


F. Multi Criteria Decision Analysis (MCDA)




Multi Criteria Decision Analysis (MCDA)

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Atlanta BeltLine Decision Support Tool Strategic Planning Session, Georgia Tech, : Atlanta, Georgia October 16, 2008

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INTRODUCTION

What is a Multi Criteria Decision Analysis (MCDA)?

- ▶ Conflicts between **criteria**
- ▶ Conflicts between different interpretations of the criteria and different sets of preferences among the **different actors**

The MCDA is a technique to help the decision makers **to choose, rank or sort alternatives** in situations of **multiple and conflicting criteria** (Peckham, 1997).



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Basic Structure of Multi Criteria Decision Analysis

INTRODUCTION

- ▶ A serial process of **defining objectives** Vision & goals
- ▶ Arranging them into **criteria** Setting
DST Strategy
- ▶ Identifying all possible **alternatives**
- ▶ Measuring **consequences** Analysis of
Plan Scenarios
- ▶ **Aggregation** Setting
Priorities

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(1) Decision Matrix

Process of the MCDA

		Alternatives					
		A^1	A^2	...	A^i	...	A^n
Criteria	1						
	2						
	...						
	j				c_j^i		
	...						
	q						

- ▶ Preferences on consequences are captured as values: $v_j(A^i) = f_j(c_j^i)$

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Process of the MCDA

(2) Standardization of Consequences

Since the unit of each criterion is different, the consequence data of all criteria should be transformed **to unify the unit and sign**.

- ▶ Standardization (1)

For negative criteria:
$$v_j^i = \frac{\min_{A^i \in A}(c_j^i)}{c_j^i}$$

For positive preference criteria:
$$v_j^i = \frac{c_j^i}{\max_{A^i \in A}(c_j^i)}$$

- ▶ Standardization (2)

$$v_j^i = \frac{(c_j^i - \min(c_j^i))}{(\max(c_j^i) - \min(c_j^i))}$$

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Process of the MCDA

(3) Preferences on Criteria

- ▶ Expressions of the relative importance of criteria: $w_j > 0$ and $\sum w_j = 1$.
A weight vector is denoted as $w = (w_1, w_2, \dots, w_j, \dots, w_q)$.
- ▶ A popular evaluation method is the simple additive weighting (SAW) method: $V(A^i) = \sum w_j \cdot v_j(A^i)$
Where $V(A^i)$ is the overall evaluation of alternative A^i .
- ▶ Sensitive analysis: the role of each criterion on the selection of alternatives can be understood by changing weights of different criteria.

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Process of the MCDA

(4) Other aggregation models in MCDA

- ▶ Maximin Criterion
- ▶ Maximax Criterion
- ▶ Hurwicz Criterion
- ▶ Bayes Criterion
- ▶ Concordance & Disconcordance Analysis

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Selecting Priority of the Individual Projects

(1) Consequence data

Categories	Criteria	Plan Scenarios for Economic Development			
		ALT1	ALT2	ALT3	ALT4
1. Population & Economic impacts	Density (POP/sq2) (+)	3161	5000	2000	1000
	Growth Rate (POP) (+)	0.05	0.2	0.15	0.1
	Retail Growth Rate (+)	0.15	0.3	0.25	0.2
2. Transportation & Infrastructure impacts	Income (Revenue) (million \$) (+)	2	15	10	7
	Traffic (Commuting time) (-)	25	23	21	19
	Sewer capacity (M-gallon) (+)	4.2	6	3	2
3. Environmental impacts	Water Quality Index (+)	50	48	53	55
	Air Quality Index (+)	0.68	0.65	0.75	0.81
⋮	⋮	⋮	⋮	⋮	⋮

- ▶ Suppose that there are 4 plan scenarios (proposals) for the economic development.

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(2) Standardization

Categories	Criteria	Plan Scenarios for Economic Development			
		ALT1	ALT2	ALT3	ALT4
1. Population & Economic impacts	Density (+)	0.63	1.00	0.40	0.20
	Growth (+)	0.25	1.00	0.75	0.50
	Retail growth (+)	0.50	1.00	0.83	0.67
	Income (Revenue) (+)	0.13	1.00	0.67	0.47
2. Transportation & Infrastructure impacts	Traffic (+)	0.76	0.83	0.90	1.00
	Sewer capacity (+)	0.70	1.00	0.50	0.33
3. Environmental impacts	Water quality (+)	0.91	0.87	0.96	1.00
	Air quality (+)	0.84	0.80	0.93	1.00
⋮	⋮	⋮	⋮	⋮	⋮

► The consequence data should be transformed to unify the unit and sign.

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(3) Setting weights

CRITERIA									
Density	Growth	Retail growth	Income (Revenue)	Traffic	Sewer capacity	Water quality	Air quality	...	Sum
0.06	0.10	0.12	0.20	0.16	0.12	0.14	0.10	...	1.00

► Assume that “Revenue” has the highest weight (0.20) and “Density” the lowest (0.06) according to the survey results.

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(4) Aggregation

Categories	Criteria	Plan Scenarios for Economic Development				
		W	ALT1	ALT2	ALT3	ALT4
1. Population & Economic impacts	Density	0.06	0.038	0.060	0.024	0.012
	Growth	0.10	0.025	0.100	0.075	0.050
	Retail growth	0.12	0.060	0.120	0.100	0.080
	Income (Revenue)	0.20	0.027	0.200	0.133	0.093
2. Transportation & Infrastructure impacts	Traffic	0.16	0.122	0.132	0.145	0.160
	Sewer capacity	0.12	0.084	0.120	0.060	0.040
3. Environmental impacts	Water quality	0.14	0.127	0.122	0.135	0.140
	Air quality	0.10	0.084	0.080	0.093	0.100
⋮	⋮	⋮	⋮	⋮	⋮	⋮
SUM ($V(A^i) = \sum w_j \cdot v_j(A^i)$)			0.566	0.935	0.765	0.675



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(5) Other aggregation methods

	Population & Economic Impacts				Transportation & Infrastructure Impacts		Environmental Impacts		SUM	MAX	MIN
	Density	Growth	Retail Growth	Income	Traffic	Sewer Capacity	Water Quality	Air Quality			
ALT1	0.038	0.025	0.060	0.027	0.122	0.084	0.127	0.084	0.566	0.127	0.025
ALT2	0.060	0.100	0.120	0.200	0.132	0.120	0.122	0.080	0.935	0.200	0.060
ALT3	0.024	0.075	0.100	0.133	0.145	0.060	0.135	0.093	0.765	0.145	0.024
ALT4	0.012	0.050	0.080	0.093	0.160	0.040	0.140	0.100	0.675	0.160	0.012

- ▶ **Maximin Criterion (conservative approach):**
ALT2 (0.060) > ALT1 (0.025) > ALT3 (0.024) > ALT4 (0.012)
- ▶ **Maxmax Criterion:**
ALT2 (0.200) > ALT4 (0.160) > ALT3 (0.145) > ALT1 (0.127)



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► Hurwicz α Criterion:

ALT2 (0.165) > ALT4 (0.123) > ALT3 (0.115) > ALT1 (0.102)

Strategy = $\alpha(\text{Min}) + (1 - \alpha)(\text{Max})$

If we assume that $\alpha=0.75$,

ALT1 = $0.75*(0.025)+0.25*(0.127)= 0.102$
ALT2 = $0.75*(0.060)+0.25*(0.200)= 0.165$
ALT3 = $0.75*(0.024)+0.25*(0.145)= 0.115$
ALT4 = $0.75*(0.012)+0.25*(0.160)= 0.123$

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► Bayes Criterion:

- If the decision maker supposes that each criteria has a same weight,

$$V(A^i) = \frac{\sum v_j(A^i)}{N}$$

ALT2 (0.938) > ALT3 (0.743) > ALT4 (0.646) > ALT1 (0.591)

- If the decision maker consider the weights of all criteria,

$$V(A^i) = \sum w_j \cdot v_j(A^i)$$

ALT2 (0.935) > ALT3 (0.765) > ALT4 (0.675) > ALT1 (0.566)

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► Concordance Analysis

	1. Density	2. Growth	3. Retail growth	4. Income	5. Traffic	6. Sewer capacity	7. Water quality	8. Air quality
ALT 1	0.038	0.025	0.060	0.027	0.122	0.084	0.127	0.084
ALT 2	0.060	0.100	0.120	0.200	0.132	0.120	0.122	0.080
ALT 3	0.024	0.075	0.100	0.133	0.145	0.060	0.135	0.093
ALT 4	0.012	0.050	0.080	0.093	0.160	0.040	0.140	0.100

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► Concordance Analysis

$$K_{ij} = \{j \mid V_{ij} \geq V_{ij'}\}$$

Concordance Set

$$C_{ij} = \{\# \text{ of } K_{ij} \text{ element}\}$$

Concordance Index

$K_{12} = \{7, 8\}$

$K_{13} = \{1, 6\}$

$K_{14} = \{1, 6\}$

$K_{21} = \{1, 2, 3, 4, 5, 6\}$

$K_{23} = \{1, 2, 3, 4, 6\}$

$K_{24} = \{1, 2, 3, 4, 6\}$

$K_{31} = \{2, 3, 4, 5, 7, 8\}$

$K_{32} = \{5, 7, 8\}$

$K_{34} = \{1, 2, 3, 4, 6\}$

$K_{41} = \{2, 3, 4, 5, 7, 8\}$

$K_{42} = \{5, 7, 8\}$

$K_{43} = \{5, 8\}$

$C_{12} = 2$

$C_{13} = 2$

$C_{14} = 2$

$C_{21} = 6$

$C_{23} = 5$

$C_{24} = 5$

$C_{31} = 6$

$C_{32} = 3$

$C_{34} = 5$

$C_{41} = 6$

$C_{42} = 3$

$C_{43} = 2$

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► Concordance Analysis

$$ALT_i = \sum C_{ii'} - \sum C_{i'1}$$
$$ALT1 = (C_{12} + C_{13} + C_{14}) - (C_{21} + C_{31} + C_{41}) = 6 - (6 + 6 + 6) = -12$$
$$ALT2 = (C_{21} + C_{23} + C_{24}) - (C_{12} + C_{32} + C_{42}) = 16 - (2 + 3 + 3) = 8$$
$$ALT3 = (C_{31} + C_{32} + C_{34}) - (C_{13} + C_{23} + C_{43}) = 14 - (2 + 5 + 2) = 5$$
$$ALT4 = (C_{41} + C_{42} + C_{43}) - (C_{14} + C_{24} + C_{34}) = 11 - (2 + 5 + 5) = -1$$

ALT2(8) > ALT3(5) > ALT4(-1) > ALT1(-12)

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SUMMARY

► Maxmin Criterion (conservative approach):
ALT2 (0.060) > ALT1 (0.025) > ALT3 (0.024) > ALT4 (0.012)

► Maxmax Criterion:
ALT2 (0.200) > ALT4 (0.160) > ALT3 (0.145) > ALT1 (0.127)

► Hurwicz α Criterion:
ALT2 (0.165) > ALT4 (0.123) > ALT3 (0.115) > ALT1 (0.102)

► Bayes Criterion:
ALT2 (0.938) > ALT3 (0.743) > ALT4 (0.646) > ALT1 (0.591)
ALT2 (0.935) > ALT3 (0.765) > ALT4 (0.675) > ALT1 (0.566)

► Concordance Analysis:
ALT2 (8) > ALT3 (5) > ALT4 (-1) > ALT1 (-12)

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Conclusions

A Multi Criteria Decision Analysis (MCDA) provides

- ▶ a structured and documented information on the problem
- ▶ a focus for discussion
- ▶ and a means for resolving conflicts

Finding a solution with MCDA implies a process of generating solutions with information on why certain solutions are better.

The information generated can provide a basis for negotiation between the different actors.

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