

χ -bounded classes

Stéphan Thomassé

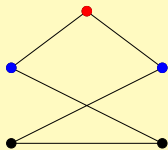
Ecole Normale Supérieure de Lyon

Triangle-free graphs with high chromatic number

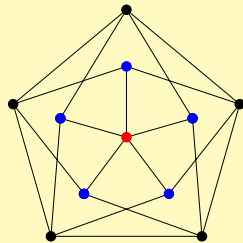
Mycielski Graphs



M2

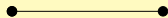


M3

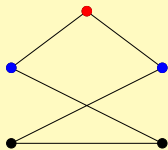


M4

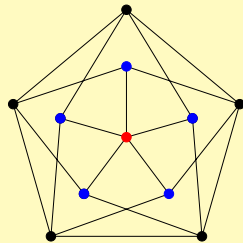
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M_k has chromatic number k .

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Arbitrary girth can be achieved.

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Chromatic number is $d + 2$.

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Chromatic number grows with n .

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Light version of SPGT?

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All open. Last two even for triangle-free.

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Scott's Conjecture: Excluding all induced subdivisions of a fixed graph F gives a χ -bounded class.

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Theorem (Mader): If H has average degree $\gg t$, it contains a subdivision of K_t .

If $\chi \gg t$, G contains an induced subdivision of any fixed graph F on t vertices.

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Cover "sparse" and "dense" aspects of Scott's conjecture.

VC-dimension

Transversal and Packing

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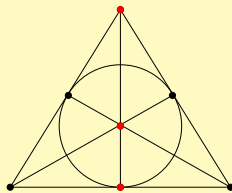
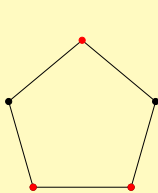
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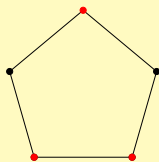
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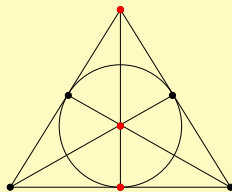
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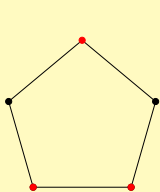
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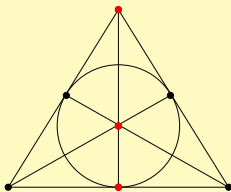
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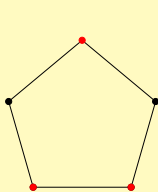
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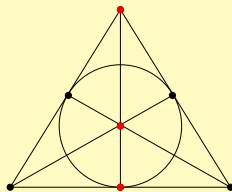
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$\nu \leq \tau$. The difference is the *duality gap*.

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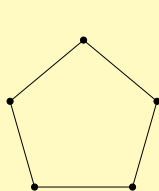
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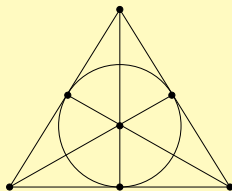
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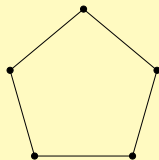
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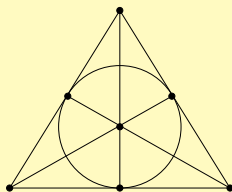
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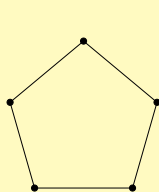
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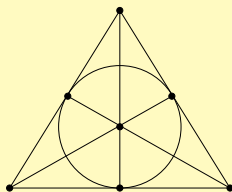
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Theorem (Ding, Seymour, Winkler): When 2-VC-dimension is bounded, the duality gap is bridged.

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- G induces all bipartite graphs of fixed size.

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- $G[A \cup S]$ is our induced 1-subdivision.

Segment graphs

Recent News

Conjecture (Erdős): Triangle-free segment intersection graphs have bounded χ .

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Scott's conjecture does not hold for 1-subdivision of K_4 .

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- Is fractional chromatic number bounded?
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- Does the neighborhood of an induced path have bounded χ ?