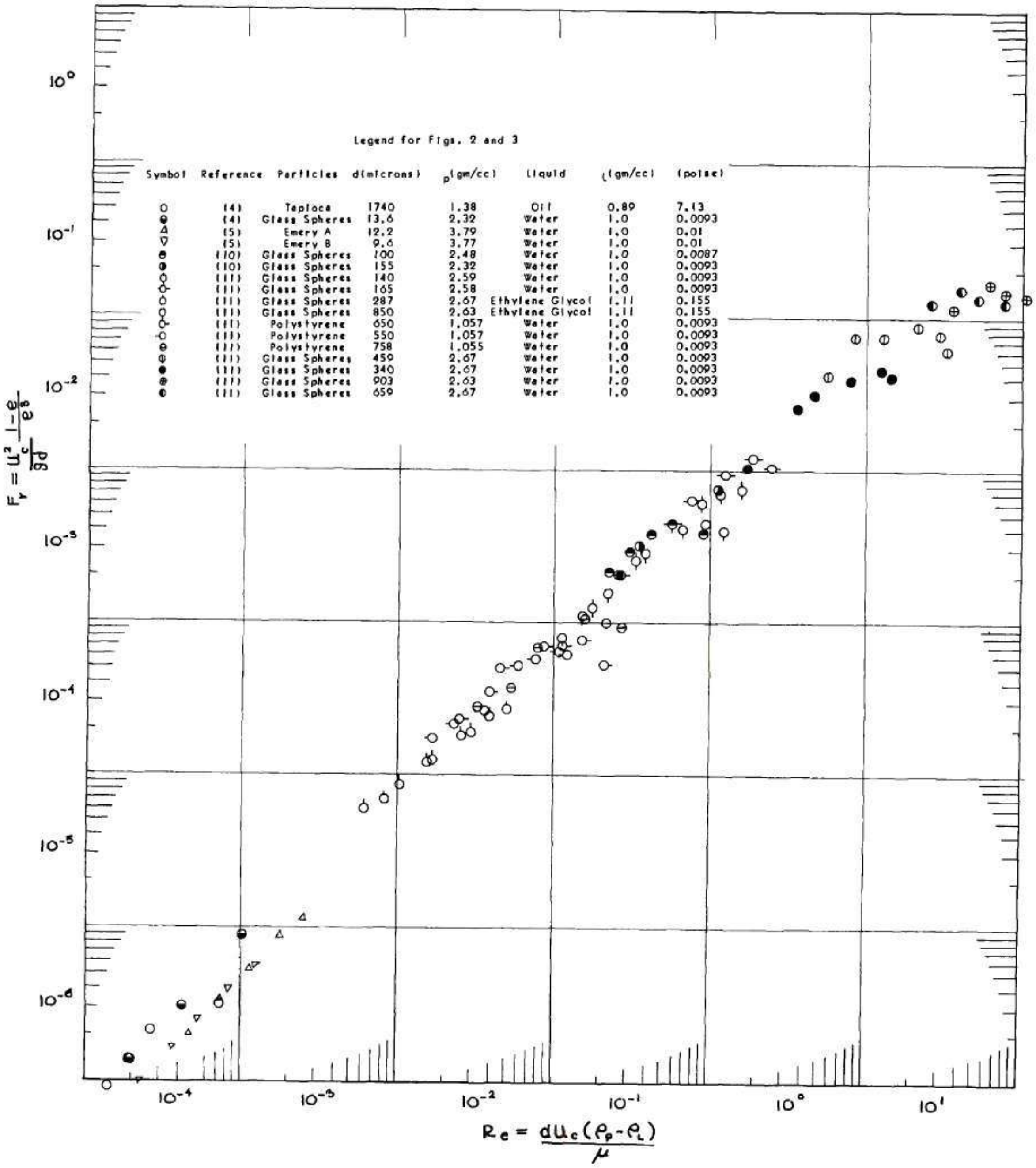

Figure 2. Froude-Reynolds Relation for Hindered Settling.

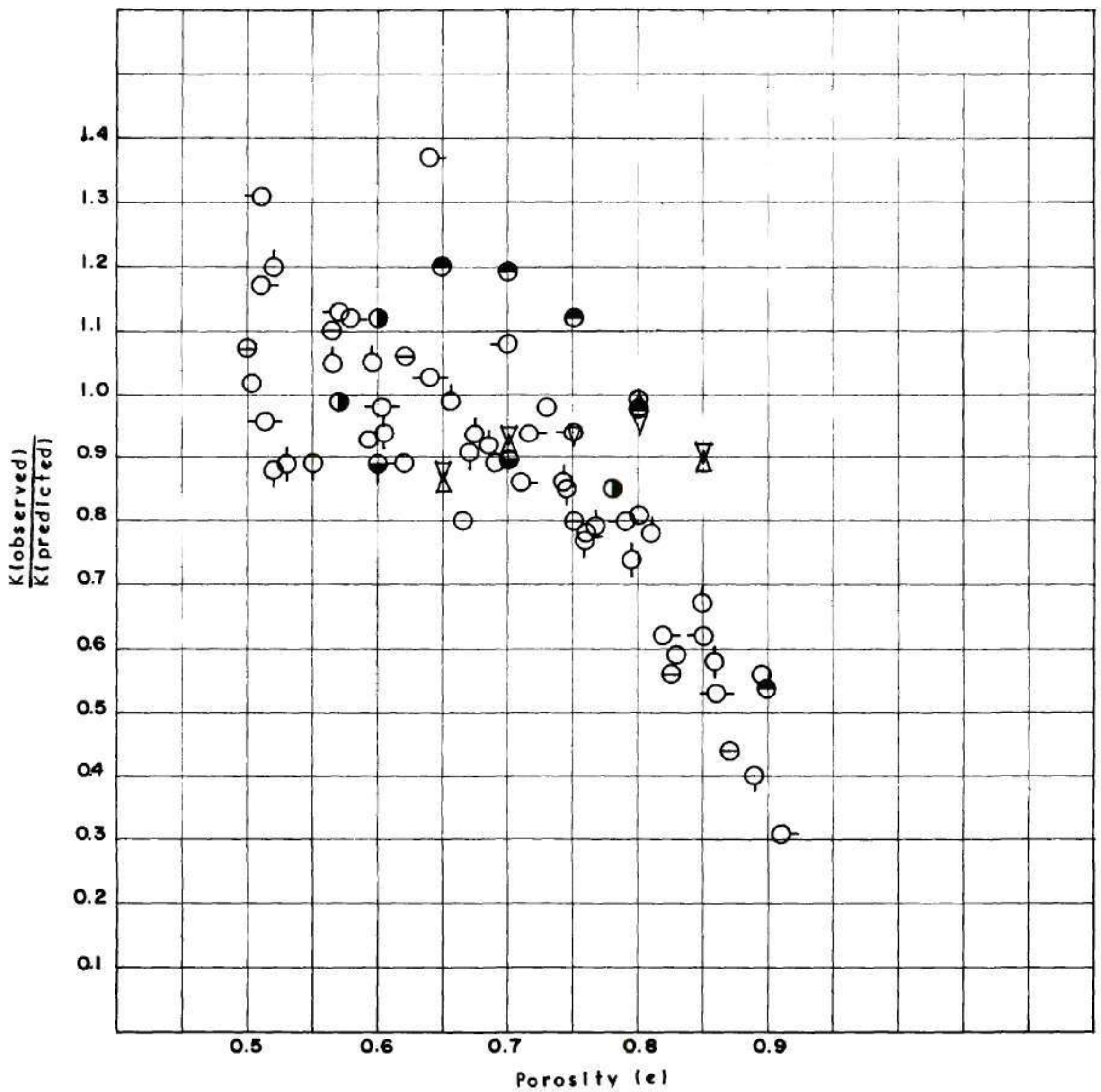

Figure 3. $\frac{K(observed)}{K(predicted)}$ Versus Porosity (e).

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