AN ECONOMIC ANALYSIS OF THE LIGHT WATER BREEDER REACTOR

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Charles Abernathy Sparrow

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AN ECONOMIC ANALYSIS OF THE LIGHT WATER BREEDER REACTOR

Approved:

Joseph D. Clement, Chairman

Ratib A. Karam

Fred A. Tarpley, Jr.

Date approved by Chairman: May 27, 19

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SUMMARY

The Light Water Breeder Reactor (LWBR) is a design variant of the conventional Light Water Reactor. It is anticipated that a light water moderated reactor using U-233 as fissile fuel and Th-232 as the fertile isotope will possess a Fissile Inventory Ratio slightly in excess of unity. Thus, given thorium in quantities sufficient to sustain the reactor, the Light Water Breeder may operate on a self-sustaining fuel cycle.

Unlike the Light Water Reactor, the LWBR system is composed of two distinct reactor types. Both are designed to be used as base load units for the generation of electrical power. The two plants are termed the Prebreeder and the Breeder. As the name implies, the Prebreeder functions before self-sustaining operation can take place. Its design is similar to that of a standard Pressurized Water Reactor. It operates as a converter whose primary conversion function is that of transforming thorium into fissile uranium.

The Breeder utilizes U-233 and thorium fuels, and the discharge quantity of U-233 is anticipated to be slightly greater than the amount charged. Since the Breeder does not utilize U-235 or U-238, it requires neither enrichment nor U_3O_8 for its operation and hence possesses attractive fuel cycle properties. The overall system, however, does use enriching services and U_3O_8 since the Prebreeder design relies heavily upon these.

It is the purpose of this investigation to determine whether the LWBR system may be justified on purely economic grounds. Since resource and service price level is influenced to some extent by the requirement for that resource or service, it is desirable to model the resource requirements as a function of consumption while maintaining the capability to vary parameters arbitrarily.

For an hypothesized system history, resource requirements may be determined through use of a standard materials and services accounting procedure. The computational implementation for this approach is available in the form of the code NUFUEL, written by Artha Jean Snyder of the Energy Research and Development Administration. This program accepts as input the description of any fuel cycle whose fissile isotopes are U-235 and fissile plutonium. Descriptions for an hypothetical Prebreeder fuel cycle were developed for input to NUFUEL and plant commitment parameter variations were performed utilizing reference power scenarios published by the Atomic Energy Commission. The choice of forecast is arbitrary; the resource commitment model allows any combination of yearly increments. Plant mix definition was facilitated by use of a series of calculations which allows the time-dependent commitment of Prebreeders to vary with the level of power addition required and with the ability of the industry to support the Prebreeder fuel cycle. The fractional rate of Prebreeder introduction is presumed to vary with the difference between the industry potential for Prebreeder support and the level of Prebreeder production. This results in a model for the Prebreeder industry whose full response to a requirement is not immediately observed.

It is seen as a result of resource requirement evaluations that the Prebreeder-Breeder system increases resource requirements for a period sufficiently long to counter any economic benefit from the introduction of Breeders. For the span which is anticipated as the useful lifetime of fission as a source of electrical energy, therefore, the Prebreeder/Breeder system appears to yield costs greater than the system composed only of Light Water Reactors.

The resource requirements approach to economic analysis relies upon the tacit assumptions that power generation requirements and prices are known. As the history of both uranium and enrichment prices shows, fuel charges may vary with the consumption. This is particularly true of U_3O_8; the same conclusion would be anticipated to hold in the event that enriching facilities were privately owned and the prices determined by a market in which there is unsatisfied demand. The consumption depends upon the mix of reactors while total cost for the nation's energy generating system depends upon the mix of reactors selected to fulfill the nuclear-electric generating requirements. An approach which allows the simultaneous determination of prices and the optimal reactor mix is based upon formulation of the energy-generating cost calculation as a variational problem. The implementation of these concepts in computational form is known as dynamic programming. An algorithm is developed in which the number of reactor types and power requirements are assumed and optimal plant commitment schedules are generated for any set of hypothesized economic conditions. The application of this algorithm to the system containing Light Water Reactors, Prebreeders, and Breeders is made and costs are generated. An approximation to industrial behavior is made by allowing specification of an arbitrary set of constraints upon plant commitment which may reflect assumptions concerning minimum and maximum levels of production in support facilities required for Prebreeder or Light Water Breeder operation.

It is concluded that the Light Water Breeder system shows no evidence of having the potential for strong economic competition with Light Water designs. The reason for this penalty is attributable to the cost differential between the assumed configurations for the Light Water Reactor and the Prebreeder. Discounting of costs tends to eliminate any advantage arising from the lower Breeder fuel cycle costs, thus also diminishing the importance of price differences between systems containing large numbers of Breeders and those composed primarily of Light Water Reactors.

It is to be noted that this analysis does not explicitly address considerations associated with waste disposal and the social costs identified with the buildup of uranium isotopes in the Light Water Breeder as opposed to the actinide element production which is characteristic of Light Water Reactors. The mechanism for approximating such a difference is available, however, in the treatment of the reprocessing function. Modifying either the unit cost associated with discharge fuel or the quantity of spent fuel serves to delineate the difference between costs of waste handling for the various reactor types.

The economic analysis of the mix of reactor types is not limited to the particular system of Light Water Reactors, Prebreeders and

Breeders for which the detailed investigation was performed. Available data for Prebreeder designs are most detailed for a design using moderately enriched uranium as fissile fuel. A design utilizing plutonium as the fissile fuel has also been proposed. Data available for this design are not sufficient to permit an economic analysis. Moreover, such an analysis would contribute little to present understanding of fuel cycle economics since the most profitable utilization of plutonium is its employment as fuel for the Liquid Metal Fast Breeder Reactor.

The use of plutonium generated by Light Water Reactors in Prebreeders as a means for generating U-233 may be considered with the objective of diminishing dependence upon the use of enriched uranium and replacing U-235 by U-233 in Light Water Reactor designs. An analysis of the use of plutonium rather than U-235 in Prebreeder reactor designs may furnish the basis for a further study. An economic analysis of such a fuel use scheme requires a precise definition of the benefits of the thorium-U-233 fuel cycle relative to those of the U-235/U-238/Pu cycle. It is anticipated that any fueling scheme for Light Water Reactors other than that which uses recycle plutonium will be less attractive economically due to the discounting of future benefits as well as to greater requirements for fuel and services.

It is found as a result of these investigations that the optimal mix of nuclear-electric facilities is relatively insensitive to price variations in any of the fuel cycle components. The costs associated with systems containing Prebreeders and Breeders in some quantity are higher than those containing only Light Water Reactors, but not sufficiently high to justify termination of the Light Water Breeder Reactor Program. Benefits are assumed herein to be limited to those associated with the generation of electric power. As Admiral Rickover (1971) has indicated, this assumption may not be valid for a program in developmental stages. It is concluded, therefore, that the cost of electric power generation is not materially affected by inclusion of an LWBR system in the energy-generating mix. Since the benefits defined for the economic analysis are limited to those associated with nuclear-electric energy generation, the ultimate effective benefit deriving from the Light Water Breeder Reactor development may be sufficient to make it yield benefits as great as those which might arise from continued pursuit of the Light Water Reactor program.

CHAPTER I

PERSPECTIVES ON NUCLEAR POWER COMMITMENT

Nuclear Power Growth

Forecasts of nuclear power growth are difficult to obtain if the intent is to use the quantities as accurate measures of the response of the consuming society to the pressures of the life style which it adopts. Power growth forecasts are useful, however, as constraints upon unit scheduling, just as enrichment, reprocessing availability, and U_3O_8 are factors which limit deployment of reactors. Table 1 gives the spectrum of forecasts of growth presented by the Atomic Energy Commission in its 1974 report "Nuclear Power Growth 1974-2000."

The four cases in Table 1 correspond to varying assumptions concerning licensing time, labor productivity, and growth rate of electricity use. Case A, the lowest of the four, assumes that delays in bringing nuclear plants on line characterize the industry. Effectively about 10 years is estimated for the licensing and construction of a generating unit. Case B assumes that present regulatory and plant construction times will be shortened somewhat, the result being that eight years' time will be consumed in the entire licensing, construction and startup effort. Case C assumes even more optimistic estimates of licensing and construction procedures. In particular, the project time would be shortened to six years by virtue of parallel processing of

Table 1. Nuclear Power Growth in the United States (GWe)

Year	Case A		Case B		Case C		Case D	
	Additions	Cumulated	Additions	Cumulated	Additions	Cumulated	Additions	Cumulated
1974	3.4	27.5	8.7	32.8	18.1	42.1	8.7	32.8
1975	15.9	43.3	14.5	47.3	9.8	52.0	14.5	47.3
1976	9.8	53.1	7.0	54.3	8.5	60.5	7.0	54.3
1977	7.4	60.5	7.0	61.3	4.9	65.4	7.0	61.3
1978	4.0	64.5	8.3	69.7	7.4	72.7	8.3	69.7
1979	7.9	71.7	11.8	80.7	14.6	86.5	11.8	80.7
1980	13.4	85.0	21.5	102.1	26.0	112.4	21.5	102.1
1981	29.2	114.2	32.2	134.3	32.0	144.4	24.9	127.
1982	28.4	142.6	28.1	162.4	30.1	174.4	28.	155.
1983	31.2	173.8	25.8	188.2	34.6	209.0	29.	184.
1984	31.2	205.0	34.7	222.9	31.	240.	32.	216.
1985	25.9	230.9	37.1	260.0	35.	275.	34.	250.
1986	34.1	265.	42.	302.	48.	323.	38.	288.
1987	35.	300.	43.	345.	57.	380.	43.	331.
1988	35.	335.	48.	393.	60.	440.	44.	375.
1989	37.	372.	52.	445.	65.	505.	48.	423.
1990	38.	410.	55.	500.	70.	575.	52.	475.
1991	42.	452.	60.	560.	72.	647.	52.	527.
1992	43.	495.	61.	621.	74.	721.	56.	583.
1193	43.	538.	65.	686.	77.	798.	57.	640.
1994	42.	580.	66.	752.	80.	878.	60.	700.
1995	40.	620.	68.	820.	82.	960.	60.	760.
1996	45.	665.	69.	889.	85.	1045.	63.	823.
1997	46.	711.	74.	963.	87.	1132.	65.	888.
1998	48.	759.	75.	1038.	89.	1221.	65.	953.
1999	46.	805.	80.	1118.	90.	1311.	67.	1020.
2000	45.	850.	82.	1200.	89.	1400.	70.	1090.

environmental and safety review reports. Standardized plant designs are assumed in order to expedite the Safety Analysis Review procedure. Case D is similar to Case B, the difference lying in the fact that the assumed electricity growth rates are slightly different. For this last case, electricity production in the near term is reduced by attenuating production from oil and gas fired plants.

The underlying assumptions for these four cases are given in Table 2. It may be observed that the electric generating capacity per capita varies significantly from case to case. Population predictions from the Census Bureau's "Series E" projection were employed to develop the energy and economic growth forecasts.

It is clear that none of these cases may be achieved without the discovery of additional uranium resources and the establishment of more enriching capacity. If uranium resources are not found and developed, neither will private investment in enriching capacity take place. Therefore, the long-term predictions are valid only when constraints upon uranium availability are removed. Similarly, the availability of capital acts as a constraint upon both the exploration and enriching sectors. A long-range economic analysis will include effects due to the price elasticity, since rates will no doubt rise as a result of substantially increased prices and costs in the nuclear fuel cycle. Further, the available nuclear generating capacity will be limited by these factors, and the expected requirements perceived by investors in the nuclear market will be changed as a result of utility investment activity. Given the constraints upon the fuel cycle, therefore, the

Table 2. Energy Forecasts for the U. S. Consumption and Generating Capacity

Parameter	Case	1960	1970	1975	1980	1985	1990	1995	2000
Energy Consumed (Million Btu/Capita)	A	247.	329.	357.	378.	401.	429.	462.	499.
	B	247.	329.	372.	428.	485.	558.	635.	719.
	C	247.	329.	376.	434.	497.	569.	650.	737.
	D	247.	329.	364.	399.	438.	494.	563.	642.
Fraction for Electricity Generation	A	0.18	0.24	0.29	0.33	0.37	0.42	0.46	0.51
	B	0.18	0.24	0.29	0.31	0.34	0.40	0.45	0.50
	C	0.18	0.24	0.29	0.34	0.38	0.43	0.49	0.54
	D	0.18	0.24	0.29	0.32	0.36	0.41	0.46	0.50
Energy Consumed for Electricity Generation (Million Btu/Capita)	A	44.2	80.3	105.	125.	148.	180.	215.	253.
	B	44.2	80.3	107.	133.	166.	220.	283.	357.
	C	44.2	80.3	111.	147.	189.	246.	316.	399.
	D	44.2	80.3	107.	129.	156.	201.	257.	324.
Apparent Capacity Factor	A	0.49	0.52	0.50	0.49	0.50	0.51	0.51	0.52
	B	0.49	0.52	0.50	0.49	0.50	0.51	0.51	0.52
	C	0.49	0.52	0.50	0.49	0.50	0.51	0.51	0.52
	D	0.49	0.52	0.50	0.49	0.50	0.51	0.51	0.52
Heat Rate (Thousands Btu/kWh)	A	10.7	10.5	10.2	10.1	10.0	9.8	9.8	9.6
	B	10.7	10.5	10.2	10.1	10.0	9.8	9.8	9.6
	C	10.7	10.5	10.2	10.1	10.0	9.8	9.8	9.6
	D	10.7	10.5	10.2	10.1	10.0	9.8	9.8	9.6
Total Electric Generating Capacity per Capita (kW/Capita)	A	0.97	1.67	2.36	2.88	3.33	4.14	4.90	5.81
	B	0.97	1.67	2.41	3.07	3.76	5.07	6.45	8.19
	C	0.97	1.67	2.50	3.38	4.25	5.67	7.27	9.22
	D	0.97	1.67	2.41	2.99	3.52	4.62	5.85	7.45

Table 2. Continued

Parameter	Case	1960	1970	1975	1980	1985	1990	1995	2000
Total Electric	A	168.	341.	510.	655.	800.	1040.	1280.	1575.
Generating	B	168.	341.	520.	700.	903.	1275.	1685.	2220.
Capacity (GWe)	C	168.	341.	540.	770.	1020.	1425.	1900.	2500.
	D	168.	341.	520.	680.	865.	1160.	1530.	2020.
Total Nuclear	A	0.02	5.8	43.3	85.0	230.9	410.	620.	850.
Generating	B	0.02	5.8	47.3	102.1	260.0	500.	820.	1200.
Capacity (GWe)	C	0.02	5.8	52.0	112.4	275.0	575.	960.	1400.
	D	0.02	5.8	47.3	102.1	250.0	475.	760.	1090.

Light Water Breeder exists as a possibly attractive alternative which allows the energy-generating sector to expand its nuclear capacity beyond that which can be supported by the enriching and fuel milling capability.

Economic Considerations

Decision problems are usually cast in the form of the minimization of some objective function under conditions of constraint. However, in the nuclear fuel cycle, the constraints are both time-varying and arbitrarily fixed. The most obvious constraints are those pertaining to uranium. First, the market for uranium is limited to defense and commercial power generation. Military purchases and the records thereof are not available to the general public. The price history of U_3O_8 indicates, however, that the purchase of uranium for military use is not a strong component of yellowcake sales. For if it were, there would not have been the price decline of the late sixties which forced the Atomic Energy Commission to purchase uranium and stockpile it for future use. Therefore, it may be assumed that the market for uranium is controlled by the requirement for uranium as a commercial fuel for electric power generation.

Requirements for uranium in a commercial power generation environment depend upon the number of reactors scheduled. It is important to observe that the number of reactors depends in turn on the base load requirements anticipated by an electric utility. Unlike many other enterprises, electric utilities cannot respond rapidly to short term perturbations in their system load requirements. Therefore, at practically all levels of their enterprise, a reserve capacity is maintained. For example, on a daily basis a spinning reserve is kept so that immediate response can be made to abrupt changes in load. Peaking units are employed to generate electricity when the system load varies on a daily basis, arising from changes in weather conditions. Excess capacity is maintained so that growth in electrical demand may be met and so that forced outages do not have a disastrous effect upon electrical power generation. Therefore, the purchaser, or class of purchasers, of commodities such as uranium, separative work, and fuel element fabrication, is confined to those whose short-term ability to respond is limited and whose long-range plans are definite. Thus, when a utility decides to build a reactor and to amortize it over a period of, say, 25 years, it correspondingly goes forth to purchase uranium to be used in that reactor for that period of time. For otherwise, the shutdown of an income-generating unit before its economic service life is filled would result in a general rate increase by the customers of that utility and in a review of management practices by the appropriate regulatory commission. Therefore, when commitment of a unit is made, the utility will attempt to purchase uranium for a substantial portion of its lifetime. A similar technique is followed with respect to long-term purchase of coal. Nowhere can this effect be seen more clearly than in the separative work contracting market, where commitments for separative work have been made as far in advance as planning permits (U 1482, 1975).

Analysis of the economic aspects of the nuclear fuel cycle should take into account the exceptionally long contracting obligation which exists by virtue of commitment of a light water reactor. Thus, the supply of uranium is a constraining factor upon deployment of nuclear plants. Each process in the nuclear fuel cycle is capital-intense. Therefore, the initial investment in any support facility of the nuclear fuel cycle will necessarily require amortization over a period sufficiently long so that the uncertainty of the industry must be a large factor in making an investment decision. And although the need for electrical energy will not diminish except in the face of a real economic disaster, it is not at all assured that presently-operating light water reactors, subject to regulation at both state and federal levels, will be allowed to continue as an effective base load energy source.

Projections of energy growth are taken as the starting point in the analysis of the nuclear-electric economy. The generation of such projections is itself a substantial exercise in economics, since it is necessary to take into account housing patterns, migration, gross national product, and population growth statistics in order to derive a realistic power requirements curve. Less elegant techniques are also used. Among the most frequently seen is the assumption of constant growth rate. This growth rate is either extrapolated from past history or else is the subjective judgment of the investigator. In many economic studies, the distribution of resources is independent of the growth rate or the level of use of those resources; thus such simply-derived curves may often be used without distorting the conclusions. It must be noted, however, that an assumed growth rate is not necessarily the same as a prediction or forecast, which should be based upon a number of factors, both social and economic. Further, there are two rates that are of interest to the energy analyst. First is the prediction of total energy consumption. The correlation between electrical consumption and economic growth has been noted (FEA, 1976). Whether this correlation will continue to hold, and whether economic growth must be maintained are issues worthy of serious consideration. However, with an industrial economy having important segments which are also energy-intensive, it must be assumed that the same mechanism which led to the energy consumption vs economic growth curve during earlier years still predominates. Second, and of direct importance to an analysis of nuclear fuel cycle economics is the consumption of electrical energy. It is anticipated that the growth in electrical energy consumption will be greater than that attributable to total energy consumption. This latter consideration derives from the availability and transportability of electrical energy as opposed to other forms. Further, it is anticipated that the supply of natural gas is sufficiently small that functions now served by natural gas will be assumed by electricity.

The question of interfuel substitution has been under study for several years. The RAND Corporation has published work which shows that the demand for both electricity and natural gas is price-elastic (Anderson, 1973). The hypothesis that fuel (or energy) utilization is affected by price is strengthened by the results of a study performed by Ciliano, Erickson and Spann (1974). In their analysis, the energy-intensive industries were examined for own-price and cross-price effects.

The results, which were statistically significant, showed clearly that fuel price effects were highly important in the energy-intensive industries, and hence that a thorough forecasting effort for the prediction of energy use should take this factor into account. Under conditions of rising relative energy cost, it is reasonable to presume that total energy growth and electrical energy growth will both diminish over the historical pattern which has been established during a period of relatively low energy cost. Further complicating the long-term behavior of utility purchases is the fact that the composition of American business is changing. It is believed that, as technology becomes increasingly more sophisticated, the commercial sector will become the dominant user of electrical energy. Significant savings of energy are possible provided that the nation decides, for economic reasons or for others, to reduce considerably its production of aluminum and plastics. If, corresponding to the decrease in the industrial sector the commercial and governmental sectors were to experience compensatory growth, the overall energy consumption would be reduced.

Three energy growth scenarios have been studied as part of the "Energy Policy Project of the Ford Foundation" (1974). The features of these scenarios are shown in Table 3. The quantities are not arbitrarily chosen, although the statement is made that work on the scenarios, as presented here, is sufficiently involved as to preclude the immediate derivation of alternatives. The historical growth rate shown here is the average growth rate of total energy consumption over the period 1950-1972: 3.4 percent. Even within the historical growth scenario, it is not well known what the distribution of energy sources will be. Regardless, nearly all published forecasts indicate that the meeting of this energy need through the year 2000 will require a substantial commitment to nuclear power. This implies a commitment to light water reactors which are fueled with slightly enriched uranium.

Table 3. Energy Growth Scenarios

Name	Historical Growth	Technical Fix	Zero Energy Growth
Annual Energy Cons. in Year	(10^{15} Btu/yr)	(10^{15} Btu/yr)	(10^{15} Btu/yr)
1970	67.4	67.4	67.4
1975	81	81	81
1985	115	96	93
2000	185	118	100

The utility decision maker, given an expected power generation requirement for his system, must assess all available data and make a decision which results in the purchase of either a fossil fuel unit or a light water reactor. The overriding variable is usually the fuel cycle cost. Coal contracts are usually written over a period of many years in order to ensure a steady supply without the accumulation by the utility of a large inventory at any time. Yellowcake contracts have not always shown this trend, partially because the uranium resources are not as extensive as those of coal and partially because utility purchasers anticipated that uranium prices would remain at the level about which the economic computations were performed. There is some evidence that this practice underwent a change during 1974 and 1975, when some of the larger utilities took steps to ensure a long-term uranium supply for their Light Water Reactors (Nuclear Industry, 1974). While much study of the costs associated with the nuclear fuel cycle has been carried out, relatively little has been put forth concerning the economic behavior, that is, the cause and effect relationships which govern prices and availabilities.

The most obvious reason for the slow advancement of analytical effort in this regard is the paucity and unreliability of data from which to draw conclusions. Generally, dynamic analyses use price as the independent variable. However, as has been shown by the behavior of the uranium market in the 1973-1976 period, prices are not necessarily an adequate reflection of the equilibrium supply-demand condition. Further, the price of separative work, though controlled in part by the United States Government, is subject to variation for reasons unrelated to market conditions. The price for the reprocessing of fuel has yet to be developed, since there is virtually no useful commercial operating experience in the reprocessing industry. It is therefore necessary to examine in some detail the underlying factors which will affect the more important fuel cycle prices, and hence activities in the nuclear market. First, it must be noted that the fuel cycle cost is a relatively small (of the order of 10 percent) part of the electric power generation market. Therefore, large variations in price can be tolerated before price becomes a factor in the choice of nuclear or an alternative energy source.

The Light Water Breeder Reactor

The Light Water Breeder Reactor has been under development by the Office of Naval Reactors for about 10 years. It was thought in the early days of reactor development that the number of neutrons per absorption, η, was too small to sustain a breeding mode. More recent measurements of the η of U-233, however, have led to the conjecture that breeding is possible in a light water moderated lattice, provided that the fuel spacing and fuel composition are properly controlled. The Light Water Breeder is designed to operate upon the thorium-U-233 fuel cycle. There are two attractive features of this fuel cycle. One is the extension of the resource base for fission reactors without dependence upon plutonium through a near-50% thorium utilization. As a corollary benefit, the use of thorium as the fertile fuel also diminishes the production of long-lived actinide elements, although the use of U-233 as the fissile fuel results in buildup of U-234. The other is the use of technology which has been proved to be effective through commercial and military use. From an economic point of view, this suggests that the Marginal Cost of bringing a Light Water Breeder System on line is considerably less than that of an alternative system of similar capacity. There is no a priori reason to believe that the Light Water Breeder system is the single most economical of all reactor designs. This claim has been made for the Liquid Metal Fast Breeder Reactor, owing to its expected high breeding ratio and to its low fuel cycle cost arising from the use of U-238 available from tails of diffusion plants to fuel the blanket.

Other designs also may be attractive from the viewpoint of conservation of energy resources. Most notable of the untested designs is the Heavy Water Breeder Reactor which promises a Fissile Inventory Ratio in excess of that possible with the Light Water Breeder. D_2O technology is not new, having been employed successfully in Canada on the natural uranium-fueled CANDU reactors. The technology of utilizing plutonium as a fuel in either a Light Water Breeder or a Heavy Water Breeder system is also a possibility which merits serious consideration; however, the Marginal Cost associated with such a design is likely to be greater than the corresponding cost based upon a uranium-only cycle. The reason for this, at least in the early years of development of breeder reactor concepts, is the additional safety and licensing effort which must be expended. Therefore, plant selection may well tend toward uranium-fueled rather than plutonium-fueled concepts if commercial considerations are to be used as controlling factors.

The objective of this research is the determination of the economic characteristics of the Light Water Breeder Reactor system. This is approached by considering the LWBR system as a subsystem of one containing conventional Light Water Reactors as well as the Prebreeders and Breeders. Data pertaining to the possible reactor design are found in the Draft Environmental Statement (ERDA-1541) for the Light Water Breeder Reactor which is scheduled for operation in the Shippingport facility for a two-year test period to determine feasibility of breeding in the Breeder part of the LWBR system. The design of the reactor does not follow an existing commercial design. It is assumed that the use of the reactor specifications contained in the Draft Environmental Statement will be sufficiently close to any commercial design so that differences in physical design will affect the economics only slightly, as the properties of the reactors cannot be vastly different from the existing feasible design and still attain the objective of U-233 production under which the preliminary analysis was performed. Further, the main technology for comparison of quantities is the existing Light Water Reactor System. Therefore, conditions which might affect the physical design of the LWBR reactors will also possibly have an effect upon the LWR system. An example is licensing restrictions which may be applied to both reactor types, since the physical processes are essentially the same. It is necessary to examine the physical resource requirements for the LWBR system under conditions of electrical demand which are anticipated to exist within the United States for several years hence. Fuel cycle resources and service requirements are computed for a specified mix of reactor types, and the totals are retained for comparison. The comparison is made with the non-LMFBR component of power generation assigned only to Light Water Reactors.

The LWBR system is composed of two reactor types. The Prebreeder is a converter reactor whose dual function is to produce power, like any other LWR, and to transform thorium into U-233 as well. The mechanism for accomplishing this is the absorption of neutrons by the fertile isotope Th-232, the beta-decay to Pa-233, and the second beta decay to U-233. Following the discharge cycle, the U-233 is separated from the thorium in a reprocessing plant. Reprocessing costs may be higher for Prebreeder fuel than for spent fuel from LWR's since two distinct fuels will necessitate two parallel separation processes. Available data for reprocessing in general are scarce; however, the estimate of twice the cost of U-Pu separation has been made for order of magnitude computations (Bethe, 1975). The benefit of an LWBR system begins to be apparent after several years of operation of the Prebreeder. When enough U-233 has been obtained to sustain a Breeder reactor, the Prebreeder may be backfitted to the Breeder configuration and a self-sustaining fuel cycle employed. Alternatively, instead of backfitting an existing plant as enough fuel becomes available, it might be more feasible to construct as part of the scheduled capacity additions a Breeder reactor, retaining the existing Prebreeder to continue to generate U-233 for subsequent capacity additions. The Prebreeder design used as a reference for computation is based upon Pressurized Water Reactor characteristics. In order to maintain the power density required for generation of one GW(e), uranium enrichment must be greater than 10 percent. For a refueling cycle of one year, an enrichment of 13.5 percent is specified for the equilibrium fuel cycle. This results in greater separative work and U_3O_8 requirements per unit energy for the Prebreeder than for the Light Water Reactor.

Because the Light Water Breeder System is still a conceptual entity, many cost data are yet unavailable. For the most part, costs obtained from the operation of Light Water Reactors may be used to estimate the expense involved in the deployment of the thorium-U-233 variant.

Physical data for the Prebreeder and Breeder are given in Tables 4 and 5, respectively. Only the requirements corresponding to the yearly refueling schedule are given.

Table 4. Prebreeder Fuel Cycle Requirements (1000 MW(e) Reference Design)

Fresh Fuel Assay (wt % U-235)	13.5
Spent Fuel Assay (wt % U-235)	7.6
Uranium Ore Supply (MT)	79,000
Thorium Ore Supply (MT)	5,900
U_3O_8 Supply (MT)	158
Natural UF_6 (MT)	199
Separative Work (MTSWU)	165
Enriched UF_6 (MT)	16.5
Enriched UO_2 (MT)	12.6
Fertile ThO_2 (MT)	29.6
Plutonium Consumed (kg)	--

Table 5. Breeder Fuel Cycle Requirements (1000 MW(e) Reference Design)

UO_2 (MT)	1.56
Fissile UO_2 (MT)	1.22
Fertile ThO_2 (MT)	80.7
Total Heavy Metal (MT)	72.4
Makeup ThO_2 (MT)	1.91

Some perspective on the resource requirements may be obtained by comparison with the equilibrium LWR fuel cycle requirements. These are given in Table 6, which may be compared with the entries in Table 4.

Table 6. LWR Fuel Cycle Requirements
(1000 MW(e) PWR Reference Design)

	Without Pu Recycle	With Pu Recycle
Fresh Fuel Assay (wt % U-235)	3.1	2.6
Spent Fuel Assay (wt % U-235)	0.90	0.75
Ore Supply (MT)	68,000	56,600
U_3O_8 Supply (MT)	135	113
Natural UF_6 (MT)	170	142
Recycled UF_6 (MT) (before enriching)	38	38
Separative Work (MTSWU)	114	88
Enriched UF_6 (MT)	39.6	39.4
Enriched UO_2 (MT)	30.4	30.3
Fuel Loading (MTU)	26.5	26.4

Fuel loadings with the plutonium recycle option are somewhat arbitrary. For the Pressurized Water Reactor, an equilibrium loading of 485 kilograms fissile plutonium has been hypothesized for the environmental study of plutonium recycle in Light Water Reactors, (NUREG-0002, 1976). Assuming a plant design in which the total heavy metal loading is 76,425 kilograms and a fissile plutonium to U-235 value of 0.9, the uranium enrichment is calculated to be approximately 2.6 weight percent U-235. The corresponding spent fuel assay may be estimated by assuming depletion of each fissile species to occur in the same fraction.

It will be observed that the yellowcake supply required for the model Prebreeder will be greater than that for the Light Water Reactor. However, the LWR fueled with U-235 or a mixture of U-235 and fissile plutonium cannot breed. Hence, given a finite uranium supply, a system consisting of Light Water Reactors alone will have a finite lifetime. On the other hand, the Prebreeder produces enough fuel to maintain a self-sustaining system after approximately 18 years. Thus, the penalty associated with a higher fuel cycle cost and greater resource utilization is offset by the availability of a power source which does not require large quantities of non-renewable resource input after its induction period.

It is noted that the envisioned system containing LWR's and Liquid Metal Fast Breeder Reactors is also eventually self-sustaining. There are a variety of systems which may be used in the economic analysis of the Light Water Breeder Reactor. The system analyzed in this research effort was chosen for compatibility with the study reported in the LWBR Draft Environmental Statement. It is assumed that the introduction of a breeder operating on the thorium-U-233 cycle will have a small impact upon the economics of a plutonium breeding system. Since, at the time of this writing, the plutonium breeder has not reached production status as a mode of energy generation, the simplified mix consisting of three reactor types is a reasonable approach to the analysis of the economic character of a Prebreeder/Breeder system. An analysis of the plutonium-fueled Prebreeder, however, would appropriately include the LMFBR as a competitor for fuel.

Fuel cycle processes for the Light Water Reactor are shown in Figure 1. The recycle of plutonium in Light Water Reactors is shown by the dashed arrow as an optional part of the fuel cycle. Bred plutonium may alternatively be stored for use in fast reactors or may be retained as waste.

Materials flow in the Prebreeder fuel cycle is shown in Figure 2. It will be noted that U-233 and ThO_2 separated from spent fuel in the reprocessing stage are not immediately put into the fuel cycle but are stored for later use by the Breeder. Reprocessing for the Prebreeder involves both the Purex and the Acid Thorex processes.

Figure 3 illustrates the Light Water Breeder Reactor materials flow. It may be immediately observed that conversion and enrichment are not elements of the LWBR fuel cycle. Thorium and uranium from Prebreeder storage are used for the LWBR fuel cycle. The Acid Thorex process alone is used for this, since the breeder product is composed primarily of thorium and U-233.

The Prebreeder fuel cycle is similar to that for the Light Water Reactor. However, for a given energy output, the Prebreeder requires more separative work, more U_3O_8, a fuel fabrication facility which permits both uranium and thorium, and a reprocessing facility which has parallel lines for the uranium-plutonium separation process and the uranium-thorium separation process.

The requirement for fuel storage arises from the need to save separated thorium and uranium for Breeder operation and from the radioactive nature of U-232 daughter products. U-232 is produced in a Prebreeder or a Breeder primarily through the $^{232}Th(n, 2n)^{231}Th$ reaction,

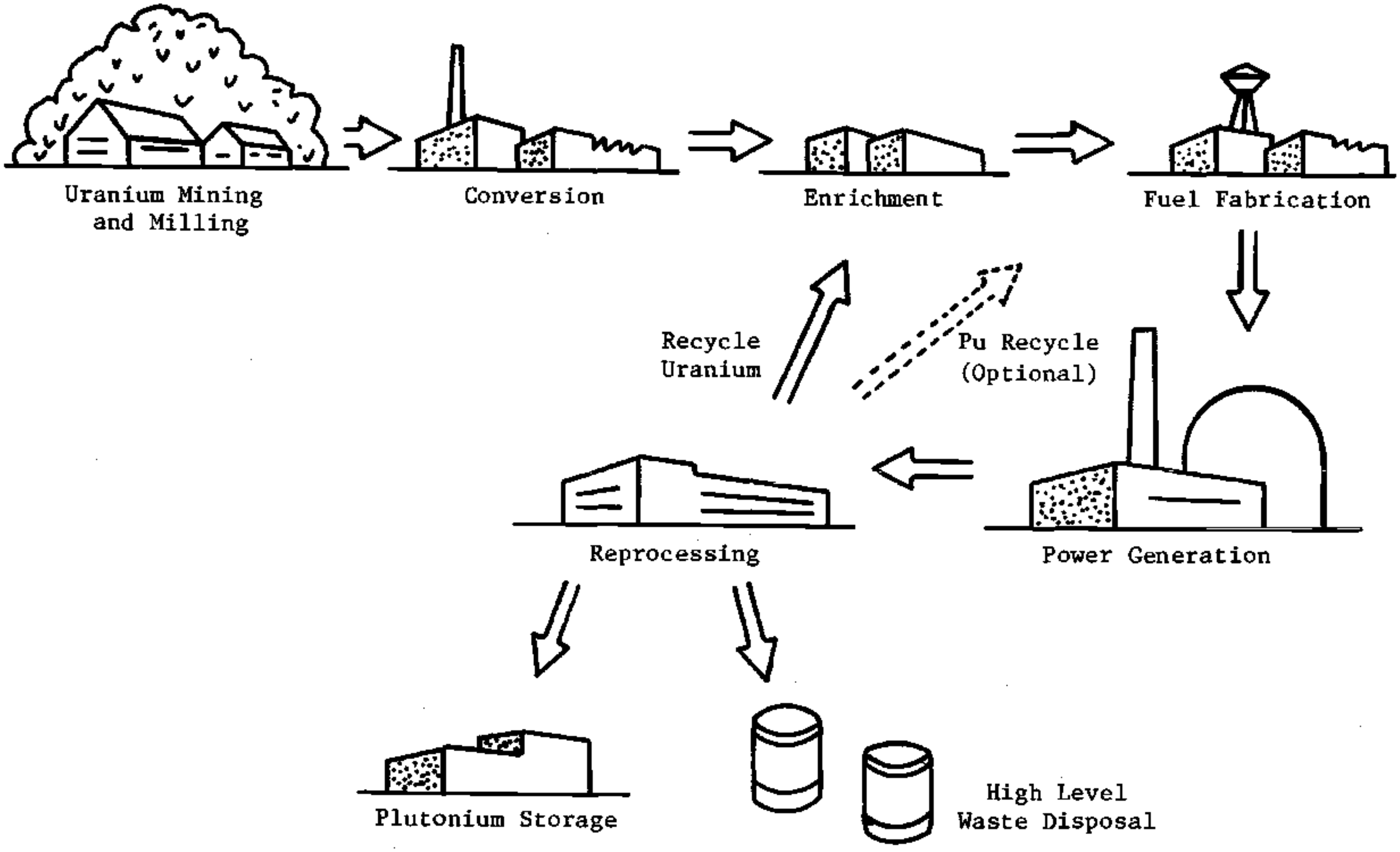

Figure 1. Material Flow in the Light Water Reactor Fuel Cycle

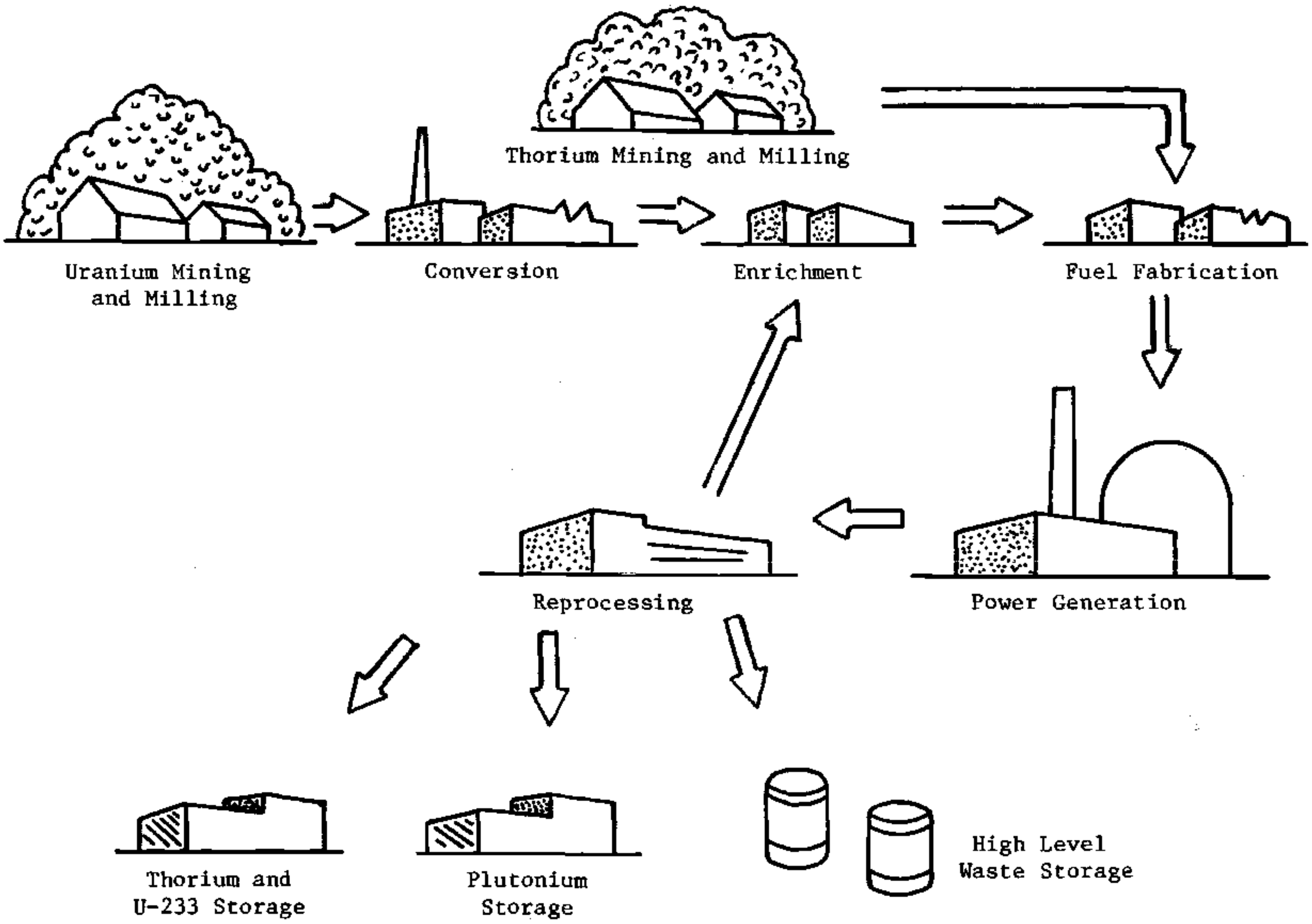

Figure 2. Material Flow in the Prebreeder Fuel Cycle

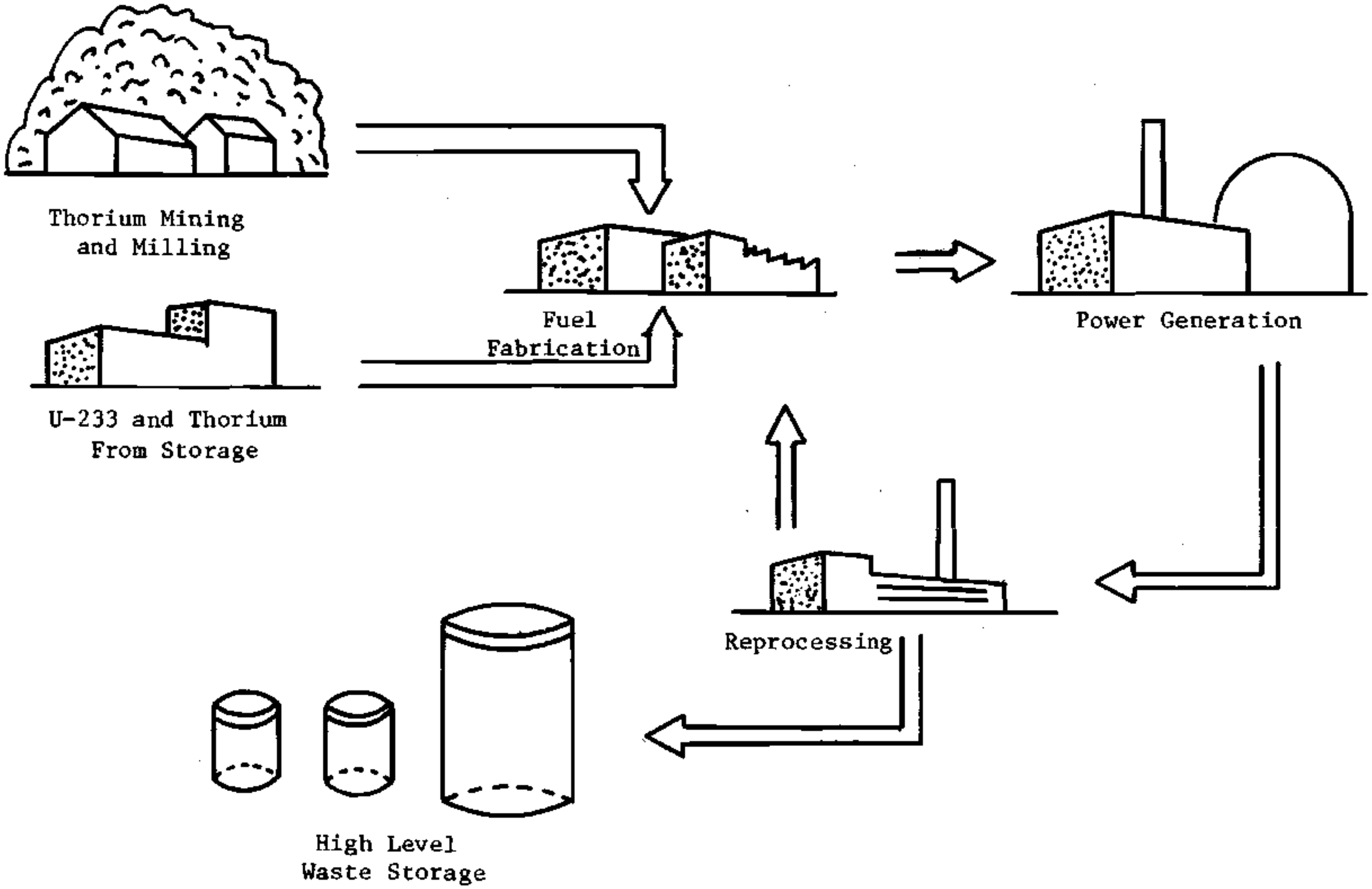

Figure 3. Material Flow in the Light Water Breeder Fuel Cycle

followed by beta decay to Pa-231 with a 25 hour half life and the absorption of a neutron yielding U-232. The decay chain for U-232 is shown in Figure 4. It may be noted that the equilibrium is determined by the half lives of the first two elements in the chain, U-232 and Th-228. Other daughter products have half lives which are short relative to that of Th-228. Storage times of 10 years or longer allow the Th-228 produced in the reactor to decay. In this span, U-232 and Th-228 reach transient equilibrium.

Fabrication of LWBR fuel is hindered by the presence of Tl-208, whose gamma ray necessitates shielding in excess of that required for LWR and Prebreeder fuel fabrication. One possible method for reducing shielding requirements is the chemical separation of uranium and thorium shortly before fabrication. Buildup of Tl-208 in the separated uranium is controlled by the 72-year and 1.91-year half lives of the first two members of the decay chain, thus reducing to a minimum the high energy gamma radiation.

It is presumed that the Prebreeder will use virgin thorium. In principle, recycle thorium could be used, but the level of gamma activity would require fuel fabrication and handling techniques more costly than those associated with the Light Water Reactor or the Prebreeder. Accordingly, the Prebreeder fuel cycle is designed to operate with freshly-mined thorium. The Breeder portion of the system assumes the additional cost associated with the handling of high level gamma emitters arising from Th-228 decay. Hence, fabrication costs are higher for the LWBR than for the Prebreeder.

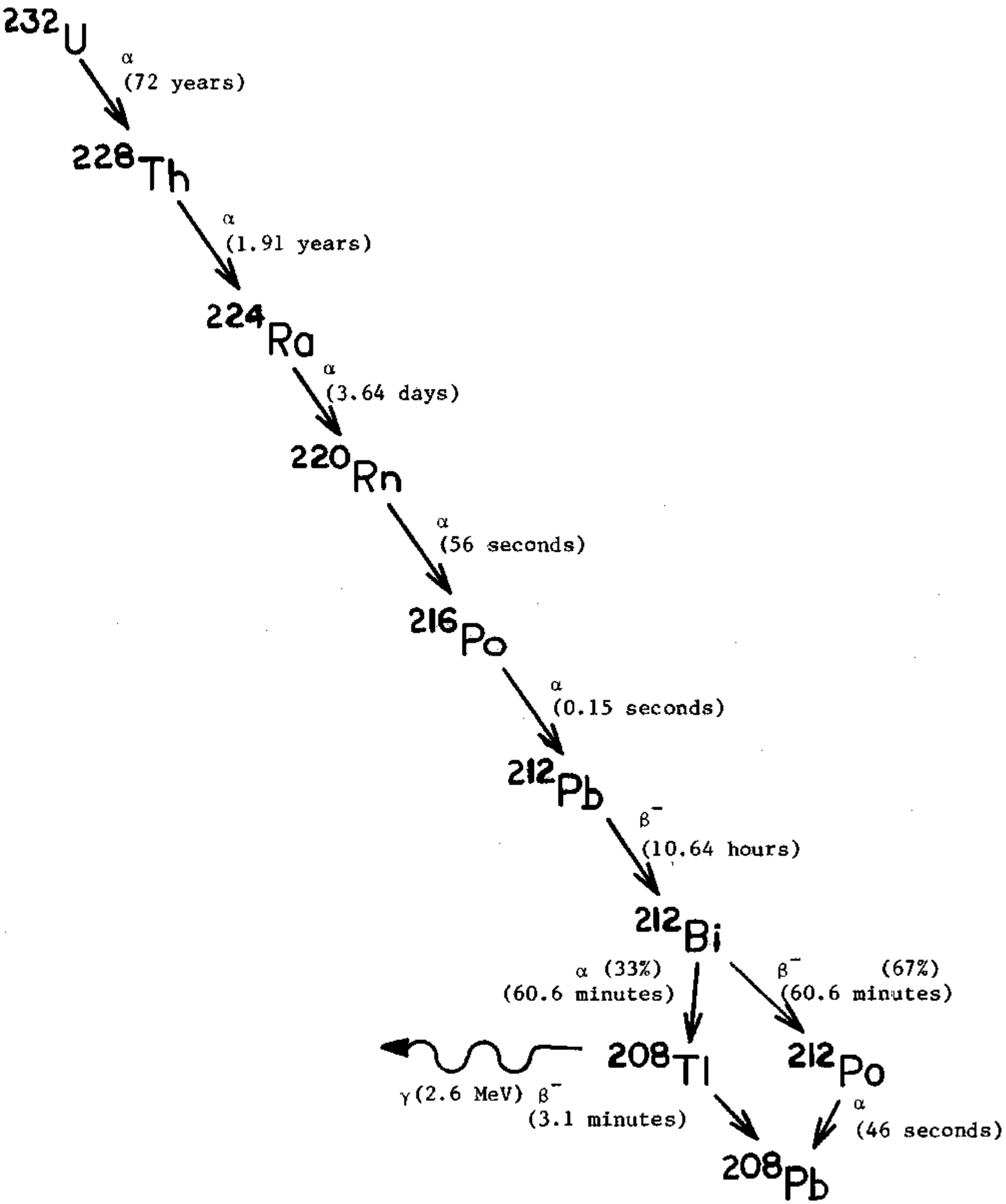

Figure 4. Uranium-232 Decay Chain

Under what scenarios, then, will the Light Water Breeder possess favorable economic properties? It would appear that the main distinction between the LWBR and the LWR is the lifetime constraint which applies to the LWR. Thus, in an environment of severely limited uranium availability, the penalty associated with the early termination of LWR's as generating units may be sufficient to justify the added front-end expense of additional fuel cycle cost to establish a non-terminable nuclear-electric economy.

Basis for a Light Water Breeder System

In addition to the Marginal Cost considerations already noted, the uranium resource constraints are stimulating interest in the converter concept as a means of extending the nuclear-electric power generation capability. Dietrich (1975) suggests that the economic evaluation of any system which utilizes U-233 should take into account the process by which the U-233 is produced. He further notes that the introduction of alternative reactor systems should not detract from existing development efforts and that the concept should be one which may be introduced on a substantial scale without undue delay. While the Light Water Breeder system is not unique in this regard, nevertheless it does meet these criteria. It is therefore appropriate that an investigation of the economics of the Light Water Breeder be made.

Within the Draft Environmental Statement for the Light Water Breeder Reactor Program, ERDA-1541, an economic analysis was performed in which two hypothetical, static reactor systems were compared. One consisted of 10 Light Water Reactors, the other was composed of 10

Prebreeders which eventually were replaced by Breeders as sufficient U-233 was produced for their fueling. Several criticisms of this technique have been put forth. Puechl (1975) suggests that the sixty-year planning period used in the Statement is long enough to include all possible benefits due to Breeder introduction. The point is made that the Prebreeder is inherently more expensive than the Light Water Reactor, and therefore in a competitive economy there is some question concerning how one would go about persuading utility interests to invest in a Prebreeder-LWBR system. The Environmental Protection Agency notes that the static nature of the analysis is not representative of the selection process, nor does it allow for the analysis of the effect of one system's requirements upon the cost of the other (Hanmer, 1975). The initial economic analysis performed for inclusion in the Draft Environmental Statement does indicate some justification for the pursuit of the Light Water Breeder Program. The economic analysis carried out herein for the LWBR system assumes a dynamic situation of mixed reactor types, thus addressing the concerns of the Environmental Protection Agency's reviewer.

In an economic analysis there are two aspects which are of concern. One is the identification of cost properties associated with a given energy source type. The other is the allocation of resources in light of growth patterns anticipated and the secondary effects which derive from selection of a certain mix of reactors. The Draft Environmental Statement tends to address the first of these considerations. The work described herein has as its main objective the analysis of the growth pattern, although the quantification of fuel cycle costs is a necessary step in the construction of the larger analysis.

Previous Economic Analyses of the LWBR

The Draft Environmental Economic Impact Statement contains a cost/benefit analysis of the LWBR reactor system. The method of evaluation is by way of a comparison with the corresponding LWR system of the same capacity. The analysis is equivalent to comparison of static systems of fixed capacity. Variation in cost derives only from the variation in fuel cycle component costs, which were varied independently of each other.

Some criticism of the structure of the analysis can be made on the basis of the treatment of reprocessing costs, wherein $50.00 per kg is used for both the thorium and U-235 costs. The cost of reprocessing for the U-233-thorium cycle is anticipated to be greater than that for the U-235-Pu cycle because of the greater amount of shielding required. Thus, in the closed portion of the fuel cycle, the existing LWR reprocessing and fabrication costs are expected to be less than the corresponding quantities for the U-233 cycle. Constraints are not given a thorough analysis in the ERDA report. The tacit assumption is made that the uranium and separative work are available no matter what the shape of the nuclear industry. In fact, the availability of services depends not only upon current levels of utilization but also upon anticipated levels of use and also upon the availability of the services necessary to support such functions as reprocessing and enrichment. In particular, the enrichment question is deserving of attention in that gaseous diffusion, the only process in existence for production, is limited by the availability of electrical power. The Tennessee Valley Authority has had to decrease power to the Oak Ridge Gaseous Diffusion Plant, and in fact the contracted power is specified so that the last increment of power for the enrichment facility is provided by the Tennessee Valley Authority on a flexible basis; that is, the power necessary to sustain production at the projected plant maximum is not guaranteed by TVA because the capacity is not guaranteed. With regard to enrichment, a further consideration is given by the power-intensive nature of the gaseous diffusion process. The recent history of price increases for separative work has been attributed to the increase in power cost to the enriching facility. Thus, a correlation can be drawn between power costs and separative work costs if gaseous diffusion technology is specified. Should gas centrifuge or Laser Isotope Separation become reality, the dependence upon the cost of electricity would disappear. This would necessarily appear in any economic model as an exogenous variable. Similarly, energy inputs to other industries in the nuclear fuel cycle should be defined to enable the analyst to specify properly the feedback term. It should be observed that the cost of electrical energy constitutes a positive or destabilizing term in the refined economic analysis. The effects of such costs may be attenuated by technological changes, as the introduction of the gas centrifuge on a large scale, displacing existing gaseous diffusion units. Therefore, in an economic model it is desirable to have a mechanism for imposing exogenous conditions as an alternative to the capability to control future actions within the market.

Purpose of Research

The technology of light water reactors is well developed. If light water reactors are to continue in their role as base-load units, the apparent limitation upon the availability of uranium makes desirable the increase in fuel utilization in these reactor types. The Light Water Breeder system provides the only approach to the problem of increasing fuel utilization beyond the limits achievable with present commercial reactor types. Unknown quantities are the Fissile Inventory Ratio and the system-wide economics of the Breeder.

In its 1973 recommendations, the Energy Research and Development Advisory Council to the Chairman of the Atomic Energy Commission indicated that continued support for the Light Water Breeder Reactor for proof-of-breeding investigation was recommended, and at the same time stated that extensive studies of both the reactor plant and fuel cycle economics were necessary before the commercial feasibility of the LWBR system could be ascertained (WASH-1281-10).

It is the purpose of this study to examine the economic features of the LWBR system within the context of the existing nuclear-electric generating capability in the United States. Because both Light Water Reactors and Prebreeders are users of U_3O_8 and enriching services, there is competition for these which may be reflected in their prices. Thus, the eventual mix of reactor types is a function of the time-dependent reactor selection process which, in turn, has an impact upon resource prices.

In this study it is assumed that prices for enriching services are unaffected by the level of demand and thus are independent of the mix of reactors. This situation obtains when long-term contracting for enriching services is employed in the commercial nuclear industry. This mode of facility commitment will be the case whether enriching facilities are held by Government or by private concerns. Variations in enriching service prices are therefore applied exogenously.

Uranium prices are assumed to follow free-market behavior in which prices rise as the commodity is exhausted. For this purpose a piecewise linear function is utilized. The exact nature of the function cannot be determined, but an indication of the effect upon costs may be obtained by choosing several representations which might reasonably be expected to represent price behavior for uranium and the economic analysis performed for each of these assumed functions.

One method of system economics evaluation is a cost-benefit computation in which the component costs are allowed to vary over prescribed ranges and cost/benefit ratios are compared. Where two systems are to be compared, the assumption of independence may neglect the coupling effect arising from demands for the same resource in a mixed environment.

The essence of the question concerning economic characteristics of the Light Water Breeder System is whether it is to the advantage of society to develop this technology. The Breeder has a lower fuel cycle cost than does the Light Water Reactor owing to its independence of enrichment and mined uranium. The cost of its deployment must include, however, the costs associated with the generation of enough U-233 to allow it to be a self-sustaining system (Dietrich, 1975). Proper discounting of costs and benefits implies that the costs associated with operation of Prebreeders are weighted more heavily than benefits deriving from Breeder use. It is not at all clear, therefore, whether the Light Water Breeder System may be defended entirely upon the basis of arguments associated with the commercial power generation function.

For a reactor of type r at time t, the total cost for fuel cycle services $Z_r(t)$ may be given by

$$Z_r(t) = \sum_{i=1}^{N_S} \overline{C}_{ir}(t)\ R_{ir}(t) \tag{1}$$

where

$\overline{C}_{ir}$ is the unit cost for fuel cycle service i associated with reactor type r

R_{ir} is the resource requirement of fuel cycle service i for one GWe of energy generation capacity from reactor type r

N_s is the number of fuel cycle services.

The fuel cycle services which would normally be dependent upon reactor type are fuel fabrication and fuel reprocessing. These may be equivalently enumerated as distinct fuel cycle services to reflect the processes associated with the treatment of fuel containing U-235, ThO_2, and U-233. Equation (1) may thus be written

$$Z_r(t) = \sum_{i=1}^{N_T} C_i(t)\ R_{ir}(t) \tag{2}$$

where

$C_i(t)$ is the unit cost associated with a particular fuel cycle service and technology

N_T is the total number of fuel cycle service and technology components.

Evaluation of the economic merit of the Light Water Breeder Reactor system may be accomplished by determining the optimum mix of Prebreeders and Light Water Reactors in the nuclear-electric sector. A specific requirement for nuclear-electric generating capacity is assumed and the optimal commitment for reactor types is computed based upon economic assumptions. The objective function may reflect any of several criteria, defining the nature of the parameter to be maximized or minimized. For this analysis, the objective function is taken to be the total discounted cost to society. The optimization problem to be solved is thus the minimization of this discounted cost. Since no firm cost estimates exist for any of the nuclear fuel cycle services save for conversion, it is necessary to assume several sets of economic quantities and evaluate system behavior under each combination of assumptions.

In the particular case of the Light Water Reactor, the costs associated with U_3O_8 procurement, enrichment and reprocessing dominate fuel cycle expenditure. As a first step in the evaluation of cost centers and their importance, the requirements of the Prebreeder for these services are evaluated and compared with those of the Light Water Reactor. The results of this comparison are shown in Figures 5-16 for arbitrarily-assumed values of market penetration. In this comparison a certain fraction of the market from 1980 to 2000 was assumed to be

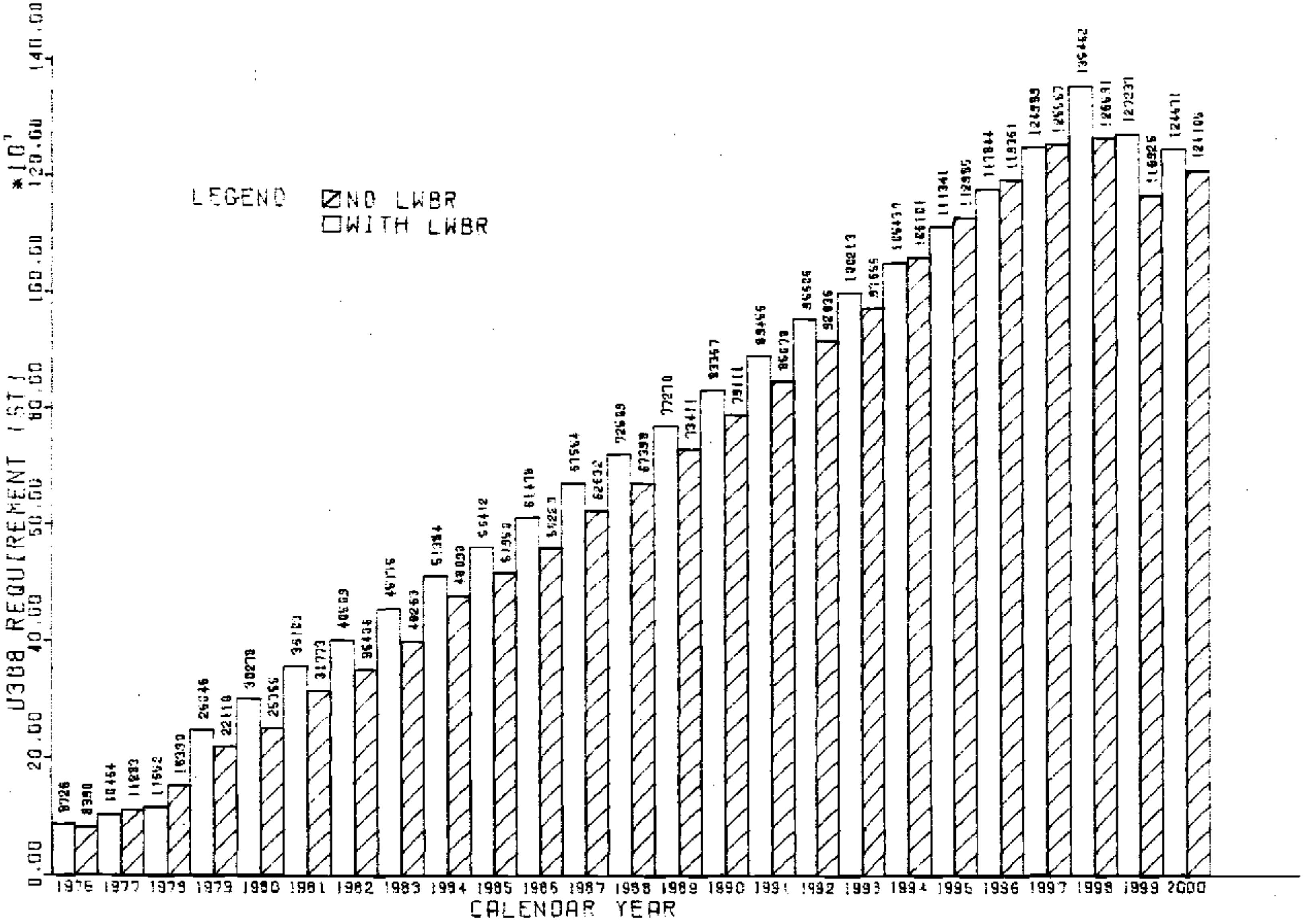

Figure 5. U_3O_8 Requirement, Logarithmic Increase in Prebreeder-to-LWR Ratio

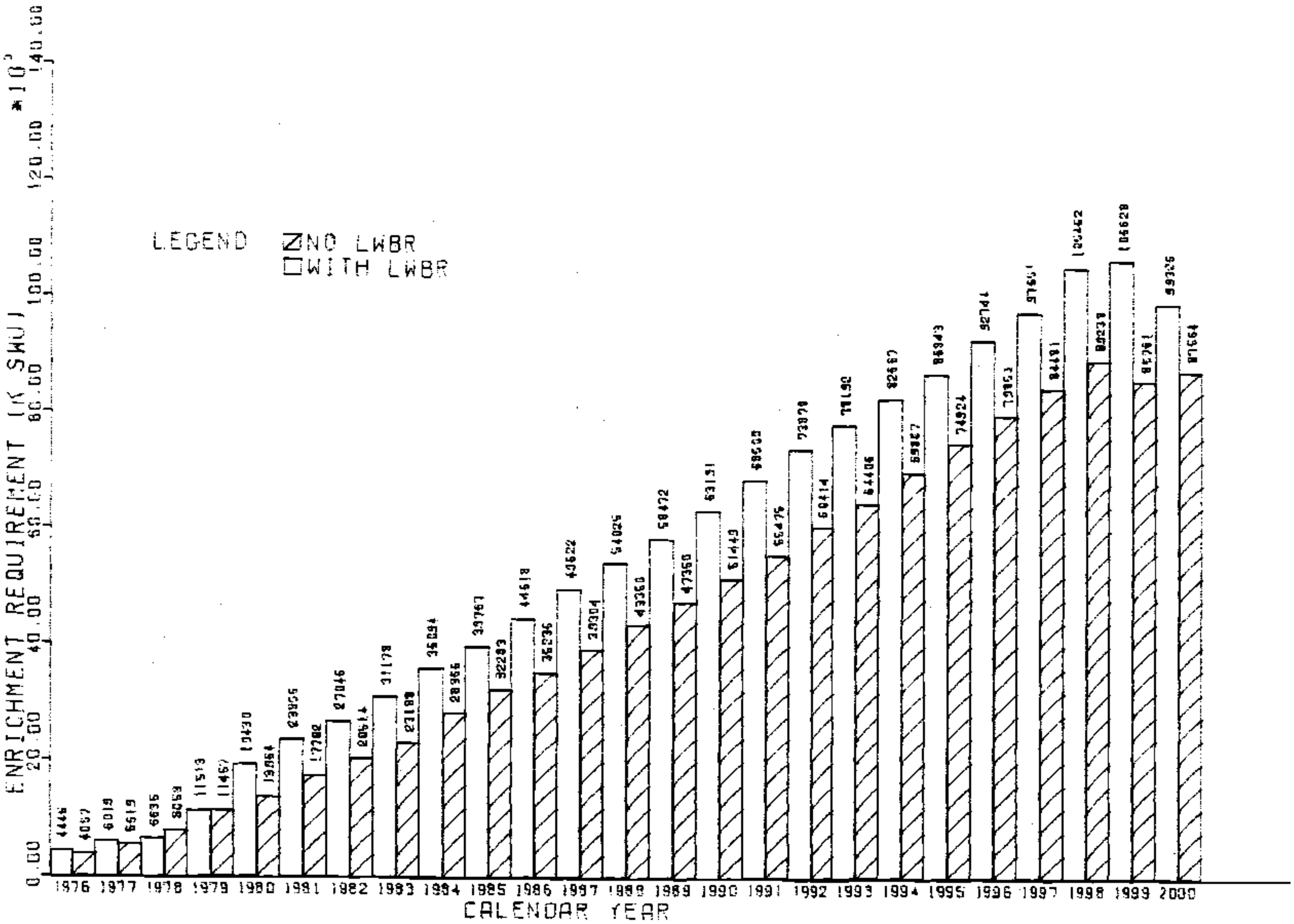

Figure 6. Separative Work Requirement, Logarithmic Increase in Prebreeder-to-LWR Ratio

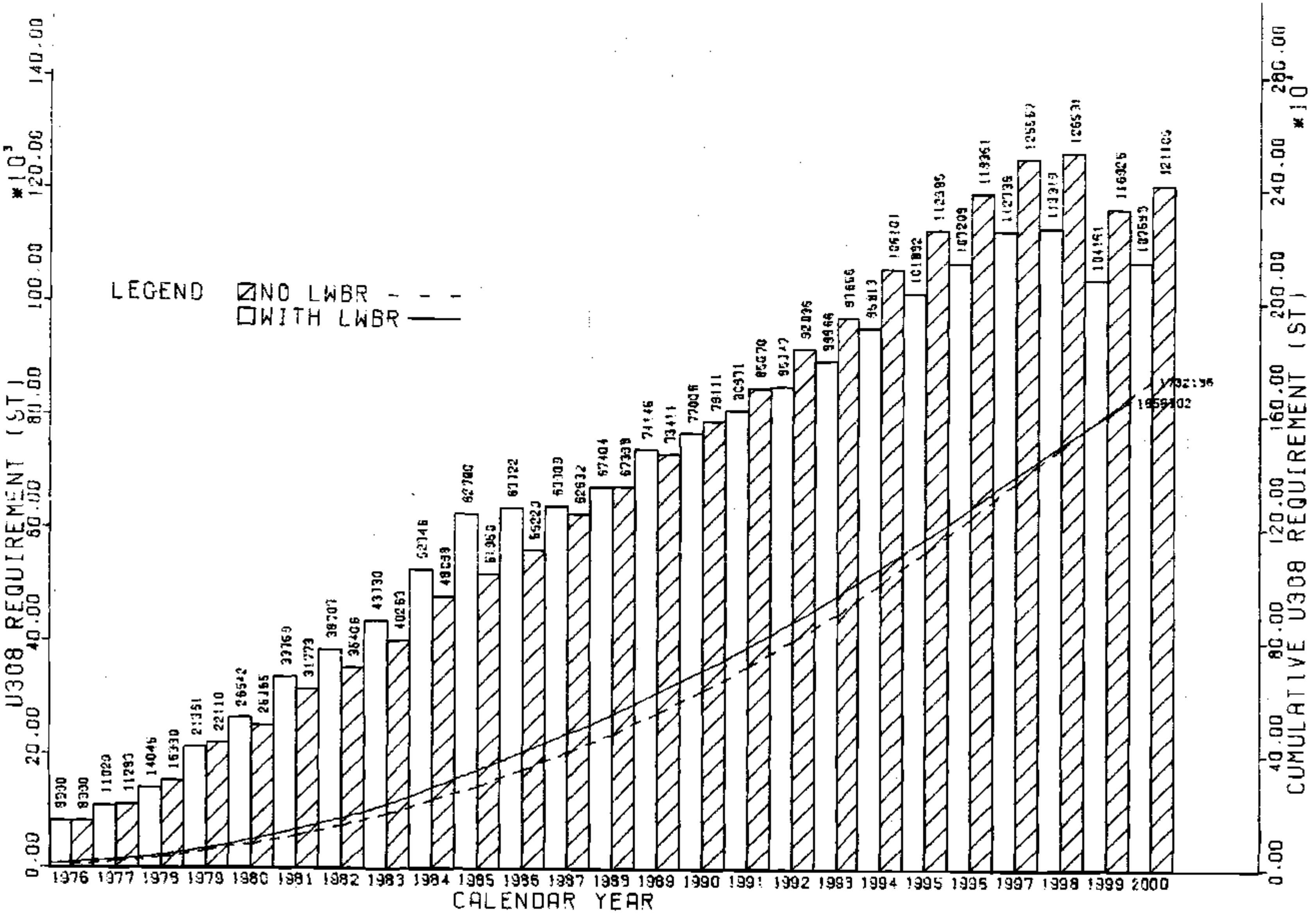

Figure 7. U_3O_8 Requirement, Finite Period for Prebreeder Commitment

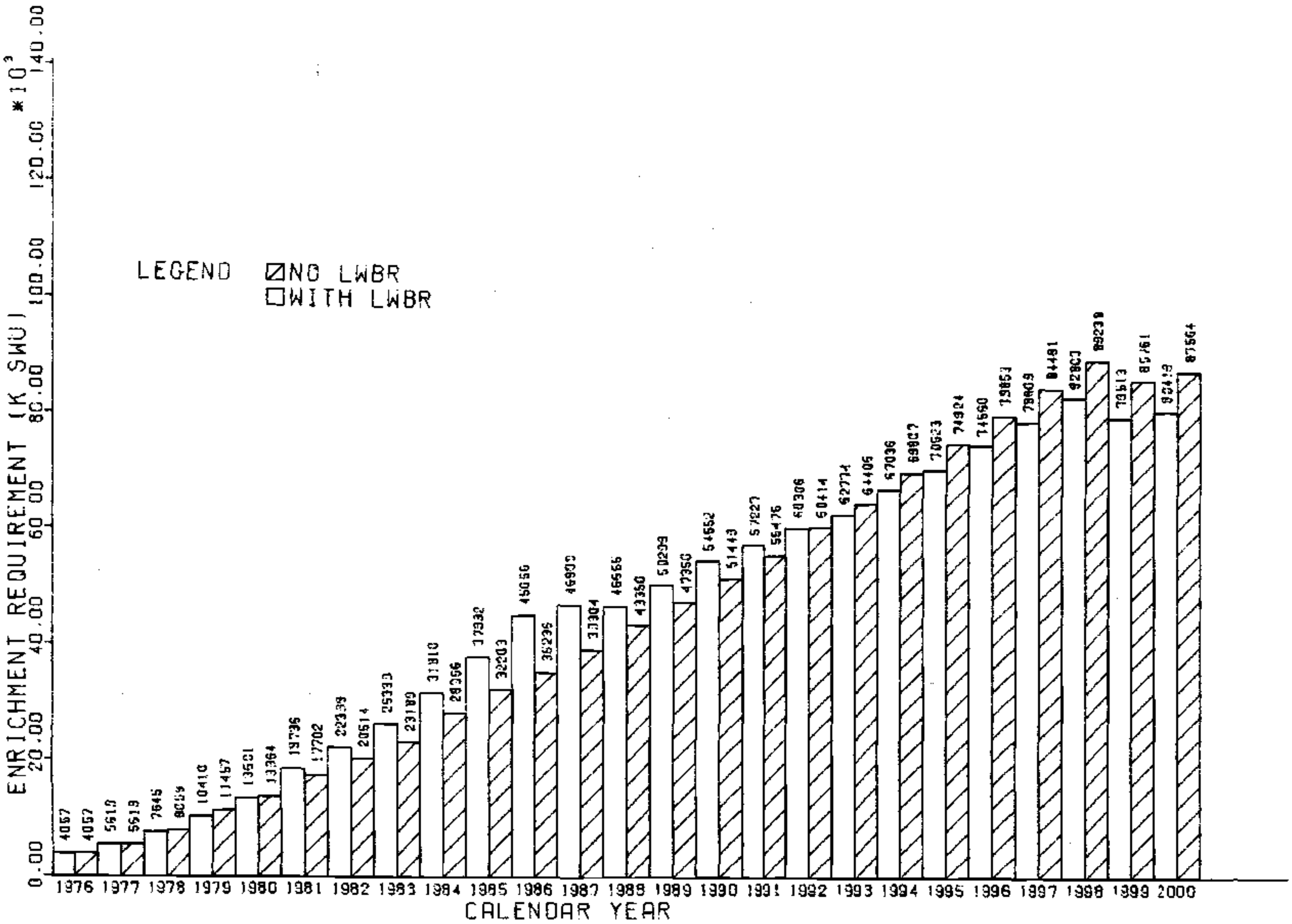

Figure 8. Separative Work Requirement, Finite Period for Prebreeder Commitment

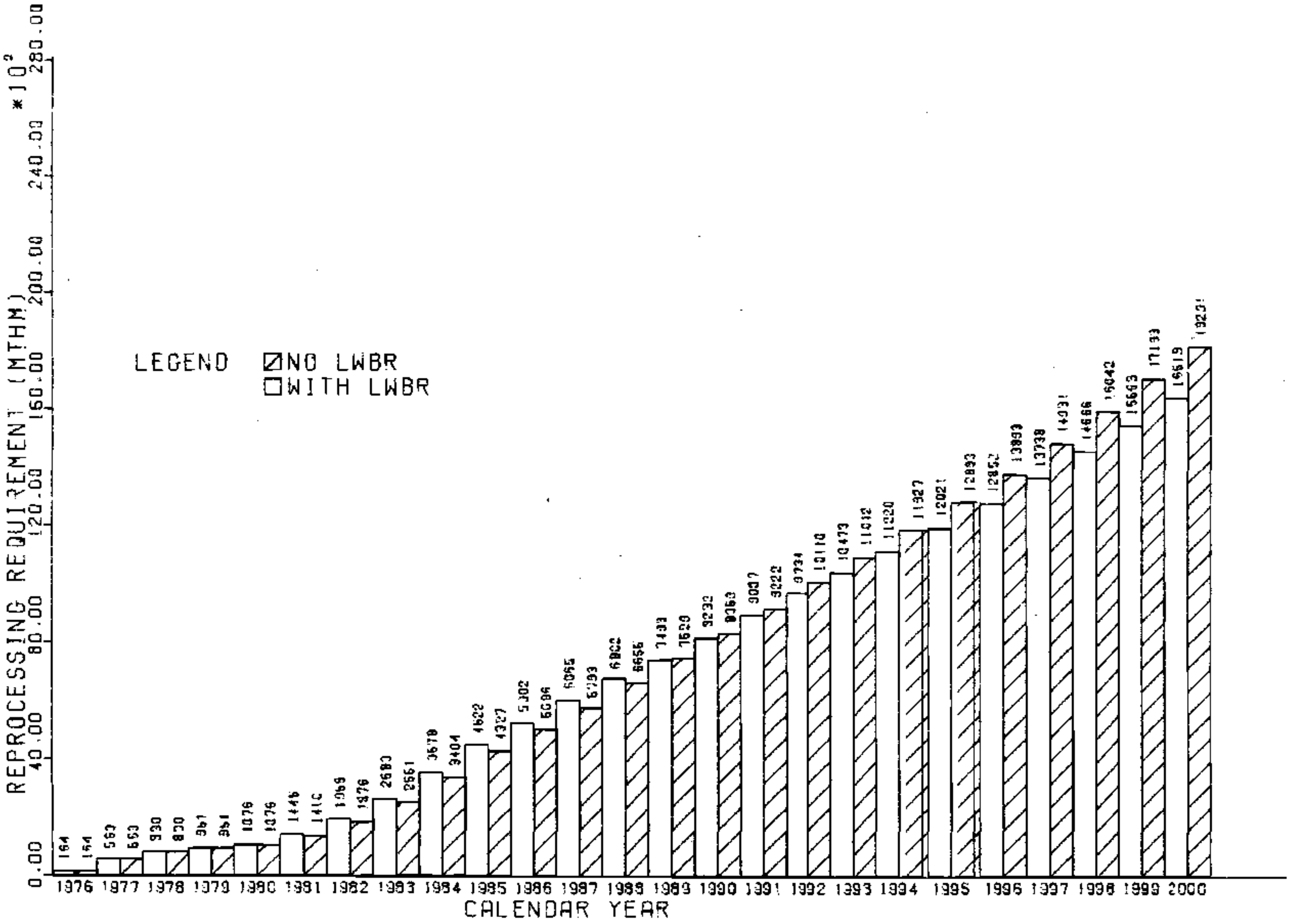

Figure 9. Reprocessing Requirement, Finite Period for Prebreeder Commitment

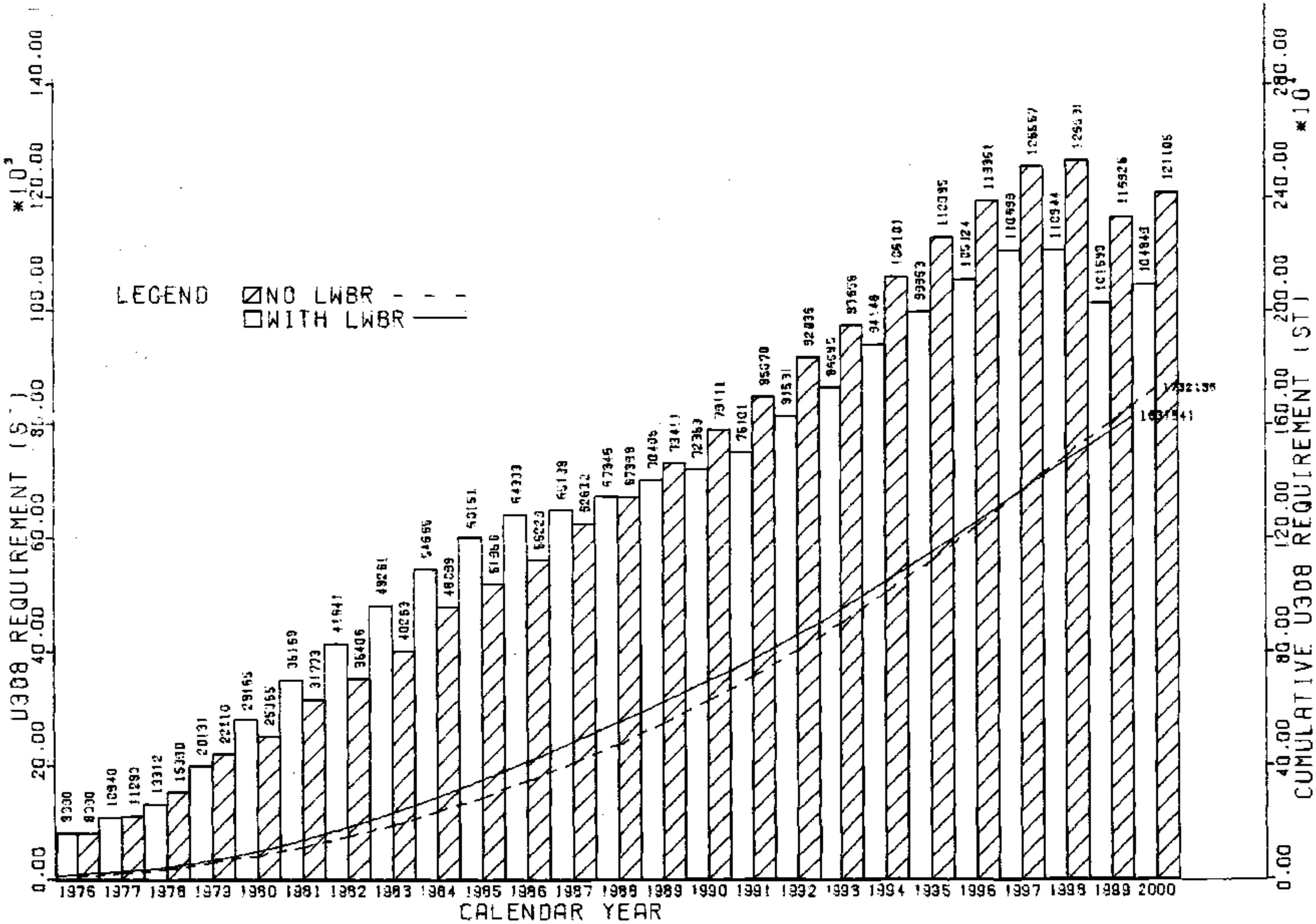

Figure 10. U_3O_8 Requirement, Vigorous Commitment of Prebreeders

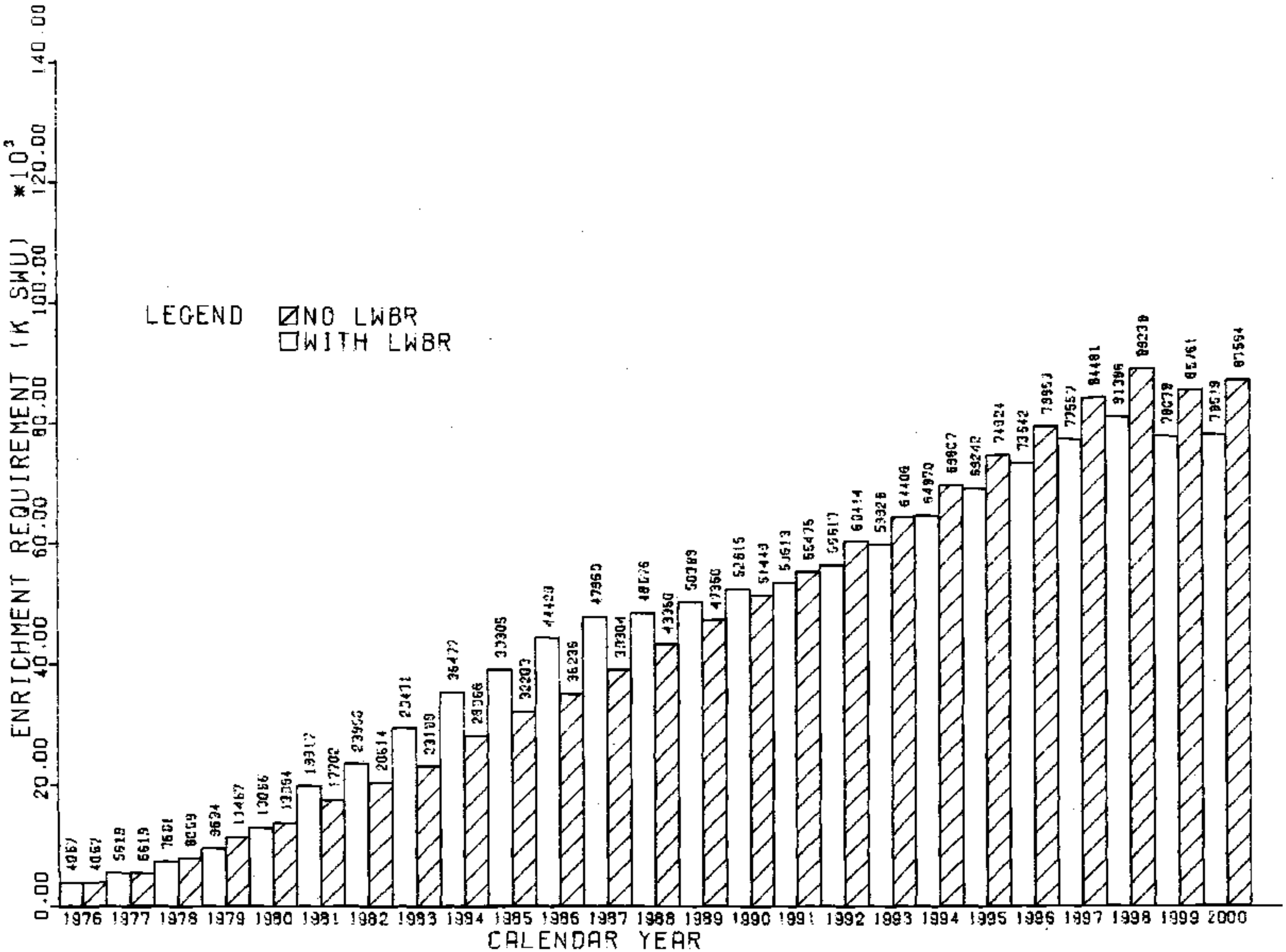

Figure 11. Separative Work Requirement, Vigorous Commitment of Prebreeders

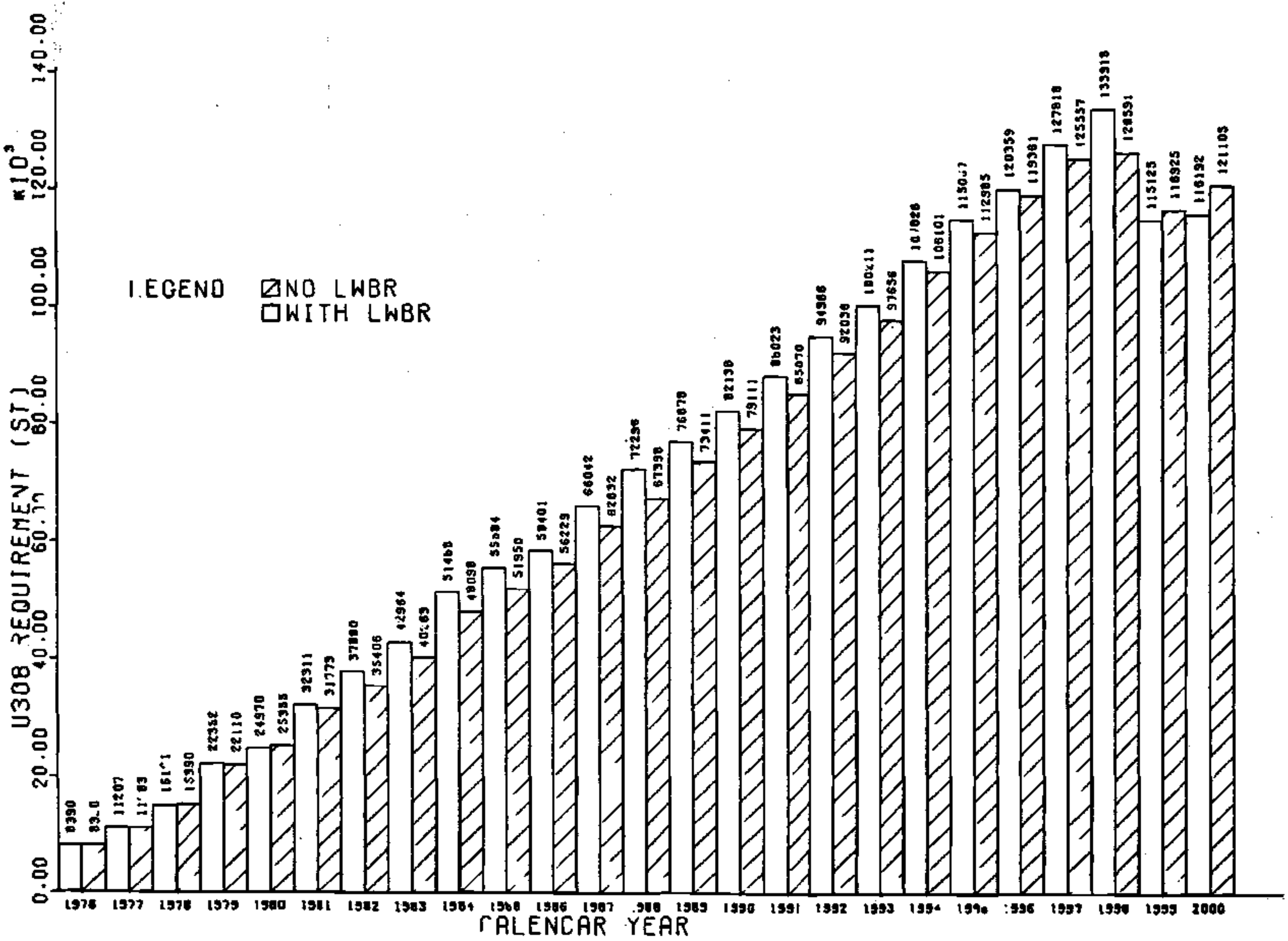

Figure 12. U_3O_8 Requirement, 25% Asymptotic Market Penetration for Prebreeders

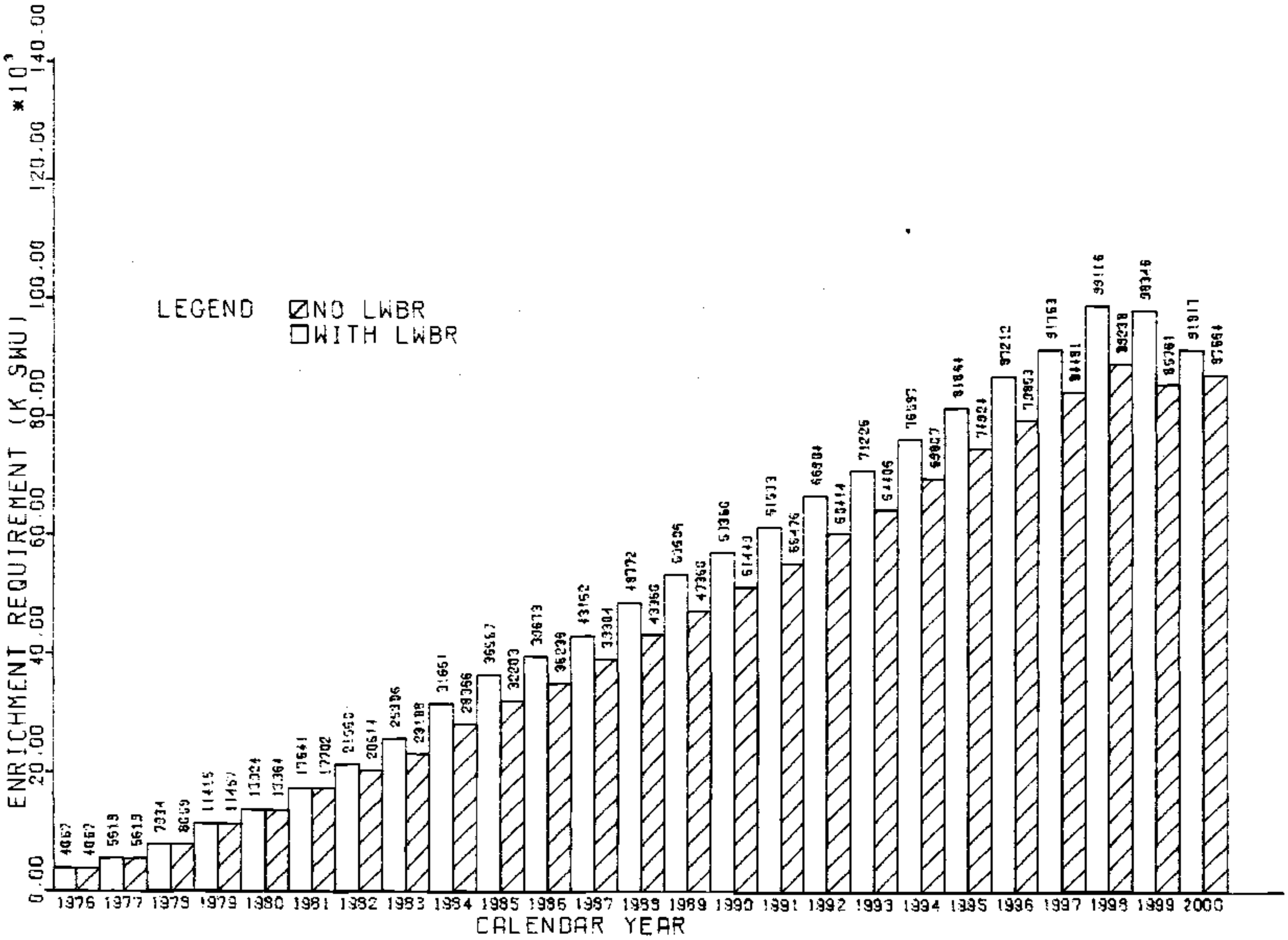

Figure 13. Separative Work Requirement, 25% Asymptotic Market Penetration for Prebreeders

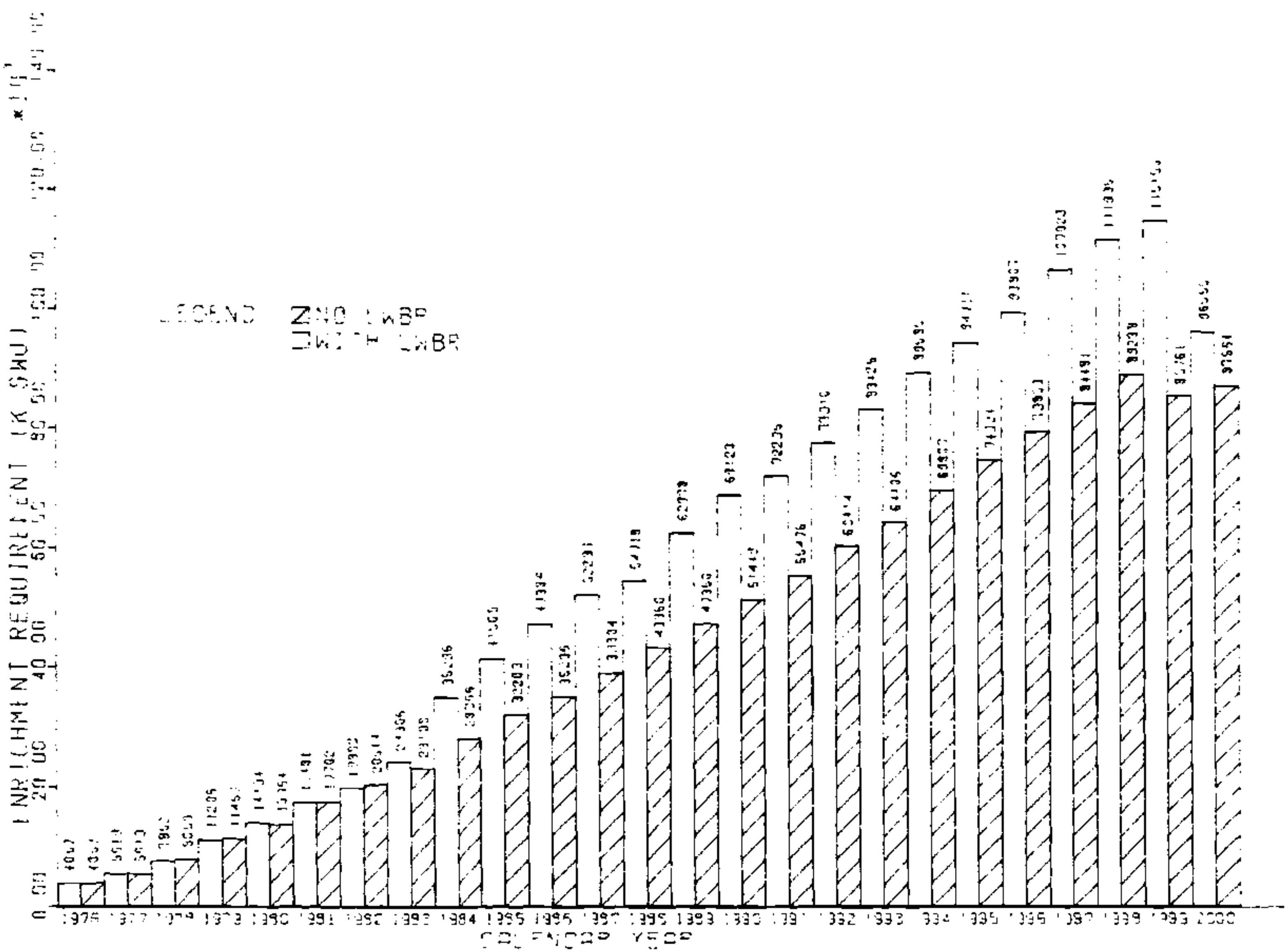

Figure 14. Separative Work Requirement, 75% Asymptotic Market Penetration for Prebreeders

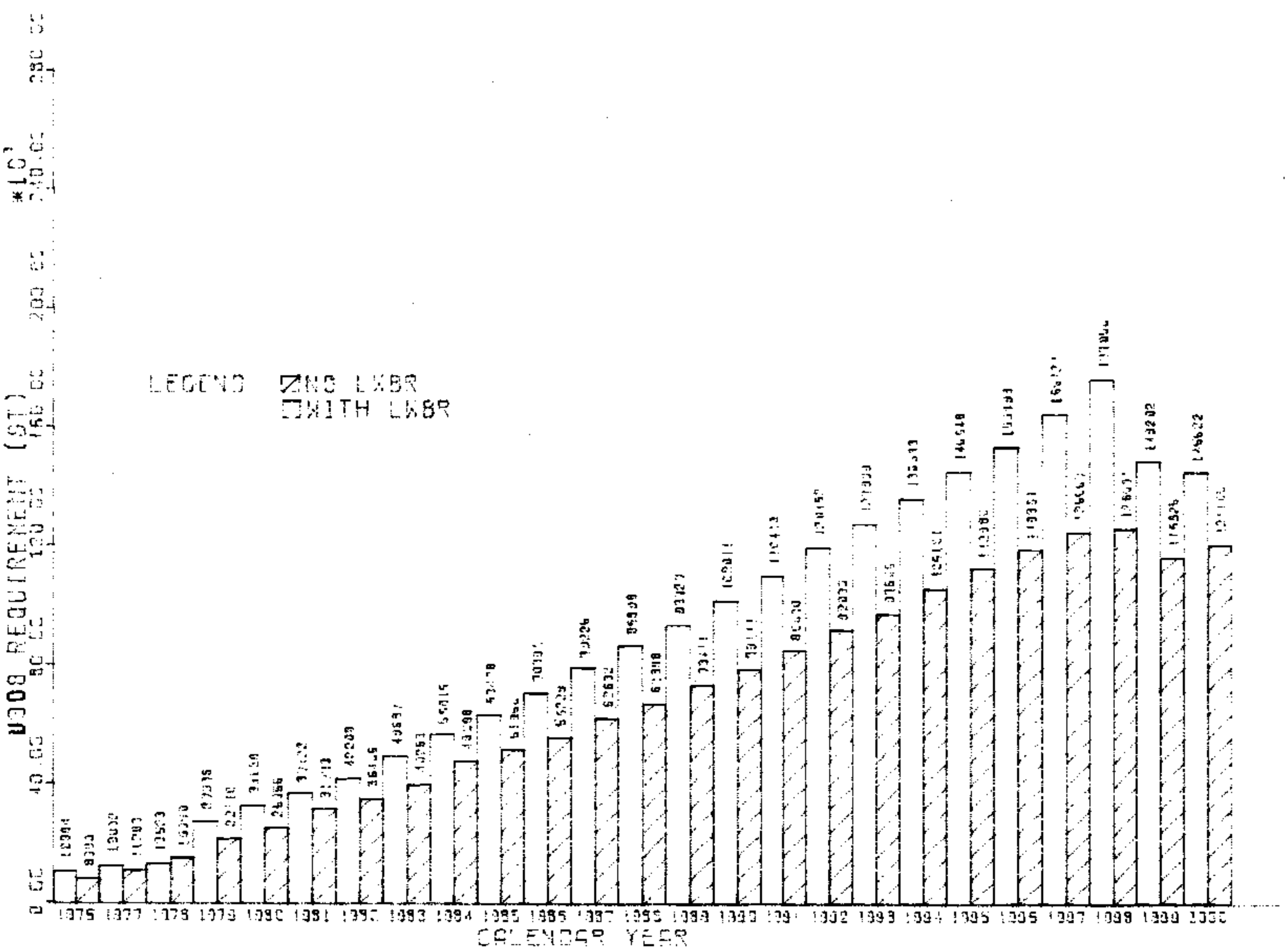

Figure 15. U_3O_8 Requirement, 100% Asymptotic Market Penetration for Prebreeders

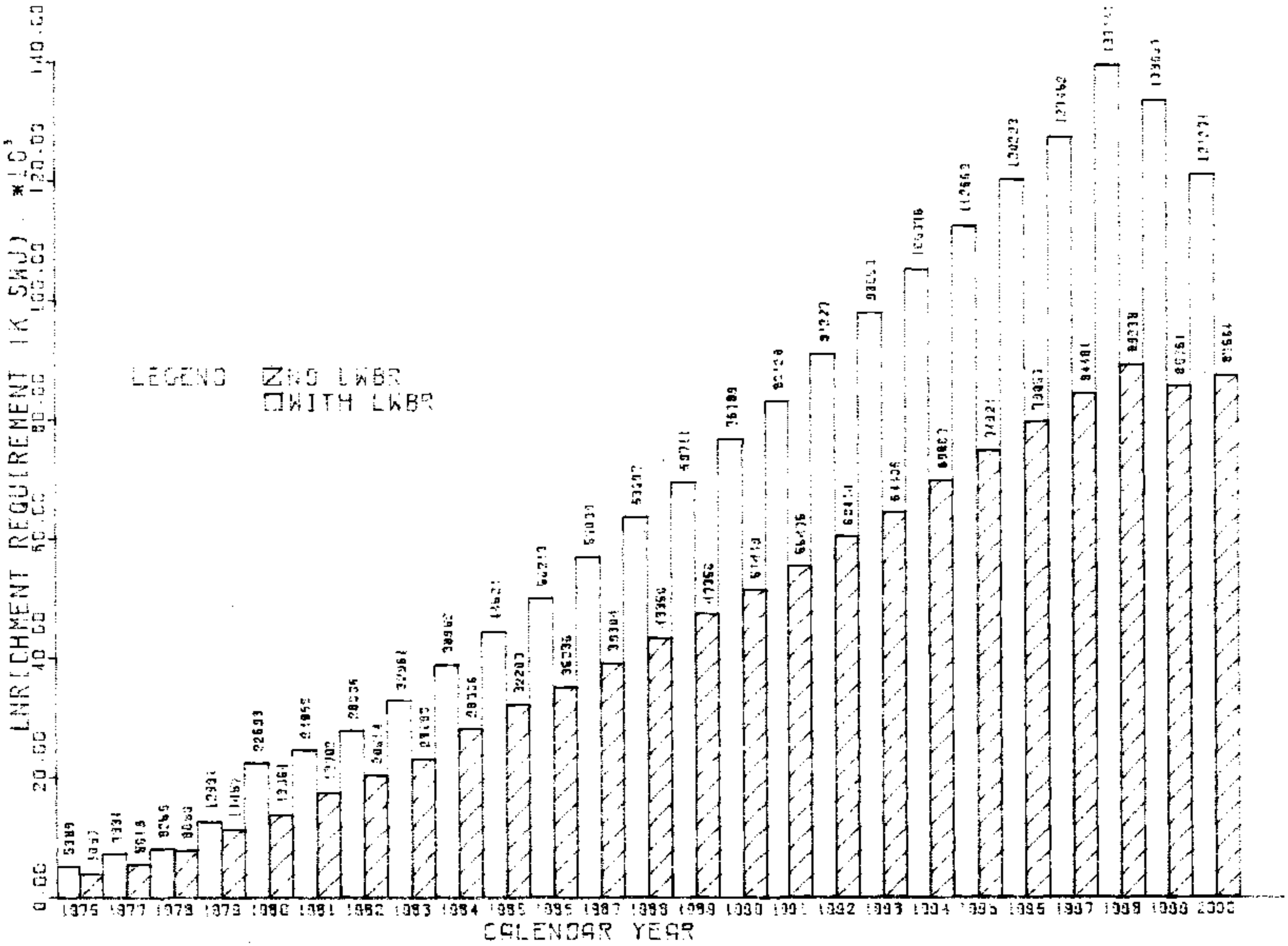

Figure 16. Separative Work Requirement, 100% Asymptotic Market Penetration for Prebreeders

given to the LWBR system. The natural comparison to be made is that with the LWR-only system which generates the same amount of electricity. Case A from WASH-1139-74 was used as the power commitment constraint. It is seen from examination of these figures that the Prebreeder/Breeder system is more intensive with respect to both the U_3O_8 and enriching services requirements than is the LWR system. Thus the optimization problem would be anticipated to be most sensitive to these fuel cycle components, just as is the fuel cycle cost of the Light Water Reactor. Inasmuch as the U_3O_8 price is a function of the reactor mix, it is observed that this parameter may play an important role in the determination of an optimal schedule. For given the two reactor types (denoted by subscripts LWR and Pre for Light Water Reactor and Prebreeder, respectively) the cost difference is determined from Equation (2) as

$$Z_{LWR}(t) - Z_{Pre}(t) = \sum_{i=1}^{N_T} C_i(t)[R_{i,LWR}(t) - R_{i,Pre}(t)] \qquad (3)$$

Since both the cost C_i for U_3O_8 and the resource requirements difference are nonnegligible, it is anticipated that the U_3O_8 cost schedule and time dependence will have an important effect in the optimization process.

Prebreeder/Breeder Scheduling

Two modes of deployment are available for the Breeder. If maximization of the number of Breeders is unimportant, the Prebreeder may be replaced by a Breeder when sufficient U-233 fuel is available. This has the effect of attenuating the ability to produce U-233 for further

Breeders and hence limits the number of Breeders possible.

A second mode of Breeder deployment is perhaps more realistic for a growth economy. The Breeder enabled by production of U-233 is utilized to satisfy requirements implied by the assumed commitment schedule. In effect, the Breeders thus constructed replace Light Water Reactors which would otherwise be commissioned. While each Breeder plant requires the same number of Prebreeder-years operation in order to generate sufficient U-233 fuel, time dependence of costs, in particular the U_3O_8 costs, suggests that the earlier Breeders are deployed the better, and hence that one would wish to maximize the number of Breeders to achieve an optimal mix. The penalties associated with Breeder deployment are the Prebreeder costs which are in excess of those for the Light Water Reactor. Time dependence of costs, and in the case of U_3O_8, the dependence of costs upon the mix of units, makes necessary detailed computation and comparison of specific schedules.

Since U_3O_8 cost depends upon cumulative consumption, the fraction of Prebreeders depends in part upon the growth characteristics anticipated for the nuclear-electric generating sector. A low-growth scenario would be expected to favor the Light Water Reactor, whereas a heavy growth case might favor those plant types whose characteristics include resource conservation.

The uncertainty associated with predictions of resource utilization is demonstrated in Figure 17, which compares the scheduled reactor additions to the electricity generating sector with those predicted by the Case A assumptions released by the Atomic Energy Commission early in

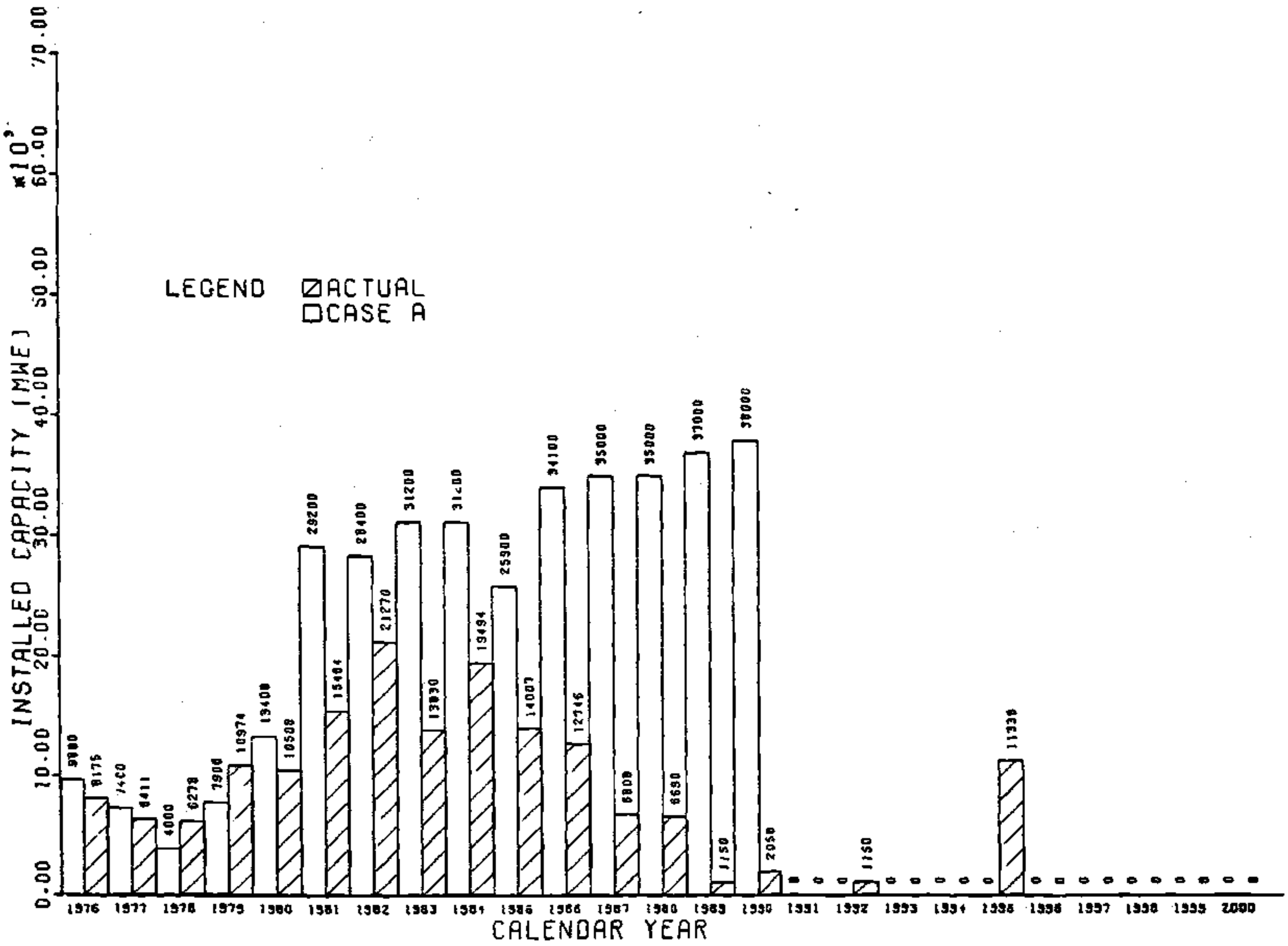

Figure 17. Reactor Commitment Schedule

1974. This is the lowest growth rate case of the four. The assumptions differed mainly in the speed of licensing and the expansion of the Gross National Product. Owing to the length of time required for licensing and startup, it is not possible to improve upon plant commitment in years 1980, 1981 and 1982 in order to bring actual construction in line with prediction. Whether there will be a period of heavy reactor ordering which will compensate in the mid-1980's for the unanticipated slowdown is largely a matter of speculation. The comparison data, taken from Nuclear News (1976), do indicate the substantial uncertainty which surrounds estimates of market penetration by nuclear plants. Therefore, in order to evaluate the economic potential of the Prebreeder/Breeder system, it is necessary to incorporate in the basic model the ability to vary the nuclear-electric commitment schedule.

The stretchout represented by the data of Figure 17 suggests that the level of U_3O_8 prices could suffer a reversal from their levels of 1974-76. Utilities which contracted for uranium and subsequently extended their plant construction period could conceivably possess uranium or separative work which they might wish to sell several years after initially contracting for it. In the example hypothesized for these calculations, this might occur around 1980 or 1981. The price of uranium has shown some short-term volatility, the price having risen from approximately $9.00 per pound in 1973 to nearly $40.00 per pound in mid-1976 (Metals Week). The spot market could revert from a seller's to a buyer's market, thereby depressing the short-term price. Hence, the real cost of power generation in the early years of substantial nuclear commitment is not predictable without knowledge of the commitments held by utilities.

There is therefore a broad spectrum of conditions which can possibly exist. The economic analysis technique applied must admit the possibility of variations in any of the fuel cycle service prices or requirements in order to allow representation of any anticipated economic situation. In particular, it is important to be able to allow variations in the price of U_3O_8, enriching services and reprocessing, as well as in the power requirements constraint, since these have traditionally shown substantial variability.

Treatment of Breeder Deployment

It has been indicated that there are two distinct approaches to the deployment of the Breeder reactor. It is assumed for the purposes of investigating the dynamic plant commitment problem that Prebreeders are not supplanted during the interval of study. This maximizes the number of Breeders eventually deployed. Though it might be satisfying to perform an optimization analysis which accounts for reactor deployment over a 25-year period, it is not feasible from a computational standpoint to analyze a problem with 25 distinct increments of time. Further, the uncertainty in the power requirements schedule alone would restrict such an analysis to limited usefulness.

An alternative formulation of some economic importance may be described in the following way: given a power requirements schedule representing the nuclear commitment for N years, determine the total cost of power generation over an expected period of M years, the optimal commitment of reactor plants over the first N years, and the consequence of this commitment. This is consistent with utility practice, since utilities can predict the market for only a fraction of the time necessary to recover their investment in a plant. With appropriate simplifications in this approach, the value N=10 is a tenable choice for the dynamic analysis part of the plant commitment problem with power requirements schedule given by Case A of WASH-1139(74) and constraints upon commitments of Prebreeders as may be deemed reasonable from the ability of the nuclear industry to support the Th-U-233 fuel cycle. In this analysis, a 30-year amortization period is assumed.

The effect attributable to a specific commitment schedule may be analyzed by assuming no further growth effects beyond year N. This is then the same as a terminal constraint function, which is dependent only upon the state at N stages and does not assume further effects arising from continued growth. It does, however, allow a comparison of possible deployment schedules which reflects only the conditions of the first N years. It thus provides a means of approximating the effect of various admissible decisions which does not require assumptions about the long-term market behavior.

A computational advantage of this procedure is that the calculations describing the first N years of system operation are distinct from those of the last (M-N) years. Thus, the description of possible states and their properties may be computed once and any number of sets of assumptions may be used for price behavior during the latter years. The computationally time-consuming aspect of the problem is that pertaining to the generation of all paths and states during the first N years.

Much of the effect of economic variation may be accounted for by changing the terminal constraints.

For the terminal function, the assumption is made that the Breeders replace Prebreeders as U-233 is generated to sustain Breeders. This results in an exponential decrease of the number of Prebreeders. Since no growth is assumed, it is more favorable for the Breeder to replace the Prebreeders at the earliest opportunity, since the Prebreeder has a higher fuel cycle cost than the Light Water Reactor. This accounting for the disposition of U-233 is the same as that used in the economic analysis of the Draft Environmental Statement.

CHAPTER II

WORLD URANIUM RESOURCES

There are four major uranium producers whose resource estimates are publicly available. Australia, Canada, South Africa, and the United States dominate the world uranium supply market. Uranium contracts with Canada are already in existence, and the policy of Australia may eventually lend itself to the formation of an export market in yellowcake. Estimates of world resources are given in Table 7 (Symonds, 1975).

Table 7. Estimated World Resources of Uranium (Quantities Given in Tonnes)

Country	Cost Range: < \$10/lb U_3O_8		Cost Range: \$10-15/lb U_3O_8	
	Reasonably Assured	Estimated Additional	Reasonably Assured	Estimated Additional
Australia	184,000	32,000	60,000	46,000
Canada	185,000	190,000	122,000	219,000
France	37,000	24,000	20,000	25,000
Gabon	20,000	5,000	Nil	5,000
Niger	40,000	20,000	10,000	10,000
South Africa	202,000	8,000	62,000	26,000
Sweden	Nil	Nil	270,000	40,000
U.S.A.	242,000	738,000	81,000	438,000

The most uncertain of these figures is that pertaining to estimated additional resources in Australia, where deposits have been discovered only recently. Therefore, substantial further explanation is in order. It should be observed that the estimate utilized by Symonds is somewhat at variance with the levels published by the Canadian Ministry of Energy, Mines and Resources. Estimates developed in 1974 show 184,000 short tons Reasonably Assured reserves and 237,000 short tons in the Estimated Additional category (Merlin, 1975). The world resources might well be anticipated to be less than the total shown in Table 7, thus intensifying the problems associated with a shortfall of U_3O_8. Problems associated with uranium supply have been forcefully delineated by several observers of the nuclear industry. McGee, in testimony before the Joint Committee on Atomic Energy (1974), has succinctly indicated the level of uranium price necessary to stimulate exploration activities on the part of private enterprise. The Energy Research and Development Administration has established the National Uranium Resources Evaluation (NURE) program to determine the extent of the economically recoverable uranium reserves, and has at the same time established a program to encourage exploration by smaller companies with the objective of enlarging the known reserves (GJO-111, 1976).

Uranium Resources

Although considerable uncertainty exists concerning domestic uranium resources, an effort has been put forth to obtain, on a regular basis, reliable data concerning the location and nature of uranium reserves in the United States. Two categories of resources are defined (GJO-105). The proven reserves, or reserves are estimates of uranium which exist in known deposits. This term is similar to the classification "reasonably assured resources used by the International Atomic

Energy Agency. The less certain resource category has been known in the United States by the term potential resources. Potential resources are those which are believed to exist based upon known geological data, but which lie in underground or unexplored areas. This latter category refers, for example, to uranium deposits which might exist in areas adjacent to currently-producing uranium mines. An understanding of the history of uranium production and pricing in the United States is essential to the correct formulation of an economic model based upon past data. Even if no interest is shown in using price and consumption values from past years, it is interesting to revisit the situations which have been encountered by the uranium industry during its development.

In the early days of uranium use the Manhattan Project used nearly 12,000 tons of concentrates. These supplies came not only from the Colorado Plateau, the major producing area at this time, but also from the Belgian Congo and from Canada. At the end of 1948, uranium reserves totaled only 2,200 tons U_3O_8. Domestic production of U_3O_8 recorded for 1948 was only 83 tons (Appelin, 1973). Pricing policy also grew in a somewhat uneven fashion. Some insight into the approach to pricing is given by John Patterson (1973) who has provided historical perspective on the pricing practices of the Atomic Energy Commission, which was until 1964 the exclusive purchaser of uranium in the United States. In the early days of uranium purchase for defense and research, the price was negotiated and based upon both the assay of uranium and the quantity of vanadium in the ore. From 1962 until 1968 the fixed eight dollars/lb U_3O_8 was in effect. The price paid for ore during that period was arranged between the mining and milling interests. Observing that continued government purchases would be required to maintain the uranium producing sector until a commercial market developed, the Atomic Energy Commission implemented a program to smooth the transition between government and private purchases. U_3O_8 deliveries scheduled during the 1963-1966 period were rescheduled to maintain mining and refining activity until 1968, and additional uranium equal to the amount deferred was purchased for the 1969-1970 time period. Prices paid were derived from production costs during the previous 1963-68 period. The average price for yellowcake during the 1969-70 period was $5.78 per pound U_3O_8. The complex effect of even a simple tactic such as the stretchout and industry maintenance program may be seen in the apparent hesitancy of the mining industry to engage in large scale exploration once commitment to nuclear power had been made by utilities. From the various aspects of its operation, the U.S. Government in 1973 was both the purveyor of separative work and the owner of a stockpile of uranium which had come about partly as a result of purchases made in 1969-70 at prices which were near Marginal Cost. Because the capability of performing enriching services was limited, the tails assay of the enrichment plant was raised from 0.2 percent to 0.3 percent to reduce the amount of separative work necessary to produce any given quantity of enriched uranium. The contract requirement for feed material was not changed from the requirement at 0.2 percent and the extra feed was provided from the government stockpile. During this period the price of enrichment was raised, and the price of uranium also rose, thus furnishing some incentive for the producers of yellowcake to follow the market behavior. Expectations of a strong commercial market should stimulate exploration by the mining industry, an important factor in the discovery of new uranium sources. At this mode of operation, however, the U.S. Government was effectively acting as a seller of uranium, since it was providing feed material from its inventory. As perceived by the mining industry (at least one member) this action placed the government in competition with the private mining interests for utility requirements. Thus, the seemingly helpful tactic of purchasing uranium during a depressed market period has exhibited a delayed feedback effect which serves to retard the growth of the industry and to increase the uncertainty surrounding the capability of the nuclear industry to sustain itself as a commercial enterprise (McGee, testimony before the JCAE, 1974).

The forward cost concept has been adopted by the U.S. Government in identifying its uranium resource estimates. It must be observed that this is not identical with, nor is it intended to be the same as, price. Cost measures are fairly objective, based upon the forward cost (essentially the cost of production of U_3O_8). Prices, on the other hand, are subject to variation for reasons which may be related to factors peculiar to the specific company and also because market prices are often proprietary information in the transactions between buyers and sellers. Each company and each mine has its own unique set of circumstances which affect the cost, whereas price is set by market factors (Appelin, 1973). Three quotations may be cited which yield some insight into the perception of the uranium supply situation by prospective purchasers of burner reactors. From the Collins Securities Corporation (1969): "The apparent unavailability of long range domestic uranium reserves is believed causing concern among utilities as it is an important factor in the economic comparison of the respective advantages of nuclear and fossil-fueled reactors." From R. D. Nininger in 1973: "We have not attempted to assess the total resources of the country, and the lack of such an assessment has caused some confusion about uranium supply." Finally, from the fast breeder economists (Stauffer et al., 1975a), we have the following observation: ". . . it is not merely unknown, but also unknowable, whether the present uranium forecast . . . is too low—or too high."

For an economic analysis using published data, it is important to model the constraint of uranium as a scarce resource. In a direct approach, several scenarios of uranium availability may be postulated and the costs computed for each resource availability assumption. In a probabilistic model, which might be of more interest to a policy planner, the resource availability would likely be modeled as a probability distribution function of asymptotic character. Development of this function would be a major undertaking in itself. Simple analyses have already been performed using geologically-based reasoning. Probability distribution functions for the occurrence of uranium have been published (Searl, 1974). An alternative method, used in the analysis of the Liquid Metal Fast Breeder Reactor, is the assessment of rising prices with cumulative consumption (Stauffer et al., 1975a). For computational purposes, this last method is simple to use and has the advantage that the state variable may also be used in the computation of the objective function. The form of the U_3O_8 price vs cumulative consumption curve is taken to be piecewise linear. Lack of published price data makes development of this curve an uncertain task at best. The graphical form given in Stauffer et al. suggests that the break points used in the LMFBR analysis correspond to total resource estimates at $10/lb and $30/lb, respectively. These numbers, published by the Energy Research and Development Administration, are 1,130,000 and 2,490,000, respectively (WASH-1224). Limited U_3O_8 supply is modeled by allowing the slope at the resource limit to become very large relative to the slope of the function at points below the perceived resource limit. A similar approach may be taken in the modeling of the labor constraint. Given a limited availability of labor, increased production simply means moving to another isoquant at a higher capital input. Thus price variations may be employed to describe the constraints upon availability of labor in the mining industry or in any of the fuel cycle support facilities.

Low-Grade Uranium Reserves

Uranium resources which might potentially be utilized in light water reactors are the shale and granite deposits. Nininger (1973) has given an estimate of the forward cost of recovery of each grade of uranium deposit. This information is summarized in Table 8.

While these figures would seem to indicate that sufficient uranium exists for the foreseeable future, it must be borne in mind that the costs are somewhat arbitrary and that much of the low-grade uranium is in reality not available. The costs to be used should include the cost of land reclamation. Further, some of the lower grade deposits lie in areas where the population density might preclude their extraction. Finally, the risk associated with the location of a uranium mill in an area of restricted potential for production will deter uranium producers from exploiting lower grade reserves.

Table 8. Domestic Uranium Resources by Grade

Description	Cost ($)	Quantity (Million tons U_3O_8)
$10/lb ores	10	~ 0.5
$30/lb ores (including $10/lb deposits)	30	~ 2.2
60-80 ppm shale	50	~ 4.5
25-60 ppm shale	100	~ 8.0
10-20 ppm granite	200	~ 8.0
10-25 ppm shale	200 +	~ 200
4-10 ppm granite	200 +	~ 1800
0.003 ppm sea water	200 +	~ 4000

Uranium Consumption History

The history of uranium consumption in the United States is given in Table 9 (COMRATE, 1975). It is of interest to note that the Reserves and the production have been fairly stable over the period following the market depression of 1968. The Shipment to Mills category includes miscellaneous U_3O_8 receipts from sources such as in situ production and leaching residues as well as normal ore shipments. Reserve Estimates from the year 1961 forward are based upon the $8/pound category as employed by the Atomic Energy Commission.

Table 9. Uranium Ore Reserves and Production--1947 Through 1973 (Tons U_3O_8 in Ore)

Year End	Shipment to Mills	Cumulative Production	Reserve Estimation	Sum of Cumulative Production and Reserves
1947	----	----	2,200	2,200
1948	83	83	2,200	2,283
1949	502	585	2,200	2,785
1950	810	1,395	3,000	4,395
1951	1,088	2,483	5,800	8,283
1952	1,288	3,771	7,346	11,117
1953	2,315	6,086	15,203	21,289
1954	3,539	9,625	27,582	37,207
1955	4,425	14,050	67,595	81,645
1956	8,434	22,484	120,240	142,724
1957	9,837	32,321	166,300	198,621
1958	14,003	46,324	181,800	228,124
1959	17,377	63,701	197,100	260,801
1960	18,842	82,543	187,100	270,443
1961	18,513	101,056	174,200	275,256
1962	17,085	118,141	166,200	284,341
1963	14,721	132,862	160,231	293,093
1964	13,888	146,750	150,927	297,677
1965	10,578	157,328	144,702	302,030
1966	10,051	167,379	140,835	308,214
1967	10,866	178,245	147,741	325,986
1968	12,850	191,095	160,819	351,914
1969	12,595	203,690	204,080	407,770
1970	13,073	216,763	246,100	462,863
1971	13,089	229,852	273,200	503,052
1972	13,863	243,715	273,200	516,915
1973	13,787	257,502	276,700	534,202

SOURCE: AEC data, published in Mineral Resources and the Environment.

Thorium Resources

The supply problem for uranium does not appear to exist for thorium except in the very long term. For the utilization of thorium in a light water reactor is considerably greater than the utilization of uranium; hence the mining requirement per unit energy generated is less for thorium than for uranium. Thorium resources are not well known because no strong identifiable market has been stimulated by the presence of an end use. However, enough is known concerning existing deposits so that thorium-based fuel cycles may be investigated without fear of discovering that the primary fuel is insufficient. Table 10 gives the estimated resources of thorium in the United States as known in the last decade. Most of the thorium is found in Idaho and Montana, although some thorium is recovered as a byproduct of mining operations in Georgia and Florida.

Table 10. Thorium Resources in the United States (Short Tons ThO_2)

Production Cost ($ per lb ThO_2)	Reasonably Assured	Estimated Additional	Total
10.00	65,000	335,000	400,000
30.00 (or less)	200,000	400,000	600,000
50.00 (or less)	3,200,000	7,400,000	10,600,000
100.00 (or less)	11,200,000	24,400,000	35,600,000

Foreign reserves of thorium are plentiful, the major deposits lying in South Africa, India, and Canada. Current projected demands for thorium for uses other than the Light Water Breeder Reactor are negligibly small, deriving mainly from the anticipated introduction of the Gas Cooled Fast Breeder Reactor, a concept which also may have technical and economic merit in the face of rising energy prices and constrained uranium reserves. In addition to the availability of thorium within the borders of the United States, the price is likely to remain stable for two reasons. First the cost of mining thorium can be relatively small if beach sands are used. Thus, existing industry can be expanded in the beginning to recover thorium at reasonable cost. Further, the abundance of thorium in the United States and the suggestion that even more exists should engender a competitive market in which price will tend toward the marginal cost of production. The situation is different from the uranium scene in that it is anticipated that only a fraction of existing reactors will be thorium users. Even when the fraction becomes large enough to cause some enlargement of milling capacity, the prospect of imports from one of the thorium-rich countries will tend to maintain price at a level approximating Marginal Cost.

Among the questions which must be answered in an economic analysis is that of the cost of establishing a thorium mining and milling industry. The cost of this industry will depend upon the processes required for the recovery of the metal from its parent ore, and the incentive for its establishment will derive from the probability of favorable rate of return on equity capital. Here the economic laws which favored the enforcement of Marginal Cost pricing act to retard expansion of the mining and milling industry. The reprocessing cost for thorium-U-233 fuel is estimated to be a factor of two higher than the charge for LWR fuel. Chemical requirements for the reprocessing facilities are not significantly different, thus one would not anticipate a great advantage to one fuel cycle or the other as a consequence of increased materials costs. It is interesting to observe that the breeder segment of the LWBR system is less energy intensive in the reprocessing component than is the Prebreeder segment. Table 11 shows some of the natural resource requirements arising from the reprocessing function associated with the LWBR system.

Table 11. Annual Natural Resource Requirements (Assumed LWBR Reprocessing Facilities)

Resource	Prebreeder Facility	Breeder Facility
Land (acres)		
Temporarily Committed	2,940	2,960
Undisturbed	2,670	2,780
Disturbed	270	180
Permanently Committed	60	40
Total Committed	3,000	3,000
Water (million gallons)		
Total Input	320	180
Released to Atmosphere	280	150
Released to Watershed	28	15
Released to Ground	7.5	8.6
Process Use	3.7	5.5
Energy/Fuel		
Offsite Electrical Power (GWhr)	80	54
Coal for Electrical Power (thousand MT)	17.9	12
Natural Gas for Steam Power (million SCF)	1,680	647

It is seen that the hypothetical Breeder reprocessing facility requires less than half the natural gas than does the Prebreeder reprocessing facility. The electrical energy requirements, while not quite as drastically reduced, are nevertheless significantly lower for the Breeder reprocessing facility than for the Prebreeder counterpart. Therefore, the cost of reprocessing LWBR fuel is not quite as sensitive to energy costs as is the LWR fuel.

Economic Aspects Affecting Nuclear Energy Growth

It has been accepted as a postulate for several years that nuclear energy would form the base load (or the majority of it) by the year 2000. Predictions were generated upon the basis of two assumptions which should be examined seriously. The first of these is the expectation that necessary uranium would be available to fuel the committed reactors. Older predictions show that about two million tons U_3O_8 would need to be mined before the year 2000 in order to fulfill the anticipated energy needs. The uranium supply situation has already been addressed, the result being that the uncertainties are being reduced but that the estimates of available retrievable quantities are considerably lower than might have been the case in 1970. A second assumption which must be called into question is the requirement for power itself. The correlation has been made and accepted that energy is necessary for economic growth, and that economic growth is necessary in an industrial society which seeks to provide for the welfare of its citizens in a manner consistent with the level of its technology. Economic growth is generally measured in terms of the Gross National Product, and is a fair measure of the ability of the society to meet the society's goals of providing employment for the able-bodied and comfort for the average member thereof. It is necessary to examine the assumption that continued growth is a necessary and desirable thing, as well as to observe that such growth is by no means assured. Retreating to the accounting basis for computation of the GNP, it may be seen that the goods produced are tied to the primary metals industries. Thus, a growth component of the GNP is due to the conversion of an exhaustible non-renewable resource such as iron rather than the development of a process which adds value to a resource without disturbing its availability. Where the resource is unbounded, the concept of exhaustibility does not apply. However, as has been witnessed in the case of petroleum, resources which were once viewed as inexhaustible are no longer regarded so. Therefore, the mathematical treatment would be expected to differ. It has been seen, as a result of increased trade with the Middle East nations, that the United States no longer possesses the economic power in the world that was once its claim. This turn of events has resulted in an outflow of money from the country's resources, and thus an attenuation of actual growth as well as a sobering review of the ability of the financial institutions of the country to meet the more heavily-capitalized needs such as investment for gaseous diffusion. The economic forces which were the underpinning for deficit financing and therefore growth, are no longer operative, as Roosa (1975) has recently observed. Thus, it can no longer be accepted as a postulate that energy growth is required to maintain the lifestyle to which American society has become accustomed, since the shift of economic power brought about by recognition of the world distribution of resources and the successes and failures of American foreign policy. What is true is the statement that growth is necessary to maintain the way of life as it existed in 1965; it is not necessarily a valid proposition to claim that economic and energy growth is required for stability of the society henceforth. And in fact, that society may not demand growth of services as once it did; i.e. its preferences may change. Thus, predictions of the past are based upon premises which should be subjected to renewed scrutiny in light of an apparent change in the preferences of society.

If one holds in abeyance the proposition that growth is required, the range of possible and realistic scenarios is enlarged. This type of study has been undertaken by the Ford Foundation (1974), which examined the prospect of several energy growth scenarios, including one entitled Zero Energy Growth. This scenario emphasized conservation and rearrangement of priorities of society. Combined with the availability of coal and the fear of plutonium contamination, it is realistic to consider the case in which nuclear additions, instead of increasing with time, are minimally constant or are decreasing. Thus, the market anticipated for services such as enrichment, conversion, and yellowcake are not necessarily guaranteed to return profits to those which might invest in these enterprises. Therefore, such investment will in all likelihood not materialize, and hence the industry will have its growth attenuated from within as well as without. The turn of events just described would not necessarily have a disadvantageous effect upon the prospects for the Light Water Breeder Reactor. If enrichment capacity is limited, and if uranium supplies are similarly limited, the Light Water Breeder may well be the optimum interim solution to the mid-range energy supply question because of its low Marginal Cost and its perceived low social cost arising from the absence of actinide elements in discharge fuel. Further, if uranium prices continue to rise, the breeder will be an advantageous system because of the independence of natural uranium supply. Items of critical importance to a breeder operation are reprocessing and fabrication capability; hence the imposition of constraints upon these segments of the fuel cycle could result in the attenuation of effort to develop breeders in general.

Econometric Analyses

The demand for energy in the United States is not an easily predicted function. To some extent, the form of energy production is dictated by the composition of the activities carried out. Recent estimates from the Shell Chemical Company are shown in Figures 18 and 19. The first of these shows the distribution of energy in terms of the source. The second shows the distribution according to the sector which uses it. It is observed that the commercial and residential sectors are smaller consumers of energy than the transportation, utilities, and industrial areas. Since it might reasonably be assumed that the industrial and transportation sectors use a substantial portion of energy in forms other than electrical, it may be concluded that generalizations about energy demand do not necessarily apply to the electricity-generating and -consuming process. Among the interesting features of the projection is that the residential demand for energy is near constant through 1990. The

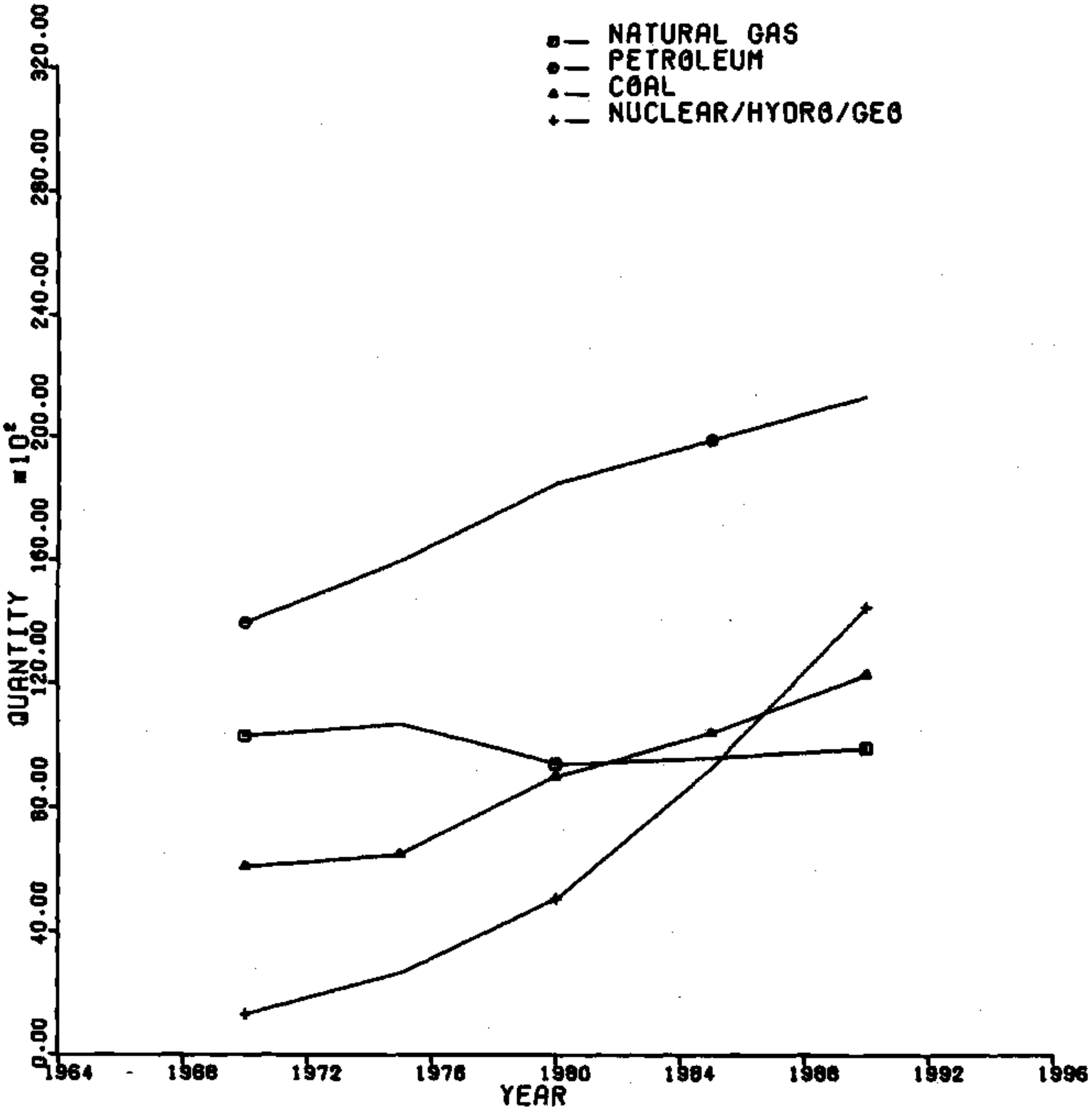

Figure 18. Energy Demand Forecast--Fuel Requirements

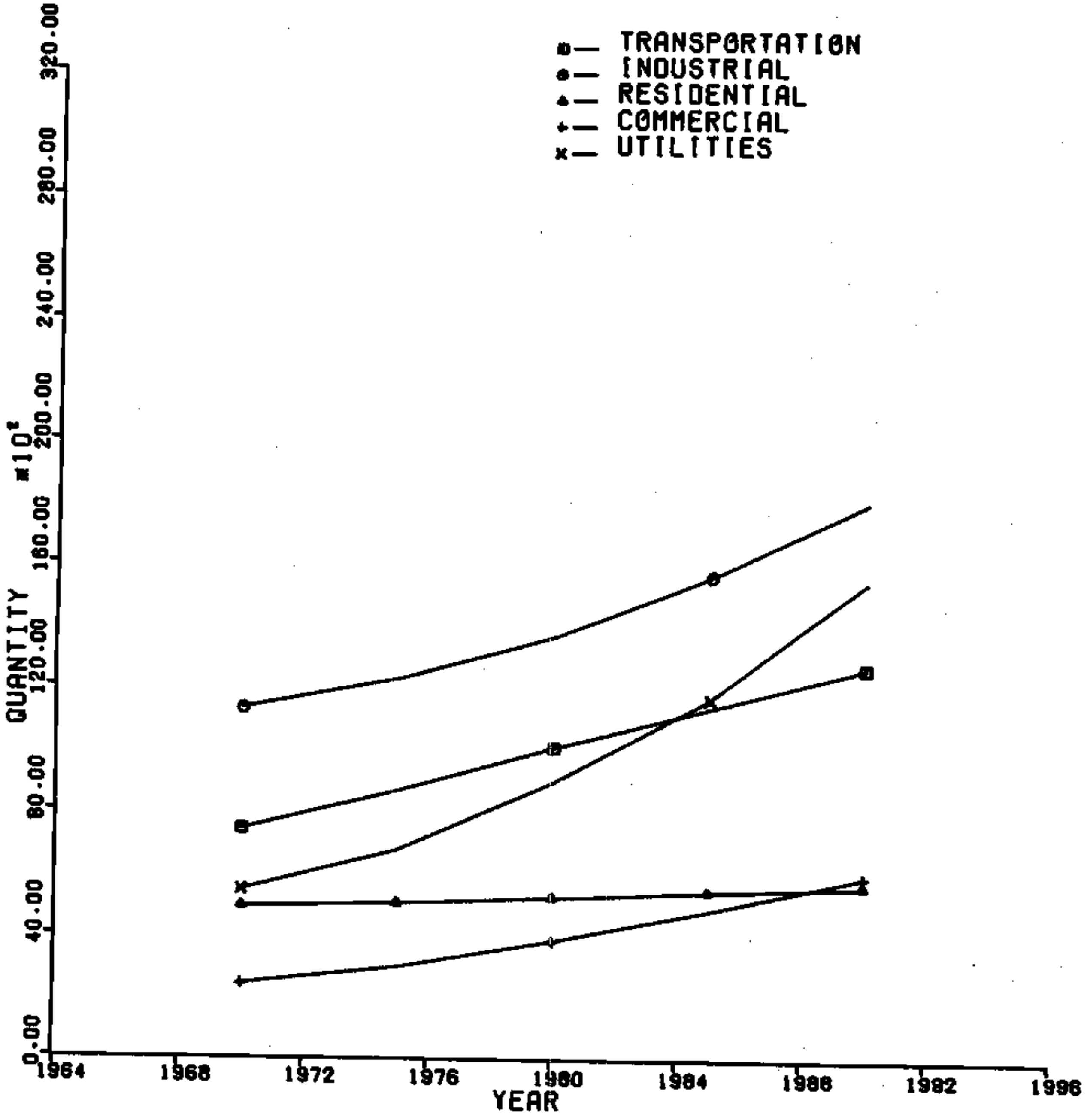

Figure 19. Energy Demand Forecast--End Use

commercial, industrial and utility sectors are predicted to exhibit more pronounced growth. Thus, economic effects which would retard industrial or commercial growth would be expected to have a pronounced effect upon total energy consumption. Since there is very little growth projected for the residential sector, it is anticipated that the majority of pressures upon primary energy producers will come from the commercial and industrial sectors. Electrical power demand through the year 2000 is shown in Figure 20. The source of these figures is the National Petroleum Council and the RAND Corporation (Morris, 1972). These projections were made somewhat in advance of the electricity price increases which have received widespread attention. It is of interest to note the effect of a lower growth rate and a price increase. In this case, the lower growth rate refers to one which is based upon gradual slowing of population growth to zero in the second quarter of the twenty-first century and a decrease of GNP expansion rate to 2.5 percent by the end of the twentieth century.

Econometric analyses of the demand for electricity are most useful when performed upon a sector-by-sector basis. The characteristics of the residential, commercial, industrial, and service sectors would normally be expected to differ. The problem is further complicated by the transition taking place within the society, which sees an increasing trend toward service functions and away from production activities.

Energy Intensity Models

Among the earlier relevant applications of econometric analysis to the problem of estimating electrical demand in the United States is

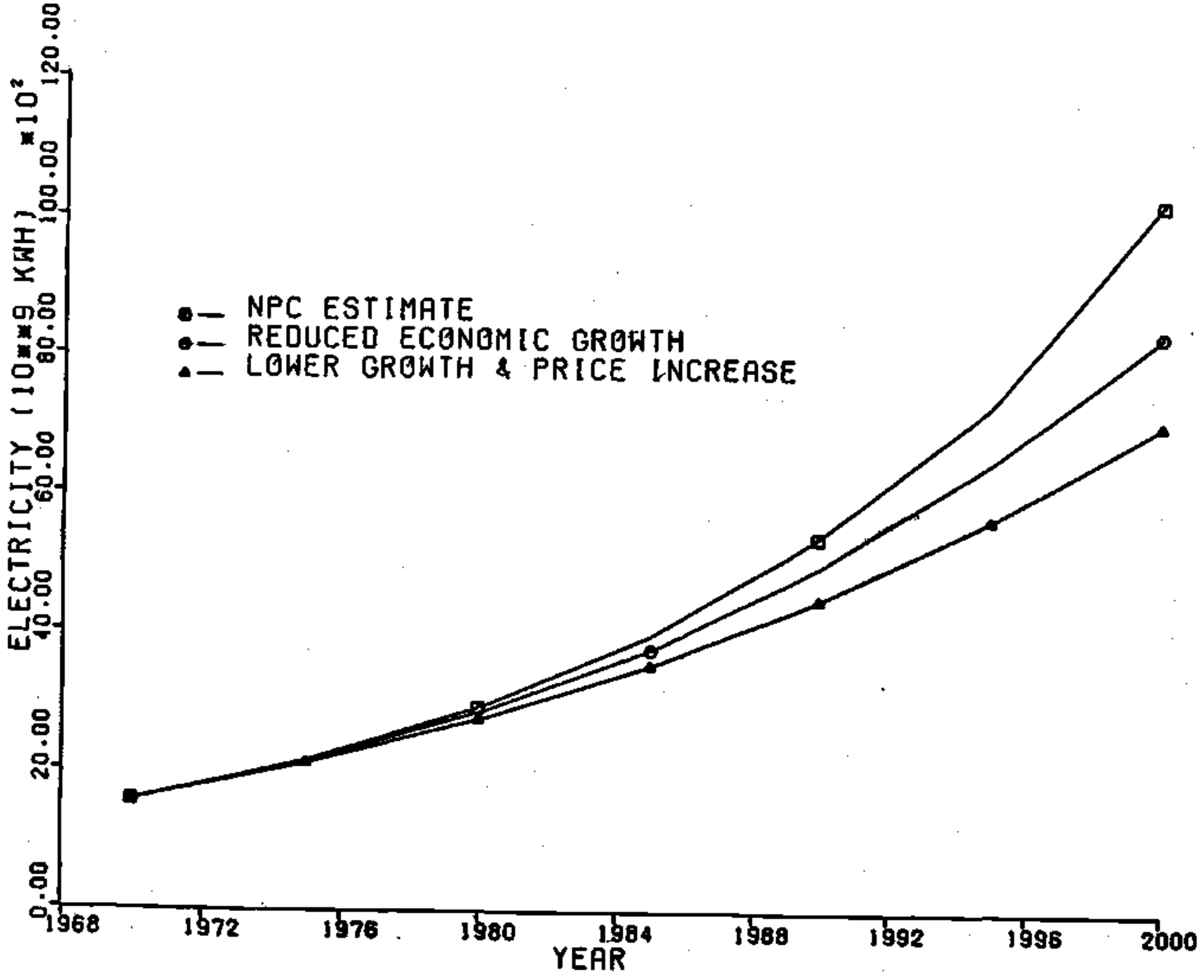

Figure 20. Electricity Demand Estimates

that of Fisher and Kaysen (1962). The basic notions used by these authors have been used by others in later and slightly different analyses. Fisher and Kaysen pointed out that the demand for electricity was derived from the demand for services which could be provided by electrical appliances. To these electrical appliances, whether washing machines or shavers, water heaters or light bulb fixtures, they gave the name "white goods." Traditional use of this term refers only to major appliances; these authors included minor ones as well. The model of Fisher and Kaysen uses the concept of energy-intensiveness. Basically, it says that the demand for electricity is the sum of the demands from each type of electrical appliance. Letting the subscript t refer to the t^{th} discrete time period and the subscript i refer to the type of white good, the demand function D may be written

$$D_t = \sum_{i=1}^{n} K_{it} W_{it} \qquad t = 1, \ldots, T \qquad (4)$$

where

T is the number of time periods

W_{it} is the average stock of the i^{th} white good owned by users in period t

K_{it} is the intensity of use of the stock W_{it} in kWh/time period/unit of white good

n is the number of appliance types.

In this early study, it was assumed that the economic factors influencing demand were personal income, price of electricity, price of natural gas, and the price of substitute services. In the short run, these latter two have negligible effect due to the time required to change a function from electricity to natural gas and because items such as electric washers and dryers represent sunk costs. Competing services would therefore have little effect upon the utilization of already-purchased white goods in the short run. The assumption was further made that the intensity functions were log-linear in form. Thus,

$$K_{it} = F^{i}(P_t, Y_t) = A_i \, P_t^{\alpha_i} \, Y_t^{\beta_i} \tag{5}$$

where

P_t is the average price of electricity to households

Y_t is per capita personal income in time period t

A_i is a constant

α_i and β_i are assumed constant and they are the price and income elasticities, respectively, of the intensity of use for the i^{th} white good.

By adopting the convention that a unit of white good is that quantity which consumes one kilowatt-hour in one hour of normal use, the authors arrived at the equivalent expression for short run demand.

$$D_t = \sum_{i=1}^{n} C_i (P_t/\overline{P})^{\alpha_i} (Y_t/\overline{Y})^{\beta_i} W_{it} \qquad t = 1, \ldots, T \tag{6}$$

where

C_i is a constant for the i^{th} good

$\overline{P}$ and $\overline{Y}$ are the averages of price and per capita personal income over the T periods of observation.

Even if data were available, estimation of individual elasticities for each of the n types of white good would be extremely taxing if not impossible. Since the objective was the estimation of total electrical demand, rather than the demand functions for the individual goods, Fisher and Kaysen made the assumption that the residential demand for electricity would be written approximately as

$$D_t = C(P_t/\overline{P})^{\alpha} (Y_t/\overline{Y})^{\beta} \sum_{i=1}^{n} W_{it} \tag{7}$$

This is equivalent to the assumption that the elasticities of demand are equal for each of the white goods.

Defining $W_t = \sum_{i=1}^{n} W_{it}$ and writing the demand equation as

$$D_t = C\overline{P}^{-\alpha} \overline{Y}^{-\beta} P_t^{\alpha} Y_t^{\beta} W_t = A P_t^{\alpha} Y_t^{\beta} W_t \tag{8}$$

where

$A = C\overline{P}^{-\alpha} \overline{Y}^{-\beta}$ and taking natural logarithms, the equation for estimation is obtained.

Including an error term u_t which varies with time, the following equation results

$$\ln D_t = \ln A + \alpha \ln P_t + \beta \ln Y_t + \ln W_t + u_t \tag{9}$$

Because this was a time series problem, first differences of both sides were used in the analysis to remove serial correlation problems.

Results of the Fisher-Kaysen short-run demand analysis, while showing little significance on an individual state basis, did reveal that groupings of states showed good statistical behavior. Price elasticities for data from years 1946-57 ranged from -0.1623 to -0.9974. In general, the economically younger states showed higher absolute values for elasticity than did the more urban states in the North and East. There is no entirely satisfactory explanation for this. Since the model used for estimation was built on the basis of the stock of white goods, it might be expected that the poorer states would have relatively more low-elasticity electricity-consuming durables such as refrigerators and fewer appliances of elastic usage. The conclusion reached by the authors was that the difference between the more economically mature states and the less mature ones was attributable to a fundamental difference in the demand functions for the two regions. An hypothesis which explains in part the behavior of the results is that in economically newer regions, manual labor is a substitute for electricity usage to a greater extent than in the more urban regions, and thus the response to price changes would be more pronounced. The long run residential consumption of electricity being a function of the stock of electrical appliances, the Fisher-Kaysen analysis used the following log-linear model:

$$\frac{S_{it}}{S_{i(t-1)}} = A_i \left(\frac{Y_t^E}{Y_{t-1}^E}\right)^{\eta_{i1}} (Y_t)^{\eta_{i2}} (E_{it})^{\eta_{i3}} (G_{it})^{\eta_{i4}} \left(\frac{H_t}{H_{t-k}}\right)^{\eta_{i5}} \tag{10}$$

$$\times \left(\frac{F_t}{F_{t-1}}\right)^{\eta_{i6}} (M_t)^{\eta_{i7}} (P_t^E)^{\eta_{i8}} (R_{it})^{\eta_{i9}} (Y_t^E)^{\eta_{i10}} U_{it}$$

$$i = 1, \ldots, n; \; t = 1, \ldots, T$$

where

S_{it} is the number of physical units of white good i owned by the community in year t

A_i is a constant

Y_t^E is a moving average of real personal income per capita

Y_t is current income per capita in period t

E_{it} is the price of white good i in time period t

G_{it} is the price of a natural-gas-using substitute for good i

H_t is the number of customers for electricity per capita

F_t is the population of the community

M_t is the average of the number of marriages in periods t and t-1

P_t^E is the anticipated price of electricity (a 3-year moving average was used in the Fisher-Kaysen analysis for this quantity)

R_{it} is the consumption ratio of the appliance measured in kWh per hour of average use

V_t^E is the anticipated price of natural gas as measured by a 3-year moving average

U_{it} is an error term.

The $\eta_{ij}, j = 1, \ldots, 10$ are the elasticities of demand for the specified variables. For convenience, they are

η_{i1} is elasticity of demand with respect to change in long run income

η_{i2} is elasticity of demand with respect to current income

η_{i3} is elasticity of demand with respect to price of appliance i

η_{i4} is elasticity of demand with respect to price of gas-using substitute for white good i

η_{i5} is elasticity of demand with respect to change in number of wired households per capita

η_{i6} is elasticity of demand with respect to population change

η_{i7} is elasticity of demand with respect to number of marriages

η_{i8} is elasticity of demand with respect to price of electricity

η_{i9} is elasticity of demand with respect to consumption rate

η_{i10} is elasticity of demand with respect to price of natural gas.

It would be reasonable to expect that for some given appliance type, say for electrical washing machines, some of the terms in the above equation would not affect the stock of goods. Further, appliances such as washing machines do not have gas-using substitutes, so that the 4th and 10th terms are replaced by unity in the stock equation. It might be speculated that the number of washing machines purchased depends very little, if any, upon the consumption factor. Indeed, this is borne out in the study performed, which showed in a broad sense that the operating parameters were those associated with income and number of available households. For this analysis, the country was separated into eight economic regions. Regression analyses were carried out on data for washing machines, refrigerators, ironing machines, and electric ranges. One of the conclusions of the study is that the demand for major electric appliances is affected by the price of electricity only where the price of electricity is high and where there is competition from the natural gas sector. The implications of this study for the electric power generation industry would appear to be important. The decomposition of elasticity into short run and long run components reveals that price has little long term effect upon consumers' stocks. However, there is a price effect attributable to the change in intensity of use of electrical appliances. For long range forecasting, therefore, the dominant factors affecting residential electricity consumption are those of income and population. For the industrial sector, values of price elasticity substantially higher in absolute value were found than those for the household case. Of course, the elasticities varied widely depending upon the electrical intensiveness of the industry. The Chemical Products and the Electrical Machinery Industries showed price elasticities of 2.5976 and 1.8209, respectively. (Data were for the time period 1950-56.) Since the industrial sector is an important consumer of electricity, it is of some importance to include price effects in forecasts of long term demand. For industries having high operating costs may find it profitable to convert part of their energy source from electrical to natural gas, coal, or fuel oil, depending upon anticipated prices. The residential consumer, on the other hand, normally faces operating costs which are small in relation to the fixed cost of his appliances.

The phenomenon of rising energy prices is a relatively new development. More recent studies by The RAND Corporation have contributed to the understanding of the nature of residential energy demand. There are two aspects to the energy question. One is the effect of price upon total electrical demand. The other is the effect upon the mix of units that will be commited. Morris (1972) concludes that price changes of 50 percent or less would have small impact upon the quantity of electricity demanded. He estimates that a change of that sort would have a diminishing effect upon electricity consumption only 5 to 10 percent by the year 2000. Even less sensitive is the share of energy generation distributed among coal, oil, gas, and electricity. He estimates, for example, that a 50 percent increase in the price of one resource relative to the others would result in a decrease in the use of only about four percent.

Anderson's study (1973), also a RAND Corporation analysis, is based upon utilizing a cross-section analysis. Like the study of Fisher and Kaysen, estimates were made of both energy and stock equations having the log-linear form. Instead of using time series data, numbers from the two census years 1960 and 1970 were utilized. Regression analyses were performed upon the stock equations with ratios of services as the dependent variable. The energy equation resulted in the computation of elasticities for single parameters, such as electrical energy consumption or natural gas energy use. Anderson's formulation of the stock equation includes the concept of a retention ratio. Between distinct observations, separated in time, old units will be scrapped or replaced with new ones and additions will be made. In the formulation which follows, the time periods will be denoted by t and t - n. The initial equation says that the number of consumers who employ energy type i (electricity, for example) to perform a certain function depends upon the number who were using this energy for the specified work in period (t - n) and the number who converted or added appropriate installations in the intervening n periods. The determinants of this decision to add or change energy types are of primary importance in the analysis by Anderson.

For m energy types, the basic equation is

$$N_{it} = r_i N_{i(t-n)} + s_i (N_t - \sum_{i=1}^{m} r_i N_{i(t-n)}) \qquad (11)$$

where the subscript i refers to energy type such as electricity, natural gas, or fuel oil. Other parameters are defined as follows:

N_{it} is the number of households in period t using energy type i

N_t is the total number of households

r_i is the fraction of households keeping existing equipment

s_i is the fraction of new installations and replacements bebween periods t - n and t which use energy type i.

r_i is defined in terms of a retention ratio as follows:

$$r_i = \begin{cases} \rho_i \, N_{it}/N_{i(t-n)} & N_{it} \leq N_{i(t-n)} \\ \\ \rho_i & N_{it} > N_{i(t-n)} \end{cases} \qquad (12)$$

Thus, a decrease in the number of consuming units is properly treated in the equations.

Given the retention ratio, the sales share fraction s_i may be calculated. The problem then is one of finding some functional relationship which explains the variation in s_i over the sample points, the states, for the time interval taken. A constraint is that the sum of the s_i over all energy types must be equal to unity, since the s_i are fractions. The form assumed is

$$s_i = s_i(p,y,v,z,u) = \frac{a_i\ p_i^{b_i}\ y^{d_i}\ v_i^{c_i}\ z_i}{\sum_{i=1}^{m} a_i\ p_i^{b_i}\ v_i^{c_i}\ y^{d_i} z_i}\ u_i, \qquad i = 1, \ldots, m \quad (13)$$

where

p_i is the price of the ith energy source

v_i is the purchase price representative of the class of devices which perform the function

y is the household income

u_i is an error term

z_i represents other household variables.

With this form, the ratio s_i/s_m can be expressed. It is this ratio which is used in the estimating procedure. Any energy source other than the ith one can be used, but in this type of analysis the elasticities will be different for each ratio. The derived form of the stock equation is thus

$$\frac{s_i}{s_m} = \frac{a_i}{a_m}\ p_i^{b_i}\ p_m^{-b_m}\ v_i^{c_i}\ v_m^{-c_m}\ y^{(d_i-d_m)}\ \frac{z_i}{z_m}\ \frac{u_i}{u_m} \qquad i = 1, \ldots, m-1 \quad (14)$$

For the RAND analysis, the household variables and their respective units represented above as z_i are household size (persons/household) and mean December temperature (degrees F). The form of the equations actually estimated is

$$\ln \frac{s_i}{s_j} = a_{ij} + b_i \ln P_i + b_j \ln P_j + C^1_{ij} \ln Y + C^2_{ij} \ln HS \qquad (15)$$

$$+ C^3_{ij} SHU + C^4_{ij} NUHU + C^5_{ij} WTEMP + U_{ij},$$

$$i = 1, . . ., 8$$

$$i \neq j$$

where the variables which appear in capital letters denote the following quantities:

HS is the household size

SHU is the fraction of single housing units

NUHU is the fraction of non-urban housing units

WTEMP is the average December temperature.

Taking eight energy forms, the index j was fixed (denoting a reference energy source) and seven equations were estimated using the techniques of generalized least squares. For Anderson's analysis, the energy forms are natural gas, fuel oil, coal, electricity, bottled gas, wood, other fuels, and none. Energy use functions examined include space heating, water heating, cooking, food freezing, and washing and drying. The energy use equation is formulated in a similar fashion. Price and income effects are log-linear in form and a retention ratio concept was used to account for stock adjustment in the ten years between observations. The equations estimated had the form

$$\begin{aligned}\ln X = A_0 + A_1 \ln PELEC + A_2 \ln PGAS + A_3 \ln PCOAL \\ + A_5 \ln PBGAS + A_6 \ln YPH + A_7 \ln HS + A_8 \ln SHU \\ + A_9\ NUHU + A_{10}\ WTEMP + A_{11}\ STEMP + U\end{aligned} \tag{16}$$

The dependent variable X can be consumption per customer or consumption per household. Variables heretofore undefined are

PELEC is price of electricity
PGAS is price of natural gas
PCOAL is price of coal
PBGAS is price of bottled gas
STEMP is mean July temperature (degrees F).

From the standpoint of demand for electrical energy, the coefficients of primary interest are the price elasticity estimates. The consumption equations yield separate estimates based upon both the set of data used and the model chosen. The "dynamic" model measures the ratio of the change in consumption and the change in consuming households; this is approximately a first derivative. The "static" model measures the ratio of the change in consumption to the number of households in the latter period. Table 12 shows the coefficients of elasticity for eight conditions of estimation. In each case there were 38 degrees of freedom, so that the values of elasticity thus presented are highly significant. At 40 degrees of freedom, for example, a t-ratio of 5.77 corresponds to a significance level of 10^{-6}, while more general conclusions were not as strongly supported by the results of the analysis, it is a useful result that, for residential demand, price

Table 12. Price Elasticity of Residential Electrical Energy Consumption (t-ratios in parentheses)

Condition of Estimation		Dependent Variable	Own-Price Elasticity	
Model	Data Set			
	1960	kWh/customer-year	-1.07	(-6.70)
	1960	kWh/household-year	-0.99	(-6.20)
	1970	kWh/customer-year	-1.26	(-6.80)
	1970	kWh/household-year	-1.12	(-6.00)
static, retention ratio = 0.5	1960	kWh/household-year	-0.99	(-6.17)
	1970			
dynamic retention ratio = 0.5	1960	kWh/household-year	-0.95	(-6.91)
	1970			
static, retention ratio = 0.75	1960	kWh/household-year	-1.03	(-5.39)
	1970			
dynamic, retention ratio = 0.75	1960	kWh/household-year	-0.91	(-6.20)
	1970			

effects may not be ignored. Estimates of elasticities for the commercial sector are more difficult to obtain because the various sources of data do not follow a consistent pattern of organization. A model of the demand for commercial electricity has been put forth by Mooz and Mow (1973). In a manner similar to that of Fisher and Kaysen, the concept of energy intensity was used. The demand function obtained as a function of real Gross State Product is

$$\hat{D}'_t = [0.235 + 0.0204(t - 1955)]\ GSP_t \tag{17}$$

In a slight departure from the residential analysis, a lagging term is included in the expression which includes price effects. The energy demand becomes

$$D_t = \hat{D}_t \left[\lambda \left(\frac{P_{e_t}}{P_{e_o}}\right)^{E_c} \left(\frac{P_{g_t}}{P_{g_o}}\right)^{E_g} + (1-\lambda)\ A_{t-1}\right] \tag{18}$$

where

A_t is an adjustment factor for period t

λ is a factor which allows accounting for lag effects due to long conversion times

P_{e_o}, P_{e_t} are the real price of electricity in periods 0 and t

P_{g_o}, P_{g_t} are the real prices of gas in the base year and the year t

E_c, E_g are price elasticities for electricity and natural gas, respectively.

The Analysis of Mount, Chapman, and Tyrrell

An analysis by Mount, Chapman, and Tyrrell (1973) uses fewer explanatory variables than does the Fisher-Kaysen study, but the concept of variable elasticity is investigated. The equations are similar in structure to those used by Anderson in his study of residential demand for electricity, the main difference being that the Mount-Chapman-Tyrrell study (hereafter called the MCT analysis) employs a consumption term lagged by one period and uses both cross-sectional and time-series data. The three models employed are the following:

1. Constant Elasticity

$$Q_{it} = A\, Q^{\lambda}_{i,t-1}\, V^{\beta_i}_{1it} \cdots V^{\beta_N}_{Nit} \qquad (19)$$

where the subscript

i denotes state

t denotes period

λ is the response term

A is a constant

β_j are elasticity coefficients

V_{jit} are levels of use for the explanatory variables.

2. Variable Elasticity Model A

$$Q_{it} = A\, Q^{\lambda}_{i,t-1}\, V^{\beta_1}_{1it} \cdots V^{\beta_N}_{Nit}\, e^{\gamma_1/V_{1it}} \cdots e^{\gamma_N/V_{Nit}} \qquad (20)$$

where γ_j are constants to be determined from the regression analysis.

3. Variable Elasticity Model B

$$Q_{it} = A \exp(\delta_o/D_{it}) Q^{\lambda}_{i,t-1} V_{1it}^{\beta_i+\delta_1/D_{it}} \cdot \cdot \cdot V_{Nit}^{\beta_N+\delta_N/D_{it}} \qquad (21)$$

$$\exp(\gamma_1/V_{1it}) \cdot \cdot \cdot \exp(\gamma_N/V_{Nit})$$

where

D_{it} is the level of the shift variable

δ_k are unknown constants to be determined from the regression.

Examples of shift variables are mean January temperature and degree of urbanization. Estimation of parameters in the Variable Elasticity Models was accomplished utilizing both the technique of Ordinary Least Squares and that of Instrumental Variables. For the Instrumental Variables approach, the partial elasticities for the commercial, residential and industrial sectors are given in Table 13. It is interesting to observe that in all three sectors, the price elasticity is greater in magnitude than unity.

It is of interest to compare the results for the commercial sector with the formulation assumed by Mooz and Mow in which the income term was assumed to have a linear multiplicative effect. The income effect as determined by the MCT analysis is the income factor raised to the power 0.88, for which a linear approximation might well be a valid simplification in many cases. The authors of the MCT analysis suggest that the price elasticity of electrical demand may attenuate the expected consumption of electricity if costs associated with energy generation should rise. The long-term effects of a reduction in consumption would be a reduction in the purchase of new power plants and in the aggregate requirement for fuel. It is of some interest to examine the system composed of raw materials suppliers, necessary services, power generation, and consumers to determine the properties of short-term response and stability.

Table 13. Mount-Chapman-Tyrrell Study--Econometric Analysis of Electricity Demand, Variable Model

Class	Factor	Mean Level of Elasticity
Residential	Population	0.95
Residential	Income	0.21
Residential	Price of Electricity	-1.24
Residential	Price of Gas	0.13
Residential	Appliance Price	-0.74
Commercial	Population	0.98
Commercial	Income	0.88
Commercial	Price of Electricity	-1.45
Commercial	Price of Gas	0.04
Industrial	Population	1.05
Industrial	Income	0.65
Industrial	Price of Electricity	-1.74
Industrial	Price of Gas	0.06

Using the Ordinary Least Squares estimates of the elasticities, Tyrrell (1973) has published a study which exhibits the effect of hypothesized price variations. It is of interest to note that the analysis was performed before the sharp increase in petroleum prices. By way of exhibiting the anticipated behavior due to "normal" events, the projections of electrical energy consumption for the year 1990 are compared with three others of some stature. The results are displayed in Table 14, which shows the effects of the elasticity of the electric power consuming sector. The long-term elasticity is the effective parameter; in the MCT analysis, the short-term elasticity is given by Tyrrell as -0.145.

Table 14. Electricity Demand Predictions for 1990 (trillion kWhr)

Source	Residential Demand (1970 = 0.45)	Total Generation (1970 = 1.52)
Electrical World	1.79	5.93
Federal Power Commission	1.41	5.83
Cornell-NSF Workshop	5.38	
Tyrrell, Base Case	1.20	3.88

Wilson's Approach

John W. Wilson (1971) has performed an analysis based upon cross section data from 77 cities. He points out that the Fisher-Kaysen results may be affected by the use of statewide averages. Wilson cites the state of New York as an example, mentioning Buffalo, where rates are low and consumption is high, and New York City where rates are extremely high and consumption is correspondingly low. In the Fisher-Kaysen analysis, the effects of these two distinct demographic areas are obscured by the statewide averaging process. In the Wilson analysis, two forms of the demand equation are estimated. The determinants of residential electricity consumption are assumed to be

1. price of electricity (P)
2. average price of natural gas (G, cents/therm)
3. median annual family income (Y)
4. average size of housing units (R)
5. climate (degree-days).

For the 77-city cross section, the first three of the factors listed above were found to be the most important. An estimation of the log-linear form of the demand equation yields

$$\ln Q = 10.25 - 1.33 \ln P + 0.31 \ln G - 0.46 \ln Y + 0.49 \ln R - 0.04 \ln c \qquad (22)$$

The R-square value resulting from this analysis is 0.566. In the log-linear form, the coefficients of the logarithmic terms are the elasticities. Thus, the price elasticity as derived by Wilson from cross section data is 1.33. Estimates of coefficients of electricity price and income are statistically significant at the 0.001 confidence limit; the gas price coefficient is significant at the 0.01 confidence limit. The housing unit size and climate coefficients are statistically significant at the 0.10 confidence limit. An analysis of stock equations in which the stock of an appliance type is modeled as a function of electricity price, gas price, income and climate yields similar dependence upon price. Of importance is the price elasticity of electric ranges and water heaters, since close substitutes for these exist. As would be expected, a strong positive effect of the price of gas is seen.

Unlike the Fisher-Kaysen results, however, the stock of electric ranges is seen to be sensitive to the price of electricity, the price elasticity of demand being 1.98. (The exponent in the stock equation is thus -1.98.) Fisher and Kaysen, using an admittedly approximate formulation, obtained a positive exponent for seven out of eight of the sections defined by their pooled data. Their conclusion, using time-series data, was that the price of electricity and the price of gas have no significant effect upon the stock of electric ranges (1962).

One of the fundamental differences between the Wilson model and that of Fisher and Kaysen is that the Fisher-Kaysen formulation takes into account the price of the appliance and the price of its substitute. These two variables are not included in the Wilson analysis. It may be noted that the Fisher-Kaysen study was based upon data for the years 1946-49 and 1951-57, and that the Wilson regression used data from 1966. During the earlier period there was an expansion of real income and many purchases were stock additions rather than replacements. By 1966, it is possible that the purchase of items as essential as electric (or gas) ranges was primarily for the replacement of existing equipment. Thus, the stock would be less sensitive to the purchase price.

One aspect of Wilson's work of significance for the utility industry and the environmentally-conscious organizations is the suggestion of the use of marginal cost pricing in the sale of electricity. That is, that the electric utility should be forced to adopt a rate structure which would make the price of electricity identical to the price which would exist in perfect competition. If a utility is operating in a region which is characterized by decreasing Marginal Cost and Average Cost, Wilson argues that marginal cost pricing can be used to reduce considerably the welfare loss. The implications of this position deserve serious and thoughtful consideration.

First, Wilson and other investigators of recent times have shown that the demand for electricity is price-elastic. This means that a reduction in the price of electricity will result in a more-than-compensating increase in its use so that the total revenue of a utility is increased. A conflicting view is reported by Myhra (1974), who notes that utility executives believe that electricity demand is not sensitive to rate increases. Figure 21 puts the problem in reasonable perspective. In this example it is assumed that the Marginal Cost of electricity is constant and the Average Cost is declining. D1-D1 is the demand function whose price elasticity is greater than unity. In the region of interest, the Marginal Cost curve MCZ is assumed to be constant and less than the Average Cost, which is represented by the curve AC. The evaluation criterion is the welfare loss, a term used to indicate a nonrecoverable loss in benefit to society due to a price increase. Wilson implies that the criterion of optimality is minimization of the welfare loss. A firm in perfect competition will produce up to the point where its Marginal Cost equals Price, which is equal to Marginal Revenue it receives for its goods. An unconstrained monopolist will also produce until Marginal Revenue is equal to Marginal Cost. Unlike the firm in perfect competition, the monopolist has no competitors and

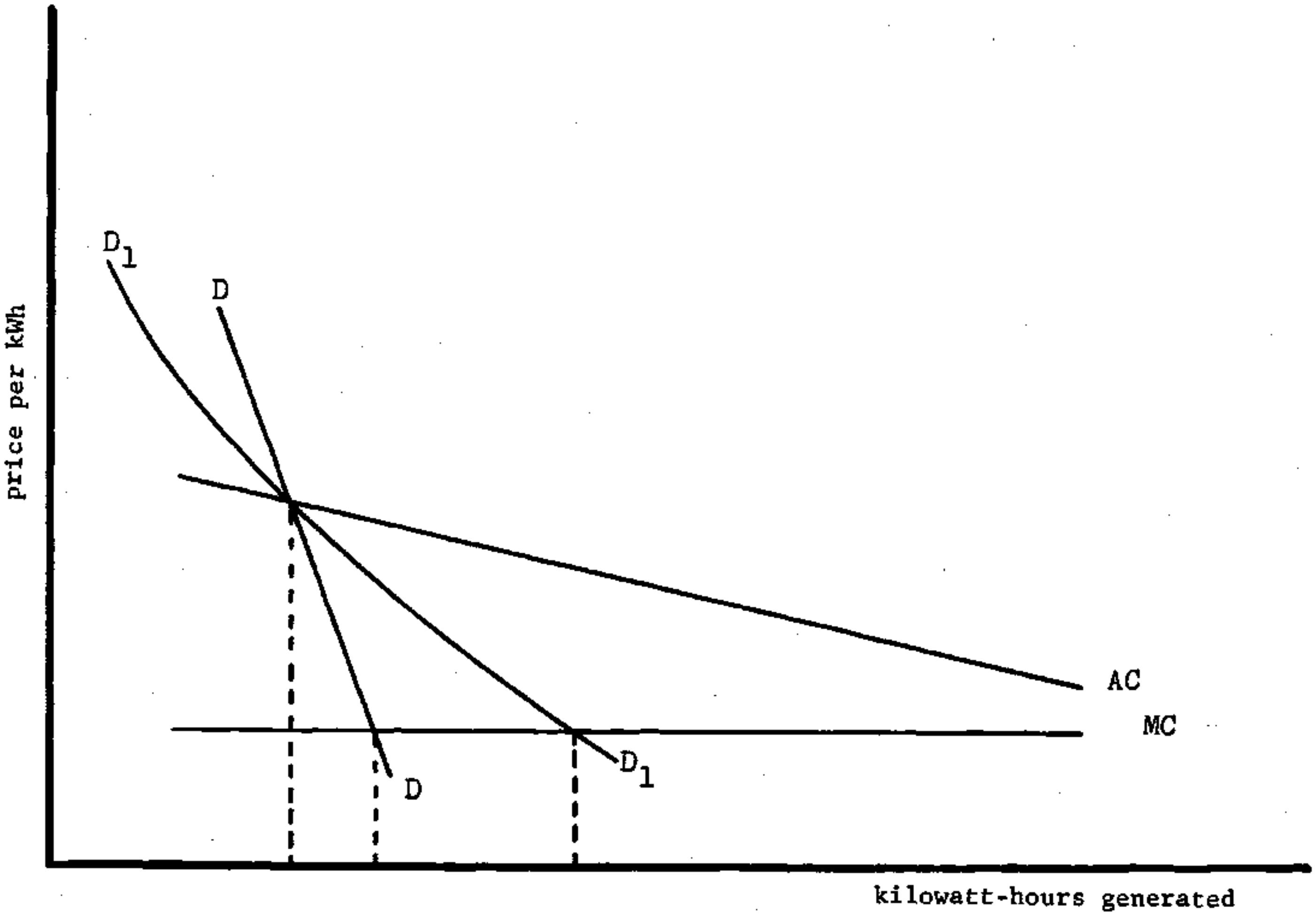

Figure 21. Welfare Loss Under Conditions of Average Cost Pricing

thus the demand curve for the industry is the demand curve for the firm. The producer of electricity who sets his rates controls the amount of electricity that is consumed. A firm in perfect competition must take the market price and produce as much as it profitably can; a single such producer does not affect the total quantity purchased by the consuming sector. The effect of pricing and pricing criteria upon the consumption of electricity is of importance if the demand is price-elastic. When the demand is price-inelastic, as Myhra suggests, the effect of pricing criteria is diminished. Referring to Figure 21, the welfare loss is represented by the area bounded by the demand curve, the Marginal Cost curve, and the vertical drawn to intersect the demand curve at the sale price. In the case where price is inelastic, the Marginal Revenue curve is reasonably close to the demand curve and thus the welfare loss due to noncompetitive pricing is small relative to that which obtains for the case in which there is price elasticity.

Since Marginal Cost pricing is being proposed as a possible mechanism to increase the benefit to the consumer, a brief look at the underlying assumptions is in order. The primary assumption is that Marginal Cost is constant and lies below Average Cost. In Wilson's analysis, the assumption is made that the customers are already using electricity from the system. The behavior does not deal with system expansion brought about by an increase in the customer base. If the utility is operating its base load units at a level below their rated capacity, the increased cost to the utility due to an increase in utilization would consist primarily of fuel cost, and the assumption of constant Marginal Cost is appropriate. Should the base load units be fully committed, however, it would be necessary to bring peaking units on line, and the Marginal Cost would then assume the form of a step function. Another consideration to be taken into account is the effect of pricing strategy upon the allocation of resources. If Marginal Cost pricing schemes can be employed to make the price of electricity used at night cheaper than that used during the daylight hours, the result would be a flattening of the daytime peak. However, if the Marginal Cost pricing strategy reduced the daytime rate as well, the consumption of electricity would be greater. In the case of a price-elastic demand, this would lead to expenditure on more base-load units such as nuclear plants. This in turn would increase the requirement for uranium and for fuel cycle services.

Econometric Analysis: Summary

The most compelling reason for performing economic studies in the engineering discipline is that of prediction. By calculating or assigning monetary values to each component of a process and to its effects, a comparison of alternatives may be made. The purpose of an econometric analysis is the estimation of parameters included in a given model and, as an outgrowth of this, an assessment of the usefulness of the model. It can be concluded from the Wilson, MCT, and the RAND studies that there is a definite price-elastic behavior for the consumption of electrical energy in the residential sector. It is assumed by Mooz and Mow that the magnitude of this effect is the same in the commercial sector of an economically well-developed state. In utilizing the results of these studies, it is necessary to observe that prices of competing energy types were treated as independent variables. For the data used, this assumption is probably very good. However, since the time of the substantial increase in the prices of fuel oil, coal, and U_3O_8, the cost of electricity will be more strongly correlated with the cost of competing energy sources. If the cost of electrical energy increases as a result of a uniform increase in the cost of all fuels, there will be a decrease in use due to a lessened intensity but no change in the mix of energy-consuming goods.

In his study of the residential sector, where more than one energy source exists, Anderson (1973) finds that about one third of the price effect presents itself in the form of conservation and about two thirds in the form of interfuel substitution. Therefore, if the substitution effect is suppressed, the magnitude response to a price increase should be diminished.

It is of interest to observe that techniques in economic analysis are as advanced as in other disciplines. Problems arise because of incomplete or ambiguous historical data and because of approximations or omissions in the model. An interesting and useful approach has been taken by Pindyck (1973), who has implemented a state variable formulation of a simple national economy model. His model consisted of nine behavioral equations together with income and tax relations. Coefficients were estimated using a two-stage least squares technique in combination with a transformation designed to remove first-order serial correlation effects. After developing the model and estimating the coefficients, simulation studies were run and compared with observed data over the period 1955-1968. The overall results of the simulation agreed rather well with historical data. Pindyck then transformed the system into a system of difference equations having single period lags and applied dynamic programming techniques, assuming a quadratic form for the cost functional. Because the problem is relatively simple, it can be attacked effectively with dynamic programming. This approach allows one to obtain not only the optimal path but also the suboptimal paths. By studying both the optimal solution and the suboptimal ones, the effect of the control variables may be analyzed. In the analysis of a real-world problem, this approach permits a decision-making body such as the U.S. Government to analyze the potential effect of changing its policy. For the nuclear industry, the suggestion is that if linear difference equations can be written which specify the behavior of the most important parameters, then the effect of policies may be evaluated. This approach is potentially of great use in analyzing the effect of government policy concerning imports and exports of U_3O_8 and separative work.

CHAPTER III

METHODOLOGY

Resource Requirements Computation

Computations of resource requirements for a stated system of reactors are carried out using the version of the NUFUEL code (Snyder, 1974) adapted for use on the CYBER 74 system at the Georgia Institute of Technology. For input fuel cycle loading and discharge patterns and reactor commitments specified by the user, fuel cycle requirements are computed. The NUFUEL routine carries out its computations in quarterly stages and hence is an excellent economic planning tool. It is designed to be used when the input data may be specified with precision. Figures 5 through 16 were produced using assumed Prebreeder and Breeder fuel cycles. The Light Water Reactor fuel cycle properties were taken to be those supplied in a sample data set with the NUFUEL code.

Fuel cycles for the Prebreeder and Breeder were developed from information given in the Draft Environmental Statement on the Light Water Breeder Reactor. Assuming an equilibrium fuel cycle requiring annual charges of 11157 kg U at 13.5 percent enrichment, and discharge at 7.6 percent enrichment, a fuel cycle having the same approach to equilibrium as the advanced Pressurized Water Reactor design was used. Detailed fuel cycle data are not available. Since the dominant contributors to fuel costs are the U_3O_8 and enrichment sectors and since there is uncertainty in prices for both these components, it is assumed that the uncertainty in the reactor loading is a small contributor to error in the analysis of LWBR system economics. The 13.5 percent enrichment, in particular, implies not only considerably more separative work but also more natural uranium feed to the enriching process than is the case for the Pressurized Water Reactor. The question is then whether the Prebreeder produces U-233 at a great enough rate to make the LWBR system a more favorable enterprise than the LWR. Given the substantial uncertainty in future U_3O_8 and enrichment prices, the uncertainty associated with specification of the Prebreeder fuel cycle is indistinguishable from the variation in prices, since these terms appear as a product wherever they occur in an economic analysis.

The fuel cycle cost assumed for the Breeder should be specified with some care. While the Breeder has a lower fuel cycle cost than either the Light Water Reactor or the Prebreeder, the cost of reprocessing is a significant contributor to the Breeder fuel cycle cost.

Market Penetration

Market penetration of Prebreeders is modeled by assuming that the rate of introduction of Prebreeders is proportional to the difference between the eventual asymptotic penetration fraction f and the existing Prebreeder level g(t). The growth equation for Prebreeders thus becomes

$$g'(t) = a(f - g(t)) \tag{23}$$

which has the solution

$$g(t) = f(1 - e^{-at}) \tag{24}$$

In this formulation, a is a rate constant for Prebreeder introduction.

The boundary condition is that of no Prebreeders at the initiation of the study

$$g(0) = 0 \tag{25}$$

Knowing the number of Prebreeders which may be expected to be acquired, it is necessary to determine the time rate of introduction of the Prebreeder. The model for this utilizes an input power scenario and creates a power generation history which may be utilized as input data to a resource requirements algorithm. Two algorithms are possible for the cumulative commitment of Prebreeders. The first assumes that the Breeders engendered by Prebreeder deployment replace Light Water Reactors which would be built were the Breeder not available. For a finite period T, this procedure maximizes the number of Breeders which may be deployed. The assumption is made that no Prebreeders are retired or converted during this period. A second approach to the commitment of power generating units is the immediate replacement of Prebreeders by a Breeder reactor when sufficient U-233 becomes available. As a result, there is an attenuation of the Prebreeder level. For time intervals where no Prebreeders are added, the Prebreeder level decreases in an exponential fashion. The rate of replacement is governed by the cumulative amount of U-233 available, the rate of U-233 production diminishing as Prebreeders are removed from the system.

Variational Approach to the Problem

Consider a system composed of three reactor types. Define the pertinent parameters as follows:

$N_j(t)$ is the number of reactors of type j, j=1,2,3 in place at time t

v_j is the cost coefficient per reactor of type j

$C_j(t)$ is the cost at time t due to reactors of type j

e_j is the enrichment requirement for a generating unit of type j

E is the total capacity of enriching plants.

Constraint: Assume that enrichment capacity is saturated. That is, that sufficient reactors are built so that no further enriching capacity is available.

$$\sum_{j=1}^{3} N_j(t)e_j = E \tag{26}$$

Type 3 is assigned to the Light Water Breeder; thus $e_3 = 0$ and

$$\sum_{j=1}^{3} N_j(t)e_j = \sum_{j=1}^{2} N_j(t)e_j = E \tag{27}$$

Objective Function: The discounted total cost C_{total}, written

$$\text{Minimize } C_{total} = \int_0^T e^{-\alpha t} \sum_{j=1}^{3} C_j(t)\, dt \tag{28}$$

where α is the discount rate.

Breeder-Prebreeder Relationship

Assume no retirement of Prebreeders. The number of Breeders which may be supported is a function of the number of Prebreeder-years which have accumulated. Assuming a constant γ kg U-233 per Prebreeder unit, the number of Breeders which may be supported is

$$N_3(t) = \frac{1}{R} \gamma \int_0^{t-1} N_2(t)\ dt \tag{29}$$

where R is the U-233 requirement, in kg, for one Breeder unit. The upper limit is t-1, assuming one year for processing and fabrication of U-233 fuel.

Now $N_2(t)$ may be assumed to have the form $N_2(t) = f[1 - e^{-at}]$. This function is easily integrated to yield

$$\int_0^{t-1} N_2(\tau)\ d\tau = \int_0^{t-1} f[1 - e^{-a\tau}]\ d\tau = f\ \tau\Big|_0^{t-1} - f \int_0^{t-1} e^{-a\tau} d\tau \tag{30}$$

$$\int_0^{t-1} N_2(\tau)\ d\tau = f(t-1) - f\left[-\frac{1}{a}\ e^{-a\tau}\right]_0^{t-1} = f(t-1) + \frac{f}{a}\ [e^{-a(t-1)} - 1]$$

$$= f\left[t-1 + \frac{1}{a}\ e^{-a(t-1)} - \frac{1}{a}\right]$$

Now from the Constraint Equation

$$N_1(t) = \frac{1}{e_1}\ [E - f(1 - e^{-at})e_2] \tag{31}$$

Substituting these relationships into the Objective Function,

$$C_{total} = \int_0^T e^{-\alpha t} \left\{ \frac{1}{e_1} \left[E - f(1 - e^{at}) e_2 \right] v_1 + f(1 - e^{-at})\, v_2 + \gamma f \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) v_3 \right\} dt \tag{32}$$

$$C_{total} = \int_0^T \frac{E}{e_1} v_1(t)\, e^{-\alpha t}\, dt + \int_0^T \left[-f \frac{e_2}{e_1} v_1(t) (1 - e^{-at}) + f(1 - e^{-at})\, v_2(t) + f\gamma v_3(t) \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) \right] e^{-\alpha t} dt \tag{33}$$

Now it is seen that if f is constant, the Objective Function may be written

$$C_{total} = \int_0^T \frac{E}{e_1} v_1(t) e^{-\alpha t}\, dt + f \int_0^T e^{-\alpha t} \left[- \frac{e_2}{e_1} v_1(t) (1 - e^{-at}) + (1 - e^{-at})\, v_2(t) + \gamma v_3(t) \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) \right] dt \tag{34}$$

$$C_{total} = I_1(T) + f\, I_2(T) \tag{35}$$

When regarded as a function of f, the Objective Function is minimized by choosing $f = 1$ if $I_2(T)$ is negative, or by selecting $f = 0$ if $I_2(T)$ is positive.

Now in general the commitment pattern for Prebreeders is not selected in advance. Therefore, it is desired that the time-dependent mix of reactors $[N_1(t), N_2(t), N_3(t)]$ be chosen to minimize the Objective Function independently of any particular algorithm. The relationship between the number of Breeders and the Prebreeder history is given, from

Equation (29), by

$$\dot{N}_3(t) = \frac{\gamma}{R}\,\dot{N}_2(t-1) \tag{36}$$

where the dot denotes differentiation with respect to time.

From the Constraint Equation,

$$N_1(t)e_1 + \frac{R}{\gamma}\,\dot{N}_3(t+1) = E \tag{37}$$

$$N_1(t) = \frac{1}{e_1}\left[E - \frac{R}{\gamma}\,\dot{N}_3(t+1)\right] \tag{38}$$

Substituting this in the cost functional

$$\begin{aligned} C_{total} = \int_0^T e^{-at}\Big[\frac{1}{e_1}\Big(E(t) - \frac{R}{\gamma}\,\dot{N}_3(t+1)\Big)v_1(t) + \frac{R}{\gamma}\,\dot{N}_3(t+1)v_2(t) \\ + N_3(t)v_3(t)\Big]\,dt \end{aligned} \tag{39}$$

Writing $y(t) = \frac{R}{\gamma}\,N_3(t)$ and expanding $\dot{y}(t+1)$ in a Taylor series about $\dot{y}(t)$, Equation (39) becomes, after truncating,

$$\begin{aligned} C_{total} = \int_0^T e^{-\alpha t}\Big\{\frac{1}{e_1}\,[E(t) - \dot{y}(t) - \ddot{y}(t)]\;v_1(t) \\ + v_2(t)[\dot{y}(t) + \ddot{y}(t)] + \frac{\gamma}{R}\,y(t)v_3(t)\Big\}\,dt \end{aligned} \tag{40}$$

Now the costs $v_1(t)$, $v_2(t)$ and $v_3(t)$ are dependent upon the commitment history, since U_3O_8 price may be modeled as a function of cumulative consumption.

An effective method for the attack of problems where constraints limit the range of admissible functions is that of dynamic programming. In the case of Prebreeder commitment, such an approach is tractable since the ability of the nuclear industry to support the U-233-Th fuel cycle is limited by lack of reprocessing facilities. Large capital requirements for construction of reprocessing plants would normally be assumed to act as a limiting influence upon the rate of growth of reprocessing facilities.

As a means of allowing for consideration of limitations upon production capability, the Dynamic Programming formulation of the minimization problem allows the input of arbitrary limits upon the system mix. These constraints reflect assumptions upon the capacity of the nuclear service sector to meet needs arising from various proportions of Light Water Reactors, Prebreeders, and Breeders. Alternatively, they could reflect policies adopted by either the Government or industry groups to support Light Water Breeder Reactor development. This method of attack yields an effective approach to the minimization problem defined by Equations (26) and (28). The equivalence between the calculus of variations and dynamic programming is established in Appendix B.

It is assumed that a specified power requirement for nuclear-generated electricity is followed. Any nuclear-electric commitment scheme may be chosen; for consistency, the results of the analysis reported upon in this work are based upon Case A from "Nuclear Power Growth 1974-2000" (WASH-1139(74)). The tunnel constraints limit the market penetration of Prebreeders, defined in this analysis as the ratio

$$MP(t) = \frac{P(t)}{P(t) + L(t)} \quad (41)$$

where P(t) and L(t) are the numbers of Prebreeders and Light Water Reactors, respectively, at time t.

Dynamic Formulation for Reactor Plant Selection

It is desired to determine the optimal path for reactor commitment based upon time-dependent values for capital cost, operating cost, and fuel cycle cost. The stage variable in this analysis is time; the state is the vector consisting of the reactors of various types.

$$\underline{x} = [X_1, X_2, X_3] \quad (42)$$

where X_n is the commitment of reactor type n.

The control $\underline{u}$ applied at time t is the vector of plant additions. For convenience, the elements of the $\underline{x}$ and $\underline{u}$ vectors shall be taken to be GW(e) of nuclear-electric additions. The analysis may be expanded to include plants of other types such as coal-fired and natural gas facilities.

We adopt constraints to help reduce the computing time of the problem at hand. In the first case, we are limited by uranium ore availability. Thus, the number of uranium-consuming reactors is limited. That is, the yearly consumption of uranium must be consistent with the capacity of the industry to provide it. A second constraint concerns availability of enriching services. The separative work required in any year must be less than or equal to available or projected available capacity. Fuel fabrication capability may also be employed as a constraint. However, it is generally taken to be a simplifying assumption that fabrication capability is not a limiting restriction. It is probably more precise to say that enriching services are a more restrictive constraint than is the availability of fuel fabrication facilities. Reprocessing availability will be a constraint for some reactor types but not for others.

The standard Light Water Reactors, the PWR and the BWR, are not entirely dependent upon a closed loop fuel cycle for their operation. On the other hand, the Light Water Breeder Reactor System is dependent upon reprocessing capability, and it should be observed that a rational decision-maker would not commit his company's resources to a breeder unless he were certain of reprocessing availability. The arguments may be extended to include various measures of social cost and adverse environmental effects as elements of cost computation or constraint formulation. The only requirement in such a case is that one have quantifiable relationships between costs and benefits, or more precisely, between the measurable impacts and the resulting component of cost. Such an undertaking is frequently impossible when dealing with matters relating to social costs, and even when such attempts are made, the results may not be accepted by the other scholars in the area. As a simplification for a dynamic programming approach, it is the case here that the social costs are taken to be equal for all reactor types.

We wish to minimize the total discounted cost to society due to the installation of nuclear-electric power. If we denote by $v(t)$ the total cost of power to society at time t, we may use Bellman's principle of optimality to establish the iterative equation of Dynamic Programming (Beckmann, 1968):

$$v(t) = \min_{0 \leqq \tau \leqq t} [k(\tau,t) + a(\tau)\Delta t + e^{-\alpha\Delta t} v(\tau + \Delta t)] \tag{43}$$

Reducing this to the discrete form

$$v(t) = \text{Min } [k(t) + a(t)\Delta t + \exp(-\alpha t)v(t + \Delta t)] \tag{44}$$

where

$k(t)$ is the cost of adding new units

$a(t)$ is the output rate of the current units.

The discount factor is the reciprocal of $(1 + i)$, where i is the interest rate. For quantized t and unit value of Δt, $e^{-\alpha}$ equals $(1 + i)^{-1}$. This equation says that, if one knows the ideal combination of units to generate power from year $t + 1$ to T, the terminal year in the analysis, then it is desired to determine the ideal combination of units which takes the system from the beginning to the final year. Now it is to be observed that the function $v(t + \Delta t)$ is a real-valued function of the vector $\underline{x}$, as we are ultimately attempting to determine the values of the state variable $\underline{x}$ and the control variable $\underline{k}$ as well as that of the minimum cost.

Constraints upon the problem may assume a number of forms, and in the simple analysis to be performed initially it shall be taken that the constraints be formulated in terms of state-vector quantities rather than control vector quantities. Denoting the enrichment constraint by e(t), a real quantity, and the average enrichment requirement for each reactor by the vector E, the first constraint equation may be written $E'X \leqq e(t)$ where the prime denotes transpose. Similarly, if the vector u denotes U_3O_8 production required to support each reactor type, the formulation $U'X \leq u(t)$ may be used to specify the uranium production constraint. The basic equation of the reactor deployment model is

$$\underline{x}(t+1) = \underline{x}(t) + \underline{k}(t) \tag{45}$$

which states that the capacity distribution at time t + 1 is that at time t plus any additions which might occur between t and t + 1. There is one further constraint, an equality constraint, which states that the total capacity available in period t must be equal to the demand schedule d(t). For n plant types

$$\sum_{i=1}^{n} x_i(t) = d(t) \tag{46}$$

Now the control vector $\underline{k}(t)$ may assume an unlimited number of representations, since the mix of reactor types may be continuously represented. It is necessary to make some type of additional assumption or approximation to reduce the number of states kept during the search for an optimum in the dynamic programming algorithm. The tunnel constraint approach used by Jenkins and Joy (1974) is both effective and realistic. In this method, constraints are arbitrarily imposed upon the mix of units to eliminate those combinations which are either clearly infeasible or, for policy reasons, would not be built. In the analysis of the Light Water Breeder Reactor system, the relevant measure is taken to be the Prebreeder fraction, defined to be the ratio of Prebreeder reactors to the sum of Prebreeder and Light Water Reactors. This approach allows the user to analyze the effect of policy decisions upon the cost of electricity. Most notably, what is the economic effect of creating a subsidy which results in the deployment of a fraction of reactors as Prebreeders? The forward dynamic programming approach is particularly adaptable in this regard, as it allows many terminal conditions to be analyzed without rerunning the optimization problem. In particular, when the commitment history can be traced for an optimal path (or for any other path), the life of each unit or segment of units may be computed and a retirement cost may be attributed to each terminal state to reflect the loss incurred by termination of a Light Water Reactor due to unavailability of fuel before the planned lifetime of the plant has been fulfilled. Similarly, the value of U-233 and plutonium not utilized in power generating plants may be ascribed to obtain a net worth figure for a system expansion schedule. The benefit is thus the sum of power generation and potential energy production capability, less the investment unrealized by utility stockholders due to availability constraints.

The optimization problem does not depend for its solution upon considerations which relate to a particular energy-generation system. Thus, coal-fired, natural gas, and LMFBR units may be included in an analysis where data are available. The non-general constraint which enables the problem to be formulated in unambiguous fashion is the expected demand schedule. It must be observed that the constraint is an equality constraint, thereby restricting the controls to those distributions which result in power additions of exactly d(t) GW(e).

Dynamic Programming Approach

A stage-increment technique has been developed to solve the optimization problem utilizing techniques of Dynamic Programming. The fundamental equation suggests that we may apply each control to all existing states in order to define the states at the succeeding stage. It is therefore necessary to impose constraints upon the solution in order to reduce the number of decisions to a manageable quantity.

Two approaches were combined to yield a tractable algorithm. The first is a tunnel constraint, discussed previously, and depends for its usefulness upon the judgment of the one specifying the constraint. The other is related to the quantization of the states themselves. Where a calculated state is identical to an existing state at some stage t, the principle of optimality requires that the state having minimum cost be retained. (The states are the same; what is actually manipulated is the history reflecting how the state was approached through stages.) A state is assumed to be a slowly-varying function of its parameters; it may be assumed that a nearby state is reflective of the same properties as a given state $\underline{x}$. It must be borne in mind that the state $\underline{x}$ is an ordered n-tuple which, for the case of the Light Water Breeder Reactor system, includes as one of its components the number of Prebreeder-years. Therefore, two states which are close to each other in particular have the same or nearly the same number of Prebreeder-years, and hence have generated approximately the same quantity of U-233. The metric utilized to establish closeness is the sum of the absolute values of the deviations. In general, one might use

$$d(x,y) = \sum_{i=1}^{n} |x_i - y_i| \tag{47}$$

as the distance function for the evaluation of closeness of two states. For the system consisting of the Light Water Breeder Reactor and the Light Water Reactor, the equation reduces to

$$d(x,y) = \sum_{i=1}^{3} |x_i - y_i| \tag{48}$$

Given $\epsilon > 0$ and a point y, one may say that x is in a neighborhood of y if $d(x,y) < \epsilon$. For the dynamic programming problem, all quantities are integers and therefore the neighborhood should be specified in terms of an integer. The substitution criterion therefore becomes

$$d(x,y) < K \tag{49}$$

where K is an integer. For initial dynamic programming studies, K has been taken to be equal to 2. This approximation may result in propagated error, since the algorithm is that if

$$d(x_j,x_n) < K, \qquad j = 1, \ldots, n - 1 \tag{50}$$

then the state with minimum cost is retained and the other one is discarded. This is a sequence dependent procedure. Observe for example that the sequence of triples (12,10,11), (12,10,12), (12,10,13), (12,10,14) will result in the storing of the single state (12,10,14), while the same four states in the order (12,10,11), (12,10,14), (12,10,13), (12,10,12) will result in the retention of states (12,10,14) and (12,10,12). There is therefore no readily obvious means of evaluating the effect of propagated inaccuracy. In a problem with evenly-distributed controls, however, it might be expected that the states vacated by the approximating scheme would be redefined by transitions of slightly different states under slightly different controls. Thus, it is anticipated for the LWBR-LWR comparison that the existence of quantized states and the minimum costs associated with each of them would be approximately the same as in the more detailed problem in which the entire complement of states is preserved. Terney and Fenech (1968) have utilized this method in their dynamic programming approach to control rod management.

The forward dynamic program written for analysis of the light water nuclear economy utilizes a hashing algorithm for the storage of states. All states at a particular stage are chained together by an address pointer which is encoded along with the state, control, and cost. The pointer is the address of the next state at this stage. A pointer value of zero signifies the end of the list of states at this stage. Hashing procedures have the attractive property of independence of previous operations. It is necessary only to maintain enough space in a table so that further entries may be made. For the LWR-LWBR study, a hash table consisting entirely of central memory storage is used; where more refined approaches are required, the use of mass storage such as disc may be employed without loss of generality. A particular property of the hash algorithm is that the coefficient of the Prebreeder-year term is unity. The hashing scheme is simple. The address calculated is given by

$$\text{Address} = \text{MOD}(\text{ITRANS},\text{HASH}) \tag{51}$$

where MOD is the remainder function, HASH is a hashing constant, near 9000 for a table of about 9000 entries, ITRANS is an integer quantity calculated from the sum

$$\text{ITRANS} = \sum_{i=1}^{3} a_i x_i \tag{52}$$

where the a_i are given coefficients and the x_i are elements of the state vector $\underline{x}$. In the particular algorithm used, the coefficient a_3 is unity. Therefore, states having the same numbers of Light Water Reactors and Prebreeders and slightly different numbers for the Prebreeder-years parameter are placed in successive positions in core. This ensures that a given state will be found at some point after entry in the table, without the interposition of an empty table element. Further, this approach makes tractable the procedure for reading elements whose only variations occur in the Prebreeder-years field, a particularly important aspect of exhaustive table search techniques.

Even with the imposition of constraints and simplifying approximations, the business of finding an optimal trajectory over a suitable planning period, say 20 years, is a formidable task. The problem may then be scaled to a manageable size in order to yield, in a reasonable amount of time, a solution which will bear some resemblance to the optimal path for the more elaborate form. For example, a stage may be taken to be two years instead of one, and power additions may be assumed to be in increments of two GW(e) rather than one. It is desired to investigate the effect of policy constraints upon the cost of power, or more appropriately, the cost of nuclear-electric power. It is not necessary to utilize absolute cost values; relative costs may be used to investigate the economic properties of the reactor system. In the studies performed, power generation costs for each plant type were referenced to the cost of power generation using a Light Water Reactor. The choice of cost to use in computing the economic benefit is itself a somewhat arbitrary decision. The Light Water Reactor cost reflected in the market price for PWR or BWR units is a Marginal Cost, the costs of research and development having been paid by taxpayers in previous years under the aegis of the Naval Reactors program. Similarly, the cost of the LWBR system is composed of additional research and development cost over and above that already encountered in LWR development, a figure held to be in the neighborhood of $200,000,000. This sum is to be taken from tax revenues rather than from the direct utility payments for reactor purchase. However, it should be noted that even where the additional research and development cost is added to the system cost, the impact is small unless only a few reactor plants are assumed. However, when comparing the LWBR system with other systems under development, the cost of development should be included in both to obtain a fair comparison of policy effects. Where large amounts of nuclear-electric addition are anticipated, as in Case D from the 1974 study by the Office of Planning and Analysis, the development cost which might reasonably be sustained without affecting the Average Cost is substantially greater than $200,000,000.

Computational Method

It is necessary to determine whether there will be a net benefit to society deriving from the existence of the Light Water Breeder Reactor system. It has previously been noted that the research and development costs for the LWBR system are anticipated to be small relative to those for a reactor type not in production. These costs have already been borne by society in the form of submarine reactor research. Thus, it is quite valid to compare the Light Water Breeder system with the Light Water Reactor system, since it may be assumed that production capacity and operational characteristics are similar in the two cases. It is desired that quantification be made of the present-worthed cost differential between Light Water Reactors and the Light Water Breeder system. To determine the effect of this cost differential, a Dynamic Programming approach is employed. The objective function is the present-worthed cost of those components which vary with reactor type. In the comparison of the Light Water Reactor and the Light Water Breeder/Prebreeder systems, this economic influence is primarily attributable to differences in fuel cycle cost. There is, however, a further consideration. In an optimization scheme, an objective function is minimized or maximized over a defined period. However, benefits accrue to society in the years following the defined span of the optimization period. Therefore, a terminal condition is applied to each admissible state at the end of the period to provide proper accounting for the penalties and rewards associated with each state at the final stage. Forward dynamic programming is chosen since it provides a simple and straightforward method for examining only those states for which the Prebreeder/Breeder combination is realizable from admissible states at previous stages. It may be observed that the familiar backward dynamic programming algorithm contains no provision for ensuring that a given state $\underline{x}(t)$ may be generated by applying admissible controls at stage $t - 1$. Therefore, backward dynamic programming could result in the evaluation of a large number of states which could not possibly be generated from a given initial condition. While forward generation of states may also result in a number of states which are of no consequence, there is at least the guarantee that each state so generated is derivable from some possible physical situation at the preceding stage.

The admissibility of the state depends upon constraint conditions imposed upon the particular problem. Since it is assumed that fuel for the Light Water Breeder Reactor is produced by the Prebreeder reactor, admissibility of a state is governed not only by the constraint condition but also by the system equation. The number of Breeder reactors is limited to those which may be fueled with existing quantities of U-233, retaining enough in the fabrication and reprocessing loop for one reload core. Assuming a whole core inventory of 4500 kg and a reload core requirement of 1000 kg U-233, 5500 kg of U-233 is required for the base design Breeder reactor. Assuming once-yearly refueling of the reference design Prebreeder, 310 kg of the U-233 isotope is removed each year from the Prebreeder reactor, cooled and reprocessed. Thus, one Prebreeder may be operated a period of $5500/310 = 17.7 \cong 18$ years in order to generate a quantity of uranium sufficient to sustain a Light Water Breeder Reactor. Assuming 18 Prebreeder-years per Breeder, and a delay of at least one year for cooling, reprocessing and fabrication, the number of Breeders which may be sustained is computed from the number of Prebreeders which have been utilized. The number of Prebreeder-years may be determined by summing the contribution from the deployment of Prebreeders at each stage. Prebreeder-years are computed by integrating the commitment function for Prebreeders over the prescribed interval of interest. Denoting this interval by $[0,T]$ and Prebreeder commitment by $P(t)$,

$$\text{Prebreeder-years} = \int_0^T \gamma P(t)\,dt \tag{53}$$

The amount of uranium produced by each increment of capacity is assumed for this analysis to be constant. The general expression for fissile isotope production from m distinct generating units may be written

$$\text{U-233 produced} = \sum_{i=1}^{m} \int_0^T R_i(t)\ \gamma_i(t)\,dt \tag{54}$$

where $\gamma_i(t)$ and $R_i(t)$ are production rate of U-233 and power level of unit i.

Dynamic Programming Objective

The dynamic programming approach has as its objective the minimization of the cost functional subject to availability constraints. The cost functional is given by

$$J = \int_{t_o}^{t_f} 1[\underline{x},\underline{u},t]dt \tag{55}$$

where $1[\underline{x},\underline{u},t]$ is the cost associated with the transition from state $\underline{x}$ at stage t to state $\underline{x} + \underline{u}$ at stage $t + \Delta t$. The problem is simplified by the choice of a constant value for Δt. Further, the objective function J may be written

$$J[\underline{x};\underline{u}] = \int_{t_o}^{t_q} 1[\underline{x},\underline{u},t]dt + \int_{t_q}^{t_f} 1[\underline{x},\underline{u},t]dt \tag{56}$$

where t_q is any quantized value for an intermediate stage, $t_o < t_q < t_f$. The problem to be attacked is the evaluation of the costs of unit scheduling strategies over the stage interval $[0,t_f]$. The effect of a strategy enacted over the period of t_f stages may be approximated by considering no change to the system during the stages t_q to t_f. This is the case in which the control vector $\underline{u}(t)$ is constrained to be equal to the zero vector over the interval $t_q < t < t_f$. Thus, the integral

$$\int_{t_q}^{t_f} 1[\underline{x},\underline{u},t]dt = \int_{t_q}^{t_f} 1[\underline{x},\underline{o},t]dt \tag{57}$$

and is therefore treated as a terminal state condition. Minimization is carried out over all admissible controls applied to the integral

$$J_1[\underline{x};\underline{u}] = \int_{t_o}^{t_q} l[\underline{x},\underline{u},t]dt \tag{58}$$

The intent of the problem being the identification of optimal mixes of energy generation types under various policies, the further simplifications are made that all unit additions are brought on line at exactly the same time in each year and that each increment consists of one GW(e). The first of these may be justified by the argument that there are no firm or even tentative commitment plans for the time period during which the Breeder/Prebreeder system might be introduced. Consequently, one would wish to pick a hypothetical scenario which represents as reasonably as possible the conditions under which neither reactor type (Light Water or LWBR) might enjoy an artificial cost advantage due to scheduling. Simultaneous commitment has been chosen to represent this condition. The one GW(e) per increment is a reasonable choice based upon the reactor offerings, most of which have maximum power output of approximately one GW(e). Further, LWR fuel cycle data have been developed by ERDA for this plant size. Cost data are derived from published figures (ERDA-1541, 1975) using an algorithm based upon the Seven-Page Formula. It is assumed that the capital cost of Breeder, Prebreeder, and LWR are all equal, although this is not required in the solution of the problem. Uranium prices are specified as piecewise linear functions of cumulative U_3O_8 consumption. Prices for other fuel cycle services may be specified as stage-dependent functions.

Cost Constraint Considerations

The costs are in particular time-dependent functions. Most notable is the cost of U_3O_8, a consideration which has a strong effect upon investment in the nuclear industry. Moreover, the cost of enrichment and the cost incurred in building an enrichment facility are uncertainties, and threshold considerations increase the uncertainty faced by private investors in the enrichment process. Insofar as enrichment itself is concerned, the only established method is gaseous diffusion. This method suffers from the large unavoidable requirement for pumping power, which is about three or four percent of the electrical power furnished by the reactor fueled by enriched uranium (Rotty et al., 1975). The gas centrifuge process requires about one-tenth this much electrical power, hence the Marginal Cost of enriched uranium production is potentially less for the gas centrifuge process than for gaseous diffusion. However, it must be observed that the gas centrifuge technology has not yet been refined to a production stage. The effect is to increase the expected cost of investment in both gaseous diffusion and centrifuge installations. The increase in expected cost from gaseous diffusion arises from the possibility that the centrifuge process will become viable and competitive before a new diffusion installation is amortized. The increase in expected cost for the centrifuge is a consequence of the possibility that the centrifuge will not become a technologically feasible process within the time frame assumed by the investors.

A further contribution to the uncertainty is given by the possibility of large scale photochemical separation. This technique has an impact upon all components of the nuclear fuel cycle. It is of importance first in the area of enrichment, for the reasons cited in the discussion of the impact of the gas centrifuge. Laser Isotope Separation (LIS), however, is a technologically and economically interesting development. Not only is the anticipated cost of enrichment lower in the LIS process than in the two processes previously cited, but also the yield is greater than in either the gaseous diffusion or the gas centrifuge processes (Snaveley, 1975).

The economic implications are far-reaching. Because the LIS process is more efficient than other technologies, the potential exists for the use of depleted uranium as feed material. Thus, there could be an "instant inventory" of uranium available for commercial use at a reasonable cost from a stockpile rather than from the mining industry. Therefore, the prospect of an LIS process becoming commercially feasible will tend to diminish the investment in the uranium mining industry. Similarly, the fact that the LIS process is considerably less expensive than the current modes of enrichment will tend to decrease the cost of enriched uranium and hence make the cost of power from nuclear plants decrease after the isotope separation technique becomes commercially realizable.

Constraints upon fuel cycle services will normally derive from availability of other items or services. For example, mining, conversion and reprocessing industries are dependent upon the availability of chemicals for their production. As an interesting effect, the chemical industry is energy-intensive, and therefore increases in energy cost will have a destabilizing effect upon the cost behavior of the energy supply system as a whole. The control variables, therefore, are limited to those involving primary resources such as uranium and thorium, and to the introduction of new technologies such as Laser Isotope Separation or the Gas Centrifuge process. Sensitivity studies at Brookhaven National Laboratory have shown (Beller et al., 1974) that the fraction of total energy consumption apportioned to electrical generation is an increasing function of the price of oil. The mechanism for accomplishing this is an increased dependence upon nuclear and coal as fuels for base load plants. Where oil, coal and nuclear prices increase together, it might be anticipated that the fractional increase in electrification would not be as great, if indeed it should increase at all. The rate of energy consumption is generally modeled by using population projections in conjunction with expected per capita consumption. This latter term is a subjectively derived variable, as assumptions must be made with respect to the efficiency of technological processes in the long run and also with respect to the anticipated living patterns in the society.

There is reason to believe that an increasingly urban society offers more opportunities for energy conservation than a dispersed mode of living. Space heating in apartment buildings may be more efficiently supplied in multi-story units than, for example, in individual ranch-style houses. Whether there will be a permanent migration to such units is largely a matter of conjecture. It remains, however, that the demand for electricity in the long run depends not only upon the price alone, and thus projections which do not take into account the patterns of distribution may be seriously in error. For this reason, some attention has been given in recent years to the econometric analysis of electrical energy demand in the United States in order to find the quantitative relationship to the several quantifiable variables. The earliest work is perhaps that of Fisher and Kaysen (1962) who ascertained that there was no perceptible elasticity associated with the demand for electrical energy. This view was held widely for some time, but in the late 1960's several studies were initiated which resulted in a significantly different conclusion. Among the reasons for the difference in the conclusion is the availability of data to the later researchers which were not published at the time of the Fisher-Kaysen analysis. Further, as Fisher and Kaysen themselves note, the effects operating during the time span of their analysis may have been sufficiently more important than price to yield indefinite results. The effect of importance during the period analyzed in the Fisher-Kaysen study is that of replacement, wherein persons who had no electrical appliances acquired them at the earliest opportunity. Appliances in this category would be washing machines and electric ranges, for example.

Later studies employing Bureau of the Census data for the years 1950, 1960 and 1970 would no doubt be operating upon data which would represent the residential sector as a consumer which has acquired most of the labor-saving devices which the needs of the society require. Thus, the increase in use of any particular energy source would derive from the substitution of a new item for an old one. This area of research is currently receiving much attention in the determination of the energy needs of the United States, not only at Brookhaven but also at Massachusetts Institute of Technology (Baughman, 1974) and at the RAND Corporation (Mooz and Mow, 1972).

A constraint of greater variability is the size of labor force for each aspect of the fuel cycle. This concern applies to fossil fuel as well as nuclear. As the annual uranium requirements increase, so will the demand for labor. Uranium occurs in both sedimentary and vein-like deposits. While strip mining techniques may be mechanized to yield higher production, there is little hope that the extraction of uranium from veins may be further mechanized. To extract this high-grade ore, as will be necessary to support any nuclear-electric economy, it will be essential to employ mining personnel in greater numbers. This challenge will not be easily overcome, for despite the prospect of high wages, the working conditions of desert life are sufficient to deter many who might otherwise join the mining enterprise. Hence, a thorough economic analysis of the uranium mining industry will include the specification of labor availability as a constraint upon total production capacity. Due to the proprietary nature of data, it is impossible to derive production functions for the uranium mining industry. Aggregate statistics are compiled by the U. S. Government (GJO-100); however, these are not sufficiently specific to be of use in defining the relationship of production to inputs of labor and capital. Similar considerations apply to the other components of the nuclear fuel cycle. The physical processes must be defined before economic properties may be attributed. For the components of the fission power industry, the known possibilities for technological variation are given in Table 15. Of the considerations shown, the most important with respect to the introduction and maintenance of a Light Water Breeder Reactor power generation technology are

Table 15. Potential Sources of Technological Variation

Fuel Cycle Component	Possible Modes
Mining/Milling	1. Surface Mining 2. Drilling and Tunneling 3. Thorium Mining and Exploration 4. Recovery of Low-Grade Deposits
Conversion	No change to current processes foreseen
Enrichment	1. Gaseous Diffusion 2. Gas Centrifuge 3. Laser Isotope Separation 4. Fast Breeder Economy 5. Light Water Breeder Economy
Fabrication	1. UO_2 Pellet Fabrication 2. Mixed Oxide Fabrication 3. Thorium Oxide Fabrication 4. UO_2 Fabrication (U-233) 5. UC Fabrication
Reactor Operation	1. Optimize Plant Efficiency 2. Optimize Fissile Isotope Production
Reprocessing	1. Uranium Recovery and Recycle Only 2. U-235 and Fissile Pu Recovery 3. U-233 Recovery
Transportation	1. Unlimited Transportation of Spent Fuel 2. Regional Reprocessing, Distributed Energy Centers 3. Regional Reprocessing, Concentrated Energy Centers

the enrichment alternatives, the U-233 fabrication and reprocessing capabilities, and the thorium mining capability. Absence of U-233 handling capability would inhibit the LWBR; high costs of fabrication and reprocessing would similarly act to deny its development. Increased economy of enrichment, by the Laser Isotope Separation method, for example, would have an indeterminate effect, since such a development would benefit both the LWR and the LWBR segments.

It is obvious from a consideration of the list in Table 15 that an extensive economic analysis could be established, provided that the data were available. However, the uncertainty in many of the fuel cycle component costs will tend to render useless the conclusions which might be reached as a result of such a study. Analysis of the economic dynamics of an industry, particularly in the preliminary stages, is limited in scope to one or a few pertinent economic variables. Price is usually chosen to be the controlling variable. For an economic analysis of any alternative energy source, the fuel price alone is insufficient to model the energy-generating sector. It is necessary to specify the constraints on the system, and these may arise out of considerations of physical, rather than economic, availability. While these constraints may generally be formulated in terms of costs, the estimates may be subjective and controversial. A case in point is the availability of uranium. The quantity of uranium within the United States is not well known. Geologists do not agree upon the amount available in a reasonably well-defined area (Ellis, 1975); there is even less certainty about that which is available in the regions which have not been extensively explored.

It is known, however, that low-grade (< 60 ppm) uranium deposits exist and that there is a significant amount of uranium contained therein. The deposits of note are the Chattanooga Shales and the Conway Granites. In some analyses these are considered to be available for use. However, the cost of mining and milling is not necessarily obtained by scaling up current costs of production by the fraction of uranium in high-grade ore divided by the fraction of uranium in low-grade ore. For the present deposits of high-grade ore are located in areas which are generally arid and unsuited to most of the activities which characterize U. S. society. Thus, the economic rent of the land displaced for mining is negligible. However, in the Chattanooga Shale area (eastern Tennessee and northern Alabama), existing profitable activity would quite likely be displaced by the process necessary for the recovery of uranium from the shales in that region. Further, there might well be a social cost associated with removal of existing activities in order to recover the uranium on a scale sufficient to support light water reactors. Therefore, the real cost of shale utilization may be great enough to deter its development completely. In such a case, the constraint of a finite uranium supply within the lifetime of a reactor should be applied.

If known reserves are taken as the governing availability criterion for the nuclear industry, the constraint upon light water reactor commitment is seen to be severe indeed. The available uranium at a forward cost of $30.00 per pound or less is 640,000 short tons (GJO-111, 1976). Using as a rule of thumb 200 tons U_3O_8 per GW(e) per year and assuming that each nuclear base load unit is employed for 40 years, it is seen that the available uranium from domestic proven reserves will support approximately 80 reactors. The low growth projection given by the Atomic Energy Commission in 1974 (WASH-1139(74)) indicates a need for much more nuclear capacity than currently-known deposits will support. It is therefore necessary to inquire into the possibility of finding more uranium within domestic borders. The yield from exploratory drilling during the period 1971-1975 has not followed the pattern set by earlier prospecting efforts. Hence the utility decision-maker is confronted with the possibility that a nuclear plant committed any time after 1976 will be faced with the nonavailability of fuel before that plant has generated enough revenue for the utility to have recovered its investment. For the same reason, the prospective investor in an enrichment plant or a uranium facility should be hesitant to commit private capital until further sources of uranium are declared available.

Even if a stable nuclear-electric economy develops, the investor's problems do not vanish. For there is now, and for several years hence will be, a substantial backlog of spent fuel produced which must be reprocessed. The extracted uranium and plutonium will be recycled to fabrication plants for the production of fuel elements. In order to dispose of the backlog, some overbuilding of production facilities must take place. As Wolfe and Lambert (1975) observe, this overcommitment of resources will be sufficiently great so that by 1996 there will be insufficient flow of material to make use of the anticipated production capability. Knowing that this result may well be the case, the rational investor may avoid the commitment of capital to a project with the characteristics of fuel fabrication, since he cannot effectively stimulate the market for that service. An economic analysis of the nuclear fuel cycle should focus upon questions of availability of nuclear fuels and upon incentives for investment in supporting services such as enrichment. To fail to take into account these constraints and the corresponding mechanisms available for dealing with them is to generate information for a scenario which inadequately approximates reality. It must be observed that current energy resource data are not free from error. For example, the designation of uranium resources by the Energy Research and Development Administration into categories of "known" and "speculative" reserves has changed somewhat the demarcation which existed as a consequence of the earlier nomenclature of "reasonably assured" and "potential" resources. Such figures are the most accurate available and must be used in analytical work, however.

Fixed Cost Considerations

There are two measures used for evaluating the impact of capital costs. One is the cost per unit output, a number which is utilized by decision makers having the profitability of their firm in mind. Cost-benefit analyses, on the other hand, utilize the concept of installed capacity when forming the problem specifications. One formulation yields the evaluation of strategies which are designed to attain a specified objective; the other gives the cost associated with paths whose objective is the maximization of investment efficiency. In evaluating the Light Water Breeder System, the cost of producing a unit of output shall be used as the pertinent measure. This is the quantity which a utility executive would use in deciding between competing energy systems. The total cost of electricity generation is therefore the cost of fuel plus the capital cost plus the operating and maintenance cost. For the Light Water Breeder System, it is to be anticipated that the cost of fuel will be somewhat higher in the Prebreeders than in the Light Water Reactor designs currently in use because of the enrichment requirement (13.5 percent in a Prebreeder operating on a once-yearly refueling schedule, 3.2 percent in a PWR operating on the same schedule). Using the Seven-Page Formula of Bader, Kitzke and Nordman (1969), it is found that the fuel cycle cost of the Prebreeder is about double that of the LWR. Power generation costs will thus reflect the difference in fuel cycle costs, since the designs of the Prebreeder and the LWR are very nearly the same. It may be anticipated that the Prebreeder could be built and installed for very nearly the same amount as the conventional LWR. The initial cost of fuel, however, would be weighted more heavily in present-worth computations than the lower-cost power from the Breeder reactor. The fundamental question to be answered, then, is whether the savings brought about by installation and operation of Breeder reactors is sufficient to justify the investment in a Prebreeder by an enterprise which must maintain a favorable cash flow position. Of secondary importance is the consideration of whether the Government should subsidize the development of the Light Water Breeder. In this latter case the description of benefits may be made in many ways, thereby admitting a variety of numerical results without loss of accuracy in technique.

Economic Analysis of Reactor Types

The costs of power plant use for any type of generating unit may be broken up into three components: capital cost, operating cost, and fuel cycle cost. In general, fossil units have lower capital costs owing to shorter construction time and fewer licensing restrictions. Nuclear fuel cycle costs, evaluated over the expected period of plant operation, are usually sufficient to make nuclear installations more economical than other base load types. Normally, an individual utility will attempt to minimize its expected total cost, but at least one author (MacAvoy, 1968) has indicated that this pattern may be affected by the desire of a utility to choose a newer technology over an older one. A single decision making unit such as a utility will act as an indicator of the behavior of all other decision making units. In particular, it will base its analysis in part upon the history of prices and availabilities. Many of the input data for such decisions will arise from aggregated industry statistics, hence there will be a common input to all decision makers. Therefore, there will be pressures for all segments of the industry to move as a single unit. In particular, when nuclear power for Light Water Reactors appears to be an attractive option, each individual utility will commit funds to nuclear rather than (or in addition to) fossil fuel in order to satisfy its expansion requirements. Therefore, the prices of uranium and associated fuel cycle services will be expected to rise as a result of increased industry-wide requirements. This effect would not be forecast by any of the individual decision makers, since contemporaneous events would act to exert upward pressure upon fuel prices. Certain functions within the nuclear fuel cycle may be subject to wide variation in price and availability, while others whose demand is relatively constant will exhibit little price change. Yellowcake and separative work prices have shown marked variation, while prices for conversion have been slow to change. In an economic analysis of reactors, it is necessary to examine the fundamental process which may induce a price change in any of the fuel cycle components. Therefore, the analysis is transformed from one in which price is a measure of the state of the industry, and thus is an intermediate variable.

Economic analysis techniques vary depending upon the nature of the problem to be attacked. In many cases a single commodity is to be considered, and an analysis may be carried out using only the demand and supply functional relationships to develop the stability conditions for the system. In the nuclear fuel cycle, each component must be specified, since constraints may be applied to one segment which shows little variation with known factors. Conversion is a good example. The conversion capacity in the United States is anticipated to be stable at the level of 24,000 short tons uranium per year. The cost of conversion is dependent primarily upon the cost of materials. Thus, the flow of natural uranium hexafluoride to enrichment plants from domestic production is relatively unimpeded.

Prebreeder Resource Requirements

Both the Prebreeder and the Light Water Reactor are assumed to operate on a one-year cycle. Prebreeder average annual mass flows are given in the Draft Environmental Statement for the Light Water Breeder Reactor. For the reference design utilizing moderately enriched uranium, 12,600 kg UO_2 is required at an enrichment of 13.5 weight percent U-235.

Fuel cycle parameters for the three reactor types under consideration are given in Table 16, based upon an equilibrium cycle of one year at a capacity factor of 0.7.

Impact of the Breeder upon Plant Commitment

The impact of the Light Water Breeder may be treated in two ways where the nuclear-electric generating capacity is expanding. In one algorithm, each Breeder may be assumed to replace a Prebreeder, as was done in the Draft Environmental Statement. This has the effect of removing from the analysis that equipment capable of generating U-233. A second, perhaps more flexible, approach is to assume that as sufficient U-233 becomes available, the Breeder thus enabled will replace a Light Water Reactor in the power generation scheme and hence allow existing Prebreeders to continue production of U-233. It may be noted that the economic analysis contained in the Draft Environmental Statement did not examine a growth condition; the system was static and the Light Water Breeder necessarily replaced one of the parent Prebreeders. If the objective is maximization of the total amount of U-233 produced, the strategy should be that in which the maximum number of Prebreeders is maintained. Under this condition, then, the resulting LWBR should be utilized in lieu of a Light Water Reactor. There are counteracting economic effects. The Prebreeder is inherently more expensive than is the LWR, while at the same time it generates U-233 which may be used to reduce the fuel cycle costs in the future. Unless a very high value be assigned to U-233, there is little if any economic advantage to the strategy in which Breeders replace LWR's where short-term factors are utilized in the

Table 16. Fuel Cycle Parameters for Prebreeder, LWR, and Breeder--One Year Equilibrium Cycle

Material	Prebreeder	LWR	Breeder
Enriched UO_2 (kg)	12,600	30,400	--
Fresh Fuel Assay (wt %)	13.5	3.1	--
Spent Fuel Assay (wt %)	7.6	0.9	--
Thorium Ore (MT)	5,900	--	80.7
U_3O_8 (MT)	158	135	0
Uranium Charged (kg)	11,157.1	25,475	--
Uranium Discharged (kg)	10,923	23,805	--
Separative Work (MTSWU)	165	114	0
Fissile U Charged (kg)	1506	790	1,070
Fissile U Discharged (kg)	1140	214	1,080
Thorium Charged (MT ThO_2)	29.6	--	71
Thorium Discharged (MT ThO_2)	28.6	--	70
Enrichment Plant Tails Assay (wt %)	0.2	0.2	--
Capacity Factor	0.7	0.7	0.7

NOTE: For the first two cycles, a capacity factor of 0.65 is used. Fuel cycle characteristics are modified slightly to conform to those assumed in the writing of NUFUEL. The two initial cycles are assumed to operate at a capacity factor lower than that of the equilibrium cycle. Sixty-five percent is used for the first two cores. Plutonium returns are adjusted accordingly to conform to the lower burnups associated with exposure times of one and two years. In zone 1, the quantity of fissile Pu returned is 34 kg. For zone 2, the exposure time is two years and the fissile Pu return is 68.09 kg at a capacity factor of 65 percent.

evaluation. Limitations upon the availability of uranium ore, however, would appear to dictate that the Breeder, when committed, should be used to reduce the LWR requirements. In a growth environment there will be commitments due to the requirements for new Prebreeder units as well as the possible retirement of Prebreeders due to their replacement by Breeders. There is also growth of the Prebreeder penetration due to the expansion of the industry necessary to support the thorium-using reactors. Therefore, the commitment of Prebreeders will vary depending upon the Breeder commitment algorithm chosen and upon the anticipated market share of the Prebreeder. For the scenario in which the Prebreeder is not retired when a Breeder comes on line, the Prebreeder commitment is a function only of the ability of the fuel cycle industry to support the reactors, and of the relative costs of Prebreeder and LWR.

CHAPTER IV

COST DATA AND THEIR VARIABILITY

Enrichment Capability

Although adequate for the near term, enrichment is foreseen to be a significant constraint in the future. Owing to the high capital outlays required for construction of a separative work facility and the lack of ability to control the market, private investment in isotope separation plants has been nonexistent. The possibility of competition with government-owned facilities acts as a significant deterrent to private initiative. Efforts by the Federal government to assure the success of private enterprise in this field have met with little success. Representative Hosmer (now retired) sponsored a bill to create a United States Enrichment Corporation which would operate existing gaseous diffusion facilities. More recently, legislation was introduced which would give guarantees to private firms undertaking to provide enrichment services to the commercial market. This legislation has been delayed, however, because analysis performed for the Comptroller General has shown that the least expensive means of obtaining the next increment of generating capacity is the addition of facilities to the existing gaseous diffusion plants (RED-76,36, 1975). Enrichment uncertainty is evidenced by the operation of the Oak Ridge Gaseous Diffusion Plant at a tails assay of 0.25 percent, thus increasing the number of units of separative work required to produce the contracted product (*Nuclear Industry*, July 1975).

This change resulted from a lack of expected deliveries, and reveals that even though contractual commitments may exist, the schedules for physical delivery may be altered by delays in reactor plant startup. But the schedule changes are the simple problems, at least from the point of view of a private investor. The overriding considerations which could spell success or doom of a privately financed enrichment enterprise are the availability of uranium for use in light water reactors, the deployment of light water reactors, and the possibility of competition.

For the private investor in the enrichment process, the primary source of competition is the U. S. Government. First, there is the existing complex of separative work facilities at Portsmouth, Ohio, Paducah, Kentucky, and Oak Ridge, Tennessee. Next, there is the government-sponsored research and development effort in gas centrifuge technology. Finally, there is the Laser Isotope Separation method under study at Lawrence Livermore Laboratories which has the potential of reducing the cost of isotope separation drastically (Snavely, 1975).

In the Laser Isotope Separation method, a tunable laser of great resolution is employed to raise one isotope of uranium to an electronically excited state. A second laser ionizes this isotope, and the charged particle is then easily collected. While the development of a laser suitable for commercial use is still in the future, it is apparent that, once the laser of suitable power, linewidth and stability characteristics is found, its use for isotope separation will immediately render other processes obsolete. The private investor in the enrichment process would need to have a guarantee of accessibility to laser technology once such a development becomes a reality.

Projections of the cost of separative work are given to be \$75/kg SWU (Browne, 1975). Other estimates range as high as \$200/kg SWU, a pattern which does not seem inconceivable in light of experience with reprocessing developments. Since both the Light Water Reactor and the Prebreeder phase of the Light Water Breeder Reactor System require separative work, it is not unreasonable to assume that price variations in separative work will affect both systems more or less the same. The penetration of reactor installations by Breeders replacing Prebreeders, however, does suggest that, in a system heavy with Breeders, the long-range availability of uranium enrichment will be less of a consideration than in a system containing only Light Water Reactors.

Reprocessing

The availability of reprocessing is required for the implementation of the Light Water Breeder Reactor program. It is necessary to be able to obtain the U-233 from the thorium rods as well as the U-235 from the moderately-enriched uranium rods for recycle. Uranium-233 presents a radiation problem because of the presence of a 2.6 MeV gamma ray emitted from Tl-208. This requires shielding in the reprocessing, fuel fabrication, and fuel loading processes involving U-233 fuel. Thus, the availability of reprocessing capability determines whether, in fact, a Light Water Breeder program can be established, and in the absence of the High Temperature Gas Cooled Reactor, the LWBR system furnishes support for the U-233-Th reprocessing investment.

Subject to some variation, the expected costs of reprocessing depend in part upon the price of chemicals and in part upon the amount of shielding required for personnel protection. Estimates of reprocessing cost are easily obtained, but a systematic development of the production function for reprocessing is lacking. In part, the reason for the uncertainty in reprocessing stems from lack of regulatory action. However, a more thorough look at the reprocessing industry reveals an enterprise whose market may not be guaranteed.

The principal outputs of a reprocessing facility are uranium and plutonium. The uranium may be returned to an enrichment plant, or correspondingly may be mixed with virgin fuel in a fabrication facility. Plutonium may be used as a fuel in either a converter or a breeder plant. When the sum of the uranium and plutonium values is greater than the cost of reprocessing, then reprocessing is an economically viable approach to the effective utilization of resources. In an economy consisting only of light water reactors, the profitability of reprocessing depends upon the price of yellowcake and separative work. Where these items are or are expected to be cheap, it may be the case that the throw-away fuel cycle, in which there is no reprocessing of spent fuel for commercial use, will be more economical than its closed-loop counterpart.

In order to provide effective protection from ionizing radiation, reprocessing plants are highly automated. Therefore, the size of the available labor force is not expected to act as a constraint upon the establishment or growth of the reprocessing industry. Furthermore, the magnitude of investment in reprocessing, while large on a per unit basis, is still small relative to other important components of the nuclear fuel cycle. Dickeman (1974) estimated that for the 25-year period, 1975-1999, the nuclear industry would encounter requirements of \$15-\$20 billion for the construction of uranium mills and the development of reserves; about \$20 billion for enrichment plants, and \$5-\$6 billion for the establishment of spent fuel reprocessing facilities. The implications of these numbers are fairly important in a long-range analysis. If one observes that the commercial nuclear power industry is yet in its infancy, it may be assumed that the figures given by Dickeman are representative of the proportional costs. To make the fuel cycle more economical, therefore, one would attempt to find ways to reduce the cost of enrichment and mining before turning attention to reprocessing. The breeder fuel cycles accomplish this. A tentative conclusion is that a breeder of some sort is inevitable and that the reprocessing function (the availability of reprocessing) may be a requisite for a lower fuel cycle cost over the long term.

Fabrication

The characteristics of the fuel fabrication industry are similar to those of the reprocessing sector. Both are affected by governmental regulations which limit use of mixed oxide fuel. The market for fuel fabrication services, however, is well-defined by installed nuclear power, whereas the market for reprocessing is subject to the purchaser's economic interpretation of the price history. A utility has the option of operating on a throwaway fuel cycle and will do so if it appears to be more profitable. The operator of a reactor has no choice concerning the type of fuel element which will be used. Oxide pellets are the single fuel type in use for commercial power generation. The fuel fabrication industry has a guaranteed minimum market, and investment may proceed in an orderly and systematic fashion as reactors are deployed.

Reactor Operation

Extensive analyses have been performed to discern the best reload batch, the best fuel shuffling pattern, and the optimal mix of reactor types. It is assumed that each reactor has associated with it a pattern of fuel utilization which, if not optimal, has at least been carefully developed using the best information and techniques available. The operator of the reactor has the long-term option of choosing his fuels so as to minimize his discounted cost. Therefore, some consideration should be paid to the components of the fuel cycle under examination. In this regard, the availability of the Light Water Breeder Reactor, with backfit capability, can play an important role in the utilization of uranium and thorium resources.

The optimal fuel loading pattern within a reactor has been subject to thorough study since the advent of optimization techniques applicable to large computers. These problems are characterized by a well-defined objective function and by physically developed constraints. The corresponding problem for a system is not as specific. Formulation of the objective function is not as clear as in the case where the reactor operator (a utility) has a constraint imposed by stockholders or regulatory bodies which guides the enterprise. For a fission system, several objective functions may justifiably be used in an economic analysis. The first is that of extending resource utilization over the maximum period. This objective function may well be the subject of pected costs, substantial study efforts, as the perception of domestic energy resource limitations is refined. A second criterion, employed in the economic analysis of the Liquid Metal Fast Breeder Reactor, is the minimization of the total present-worthed cost to society. This formulation requires some care, since there is a cost incurred owing to the early termination of use of energy generating facilities. A third function might involve the removal of the discount factor in computing the cost to society, thus using the total cost to society as the criterion.

Optimization techniques have gained widespread use in the analysis of both physical and economic problems. In the nuclear industry, optimization of fuel cycle performance has been extensively studied using computational models involving dynamic programming techniques. Systems-oriented work has been carried out at the Massachusetts Institute of Technology by a group under the direction of Professor E. A. Mason. Their results have been published in part as a portion of the proceedings of the Nuclear Utilities Planning Methods Symposium (ORNL-TM-4443, 1974). The focus of this conference was upon the methods potentially available to nuclear utilities for scheduling of unit commitment, reloading, and capital commitment. The modeling of larger systems requires assumptions significantly different from those used to describe a single utility's behavior. For example, it is necessary to describe the response of the consuming sector to price, while the utility seeking to optimize its operation is regarded as a price-taker. Models of nuclear power or, in fact, of total energy systems have been developed for use in planning activities. Among the best known is the Brookhaven Reference Energy System, which uses an input-output structure to define resource requirements. The Bechtel Corporation has employed a similar model in its resource evaluation efforts (Carasso et al., 1974). Both models are extensive in their scope and reflect a thorough effort in the acquisition of input data. However, input-output models are deterministic in that they must assume availability levels for each of the constituent production sectors. One of the interesting characteristics of the nuclear fuel cycle is the role that expectations play in the determination of supply capability. Thus, projections made using an input-output model require the accurate specification of constraints in order for the answers to be meaningful. Where the side conditions derive from physical properties such as known resource levels, the input-output approach may be of use.

The Brookhaven Reference Energy System has been used (Beller, 1974) to ascertain the effects of technological change and petroleum price increases. The state of the system is observed at two specific times--1985 and 2000. For this particular study, the Reference Energy System was used in conjunction with a Linear Programming algorithm to ascertain the behavior of the substitution mechanism in face of rising price behavior of a primary energy source. Among the input data required for execution of the model is fuel cost. The choice of fuel costs in 1974 is instructive, since it should be an accurate indication of the prices prevailing at that time. The assumed resource costs are presented in Table 17. It is interesting to observe that the cost of strip-mined coal, expressed in 1970 dollars, was taken to be $6.79 per ton, and the fuel costs for the Light Water Reactor and the HTGR were both taken to be equal to 22.0 cents per million Btu. These prices may appear to be low relative to those which have prevailed since publication of the study.

Table 17. Fuel Costs Used in 1974 Interfuel Substitution Study (Beller et al., 1974) (All Costs Expressed in 1970 Dollars)

Resource	Year	
	1985	2000
Underground Coal ($/ton)	9.60	10.00
Strip-mined Coal ($/ton)	6.79	8.00
Crude Oil ($/barrel)	6.17	13.50
Natural Gas (¢/1000 ft^3)	44.3	90.0
Nuclear Fuel Cycle Cost (¢/MMBtu)		
LWR and HTGR	22.0	30.0
LMFBR	--	16.00

Financial Assumptions

Fuel cycle service price selections are somewhat arbitrary. For initial periods, prices prevailing, observed or expected during the 1974-1976 period were used, with escalation or reduction in years 11 and after specified either implicitly or explicitly. Prices utilized for computations during the initial 10-year period for each reactor are shown in Table 18.

The initial period represents that for which a utility has made firm commitments. Constant prices for uranium and separative work may be regarded as averaged costs over the period of firm commitment. The fundamental purpose for the fuel cycle cost calculation is the establishment of relative fuel cycle costs for each reactor type. In this regard, it is seen that the fuel cycle costs for the Light Water Breeder are about four-fifths those of the LWR, or about one mill/kWh less expensive.

Table 18. Coordinated Financial Assumptions for Plant Types

Reactor Type Cost Element	LWR	Prebreeder	Light Water Breeder
Cost of Capital (%)	18	18	18
U_3O_8 ($/lb)	15	15	--
Conversion ($/kg)	3	3	--
Enrichment ($/SWU)	59	59	--
Uranium Fuel Fabrication ($/kg)	90	90	175
Uranium Reprocessing			
Transportation ($/kg)	30	30	60
Recovery ($/kg)	90	100	180
Conversion ($/kg)	30	20	60
ThO_2 ($/lb)	--	10	10
Thorium Reprocessing			
Transportation ($/kg)	--	60	--
Recovery ($/kg)	--	180	--
Conversion ($/kg)	--	60	--
Thorium Fuel Fabrication ($/kg)	--	135	--
Plant Efficiency (%)	32	32	32
Bred Fuel Credit ($/gram-fissile)	20	20	20
Fuel Cycle Cost (mills/kWh)	5.119	8.099	4.255

CHAPTER V

COMPUTATIONAL RESULTS

Primary Results

The purpose of the investigation being the identification of those elements which impinge upon fuel cycle performance, several variations were run with the optimization algorithm. The optimal mix and the associated optimal path which led to this mix of energy generating units were identified. Primary variables which affect the number of energy generating units are U_3O_8 price and reprocessing cost. Surprisingly, the cost of enrichment does not appear to affect the choice of an optimum, though the costs of the system are affected. The initial costs of the Prebreeder outweigh the benefits derived from utilization of the Light Water Breeder when sufficient U-233 becomes available. Imposing growth constraints upon the system generally results in higher unit costs, again due to components incurred during the initial phases of deployment.

Factors affecting dynamic plant commitment are given in Table 19, which identifies state files for the Dynamic Programming analysis. These files consist of states, controls, costs, and links corresponding to admissible choice combinations of reactor types during the first ten years of plant selection. Terminal constraint conditions representing the integral in Equation (57) are enumerated in Table 20. A summary of scenarios evaluated using the assumed discount rate, growth, and pricing patterns is given in Table 21. A parameter of interest in the analysis is

Table 19. File Definitions

File Name	Description	Tunnel Constraints		
		Year (t)	Low Bound (F_t^{LOW})	High Bound (F_t^{HIGH})
ST0610	Ten-year file containing all admissible states and costs at a discount rate of 6%	1	0.1	0.30
		2	0.1	0.40
		3	0.15	0.50
		4	0.20	0.50
		5	0.20	0.50
		6	0.20	0.50
		7	0.175	0.50
		8	0.15	0.90
		9	0.1	0.90
		10	0.1	0.90
ST0810	Ten-year history of states and costs at a discount rate of 8%	Same as those for ST0610		
ST1010	Ten-year history of states and costs at a discount rate of 10%	Same as those for ST0610		
ST1210	Ten-year history of states and costs at a discount rate of 12%	Same as those for ST0610		
LT0610	Low limits on market penetration; modified Case A data. 6% discount rate	1	0.05	0.3
		2-10	0.1	0.3
LG0610	Ten-year study, 6% discount rate. Logarithmic growth imposed upon market penetration for Prebreeders	1	0.0	0.15
		2	0.05	0.20
		3	0.125	0.275
		4	0.175	0.325
		5	0.225	0.350
		6	0.250	0.375
		7	0.275	0.400
		8	0.300	0.425
		9	0.325	0.475
		10	0.350	0.500

Table 19. Continued

File Name	Description	Tunnel Constraints Year (t)	Low Bound (F_t^{LOW})	High Bound (F_t^{HIGH})
ZEROST	Ten-year study 6% discount rate, lower limit for Prebreeder commitment set at zero	All lower bounds for market penetration set to 0.0; high bounds the same as for case which generated ST0610		

Tunnel constraints apply to the market penetration, MP_t, in year t. Market penetration is defined, for this study, to be the fraction of burner reactors

$$MP_t = \frac{\text{Number of Prebreeders in place through period t}}{\text{Number of Prebreeders + Number of Light Water Reactors in place through period t}}$$

The lower and upper bounds given are applied to MP_t for each possible mix of reactor types. Only those for which the relation

$$F_t^{LOW} \leqq MP_t \leqq F_t^{HIGH}$$

holds are retained for further computation.

Table 20. Parametric Variations for Fuel Cycle Prices

Function	Notation	Price Formulation
U_3O_8 Price	LOW	Price a Function of Cumulative U_3O_8 Consumption x as follows: U_3O_8 price per lb U_3O_8 $\begin{cases} 15.00 + 0.1372\ x \times 10^{-4} & 0 \leqq x < 1{,}130{,}000 \\ 30.00 + 0.3676\ (x-1{,}130{,}000) \times 10^{-4} & 1{,}130{,}000 \leqq x < 2{,}490{,}000 \\ 80.00 + 0.1 \times 10^{-4}\ (x-2{,}490{,}000) & x \geqq 2{,}490{,}000 \end{cases}$
	HIGH	Price a Function of Cumulative U_3O_8 Consumption x as follows: U_3O_8 price $\begin{cases} 15.00 + 0.3488 \times 10^{-4}\ x & 0 \leqq x < 430{,}000 \\ 30.00 + 0.1724 \times 10^{-3}\ (x-430{,}000) & 430{,}000 \leqq x < 720{,}000 \\ 80.00 + 0.1 \times 10^{-4}\ (x-720{,}000) & x \geqq 720{,}000 \end{cases}$
Enrichment Price (E)	ENR100	\$59.00/SWU base price, escalated each year by the factor 1.00; that is, constant \$59.00/SWU price over the lifetime of the study
	ENR101	\$59.00/SWU base price, escalated annually by the factor 1.01 (1% escalation)
	ENR102	\$59.00/SWU base price, escalated annually by the factor 1.02 (2% annual escalation)
	ENR103	\$59.00/SWU base price, escalated annually by the factor 1.03 (3% annual escalation)
	ENR104	\$59.00/SWU base price, escalated annually by the factor 1.04 (4% annual escalation)

Table 20. Continued

<table>
<tr><th>Function</th><th>Notation</th><th>Price Formulation</th></tr>
<tr><td>Enrichment Price (E) (cont)</td><td>HENR100</td><td>$75.00/SWU base price, escalated annually by the factor 1.00 (constant $75.00 price over the lifetime of the study)</td></tr>
<tr><td></td><td>HENR102</td><td>$75.00/SWU base price, escalated annually by the factor 1.02 (2% annual escalation)</td></tr>
<tr><td></td><td>HENR104</td><td>$75.00/SWU base price, escalated annually by the factor 1.04 (4% annual escalation)</td></tr>
<tr><td>Reprocessing Price</td><td>V1</td><td>Reprocessing price following trend established by past estimates:
<table>
<tr><th>Years</th><th>Price ($/kg)</th></tr>
<tr><td>1-5</td><td>120.00</td></tr>
<tr><td>6-7</td><td>130.00</td></tr>
<tr><td>8-9</td><td>140.00</td></tr>
<tr><td>10-13</td><td>150.00</td></tr>
<tr><td>14-16</td><td>140.00</td></tr>
<tr><td>17-19</td><td>130.00</td></tr>
<tr><td>20-22</td><td>120.00</td></tr>
<tr><td>23-25</td><td>110.00</td></tr>
<tr><td>26-27</td><td>100.00</td></tr>
<tr><td>28-30</td><td>90.00</td></tr>
</table></td></tr>
<tr><td>(REP)</td><td>V2</td><td>Reprocessing price increases monotonically:
<table>
<tr><th>Years</th><th>Price ($/kg)</th></tr>
<tr><td>1-5</td><td>120.00</td></tr>
<tr><td>6-7</td><td>130.00</td></tr>
<tr><td>8-9</td><td>140.00</td></tr>
<tr><td>10-13</td><td>150.00</td></tr>
<tr><td>14-15</td><td>160.00</td></tr>
<tr><td>16-17</td><td>170.00</td></tr>
<tr><td>18-20</td><td>180.00</td></tr>
<tr><td>21-24</td><td>190.00</td></tr>
<tr><td>25-30</td><td>200.00</td></tr>
</table></td></tr>
</table>

Table 20. Continued

Function	Notation	Price Formulation
Reprocessing Price (REP) (cont)	V3	Economics of scale result in lowering of reprocessing price:

Years	Price ($/kg)
1-5	120.00
6-8	130.00
9-11	120.00
12-15	110.00
16-20	100.00
21-25	90.00
26-30	80.00

Table 21. Sensitivity Studies for LWR-LWBR-Breeder System

Optimal State	Discount Rate (%)	States Array	U_3O_8 Price Scenario	Enrichment Price Scenario	Reprocessing Price Scenario	Cost Per Reactor (Million $)	Terminal Function Cost (Billion $)
182-48-5	6	LT0610	High	ENR102	V2	435.90	40.882
200-27-8	6	ST0610	High	ENR102	V3	434.61	40.200
186-37-12	6	ST0610	High	HENR104	V3	471.42	47.960
196-30-9	6	ST0610	High	HENR102	V2	459.99	45.980
200-27-8	6	ST0610	High	ENR102	V1	438.17	41.038
196-30-9	6	ST0610	High	HENR104	V1	475.10	49.531
196-30-9	6	ST0610	High	ENR104	V2	457.20	45.325
196-30-9	6	ST0610	High	HENR102	V1	456.02	45.049
196-30-9	6	ST0610	High	HENR104	V2	479.07	50.463
200-27-8	6	ST0610	High	ENR102	V2	442.06	41.952
196-30-9	6	ST0610	High	ENR104	V3	449.61	43.541
196-30-9	6	ST0610	High	ENR104	V1	453.24	44.393
226-9-0	6	ZEROST	High	HENR102	V2	457.80	47.628
226-9-0	6	ZEROST	High	ENR102	V2	438.89	43.183
226-9-0	6	ZEROST	High	ENR102	V1	435.48	42.382
226-9-0	6	ZEROST	High	ENR102	V3	432.33	41.642
226-9-0	6	ZEROST	High	HENR102	V2	457.81	47.628
226-9-0	6	ZEROST	High	HENR102	V1	454.40	46.828
226-9-0	6	ZEROST	High	HENR102	V3	451.25	46.087

Table 21. Continued

Optimal State	Discount Rate (%)	States Array	U_3O_8 Price Scenario	Enrichment Price Scenario	Reprocessing Price Scenario	Cost Per Reactor (Million $)	Terminal Function Cost (Billion $)
201-26-8	6	ST0610	Low	HENR100	V1	420.21	36.829
201-26-8	6	ST0610	Low	HENR104	V3	450.55	43.959
201-26-8	6	ST0610	Low	HENR102	V3	431.24	39.419
201-26-8	6	ST0610	Low	HENR104	V2	457.99	45.705
201-26-8	6	ST0610	Low	HENR102	V2	438.67	41.165
201-26-8	6	ST0610	Low	HENR100	V3	416.67	35.995
201-26-8	6	ST0610	Low	ENR102	V3	413.29	35.201
201-26-8	6	ST0610	Low	HENR100	V2	424.09	37.741
201-26-8	6	ST0610	Low	HENR104	V1	454.11	44.793
201-26-8	6	ST0610	Low	HENR102	V1	434.79	40.254
201-26-8	6	ST0610	Low	ENR102	V2	420.72	36.947
201-26-8	6	ST0610	Low	ENR102	V1	416.84	36.035
200-27-8	6	ST0610	High	HENR100	V2	445.45	42.748
200-27-8	6	ST0610	High	HENR100	V3	437.99	40.996
196-30-9	6	ST0610	High	HENR102	V1	452.40	44.197
182-48-5	6	LT0610	High	ENR104	V2	449.77	44.142
182-48-5	6	LT0610	High	HENR102	V2	453.31	44.974
180-50-5	6	LT0610	High	HENR104	V2	470.90	49.088
180-50-5	6	LT0610	High	ENR104	V3	442.26	42.358

Table 21. Continued

Optimal State	Discount Rate (%)	States Array	U_3O_8 Price Scenario	Enrichment Price Scenario	Reprocessing Price Scenario	Cost Per Reactor (Million $)	Terminal Function Cost (Billion $)
182-48-5	6	LT0610	High	ENR102	V3	428.44	39.130
180-50-5	6	LT0610	High	HENR102	V3	445.83	43.196
180-50-5	6	LT0610	High	HENR104	V3	463.39	47.324
180-50-5	6	LT0610	High	ENR104	V1	445.99	43.233
182-48-5	6	LT0610	High	ENR102	V1	432.14	40.000
180-50-5	6	LT0610	High	HENR102	V1	449.55	44.071
180-50-5	6	LT0610	High	HENR104	V1	467.12	48.199
143-77-15	6	LG0610	Low	ENR102	V1	459.05	31.750
144-78-13	6	LG0610	Low	ENR102	V2	463.61	32.629

given in the first column of this table, where the three integers represent the cumulative ten-year commitment of Light Water Reactors, Prebreeders, and Light Water Breeders, respectively.

It is observed that lower costs are generally associated with combinations which have a minimum Prebreeder-to-LWR ratio. There is slight improvement in the market penetration when the U_3O_8 price assumes its high values, but in the majority of cases the penetration parameter is near the lower limit set by the tunnel constraints. Where the tunnel constraints were chosen to include the zero value for market penetration, only Light Water Reactors were chosen until the last year of the planning period.

It is seen that the most important mechanism is the U_3O_8 cost, with some effect attributable to reprocessing. This result is consistent with the findings of Allied General Nuclear Services, whose recent sensitivity analysis shows that price behavior of U_3O_8 and timing of plutonium recycle introduction are the most important factors affecting recycle economics (Cholister, 1976). The apparent lethargic behavior to the U_3O_8 and enrichment price suggests that there are not substantial gains to be made by changing national policy to support a Light Water Breeder of such marginal breeding character. This analysis omits the question of cost differences for the two fuel cycles attributable to differences in heavy element composition. It is tacitly assumed that the social cost of the U-233-Th cycle is equal to that of the U-235-Pu cycle. That is, there is neither penalty nor benefit associated with the differences between the two fuel cycles.

There are several aspects to the LWBR operation to which no value has been assigned. One is the stored thorium fuel, which may be used to fuel Breeder reactors as they come on line. A complete analysis will account for the storage of Pu, Th, and U-238. Data for these activities are incomplete at present, and the assumption is made that the cost differential between thorium storage and plutonium retention is negligibly small. The most substantial economic effects arise from the initial deployment of the Prebreeder. It is impossible to avoid the increased U_3O_8 and enrichment costs before the Breeder is brought on line. The discounting technique increases the importance of the Prebreeder penalty relative to Breeder benefits. Given that the commitment to a U-233-Th fuel cycle is made, it is important to ascertain the most efficient means of accomplishing the desired market penetration. The results of this study reveal that the best deployment of Prebreeders is one which has a maximum near the middle of the 10-year planning period. This is consistent with the real known behavior associated with construction of supporting facilities, since the full level of support may not be available when the Prebreeder is first deployed. The optimal strategy is subject to the tunnel constraints imposed.

Of particular interest is the role of enrichment. While enrichment assumptions affect the cost per unit, the optimal path is unchanged. Thus the variable of interest for this analysis is the U_3O_8 price vs cumulative consumption relationship. The marked tendency of the solution to cluster near the lower bound of market penetration suggests that even this mechanism does not exert an effect of importance. It is likely that reprocessing and fabrication costs will assume higher values for the

Prebreeder and the Breeder relative to the Light Water Reactor than those used in this analysis. This would further strengthen the position of the LWR in the energy-generating mix. Because these costs are minor compared to the uranium and enriching components, the conclusions are not modified by variation of these data elements. Prebreeder, U_3O_8, and enrichment costs have a substantial effect upon the unit cost of the reactor. This may be seen by examining the various constraint scenarios for similar U_3O_8, enrichment, and reprocessing prices. Taking, for example, a constant charge of $75.00 for enrichment and the reprocessing scenario denoted by the code V1, a comparison may be made of the unit costs for the two tunnel constraints which result in different optimal solutions. The low penetration case whose final state (at the 10-year cutoff point) is (200-27-8) has an associated cost of $441.55 million. The same fuel cycle parameters but a different mix resulting in a terminal state of (143-77-15) has an associated cost of $474.38 million.

The economic aspects of the LWBR computations are somewhat uncertain. Specifically, as Puechl points out, the value of the bred U-233 is open to question as is the mechanism for handling the storage of the fissile bred product. If, in fact, it is necessary to store fuel for a long period, the charge to Breeder operation for storage should be made, thus penalizing the LWBR system even more. This offsets the more favorable environmental aspects of the LWBR system, hence taken together would be anticipated to exert minimal effect upon the economic characteristics of the LWBR/LWR/Prebreeder system.

The major economic factors which affect any light water system are the costs of uranium and enrichment. For the comparison of the

Light Water Reactor and the Light Water Breeder, the cost of reprocessing is also a major consideration. There are no firm estimates of the relative costs for the reprocessing of spent fuel from LWBR's, Prebreeders, and LWR's. For the purpose of this study, the cost of reprocessing for the Th-232-U-233 fuel cycle is taken to be twice that of the Light Water Reactor. The result of this estimate is that the fuel cycle cost for the Light Water Breeder is about four-fifths that of the Light Water Reactor. The Prebreeder is more expensive owing not only to the reprocessing cost but also to the higher uranium and enrichment requirements.

The external economic influences attributable to policy decisions or fuel cycle service commitments are formulated in terms of constraints. For the problem at hand, these take the form of tunnel constraints which determine upper and lower bounds upon the cumulative commitments of Prebreeders in a given year. Uranium-233 fuel fabrication facilities will not be built unless there is some threshold number of Prebreeders committed; conversely, no Prebreeders will be committed at all unless there is a reasonable expectation of support from the nuclear fuel cycle service industry. The upper bound arises because the services have finite capacity. Because there is no commitment of fuel cycle facilities for LWBR support, such constraints were assumed as believed to be reasonable for the LWBR commitment problem. Several sets of constraints were assumed for the commitment of Prebreeders in order to ensure that the economic properties of the light water reactor mix were appropriately derived. The total cost of a mix is dependent upon the system constraints, since it is always possible to formulate the problem so that the unconstrained optimum is not an admissible path. For the case of a mix of several energy generating types, it is possible to bound the problem so that the least-cost solution to a given constrained formulation is significantly higher in cost than that of a different formulation.

Constraints are defined in terms of the ratio of Prebreeders committed to the number of Light Water Reactors committed. These quantities form two of the three components of the optimal state. For computational control the optimal state is taken to be the vector of LWR commitment, Prebreeder commitment, and Prebreeder-years. For presentation of results, the optimal state may equally well be taken to be the vector of LWR's, Prebreeders, and Breeders. Results are summarized in Figures 22-24 for various economic conditions. The optimal path as indicated by the Prebreeder-to-LWR ratio is shown in addition to the constraints for the particular study. Space limitations require that the headings contain brief notation describing the price assumptions. These are given in Table 19.

As a simplification, the cost of reprocessing for LWR and LWBR fuel cycles is taken to be the same after 10 years. It is possible to examine the effect of this assumption by varying the reprocessing requirement for the Light Water Breeder (or for any of the reactors). This has been done, and no variation in optimal path appeared. The major effect in the period following initial introduction of the Breeder is the rise in uranium prices due to the demand schedule implied by the mix of reactors. The optimal path is thus determined primarily by requirements for uranium and enriching services, the same factors which are primary forces in Light Water Reactor fuel cycle costs.

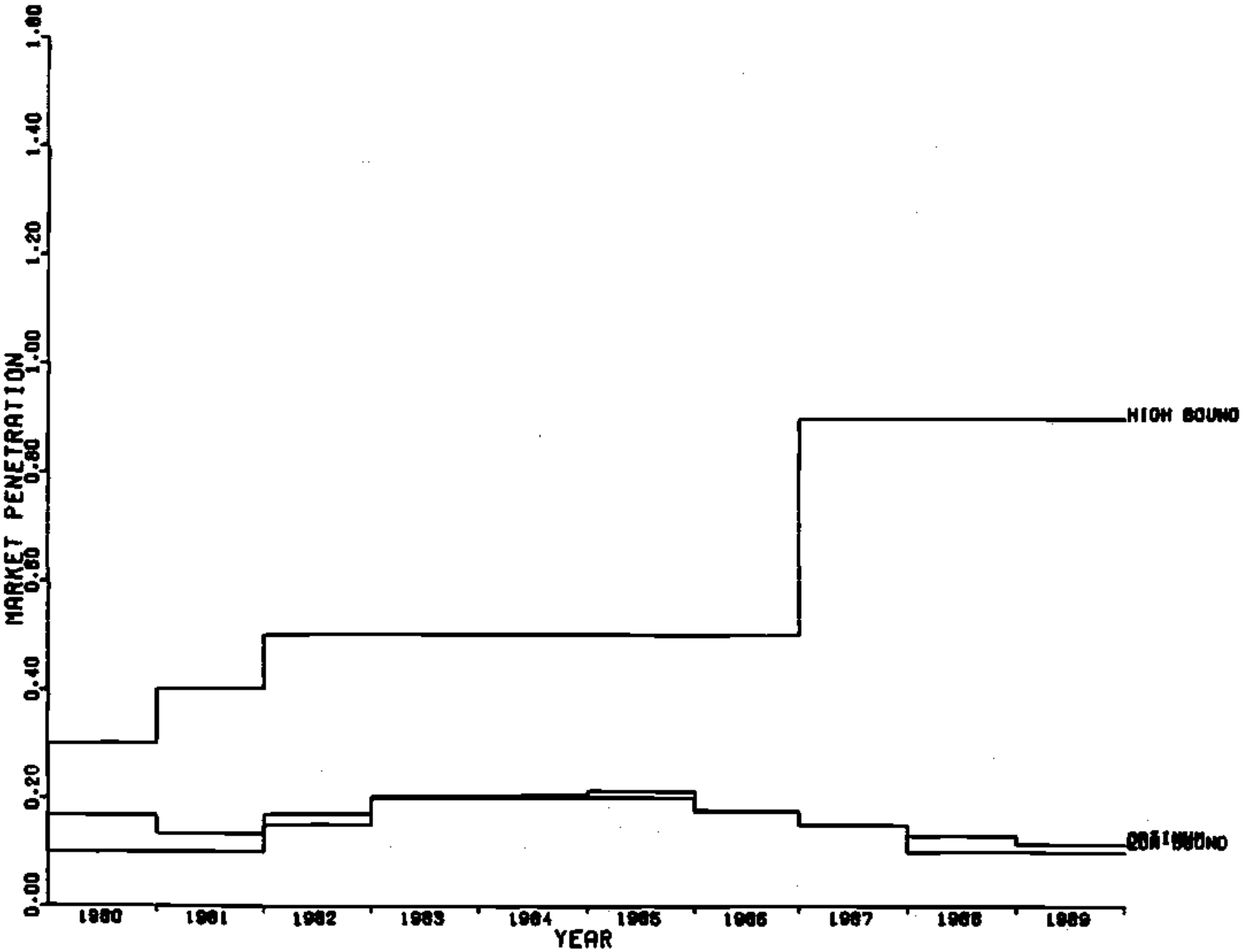

Figure 22. LWBR/LWR Optimal System Commitment--Scenario: (6%, U=LOW, ENR=HENR100, REP=V2)

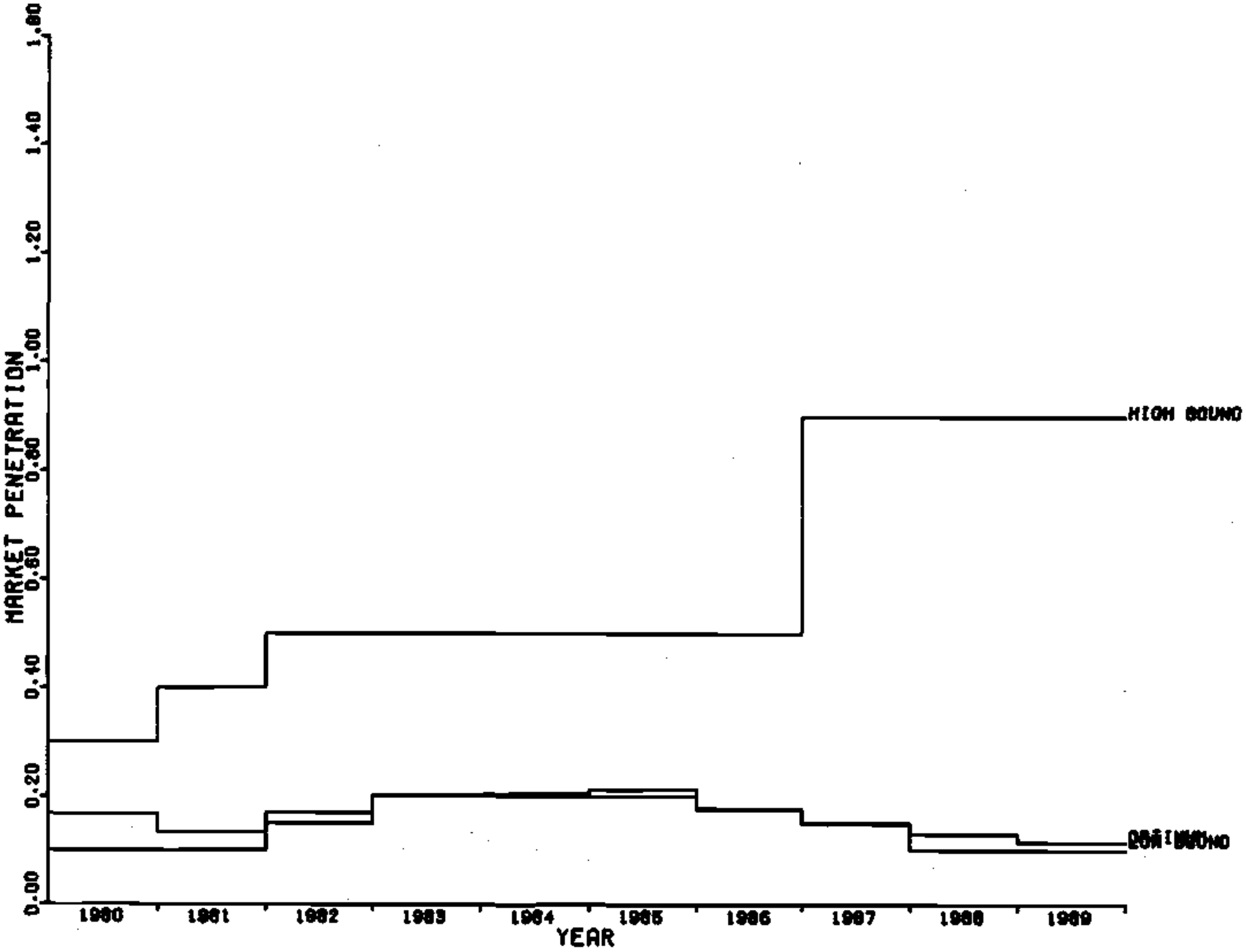

Figure 23. LWBR/LWR Optimal System Commitment--Scenario: (6%, U=LOW, ENR=HENR102, REP=V1)

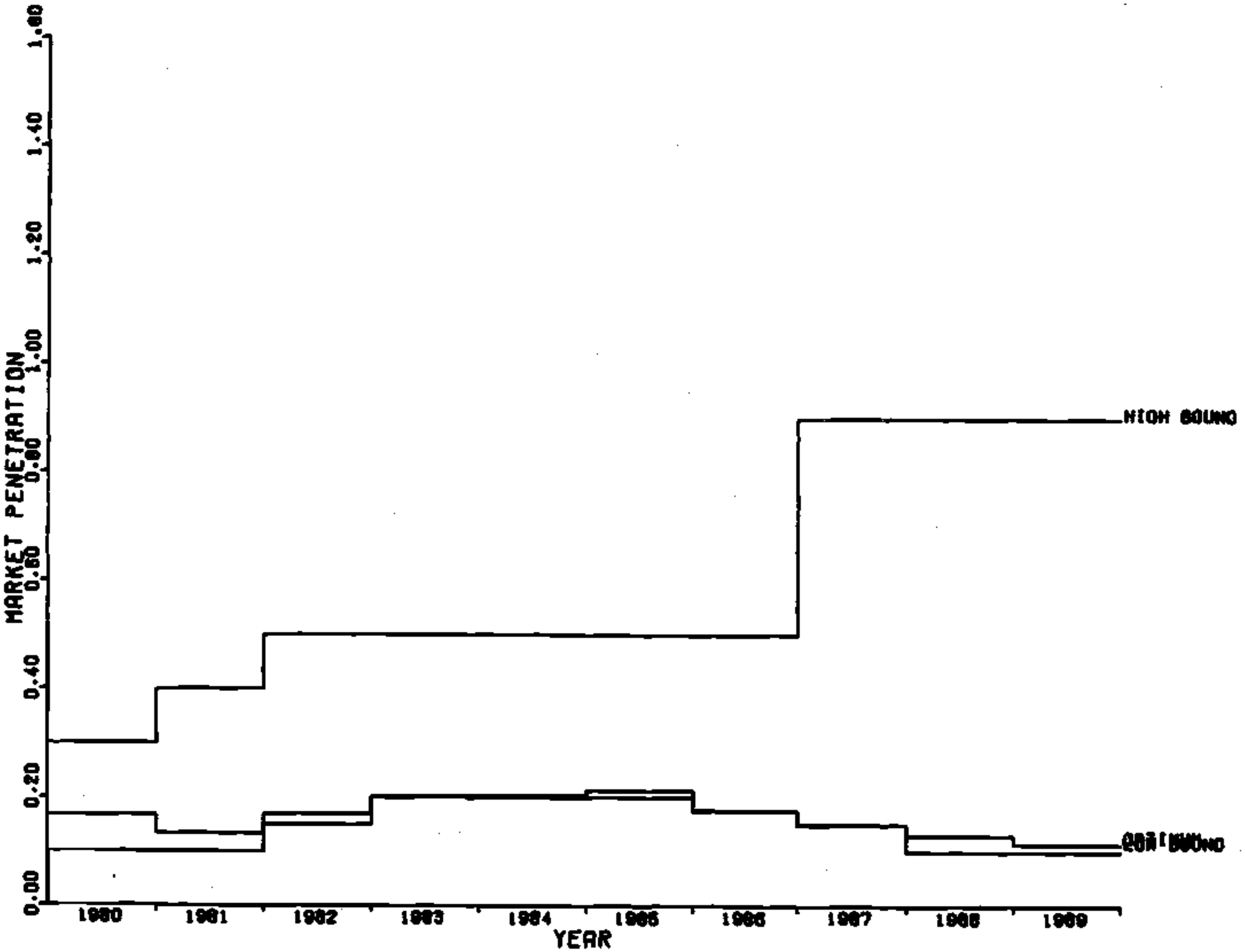

Figure 24. LWBR/LWR Optimal System Commitment--Scenario: (6%, U=LOW, ENR=HENR104, REP=V3)

The graphical presentations in Figures 22 through 24 do not present completely the effect of Prebreeder commitment, for they do not show the number of Breeders resulting from Prebreeder emplacement. During the first ten years, this number is relatively small owing to the length of time required for production of sufficient U-233 for fueling of Breeders. In the case with low constraints, only nine Breeders are engendered during the first ten years.

Enriching service requirements well in excess of those resulting from the Cascade Improvement Program and the Cascade Uprating Program are assumed to exist for Case A computations. Given the large capital investment required for an enrichment plant, it is probable that the stimulation of a commercial Light Water Breeder Program depends upon control of enriching services by an entity powerful enough to sustain the oscillations which are characteristic of a new enterprise. Private capital may be expected to wait until the risk associated with investment is relatively small. Should private ventures finance the next increment of enriching capacity, it would be anticipated that the commercial power generating sector would choose the method of electrical power generation which has least requirement for enriching services. Thus, unless enriching capacity is guaranteed, it is to be expected that the Light Water Reactor would be preferred by utilities and hence that the motivation for fuel cycle services oriented toward the Light Water Breeder System would be attenuated.

It is further apparent that some form of incentive may be required in order to induce a utility to employ a Prebreeder rather than a conventional Light Water Reactor. Given the reasonably close levelized costs for the LWR system and the LWBR system, it is possible that tax credit could be allowed for utilities which embrace the LWBR system. This mechanism, and others which subsidize utilities from public funds, are subject to the criticism of unnecessary government intervention.

A more plausible and perhaps less objectionable approach exists at the local level, where public service commissions may explicitly approve the rate structure necessary to allow utilities to recover the cost of power generation from a Prebreeder/Breeder system. In such a case, the justification would not be minimization of expected cost, but minimization of risk of losing generation capacity.

Computation of cost associated with the loss of generation capacity arising from a yellowcake shortage may be accomplished by the variation of U_3O_8 prices. For quantities of uranium in excess of the assumed reserve level, an arbitrarily large price coefficient may be attributed to the higher consumption portion of the price vs cumulative consumption function. The cost thus calculated in reality reflects two mechanisms: the cost of uranium and the risk accepted by the reactor owner that the uranium supply will be exhausted before the generating unit has produced sufficient revenue to pay for itself.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Economic Feasibility of the LWBR

Based upon data developed from that in the Draft Environmental Statement, ERDA-1541, the Dynamic Programming analysis used for system evaluation indicates that inclusion of the LWBR subsystem in the commercial energy-generating mix increases the cost of electricity production. Further, the optimal schedule is weakly affected by variations in prices for U_3O_8, separative work, and reprocessing.

Resource requirement computations show that substitution of Prebreeders for Light Water Reactors results in increased need for both yellowcake and separative work.

The simplified economic analysis found in Appendix A indicates that an alternative reactor system of deferred benefits and higher relative near-term costs may not be economically competitive. The more elaborate computational scheme yields a similar result. Hence, the LWBR subsystem is a less viable commercial entity than is the Light Water Reactor.

Implementation

Although fabrication and reprocessing may lag the commitment of Prebreeders, these functions are not anticipated to present major system problems. In many respects, fabrication of U-233-bearing fuel elements is similar to that of other oxide fuels.

Existing facilities may thus be adapted for use with the Light Water Breeder Reactor system, with modifications arising from the shielding requirements. Construction of reprocessing facilities is not critical to the inception of a Light Water Breeder program, since it is not required that U-233 be available until time for Breeder commitment, or more precisely, until time for Breeder fuel fabrication. From the standpoint of utility commitment, therefore, it is necessary that there first be a guarantee of revenue generation sufficient for cost recovery. Next, it is required that there exist a promise of enriching service availability adequate to sustain the Prebreeder requirements. Finally, there must be fabrication and reprocessing facilities capable of handling the U-233 fuel with the 2.6 MeV gamma ray emitted by Tl-208. With the exception of reprocessing, these considerations are also pertinent to decisions involving Light Water Reactor commitment, and in these instances the determinants of the commitment decision vary only in degree between the LWR and the LWBR systems. Hence there are no technological barriers which would prevent implementation of the Light Water Breeder System.

Recommendations

Because the resource requirements for the Prebreeder and the Light Water Breeder are based upon conceptual designs rather than upon commercial offerings, the fuel cycle parameters for these two reactors are subject to some uncertainty. In view of the necessarily approximate nature of the data for this analysis, there is little to be gained from a more refined study which might include secondary costs such as those attributable to environmental insult. However, as reactor designs become more precise and as the character of uranium prices becomes better known, it will be desirable to extend the analysis to include costs other than those related to the fuel cycle stages, and it will perhaps be possible to model uranium prices more accurately than is given by a simple relationship between price and cumulative consumption.

It has been the purpose of this study to examine the LWBR System as a competitor in the economic sense to the LWR. In this regard, it must be concluded that there is no indication that the LWBR System is economically preferable, owing to the assumed costs associated in the early years of Prebreeder deployment with generation of U-233. However, an examination of the "cost per reactor" figure, which is additively related to the variable cost of power production for the two systems, reveals that the addition of LWBR systems to the mix may be justifiable if the power generating sector is subjected to external constraint conditions not assumed in the foregoing analysis. The ability to use recycle thorium as Prebreeder fuel would improve the economic merit of the LWBR system relative to the LWR. Or, as has been previously indicated, the risk of exhausting uranium supplies before reactor plants are amortized may be unacceptably high either to utilities or to the bodies responsible for their regulation. Considerations of this nature may be included in the analysis, the effect possibly being that optimal and near-optimal solutions obtained in the non-restrictive case are not allowed in the more refined formulation.

The methods utilized for the analysis of LWBR economics do not entail arbitrary assumptions except in the form of tunnel constraints. These constraints exist primarily to reduce computing system resource use.

They are not intended to be used to modify the economic character of the problem under evaluation. More precisely stated, there is no requirement that tunnel constraints be used to affect the nature of the optimal solution for a given problem. The results thus reflect the economic forces which result in a cost to society. The optimum may be determined as a function of the price behavior of resources and the technological character of the energy-producing system. As more precise data become available, the same technique should be utilized for the analysis of systems consisting of more than one energy generating type.

It may be noted that the commitment history for Prebreeders often shows moderate commitment until the tenth year, when deployment appears to be uncharacteristically heavy. This is a result of the model, which divides the problem into two time periods; one in which the decision logic pertaining to system growth is employed and one in which the effects of this logic are evaluated. The computational method which provides the accounting for the evaluation of effects, or terminal constraints, is based upon the premise that a Prebreeder is retired as soon as sufficient U-233 is available to allow its replacement by a Breeder. This assumption is not employed during the first phase of the analysis in which the time-dependent growth combinations are enumerated. Therefore, there is the effect of a short lifetime for some of the Prebreeders deployed in the later years of the investigation, and since the Prebreeder has the highest fuel cycle cost of the three reactor types, the optimal path might be expected to correspond to that condition which minimizes the number of Prebreeder-years for a given number of Prebreeders. The nature of the terminal constraint formulation in this analysis yields an unusually heavy commitment during the last year of growth. It is possible that the Dynamic Programming model might be improved by an investigation of the natural boundary conditions corresponding to the variational formulation which is the basis for the LWBR study. The terminal constraint formulation might, then, be improved to include this consideration, which is expected to have the effect of smoothing the year-by-year deployment of Prebreeders as reflected in the Prebreeder-to-LWR fraction. Since the basic economic properties of the reactor types remain the same, there is no _a priori_ reason to believe that this refinement to the algorithm will result in a cumulative ten-year deployment which differs substantially from the ones given as a result of the present investigation. That is, the variation in answer due to a change in the algorithm is expected to be less severe than those variations which may be induced by utilizing different values for uranium, enrichment, and reprocessing prices.

A more accurate reflection of costs may be incorporated by specifying each reactor individually so that transportation costs and power levels may be precisely determined. This highly-refined breakdown permits a more accurate computation of fuel cycle costs at the expense of computing time. This latter consideration is of such import that the individual reactor modeling feature would usually be used for cost computation only in the vicinity of an optimal path. Such a specification would be useful in evaluating the economic potential for a nuclear energy complex in which reprocessing and fabrication were carried out in an integrated plant system with a potentially smaller cost for both reprocessing and fabrication stages. This particular concept may be advantageous from the standpoint of a mix containing LWBR systems. Among the practical planning problems, this is certainly one which has sufficient constraints to allow a reasonable solution procedure and which should be investigated from an economic standpoint.

The results of both the static and dynamic analyses do suggest, however, that close attention be given those concepts which hold promise for extending the usefulness of the nation's uranium resources. Thus, assuming the LWBR system to be among the economically more attractive alternatives to existing Light Water Reactors, it is seen that research and investment in an enriching technology such as Laser Isotope Separation can yield substantial benefits to society. Because the energy requirements for Laser Isotope Separation are significantly lower than those of either the gaseous diffusion or the centrifuge process, the prospect exists of lowering enrichment price while expanding the utility of the nation's resource base.

With respect to the consumption of U_3O_8, commitment of an LWBR system would have the effect of increasing demand for uranium resources for the period in which the presently-known resource base of approximately 640,000 tons U_3O_8 will support burner reactors. Hence, the importation of uranium would have little effect upon the desirability of an LWBR system. In fact, commercial implementation of a Light Water Breeder System would increase requirements for uranium in all but the long term. Hence, national policy should permit and encourage a uranium import program if the Light Water Breeder becomes a commercial reality.

Reprocessing is essential for the existence of any breeder reactor program. If plutonium recycle is denied the operators of Light Water Reactors, it is possible that the retrieval of U-233 may be allowed even though the recovery of Pu is not permitted. However, it is unlikely that capital investment would be made in a commercial reprocessing facility for separation of uranium and thorium before a market is guaranteed. Hence, reprocessing of plutonium and U-233 is likely a requisite for commercial introduction of the Prebreeder.

It is concluded, therefore, that policy implementations which would affect the Light Water Reactor system will also have similar effect upon the economic properties of the LWBR system. The policy toward development of the Light Water Breeder is, with the exception of reprocessing, independent of those policies which may govern the rest of the fuel cycle. Any effort to improve the economic characteristics of nuclear-electric power would center upon improvement of the LWR fuel cycle or capital cost, and would thus concentrate most heavily upon enriching and U_3O_8 costs.

Long-Range Energy Policy Analysis

Further economic investigations of energy systems, including the Light Water Breeder Reactor system, are essential to a proper comparison of alternatives. Data pertaining to the physical utilization of resources and services should be refined to a high degree when evaluating the effect of inclusion in a potential commercial system. Of particular importance in the case of the Prebreeder is the design requirement that fuel be enriched to 13.5% U-235 by weight. A further consideration is the separation of thorium- and uranium-bearing rods. This separation derives from the desire to recycle the discharge uranium and hence to maintain purity of uranium isotopic mixtures. Further, since it is desired to extract the U-233 as a separate isotope, separate fuel rods are used for U-233-producing material and for fissile U-235. It is suggested that the Prebreeder could be made more efficient by mixing the thorium and uranium as is the case in the Breeder. Recovery of U-233, U-234, and U-238 would then necessarily be accomplished by a process other than gaseous diffusion.

The dynamic programming techniques developed and employed in this analysis of the LWBR system may be used to evaluate other combinations of plant types. Further studies should include the Liquid Metal Fast Breeder Reactor, the High Temperature Gas Cooled Reactor, and the Gas Cooled Fast Reactor. For these designs, development expenditures must be included in the capital cost formulation.

The various fissile fuel loading patterns possible for the Prebreeder suggest that detailed studies of core physics be performed before commitment to any new algorithm for reactor use. Removal of the design constraints associated with current handling of the enriching function make possible a wide range of fuel loadings and thus may permit a more economical use of fuel in a reactor. When analyzing the in-core economics of fuel loadings, it is necessary to ensure that costs of fuel preparation are properly handled. Utilization of oxide fuel pellets makes the fabrication of fuel rods with enrichment or composition gradations a relatively straightforward procedure.

Further research should include a detailed system involving burnup characteristics of the various reactor types as well as economic models of the various fuel cycle services. Whereas the existence of services in the foregoing analysis is a postulated parameter, an alternative approach is the inclusion of investment criteria to model expansion of existing facilities. As a beginning, an economic model segment containing an historical relationship between electrical capacity and Gross National Product could be utilized to generate a demand for new facilities. If a correlation exists between the expected return on capital and the Gross National Product history, an estimate may be made of the rate of return needed to attract investment capital in a fuel cycle facility. From this consideration, prices for fuel cycle services may be derived and energy costs computed. While this appears to be an exhaustive undertaking, it nevertheless possesses the desirable property of establishing a coordinated relationship between prices of fuel cycle services and the scheduling of plant commitment. In this regard, it is possible that existing econometric models may be employed to yield electrical power requirements for a dynamic analysis of the energy economy.

APPENDIX A

CONSTANT COST ANALYSIS OF A SYSTEM WITH MORE THAN ONE TYPE OF ENERGY GENERATING UNIT

Enrichment subsidy would have no effect upon the mix of reactors deployed. The increased cost of enriching services would favor the Light Water Reactor. However, deployment of the Light Water Breeder would reduce the risk associated with the underutilization of enriching plants. Thus, if a risk term is included in the economic analysis, the presence of the Prebreeder acts to reduce the risk associated with plant operation by maximizing utilization of capacity. If LIS becomes a reality, the LWR would be favored. If this is the case, however, the dominant long-term economic consideration becomes the availability of uranium. Because the uranium requirement in Prebreeders is greater than that for Light Water Reactors, the cost of the individual LWR will be less than that of the Prebreeder.

The discounted benefits are given by the usual present-worthing factor applied to the uniform benefit B. The present-worthed benefit is

$$\text{Present-worthed benefit} = B(1 + \rho + \rho^2 + \ldots) \qquad \text{(A-1)}$$

For an eight percent discount rate, the present-worth factor for an infinite planning horizon is equal to 13.5. If ΔC is the annual cost difference for the production of power, the total front-end differential cost of an alternative power system may not exceed 13.5 (ΔC).

$$\Delta C = \text{Annual cost of alternate system - annual cost of Breeder} \quad \text{(A-2)}$$

The above analysis is a simplification since there is no methodical way to predict the value of the interest rate (or more appropriately, the discount factor). However, it does give some insight into the decision criteria necessary for the formulation of a plan based upon observed facts. Suppose we have a 10-year front-end period. The present-worthed commitment is then

$$\Delta C_{10}(1 + \rho + . . . + \rho^9)$$

where ΔC_{10} denotes the cost difference between two competing energy sources. Now

$$1 + \rho + . . . + \rho^9 = \frac{1 - \rho}{1 - \rho}(1 + \rho + . . . + \rho^9) = \frac{1 - \rho^{10}}{1 - \rho} \quad \text{(A-3)}$$

If $\rho = \frac{1}{1 + i}$, where i is the annual discount rate, the cumulative discounting factor for 10 years is

$$d_{10} = \frac{(1 + i)^{10} - 1}{i(1 + i)^9} \quad \text{(A-4)}$$

For an annual discount rate of 0.08,

$$\frac{(1.08)^{10} - 1}{(0.08)(1.08)^{9}} \Delta C_{10} \leqq 13.5 \; \Delta C \tag{A-5}$$

whence

$$\frac{\Delta C_{10}}{\Delta C} \leqq 1.86 \tag{A-6}$$

Thus, given a levelized system cost ΔC reflecting the annual levelized cost difference between two energy sources such as the LWR system and the LWBR system, an upper bound upon the annual cost difference between the Prebreeder and the Light Water Reactor may be estimated.

Let C denote cost as before and let the superscripts LWR, P, and LWBR indicate Light Water Reactor, Prebreeder and Light Water Breeder Reactor, respectively. Consider the economics of a simple reactor. For a 30-year period the cost of an LWR is

$$C_T = \sum_{j=0}^{29} \frac{1}{(1 + i)^j} C_j^{LWR} \tag{A-7}$$

Assume now that the fuel cycle cost is a constant

$$C_j^{LWR} = C^{LWR} \qquad j = 0, 1, \ldots, 29 \tag{A-8}$$

Thus

$$C_T = C^{LWR} \sum_{j=0}^{29} \frac{1}{(1 + i)^j} = C^{LWR} \sum_{j=0}^{29} \rho^j = \frac{1 - \rho^{30}}{1 - \rho} C^{LWR} \tag{A-9}$$

Consider the alternate plant, the Prebreeder/Breeder system. Assume exactly one reactor. Then the first 18 years of operation will be dedicated to U-233 generation as well as power generation at a cost C^P. The last 13 years will be dedicated to LWBR operation with Fissile Inventory Ratio unity. As before, all costs are assumed constant. The system cost for the Prebreeder/Breeder system is therefore given by

$$C^{P/B} = \sum_{j=0}^{17} \rho^j C^P + \sum_{j=18}^{29} \rho^j C^{LWBR} = C^P \sum_{j=0}^{17} \rho^j + C^{LWBR} \sum_{j=18}^{29} \rho^j \qquad \text{(A-10)}$$

whence

$$C^{P/B} = C^P \frac{1 - \rho^{18}}{1 - \rho} + C^{LWBR} \rho^{18} \frac{1 - \rho^{13}}{1 - \rho} \qquad \text{(A-11)}$$

For the Prebreeder/Breeder combination to be economically more attractive than the Light Water Reactor requires that

$$C^{P/B} < C_T \qquad \text{(A-12)}$$

or that

$$C^P \frac{1 - \rho^{18}}{1 - \rho} + C^{LWBR} \rho^{18} \frac{1 - \rho^{13}}{1 - \rho} < \frac{1 - \rho^{30}}{1 - \rho} C^{LWR} \qquad \text{(A-13)}$$

The Prebreeder-Breeder relationship may be written from this equation as

$$\frac{C^{LWBR}}{C^P} \frac{1 - \rho^{13}}{1 - \rho^{18}} < \frac{C^{LWR}}{C^P} < \frac{1 - \rho^{30}}{1 - \rho^{18}} - 1 \qquad \text{(A-14)}$$

For a discount rate of 0.08, or eight percent,

$$\frac{C^{LWBR}}{C^{P}} (0.8433) < \frac{C^{LWR}}{C^{P}} (1.2012) - 1 \qquad (A\text{-}15)$$

If $\frac{C^{LWR}}{C^{P}} = 0.75$, then

$$\frac{C^{LWBR}}{C^{P}} < - 0.1175 \qquad (A\text{-}16)$$

Clearly this is an impossible condition. The variability of this upper bound with discount rate is shown in Table 22.

Table 22. Bounds on Fuel Cycle Costs for Breeder Relative to Those of Prebreeder

Discount Rate (%)	Upper Bound on C^{LWBR}/C^{P} for Economic Attractiveness
2	0.1591
4	0.0311
6	- 0.0569 (Economically Infeasible)
8	- 0.1175 (Economically Infeasible)

Now let us consider the cost of operating N Prebreeders until there is no further available uranium. Assume that N is sufficiently small that each Prebreeder may be converted into a Breeder. This still implies 18 Prebreeder-years of operation per Prebreeder, and the economic aspect of the problem is unchanged.

Suppose now that N Prebreeders are used to engender M Breeders. At the termination of uranium availability, N-M plants must be replaced by non-nuclear units. This case corresponds to the computational formulation in which Breeders replace LWR's. The system cost may be expressed as follows:

$$\text{Cost} = N \cdot C^{P} \sum_{j=0}^{T-1} \rho^{j} + MC^{B} \rho^{T} \frac{1-\rho^{30}}{1-\rho} + (N-M)\ C^{A} \rho^{T} \frac{1-\rho^{30}}{1-\rho} \qquad \text{(A-17)}$$

$$+ \text{ Decommissioning Loss on } N - M \text{ plants}$$

where

C^{A} is the annual cost of an alternative power source

T is the number of years of Prebreeder operation

and 30 years of plant amortization are assumed.

As a simplifying assumption, all Prebreeders are taken to be committed simultaneously. The number of Breeders, assuming s Prebreeder-years necessary to generate enough U-233 to fuel one Breeder, is given by

$$M = \frac{TN}{s} \qquad \text{(A-18)}$$

Substituting this in the cost equation

$$f(T) = \text{Cost} = N \cdot C^{P} \frac{1-\rho^{T}}{1-\rho} + MC^{B} \rho^{T} \frac{1-\rho^{30-T}}{1-\rho} \qquad \text{(A-19)}$$

$$+ (N-M)\ C^{A} \rho^{T} \frac{1-\rho^{30-T}}{1-\rho}$$

$$f(T) = N \cdot C^P \frac{1 - \rho^T}{1 - \rho} + \frac{TN}{s} C^B \rho^T \frac{1 - \rho^{30-T}}{1 - \rho} + N\left(1 - \frac{T}{s}\right) C^A \rho^T \frac{1 - \rho^{30-T}}{1 - \rho} \tag{A-20}$$

This function of T will possess a local extremum if its derivative with respect to T at an interior point of $[0,s]$ is equal to 0.

$$\frac{d}{dT} f(T) = (\ln\rho)\rho^T \left(NC^P + \frac{NTC^B}{s} + NC^A - \frac{NT}{s} C^A\right) + \rho^T\left(\frac{NC^B}{s} - \frac{NC^A}{s}\right) - \rho^{30}\left(\frac{NC^B}{s} - \frac{NC^A}{s}\right) \tag{A-21}$$

If it is assumed, however, that the cost of an alternative C^A is greater than the cost of the Breeder C^B, and if T is restricted to be less than or equal to s, it is seen that the above expression is negative for $T \epsilon [0,s]$. For

$$\ln\rho < 0 \tag{A-22}$$

$$\rho^T > 0$$

$$\left(NC^P + \frac{NTC^B}{s} + NC^A - \frac{NT}{s} C^A\right) > 0$$

hence their product must be strictly less than 0. The second and third terms of Equation (A-21) may be written

$$\frac{N}{s}(\rho^T - \rho^{30})(C^B - C^A) \tag{A-23}$$

which is less than 0 since the assumption has been made that $T \leqq s < 30$ and $C^B < C_A$.

Since the first derivative is everywhere negative in (0,s), it may be concluded that cost is minimized by deferring the addition of more expensive plants as long as is possible.

A slightly more facile approach may be utilized to analyze the general problem of plant commitment. Suppose that, after T years, the average fuel cycle cost of the mix of plants is greater than that of the Prebreeder. As in the rest of the development, it is assumed for simplicity that operating and capital costs of all plant types are the same. In this case, the Prebreeder should be operated as long as resources permit. For a mix containing N plants, the total present-worthed cost is, from Equation

$$\text{Cost (Present-worthed)} = NC^P \frac{1 - \rho^T}{1 - \rho} + MC^B \rho^T \frac{1 - \rho^{30}}{1 - \rho} \qquad \text{(A-24)}$$

$$+ (N - M)\ C^A \rho^T \frac{1 - \rho^{30}}{1 - \rho}$$

When T = s, enough U-233 has been generated to fuel N, Breeders, hence the requirement for alternative sources of energy vanishes. It is necessary to examine the question of the time of commitment. Uranium-233 is produced at the rate such that $\frac{N}{s}$ Breeders are enabled each year. Writing

$$\alpha = \frac{C^A}{C^P} \qquad \text{(A-25)}$$

$$\beta = \frac{C^B}{C^P} \qquad \text{(A-26)}$$

and forming $g(T) = \frac{f(T)}{NC^P}$

$$g(T) = \rho^T + \beta \frac{T}{s} \rho^T (1 - \rho^{30-T}) + \alpha\rho^T (1 - \rho^{30-T}) - \frac{T}{s} \alpha\rho^T (1 - \rho^{30-T}) \tag{A-27}$$

$$g'(T) = (\ln\rho)\rho^T + \frac{1}{s} (\rho^T - \rho^{30}) \left(\beta - \alpha + \frac{s\alpha}{T}\right) \tag{A-28}$$

$$+ \frac{T}{s} \left[(\ln\rho)\rho^T \left(\beta - \alpha + \frac{s\alpha}{T}\right) + (\rho^T - \rho^{30}) \left(- \frac{s\alpha}{T^2}\right)\right]$$

$$g'(T) = (\ln\rho)\rho^T + \frac{(\beta - \alpha)}{s} \rho^T - \frac{\rho^{30}(\beta - \alpha)}{s} + \frac{(\ln\rho)}{s} T \rho^T(\beta - \alpha) \tag{A-29}$$

$$+ \alpha(\ln\rho)\rho^T$$

$$g'(T) = (\ln\rho)\rho^T + (\rho^T - \rho^{30}) \frac{\beta - \alpha}{s} + \frac{(\ln\rho)}{s} T \rho^T\beta \tag{A-30}$$

$$+ \alpha(\ln\rho)\rho^T \left(1 - \frac{T}{s}\right)$$

For $T \leqq s$ it is seen that $g'(T)$ is negative and hence that g is monotonic on $[0,s]$. The conclusion, as before, is that the effect of the discount rate is that of forcing the more expensive units to later times in the commitment schedule.

APPENDIX B

EQUIVALENCE BETWEEN DYNAMIC PROGRAMMING AND THE CALCULUS OF VARIATIONS

For backward dynamic programming, proofs of equivalence between the dynamic programming algorithm and the calculus of variations are found in texts by Beckmann (1968) and by Hadley and Kemp (1971). The proof which follows utilizes a generalization of the approach found in the latter work.

The minimum cost function $\Lambda_v(\underline{x})$ may be defined (Larson, 1968) by the equation

$$\Lambda_v(\underline{x}) = \underset{\underline{y}_{v-1}, \ldots, \underline{y}_1}{\text{Min}} \sum_{u=1}^{v} F\left(\underline{x}_u, \underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \qquad \text{(B-1)}$$

Consider the case of finding the vector-valued function $\underline{y}$ over a finite interval $[\alpha,\beta]$ which minimizes a cost functional $J[\underline{y}]$. Let it be assumed that $\underline{y}(\alpha) = \underline{a}$ and that $\underline{y}(\beta) = \underline{b}$. Let P_r be a partition of $[\alpha,\beta]$ which divides $[\alpha,\beta]$ into r finite intervals

$$P_r = \{\alpha = x_0, x_1, \ldots, x_r = \beta\} \qquad \text{(B-2)}$$

Consider an admissible function $\underline{y}(x)$ and denote by $\underline{y}_u$ the vector $\underline{y}(x_u)$. In subinterval u, $\underline{y}(x)$ may be approximated by the line segment joining the points $\underline{y}_{u-1}$ and $\underline{y}_u$. Over $[\alpha,\beta]$, $\underline{y}(x)$ is thus approximated by a piecewise linear function $\hat{y}(x)$. The derivative of $\hat{y}(x)$ in interval u is given by $\hat{\underline{y}}'(x) = (\underline{y}_u - \underline{y}_{u-1})/\Delta x_u$, where $\Delta x_u = x_u - x_{u-1}$.

Consider now the definition of the minimum cost function

$$J[y] = \int_{\alpha}^{\beta} F(x,\underline{y},\underline{y}')dx \tag{B-3}$$

denoting the fact that the cost depends not only upon the values of the components of $\underline{y}$ but also upon the rate of change $\underline{y}'$ required in order to follow the path. This integral may be approximated by

$$J[y] \cong \sum_{u=1}^{r} F\left(x_u,\underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u \tag{B-4}$$

Suppose that there exist r + 1 points $\underline{z}_0, \underline{z}_1, \ldots, \underline{z}_r$ which minimize the right-hand side of Equation (B-4). This set of points determines a polygon function $\hat{z}(x)$ which is an approximation to the optimal solution $\underline{z}(x)$ of the integral formulation. The finitely-partitioned formulation is a discrete optimization problem.

The sequence of functions $\{\Lambda_v(\xi)\}_{v=1}^{r}$ is defined as follows

$$\Lambda_v(\underline{\xi}) = \underset{\underline{y}_1, \ldots, y_{v-1}}{\text{Min}} \sum_{u=1}^{v} F\left(x_u,\underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u \tag{B-5}$$

$$\underline{y}_v = \underline{\xi}, \qquad v = 1, \ldots, r$$

$\Lambda_v(\underline{\xi})$ is thus the minimum value for $\sum_{u=1}^{v} F(x_u,\underline{y}_u,\underline{y}_u')\Delta x_u$ when $\underline{y}_v = \underline{\xi}$.

Note that no minimization is involved in the first stage.

$$\Lambda_1(\xi) = F\left(x_1, \underline{\xi}, \frac{\underline{\xi} - \underline{a}}{\Delta x_1}\right) \Delta x_1 \tag{B-6}$$

Now it may be observed that the $\Lambda_v(\xi)$ may be determined in a recursive manner.

$$\begin{aligned}
\Lambda_v(\underline{\xi}) &= \underset{\underline{y}_1, \ldots, \underline{y}_{v-1}}{\text{Min}} \sum_{u=1}^{v} F\left(x_u, \underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u && \text{(B-7)} \\
&= \underset{\underline{y}_1, \ldots, \underline{y}_{v-1}}{\text{Min}} \left[\sum_{u=1}^{v-1} F\left(x_u, \underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u \right. \\
&\qquad \left. + F\left(x_v, \underline{y}_v, \frac{\underline{y}_v - \underline{y}_{v-1}}{\Delta x_v}\right) \Delta x_v \right] \\
&= \underset{\underline{y}_{v-1}}{\text{Min}} \; \underset{\underline{y}_1, \ldots, \underline{y}_{v-2}}{\text{Min}} \left[\sum_{u=1}^{v-1} F\left(x_u, \underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u \right. && \text{(B-8)} \\
&\qquad \left. + F\left(x_v, \underline{\xi}, \frac{\underline{\xi} - \underline{y}_{v-1}}{\Delta x_v}\right) \Delta x_v \right]
\end{aligned}$$

Now it may be observed that the minimization operator $\underset{\underline{y}_1, \ldots, \underline{y}_{v-2}}{\text{Min}} [\cdot]$ does not operate upon $F\left(x_v, \underline{\xi}, \frac{\underline{\xi} - \underline{y}_{v-1}}{\Delta x_v}\right) \Delta x_v$. Thus,

$$\begin{aligned}
\Lambda_v(\underline{\xi}) = \underset{\underline{y}_{v-1}}{\text{Min}} &\left[F\left(x_v, \xi, \frac{\xi - \underline{y}_{v-1}}{\Delta x_v}\right) \Delta x_v \right. && \text{(B-9)} \\
&\left. + \underset{\underline{y}_1, \ldots, \underline{y}_{v-2}}{\text{Min}} \sum_{u=1}^{v-1} F\left(x_u, \underline{y}_u, \frac{\underline{y}_u - \underline{y}_{u-1}}{\Delta x_u}\right) \Delta x_u \right]
\end{aligned}$$

The second term is, by definition Λ_{v-1} $(\underline{y}_{v-1})$, so that

$$\Lambda_v(\underline{\xi}) = \underset{\underline{y}_{v-1}}{\text{Min}} \left[F\left(x_v, \underline{\xi}, \frac{\underline{\xi} - \underline{y}_{v-1}}{\Delta x_v}\right) \Delta x_v + \Lambda_{v-1}(\underline{y}_{v-1}) \right] \qquad \text{(B-10)}$$

$$v = 1, \ldots, r$$

$\Lambda_r(\underline{b})$ is the approximation to the integral which is sought. Thus, the fundamental statement of the variational problem gives rise to the discrete forward dynamic programming formulation.

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VITA

Charles Abernathy Sparrow was born in Charlotte, N. C. on January 17, 1941. After graduation from West Mecklenburg High School, he attended North Carolina State University in Raleigh, N. C., where he graduated in June, 1962 with a Bachelor of Science degree in Nuclear Engineering. While an undergraduate, he was active in several campus organizations and was inducted into Phi Eta Sigma, Order of 30 & 3, and Blue Key honorary societies.

Upon graduation from North Carolina State he accepted employment with the Lockheed-Georgia Company where he rose to the position of Senior Mathematician. Beginning part-time graduate study while at Lockheed, Mr. Sparrow received the Master of Science in Nuclear Science degree in June, 1969. While continuing to pursue graduate work in the School of Nuclear Engineering, he assumed employment with the Rich Electronic Computer Center where he was principal investigator for a variety of research projects. From July 1974 to September 1976 he served as a Graduate Research Assistant and Research Associate in the Schools of Nuclear Engineering and Aerospace Engineering.

Mr. Sparrow is an Associate Member of Sigma Xi and has served as a reviewer for Computing Reviews and as technical referee for the National Computer Conference 76. He is married to the former Martha Whitehead Cardwell of Oak Ridge, Tennessee. They are parents of a son, Charles, Jr., and a daughter, Jennie.

Mr. Sparrow is an Assistant Professor of Nuclear Engineering at Mississippi State University in Starkville, Mississippi, where he and his family reside.