

# Bid-Response Models for Customized Pricing

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June 2007

**Abstract:** In this paper, we study pricing situations where a firm provides a price quote in the presence of uncertainty in the preferences of the buyer and the competitive landscape. We introduce two customized-pricing bid-response models used in practice, which can be developed from the historical information available to the firm based on previous bidding opportunities. We show how these models may be used to exploit the differences in the market segments to generate optimal price quotes given the characteristics of the current bid opportunity. We also describe the process of evaluating competing models using an industry dataset as a test bed to measure the model fit. Finally, we test the models on the industry dataset to compare their performance and estimate the percent improvement in expected profits that may be possible from their use.

**Keywords:** bid-response functions, customized pricing, price optimization, bid-pricing.

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## 1. Introduction

While the majority of the previous literature in the price-optimization area focuses on the pricing of consumer goods or the optimal design of auctions, a large percentage of firms face pricing decisions in a business-to-business setting where a customer requests bids from a small set of competing firms and the firms vying for the customer's business respond with a single price quote for the product or service. When the total annual sales to the firm requesting the bid does not justify a dedicated sales person on behalf of the firm responding to the bid, many firms have started using bid-response models to provide customized pricing recommendations on what price to offer for the business being bid upon. Customized-pricing bid-response models (CPBRMs) provide a probability of winning for every possible price response, allowing a firm to balance a decreasing margin with an increasing win probability needed in a price optimization model. Examples of firms using CPBRMs include United Parcel Service (UPS) when responding to bids for from their small to medium size customers (Kniple, 2006), and BlueLinx, the largest building products distributor in the U.S, responding to requests for products from construction companies (Dudziak, 2006). (Phillips, 2005a) described a prevalent use of these models in the financial services industry when firms determine what interest rate to offer when responding to request for mortgages (prime, home equity, sub prime), credit cards, and auto-loans. The financial impact from using CPBRMs can be significant. UPS reported an increase in profits of over \$100 million per year by optimizing their price offerings using CPBRMs (Boyd et al. 2005).

In determining the winning bid probability, CPBRMs effectively determine the price segment the current bid falls in. Price segments are defined as sets of transactions, classified by customer, product, and transaction attributes, which exhibit similar price sensitivities. Customer attributes may include customer location, size of the market the customer is in, type of business the customer is in, the way the customer uses the product, customer purchase frequency, customer size, and customer purchasing sophistication. Product attributes may include product type, lifecycle stage, and the degree of commoditization. Transaction attributes may include order size,

other products on the order, channel, specific competitor, when the order is placed, and what the urgency is of the bidder. In addition, some models assume knowledge of the historical and current bid-price of competing firms participating in the bid.

A common characteristic of situations where firms employ CPBRMs is when the bidder with the lowest price does not always win the bid. Thus, markets are characterized by product differentiation where a given firm may command a positive price-premium over its competitors; dependent upon the particular customer offering the bid. Even assuming a firm collects enough historical data to perfectly derive its price premium for a given customer, there may still be some inherent amount of uncertainty in the bid winning probability due to the bid-requesting firm randomly allocating its business to different competitors to ensure a competitive market for future bids. Therefore, a firm will never be able to remove all uncertainty from the bid-price response process and must work with probabilistic models.

Another common characteristic of situations where firms have used CPBRMs is when the size of the bid opportunities is not large enough to justify a dedicated sales person for each bid opportunity. Thus, the most common alternatives to using CPBRMs is either to charge a fixed price to all customers or to have a sales agent respond to each separate bid opportunity with a customized price. Charging a fixed price leads to missed opportunities to price discriminate between different customer segments, a practice that has been well publicized for significantly increasing a firm's profit in many different industries. The other alternative, relying on a sales agent to respond to multiple bid opportunities, is also problematic. Theoretically, the sales agent should have knowledge of the market, based on a history of former bid-responses with the customer requesting the bid, allowing the sales agent to respond with a customized price that optimizes this inherent trade-off between decreasing margins, due to lowering the price, and increasing probabilities of winning the bid. In reality, sales agents often do not make good tradeoff decisions in these situations, either because of a lack of historical knowledge, the inability to process this historical knowledge into probability distributions, or mis-aligned

incentives (Garrow et al., 2006). The judicious use of CPBRMs allows firms to capture historical bid information, process it, and present non-biased price recommendations to bidding opportunities. If there is additional information available regarding the bidding opportunity that can not be captured in the CPBRMs, the CPBRM's recommended price may serve as one of possibly many inputs to the person responsible for making the bid-response decision.

To summarize, CPBRMs apply to situations where a firm selling a non-commodity product must respond to frequent request for small to medium sized bids from a number of different customers where the bid-winning criteria is not always the lowest price. To use a CPBRM, a firm must have access to their historical bid history that includes, as a minimum, the price the firm bid at each opportunity and the corresponding bid result (win or loss). Other useful historical information used in developing CPBRMs is, for each historical bid opportunity, the customer, the length of the relationship with the customer, the size of the order, delivery date requirements, competitors' bids, and any other pertinent information useful for market segmentation. When CPBRMs are used as an input to a price optimization model, there is also the implicit assumption that the actions of the competitors can be determined probabilistically and independently of the decision maker's action. If all competitors have similar analytic capabilities and jointly optimize against each other, competitive response modeling techniques such as game theory must be used.

In this paper, we evaluate two CPBRMs, namely the Logit and Power functions, which model the response of the buyer subject to the segmentation criteria described above. We demonstrate, on an industry dataset, how each model may be developed and expected improvements in profits may be estimated. By assessing two goodness-of-fit criteria for each model, we find the Logit function provides a better fit when there is limited data available on the historical bid opportunities for determining customer segments. When detailed information is available about each former bidding opportunity such as the competitors' prices and the size of the order, the Power function is a better fit on our test dataset. We demonstrate how to modify the

functions to incorporate various degrees of segmentation data available to the firm. We then test both functions on the industry dataset to analyze the relation between the nature of the segmentation information available to the firm and the potential improvements in profit generated by our approach. Finally, we observe that the model providing the better fit to the data also results in higher expected profit improvements.

The rest of the paper is organized as follows. In §2 we review the academic literature and industry practices related to the modeling of bid-price responses. In §3 we present two CPBRMs that are used in practice and show how they can be modified to be used under three different levels of availability of historical and competitive information. We discuss a step-by-step procedure for developing CPBRMs and using them for quoting customized prices. We also discuss different diagnostic measures and segmentation methods which can be used, based on the nature of data available. In §4 we present the results from applying the two CPBRMs to an industry dataset and assess their fit for un-segmented and segmented data. We then compare their performance by measuring the percent improvement in expected profits under different information levels. In §5 we summarize our observations from the numerical comparison and conclude with some limitations and managerial implications of using CPBRMs.

## **2. Literature Review**

In this section, we discuss the academic literature on bid-price response models and how CPBRMs are unique. We also discuss the motivation from industry practices related to such competitive pricing settings.

Several papers develop bid-price response models where price is the only attribute of the model. Friedman (1956) and Gates (1967) both develop models which use the historical bid information available. Morin and Clough (1969) build on their work by identifying key competitors and capturing temporal sensitivity to changes in strategy by giving recent data more importance. However, these models consider price as the sole criterion for winning a bid and only consider the objective of maximizing profits. Chapter 4 in Lilien et al. (1992) provides an

overview of competition oriented pricing where the firm makes a trade-off between margin and probability of winning the bid. This is the same trade-off the firm makes in our models. The difference, however, occurs in the estimation of the winning probabilities. In their model, the lowest bid always wins, so the probabilities are based on the number of competitors and each competitor's estimated bid-to-cost ratio. King and Mercer (1991) discuss estimation methods for determining the distributions for these ratios. The models we review are more general; they include non-price factors such as order size and continue to hold when factors other than just price are included in the buyer's decision.

Papaioannou and Cassaigne (2000) provide a detailed review of bid-price response models and develop a "*ServPrice*" model which, like CPBRMs, accommodates several firm objectives and accounts for both price and non-price attributes. However, their model relies only on the sales or pricing agents to internally make tradeoffs and analyze the historical information without providing any analytical tools for doing so. In contrast, CPBRMs help the firm obtain a non-biased input to the bid-response decision by processing the relevant historical information statistically. Lawrence (2003) develops an analytical model for providing bid-response quotes that predicts the outcome of a bid as a function of its attributes. His model requires a more extensive bid history (number of historical bids) than a typical CPBRM and uses a machine-learning approach. In addition, it doesn't exploit any additional information that is available to the firm such as order size, length of relationship, etc. CPBRMs, in contrast, can exploit this additional information to determine particular market segments. We also study the difference in improvements when CPBRMs are used with different levels of this information.

This paper is most closely related to the work presented in Chapter 11 of Phillips (2005b) and the U.S. patent of Boyd et al. (2005), who discuss the use of CPBRMs in industry and develop models using a Logit function as a bid-response function. These models capture the inherent preference uncertainty and non-price factors which play a critical role in winning a bid. Finally, Elmaghraby (2006) provides a brief review of CPBRMs in relation to business-to-

business auctions but does not describe the models or how to implement them in practice. The contributions of our work over the methods suggested by Phillips and Boyd et al. are as follows: We extend the Logit bid-response function to include the competitor's price which helps to capture the competitive dynamics. We also present another CPBRM, the Power function (sometimes found in practice) which includes a parameter of the ratio of the bidder's price to the expected bid price of the bidder's competitor(s). We numerically test both models on an industry dataset and, based on the fit and performance of each model, we provide observations on when each model may be preferred. To the best of our knowledge, this is the first academic paper to present such models and discuss their use for customized price optimization.

### **3. Customized-Price Bid-Response Models**

In this section we describe what CPBRMs are, present two CPBRMs used in practice, and discuss how they may be developed to be used in a price optimization model. CPBRMs calculate the probability of winning a bid opportunity for each possible price response given the market characteristics and competitive dynamics for a particular customer segment. The parameter values for these models are statistically estimated from historical bid information and include, at a minimum, the bidding firm's responses to previous bid opportunities and the outcome from each bidding opportunity (win or loss). Intuitively, if the price quoted by a firm is very low compared to its competitor's price, the probability of winning the bid should be close to 100%. If it is very high by comparison, the probability of winning should be close to zero. This probability of "winning the bid" should monotonically decrease with an increase in price (or price ratio). Also, the slope of the response curve should be steeper for prices close to the competitor's price as compared to prices far higher or lower than the competitor's price. Hence, the bid response curve is generally S-shaped in nature. In a single competitor setting with no price premium enjoyed by either firm, pricing equal to the competitor's price should result in a 50% chance of winning the bid opportunity. In practice however, one of the firms usually enjoys some

price-premium over the other. CPBRMs can be used to identify what this price-premium is for each customer segment.

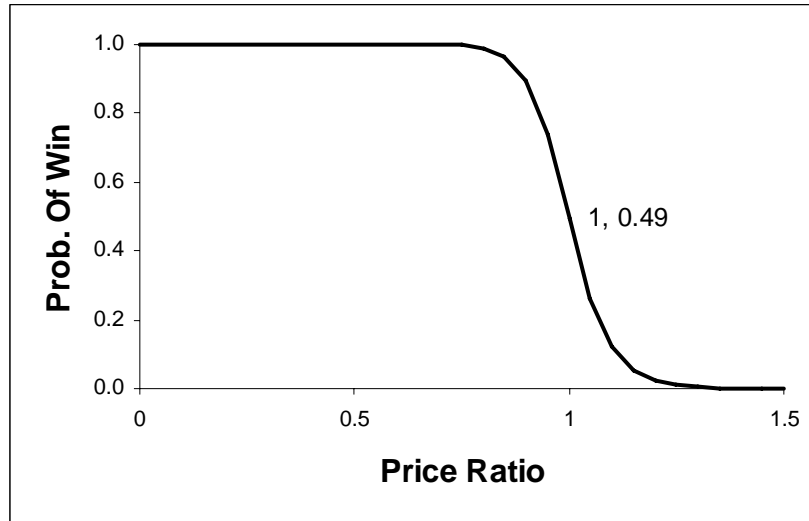


Figure 1 Bid-Response Curve

Figure 1 shows a CPBRM curve applied to one of our test case datasets. The price ratio on the x-axis is the ratio of the firm’s price relative to its competitor’s price. For the particular firm corresponding to this bid-response curve, a price equal to its competitors price (price ratio = 1) results in a probability of winning the bid of 49% (a price ratio = .99 equates to a 50% win probability for this firm). Thus, this firm has a negative price-premium and must price below its competitor’s price for an equal opportunity of winning the business of the firm offering the bid. Before presenting the two CPBRM functions reviewed in this paper, we first introduce some notation.

Table 1: Notation

$p_i$	Unit price quoted by the firm for bid opportunity $i$ (our decision variable)
$\rho(p_i)$	Bid-Response Function, i.e. probability of winning bid opportunity $i$ given a price of $p_i$
$i$	Index for bid opportunities, $i = 1, 2, \dots$
$a_j, \alpha_j$	Parameters related to non-price factors for a segment $j$
$b_j, \gamma_j$	Parameters related to price factors for a segment $j$
$j$	Index for segments, $j = 1, 2, \dots$



$c_c$	Coefficient for the price quote of the competitor
$p_{c,i}$	Unit price quoted by the competitor(s) for bid opportunity $i$
$c_q$	Coefficient for the order size
$Q_i$	Order size for bid opportunity $i$
$r(p_i)$	Price ratio = $p_i/p_{c,i}$ for bid opportunity $i$
$x_j$	Indicator variable for segment $j$ (binary)
$\varepsilon(p_i)$	Elasticity of the bid- response function
$c_p$	Marginal costs for the firm
$W_i$	Win/Loss indicator variable for a bid opportunity $i$ ( binary)
$y_i$	Actual Outcome for the $i^{\text{th}}$ bid.

### 3.1 Two Common CPBRMs

In this section we describe the two CPBRMs compared in this paper and discuss how they can be adjusted to include segmentation and competitive pricing information; which have been conjectured to significantly enhance the predictive power of a CPBRM. Bid-responses may differ based on customers, channels, or product attributes such as warranty or payment terms. We capture these possible aspects in our models through a single counting variable  $j$ , where  $j = 1, 2, \dots$  representing the number of distinct, discrete customer segments. Other factors such as the size of the order or the competitive price can often be modeled (depending on the CPBRM) on a continuous scale and may sometimes be treated separately. When distinct clusters or segments exist in the data, a discrete approach should be used.

#### ***Logit Bid-Response Function***

Phillips (2005b) & Boyd et al. (2005) both present the Logit function as their representation of a CPBRM. As discussed in Phillips (2005b, pg. 289), for a dataset with  $j$  distinct segments, the general form for the Logit function is:

$$\rho(p_i) = \frac{1}{1 + e^{\sum_j a_j + \sum_j b_j p_i}} .$$

One of the main advantages of the Logit function is the ease of adding additional segmentation factors such as the size of the order,  $Q_i$  and the competitor's price quote,  $p_{c,i}$ . If the segmenting variables can be used as continuous variables, the model may be adjusted to include these segmentations by adding coefficients such as  $c_q$  to measure the effect of order quantity segmentation and  $c_c$  to measure the effect of the competitor's price segmentation. These coefficients are multiplied by  $Q_i$  and  $p_{c,i}$  respectively:

$$\rho(p_i | Q_i, p_{c,i}) = \frac{1}{1 + e^{\sum_j a_j + \sum_j b_j p_i + c_q Q_i + c_c p_{c,i}}}.$$

Note that a relative price ratio may also be used in the Logit function by replacing  $c_c p_{c,i}$  with  $c_c (p_i / p_{c,i})$  in the equation above. In our performance test, we found little difference between these two representations so we only present the simpler form with just the competitor's price.

Using the simplest form of the Logit function:  $\rho(p) = \frac{1}{1 + e^{a+bp}}$ , the slope  $\rho'(p)$  and elasticity  $\varepsilon(p)$  of the Logit function is (Phillips 2005b pg. 284):

$$\rho'(p) = -b\rho(p)(1 - \rho(p)) \text{ and } \varepsilon(p) = bp(1 - \rho(p)).$$

### ***Power Bid-Response Function***

An alternate CPBRM sometimes used in practice is the Power function, defined in its general form as:  $\rho(p_i) = \frac{\alpha_j}{\alpha_j + r(p_i)^{\gamma_j}}$ . The main advantage of the Power function is that, compared to the Logit function, competitive price dynamics are explicitly captured. The main disadvantage is that it is more cumbersome to adjust the model for non-price, continuous variable attributes. Segmentation parameters can be added to the Power function but only through a discrete characterization. Thus, a variable such as order size must be broken into discrete intervals and captured through the parameter  $\gamma_j$ , where the subscript  $j$  now represents the discrete

intervals of the order size. Using the simplest form of the Power model with no segmentation,

$$\rho(p) = \frac{\alpha}{\alpha + r(p)^\gamma}, \text{ the slope and elasticity of the Power function is}$$

$$\rho'(p) = -\frac{\gamma}{p} \rho(p)(1 - \rho(p)) \text{ and } \varepsilon(p) = \gamma(1 - \rho(p)).$$

For a CPBRM to be a strictly decreasing function in  $p$ , the price dependent parameters must be strictly greater than zero. More specifically, for the Power function:  $\gamma > 0$ . The parameter  $\gamma$  is a measure of the price sensitivity of the buyer where higher values of  $\gamma$  imply greater price sensitivity. The effect of the parameter  $\gamma$  on the probability of winning is shown in Figure 2.

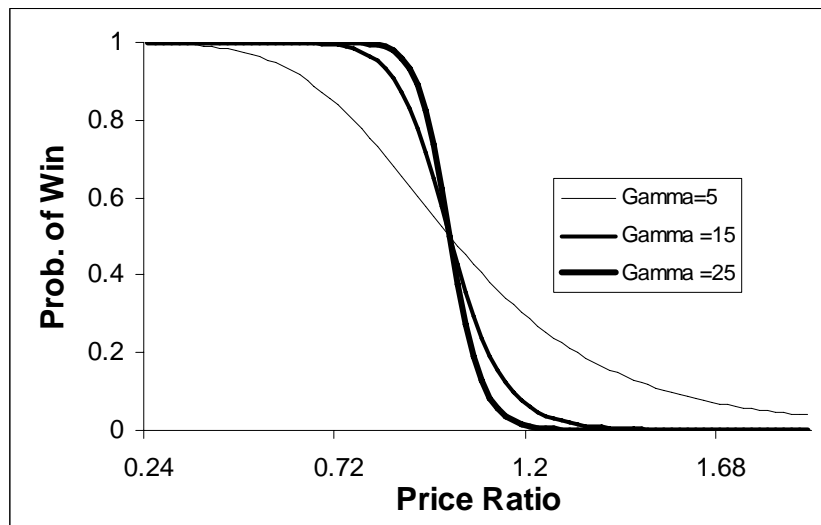


Figure 2: Effect of  $\gamma$  (Price Sensitivity) on the probability of winning

The parameter  $\alpha$  is a measure of the price premium the firm enjoys, with a higher value of  $\alpha$  implying a larger price premium on the market. Thus, an increase in the value of  $\alpha$  allows the firm to charge a higher price for the same probability of winning. The effect of the parameter  $\alpha$  on the probability of winning is shown in Figure 3.

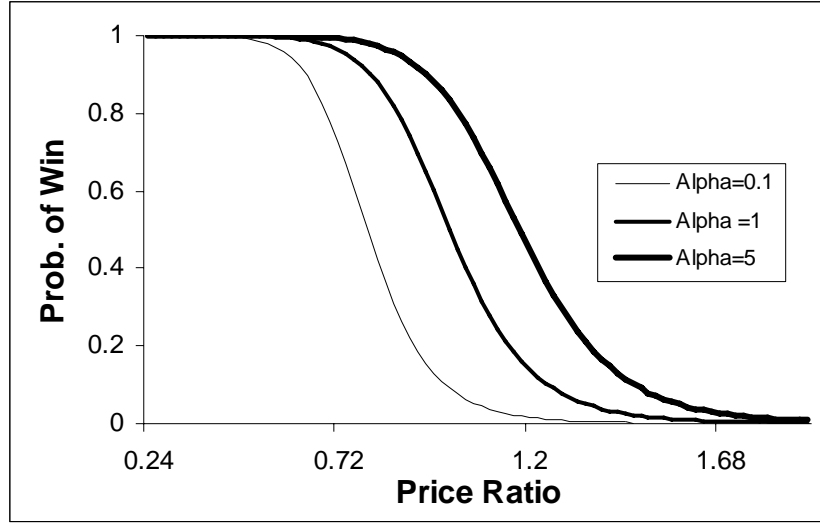


Figure 3: Effect of  $\alpha$  (Price Premium) on the probability of winning

### 3.2 Estimation of Parameter Values

The parameter values of a CPBRM can be estimated statistically by fitting a curve to the available bid-history data based on minimizing the squared errors or using maximum-likelihood estimates. Before estimating the parameter values of the models however, it is important to divide the dataset into two segments; one for estimating the parameter values and the other for measuring the fit. Similar to time-series forecasting models, measuring the goodness-of-fit on the same data as the parameter values are estimated on may result in a misleadingly close fit as compared to testing the model on a holdout sample. We briefly describe two estimation methods below, using the Power function as the CPBRM of reference. (Phillips 2005b, pg. 285) describes how each estimation method is applied to the Logit function. The two methods are:

a. Minimize the squared error residuals:  $Minimize_{\alpha, \gamma} \sum_i [\rho(p_i | \alpha, \gamma) - W_i]^2$

b. Maximize likelihood estimates:  $Maximize_{\alpha, \gamma} \sum_i \ln[\rho(p_i | \alpha, \gamma)W_i + [1 - \rho(p_i | \alpha, \gamma)](1 - W_i)]$ .

### 3.3 Segmentation Methods

Many different approaches to segment data exist. The number and type of segments can be determined in advance (*a-priori*) or can be determined on the basis of data analyses (*post-hoc*).

Predictive methods where one set consists of dependent variables to be predicted by the set of independent variables can also be used. Some of the more popular methods are non-overlapping and overlapping clustering methods, classification and regression trees, and Expectation Maximization algorithms. A detailed analysis of these methods is beyond the scope of this paper but we refer the reader to Wedel & Kamakura (1998) for a detailed overview. In general, the number of bid attributes (segments) that can be accurately estimated depends on the amount of historical bid-information available. If extensive information is available, greater degrees of segmentation can be achieved without compromising the accuracy and robustness of the statistical estimation of the parameter values.

### 3.4 Diagnostic Measures

After the segments have been determined and the parameters of the model have been estimated, the goodness-of-fit of the response function should be assessed using the holdout sample of the dataset. Various diagnostic measures are available to check the fit of the model to the data. Some of the more common include the Hosmer & Lemeshow tests (*H & L tests*), the very similar Pearson Chi-Squares tests, and the Akaike Information Criteria (AIC). Hosmer and Lemeshow (1989) discuss the *H & L tests* and the Pearson Chi-Squares tests and, Burnham and Anderson (1998) discuss the AICc estimates in detail. For our analysis, we use *H & L tests* and AICc estimates to assess the goodness-of-fit of our models to the holdout sample.

*H & L tests* are a popular diagnostic measure for logistic regression models. The observations are partitioned into 10 equal segments based on the estimated probabilities and the diagnostic measure is then calculated using

$$X^2 = \sum_{k=1}^{10} \frac{(O_k - E_k)^2}{E_k (1 - E_k / n_k)},$$

where  $O_k = \sum_i y_{ik}$  = Sum of the actual outcomes for each segment  $k$ ,  $n_k$  is the number of

records in segment  $k$ , and  $E_k = \sum_k \rho_{ik}$  = Sum of the estimated probabilities for each segment  $k$ . If

the fitted model is appropriate, the distribution of  $X^2$  is well approximated by the chi-square distribution with  $(k-2)$  degrees of freedom. Thus, a p-value can be calculated from the chi-square distribution using  $k$  degrees of freedom to test the fit of the model to the data. If the p-value from the  $H$  &  $L$  tests is 0.05 or less, we can accept the null hypothesis at the 95% confidence level that the model does not predict values consistent with the observed values. However, if the p-value is greater than 0.05 we can't reject the null hypothesis that there is no difference between the observed values and the model predicted values, implying that the model's estimates fit the data at an acceptable level. Pearson Chi-Square estimates can be calculated in a similar way, but the grouping is based on the predictor variables.

AIC estimates compare two alternative systems using the number of parameters estimated ( $K$ ), the number of observations ( $N$ ) and the residual sum of squares ( $SS$ ). The estimate is calculated as follows:  $AIC = N \ln\left(\frac{SS}{N}\right) + 2K$ . If the number of observations is small, a correction factor is often added to the Akaike's Information Criteria:  $AIC_c = AIC + \frac{2K(K+1)}{N+K-1}$ .

If  $N$  is much greater than  $K$  (as in our dataset), then corrected the  $AIC_c$  is approximately equal to the AIC estimate. A model with a lower  $AIC_c$  estimate is more likely to be a better fit for the data.

After estimating the parameter values of a CPBRM using historical bid data and determining the CPBRM fits the data well using the holdout sample, the CPBRM can now be used to determine the optimal bid-response price for an upcoming bid opportunity. This process is described in the next section.

### **3.5 Use of a CPBRM in Price Optimization**

We now look at how CPBRM curves can be used in price optimization. For the following discussion, we use the objective of maximizing expected profits. However, other strategic or operational objectives can be easily accommodated such as increasing market shares or including constraints on capacity, inventory, price or margin. The price optimization problem for bid opportunity  $i$  is

$$\text{Max}_{p_i} \pi(p_i) = \rho(p_i) \times (p_i - c_p) \times Q_i.$$

Note, the margin  $(p_i - c_p)$  is strictly increasing in price (Figure 4) but the probability of winning the bid is strictly decreasing in price (Figure 5). Therefore, the expected profit is a unimodal function as shown in Figure 6.

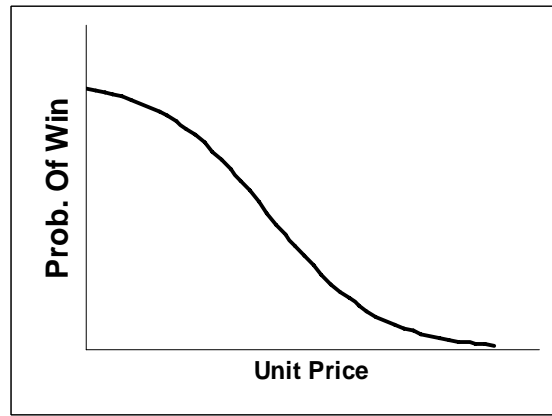
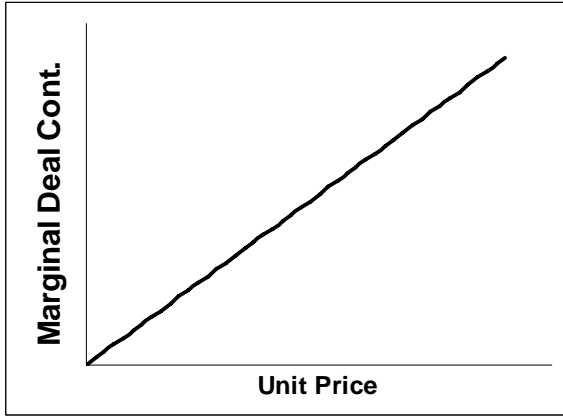


Figure 4: Marginal Deal Contribution vs. Unit Price

Figure 5 Prob. Of Win vs. Unit Price

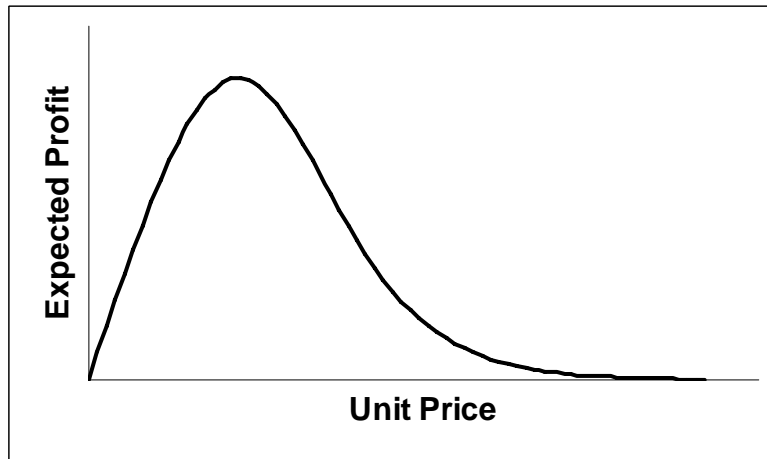


Figure 6 Expected Profit vs. Unit Price

Determining the optimal price involves finding a global maxima for the expected profit which is unimodal in nature. The profit-maximizing price occurs where the elasticity of the expected profit function is equal to the inverse of the marginal contribution ratio,

$$\varepsilon(p_i) = \frac{p_i}{p_i - c_p}.$$

The derivation is available from the authors by request.

We have described two CPBRMs and explained how they can be used to find an optimal price response for a specific bid opportunity. In the next section we demonstrate how to apply the CPBRMs to historical bid data and test them on two industry datasets corresponding to two extremes of historical information available to the user.

## **4. Numerical Comparisons of CPBRMs on Industry Data**

In this section, we compare the performance of the two CPBRMs described in the previous section using a bid-history industry dataset. The dataset contains a single-competitor setting where extensive bid history is available including the competitor's price at each bid opportunity. We test the two CPBRMs under a wide set of scenarios pertaining to: 1) the amount of knowledge of the competitors' price response to the current bid request, and 2) the amount of segmentation included in the models based on the size of the order in each bid opportunity.

### **4.1 Test Case Scenarios**

The firm providing our dataset manufactures and sells medical testing equipment to laboratories at hospitals, clinics, and universities across North America. One of their popular products is a gas chromatograph refill cartridge that has a list price of \$11.85. The marginal cost associated with each unit is \$6.00. The refill cartridges are ordered in batches ranging in size from 100 to over 1000. Orders for fewer than 200 units are handled through the company's website or through resellers with no associated discount from the list price. At the other extreme, the company receives about 100 orders per year for more than 1000 units. These large deals are negotiated by a national account manager, usually as part of a much larger sale. Orders for 200-1000 units are handled by a regional sales staff that has considerable leeway with regard to discounting. We only look at this middle-size segment to apply the CPBRMs. The requested size of the order for each bid opportunity is also recorded, allowing us to test both segmented and unsegmented versions of the CPBRMs. Because of the specialized nature of the product, the firm has only one significant competitor and they are able to capture their competitor's price after each bid



opportunity. Their bid history information is exhaustive, with approximately 2400 records of previous bid opportunities. A snapshot of this dataset is shown in Table 2.

Table 2: Bid History for a Medical Device Company

Bid Number	Win	Firm's Bid	Competitor's Bid	Order Size
1	Y	\$8.44	\$10.92	353
2	N	\$11.88	\$9.99	773
3	N	\$11.29	\$10.59	974
4	Y	\$9.78	\$11.52	857
5	Y	\$9.28	\$11.47	283

For this application, the probability of winning the bid at a price equal to the competitor's price (i.e. price ratio is one) is 51%. This percentage implies the firm doesn't enjoy any significant positive or negative price premium compared to its competitor.

### **Knowledge Level of Competitors' Pricing**

We tested the two CPBRMs under three different levels of knowledge a firm may possess regarding its competitors' pricing, i.e. worst, medium, and best cases. Historical competitive bid-price information is often available in many B2B applications through either formal or informal channels, depending on the relationship the bidder shares with the buyer. UPS, for example, obtains competitors' bids in approximately 40% of the parcel shipping bid opportunities they participate in (Kniple, 2006). In some business-to-business scenarios, a firm may even be provided with the competitors' bids and asked to respond with a quote of their own (note that for reasons explained earlier, providing the lowest bid does not always guarantee a win in these situations). In many B2C markets such as loan and insurance quotes, information about the competitor's price may be available from a simple web-page search.

**Worst Case: No Price Information Case:** In this case, the firm has no historical price information on its competitors, nor does it have any information about how its competitors will price for the current bid opportunity. This scenario is rare in practice but, for our analysis, serves

as a lower bound on the knowledge of competitors' pricing. With no competitor price information, the Logit function is the only CPBRM available, as the Power function requires an estimate of the competitor's price in the current period (via the price ratio).

**Medium Case: Naïve Price Estimation Case:** In this medium case, the firm has no information about how its competitors will price in the current period except for the price history of its competitors on past bidding opportunities. Thus, the firm can estimate its competitors' prices for the current period through some type of forecasting or regression model. In our analysis, we use a simple 10-period moving average to predict the competitor's price in the current period. We experimented with moving averages of different numbers of periods but found the 10-period moving average resulted in the most accurate and least biased estimates for the future competitor's bid. By estimating the competitor's price response, we can now test both the Logit and Power functions.

**Best Case: Perfect Competitive Price Knowledge:** In this best case, the firm knows exactly what its competitors' bids will be in the current period. This can be considered an upper bound on the firm's forecasting capabilities. It also applies to cases where the buyer provides competitors' bids before requesting a bid from the firm or in applications where a firm can check its competitors' prices (possibly via their web pages) before responding with its own price quote.

The chart below summarizes which knowledge levels were tested for each CPBRM.

	Logit	Power
Worst Case	✓	✗
Medium Case	✓	✓
Best Case	✓	✓

The next section describes the procedure we used to develop and test the two CPBRMs on the dataset and scenarios described above.

#### 4.2 Procedure for Testing CPBRMs

1. We divided the dataset into two distinct sets; the first for estimation of the model parameter values and the second for performance evaluation (the holdout sample). We used the first 90% of the historical bid records as our *estimation data* and the remaining

10% as our *performance test data*. While the choice of 10% for the performance test may seem arbitrary, it is a common choice for holdout samples in forecast methods evaluations. Sensitivity test with different percentages of the historical data used for measuring performance were also performed. The changes in the parameter estimates and performance results on the dataset were insignificant when tested over a range of 10% - 20% for the performance test dataset.

- Using the estimation data, we calculated the parameter values for both the Logit and Power functions using ordinary least squares and maximum likelihood estimators. We found little difference in the fit of the models between the two estimation methods so we present the values found using least squares. The parameter values from the unsegmented analysis of the dataset are:

Table 3: Parameter Estimates for Unsegmented Analysis

	Logit Model			Power Model	
	$a$	$b$	$c_c$	$\alpha$	$\gamma$
Worst Case	-8.2055	0.8186	NA	NA	NA
Medium Case	-0.2413	1.0579	-1.036	1.0198	10.3794
Best Case	-0.2413	1.0579	-1.036	1.0198	10.3794

Because the Power function requires an estimate of the competitor's price, it could not be used under the Worst information case. The reason the parameter values are the same for the Medium and Best information cases is because past bid opportunities are used for estimating the parameter values when the competitor's price is known with certainty (note the information cases pertain to knowledge of the competitors' price in the current period; the past prices are assumed to be known with certainty). In the next section, we describe how we also used the order size as a segmentation variable.

- After estimating the parameter values for each model, we measured the goodness-of-fit of the models using both the *H & L tests* and the  $AIC_c$ .
- After selecting the model that provided the best fit for the holdout sample data, we used the CPBRM to optimize the bid-prices for all the bids in the performance test data subset.

5. Finally, we computed the percent improvements over expected profits and over actual profits as explained in section 4.3. This provided us with two metrics of performance.

The incorporation of the competitor's price is only one possible input to CPBRMs (although for the Power function it is a required input). Another common input is the size of the order request. It is reasonable to assume the price sensitivity of customers only ordering a few units will differ significantly from customers ordering large quantities. Thus, we describe how we segmented the bids based on the size of the order below.

### Segmentation Based on Order Size

In our dataset, order quantities range between 200 and 1000. For segmentation based on the order size using the Logit function, we used discrete segmentation based on the order size and estimated the model parameter values by fitting the following model to the estimation data:

$$\rho(p_i | Q, p_c) = \frac{1}{1 + e^{a + p_i \sum_j b_j x_j + c_c p_c}}$$

For discrete order size segmentation, a separate parameter must be estimated for each segment while for continuous order size segmentation, the estimation of only one parameter is required. Therefore, it is easier to use a continuous variable if the model allows it. For segmenting based on the order size using the Power model however, a discrete approach is the only option. The bid response for the Power function was calculated by estimating a different value of  $\gamma_j$  (our estimates for  $\alpha$  did not change and was thus, held constant) for each order size segment  $q$  by fitting the following model to the estimation data:

$$\rho(p_i | q) = \frac{\alpha}{\alpha + r(p_i)^{\sum_j \gamma_j x_j}}$$

The positive correlation between the size of the order and the estimate of  $\gamma$  (a measure of price sensitivity) indicates that with an increase in order size, the price sensitivity increases. To test if there were any inherent order size segments in our dataset, we tried various clustering algorithms

and segmentation techniques. These efforts did not expose any naturally occurring order size segments, so we used an *a-priori* approach with the following segmentation:

Order Size Between	200-299	300-399	400-499	500-599	600-699	700-799	800-899	900-999
Segment	2	3	4	5	6	7	8	9

To estimate the parameter value for each segment, we used a binary indicator variable  $x_j$ ,  $j = 2, 3, \dots, 9$ , which was assigned a value of one for the order size segment a specific bid fell under.

This classification scheme is demonstrated in the table below:

Bid Number	Win	Firm's Bid	Competitor's Bid	Order Size	Indicator Variables for Order Size Segments							
					2	3	4	5	6	7	8	9
1	1	\$8.44	\$10.92	353	0	1	0	0	0	0	0	0
2	0	\$11.88	\$9.99	773	0	0	0	0	0	1	0	0

Based on least squares fits, Table 4 provides the estimated parameter values for each segment.

Table 4: Parameter Estimates for Segmented Analysis

Knowledge of Comp. Price	Logit										
	$a$	$c_c$	$c_{q,j}$								
			2	3	4	5	6	7	8	9	
Worst Case	8.241	NA	0.833	0.818	0.802	0.807	0.809	0.836	0.832	0.837	
Medium Case	0.258	-1.038	1.049	1.055	1.052	1.062	1.052	1.070	1.069	1.082	
Best case	0.258	-1.038	1.049	1.055	1.052	1.062	1.052	1.070	1.069	1.082	
	Power										
	$\alpha$	$\gamma_j$									
		2	3	4	5	6	7	8	9		
Medium Case	1.038	8.925	8.695	8.611	7.663	7.687	14.417	17.188	14.788		
Best Case	1.038	8.9253	8.695	8.611	7.663	7.687	14.42	17.19	14.79		

Although each order size segment had approximately the same number of bids, the price sensitivity parameters ( $c_{q,j}$  and  $\gamma_j$ ) did not increase monotonically with the order size. This is an interesting observation as our intuition led us to predict otherwise (demand becomes more sensitive to price, on a continuous scale, as the size of the order increases). It is possible however, that this is just an artifact of our dataset.

In summary, we built models for the Logit and Power CPBRMs using an estimation data subset, three levels of knowledge of the competitors' prices, two levels of segmentation on the

order size, and measured performance using two performance measures. Thus, we had a total of 12 scenarios to base our observations. Table 5 summarizes the various scenarios.

Table 5: Summary of Test Scenarios

<b>Knowledge of Competitors' Price</b>	<b>Segmentation on Order Size</b>	<b>Performance Measure</b>
Worst Case	Segmented	Actual Profits
Medium Case	Unsegmented	Expected Profits
Best Case		

### 4.3 Diagnostic Measures:

For each model and case scenario, we measured the goodness-of-fit for the dataset and identified the better fitting model. We analyzed the models using *H & L tests* and  $AIC_c$  estimates. For the worst case of information i.e. (no information about the competitor's price), only the Logit model could be used. Therefore, for this case, we did not use the  $AIC_c$  estimates as they can only be used to compare two competing models. The *H & L tests* however, were used to assess the fit of the Logit model to the data. The results from this test for the worst information case scenarios are summarized in Table 6.

Table 6: *H & L Test* Results for Worst Case Scenarios

Worst Case: No Competitive Information		
Logit Model		
	$\chi^2$	p-value
Unsegmented	7.9954	0.4339
Segmented	7.6952	0.4638

The p-values are large enough to indicate the model is a good fit for the dataset in these scenarios. The models developed for the medium and the best information cases only differ in the competitor price estimates used during the price optimization sub-problem. Therefore, the model developed for both cases are identical. For reporting purposes, we refer to both cases together as *Competitive Information Cases*. The results for the Logit and the Power model for the unsegmented and the segmented cases using *H & L tests* are in Table 7. Note that larger p-values indicate a better fit for the *H & L tests*.

Table 7: *H & L Test* Results and  $AIC_c$  Estimates for Medium & Best Case Scenarios

<b>Competitive Inf.</b> <b>Cases</b>	<b>H &amp; L tests:</b>		<b>AIC<sub>c</sub></b> <b>Estimates</b>
	$\chi^2$	p-value	
<b>Logit</b>			
Unsegmented	10.877	0.2087	-4001.27
Segmented	12.984	0.1125	-4000.92
<b>Power</b>			
Unsegmented	12.2822	0.1391	-3992.3
Segmented	10.785	0.2188	-4020.22

According to the results from the *H & L tests*, for the unsegmented scenario, the Logit model provides a better fit. Also, the  $AIC_c$  estimate is smaller than the estimate for the Power model. Therefore, the Logit is a better fitting model for this scenario. Similarly, for the segmented scenario, both the diagnostic tests indicate the Power model provides a better fit to the data. One final observation is that the segmented models provide a better fit for the data than the unsegmented models. In the next two sections, we determine if a better fit also leads to better performance.

#### 4.4 Measures of Percent Improvement in Actual and Expected Profits

To test the impact of using CPBRMs on the industry datasets, we used two performance metrics: percent improvement in profits over un-optimized actual profits and percent improvement in profits over un-optimized expected profits. To understand the difference between the two performance metrics, consider the following numerical example from our dataset:

Bid	Win	Order Size	Original Bid	Optimal Bid	Probability of Win at Original Bid	Probability of Win at Optimized Bid
1	Y	353	\$ 8.44	\$ 9.35	0.79	0.64

Applying the unsegmented, worst information case, Logit CPBRM with the parameter values

obtained through the procedure outlined in section 4.2 we get:  $\rho(p) = \frac{1}{1 + e^{-8.2055 + 0.8185p}}$ .

Substituting in the original bid price of \$8.44, we calculate the probability of winning for the un-optimized bid =  $1/(1 + e^{-8.2055 + 0.8185 * \$8.44}) = 0.7852$ . Applying the optimization procedure

described in section 3.3, we calculate that the optimal bid price for this bid opportunity should have been \$9.35. Substituting in this price results in a probability of winning for the optimized bid =  $1/(1 + e^{-8.2055+0.8185*\$9.35}) = 0.6350$ .

The actual profit from this bid opportunity is = (Original Unit Price- Marginal Cost)\* Order Size\* Win/Loss Indicator Variable =  $\$(8.44-6)*353*1 = \$ 861.32$ . If the original bid had resulted in a loss, the actual profit would be zero. The original bid expected profit = (Original Unit Price-Marginal Cost)\* Order Size \* Probability of Win at the Original Bid Price =  $\$(8.44-6)*353*0.7852 = \$ 676.37$ . Note that the expected profit is always smaller than the actual profit when the bid was won, and is always larger when the bid was lost. The optimized bid expected profit = (Optimized Price- Marginal Cost)\*Order Size\* Probability of Win at the Optimized Bid Price =  $\$(9.35-6)*353*0.64 = \$ 750.37$ .

We now compare the percent improvement of the latter case over the first two:

- Percent Improvement in Optimized Bid Expected Profits over Un-Optimized Bid Actual Profits =  $(\$ 750.37-\$861.32)/\$861.32 = -0.128 = -12.8 \%$
- Percent Improvement in Optimized Bid Expected Profits over Un-Optimized Bid Expected Profits =  $(\$750.37-\$676.37)/\$676.37 = 0.1101 = 11.01\%$ .

The calculations above were performed for every bid opportunity in the performance test data subset and the average of each measure (over each bid opportunity in the performance test data subset) was used as the performance metrics presented in the next section.

#### **4.5 Model Performance**

For our dataset, the un-optimized bid actual and expected profits over the performance test subset of our dataset were exactly the same. Therefore, we only present the percent improvements in actual profits for each scenario (segmented & unsegmented and three levels of information). The percent improvements are summarized in Figure 7. For the unsegmented case, the results are from the Logit model only. For the segmented case, we used the Logit model for



the worst case and the Power model for the medium and best information cases, where the firm has some knowledge about the competitor’s pricing strategy.

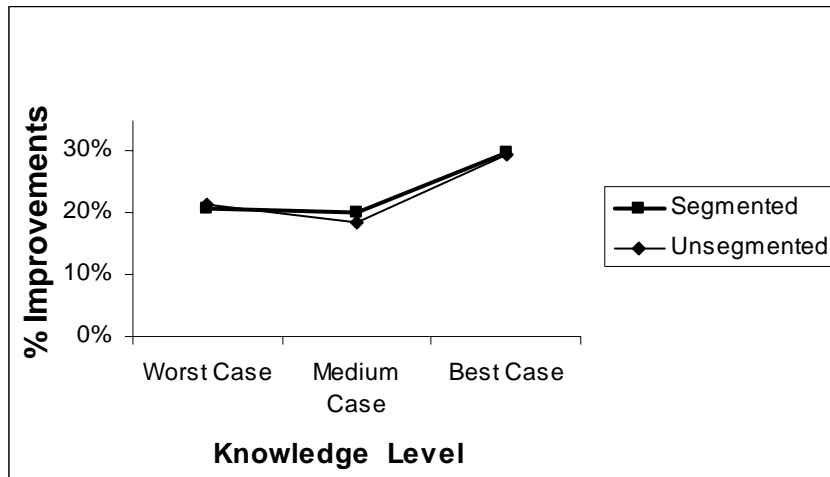


Figure 7: Percent Improvement in Actual Profits

## 5. Observations from Numerical Comparisons and Conclusions

In this section, we summarize our observations based on our numerical comparisons and attempt to answer the question: “Given a particular set of conditions, which CPBRM should a firm use to optimize prices?” We then summarize our work and provide areas for future research. Our observations come with the following caveats; they are based purely on the performance of the models on our available dataset and may not be generalizable to applications different than the ones tested. Thus, a firm should rigorously test the models using their own bid history data before drawing conclusions on the suitability of a particular model for their specific application. Based on the performance on our two industry datasets, we provide four main observations:

**Observation 1.** *If enough historical bid data is available to segment based on the order size, the segmented Power function, adjusted for each discrete customer segment, provides a better fit to the performance data than the segmented Logit function.*

This observation is evident from the results of the goodness-of-fit tests presented in Table 7. If the firm has no knowledge about the competitors’ prices for the current bid opportunity however, the Power function can not be used.

**Observation 2.** *If the firm does not segment based on the order size, the Logit function provides a better fit to the performance data than the Power function.*

This observation is again evident from the results of the diagnostic measures in Table 7.

**Observation 3.** *The model which provides a better fit for the data also provides higher improvements in profits. Also, the order size segmented models fit the data better and resulted in larger profit improvements than the unsegmented models.*

Table 8 summarizes the best fitting model and the corresponding percent improvement in expected profits for both models. One would expect a better fitting model to provide better performance and this is confirmed by the percent improvements in Table 8. It is also evident that the segmented models outperformed the unsegmented models.

Table 8: Comparison of % Improvements

	Model with Better Fit	% Improvements	
Unsegmented Analysis		Logit	Power
Medium Case	Logit	<b>18.45%</b>	16.90%
Best Case	Logit	<b>29.36%</b>	28.76%
Segmented Analysis		Logit	Power
Medium Case	Power	16.94%	<b>20.21%</b>
Best Case	Power	27.46%	<b>29.98%</b>

**Observation 4.** *Incorporating historical competitor prices into a CPBRM does not ensure better performance.*

This observation is evident in Figure 7. In both the unsegmented and segmented results, the ability to perfectly forecast information about the competitor’s pricing results in the largest improvements. Having access to historical data on the competitor’s bids can sometimes make a company worse off however, as evidenced by the worst information case (no historical competitors’ price data) outperforming the medium information case (10 period moving average of competitors’ prices). A quick check of our 10-period moving average forecast (used to provide the competitor’s price estimate) showed the forecast was unbiased but had a large variance in the forecast error. Thus, for this dataset, the past historical competitor’s bids were poor indicators of how the competitor would bid in the future. The use of these poor estimates

led to worse performance using the CPBRMs than if no estimate of the competitor's price was used at all.

## **Conclusions**

A firm adopting CPBRMs for price optimization needs to be aware of their limitations. CPBRMs assume the bid opportunities are exogenous and are not affected by the bid responses suggested through the optimization model. In reality, a firm's pricing strategy may have a significant impact on customer retention, especially if the optimization model recommends consistently pricing higher than the competition for a particular customer class. Also, CPBRMs, and their corresponding optimization models, do not assume any response from the firm's competitors. Instead, they assume the actions of competitors can be determined probabilistically and independently of the decision maker's actions. In reality, competitors may react to a firm's new pricing strategy causing the historical bid opportunity data to be unrepresentative of future bid price responses. To help detect these possibilities, mechanisms should be put in place to monitor and evaluate the performance of the CPBRMs over time. If competitors change their bid-pricing behavior due to the implementation of a CPBRM, more involved models using concepts from game theory should be employed.

In summary, we present two CPBRMs used in practice and explain how they may be used to calculate optimal bid-response prices, discuss how they may be adjusted to accommodate segmentation based on the different levels of information available, and describe how to measure their fit and performance. We illustrate the procedure through a numerical analysis on an industry dataset and, based on these results, offer a set of recommendations about the type of CPBRM a firm should use depending on the availability of past information and the level of competitive knowledge available to a firm.

## Acknowledgments:

The authors wish to thank Loren Williams for his insightful comments, and Robert Phillips for providing the industry data set and for informing us of the Power model. We also thank the participants from presentations at University of Notre Dame and the University of Maryland for their helpful suggestions.

## References

- Aldrich, J. and D. Nelson. 1984. "Linear Probability, Logit, and Probit Models." *Series: Quantitative Applications in Social Sciences*.
- Boyd, D, M. Gordon, J. Anderson, C. Tai, F. Yang, A. Kolamala, G. Cook, T. Guardino, M. Purang, P. Krishnamurthy, M. Cooke, R. Nandiwada, B. Monteiro and S. Haas. 2005. *Manugistics. Target Pricing System*. PatentNo: US 6963854 B1
- Burnham, K. P., and D. R. Anderson, 2002. *Model Selection and Multimodel Inference: A Practical-Theoretic Approach*, 2nd ed. Springer-Verlag.
- Bussey, P, N. Cassaigne, and M. Singh. 1997. "Bid Pricing - Calculating the Possibility of Winning", *IEEE SMC, Orlando USA*, 12-15.
- Dudziak, Bill. 2006. Senior manager in the planning and analysis group of BlueLinx. Panelist in non-traditional industries, Georgia Tech 2<sup>nd</sup> annual workshop on Price Optimization and Revenue Management, May 18<sup>th</sup>.
- Edelman, F. 1965. "Art and Science of Competitive Bidding," *Harvard Business Review* (July/August), 53-66.
- Elmaghraby, W. J. (2006) "Pricing and Auctions in EMarketplaces," forthcoming in the Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era, edited by David Simchi-Levi, S. David Wu, and Z. Max Shen, International Series in Operations Research and Management Science, Kluwer Academic Publishers, Norwell, MA.
- Friedman, L.1956. "A Competitive - Bidding Strategy". *Operations Research* **4**:104-112.
- Gates, M. 1967. "Bidding strategies and probabilities". *The Journal of Construction Division* **93**: 75-103.

- Garrow, L., M. Ferguson, P. Keskinocak, and J. Swann. 2006. "Expert Opinions: Current Pricing and Revenue Management Practice across U.S. Industries." To appear in *The Journal of Revenue and Pricing Management*.
- Hosmer, D. and S. Lemeshow. 1989. Applied Logistics Regression. Wiley Series in Probability & Statistics.
- King, M. and A. Mercer. 1991. "Distributions in Competitive Bidding," *Journal of the Operational Research Society*, **42**(2), 151-155.
- Kniple, Joe. 2006. Director of pricing strategy and solutions at UPS. Panelist in non-traditional industries, Georgia Tech 2<sup>nd</sup> annual workshop on Price Optimization and Revenue Management, May 18<sup>th</sup>.
- Lawrence, R. 2003. "A Machine-learning approach to Optimal Bid-Pricing" Proceedings of the Eighth INFORMS Computing Society Conference on Optimization and Computation in the Network Era, Chandler, Arizona.
- Lilien, G., P. Kotler, and K.S. Moorthy. 1992. Marketing Models, Prentice Hall
- Morin, T.L. and R.H. Clough 1969. "OPBID: Competitive Bidding Strategy Model," *The Journal of Construction Division*: 85-106.
- Papaoannou, V. and N. Cassaigne. 2000. "A Critical Analysis of Bid Pricing Models and Support Tool". IEEE International Conference on Systems, Man, and Cybernetics 3: 2098–2103.
- Phillips, R. 2005a. "Pricing Optimization in Consumer Credit" Presentation at the 2005 INFORMS Annual Meeting, San Francisco, CA
- Phillips, R. 2005b. Pricing & Revenue Optimization. Stanford University Press
- Skitmore, M. 2002. "Predicting the Probability of winning Sealed Bid Auctions: A Comparison of Models". *Journal of the Operational Research Society*. 53: 47-56
- Stark, R. 1971. "Competitive Bidding: A Comprehensive Bibliography," *Operations Research*. 19: 484-490
- Wedel, M. and W. Kamakura. 1998. Market Segmentation: Conceptual and Methodological Foundations. International Series in Quantitative Marketing. Kluwer Academic Publishers.