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A DYNAMO SIMULATION OF A
COMPLEX MILITARY TACTICAL MODEL

A THESIS

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Donald Leonard Meyer

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A DYNAMO SIMULATION OF A

COMPLEX MILITARY TACTICAL MODEL

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TABLE OF CONTENTS

TABLE OF CONTENTS (CONTINUED)

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (CONTINUED)

CHAPTER I

INTRODUCTION

Nature of the Problem

Computer simulation is used extensively in the analysis and synthesis of complex systems. The methodology of simulation is based on experimentation with a model representing a complex real-world situation. For this reason, simulation is often referred to as indirect experimentation. In the evaluation of such diverse military problems as optimal unit organization structures, new tactical doctrines, new weapons systems, and maintenance techniques, to name but a few, computer simulation has proved to be an excellent tool of research.

The development of military applications of computer simulation is not progressing as rapidly as it could. The foregoing statement is based on the author's conversations with members of such agencies as the U. S. Army Strategy and Tactics Analysis Group, the Combat Development Command and the Office of the Vice Chief of Staff of the Army. The problems voiced were essentially twofold. First, there is a shortage of qualified military personnel; second, it is difficult to "sell" the results of simulation to decision makers at higher levels.

To build a program in the languages presently used requires specialized training. To understand the program also requires special training. Because of the lack of qualified personnel, the Army has had to rely on a number of civilian research agencies for the development of many of its simulation models. These organizations have qualified research and computer simulation personnel, but in most cases these people lack the desired military expert knowledge. Some very costly research projects have resulted in massive reports that proved or said absolutely nothing.

Most of the military decision makers have not had the opportunity to receive specialized training in the computer languages presently used. As a result, the programmer or action officer has difficulty in explaining to them how the computer equations produced the results obtained. In view of some monumental blunders of the past, it is understandable that the decision-maker would be hesitant to commit himself. It would not be easy for a responsible person to make a critical decision based on someone else's inadequate interpretation of a computer run.

Resolution of the existing problems appears to lie in reducing the degree of expert knowledge required by military personnel to use and understand computer simulation. This could be accomplished by replacing the presently used FORTRAN with some simpler special-purpose language. It is easier and more logical to teach the military man a simple computer language than it is to try to infuse years of military experience into a civilian programmer.

Literature Search

A search of the literature revealed that the majority of the simulation programs developed by or for the Army were done in either FORTRAN or SIMSCRIP. Davis (5) gives an excellent summary of the more important programs in his thesis. The reason that FORTRAN and SIMSCRIP are the primary languages utilized is that the hardware in the Army inventory has been furnished by IBM and both of these languages are compatible with the IBM hardware. The majority of the models developed involve some aspect of a tactical problem. Obviously, by its very nature, a military tactical problem represents a dynamic system. For the simulation of this type of problem, the FORTRAN language was used. FORTRAN lends itself to flow diagramming, but the diagram is not self-explanatory or easily understood.

The first tactical simulation model developed by the Army was called Carmonette (26). This model simulated a platoon-level problem. It is still being used, but has been expanded so that it is capable of simulating a company-level problem.

As a follow-up to Carmonette, Centaur (2) was developed in 1962. This model was capable of simulating a brigade-level problem. As an aggregation of Centaur, Legion (3) was recently developed to simulate a division-level problem. Legion is an extremely sophisticated model. It requires two groups of players, each representing one of the division staffs. It is in essence a differential game. All of the aforementioned models are capable of simulating the effects of terrain, movement of the units, and any type of tactical situation.

Of the special-purpose languages that are compatible with the IBM system, the language that appears to promise the best results as a replacement for FORTRAN is the DYNAMO language. This language was developed by Dr. Phyllis Fox (Mrs. George Sternlieb) and Alexander L. Pugh (20). The language lends itself to diagramatic representation that is easily understood and explained. The language itself consists of a number of set equation forms that may be interconnected to build the model. The rudiments of DYNAMO may be learned in a few hours;

however, to become skilled in the use of the language takes time and practice.

DYNAMO has been used to simulate a number of diverse and complicated dynamic systems. For example, Schlager (21) used the language to simulate a number of manufacturing companies in Milwaukee, Wisconsin. Zymelman (27) used DYNAMO in connection with a stabilization problem faced by the cotton gray goods industry. Holland and Gillespie (15) used DYNAMO to simulate the economy of an underdeveloped country.

DYNAMO has not been used extensively to simulate military problems. All of the published work in this area has been done at the Georgia Institute of Technology. Davis (5) used the language to determine the proper employment of crossing vehicles and bridging materials for a division assault river crossing. Faulkender (13) used DYNAMO to simulate a counterinsurgency operation. Abele (1) and Krol (18) used the language to simulate a simple tactical combat situation. The Abele model (1) represents the first attempt to apply DYNAMO to a military tactical situation. The model simulates a one-sided combat problem where the simulated force is defending itself from a hypothetical attacking force.

The Krol model (18) is an extension of the Abele model (1). The Krol model simulates a two-sided combat problem and adds the effect of indirect fire. It assumes that the forces are stationary and that the engagement is fought on a flat table-top type of terrain. This configuration represents the type of tactical combat prevalent up to the mid 1800's (12). It may be of interest to note that it was not until 1916 that Lanchester (19) proposed the analytical formulation of the simple two-sided combat, and it was not until 1959 that Weiss (25) added the effect of supporting weapons. The Krol model uses computer simulation to combine Lanchester's and Weiss's approach with the theory of differential games proposed by Isaacs (16).

Analysis of the simulated tactical problems developed to date appeared to indicate a potential for DYNAMO as a substitute for FORTRAN. However, if DYNAMO is to replace FORTRAN, it must be capable of simulating the movement of the units under all tactical situations and representing the influences of the factors of terrain. The simulation models presently used by the Army are able to incorporate the above features. The methods used to represent terrain in Carmonette, Centaur, and Legion appear to be readily adaptable to DYNAMO. The problem then resolves itself as to whether or not DYNAMO can represent the movement of two opposing forces.

Statement of Objective

The purpose of this study is to develop a DYNAMO simulation of two-sided combat in which the behavior of the opposing forces is affected by their movement, exchange of fire, ammunition constraints, and casualty rates.

Method of Procedure

The investigation was conducted in four phases:

(1) A valid DYNAMO model was developed that was capable of representing the movement of two forces under all tactical circumstances.

(2) The Abele and Krol models (1 , 18) were improved upon by the more realistic simulation of the effects of small arms fire, casualty assessment, and

the addition of an ammunition consideration.

(3) The models developed in phases one and two were combined to represent a dynamic tactical model.

(4) Experiments were conducted with the final model to determine its utility, validity, and flexibility.

Certain constraints were placed on the tactical model. Weapons were limited to small arms direct fire weapons, and the size of the unit was limited to the small unit level (company and platoon).

The following guidelines were established for the model's development:

(1) The model should be as simple as possible to accomplish its objective.

(2) The model should not require major modification or revision for different tactical situations.

(3) The model should be easily expandable without requiring major modification or revision of the previously developed portions of the model.

(4) Exogeneous values imposed on the model as initial conditions or constants should be kept to a minimum and represent factors that are the results of the tactical situation or the decisions of the commanders.

(5) Endogeneous decisions should be made logically and realistically by the model at the correct time during the model run and not through the artificiality of exogeneous functions.

Two assumptions were made for the development of the model and the explanations of the model's development that are covered in later chapters. First, it was assumed that the effect of terrain on the speed of troop movement is uniform so that the maximum speed at which a unit may move is a constant. Second, it was assumed that the reader has a familiarity with the DYNAMO simulation language developed by Pugh (20).

The basic approach to the problem was a combination of analysis and synthesis. The problem was analyzed as a whole and the various major factors of interest were identified. These factors are termed sections in the model. The sections were analyzed and their sub-elements were identified in greater detail. These sub-elements in the model are termed modules; they constitute the building blocks of the model. Each module was developed as a separate entity and then interconnected to form the section. The sections were in turn connected to form the model. This procedure can be recognized as the synthesis approach to dynamic model building as discussed by Cohen and Cyert (4) and Krol (17) rather than the analysis approach advocated by Forester (14).

An advantage of the synthesis approach to model building is that the validity of each module and section may be tested as it is developed and expanded. If each module and section passes the tests for validity conducted upon it, there is an extremely high assurance that the final composite model will meet the criteria for validity. There is never, of course, complete assurance that the final composite model will be valid even if the tests hold for the sub-elements; but the synthesis approach should produce a higher probability of successful simulation than the analysis approach.

As previously mentioned, the problem was analyzed and the major factors of interest were identified. The factors of interest in the model, that is the subject of this study, are as follows:

(1) Troop movement.

(2) Unit strength and casualties.

(3) Direct fire.

(4) Ammunition supply and resupply.

It is recognized that other factors bear on the problem and would be of interest in a further expansion of the model, e.g., terrain and indirect fire. However, this model has been restricted to consideration of only the four factors previously mentioned.

Overall Organization

The step by step development of the model is described in succeeding chapters. A schematic diagram of sections and modules of the final composite model is shown in Figure 1. It should be noted that the final model consists of 18 modules. Ten of these modules make up the movement section. Four modules compose the strength and casualty section. Two modules represent the direct fire section and two modules represent the ammunition constraints.

Chapter II develops the movement modules M1 and M2 for the red and blue forces, respectively. It also develops the module M3 for representing an attack/defense/withdrawal problem. It is capable of simulating the situation where:

(1) Both forces are defending.

(2) Both forces are attacking.

(3) One force is attacking and one force is defending.

(4) One force is withdrawing or retiring and the other force is either

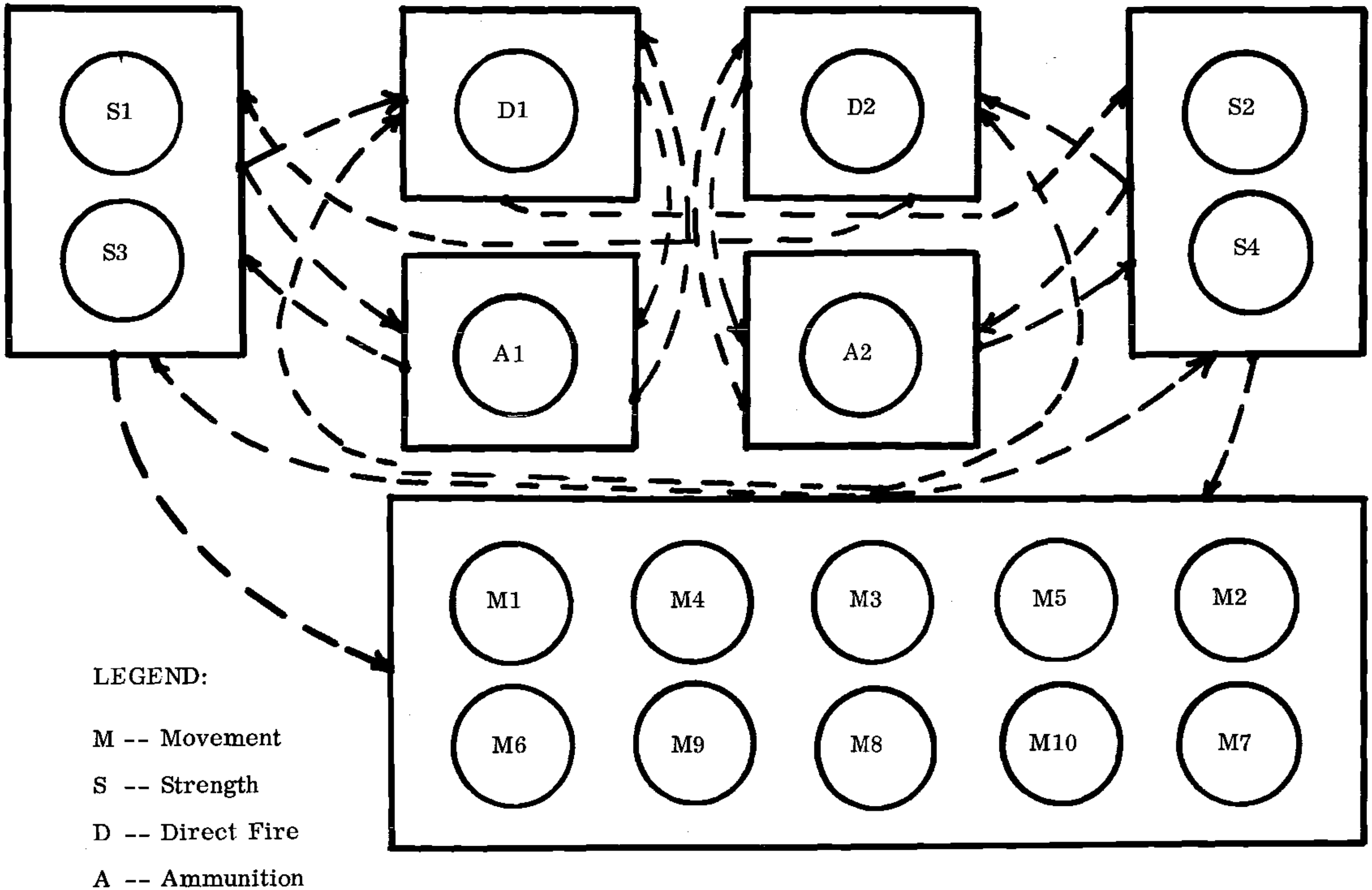

Figure 1. Components of the Model

attacking or defending.

Movement away from an opponent is termed a retrograde movement. Army field manuals (6, 9) identify three types of retrograde movement as being a retirement, a withdrawal, or a delaying action. A retirement is an orderly withdrawal of a unit according to its own plan without pressure by the enemy. A withdrawal may be forced or voluntary and may be executed during daylight or at night. The withdrawal normally involves disengagement from the enemy. An historical example of a withdrawal/retirement is the operation conducted by Kutuzov against Napoleon before and after the battle of Borodino (22, 24). Module M3 is capable of simulating a withdrawal or retirement but cannot simulate a delaying action when the defender acts as a covering force and delays on successive positions.

Chapter III develops the movement modules M4 and M5 for a delaying action problem when one force acts as a covering force and the other force acts as the attacker.

Some tactical situations require that time be gained by a commander; in these cases, he is willing to give up terrain to the enemy in order to obtain the time he needs to accomplish some other objective. A commander, for example, may need time to:

(1) Group and consolidate his forces so that he may assume the offensive.

(2) Prepare or improve a strong defensive position.

In order to gain the time required by the commander, some unit is assigned the mission of acting as a covering force and delaying the enemy on successive defensive positions. The unit that acts as the covering force does not become decisively engaged with the enemy. Instead, it inflicts the maximum number of casualties on the enemy through the application of fire power and then withdraws to the next defensive position. The number of successive positions occupied by the defender varies in each case, and the distance between the successive positions also varies. Both of the foregoing are dependent on the situation and the terrain.

Chapter IV develops five movement selection modules. The problem selection modules M6 (for the red force) and M7 (for the blue force) determine whether the simulation run is to represent an attack, defense, withdrawal, or delaying action problem. The distance selection module M8 specifies the initial distance between the forces and the distances the covering force will move if the simulation is a delaying action problem. In addition, the module keeps track of the distance between the forces as the problem progresses so that this information is available to other sections of the model. The velocity selection modules M9 (for the red force) and M10 (for the blue force), keep track of whether the unit is moving or is stationary as the problem progresses. This information is then made available to other sections of the model.

Chapter V develops the strength/casualty section (which consists of two modules for each force), the small arms direct fire module of the direct fire section, and the small arms ammunition module of the ammunition section. The strength modules, S1 (for the red force) and S2 (for the blue force), are the same as those developed by Abele (1). The small arms casualty modules S3 (for the red force) and S4 (for the blue force), determine the casualties that are inflicted on the force as a result of enemy small arms fire. The modules are reactive to various degrees of protection afforded to the forces.

The portion of the model pertaining to small arms fire was designated as a module of the direct fire section in recognition of the fact that tank direct fire could be a consideration. However, for the model developed, neither force is considered to possess an armor capability. The small arms fire modules D1 (for the red force) and D2 (for the blue force), are an extension of the Krol model (18) and the Abele model (1). The amount of small arms fire being directed against the enemy at any time is determined by these modules.

The portions of the model pertaining to small arms ammunition were designated as modules of the ammunition section in recognition of the fact that tank ammunition and indirect fire ammunition (mortar, artillery) could be a consideration in an expansion of the present model. However, only small arms ammunition is considered in the present model. The small arms ammunition modules A1 (for the red force) and A2 (for the blue force), determine the rate of ammunition expenditure and resupply.

Chapter VI discusses some of the experiments conducted with the final composite model and the results of these experiments.

Chapter VII contains conclusions and recommendations.

CHAPTER II

MOVEMENT SECTION: MODULES M1, M2, AND M3

General Considerations

This chapter is concerned with the development of the movement modules M1 and M2 for the opposing forces and with the module M3 for representing the type of tactical situation which may be identified as attack, defense, or withdrawal.

The Movement Modules M1 and M2

The first factor of interest considered for simulation was troop movement. The starting point for the synthesis of the movement section was the creation of separate modules capable of simulating the movement of two opposing forces. The criteria established for the development of these modules required that they be capable of:

(1) Simulating different speeds of troop movement.

(2) Simulating the forward and backward movement of a unit.

(3) Allowing troop movement to begin at any time during the simulation run.

(4) Allowing the movement of the opposing forces to begin at different times.

Two factors dictated the selection of the above criteria: the guidelines established in the previous chapter and the peculiarities inherent in any military tactical situation or problem. These reasons will be amplified and clarified as the modules are developed. Figures 2 and 3 show the schematic diagrams for the red and blue movement modules M1 and M2.

A convention that is used in the movement modules and throughout the rest of the model is that of letter-coding the variables. Five letters are used to designate each variable. The first letter of the variable indicates the section to which it belongs. For variables that belong to the movement section, the first letter is M. The second letter in the variable is R if it concerns the red force, B if it concerns the blue force, and any letter other than R or B if it is a variable of concern to both forces. The last three letters are chosen so as to give some indication of what the variable represents. For example, in Figure 2, the variable MRDEV represents the red force's desired velocity.

It may be noted in Figures 2 and 3 that the two modules M1 and M2, except for the second letter of the variable, are mirror images of each other. Throughout the remainder of this study, when the red and blue modules are the same except for the one letter difference in the variables, only the diagram for the red module will be shown.

It is recognized that the speed at which a unit may move is dependent on the terrain and obstacles of natural or man-made origin. For example, the maximum speed possible for a unit would be different when moving across an open field than it would be when moving through a swamp. However, the effects of terrain and man-made and natural obstacles were not considered in the model. Since terrain was not considered as a factor, the maximum speed MRMXV was taken to be a

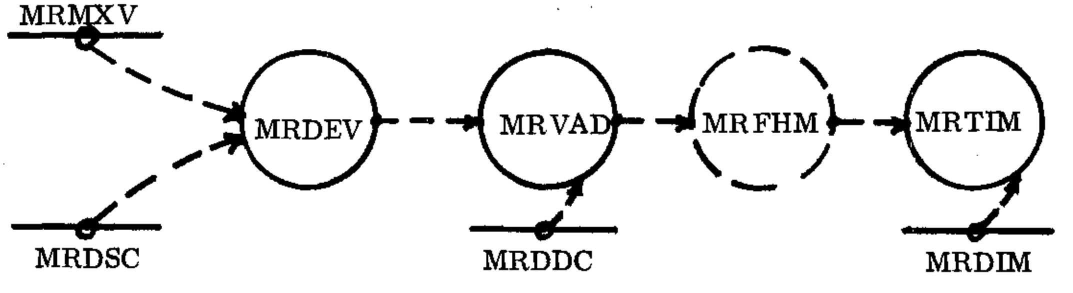

Figure 2. Red Movement Module M1

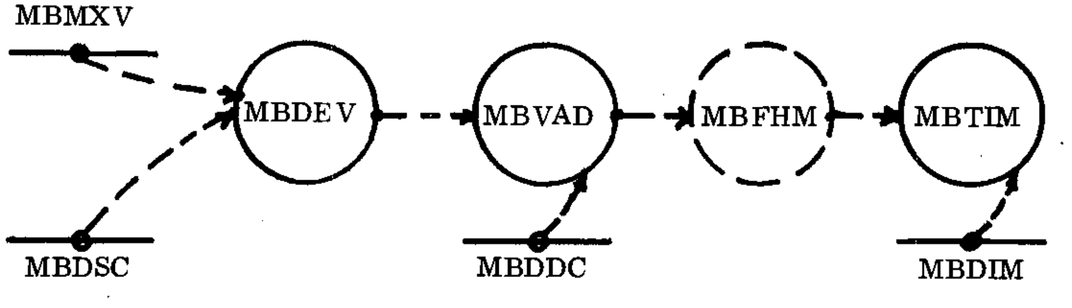

Figure 3. Blue Movement Module M2

constant and chosen as three kilometers per hour.

The commander may desire to move at his maximum possible speed or at some fraction thereof. As an example, he might desire to move at 1/2 kilometer per hour because he feels that he cannot maintain the desired control of his unit if the unit moves at a faster rate. A tactical situation wherein control is more important than speed would be a night attack. On the other hand, control may be sacrificed to speed, and the commander may wish the unit to move at as rapid a rate as possible. The tactical situation governing this decision might be that the unit must cross an open area where the unit can be observed by the enemy and is in range of the enemy weapons. The faster the unit can move across this open area, the fewer casualties it is likely to suffer. The decision of the commander as to the speed at which the unit is to move is applied to the module through MRDSC (the desired speed coefficient).

MRDEV (the desired velocity) is the product of MRMXV and MRDSC and is the actual speed at which the unit will move, when it is moving:

12A MRDEV.K = (MRDSC)(MRMXV)

The commander receives orders which determine whether he is to move forward (attack), backward (withdraw), or remain stationary (defend). The direction of movement is entered into the module through MRDDC (the desired direction coefficient). If the unit is to move forward, the value of the coefficient is +1. If the unit is to move backward, the value of the coefficient is -1. If the unit remains stationary, the value of the coefficient is 0. The product of MRDDC and MRDEV yields MRVAD (the velocity and direction):

12A MRVAD.K = (MRDDC)(MRDEV.K)

MRFHM (function to halt movement when the strength of the unit = 0) is shown as a dotted circle. This function cannot be added until the strength module is developed.

In a tactical simulation model, there are a number of circumstances where it may not be appropriate for the unit to begin moving at the time the simulation run commences. For example:

(1) To allow for preparatory fires.

(2) To allow the run to begin with a logistics exercise.

(3) To allow the opposing forces to commence movement at different times.

Of the foregoing, only the last example is obviously germane to the model being developed. The other two examples require further amplification.

Before an attack, it is normal for preparatory fires to be employed by the attacker. The purpose of preparatory fires is to "soften up" the enemy so as to make it easier for the attacker to take his objective. Preparatory fires may be fired by all types of weapons. However, since the opposing forces are usually out of range of small arms weapons, preparatory fires are normally fired only by mortars and artillery pieces. Regardless of the type of weapon or weapons furnishing the preparatory fires, the requirement exists for allowing movement to begin at some time after the run has commenced. This is also in keeping with the guidelines established in Chapter I for the development of the model. Further additions of an indirect fire section or a tank fire module would not require any revision of the movement module.

The same line of reasoning pertains to the second example. Many actual army map exercises and field maneuvers begin with a logistics exercise. The exercise may run for some length of time prior to any tactical unit movement or maneuver. The only logistics factor of concern in the model being developed is small arms ammunition. However, in the event that other logistics modules were added and it was desired to begin the simulation run with a logistical exercise, there would be no necessity for a modification to the movement module.

Two variables determine the time of initiation of movement of the unit. MRDIM (the time delay in initiating movement) is the time after the run commences that the unit will or can move. For example, if MRDIM is set equal to 1, movement may begin one hour after the beginning of the run. MRTIM is a step function and is the function which activates the movement of the unit. It specifies that prior to the time value assigned to MRDIM there will be no movement and that at time MRDIM movement may be in the direction and at the velocity represented by MRVAD:

45A MRTIM.K = STEP (MRVAD.K, MRDIM)

Through the movement module, the speed, direction, and time of initiation of movement of the unit are determined for the model. The only limitation in the module is the fact that the maximum possible speed of movement of the unit is taken to be a constant value. This limitation can be removed by the addition of a section representing terrain.

The Attack/Defense/Withdrawal Module M3

The next step in the synthesis approach to the formulation of the movement section was to develop the attack/defense/withdrawal module. The flow diagram for this module is shown in Figure 4. For this figure and flow diagrams of modules and sections that appear in later chapters, the symbols are shown for each variable of the module being discussed. If the module under discussion is affected by or affects another section or module, all of the symbols for the variables of these other modules or sections are not shown. Instead, the affected or affecting sections and modules are represented by labelled dotted rectangles, and only the symbols for the affected or affecting variables are shown within the rectangles. In Figure 4, the variables MRTIM and MBTIM of the red and blue movement modules affect the attack/defense/withdrawal module. It may be noted in Figure 4 that the second letter of the variables in module M3 is neither R nor B. As mentioned previously, this coding indicates that the variables of the module influence or are influenced by both the red and blue forces.

MCVRB (combined velocity for red and blue forces) is the algebraic sum of the individual speeds of movement. If the red force is moving backward, its speed will be a negative value. If the red force is moving forward, its speed will be a positive value. If the red force is stationary, its speed will be zero. The same situation holds for the speed of the blue force. MCVRB, therefore, represents the positive or negative rate of change of the distance between the two forces:

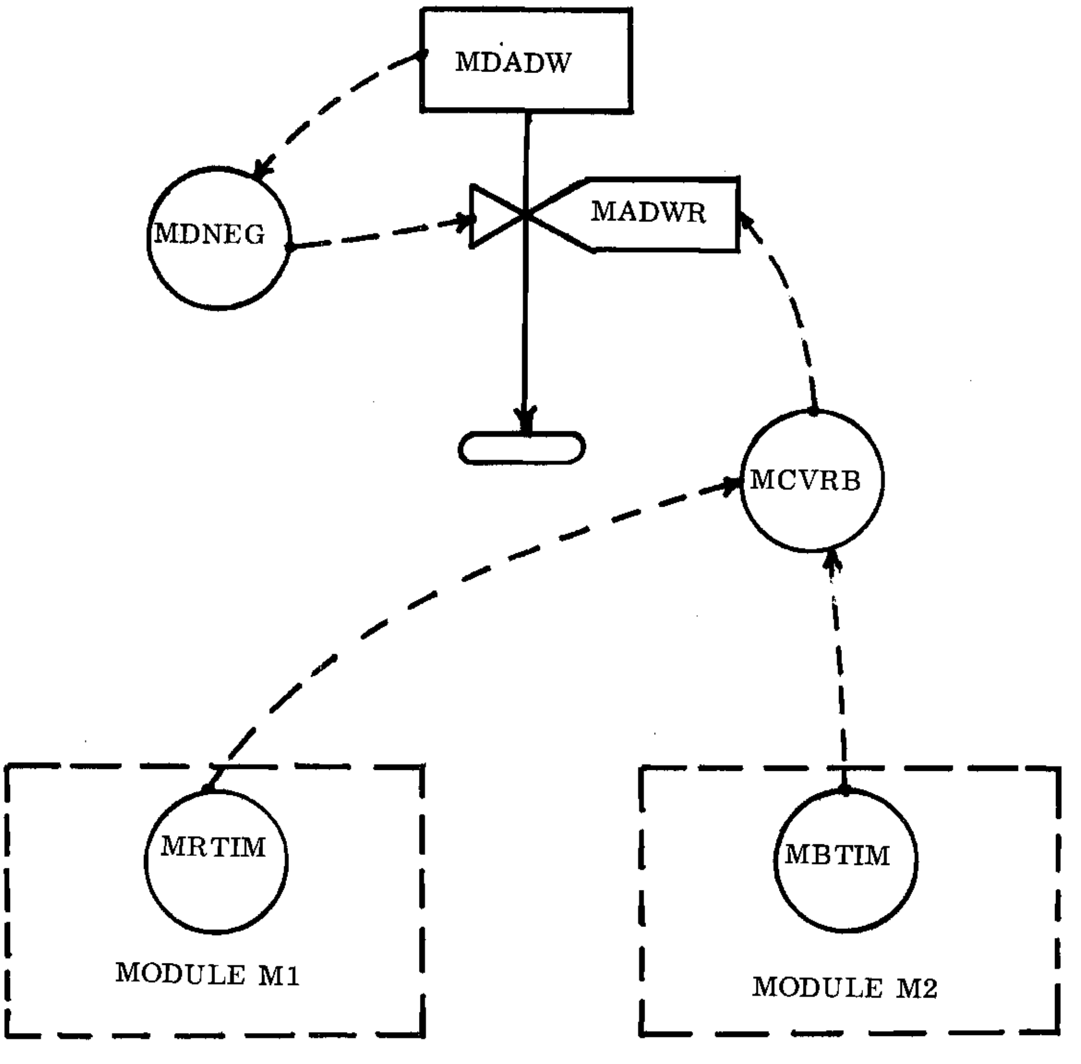

Figure 4. Attack/Defense/Withdrawal Module M3

7A MCVRB.K = MRTIM.K + MBTIM.K

Two forces opposing each other are at some specific distance apart at the beginning of any problem. As the forces move, this distance either increases or decreases unless the forces are moving in the same direction at the same speed. It was determined that the best method for representing the distance between the opposing forces at any point in time was through a level equation. MDADW is the variable that represents this distance. Since each level equation requires an initial condition value, this value is taken as the distance between the forces at the beginning of the problem. The value of MDADW would change as the forces moved. The change would be equal to the value of MCVRB. If MCVRB were positive, MDADW would decrease; if MCVRB were negative, MDADW would increase.

Upon experimenting with the module as formulated thus far, it was found that MDADW could assume negative values. This meant that if the two forces were approaching each other, they would pass through each other and continue their movement rather than being stopped when the distance between them was zero. This was not realistic. As two forces meet, they should prevent further movement until one force is overwhelmed. To rectify this shortcoming, two additional variables, MDNEG and MADWR, were introduced.

MDNEG (function to keep MDADW ≥ 0) was defined as an auxiliary by setting it equal to the distance at time K per increment of delta time:

20A MDNEG.K = MDADW.K/DT

The problem thus resolves itself into choosing the smaller value between

MDNEG and MCVRB. This is accomplished thru MADWR (rate of change of distance between red and blue), which is a minimum function and defined as a rate. Since all rate equations require an initial value, the initial condition for MADWR is set equal to zero:

54R MADWR.KL = MIN(MDNEG.K, MCVRB.K)

The level equation for the variable MDADW becomes:

1L MDADW.K = MDADW.J + (DT)(0–MADWR.JK)

In keeping with procedures established in the previous chapter for the development of the model, a number of experiments were conducted to validate the model as developed thus far. The model was found to be capable of simulating all types of tactical situations except the situation where one force conducts a delaying action. The problem of simulating a delaying action is discussed in Chapter III.

CHAPTER III

MOVEMENT SECTION: MODULES M4 AND M5

General Considerations

As mentioned in the previous chapter, modules M1, M2, and M3 were not capable of simulating a delay action tactical situation. Therefore, it was necessary to develop the additional modules which are the subject of this chapter. The reasons for conducting a delaying action and some of the tactical considerations of the commander in conducting the operation were discussed in Chapter I. The tactics will be further amplified in this chapter as portions of the modules are developed. This explanation will furnish the reader with a broader basis for understanding the reasons for the inclusion of certain variables and relationships built into the modules.

Analysis of the problem of simulating a delaying action resulted in establishment of the following criteria for the development of the modules:

(1) The time of withdrawal of the covering force should be determined endogenously not exogenously.

(2) The distance of each withdrawal should be the result of the commander's decision based on his analysis of the terrain.

(3) Four defensive delaying positions should be capable of being simulated by the modules.

Initial analysis of the problem appeared to indicate that use of a boxcar function, in conjunction with other functions, would be the best solution. The loads for the boxcars would represent the withdrawal distances. The boxcar shift time would be a variable that would be determined endogenously as specified in the first criterion for the modules. A module for the red force was constructed, employing a boxcar function which should have met all of the criteria. However, experimentation with the module proved a failure. The Burroughs 5500 compiler is not programmed to handle a boxcar function with a variable time shift. It was therefore necessary to take another approach to the problem. This resulted in the development of a more complicated and complex module than would otherwise have been necessary.

The Red Delaying Action Module M4

The flow diagram for the red delaying action module as finally developed is shown in Figure 5. In keeping with the convention initiated in Chapter II, only the red delaying action module M4 is shown. The blue delaying action module M5 differs from M4 only in the second letter of the variables.

It is difficult to visualize the military tactical situation simulated by Figure 5 by merely looking at the module. Therefore, Figure 6,which depicts graphically the tactical situation simulated by the module M4, has been included in the chapter. Both figures will be used extensively in the discussion of the module.

As in module M3, a level was used to simulate the distance between the opposing forces. MRDCF (distance between red and blue when red acts as a covering force) is the variable that reflects the distance between the forces at any time K. The initial value of MRDCF is set equal to the distance the forces are apart at the beginning of the problem. In Figure 5 and the equation below, it may be noted that

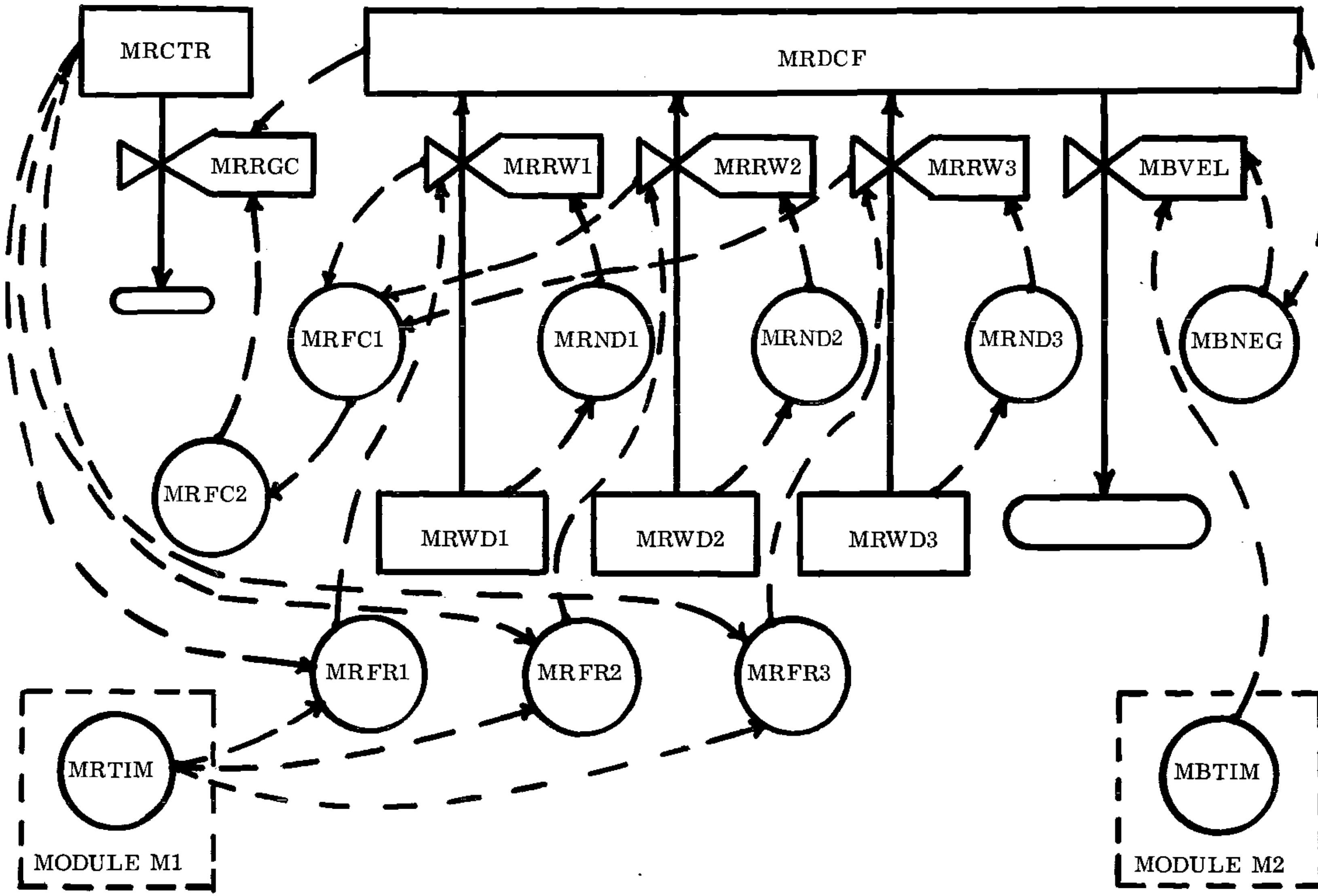

Figure 5. Red Delaying Action Module M4

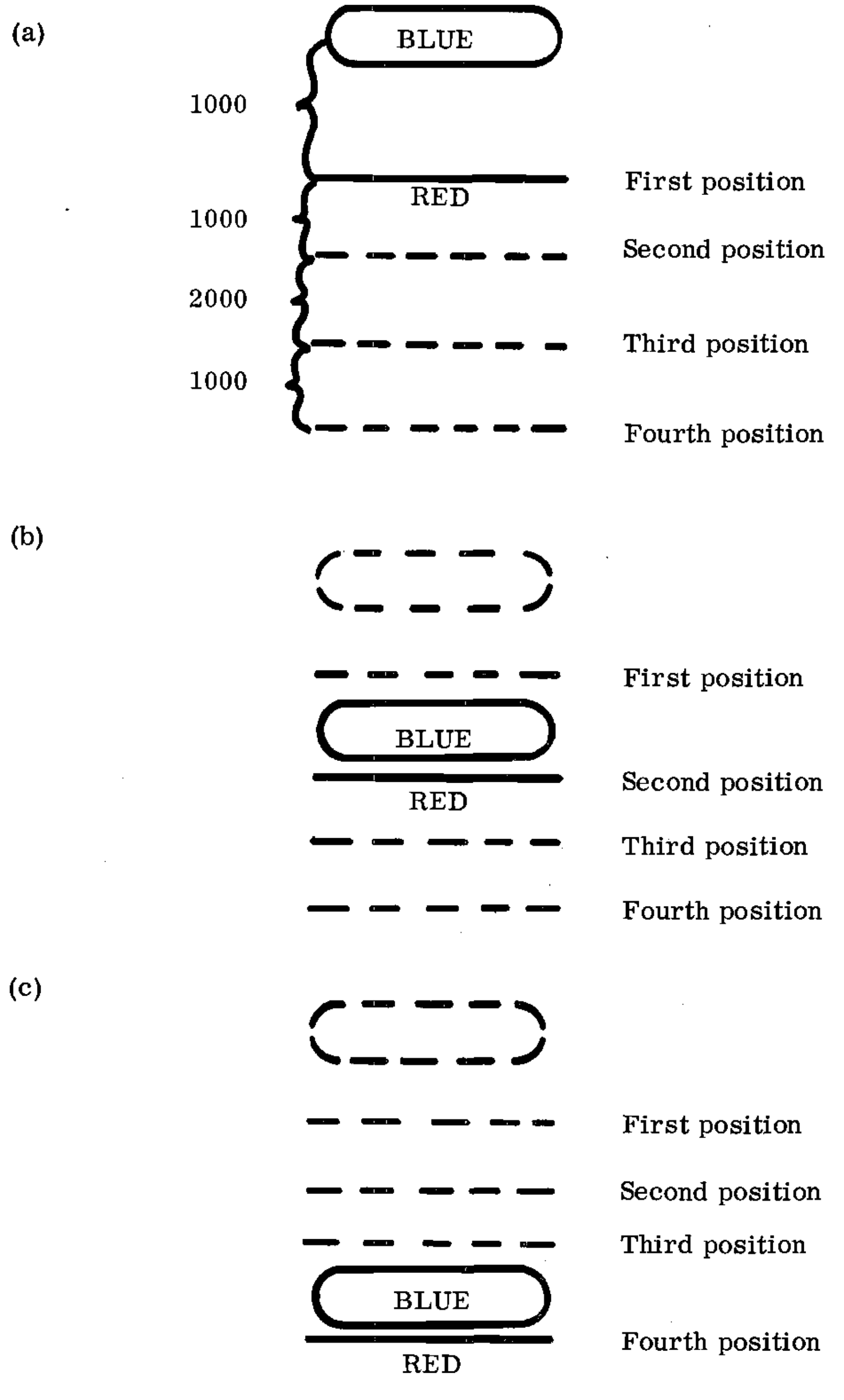

Figure 6. Red Delaying Action Problem

the value of MRDCF can be affected by four rate equations:

52L MRDCF.K = MRDCF.J + (DT)(-MBVEL.JK+MRRW1.JK+MRRW2.JK + MRRW3.JK)

Levels were used to represent the withdrawal distances. These variables are defined by MRWD1 (first withdrawal distance), MRWD2 (second withdrawal distance), and MRWD3 (third withdrawal distance). The initial values for these variables are the distances between the delaying positions. For example, MRWD1, MRWD2, and MRWD3 are equated to the distance values shown in Figure 6 (a). The distance between the first delay position and the second is 1000 meters, between the second and the third is 2000 meters, and between the third and the fourth is 1000 meters. These distances are determined as a result of the commander's analysis of the terrain.

It is very unlikely that any covering force, in an actual situation, would be required to occupy more than four successive positions. For this reason, the third criterion for the module was set at this number. If a problem were presented that required occupation of more than four positions, the module could be easily expanded to include any number of positions. A more logical situation is that the covering force would be required to occupy less than four positions. This presents no problem whatsoever. The initial value for the variable or variables representing the unused withdrawal distance or distances is merely set equal to zero. If only three positions are to be defended, MRWD3 would be set equal to zero. If two positions are to be defended, both MRWD2 and MRWD3 would be set equal to zero.

Because of the structure of the module, one precaution must be observed. The withdrawal distances must be assigned to the variables, starting with MRWD1, in the order in which they will be executed during the problem run. The reason for this precaution will become obvious later in the chapter as the development and purpose of the remaining variables are discussed. The level equations for the withdrawal distances are as follows:

1L MRWD1.K = MRWD1.J + (DT)(0-MRRW1.JK)

1L MRWD2.K = MRWD2.J + (DT)(0-MRRW2.JK)

1L MRWD3.K = MRWD3.J + (DT)(0-MRRW3.JK)

Figure 6 (a) shows the tactical situation at the beginning of the exercise. Red is acting as the covering force. Blue is the attacker and begins moving in the direction of the red force, at the speed and at the time determined by the movement module M2. The movement value is furnished by MBTIM (refer to Figure 5) to MBVEL (blue velocity when red acts as a covering force). The distance between the forces (MRDCF) decreases at a rate determined by MBVEL. As was the case with module M3, it is necessary to insure that the blue force will be halted when MRDCF becomes zero. The variable MBNEG (function to keep MRDCF positive or zero when red acts as covering force) was introduced, and a minimum function was used for MBVEL, as shown below:

20A MBNEG.K = MRDCF.K/DT

54R MBVEL.KL = MIN(MBNEG.K, MBTIM.K)

At some time during the course of the problem, a decision will be made for the red force to move to the next position. The rates MRRW1 (rate of withdrawal from first position), MRRW2 (rate of withdrawal from second position), and MRRW3 (rate of withdrawal from third position) are activated as required. The value of MRDCF is increased at a rate equal to MRRW1, MRRW2, or MRRW3 until the value of the respective level MRWD1, MRWD2, or MRWD3 is exhausted and equal to zero. The levels representing the withdrawal distances cannot be allowed to assume negative values because this would mean that the force had reversed its direction of movement and was attacking when it should be withdrawing. Therefore, the variables MRND1 (function to keep MRWD1 positive or zero), MRND2 (function to keep MRWD2 positive or zero), and MRND3 (function to keep MRWD3 positive or zero) are introduced:

20A MRND1. K = MRWD1. K/DT

20A MRND2. K = MRWD2. K/DT

20A MRND3. K = MRWD3. K/DT

The rate equations associated with MRDCF and the withdrawal distance levels are defined by minimum functions. This insures that the withdrawal distance levels will never be less than zero. For example, when MRRW1 is activated, it will always assume the smaller value between MRTIM (as furnished through MRFR1) and MRND1:

54R MRRW1. KL = MIN(MRND1. K, MRFR1. K)

54R MRRW2.KL = MIN(MRND2.K, MRFR2.K)

54R MRRW3.KL = MIN(MRND3.K, MRFR3.K)

The red commander must decide at what time or under what circumstances the withdrawal will begin from each position to the next delay position. Realistically, the red commander would have been given an order by his superior to keep the blue force forward of a specific position for a specified time. For the purpose of the validating runs, it was assumed that the red commander must keep the blue force forward of the fourth delay position for twelve hours after the start of the problem. If he is able to gain the twelve hours, he has accomplished his mission; if not, he has failed to accomplish his mission. If he can keep the blue force forward of the fourth position for a time longer than twelve hours and without suffering unnecessary casualties, this will most likely be accepted as a welcome bonus by the red commander's superior.

The red commander does not know when or how rapidly the blue force will advance and does not want to withdraw from any position until it becomes necessary. It is therefore impossible to specify the time for the withdrawal in advance, and the withdrawal time becomes a variable rather than a constant. The decision to withdraw is based on how close the enemy is to the red position. The red commander does not wish to become decisively engaged unless he is occupying his last position and has not delayed the blue force the length of time required by his superior. If the red commander allows the enemy to approach too close, he may not be able to issue the order and execute the withdrawal. Therefore, the time for executing the withdrawal is a variable that is a function of the distance between the opposing forces.

The decision as to how close the enemy will be allowed to approach is dependent on the terrain and whether the covering force is mounted in vehicles or afoot. For the case where the terrain is open and flat and the troops are afoot, the decision to execute the withdrawal would be initiated earlier than is normally the case.

To clarify the situation further, let us refer to Figure 6. The situation at the start of the problem is shown in Figure 6(a). As the blue force moves forward and reaches a point 600 meters from the red force's position, the red force withdraws to the second position. This situation is shown in Figure 6(b). The same sequence occurs for the second and third position. If the twelve-hour period has elapsed prior to the occupation of the fourth position, the mission of the red force has been accomplished. A continuation of the problem would determine how much bonus time had been gained by the red force. In the event that the specified time, in this case twelve hours, has not been gained and the fourth position has been occupied, the situation would be as shown in Figure 6(c). The red force, to gain the required time, would be forced to conduct a defense of this position. If the blue force has annihilated the red force prior to the lapse, in this example, of twelve hours, the red force has failed to accomplish its mission. On the other hand, if any of the red force still exists or has succeeded in annihilating the blue force, the red force has accomplished its mission.

In view of the foregoing, the next step in the formulation of the module was to develop a method for activating the rates MRRW1, MRRW2, and MRRW3, one at a time and at the time desired. The method used was to employ a counter and three auxiliaries. The counter consists of a level, MRCTR (counter when red acts as covering force); a rate, MRRGC (rate governing counter); and two auxiliaries, MRFC1 and MRFC2 (function to control counter).

MRCTR stores the current value of the counter as defined below:

1L MRCTR.K = MRCTR.J + (DT)(MRRGC.JK+0)

The initial value was set equal to -1. However, any value could have been chosen as long as this initial value was considered in the formulation of other equations developed later in the chapter.

MRRGC determines the rate of increase of MRCTR during any increment of delta time. MRRGC represents an endogenous decision point in the module. Its value is dependent on MRDCF and MRFC2.

The decision to execute the withdrawal is made by the red commander when the forces are 600 meters apart. This distance allows for a delay in issuing and executing the order. As long as the distance between the opposing forces is greater than or equal to 600 meters, the value of MRRGC is equal to zero. When MRDCF becomes less than 600 meters, MRRGC is set equal to MRFC2. The function used to define this relationship was a clip function:

51R MRRGC.KL = CLIP(0, MRFC2.K, MRDCF.K, 600)

MRFC2 is another endogenous decision point in the module. The value of MRFC2 at time K is dependent on the value of MRFC1 at time K. If MRFC1 is equal to zero, MRFC2 is equal to eight. If MRFC1 is not equal to zero, the value of MRFC2 is set equal to zero. The function used to define this relationship was a switch function:

49A MRFC2. K = SWITCH(8, 0, MRFC1. K)

There is nothing magical about the number 8 in the above equation. Eight was chosen because the problem increment of delta time is one-eighth of an hour and it was desired that the counter, MRCTR, accumulate values in an integer form rather than as a decimal.

MRFC1 is defined as the sum of the rates MRRW1, MRRW2, and MRRW3:

8A MRFC1. K = MRRW1. JK + MRRW2. JK + MRRW3. JK

It should be noted that in the above equation two conditions can occur. If the red force is moving, MRFC1 will have a value equal to MRRW1, MRRW2, or MRRW3. If the red force is not moving, the values of all withdrawal rates (MRRW1, MRRW2 and MRRW3) will be zero, and consequently the value of MRFC1 will be zero.

The withdrawal rates are activated by MRFR1 (function to activate MRRW1), MRFR2 (function to activate MRRW2), and MRFR3 (function to activate MRRW3). These functions, representing endogenous decision points in the module, were simulated by means of clip functions:

51A MRFR1. K = CLIP(-MRTIM. K, 0, MRCTR. K, 0)

51A MRFR2. K = CLIP(-MRTIM. K, 0, MRCTR. K, 2)

51A MRFR3. K = CLIP(-MRTIM. K, 0, MRCTR. K, 4)

The variables MRFR1, MRFR2, and MRFR3 can assume a value equal to -MRTIM or zero. The value of the variable is dependent on the current value of the counter, MRCTR. This relationship is more clearly seen in the examples below:

$$\begin{aligned} MRFR1 &= -MRTIM \quad \text{if } MRCTR \geq 0 \\ &= 0 \qquad\qquad \text{if } MRCTR < 0 \\ MRFR2 &= -MRTIM \quad \text{if } MRCTR \geq 2 \\ &= 0 \qquad\qquad \text{if } MRCTR < 2 \\ MRFR3 &= -MRTIM \quad \text{if } MRCTR \geq 4 \\ &= 0 \qquad\qquad \text{if } MRCTR < 4 \end{aligned}$$

The variables were set equal to -MRTIM because of the convention previously established for the movement module. It will be recalled that when a force moves backward, the speed is a negative quantity. The speed of movement must be changed to a positive quantity in order for MRDCF to increase. This change can be accomplished through the withdrawal rates or their auxiliaries. It was felt that less confusion would result if it were accomplished through the auxiliaries. The values of the counter that changed the value of the auxiliaries from 0 to -MRTIM were chosen as 0, 2, and 4. These values were chosen because of the initial value assigned to the counter and because of a delay that was built into the module.

In order to maintain realism in the simulation, a delay was considered to exist between the decision to execute the withdrawal and the actual beginning of the withdrawal. One increment of delta time, which is equal to 7.5 minutes, was considered an appropriate delay time.

The importance of the restrictive conditions placed on the rate MRRGC now become apparent. Without the restrictions placed on it by MRFC2, all of the withdrawal rates can be activated successively and can be functioning during the same period of delta time.

To further clarify the operation of the module, let us now consider specifically how the withdrawal rates are activated. When MRDCF becomes less than 600 meters, this condition allows MRCTR to increase from -1 to 0. This result, in turn, allows MRFR1 to assume the value of - MRTIM and MRRW1 to assume the value of MRFR1. During this increment of delta time, the value of the counter, because of the delay, is increased from zero to one. During the next and succeeding increments of delta time, MRDCF is increased at a rate equal to MRRW1 until MRWD1 becomes zero. While MRRW1 is active, MRRGC becomes zero, due to the restrictions imposed by MRFC1 and MRFC2, and the value of MRCTR cannot increase. When MRWD1 and MRRW1 become zero and MRDCF becomes less than 600 meters, MRRGC is again activated and MRCTR is increased to 2. This increase in MRCTR permits activation of the second withdrawal rate. The same sequence is followed for the activation of the third withdrawal rate.

The modules developed in this chapter also were validated by means of a number of test runs.

At this stage of the model's development, the movement section is capable of representing the movement of two opposing forces under all possible tactical situations. However, it has the disadvantage of requiring the interchange of the modules M3, M4, and M5 after the decision has been made as to what type of tactical problem is to be run. This is a violation of the guidelines established initially for the model's development in Chapter I. To overcome this deficiency, the movement section was further expanded to include certain selection modules, which are the subject of the next chapter.

CHAPTER IV

MOVEMENT SECTION: MODULES M6, M7, M8, M9, AND M10

General Considerations

This chapter develops the selection modules for the movement section. The addition of these modules precludes the necessity for interchanging the problem modules. Further, the additions make the value of the velocity of the forces and the value of the distance between the forces available to other sections of the model.

The Problem Selection Modules M6 and M7

In actual combat, a higher commander issues an order which assigns missions to his subordinate commanders. The mission determines the type of tactical problem. In computer simulation, the determination of the type of problem that is to be simulated is made prior to the beginning of the run and is therefore an exogenous decision. The problem thus presented was to determine some manner of simulating this decision so that it would be unnecessary to physically connect the appropriate problem module for each run. The model should endogenously determine which problem module will be activated. The resolution of the problem was accomplished through the development and addition to the movement section of a red (M6) and a blue (M7) problem selection module. The flow diagram for the red module is shown in Figure 7. The blue module is essentially the mirror image of

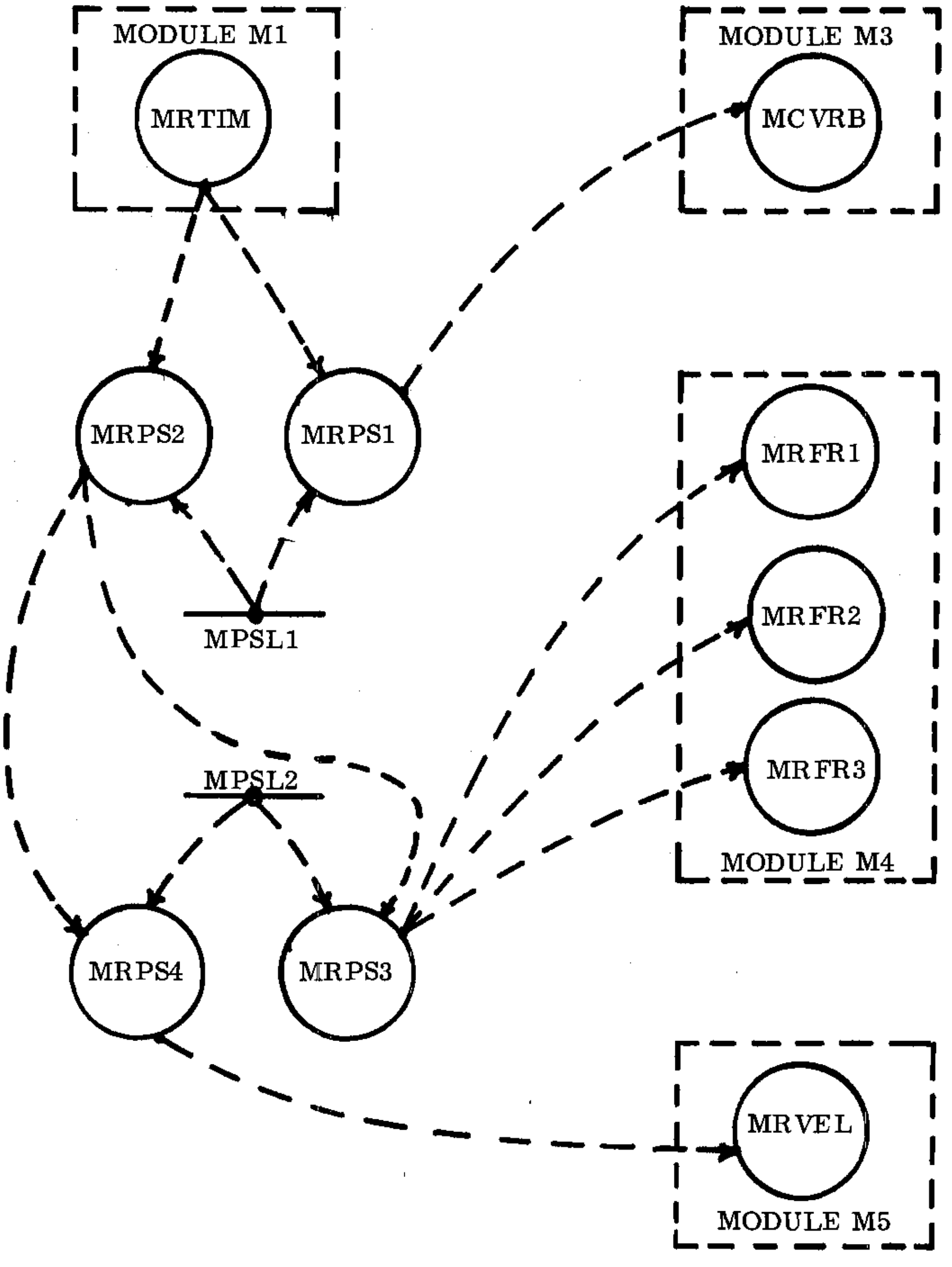

Figure 7. Red Problem Selection Module M6

the red module.

In Figure 7, the two functions that represent the type of problem to be run are MPSL1 (problem selector 1) and MPSL2 (problem selector 2). These functions can have only the values zero or one. In order for the module to operate properly, a value must be assigned to both functions prior to the start of the run. The relationship between the variable, the value assigned to the variable, and the problem module is shown below:

Variable	Value Assigned	Problem Module
MPSL1	0	Attack/defense/ withdrawal
MPSL1	1	Red or blue acts as a covering force
MPSL2	0	Red acts as a covering force
MPSL2	1	Blue acts as a covering force

As a result of assigning one of two possible values to two constant functions, the values for four red auxiliaries and four blue auxiliaries are determined. These eight auxiliaries in turn determine which of the three problem modules will be activated for the run.

MRPS1 (red problem selector 1), MRPS2 (red problem selector 2), MRPS3 (red problem selector 3), and MRPS4 (red problem selector 4) are endogenous decision functions. In essence, MRPS1, MRPS2, MRPS3, and MRPS4 act as the connecting link between the movement modules and the problem modules. They furnish the value for speed and direction of movement to the proper problem module. The relationships established by the red auxiliaries and their equations are shown below:

Variable	Value	Problem Selector Constant
MRPS1	MRTIM	MPSL1 = 0
MRPS1	0	MPSL1 = 1
MRPS2	0	MPSL1 = 0
MRPS2	MRTIM	MPRL1 = 1
MRPS3	MRPS2	MPSL2 = 0
MRPS3	0	MPSL2 = 1
MRPS4	0	MPSL2 = 0
MRPS4	MRPS2	MPSL2 = 1

49A MRPS1. K = SWITCH(MRTIM. K, 0, MPSL1)

49A MRPS2. K = SWITCH(0, MRTIM. K, MPSL1)

49A MRPS3. K = SWITCH(MRPS2. K, 0, MPSL2)

49A MRPS4. K = SWITCH(0, MRPS2. K, MPSL2)

The insertion of the problem selector modules required modification of the equations for MCVRB, MRFR1, MRFR2, MRFR3, MBVEL; and MBFR1, MBFR2, MBFR3, MBVEL. These new equations were substituted in modules M3, M4, and M5, respectively. The new equations for modules M3 and M4 are shown below:

7A MCVRB.K = MRPS1.K+MBPS1.K

51A MRFR1.K = CLIP(-MRPS4.K, 0, MRCTR.K, 0)

51A MRFR2.K = CLIP(-MRPS4.K, 0, MRCTR.K, 2)

51A MRFR3.K = CLIP(-MRPS4.K, 0, MRCTR.K, 4)

54R MRVEL.KL = MIN(MRNEG.K, MRPS4.K)

The Distance Selection Module M8

The values generated by the movement section will influence other sections of the model. For example, the distance between the forces will influence the rate of fire and casualty assessment. Two methods for making the values of the distance between the forces available to other sections of the model were examined. The first method involved connecting the variables of other sections to the appropriate module of the movement section. This method proved to be very cumbersome and unnecessarily complicated. The second method, which was finally adopted, was to develop selective modules similar to the problem selection module.

Figure 8 shows the flow diagram for the distance selection module M8. Through two auxiliary equations, the value of the distance between the forces at time K is made available to other sections of the model. The reader may recall that MPSL1 and MPSL2 determine the type of problem that is to be simulated. MDBRB (distance between red and blue forces) will equal MDADW or MDSL1 (distance selector), depending on the value of MPSL1. MDSL1 will equal MRDCF or MBDCF, depending on the value of MPSL2. The relationships and equations are shown below:

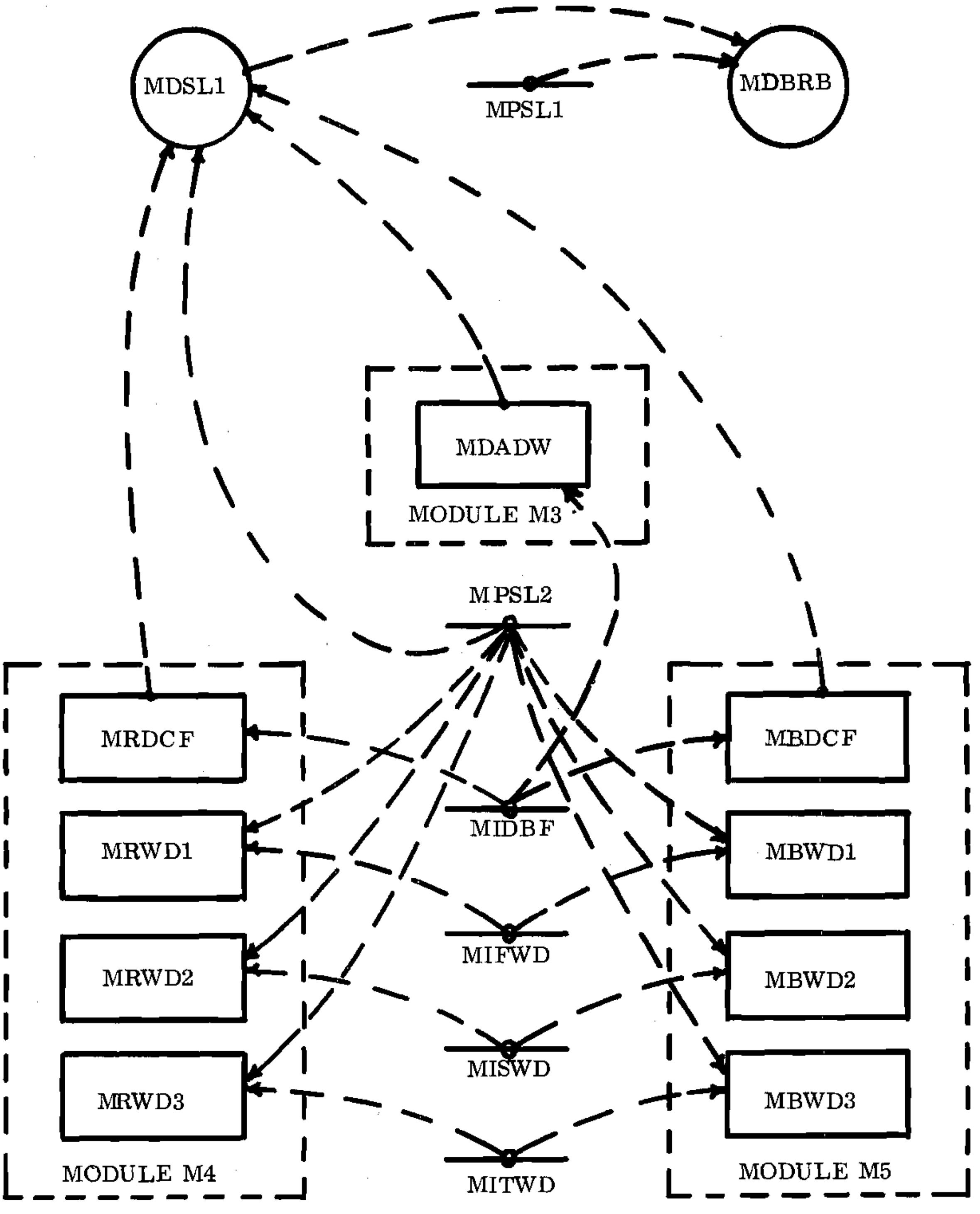

Figure 8. Distance Selection Module M8

Variable	Value of Variable	Problem Selector
MDBRB	MDADW	MPSL1 = 0
MDBRB	MDSL1	MPSL1 = 1
MDSL1	MRDCF	MPSL2 = 0
MDSL1	MBDCF	MPSL2 = 1

49A MDBRB. K = SWITCH(MDADW. K, MDSL1. K, MPSL1)

49A MDSL1. K = SWITCH(MRDCF. K, MBDCF. K, MPSL2)

In the process of developing module M8, a simplified method was devised for assigning the initial condition values to the distance level equations. This method involved the addition of four constants to the distance selection module and changing nine equations in the problem modules. This approach helps to insure that a DYNAMO error will not be made because the user inadvertently neglects to assign an initial condition to all the distance levels of the problem modules.

The constant MIDBF (initial distance between forces) determines the initial value of MDADW, MRDCF, and MBDCF at the start of the problem. The new initial condition equations for the levels are shown below:

6N MDADW = MIDBF

6N MRDCF = MIDBF

6N MBDCF = MIDBF

The constants that represent the values of the withdrawal distances are MIFWD (initial value first withdrawal distance), MISWD (initial value second withdrawal distance), and MITWD (initial value third withdrawal distance). The value of the problem selector MPSL2 determines whether the values of MIFWD, MISWD and MITWD are respectively assigned to MRWD1, MRWD2, and MRWD3 or to MBWD1, MBWD2, and MBWD3. The initial condition equations for the levels are changed from a type 6 to a type 49. The new equations for the red module are shown below:

49N MRWD1 = SWITCH(MIFWD, 0, MPSL2)

49N MRWD2 = SWITCH(MISWD, 0, MPSL2)

49N MRWD3 = SWITCH(MITWD, 0, MPSL2)

The Velocity Selection Modules M9 and M10

The determination of the degree of protection from enemy fire is influenced by whether a unit is moving or stationary. The value of the velocity of the units should therefore be capable of being furnished to other sections of the model. Figure 9 shows the red velocity selection module M9. The blue module M10 is similar to M9.

The speed and direction of movement of the red force is determined and made available to other sections through the media of MRVS1 and MRVS2 (functions to determine red velocity). Depending on the value of MPSL1, MRVS1 is set equal to MRTIM or MRVS2. The value of MRVS2 is set equal to MRFC1 or MRVEL, as determined by the value of the constant MPSL2. The relationships of the two auxiliaries and their equations are shown below:

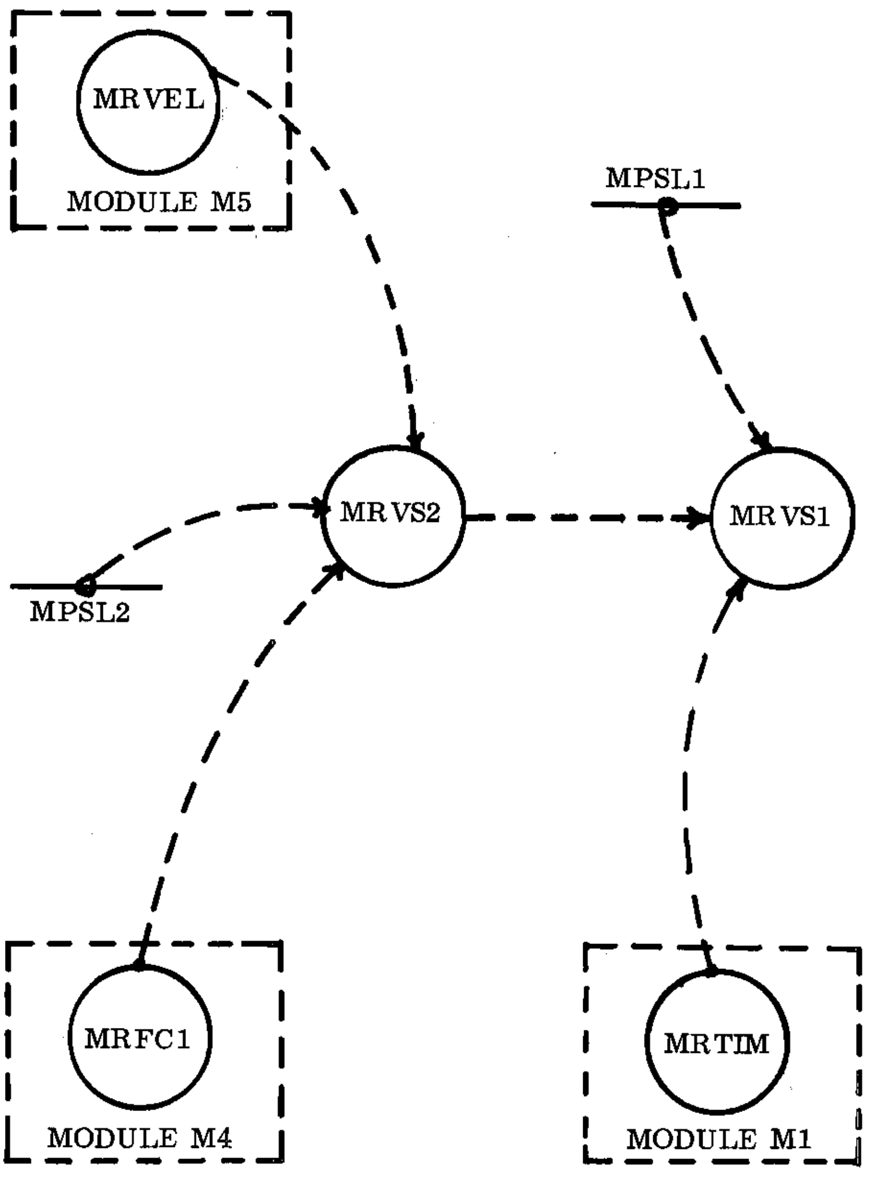

Figure 9. Red Velocity Selection Module M9

Variable	Value of Variable	Problem Selector Constant
MRVS1	MRTIM	MPSL1 = 0
MRVS1	MRVS2	MPSL1 = 1
MRVS2	MRFC1	MPSL2 = 0
MRVS2	MRVEL	MPSL2 = 1

49A MRVS1.K = SWITCH(MRTIM.K, MRVS2.K, MPSL1)

49A MRVS2.K = SWITCH(MRFC1.K, MRVEL.JK, MPSL2)

The addition of the problem, distance, and velocity selection modules completes the development of the movement section. A list of all of the variables, with an explanation of their meaning and the DYNAMO equations for the complete movement section are shown in the Appendix.

Movement Section Validating Runs

To insure that the section performed correctly, a number of experimental validating runs were performed. The results of two of these runs are shown in Figures 10 and 11.

Figure 10 represents the tactical situation where the red force is defending and the blue force is attacking. The blue force begins moving at the problem time of +1 hour at a speed of 1/2 kilometer per hour. MPSL1 is set equal to 0. MPSL2 can be assigned the value of 0 or 1, but it must be assigned some value in order for the model to function properly. The initial distance between the forces is 5000 meters; therefore, MIDBF is set equal to 5000. In order for the model to function properly, values must be assigned to MIFWD, MISWD, and MITWD. It may be noted in

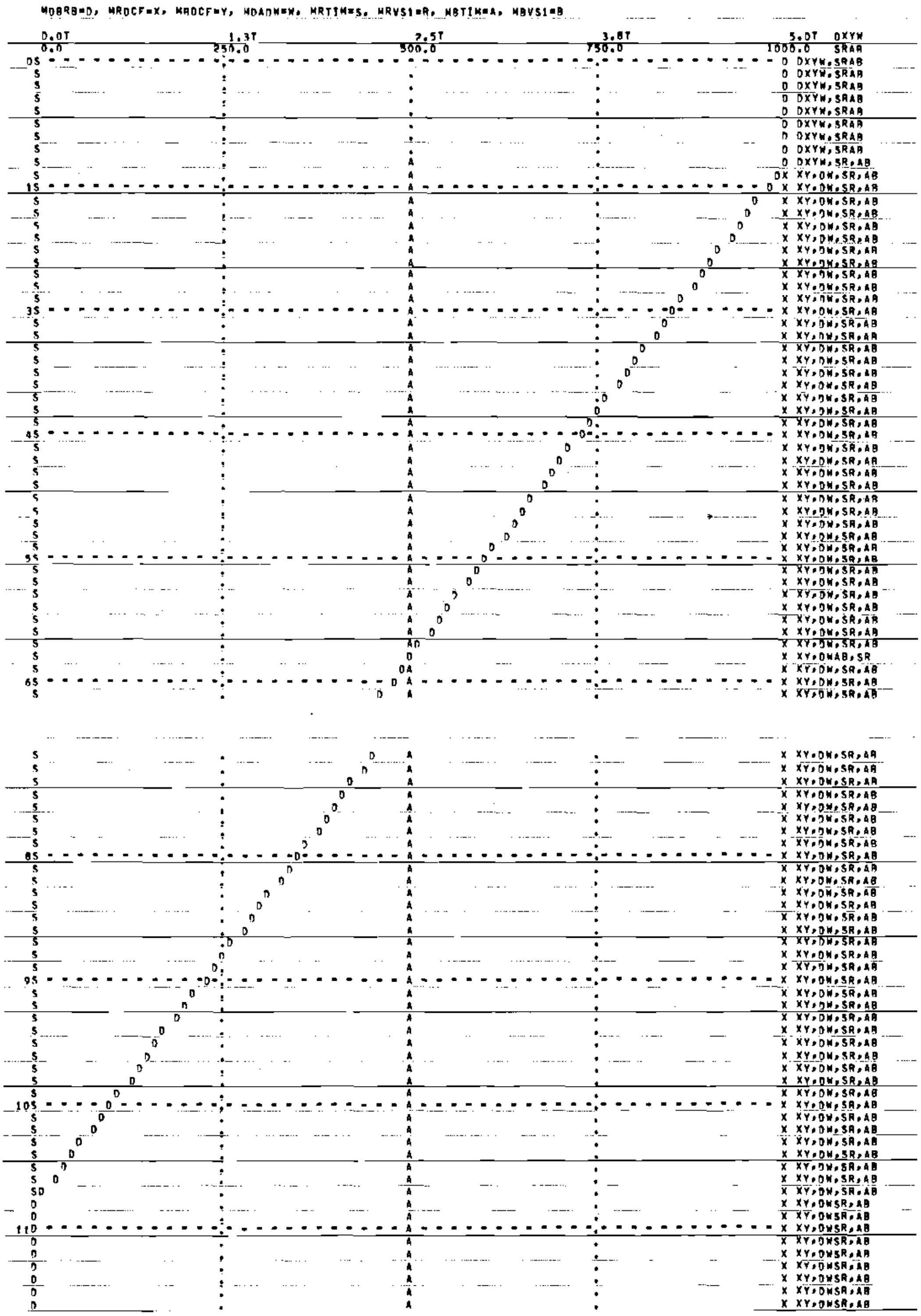

Figure 10. Movement Section Validating Run (Defense)

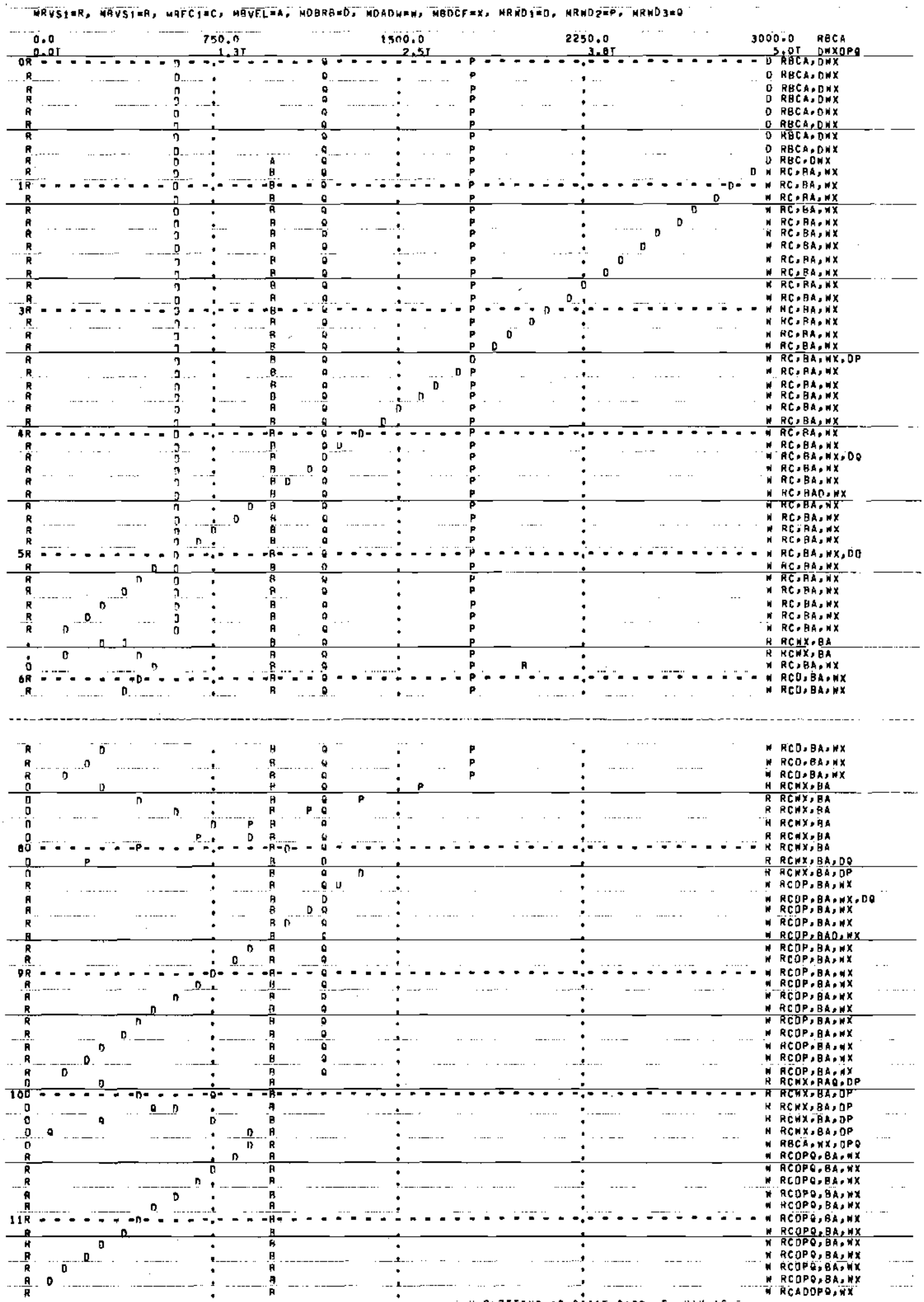

Figure 11. Movement Section Validating Run (Delaying Action)

Figure 10 that:

(1) MRVS1 and MBVS1 correctly reflect the value of MRTIM and MBTIM, respectively.

(2) MDBRB, MDADW, MRDCF, and MBDCF all initially equal MIDBF.

(3) The values of MDBRB and MDADW change during the problem run, whereas the values of MRDCF and MBDCF do not change.

(4) MDBRB is always equal to MDADW.

The plot shows conclusively that the attack/defense/withdrawal module was the only one activated. It further demonstrates that the correct values for the distance between the forces and the velocities of the forces at any time K may be obtained from MDBRB, MRVS1, and MBVS1.

A deficiency in the Burroughs compiler is found in all plots where delta time is a fraction. The horizontal scale divisions are not correctly shown, nor do they correspond to the plotted values. The plotted values are correct and correspond to the values obtained from the print-out.

Figure 11 represents the tactical situation where red is acting as a covering force and the blue force is the attacker. The blue force begins moving at the problem time of +1 hour at a speed of 1 kilometer per hour. MPSL1 is set equal to 1 and MPSL2 is set equal to 0. The initial distance between the forces is assumed as 5000 meters. The distance to the second, third, and fourth delay positions are 1000, 3000, and 2000 meters, respectively. It may be noted in the figure that:

(1) MRVS1 and MBVS1 correctly reflect the red velocity (MRFC1) and the blue velocity (MBVEL).

(2) The values of MRDCF and MDBRB change as the problem progresses, while MDADW and MBDCF remain unchanged.

(3) The withdrawal distances MRWD1, MRWD2, and MRWD3 are activated successively at the proper time.

The plot further demonstrates that the selection modules function properly and that the movement section performs as it was designed to perform.

CHAPTER V

COMBAT SUBMODEL: STRENGTH/CASUALTY, DIRECT FIRE, AND AMMUNITION SECTIONS

General Considerations

This chapter describes the development of the eight modules of the combat submodel. The strength/casualty section consists of modules S1, S2, S3, and S4. Modules S1 and S2 are concerned with the unit strength of the red and blue force, respectively. Modules S3 and S4 deal with red and blue casualties. The direct fire section consists of modules D1 and D2, which represent small arms fire of the opposing forces. Finally, the ammunition section, which consists of modules A1 and A2, deals with the ammunition supply and resupply considerations.

The Strength Modules S1 and S2

Figure 12 shows the flow diagram for the red strength module S1. The blue strength module S2 is similar to S1. Modules S1 and S2 are essentially the same as the representation used by Abele (1) and Krol (18). It may be noted that the first letter of the variable is S. All variables for the strength/casualty section start with the letter S.

The current Army Table of Organization and Equipment for an Infantry Rifle Company (11) was used as the basis for determining the initial strengths of the red and blue forces. Certain modifications were made in the organization for

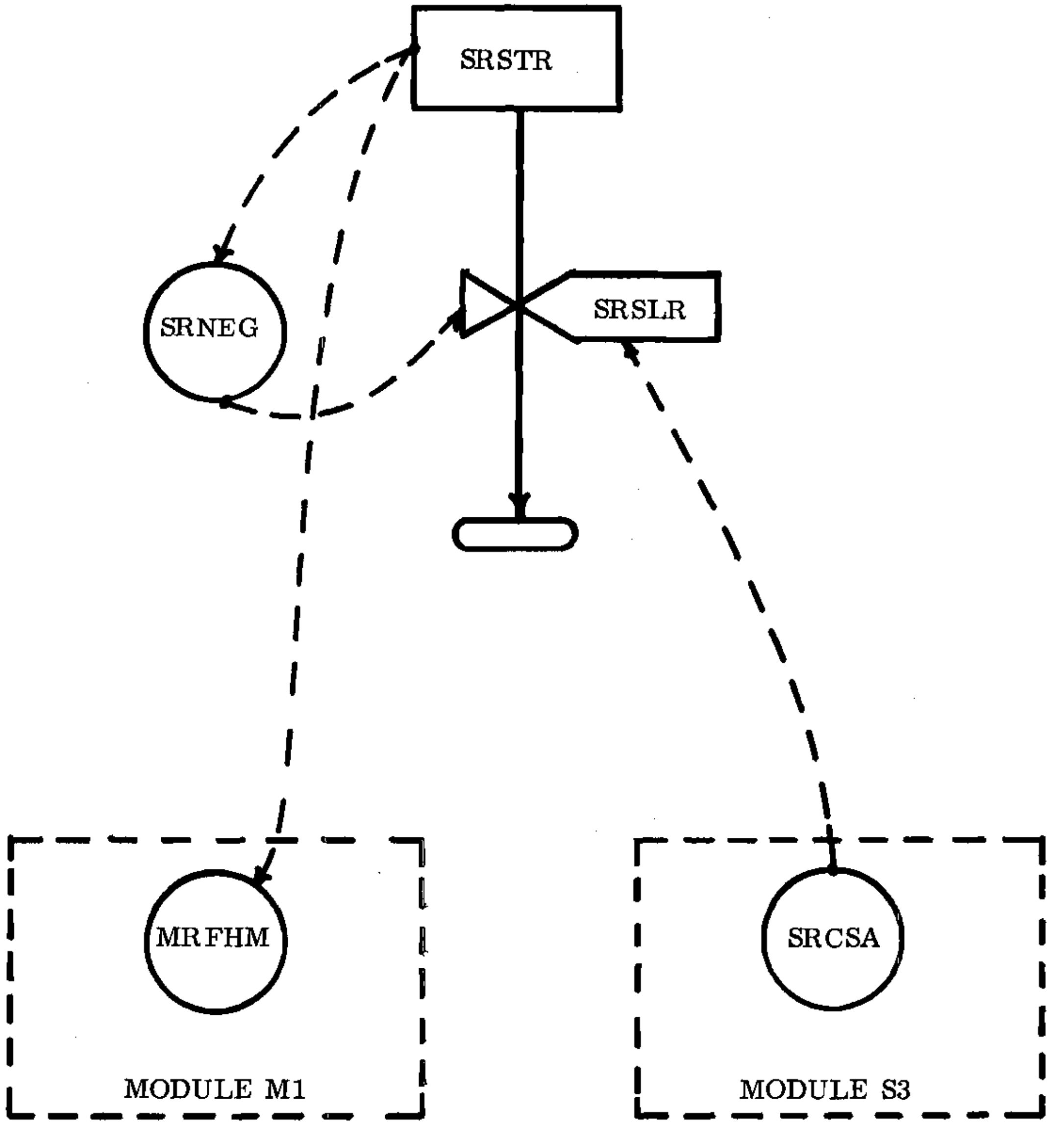

Figure 12. Red Strength Module S1

the purpose of the model. The strength of a platoon was taken as 40 men instead of 44 and the strength of the company was taken as 120 instead of 170. The antitank weapons and crews were eliminated from the weapons squads of the rifle platoons since armor was not a consideration. For the company, only the strength of the three rifle platoons was considered. The weapons platoon of the company was eliminated because indirect fire and armor were not a consideration. The company headquarters section, though necessary for command and control, would add nothing to the firepower of the unit. It too, therefore, was eliminated.

In Figure 12, SRSTR (red unit strength) reflects the strength of the unit at any time K. SRSLR (red strength loss rate) determines the rate at which casualties are assessed on the unit by decreasing the value of SRSTR. SRCSA (red casualties from small arms fire) is determined by the red casualty module, the development of which is discussed later. SRNEG (function to keep SRSTR positive or zero) is defined as an auxiliary. SRSLR assumes the smaller value between SRNEG and SRCSA so as to insure that SRSTR will not assume negative values. The equations for the red strength module S1 are as follows:

1L SRSTR.K = SRSTR.J+(DT)(0-SRSLR.JK)

20A SRNEG.K = SRSTR.K/DT

54R SRSLR.KL = MIN(SRNEG.K, SRCSA.K)

It may be recalled that in Figure 2 the symbol for MRFHM was shown as a dotted circle. The equation for this variable could not be determined until the strength section was developed. When the strength of the unit is zero, the speed of movement of the unit should realistically be zero. To incorporate this condition in the model, a switch function was added to modules M1 and M2. The equation which was added to the movement module M1 is shown below:

49A MRFHM. K = SWITCH(0, MRVAD. K, SRSTR. K)

The Casualty Modules S3 and S4

Since the strength modules are affected by the casualty modules, the latter were the next modules developed. The flow diagram for the red casualty module S3 is shown in Figure 13. The blue casualty module S4 is similar to the red module. Modules S3 and S4 are an expansion and improvement of the Abele (1) and Krol (18) models.

The Abele and Krol models simulated one and two stationary forces, respectively. Both models assumed that the forces were unprotected and assessed casualties only as a result of the amount of enemy fire. In the module being developed in this paper, casualty assessment also is based on the amount of enemy fire. However, the assessment is modified as a result of the effectiveness of the enemy fire, which is a function of the distance between the opposing forces. The assessment is further modified as a result of the degree of protection from enemy fire afforded to the unit.

In Figure 13, SRCTO (number of rounds to produce casualties among red troops in the open) and SRTDC (table to determine red casualties) represent, at any time K, the number of rounds to produce casualties as a function of the distance between the forces. This relationship is shown by the graph in Figure 14.

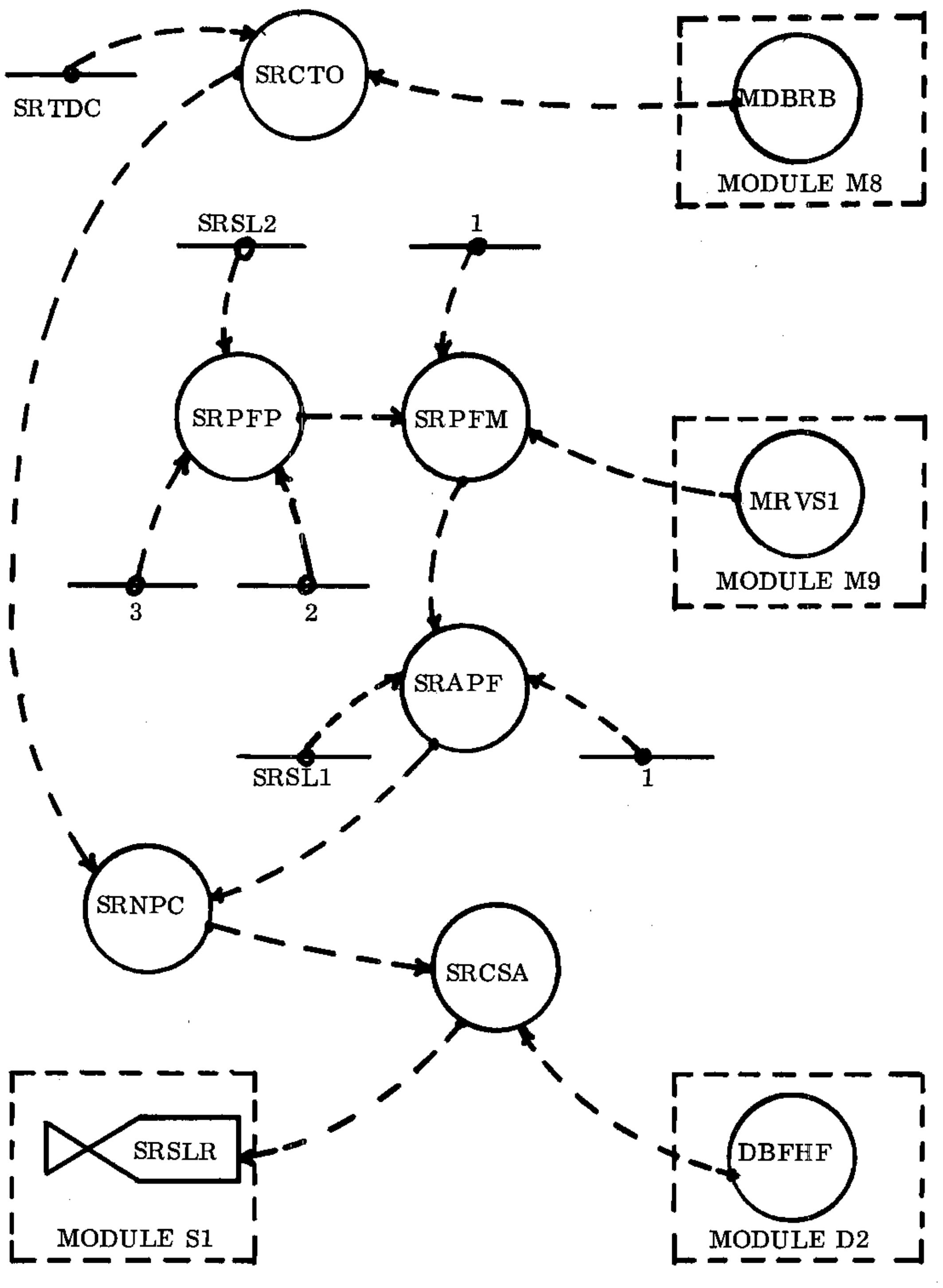

Figure 13. Red Small Arms Casualty Module S3

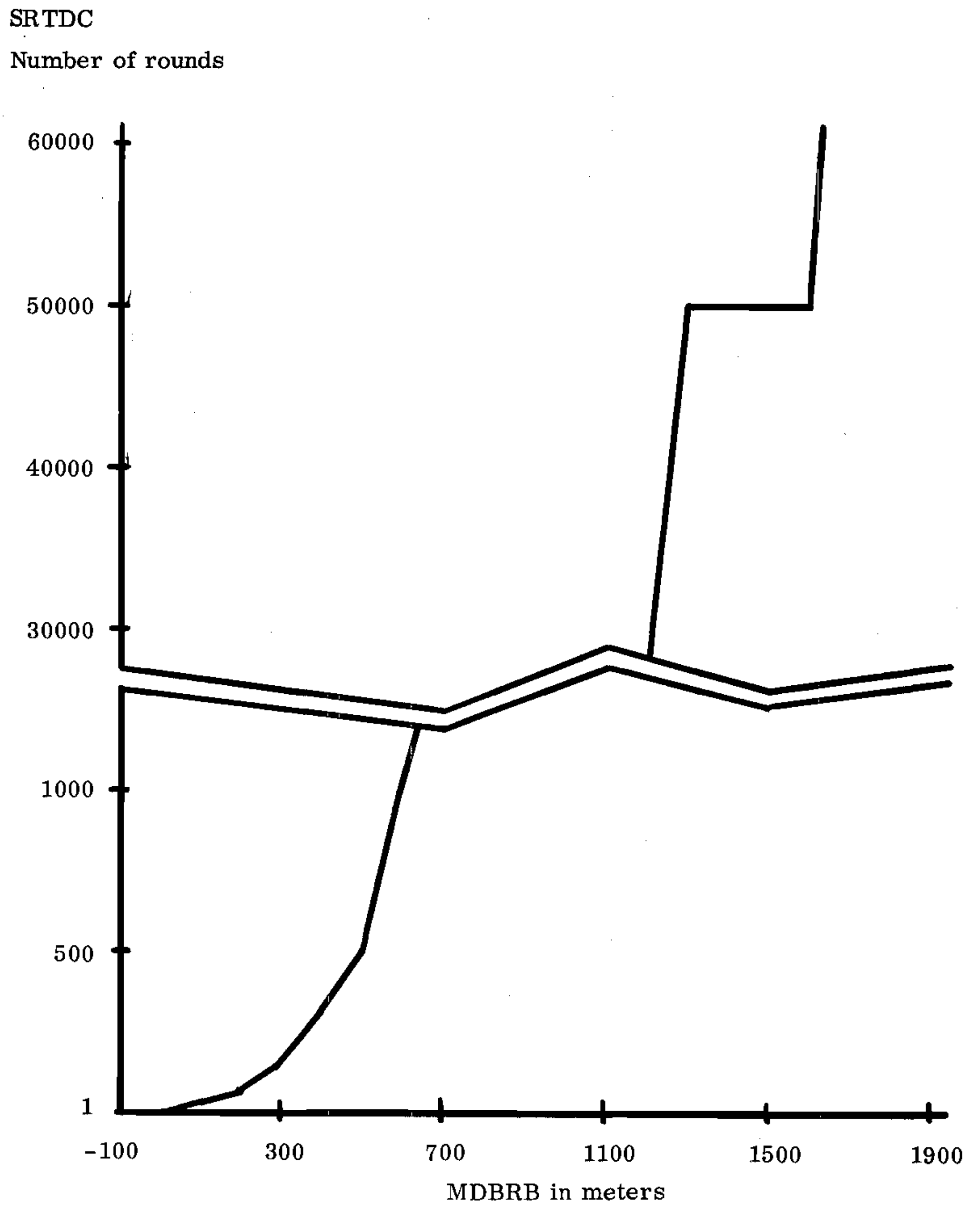

Figure 14. Number of Rounds to Produce a Casualty

No valid data are available on which to base the values for the graph. Therefore, values were chosen that were considered to be reasonable. When, and if, the statistical data became available, they could be inserted into the module. Even with valid statistical data, the values for the graph would vary. This variation, for example, could be caused by the terrain, the weather, and the degree of proficiency of the unit. It may be noted in Figure 14 that when the units are out of range of each other, the graph values are so large as to preclude any casualties being assessed. As the forces come within range of each other, it becomes increasingly feasible for a casualty to be assessed. The equations are shown below:

58A SRCTO.K = TABHL(SRTDC, MDBRB.K, -100, 2000, 100)

C SRTDC* = 1/1/20/50/150/300/500/1000/5000/10000/10000/10000/10000/
50000/50000/50000/50000/100000/100000/100000/100000/100000

Figure 14 represents the number of rounds to produce casualties among troops in the open. If a defender has prepared his defensive position by digging "foxholes" or building bunkers, the number of rounds it takes to produce a casualty will be increased by some factor. Three degrees of protection were considered for the forces. The lowest degree exists where the forces are unprotected and in the open. The second degree of protection is afforded by hasty positions. The highest degree of protection is achieved by a more deliberately prepared position such as a bunker. The two types of defensive positions are discussed in detail in FM 21-75 (8).

Based on studies made in World War II, tables were developed for the ammunition requirements to attack the different types of defensive positions (10). Using the aforementioned tables as a guide, it was assumed that it took twice as many rounds to produce a casualty among troops in hasty positions versus troops in the open and three times as many rounds for troops in bunkers versus troops in the open. The protection factor for the troops is entered into the module through SRSL1 and SRSL2 (troop protection selectors). The relationship of the value assigned to the constants and the type of position is shown below:

Constant	Value of Constant	Type of Position
SRSL1	0	In open
SRSL1	1	Hasty or deliberate position
SRSL2	0	Hasty position
SRSL2	1	Deliberate position

Based on the value of SRSL2, SRPFP (protection factor for protected troops) assumes a value of 2 or 3. This relationship and the corresponding equation are shown below:

Variable	Value of Variable	Value of SRSL2
SRPFP	2	0
SRPFP	3	1

49A SRPFP.K = SWITCH(2, 3, SRSL2)

When the situation simulated is a delaying action problem, the delaying force may be afforded some protection while occupying the delay positions but will have the same protection as troops in the open when moving afoot from one position to another. To account for this situation, the variable SRPFM (protection factor if red is moving) is introduced. The information as to whether the force is moving or not moving is available from MRVS1. The relationship and the corresponding equation are shown below:

Variable	Value of Variable	Value of MRVS1
SRPFM	SRPFP	$\neq$ 0
SRPFM	1	0

49A SRPFM.K = SWITCH(SRPFP.K, 1, MRVS1)

SRAPF (actual protection factor) reflects the value of the protection factor that is utilized in the simulation during any increment of delta time. If the force is in the open throughout the run, it does not matter what values SRPFM assumes. If the force is protected at any time, SRAPF assumes the value of SRPFM. This relationship and the corresponding equation are shown below:

Variable	Value of Variable	Value of SRSL1
SRAPF	1	0
SRAPF	SRPFM	1

49A SRAPF.K = SWITCH (1, SRPFM.K, SRSL1)

SRNPC (number of rounds to produce a casualty) is the product of SRCTO and SRAPF. If the force is in the open for the entire problem, SRNPC equals SRCTO. If the force is protected at any time during the run, SRNPC is some multiple of SRCTO during the period of time the force is protected. The equation is:

12A SRNPC. K = (SRCTO. K)(SRAPF. K)

SRCSA (casualties from small arms) is the ratio of SRNPC to DBFHF (number of rounds being fired by blue at red). The value for DBFHF is determined through the blue small arms fire module. SRCSA reflects the maximum number of casualties that may be assessed against the red force during any increment of delta time. This variable is one of the values that was used by SRSLR in Figure 12 to determine how many casualties can be assessed against the red force during any increment of delta time. This variable is one of the values that was used by SRSLR in Figure 12 to determine how many casualties can be assessed against the red force during any increment of delta time.

The Small Arms Fire Modules D1 and D2

The red small arms fire module D1 is shown in Figure 15. It may be noted that the first letter of all the variables in this module is D, in conformance with the convention established in Chapter I. The blue module D2 is essentially the same as D1. DRMFP (red maximum fire power) reflects the red force's available firepower at time K. The initial value of DRMFP is the sum of the product of DRNOR (number of rifles) times 1200 rounds per hour (sustained rate of fire) and the product of DRNMG (number of machine guns) times 6000 rounds per hour (sustained rate of fire). The sustained rates of fire used are for the M16 rifle and the M60 machine gun (23). The equations are as follows:

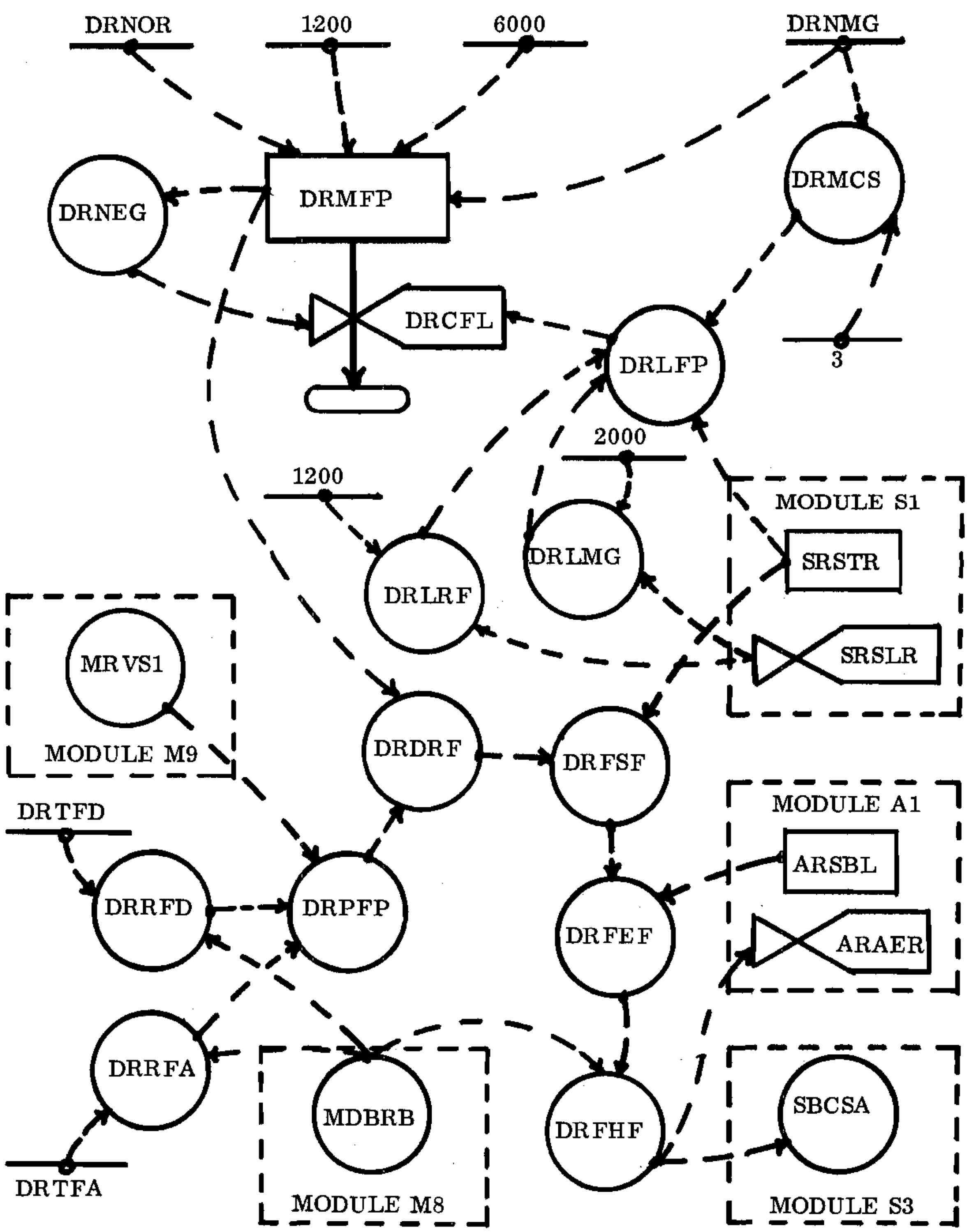

Figure 15. Red Small Arms Fire Module D1

1L DRMFP. K = DRMFP. J+(DT)(0-DRCFL. JK)

15N DRMFP = (DRNOR)(1200)+(DRNMG)(6000)

DRCFL (function to control loss of firepower) reflects the rate of loss of firepower. The value of DRCFL is determined by DRNEG (function to keep DRMFP positive or zero) and DRLFP (loss of firepower). DRCFL selects the smaller value between DRNEG and DRLFP. This formulation precludes DRMFP from assuming negative values. The equations are the following:

20A DRNEG. K = DRMFP. K/DT

54R DRCFL. KL = MIN(DRNEG. K, DRLFP. K)

DRLFP reflects the loss of firepower of the unit during any increment of delta time. This loss of firepower results from casualties suffered by the unit. Since there are two different weapons (rifles and machine guns) with two different rates of fire, the loss of a rifleman would affect the firepower differently than would the loss of a machine-gunner.

DRLRF (loss of rifle firepower) is a function of the casualty rate SRSLR during any period of delta time. For each rifle casualty, there will be a firepower loss of 1200 rounds per hour. DRLMG (loss of machine gun firepower) also is dependent on SRSLR. Since each machine gun crew consists of three men, then for each machine gun casualty that is not replaced there is a loss of one-third of the firepower of the machine guns, i.e., 2000 rounds per hour. The equations are the following:

12A DRLRF. K = (1200)(SRSLR. JK)

12A DRLMG. K = (2000)(SRSLR. JK)

Since the machine gun is a more valuable weapon than a rifle, the members of a unit are trained to replace the casualties among the machine gun crews at the expense of rifle firepower. If a machine-gunner becomes a casualty, a rifleman takes his place. Therefore the loss of firepower will be rifle firepower until only the machine gun crews are left. Thereafter, the assessment will reflect loss of machine gun firepower. The aggregate machine gun crew strength is determined by DRMCS:

12A DRMCS. K = (DRNMG)(3)

As long as SRSTR is greater than or equal to DRMCS, the firepower loss will equal DRLRF. When SRSTR is less than DRMCS, the loss will equal DRLMG. This determination is made through the equation for DRLFP:

51A DRLFP. K = CLIP(DRLRF. K, DRLMG. K, SRSTR. K, DRMCS. K)

DRMFP reflects the maximum firepower available. The commander may wish to apply his full firepower, part of it, or none of it. For instance, if he is out of range of the enemy, there is no logic in firing at all. He wishes, therefore, to use his available firepower to the best advantage to accomplish his mission. An attacker will desire to conserve his fire until he gets reasonably close to the enemy. As the distance shortens, he will desire to increase his fire. This feature is incorporated into the module through DRRFA (rate of fire for attacker) and DRTFA (table of fire for attacker). Figure 16 shows the graph for the attacker's application of firepower; the equations are shown below:

58A DRRFA. K = TABHL(DRTFA, MDBRB. K, 0, 1200, 100)

C DRTFA* = 1/1/.5/.1/.1/.05/.05/.05/.01/0/0/0

The defender will also wish to conserve his fire until the enemy is within range; but at a given range, the percentage of available firepower utilized by the defender will be greater than that utilized by the attacker. The application of firepower by the defender is shown in Figure 17, and the equations are shown below:

58A DRRFD. K = TABHL(DRTFD, MDBRB. K, 0, 1200, 100)

C DRTFD* = 1/1/.7/.5/.3/.3/.1/.1/.1/.05/.05/0/0/

To account for the circumstances inherent in a delaying action problem, DRPFP (portion of firepower utilized) is introduced into the module. In the event that red is acting as a covering force, while it is defending, DRPFP assumes the value of DRRFD; while the red force is moving to the next position, DRPFP assumes the value of DRRFA. The relationship and the corresponding equation are shown below:

Variable	Value of Variable	Value of MRVS1
DRPFP	DRRFD	0
DRPFP	DRRFA	$\neq 0$

49A DRPFP. K = SWITCH(DRRFD. K, DRRFA. K, MRVS1. K)

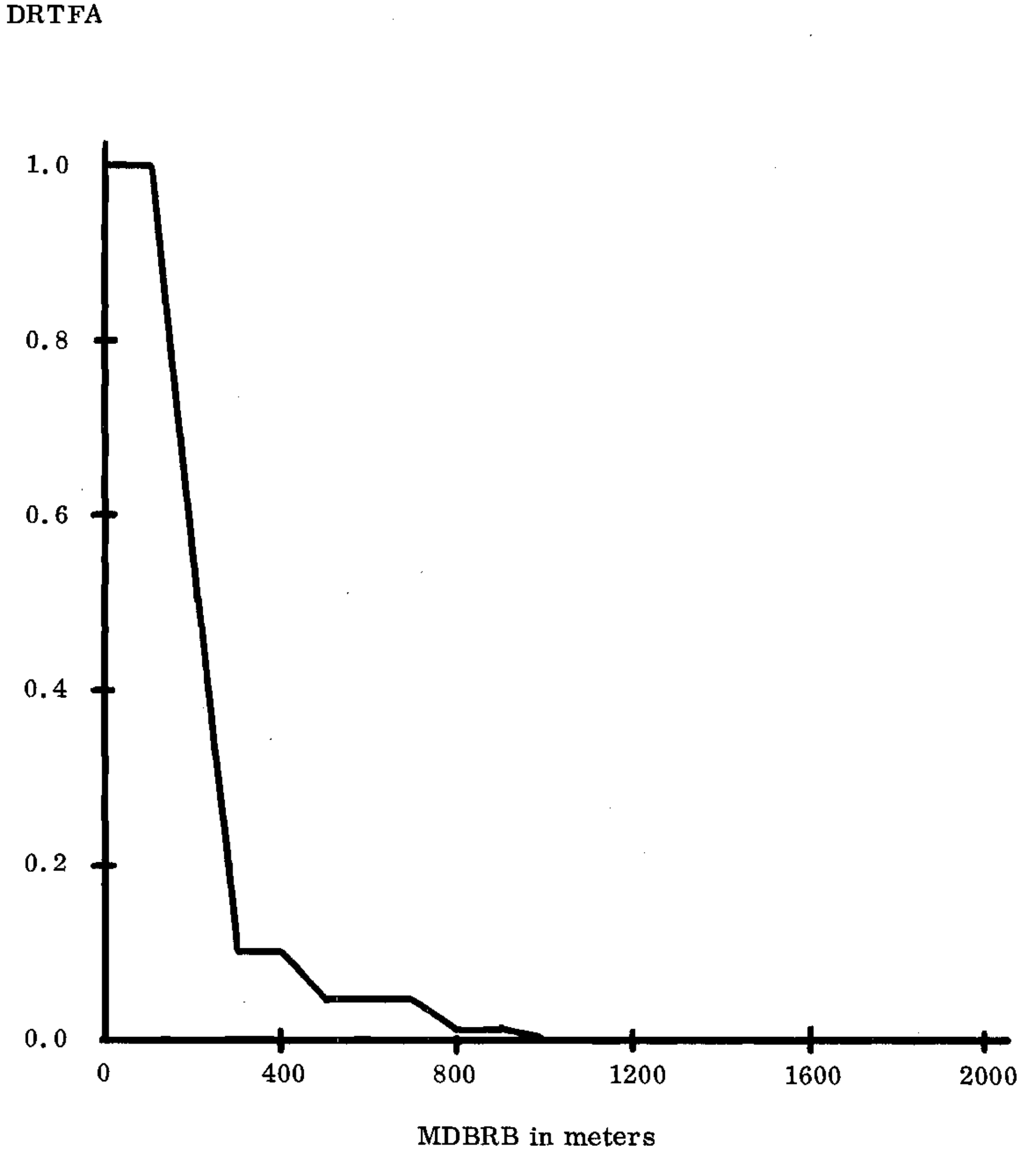

Figure 16. Application of Firepower (Attacker)

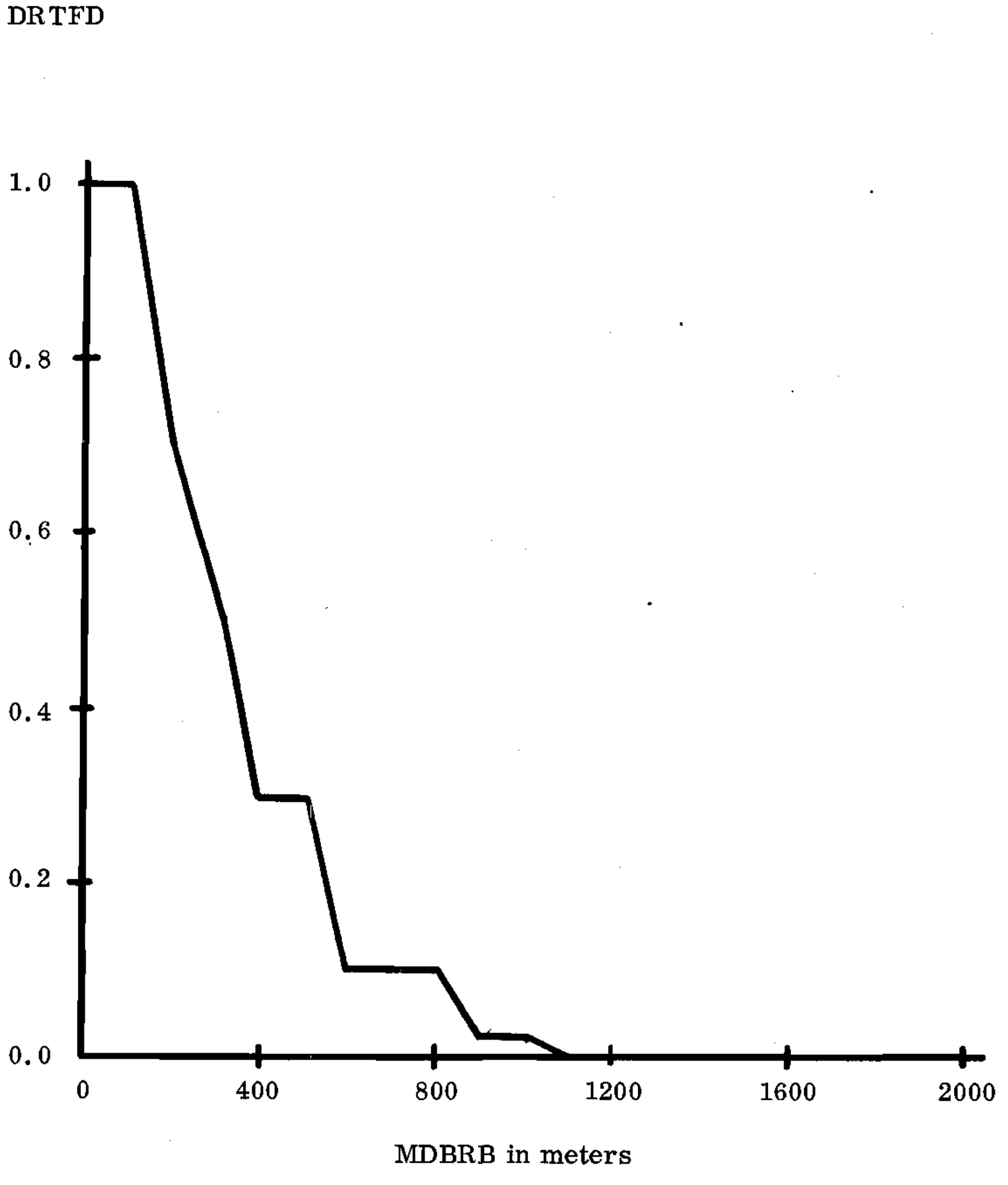

Figure 17. Application of Firepower (Defender)

DRDRF (desired rate of fire) is the product of DRMFP and DRPFP:

12A DRDRF. K = (DRMFP. K)(DRPFP. K)

Realistically, if the strength of the unit is zero, or the distance between the units is zero, or all of the ammunition has been expended, firing should cease. These are endogenous decisions and are entered into the module through DRFSF (function to halt fire when SRSTR = 0), DRFEF (function to end fire when ARSBL = 0) and DRFHF (function to halt fire when ADBRB = 0). The relationships and the corresponding switch function equations are shown below:

Variable	Value of Variable	Restriction	Value of Restriction
DRFSF	DRDRF	SRSTR	$\neq$ 0
DRFSF	0	SRSTR	0
DRFEF	DRFSF	ARSBL	$\neq$ 0
DRFEF	0	ARSBL	0
DRFHF	DRFEF	MDBRB	$\neq$ 0
DRFHF	0	MDBRB	0

49A DRFSF. K = SWITCH(0, DRDPF. K, SRSTR. K)

49A DRFEF. K = SWITCH(0, DRFSF. K, ARSBL. K)

49A DRFHF. K = SWITCH(0, DRFEF. K, MDBRB. K)

The Small Arms Ammunition Modules A1 and A2

Two small arms ammunition modules (A1 and A2) were added to the model.

Figure 18 shows the flow diagram for the red module A1. It may be noted that in conformance with the convention established in Chapter I that the first letter of all the variables in this module is the letter A.

A certain amount of all the various types of ammunition is required to be on hand within a unit at all times. This is called the prescribed load or the basic load. The basic load of the platoon is carried by the individual rifleman or the machine gun crew. A reserve for resupplying the platoon is carried on the company vehicles, and a reserve for resupplying the company is maintained by the battalion. The basic load for a unit is established by higher authorities and may vary. Normally, however, it will be very similar to tables found in FM 101-10 (10). These tables were the source for determining the basic load data used in modules A1 and A2. The basic load per rifle is equal to 100 rounds. The basic load per machine gun is equal to 1000 rounds. Since there are 34 riflemen and two machine guns in the platoon, ARIBL (initial basic load) was set at 5400 rounds for the red force. This was the initial condition value for ARSBL (small arms basic load). ARSBL is affected by two rates, ARARR (ammunition resupply rate) and ARAER (ammunition expenditure rate):

1L ARSBL.K = ARSBL.J + (DT)(ARARR.JK-ARAER.JK)

ARAER is influenced by two variables, ARNEG (function to keep ARSBL positive or zero) and DRFHF. ARAER assumes the value of DRFHF if the desired rate of fire does not exceed the number of rounds available. If DRFHF exceeds the ammunition available, ARAER assumes the lesser value ARNEG:

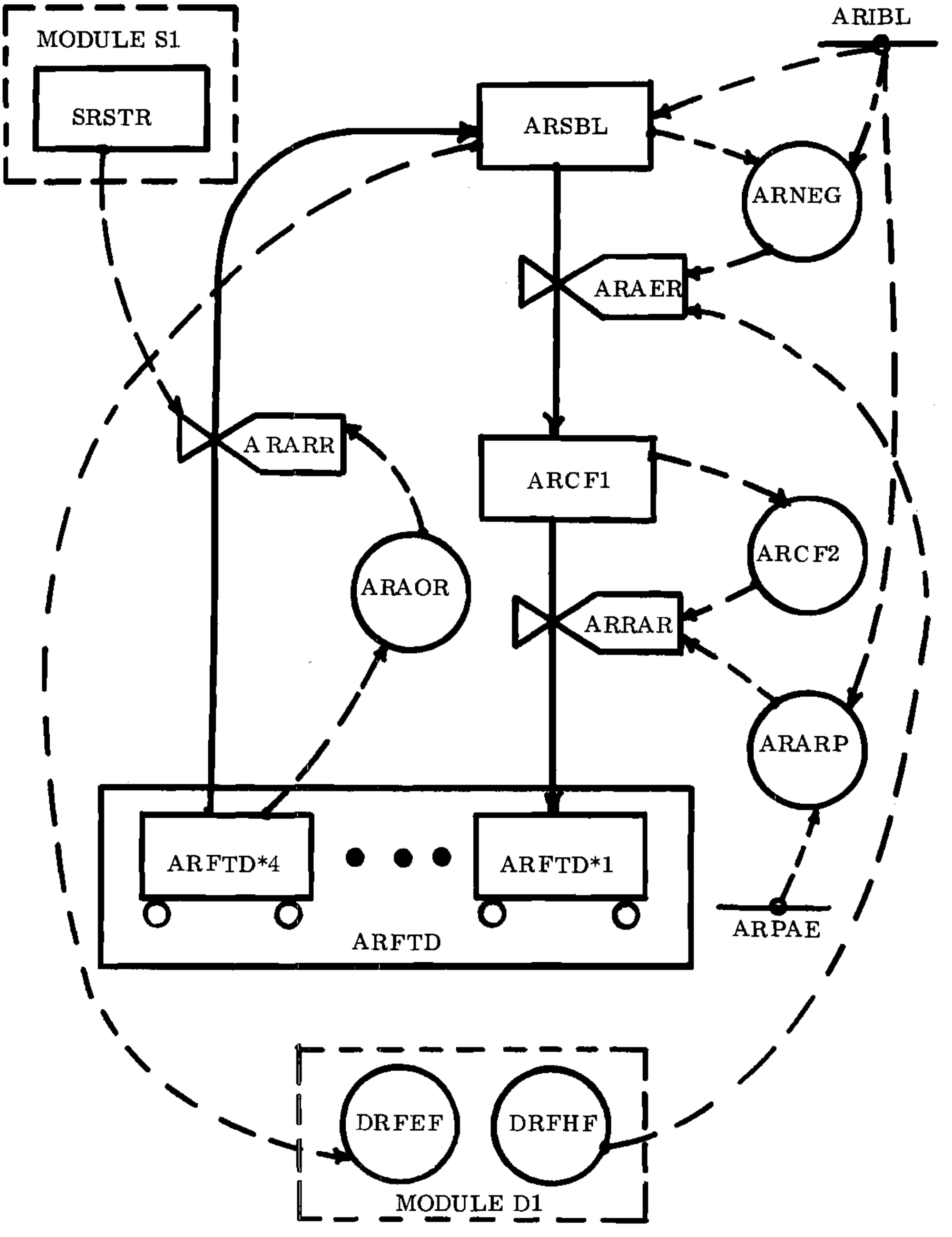

Figure 18. Red Small Arms Ammunition Module A1

54R ARAER. KL = MIN(ARNEG. K, DRFHF. K)

20A ARNEG. K = ARSBL. K/DT

In actual practice, small arms ammunition may be replenished during a lull in the fighting or when the level on hand is considered too low. The minimum percentage of the basic load that may be expended before resupply action is initiated is determined by standard operating procedures. The constant ARPAE represents the percentage of ammunition that may be expended before resupply action is initiated. ARARP (reorder point) is the product of ARIBL and ARPAE and represents the number of rounds that can be expended before resupply action is initiated:

12A ARARP. K = (ARPAE)(ARIBL)

As the ammunition is expended, it is accumulated in ARCF1 (control for reorder rate). When ARCF1 is equal to or greater than ARARP, ARRAR (ammunition request rate) is activated. The value of ARRAR, during the period of delta time it is active, is equal to ARCF2 (control for reorder rate). The relationship for ARRAR and the equations for ARRAR, ARCF2, and ARCF1 are shown below:

Variable	Value of Variable	Value of ARCF1
ARRAR	ARCF2	≥ ARARP
ARRAR	0	< ARARP

51R ARRAR. KL = CLIP(ARCF2. K, 0, ARCF1. K, ARARP. K)

20A ARCF2.K = ARCF1.K/DT

1L ARCF1.K = ARCF1.J + (DT)(ARAER.JK - ARRAR.JK)

When the resupply request is sent to the company, battalion, or other agency, the ammunition is sent to the unit. Resupply is not accomplished instantaneously, however. A time delay must be accounted for. This situation is simulated through a boxcar function and ARARR (ammunition resupply rate). The symbols used in Figure 18 to represent the boxcar function were substituted for the more universally accepted ones. They may be recognized by the reader as the standard military symbols for unit trains (7). It is felt that the new symbols more clearly convey the action of the function than do the symbols invented by Pugh (20).

ARFTD (function to represent delay between request and receipt) simulates a pipeline delay. The amount of ammunition that was accumulated in ARCF1 is placed in the first boxcar (ARFTD*1) when ARRAR is activated. The time shift is set equal to delta time or 7.5 minutes. Four boxcars are used. This arrangement represents a 30-minute delay for resupply. If a longer or shorter time is desired, the number of boxcars can be changed accordingly. When ARFTD*1 reaches ARFTD*4 (position 4), the value of the boxcar is added to ARSBL through the rate ARARR. To preclude ammunition being added to the basic load when the unit had been destroyed, ARAOR (amount of ammunition requested) is introduced into the module. The equations for the boxcar function, ARARR, and ARAOR are shown below:

37B ARFTD = BOXLIN (4, 0.125)

1L ARFTD*1.K = ARFTD*1.J + (DT)(0+ARRAR.JK)

20A ARAOR.K = ARFTD*4.K/DT

49R ARARR.KL = SWITCH(0,ARAOR.K, SBSTR.K)

C ARFTD.K = 0/0/0/0

All of the modules developed in this chapter were tested individually. Where a value was determined by or dependent on another module, the function was assigned a constant value for the validating run. Each of the modules performed satisfactorily.

CHAPTER VI

EXPERIMENTATION

General Considerations

Figure 19 shows the flow diagram for the final model. The variables and DYNAMO equations for the complete model may be found in the Appendix. The model consists of 206 DYNAMO equations and represents a nineteenth order system of difference equations. As will be shown later, millions of experiments could be performed with the model. For this reason, the validation of the model was accomplished by means of a series of diversified experiments.

Two sample printouts are shown in Figures 20 and 21. Figure 20 is one of the control runs for the machine gun variation problem. For this run, the speed of advance of the blue force is 1/2 kilometer per hour and the red force is protected by bunkers. The reader may note how the plot of the variables of interest reflects the effects of the interaction of movement, the exchange of fire, the ammunition constraints, and the casualty rates. Figure 21 is one of the runs associated with the variation of the time of withdrawal experiment. It may be noted that, as in Figure 20, the plot presents a very detailed picture of what occurred during the engagement. An analysis of the plot reveals the effects of and on the variables of interest resulting from their interaction.

The model as finally developed is a tremendous tool for experimentation. It will be shown that at least n^{26} experimental runs could be conducted with the

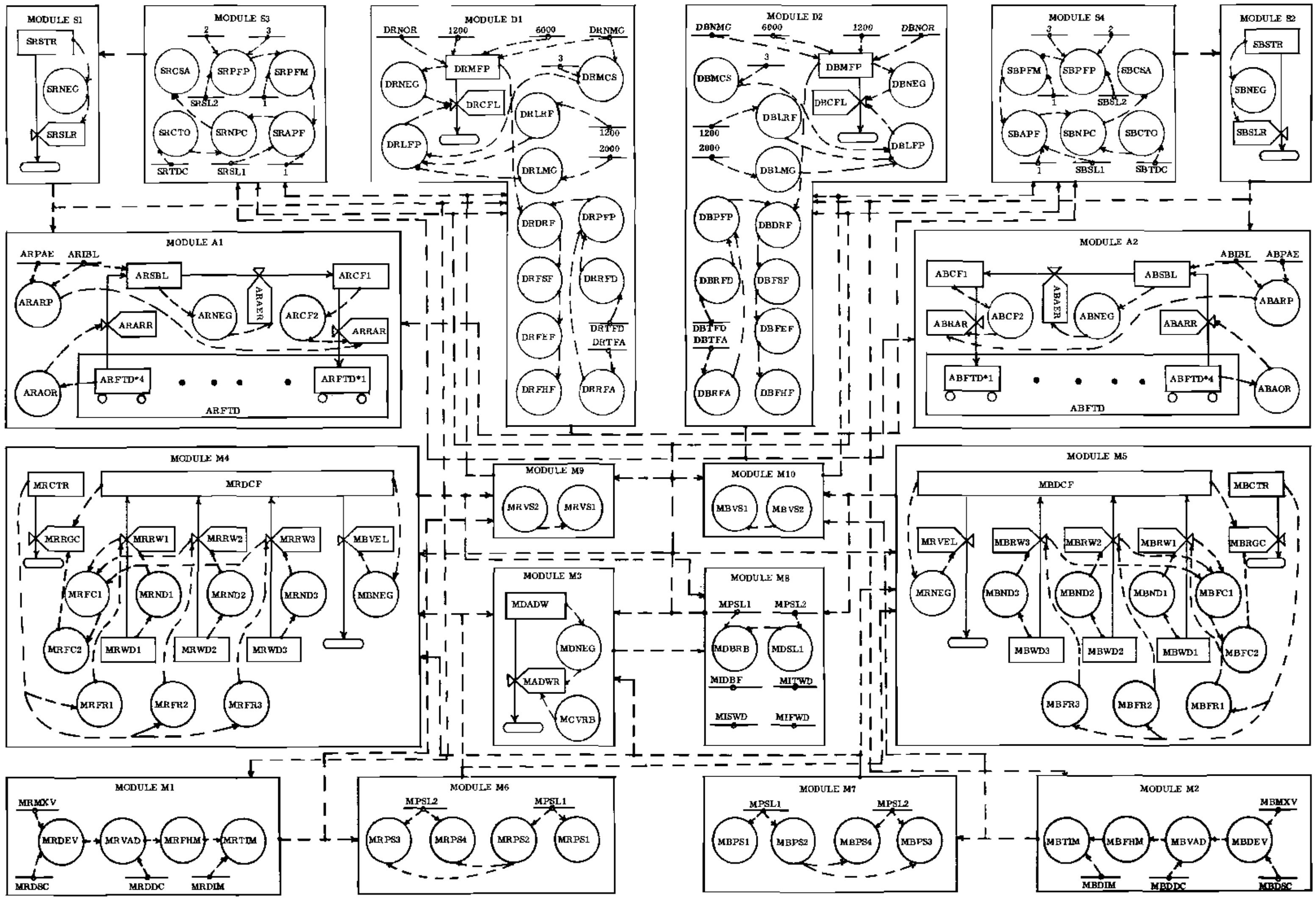

Figure 19. Final Model

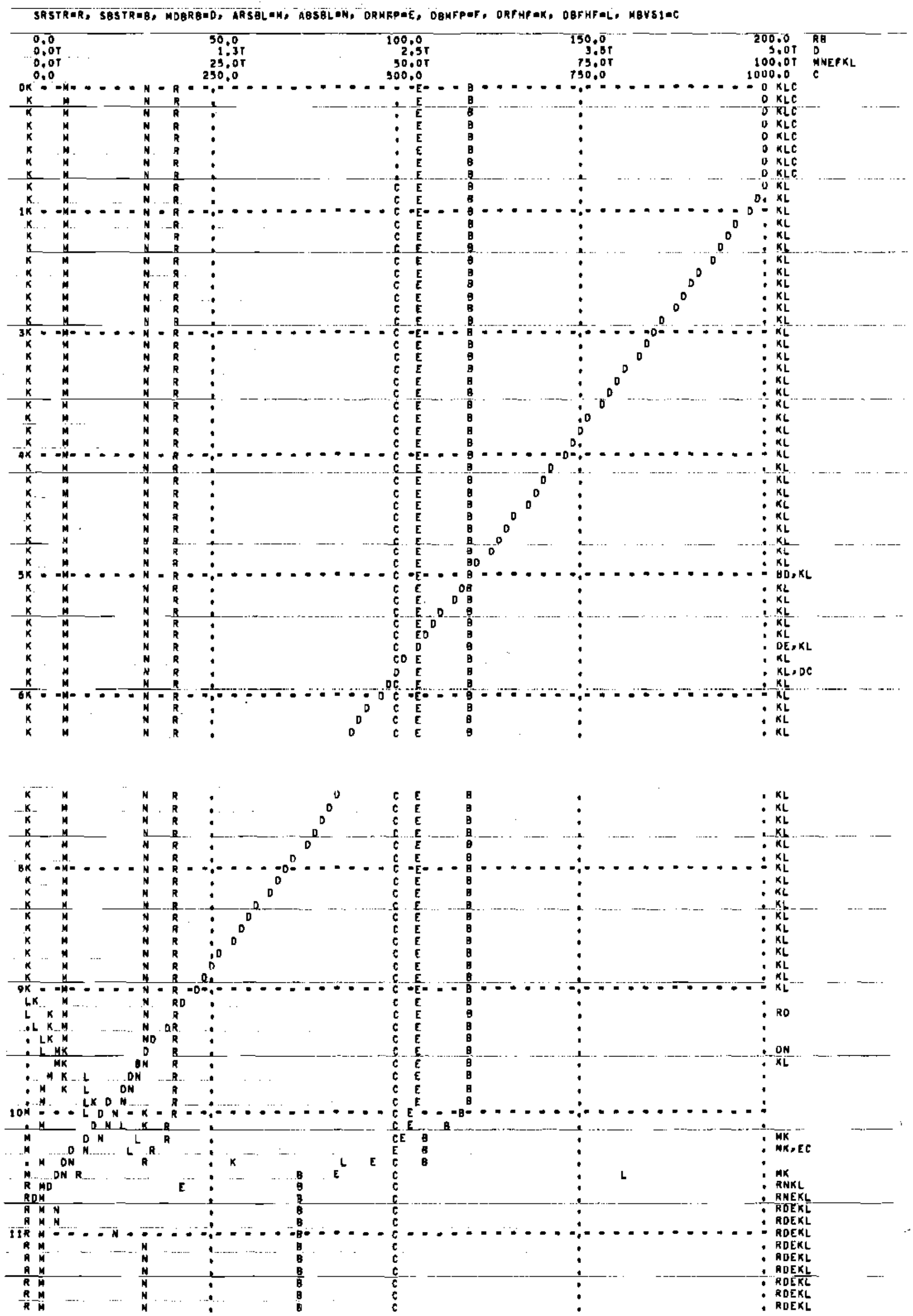

Figure 20. Final Model Validating Run (Defense)

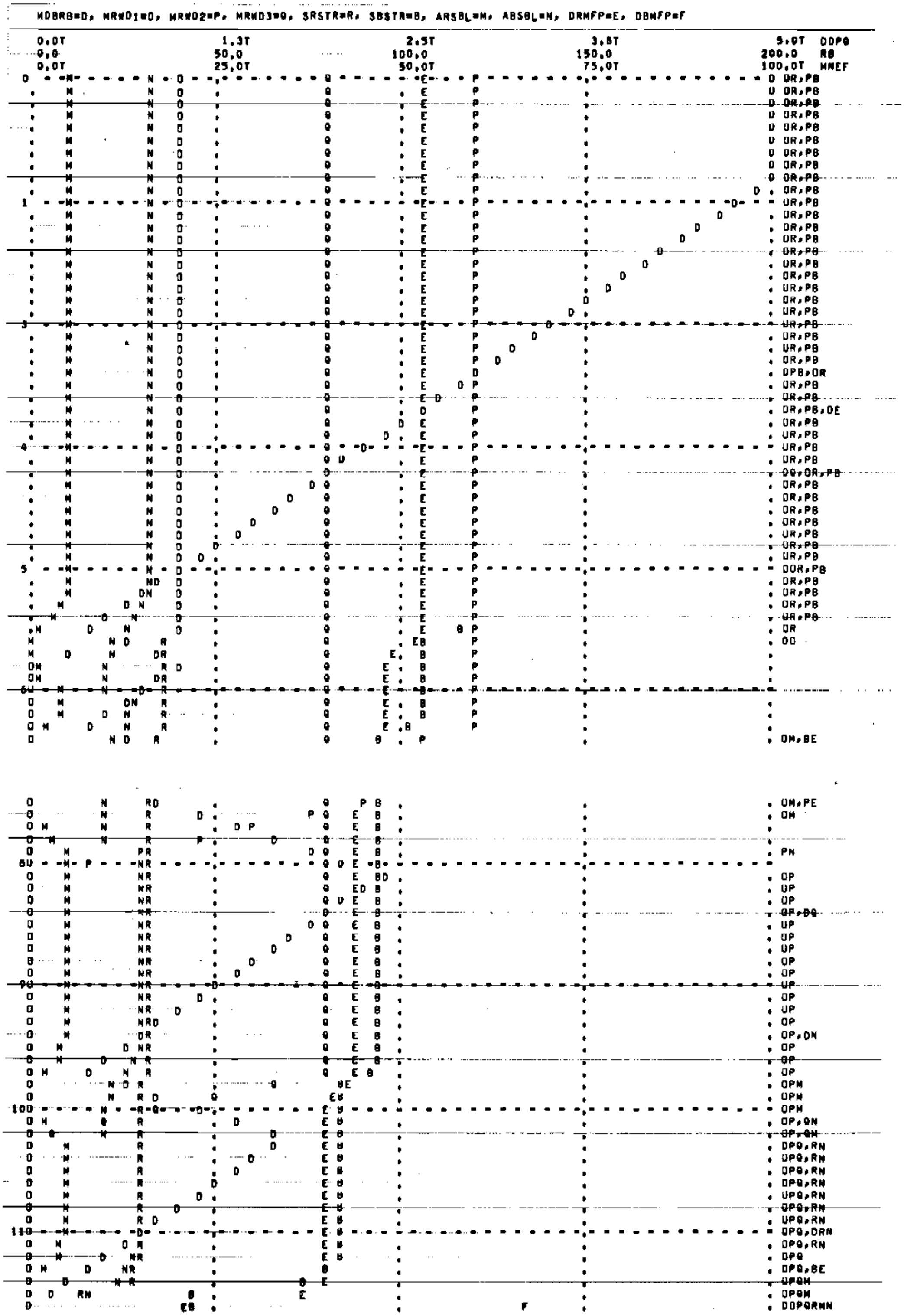

Figure 21. Final Model Validating Run (Delaying Action)

model, where 26 represents the number of variables and n the number of different values assigned to each variable.

The reader may note in the Appendix that there are 32 constants whose values may be changed. Six of the constants may be assigned only the values of one or zero. These latter constants are the ones which select the problem type and determine the protection factor of the forces. Even without considering the latter six constants, there are 26 remaining constants which could be varied. If one were to select only three different values for each of the 26 constants, this would result in 3^{26} or 2,541,865,828,329 computer runs. The number of runs could be further increased by changing some of the initial condition values.

Two conclusions are readily apparent with regard to the model. First, a great number of experiments could be made with the model; second, there is a need for shortcuts leading to the reduction of computer time. In the process of conducting experiments with the model, a procedure was developed to save computer printer time. For the initial experimental runs, only the plots were printed. Upon analysis of the plots, the critical runs were identified. The critical runs were then rerun and a printout obtained for the variables of interest for each increment of delta time. This approach allowed a more detailed analysis to be made of these runs. With the aid of this procedure, it was possible to evaluate expeditiously the results of many different configurations and to rapidly identify problem areas.

The overall objective of the experiments conducted with the model was to demonstrate the utility and validity of the model. Four representative problems of sufficient diversity were designed to satisfy this objective. The problems selected illustrate the following applications:

(1) Experimentation with the time of withdrawal for different values of MDBRB (distance between the red and blue force).

(2) Experimentation with the number of machine guns in the rifle platoon.

(3) Experimentation with the model from the standpoint of the theory of differential games.

(4) Experimentation for investigating a realistic Army research and development problem.

Experimentation with the Time of Withdrawal

When the delaying action modules M4 and M5 were developed in Chapter III, it was assumed that the commander would consider executing the withdrawal at the time when the distance between the opposing forces was 600 meters. The reader may recall that this figure was predicated on the forces being afoot.

This series of experimental runs was conducted to determine the effect on the outcome of the engagement if the red commander allowed the blue force to approach closer than 600 meters. While the red force was occupying the delay positions, it was afforded three different degrees of protection. The red force consisted of one platoon, and the blue force had three platoons. As a further enlargement of the experiment, for some runs, the red force was considered to be afoot, and for other runs, the red force was considered to have been augmented with infantry personnel carriers.

For the reader who is unfamiliar with the infantry personnel carrier, a brief explanation may be in order. The carrier is a tracked vehicle that might be attached to a unit conducting a delaying action. The carrier affords some protection to the troops from enemy fire. For the experiments involving attachment of carriers to the red force, the force was considered to be afforded the same degree of protection that would be afforded by hasty positions. Of course, this degree of protection applied only during the period of time the force was moving from one delay position to the next delay position.

A summary of the experimental runs is shown in Table 1. The result of the experiments verified that the red covering force could become too heavily engaged if the blue force were allowed to approach closer than 600 meters before the red commander decided to execute the withdrawal. In all runs, the blue force was the winner. The difference in the outcomes was reflected by the residual strength of the blue force and the amount of time the red force succeeded in delaying the blue force. When the decision to withdraw was made at 300 meters for the red force afoot, the red force never reached its second delay position regardless of the degree of protection it was afforded. When the decision to withdraw was made at 500 meters and with the red force afoot, the same result occurred as for the decision at 300 meters. With the red force afoot and the decision to withdraw made at 600 meters, a significant change was noted. The red force succeeded in delaying the blue force for a longer period of time, but there was no noteworthy change in the residual strength of the blue force. When the red force was unprotected, it was destroyed while enroute to the third position. When protected by hasty positions, the red force was destroyed while enroute to the fourth position. With the protection afforded by bunkers, the red force managed to reach the fourth position

Table 1. Variation of the Time of Withdrawal

Blue Force Residual Strength	Value of MDBRB	Protection of Red Force	Problem Time When Red Force Is Destroyed	Red Force Method of Movement
91	500	Bunkers	8.125	Carrier
99	500	Hasty positions	6.250	Carrier
104	500	None	4.375	Carrier
34	600	Bunker	12.000	Carrier
45	600	Hasty positions	11.625	Carrier
62	600	None	10.625	Carrier
66	300	Bunkers	4.625	Afoot
91	300	Hasty positions	4.500	Afoot
105	300	None	4.375	Afoot
99	500	Bunkers	4.625	Afoot
101	500	Hasty positions	4.500	Afoot
104	500	None	4.375	Afoot
105	600	Bunkers	10.125	Afoot
107	600	Hasty positions	8.250	Afoot
111	600	None	6.375	Afoot

before it was annihilated.

Analysis of the runs in which the red force was afoot indicated that the covering force appeared to be most vulnerable as it withdrew from one delay position to the next delay position. This vulnerability might be overcome or lessened if infantry personnel carriers were made available to the covering force. With the attachment of personnel carriers, the withdrawal could be executed more rapidly and some protection would be afforded from the blue force's fire.

A series of runs were made with carriers attached to the red force. The significant change in the results occurred when the decision to withdraw was made at 500 meters and at 600 meters. For the 500 meters decision, the red force never reached the fourth delay position. For the 600 meters decision, the red force managed to reach the fourth position regardless of the degree of protection it was afforded while occupying the delay positions. For these latter runs, the red force was destroyed at the fourth position; however, realistically, it would have succeeded in accomplishing its mission. The residual strength of the blue force for these runs was so low that it would have ceased to be an effective unit.

Four conclusions may be drawn from this experiment with regard to delaying action problems simulated with the model:

(1) The model is capable of simulating the interaction of the forces in a delaying action problem.

(2) For a flat, tabletop type of terrain, the decision to execute the withdrawal should be made by the covering force commander when the distance between the opposing forces is 600 meters.

(3) Infantry personnel carriers will increase the effectiveness of the covering force and influence the outcome of the engagement.

(4) The greater the degree of protection afforded the covering force while occupying the delay positions, the greater the effectiveness of the unit.

Experimentation with the Number of Machine Guns

This series of experimental runs was conducted to determine how the outcome of an attack/defense engagement would be affected if the organization of the unit was changed. The test unit was the red force The strength of the red platoon was kept at 40 men. The number of machine guns was increased from two to four and to six. For each machine gun added, there was a decrease of three men in the rifle strength. An appropriate adjustment was made in the basic load of the unit for each organizational change. For example, the net basic load change for each addition of two machine guns was plus 1400 rounds, because 600 rounds were lost by decreasing the rifle strength by six, and 2000 rounds were gained by adding the two machine gun crews and weapons. Runs made with the red force afforded three different degrees of protection for each of the three different numbers of machine guns. The speed of advance of the blue force was 1/2 kilometer per hour.

The outcome of all experimental runs resulted in the destruction of the red force. During the course of every run, the ammunition supply of the red force either became depleted or was exhausted. The greater the number of machine guns in use, the sooner the shortage occurred. It was of interest to note that the residual strength of the blue force was inversely proportional to the number of machine guns in the red force. Furthermore, when the red force had six machine guns and was afforded the protection of bunkers or hasty positions, the red force would have been the winner if it had not run out of ammunition.

The conclusions that may be drawn from the experiment are that in an attack/defense situation, when the attacker is advancing at a speed of 1/2 kilometer per hour:

(1) The model is capable of simulating the interaction of the forces in an attack/defense problem.

(2) Adding machine guns to the defensive unit at the expense of rifle strength (with an appropriate adjustment of the basic load) will not affect the outcome.

(3) Tripling the number of machine guns in the defensive unit (with a sufficient increase in the basic load to compensate for the increased fire of the machine guns and with protection of the defensive unit by bunkers or hasty positions) will result in the red force being the winner instead of the blue force.

Experimentation from the Standpoint of the Theory of Differential Games

An interesting experiment was suggested by Isaac's "Battle of the Bunker Hill Problem" (16). The general setting for the problem is two forces approaching each other. Each force is constrained in the amount of ammunition on hand. The problem is to find the best distribution of firepower.

Isaac's problem was modified to represent an attack/defense situation. For the experiments, the red strength was considered to be one platoon and the blue strength to be three platoons. The blue force was the attacker and the red force the defender. It was assumed that both forces had only the required basic load and could not be resupplied during the course of the problem. Two additional factors that were not considered in Isaac's statement of his problem were included in the experiments. First, as in the previous experiment, the red force was considered to possess three different degrees of protection; second, as it was recognized that the speed of advance of the attacker perhaps could influence the outcome, the speed of closure of the blue force was varied between 1/2 and 1 kilometer per hour.

The problem defined for the experiment had of necessity to be more limited than the problem postulated by Isaacs. For the experiment, it was assumed without proof that the best application of firepower by the defender was to gradually commit his firepower. As the distance between the forces decreased, the firepower was increased. The reader may recognize this premise as the situation represented by the graph in Figure 17. The problem then resolved itself into the question as to when the defender should begin to commit his firepower to the best advantage in view of his ammunition constraint.

The application of firepower as represented by Figure 17 was taken as the control. Runs were made with the three different degrees of protection for the red force and three different speeds of advance for the blue force. The control runs simulated the initial commitment of a small amount of firepower as the blue force first came within range. As the distance decreased, the intensity of the fire increased until eventually the defender was firing at his maximum sustained rate of fire.

In view of the ammunition constraint, it might be to the advantage of the defender to wait until the attacker was closer to the defender before committing any of its firepower. To test this hypothesis, 45 runs were made with the computer, using the shortcut procedure described earlier in the chapter. The runs simulated the red commander's decision to commit his firepower when the value of MDBRB (distance between the red and blue forces) was 400, 350, 300, 250, and 200 meters, respectively. A summary of the results of the critical test runs is shown in Table 2.

It was of interest to note that in the control runs and in the test runs, when the decision to commence firing was made with MDBRB greater than or equal to 400 meters, the red force ran out of ammunition. Furthermore, for each combination of speed of advance for the blue force and each different degree of protection for the red force, there appeared to be an optimum distance or set of distances when the red commander should decide to commit his firepower. One result at first glance appeared rather startling, but on further analysis the logic of the outcome was revealed. When the red force was unprotected, the best time for the red commander to decide to open fire was when MDBRB equaled 400 meters. This decision was optimal in spite of the fact that the red force ran out of ammunition. If the red commander waited any longer, the casualties suffered by the red force were so heavy that the ammunition constraint ceased to be a problem.

Three conclusions may be drawn from this experiment:

(1) When a defending force is restricted to the amount of ammunition on hand, there is an optimal distance or distances at which the commander should decide to begin firing.

(2) The model produced a realistic and feasible solution to the problem.

(3) The model demonstrated its applicability to the theory of differential games.

Table 2. Optimum Distance for Red Force to Decide to Open Fire

Optimum MDBRB	Red Force Protection	Blue Force Speed	Blue Force Residual Strength
350	Bunkers	0.50	0
400	Hasty positions	0.50	60
400	None	0.50	75
350	Bunkers	0.75	40
400	Hasty positions	0.75	45
400	None	0.75	55
400 to 200	Bunkers	1.00	0
400 to 200	Hasty positions	1.00	0
400	None	1.00	75

Experimentation with an Army Research and Development Problem

The Army is constantly attempting to find ways to improve the combat effectiveness of its units. One method is through the replacement of existing weapons with new and better weapons. The cost of this changeover must be weighed against the amount of increase in combat effectiveness. If the increase in effectiveness is not considerable, there is no justification for changing the weapons inventory.

The situation hypothesized for the problem was that a replacement for the present machine gun was being considered for development. Before development of cost figures, it was desired that the increase in combat effectiveness of the unit be examined when this proposed weapon was substituted for the present weapon. The measure of combat effectiveness was taken as the residual strength of the units. The state of the art was such that a machine gun could be developed to fire at a sustained rate of fire of either twice or three times the rate of the present weapon.

For the problem, the red force had an initial strength of 40 men. The blue force had an initial strength of 120 men. The red force was the test unit. For all the experimental runs, the blue force was the attacker. As was the case with the previous problems, the red force was considered to possess three different degrees of protection and because of the previously demonstrated effects of a variation of the speed of closure of the blue force, the speed of advance of the blue force was varied between 1/2 and 1 kilometer per hour. Nine runs were made for each of the three different rates of sustained fire for the machine gun.

In keeping with what is commonly done in an actual situation, it was assumed that extra ammunition had been stacked on position for the red machine guns. This additional ammunition resulted in 2000 rounds being on hand for each red machine gun instead of 1000 rounds. For the first series of 27 runs, a resupply capability was considered for both forces. Table 3 summarizes the result of these runs.

In all of the runs, the red force was either out of or short of ammunition at least once during the experimental run. As one would expect, this shortage occurred more often as the rate of fire of the machine gun was increased. One might further expect that, as a general rule, the residual strength of the blue force would decrease as the sustained rate of fire of the machine guns was increased. However, as may be noted, this was not always the case. A more detailed analysis of the results for these runs indicated that the shortage or lack of ammunition for the red force occurred at very critical points in time.

It is apparent from Table 3 that the effectiveness of the red unit may be increased by increasing the sustained rate of fire of the machine gun. However, as the rate of fire is increased, ammunition supply and resupply become more critical.

To explore the effects of a further increase in the ammunition on hand, a second series of 27 runs was made wherein the ammunition available for each red machine gun was increased by 50 percent. The results of these runs are summarized in Table 4. It will be noted that there is a decided change in the outcome. In fact,

Table 3. Variation of Sustained Rate of Fire With Double Ammunition Allowance for Red Machine Guns

Machine Gun Rate of Fire	Blue Force Residual Strength	Blue Force Speed	Red Force Degree of Protection*	Winner
Normal	0	0.50	3	None
Normal	10	0.50	2	Blue
Normal	80	0.50	1	Blue
Normal	80	0.75	3	Blue
Normal	80	0.75	2	Blue
Normal	80	0.75	1	Blue
Normal	60	1.00	3	Blue
Normal	60	1.00	2	Blue
Normal	60	1.00	1	Blue
Double	60	0.50	3	Blue
Double	60	0.50	2	Blue
Double	75	0.50	1	Blue
Double	35	0.75	3	Blue
Double	40	0.75	2	Blue
Double	40	0.75	1	Blue
Double	105	1.00	3	Blue
Double	105	1.00	2	Blue
Double	105	1.00	1	Blue
Triple	0	0.50	3	None
Triple	0	0.50	2	None
Triple	70	0.50	1	Blue
Triple	15	0.75	3	Blue
Triple	20	0.75	2	Blue
Triple	25	0.75	1	Blue
Triple	100	1.00	3	Blue
Triple	100	1.00	2	Blue
Triple	100	1.00	1	Blue

*Red Protection:
3 = Bunkers
2 = Hasty positions
1 = None

Table 4. Variation of Sustained Rate of Fire With Triple Ammunition Allowance for Red Machine Guns

Machine Gun Rate of Fire	Blue Force Residual Strength	Blue Force Speed	Red Force Degree of Protection*	Winner
Normal	0	0.50	3	None
Normal	0	0.50	2	None
Normal	60	0.50	1	Blue
Normal	40	0.75	3	Blue
Normal	40	0.75	2	Blue
Normal	45	0.75	1	Blue
Normal	0	1.00	3	None
Normal	0	1.00	2	None
Normal	50	1.00	1	None
Double	0	0.50	3	None
Double	0	0.50	2	None
Double	42	0.50	1	Blue
Double	0	0.75	3	None
Double	0	0.75	2	None
Double	20	0.75	1	Blue
Double	0	1.00	3	None
Double	0	1.00	2	None
Double	58	1.00	1	Blue
Triple	0	0.50	3	Red
Triple	0	0.50	2	Red
Triple	24	0.50	1	Blue
Triple	0	0.75	3	None
Triple	0	0.75	2	None
Triple	0	0.75	1	None
Triple	0	1.00	3	None
Triple	0	1.00	2	None
Triple	45	1.00	1	Blue

*Red Protection:
3 = Bunkers
2 = Hasty positions
1 = None

when the rate of sustained fire was tripled, the red force was the winner when it was afforded any degree of protection.

The result of the experiments would appear to indicate that there is a necessity for increasing the basic load of the weapon if the rate of sustained fire is increased. Since weight is the governing factor in the determination of the basic load, this increase of the basic load would entail development of a new lightweight cartridge to be used with the new weapon.

Two conclusion may be drawn from the experimental runs performed with the model:

(1) Increasing the rate of sustained fire for the machine gun with an appropriate increase in the basic load will enhance the effectiveness of a rifle platoon in the defense.

(2) The model is capable of furnishing answers to a realistic Army research and development problem.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

General Comments

The purpose of this study was to develop a DYNAMO simulation of two-sided combat in which the behavior of the opposing forces is affected by their movement, exchange of fire, ammunition constraints, and casualty rates. This objective was accomplished by means of the nineteenth order dynamic model consisting of 206 DYNAMO equations, which is shown in Figure 19. The model is admittedly a complex one, but when consideration is given to the complexity and number of situations which it is capable of simulating, its structure is extremely simple by comparison. It may be recalled that if one were to select only three different values for each of 26 constants, the result would be 2,541,865,828,329 computer runs.

The model, as finally developed, represents a closed system. Decisions that would normally be made during the course of a combat engagement are realistically made within the system during each experimental run. Conversely, decisions that would logically be made by the commander or be dictated by the initial situation at the start of the engagement are realistically entered into the model as exogenous decisions prior to the experimental run.

During the initial investigation and conceptual phase of the model's development, both an analysis and synthesis approach were considered. The author was at a loss to determine how the model could be developed using the analysis approach advocated by the philosophy of industrial dynamics. As a result, the synthesis approach was the method selected. The fact that the synthesis approach worked so successfully in this case is not proof that it will work in every case, but it most certainly shows that the synthesis approach has as much merit as the analysis approach.

As an outgrowth of the synthesis approach, the modular concept was conceived. The 18 modules that make up the final model are the building blocks that were used to form the sections which, in turn, were connected to form the final model. The modular approach lends itself effectively to the development of military tactical models and would probably be suitable to other types of model construction.

Conclusions

The conclusions of this study may be summarized as follows:

(1) The model developed in this study is original in the sense that it adds the effects of movement of troops to the effects of combat. Whereas the structure of the combat submodel was based on the work of earlier investigators, the movement section of the model represents the author's own contribution.

(2) The first phase of the development of the model resulted in a movement section consisting of 10 modules. The validating runs made with the section, as exemplified by Figures 10 and 11, proved that the movement section was capable of simulating the movement of two opposing forces under all tactical circumstances. The section can simulate movement of the opposing units in an attack/defense or a

retrograde movement (i.e., withdrawal, retirement, or delaying action).

(3) The second phase of the model's configuration resulted in the development of the combat submodel. The six modules simulating unit strength, casualty assessment, and the effects of small arms fire were based on the earlier work of Abele and Krol. The ammunition modules were developed by the author. These modules incorporate the boxcar function for the first time in any military simulation model. The use of the boxcar function expands the capability of the model to represent the logistical aspects of a military tactical situation. Logistical considerations may have a vital influence on the outcome of the engagement. This influence is exemplified by the effect of ammunition supply and resupply on the outcome of the experimental runs.

(4) The third phase of the model's construction interconnected flows of information between the 18 modules developed in phases one and two to produce the dynamic tactical model shown in Figure 19 and in the Appendix. The validity of the model in simulating troop movement, ammunition expenditure and resupply, casualty assessment, small arms fire, and the interaction of the foregoing upon each other was demonstrated by the experiments conducted in phase four.

(5) The fourth phase of this study was concerned with the investigation of four diverse experimental problems by means of over 250 experimental runs. The first two problems demonstrated the model's ability to furnish answers to realistic tactical problems. The third problem demonstrated the model's ability to be used in an applied differential game problem. The fourth problem demonstrated the use of the model to furnish answers to a realistic army research and development

problem. Upon analysis, the results of the experiments were found to be logical and consistent with military experience. The variety of problems showed the utility and flexibility of the model.

(6) It is felt that the model developed in this study shows that DYNAMO can be an excellent military tactical simulator. The flow diagrams used to illustrate the development of the modules are easily understood and readily explainable. The formulation of the equations is logical, and their use and application can be easily comprehended.

Recommendations

The following are recommended as areas for further study:

(1) The next expansion of the model developed in this thesis should be the addition of the terrain factors. The terrain section of the enlarged model should be capable of simulating the effects of cover, concealment, fields of fire, obstacles, observation, critical terrain, and avenues of approach. The addition of the terrain section to the present model would demonstrate conclusively the capability of DYNAMO as a military tactical simulator.

(2) The present model consists of 206 DYNAMO equations, whereas the computer can handle 2000 equations. This unused computer capacity suggests the possibility for an expansion of the model to represent a combat problem where more than two units may be simulated. For example, an expansion of the model could simulate all of the platoons of the red force and all of the companies of the battalion for the blue force.

(3) A number of experiments were conducted with the model which were not

reported in this study because of space limitations. The 100 runs made for the differential game problem were of particular interest. It is felt that without any additions to the model some very interesting research could be conducted with it to explore the theory of differential games.

APPENDIX

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NOTE                    A DYNAMO SIMULATION OF A
NOTE                  COMPLEX MILITARY TACTICAL MODEL
NOTE                       BY DONALD L MEYER

NOTE                  MODEL VARIABLES ARE AS FOLLOWS

NOTE  ABAER = BLUE SMALL ARMS AMMUNITION EXPENDITURE RATE
NOTE  ABAOR = AMOUNT OF AMMUNITION REQUESTED
NOTE  ABARR = BLUE SMALL ARMS AMMUNITION RESUPPLY RATE
NOTE  ABARP = BLUE SMALL ARMS AMMUNITION REORDER POINT
NOTE  ABCF1 = FUNCTION TO CONTROL BLUE SMALL ARMS AMMUNITION REORDER
NOTE          RATE
NOTE  ABCF2 = FUNCTION TO CONTROL BLUE SMALL ARMS AMMUNITION REORDER
NOTE          RATE
NOTE  ABFTD = FUNCTION TO REPRESENT THE TIME DELAY BETWEEN BLUE
NOTE          REQUESTING AND RECEIVING AMMUNITION RESUPPLY
NOTE  ABIBL = BLUE INITIAL SMALL ARMS AMMUNITION BASIC LOAD
NOTE  ABNEG = FUNCTION TO KEEP ABSBL≥0
NOTE  ABPAE = PERCENT OF SMALL ARMS AMMUNITION THAT WILL BE EXPENDED
NOTE          BEFORE RESUPPLY REQUESTED
NOTE  ABRAR = BLUE SMALL ARMS AMMUNITION REQUEST RATE
NOTE  ABSBL = BLUE SMALL ARMS BASIC LOAD
NOTE  ARAER = RED SMALL ARMS AMMUNITION EXPENDITURE RATE
NOTE  ARAOR = AMOUNT OF AMMUNITION REQUESTED
NOTE  ARARP = RED SMALL ARMS AMMUNITION REORDER POINT
NOTE  ARARR = RED SMALL ARMS AMMUNITION RESUPPLY RATE
NOTE  ARCF1 = FUNCTION TO CONTROL RED SMALL ARMS AMMUNITION REORDER
NOTE          RATE
NOTE  ARCF2 = FUNCTION TO CONTROL RED SMALL ARMS AMMUNITION REORDER
NOTE          RATE
NOTE  ARFTD = FUNCTION TO REPRESENT THE TIME DELAY BETWEEN RED
NOTE          REQUESTING AND RECEIVING AMMUNITION RESUPPLY
NOTE  ARIBL = RED INITIAL SMALL ARMS AMMUNITION BASIC LOAD
NOTE  ARNEG = FUNCTION TO KEEP ARSBL≥0
NOTE  ARPAE = PERCENT OF SMALL ARMS AMMUNITION THAT WILL BE EXPENDED
NOTE          BEFORE RESUPPLY REQUESTED
```

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NOTE  ARRAR = RED SMALL ARMS AMMUNITION REQUEST RATE
NOTE  ARSBL = RED SMALL ARMS BASIC LOAD
NOTE  DBCFL = BLUE FUNCTION TO CONTROL FIREPOWER LOSS
NOTE  DBDRF = BLUE DESIRED RATE OF FIRE
NOTE  DBFEF = FUNCTION TO HALT FIRE WHEN ABSBL=0
NOTE  DBFHF = BLUE FUNCTION TO HALT FIRE WHEN ADBRB=0
NOTE  DBFSF = BLUE FUNCTION TO HALT FIRE WHEN SBSTR=0
NOTE  DBLFP = BLUE LOSS OF FIREPOWER
NOTE  DBLMG = BLUE LOSS OF MACHINEGUN FIREPOWER
NOTE  DBLRF = BLUE LOSS OF RIFLE FIREPOWER
NOTE  DBMCS = BLUE MACHINE GUN CREW STRENGTH
NOTE  DBMFP = BLUE MAXIMUM FIREPOWER
NOTE  DBNEG = BLUE FUNCTION TO KEEP DBMFP≥0
NOTE  DBNMG = BLUE NUMBER OF MACHINE GUNS
NOTE  DBNOR = BLUE NUMBER OF RIFLES
NOTE  DBPFP = PORTION OF FIREPOWER UTILIZED
NOTE  DBRFA = BLUE RATE OF FIRE FOR THE ATTACK
NOTE  DBRFD = BLUE RATE OF FIRE FOR THE DEFENSE
NOTE  DBTFA = BLUE TABLE FOR ATTACKER FIREPOWER
NOTE  DBTFD = BLUE TABLE FOR DEFENDER FIREPOWER
NOTE  DRCFL = RED FUNCTION TO CONTROL FIREPOWER LOSS
NOTE  DRDRF = RED DESIRED RATE OF FIRE
NOTE  DRFEF = FUNCTION TO END FIRE WHEN ARSBL=0
NOTE  DRFHF = RED FUNCTION TO HALT FIRE WHEN MDBRB=0
NOTE  DRFSF = RED FUNCTION TO HALT FIRE WHEN SRSTR=0
NOTE  DRLFP = RED LOSS OF FIREPOWER
NOTE  DRLMG = RED LOSS OF MACHINEGUN FIREPOWER
NOTE  DRLRF = RED LOSS OF RIFLE FIREPOWER
NOTE  DRMCS = RED MACHINE GUN CREW STRENGTH
NOTE  DRMFP = RED MAXIMUM FIREPOWER
NOTE  DRNEG = RED FUNCTION TO KEEP DRMFP≥0
NOTE  DRNMG = RED NUMBER OF MACHINE GUNS
NOTE  DRNOR = RED NUMBER OF RIFLES
NOTE  DRPFP = PORTION OF FIREPOWER UTILIZED
NOTE  DRRFA = RED RATE OF FIRE FOR THE ATTACK
NOTE  DRRFD= RED RATE OF FIRE FOR THE DEFENSE
NOTE  DRTFA = RED TABLE FOR ATTACKER FIREPOWER
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NOTE  DRTFD = RED TABLE FOR DEFENDER FIREPOWER
NOTE  MADWR = ATTACK/DEFENSE WITHDRAWAL MODEL RATE OF CHANGE OF
NOTE          DISTANCE BETWEEN RED AND BLUE
NOTE  MBCTR = COUNTER WHEN BLUE ACTS AS COVERING FORCE
NOTE  MBDCF = DISTANCE BETWEEN RED AND BLUE WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBDDC = BLUE DESIRED DIRECTION COEFFICIENT
NOTE  MBDEV = BLUE DESIRED VELOCITY
NOTE  MBDIM = BLUE TIME DELAY IN INITIATING MOVEMENT
NOTE  MBDSC = BLUE DESIRED SPEED COEFFICIENT
NOTE  MBFC1 = FUNCTION TO CONTROL COUNTER WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBFC2 = FUNCTION TO CONTROL COUNTER WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBFHM = FUNCTION TO HALT MOVEMENT WHEN SBSTR=0
NOTE  MBFR1 = FUNCTION TO ACTIVATE MBRW1 WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBFR2 = FUNCTION TO ACTIVATE MBRW2 WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBFR3 = FUNCTION TO ACTIVATE MBRW3 WHEN BLUE ACTS AS COVERING
NOTE          FORCE
NOTE  MBMXV = BLUE MAXIMUM VELOCITY POSSIBLE
NOTE  MBND1 = FUNCTION TO KEEP MBWD1≥0 WHEN BLUE ACTS AS COVERING FORCE
NOTE  MBND2 = FUNCTION TO KEEP MBWD2≥0 WHEN BLUE ACTS AS COVERING FORCE
NOTE  MBND3 = FUNCTIO TO KEEP MBWD3≥0 WHEN BLUE ACTS AS COVERING FORCE
NOTE  MBNEG = FUNCTION TO KEEP MRDCF≥0 WHEN RED ACTS AS COVERING FORCE
NOTE  MBPS1 = BLUE PROBLEM SELECTOR 1 TO SELECT ATTACK/DEFENSE
NOTE          WITHDRAWAL MODULE
NOTE  MBPS2 = BLUE PROBLEM SELECTOR 2 TO SELECT COVERING FORCE MODULES
NOTE  MBPS3 = BLUE PROBLEM SELECTOR 3 TO SELECT RED COVERING FORCE
NOTE          MODULE
NOTE  MBPS4 = BLUE PROBLEM SELECTOR 4 TO SELECT BLUE COVERING FORCE
NOTE          MODULE
NOTE  MBRGC = RATE GOVERNING COUNTER WHEN BLUE ACTS AS COVERING FORCE
NOTE  MBRW1 = RATE OF WITHDRAWAL FROM FIRST POSITION WHEN BLUE ACTS
NOTE          AS COVERING FORCE
NOTE  MBRW2 = RATE OF WITHDRAWAL FROM SECOND POSITION WHEN BLUE ACTS
```

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NOTE               AS COVERING FORCE
NOTE      MBRW3 =  RATE OF WITHDRAWAL FROM THIRD POSITION WHEN BLUE ACTS
NOTE               AS COVERING FORCE
NOTE      MBTIM =  BLUE TIME OF INITIATION OF MOVEMENT CONTROL FACTOR
NOTE      MBVAD =  BLUE VELOCITY AND DIRECTION
NOTE      MBVEL =  BLUE VELOCITY WHEN RED ACTS AS COVERING FORCE
NOTE      MBVS1 =  FUNCTION TO DETERMINE BLUE VELOCITY
NOTE      MBVS2 =  FUNCTION TO DETERMINE BLUE VELOCITY
NOTE      MBWD1 =  FIRST WITHDRAWAL DISTANCE WHEN BLUE ACTS AS COVERING FORCE
NOTE      MBWD2 =  SECOND WITHDRAWAL DISTANCE WHEN BLUE ACTS AS COVERING FORCE
NOTE      MBWD3 =  THIRD WITHDRAWAL DISTANCE WHEN BLUE ACTS AS COVERING FORCE
NOTE      MCVRB =  COMBINED VELOCITY FOR RED AND BLUE ATTACK/DEFENSE
NOTE               WITHDRAWAL MODEL
NOTE      MDADW =  DISTANCE BETWEEN FORCES FOR ATTACK/DEFENSE
NOTE               AND WITHDRAWAL MODEL
NOTE      MDBRB =  DISTANCE BETWEEN RED AND BLUE FORCE
NOTE      MDROC =  DISTANCE RATE OF CHANGE BETWEEN RED AND BLUE
NOTE      MDNEG =  FUNCTION TO KEEP MDADW≥0 FOR ATTACK/DEFENSE WITHDRAWAL
NOTE               MODEL
NOTE      MDSL1 =  DISTANCE SELECTOR 1 TO SELECT RED COVERING FORCE
NOTE               MODULE(0) OR BLUE COVERING FORCE MODULE(1)
NOTE      MIDBF =  INITIAL DISTANCE BETWEEN FORCES
NOTE      MIFWD =  INITIAL VALUE FIRST WITHDRAWAL DISTANCE
NOTE      MISWD =  INITIAL VALUE SECOND WITHDRAWAL DISTANCE
NOTE      MITWD= INITIAL VALUE THIRD WITHDRAWAL DISTANCE
NOTE      MPSL1 =  PROBLEM SELECTOR 1 ATTACK/DEFENSE WITHDRAWAL(0) COVERING
NOTE               FORCES(1)
NOTE      MPSL2 =  PROBLEM SELECTOR 2 RED COVERING FORCE(0) BLUE COVERING
NOTE               FORCES(1)
NOTE      MRCTR =  COUNTER WHEN RED ACTS AS COVERING FORCE
NOTE      MRDCF =  DISTANCE BETWEEN RED AND BLUE WHEN RED ACTS AS COVERING
NOTE               FORCE
NOTE      MRDDC =  RED DESIRED DIRECTION COEFFICIENT
NOTE      MRDEV =  RED DESIRED VELOCITY
NOTE      MRDIM =  RED TIME DELAY IN INITIATING MOVEMENT
NOTE      MRDSC =  RED DESIRED SPEED COEFFICIENT
NOTE      MRFC1 =  FUNCTION TO CONTROL COUNTER WHEN RED ACTS AS COVERING
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```
NOTE          FORCE
NOTE   MRFC2 = FUNCTION TO CONTROL COUNTER WHEN RED ACTS AS COVERING
NOTE          FORCE
NOTE   MRFHM = FUNCTION TO HALT MOVEMENT WHEN SRSTR=0
NOTE   MRFR1 = FUNCTION TO ACTIVATE MRRW1 WHEN RED ACTS AS COVERING
NOTE          FORCE
NOTE   MRFR2 = FUNCTION TO ACTIVATE MRRW2 WHEN RED ACTS AS COVERING
NOTE          FORCE
NOTE   MRFR3 = FUNCTION TO ACTIVATE MRRW3 WHEN RED ACTS AS COVERING
NOTE          FORCE
NOTE   MRMXV = RED MAXIMUM VELOCITY POSSIBLE
NOTE   MRND1 = FUNCTION TO KEEP MRWD1≥0 WHEN RED ACTS AS COVERING FORCE
NOTE   MRND2 = FUNCTION TO KEEP MRWD2≥0 WHEN RED ACTS AS COVERING FORCE
NOTE   MRND3 = FUNCTION TO KEEP MRWD3≥0 WHEN RED ACTS AS COVERING FORCE
NOTE   MRNEG = FUNCTION TO KEEP MBDCF≥0 WHEN BLUE ACTS COVERING FORCE
NOTE   MRPS1 = RED PROBLEM SELECTCTOR 1 TO SELECT ATTACK/DEFENSE
NOTE          WITHDRAWAL MODULE
NOTE   MRPS2 = RED PROBLEM SELECTOR 2 TO SELECT COVERING FORCE MODULES
NOTE   MRPS3 = RED PROBLEM SELECTOR 3 TO SELECT RED COVERING FORCE
NOTE          MODULE
NOTE   MRPS4 = RED PROBLEM SELECTOR 4 TO SELECT BLUE COVERING FORCE
NOTE          MODULE
NOTE   MRRGC = RATE GOVERNING COUNTER WHEN RED ACTS AS COVERING FORCE
NOTE   MRRW1 = RATE OF WITHDRAWAL TO FIRST POSITION WHEN RED ACTS AS
NOTE          COVERING FORCE
NOTE   MRRW2 = RATE OF WITHDRAWAL TO SECOND POSITION WHEN RED ACTS AS
NOTE          COVERING FORCE
NOTE   MRRW3 = RATE OF WITHDRAWAL TO THIRD POSITION WHEN RED ACTS AS
NOTE          COVERING FORCE
NOTE   MRTIM = RED TIME OF INITIATION OF MOVEMENT CONTROL FACTOR
NOTE   MRVAD = RED VELOCITY AND DIRECTION
NOTE   MRVEL = RED VELOCITY WHEN BLUE ACTS AS COVERING FORCE
NOTE   MRVS1 = FUNCTION TO DETERMINE RED VELOCITY
NOTE   MRVS2 = FUNCTION TO DETERMINE RED VELOCITY
NOTE   MRWD1 = FIRST WITHDRAWAL DISTANCE WHEN RED ACTS AS COVERING FORCE
NOTE   MRWD2 = SECOND WITHDRAWAL DISTANCE WHEN RED ACTS AS COVERING
NOTE          FORCE
```

```
NOTE  MRWD3 = THIRD WITHDRAWAL DISTANCE WHEN RED ACTS AS COVERING FORCE
NOTE  SBAPF = ACTUAL PROTECTION FACTOR FOR BLUE FORCE TROOPS
NOTE  SBCSA = BLUE CASUALTIES FROM SMALL ARMS
NOTE  SBCTO = NUMBER OF ROUNDS TO PRODUCE CASUALTIES AMONG BLUE
NOTE          TROOPS IN THE OPEN
NOTE  SBNEG = FUNCTION TO KEEP SBSTR≥0
NOTE  SBNPC = NUMBER OF ROUNDS TO PRODUCE CASUALTIES AMONG BLUE FORCE
NOTE  SBSL1 = BLUE TROOP PROTECTION SELECTOR NO PROTECTION (0) WITH
NOTE          PROTECTION (1)
NOTE  SBSL2 = BLUE TROOP PROTECTION SELECTOR MEDIUM PROTECTION(0)
NOTE          EXCELLENT PROTECTION (1)
NOTE  SBPFM = PROTECTION FACTOR IF BLUE FORCE IS MOVING
NOTE  SBPFP = PROTECTION FACTOR FOR PROTECTED TROOPS
NOTE  SBSLR = BLUE STRENGTH LOSS RATE
NOTE  SBSTR = BLUE UNIT STRENGTH
NOTE  SBTDC = TABLE TO DETERMINE BLUE CASUALTIES
NOTE  SRAPF = ACTUAL PROTECTION FACTOR FOR RED FORCE TROOPS
NOTE  SRCSA = RED CASUALTIES FROM SMALL ARMS
NOTE  SRCTO = NUMBER OF ROUNDS TO PRODUCE CASUALTIES AMONG RED
NOTE          TROOPS IN THE OPEN
NOTE  SRNEG = FUNCTION TO KEEP SRSTR≥0
NOTE  SRNPC = NUMBER OF ROUNDS TO PRODUCE CASUALTIES AMONG RED FORCE
NOTE  SRSL1 = RED TROOP PROTECTION SELECTOR NO PROTECTION (0) WITH
NOTE          PROTECTION (1)
NOTE  SRSL2 = RED TROOP PROTECTION SELECTOR MEDIUM PROTECTION (0)
NOTE          EXCELLENT PROTECTION (1)
NOTE  SRPFM = PROTECTION FACTOR IF RED FORCE IS MOVING
NOTE  SRPFP = PROTECTION FACTOR FOR PROTECTED TROOPS
NOTE  SRSLR = RED STRENGTH LOSS RATE
NOTE  SRSTR = RED UNIT STRENGTH
NOTE  SRTDC = TABLE TO DETERMINE RED CASUALTIES
```

```
NOTE                    MODEL EQUATIONS

NOTE                    MOVEMENT SECTION

NOTE                    RED MOVEMENT MODULE
20N     MRMXV=125/DT
12A     MRDEV.K=(MRDSC)(MRMXV)
12A     MRVAD.K=(MRDDC)(MRDEV.K)
45A     MRTIM.K=STEP(MRFHM.K,MRDIM)
49A     MRFHM.K=SWITCH(0,MRVAD.K,SRSTR.K)
C       MRDIM=1
C       MRDDC=-1
C       MRDSC=0

NOTE                    BLUE MOVEMENT MODULE
45A     MBTIM.K=STEP(MBFHM.K,MBDIM)
49A     MBFHM.K=SWITCH(0,MBVAD.K,SBSTR.K)
12A     MBVAD.K=(MBDDC)(MBDEV.K)
20N     MBMXV=125/DT
12A     MBDEV.K=(MBMXV)(MBDSC)
C       MBDIM=1
C       MBDDC=1
C       MBDSC=0.5

NOTE                    ATTACK/DEFENSE/WITHDRAWAL MODULE
1L      MDADW.K=MDADW.J+(DT)(0-MADWR.JK)
6N      MDADW=MIDBF
20A     MDNEG.K=MDADW/DT
54R     MADWR.KL=MIN(MDNEG.K,MCVRB.K)
6N      MADWR=0
7A      MCVRB.K=(MRPS1.K+MBPS1.K)

NOTE                    RED DELAYING ACTION MODULE
54R     MRRW3.KL=MIN(MRND3.K,MRFR3.K)
6N      MRRW3=0
54R     MRRW2.KL=MIN(MRND2.K,MRFR2.K)
6N      MRRW2=0
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```
54R    MRRW1.KL=MIN(MRND1.K,MRFR1.K)
6N     MRRW1=0
8A     MRFC1.K=MRRW1.JK+MRRW2.JK+MRRW3.JK
49A    MRFC2.K=SWITCH(8,0,MRFC1.K)
51R    MRRGC.KL=CLIP(0,MRFC2.K,MRDCF.K,600)
6N     MRRGC=0
1L     MRCTR.K=MRCTR.J+(DT)(MRRGC.JK+0)
6N     MRCTR=-1
20A    MBNEG.K=MRDCF.K/DT
54R    MBVEL.KL=MIN(MBNEG.K,MBPS3.K)
6N     MBVEL=0
52L    MRDCF.K=MRDCF.J+(DT)(-MBVEL.JK+MRRW1.JK+MRRW2.JK+MRRW3.JK)
6N     MRDCF=MIDBF
1L     MRWD1.K=MRWD1.J+(DT)(0-MRRW1.JK)
49N    MRWD1=SWITCH(MIFWD,0,MPSL2)
1L     MRWD2.K=MRWD2.J+(DT)(0-MRRW2.JK)
49N    MRWD2=SWITCH(MISWD,0,MPSL2)
1L     MRWD3.K=MRWD3.J+(DT)(0-MRRW3.JK)
49N    MRWD3=SWITCH(MITWD,0,MPSL2)
51A    MRFR1.K=CLIP(-MRPS3.K,0,MRCTR.K,0)
51A    MRFR2.K=CLIP(-MRPS3.K,0,MRCTR.K,2)
51A    MRFR3.K=CLIP(-MRPS3.K,0,MRCTR.K,4)
20A    MRND1.K=MRWD1.K/DT
20A    MRND2.K=MRWD2.K/DT
20A    MRND3.K=MRWD3.K/DT

NOTE                   BLUE DELAYING ACTION MODULE
54R    MBRW3.KL=MIN(MBND3.K,MBFR3.K)
6N     MBRW3=0
54R    MBRW2.KL=MIN(MBND2.K,MBFR2.K)
6N     MBRW2=0
54R    MBRW1.KL=MIN(MBND1.K,MBFR1.K)
6N     MBRW1=0
8A     MBFC1.K=MBRW1.JK+MBRW2.JK+MBRW3.JK
49A    MBFC2.K=SWITCH(8,0,MBFC1.K)
51R    MBRGC.KL=CLIP(0,MBFC2.K,MBDCF.K,600)
6N     MBRGC=0
```

```
1L    MBCTR.K=MBCTR.J+(DT)(MBRGC.JK+0)
6N    MBCTR=-1
20A   MRNEG.K=MBDCF.K/DT
54R   MRVEL.KL=MIN(MRNEG.K,MRPS4.K)
6N    MRVEL=0
52L   MBDCF.K=MBDCF.J+(DT)(-MRVEL.JK+MBRW1.JK+MBRW2.JK+MBRW3.JK)
6N    MBDCF=MIDBF
1L    MBWD1.K=MBWD1.J+(DT)(0-MBRW1.JK)
49N   MBWD1=SWITCH(0,MIFWD,MPSL2)
1L    MBWD2.K=MBWD2.J+(DT)(0-MBRW2.JK)
49N   MBWD2=SWITCH(0,MISWD,MPSL2)
1L    MBWD3.K=MBWD3.J+(DT)(0-MBRW3.JK)
49N   MBWD3=SWITCH(0,MITWD,MPSL2)
51A   MBFR1.K=CLIP(-MBPS4.K,0,MBCTR.K,0)
51A   MBFR2.K=CLIP(-MBPS4.K,0,MBCTR.K,2)
51A   MBFR3.K=CLIP(-MBPS4.K,0,MBCTR.K,4)
20A   MBND1.K=MBWD1.K/DT
20A   MBND2.K=MBWD2.K/DT
20A   MBND3.K=MBWD3.K/DT

NOTE                 RED PROBLEM SELECTOR MODULE
49A   MRPS1.K=SWITCH(MRTIM.K,0,MPSL1)
49A   MRPS2.K=SWITCH(0,MRTIM.K,MPSL1)
49A   MRPS3.K=SWITCH(MRPS2.K,0,MPSL2)
49A   MRPS4.K=SWITCH(0,MRPS2.K,MPSL2)
C     MPSL1=0
C     MPSL2=0

NOTE                 BLUE PROBLEM SELECTOR MODULE
49A   MBPS1.K=SWITCH(MBTIM.K,0,MPSL1)
49A   MBPS2.K=SWITCH(0,MBTIM.K,MPSL1)
49A   MBPS3.K=SWITCH(MBPS2.K,0,MPSL2)
49A   MBPS4.K=SWITCH(0,MBPS2.K,MPSL2)

NOTE                 DISTANCE SELECTION MODULE
49A   MDBRB.K=SWITCH(MDADW.K,MDSL1.K,MPSL1)
49A   MDSL1.K=SWITCH(MRDCF.K,MBDCF.K,MPSL2)
```

```
C      MIDBF=5000
C      MIFWD=1000
C      MISWD=1000
C      MITWD=1000

NOTE                    RED VELOCITY SELECTION MODULE
49A    MRVS1.K=SWITCH(MRTIM.K,MRVS2.K,MPSL1)
49A    MRVS2.K=SWITCH(MRFC1.K,MRVEL.JK,MPSL2)

NOTE                    BLUE VELOCITY SELECTION MODULE
49A    MBVS2.K=SWITCH(MBVEL.JK,MBFC1.K,MPSL2)
49A    MBVS1.K=SWITCH(MBTIM.K,MBVS2.K,MPSL1)

NOTE                    RED STRENGTH MODULE
1L     SRSTR.K=SRSTR.J+(DT)(0-SRSLR.JK)
6N     SRSTR=40
20A    SRNEG.K=SRSTR.K/DT
54R    SRSLR.KL=MIN(SRNEG.K,SRCSA.K)
6N     SRSLR=0

NOTE                    BLUE STRENGTH MODULE
1L     SBSTR.K=SBSTR.J+(DT)(0-SBSLR.JK)
6N     SBSTR=120
20A    SBNEG.K=SBSTR.K/DT
54R    SBSLR.KL=MIN(SBNEG.K,SBCSA.K)
6N     SBSLR=0

NOTE                    RED SMALL ARMS CASUALTY MODULE
20A    SRCSA.K=DBFHF.K/SRNPC.K
58A    SRCTO.K=TABHL(SRTDC,MDBRB.K,-100,2000,100)
C      SRSL1=1
C      SRSL2=1
49A    SRAPF.K=SWITCH(1,SRPFM.K,SRSL1)
49A    SRPFP.K=SWITCH(2,3,SRSL2)
12A    SRNPC.K=(SRCTO.K)(SRAPF.K)
49A    SRPFM.K=SWITCH(SRPFP.K,1,MRVS1.K)
C      SRTDC*=1/1/20/50/150/300/500/1000/5000/10000/10000/10000/10000/500
```

```
X1      00/50000/50000/500000/100000/100000/100000/100000/100000

NOTE                  BLUE SMALL ARMS CASUALTY MODULE
20A     SBCSA.K=DRFHF.K/SBNPC.K
58A     SBCTO.K=TABHL(SBTDC,MDBRB.K,-100,2000,100)
C       SBSL2=0
C       SBSL1=0
49A     SBAPF.K=SWITCH(1,SBPFM.K,SBSL1)
12A     SBNPC.K=(SBCTO.K)(SBAPF.K)
49A     SBPFM.K=SWITCH(SBPFP.K,1,MBVS1.K)
49A     SBPFP.K=SWITCH(2,3,SBSL2)
C       SBTDC*=1/1/20/50/150/300/500/1000/5000/10000/10000/10000/10000/500
X1      00/50000/50000/500000/100000/100000/100000/100000/100000

NOTE                  RED SMALL ARMS FIRE MODULE
C       DRNMG=2
C       DRNOR=34
1L      DRMFP.K=DRMFP.J+(DT)(0-DRCFL.JK)
15N     DRMFP=(DRNOR)(1200)+(DRNMG)(6000)
20A     DRNEG.K=DRMFP.K/DT
54R     DRCFL.KL=MIN(DRNEG.K,DRLFP.K)
6N      DRCFL=0
12A     DRMCS.K=(DRNMG)(3)
51A     DRLFP.K=CLIP(DRLRF.K,DRLMG.K,SRSTR.K,DRMCS.K)
12A     DRLRF.K=(1200)(SRSLR.JK)
12A     DRLMG.K=(2000)(SRSLR.JK)
12A     DRDRF.K=(DRMFP.K)(DRPFP.K)
49A     DRFSF.K=SWITCH(0,DRDRF.K,SRSTR.K)
58A     DRRFD.K=TABHL(DRTFD,MDBRB.K,0,1200,100)
58A     DRRFA.K=TABHL(DRTFA,MDBRB.K,0,1200,100)
C       DRTFA*=1/1/.5/.1/.1/.05/.05/.05/.01/.01/0/0/0
C       DRTFD*=1/1/.7/.5/.3/.3/.1/.1/.1/.05/.05/0/0
49A     DRFHF.K=SWITCH(0,DRFEF.K,MDBRB.K)
49A     DRFEF.K=SWITCH(0,DRFSF.K,ARSBL.K)
49A     DRPFP.K=SWITCH(DRRFD.K,DRRFA.K,MRVS1.K)

NOTE                  BLUE SMALL ARMS FIRE MODULE
```

```
C       DBNMG=6
C       DBNOR=102
1L      DBMFP.K=DBMFP.J+(DT)(0-DBCFL.JK)
15N     DBMFP=(DBNOR)(1200)+(DBNMG)(6000)
20A     DBNEG.K=DBMFP.K/DT
54R     DBCFL.KL=MIN(DBNEG.K,DBLFP.K)
6N      DBCFL=0
12A     DBMCS.K=(DBNMG)(3)
51A     DBLFP.K=CLIP(DBLRF.K,DBLMG.K,SBSTR.K,DBMCS.K)
12A     DBLRF.K=(1200)(SBSLR.JK)
12A     DBLMG.K=(2000)(SBSLR.JK)
12A     DBDRF.K=(DBMFP.K)(DBPFP.K)
49A     DBFSF.K=SWITCH(0,DBDRF.K,SBSTR.K)
49A     DBFHF.K=SWITCH(0,DBFEF.K,MDBRB.K)
49A     DBFEF.K=SWITCH(0,DBFSF.K,ABSBL.K)
49A     DBPFP.K=SWITCH(DBRFD.K,DBRFA.K,MBVS1.K)
58A     DBRFD.K=TABHL(DBTFD,MDBRB.K,0,1200,100)
58A     DBRFA.K=TABHL(DBTFA,MDBRB.K,0,1200,100)
C       DBTFD*=1/1/.7/.5/.3/.3/.1/.1/.1/.05/.05/0/0
C       DBTFA*=1/1/.5/.1/.1/.05/.05/.05/.01/.01/0/0/0

NOTE                    RED SMALL ARMS AMMUNITION MODULE
1L      ARSBL.K=ARSBL.J+(DT)(ARARR.JK-ARAER.JK)
6N      ARSBL=ARIBL
C       ARIBL=5400
20A     ARNEG.K=ARSBL.K/DT
54R     ARAER.KL=MIN(ARNEG.K,DRFHF.K)
1L      ARCF1.K=ARCF1.J+(DT)(ARAER.JK-ARRAR.JK)
6N      ARAER=0
6N      ARCF1=0
20A     ARCF2.K=ARCF1.K/DT
51R     ARRAR.KL=CLIP(ARCF2.K,0,ARCF1.K,ARARP.K)
C       ARPAE=0.25
6N      ARRAR=0
12A     ARARP.K=(ARPAE)(ARIBL)
37B     ARFTD=BOXLIN(4,0.125)
C       ARFTD*=0/0/0/0
```

```
1L      ARFTD*1.K=ARFTD*1.J+(DT)(0+ARRAR.JK)
49R     ARARR.KL=SWITCH(0,ARADR.K,SRSTR.K)
6N      ARARR=0
20A     ARADR.K=ARFTD*4.K/DT

NOTE                    BLUE SMALL ARMS AMMUNITION MODULE
1L      ABSBL.K=ABSBL.J+(DT)(ABARR.JK-ABAER.JK)
6N      ABSBL=ABIBL
C       ABIBL=16200
20A     ABNEG.K=ABSBL.K/DT
54R     ABAER.KL=MIN(ABNEG.K,DBFHF.K)
1L      ABCF1.K=ABCF1.J+(DT)(ABAER.JK-ABRAR.JK)
6N      ABAER=0
C       ABPAE=0.25
6N      ABCF1=0
20A     ABCF2.K=ABCF1.K/DT
51R     ABRAR.KL=CLIP(ABCF2.K,0,ABCF1.K,ABARP.K)
6N      ABRAR=0
12A     ABARP.K=(ABPAE)(ABIBL)
37B     ABFTD=BOXLIN(4,0.125)
C       ABFTD*=0/0/0/0
1L      ABFTD*1.K=ABFTD*1.J+(DT)(0+ABRAR.JK)
49R     ABARR.KL=SWITCH(0,ABADR.K,SBSTR.K)
6N      ABARR=0
20A     ABADR.K=ABFTD*4.K/DT

SPEC    DT=0.125/LENGTH=12/PRTPER=0.125/PLTPER=0.125
```

(NOTE: PRINT and PLOT cards are not listed because they will vary with each type of experimental problem.)

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