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A handwritten signature in black ink, written over a horizontal line. The signature is cursive and appears to be "W. J. ...".

3/17/65

b

JUDGMENT ESTIMATES OF THE MOMENTS OF
HYPOTHETICAL PERFORMANCE TIME DISTRIBUTIONS

A THESIS

Presented to

The Faculty of the Graduate Division

by

Edward Grady Rodgers

In Partial Fulfillment
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JUDGMENT ESTIMATES OF THE MOMENTS OF
HYPOTHETICAL PERFORMANCE TIME DISTRIBUTIONS

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SUMMARY

One objective of this study was to show whether estimates of certain percentage points of various hypothetical time distributions, rather than their end points, will lead to more accurate estimates of the moments of the distributions.

Another purpose was to provide an improved basis for the calculation of the moments of performance time distributions as used in Program Evaluation and Review Technique (PERT).

The PERT equations are approximations of the mean and variance of a beta distribution with a range of six standard deviations. It was the assumption of the beta distribution, together with the estimates of the end points used in the PERT equations, that was questioned in this study.

The problem was approached in two steps. The first step challenged the use of the beta distribution. The range in standard deviations was noted at the zero and 100 percentiles and at various intermediate percentile points along five distributions. The results were compared to see if at some percentile the ranges for all the distributions were approximately the same for all positions of the mode of the distribution. If so, at this percentile the assumption of shape of the distribution is not important.

The second step challenged the use of the end point estimates in the PERT equations. An experiment was used to test an individual's ability to estimate the mean, mode, end points, 5 per cent points, and 10 per cent points along a distribution known only to the investigator.

The results of the experiment were analyzed to see if the subjects participating in the test submitted estimates which yielded significantly more accurate calculations of the moments of the distributions than with the normal PERT estimates.

The experiment was in the form of a test which was given to individuals with varying degrees of knowledge of PERT.

Results of comparing different distributions indicated that at the end points the range in standard deviations varies considerably from one distribution to another. Thus, if the activity duration times in PERT are not beta-distributed, incorrect calculations of the moments may be obtained from the PERT equations.

At the 5 and 10 per cent points the range in standard deviations for all the distributions studied were almost identical for all positions of the mode. Thus, the PERT equations were changed to accept the 5 and 10 per cent points for use in the experiment.

Results of the experiment indicated that the means calculated from estimates of the end points, 5 per cent points, and 10 per cent points were identical. Each test subject group performed equally as well in submitting estimates of the mean.

The 5 per cent method was significantly more accurate in calculating the variance than the usual PERT method or the 10 per cent method. As might be expected, the PERT trained subjects and technically trained subjects submitted more accurate estimates than the non-technically trained subjects.

Thus, the author suggested that the following equations be used to calculate the moments of the activity time distributions in PERT.

$$\text{Mean, } t_e = (a_{.05} + 4m + b_{.95})/6.0$$

$$\text{Variance, } V_e = (b_{.95} - a_{.05})^2/10.2$$

where $a_{.05}$ = the activity time which would be improved upon only
5 per cent of the time

m = the most likely activity time

$b_{.95}$ = the activity time which would be exceeded only 5
per cent of the time

CHAPTER I

INTRODUCTION

Objective

One objective of this study is to show whether estimates of certain percentiles of various hypothetical performance time distributions, rather than their end points, will lead to more accurate estimates of the moments of the distributions. Another purpose is to provide an improved basis for calculating the moments of performance time distributions as used in Program Evaluation and Review Technique (PERT).

Background

In recent years several scheduling techniques based on network models have been developed to aid in planning and controlling complex projects. One of the most widely used techniques is PERT. PERT was designed specifically as an aid in the development of the Polaris missile system by the Bureau of Naval Weapons of the U. S. Navy in 1958.

Since then PERT has been applied to numerous military and non-military projects.

In applying PERT, the project is depicted as a network of activities which show definite inter-dependence relationships. The operations that make up the project may be broken into as many activities as is desired, so long as each activity has a definite beginning and ending point.

Definition of PERT

PERT is a management control system based on a network model which considers the statistical treatment of uncertainty in activity performance times (1). The distributions of the activity performance times are hypothetical since no statistical sampling of data is involved. Instead, they are developed from only three points in the distributions (see Figure 1).

These three points are obtained for each activity in the form of estimates by an individual who is responsible for, or who best understands the performance of the activity (2):

a = shortest possible activity duration time

m = most likely activity duration time

b = longest possible activity duration time

These times are based on definite assumptions of the personnel and equipment employed to perform the activity.

From these estimates of a, m, and b estimates of the mean, t_e , and variance, V_e , of the distributions are determined as follows (3):

$$t_e = (a + 4m + b)/6 \quad (1)$$

$$V_e = (b - a)^2/36 \quad (2)$$

These estimates of the mean and variance for each activity in the network are then combined, using the Central Limit Theorem, to obtain the mean and variance of the overall project duration time. From these values

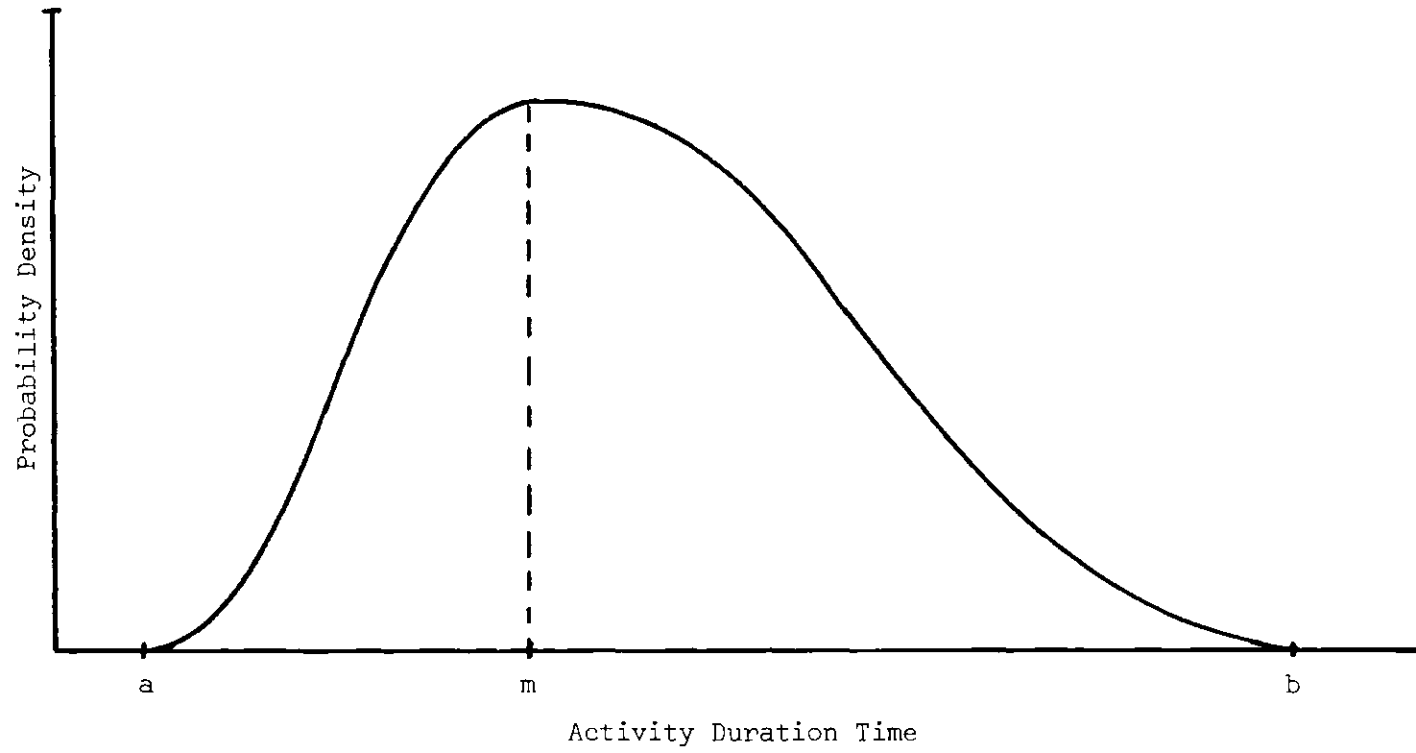


Figure 1. Activity Time Distribution Curve.

the probability of meeting a specific project duration time can be determined (4).

Statement of the Problem

One of the basic assumptions of PERT is that the activity duration times are beta-distributed (1), (5). It is further assumed that the beta distributions for these times have a range of six standard deviations (6). Equations (1) and (2) are approximations of the mean and variance of a beta distribution with a range of six standard deviations. It is the assumption of the beta distribution, together with the estimates of the end points a and b in Equations (1) and (2), that is questioned in this study.

Thus, the problem is to see if calculations made by using percentile estimates are different from the usual end point estimates.

CHAPTER II

REVIEW OF THE LITERATURE

A review of the literature on PERT yielded several sources of criticism of the PERT assumptions.

Charles E. Clark (2) in an article submitted to clarify statements in reference (7) comments on the lack of soundness in the PERT assumptions. He feels that reference (1) is somewhat misleading when it mentions beta-distributed activity times and then calculates moments of the distributions with only beta-approximation equations. Clark also feels that he has no information on the distribution of activity times and he does not suggest that the beta or any other distribution is appropriate.

Frank E. Grubbs (8) indicates his concern not only for the basis of the PERT assumptions, but also for the subjective nature of the three PERT estimates.

Grubbs shows that Equations (1) and (2), approximations of Equations (3) and (4), are valid only if $\alpha + \gamma = 4$ or $\alpha + \gamma = 6$ in the following equations for the mean and variance of a beta distribution:

$$\text{Mean} = a + (b-a)[(\alpha+1)/(\alpha+\gamma+2)] \quad (3)$$

$$\text{Variance} = (b-a)^2(\alpha+1)(\gamma+1)/[(\alpha+\gamma+3)(\alpha+\gamma+2)^2] \quad (4)$$

The conditions for α and γ can only be satisfied as follows:

1. $\alpha = 2 + \sqrt{2}$, $\gamma = 2 - \sqrt{2}$
2. $\alpha = 2 - \sqrt{2}$, $\gamma = 2 + \sqrt{2}$
3. $\alpha = \gamma = 3$

Thus Grubbs shows that beside the end points, the PERT assumptions restrict the activity times to only three possible beta distributions.

Grubbs also feels that since a , m , and b are subjective estimates, they may not necessarily fit the actual time distribution in the ordinary statistical sampling sense. He states that there could possibly be no connection between the estimates and the true distribution. Furthermore, he states that even if we were dealing with a known sample, estimating end points is a tricky and dangerous business.

MacCrimmon and Ryavec (5), in their discussion on the possible errors introduced by the PERT assumptions, state:

The true distribution of an activity and its mean and standard deviation are not known. . . . If the actual activity distribution possesses the aforementioned three properties (i.e., unimodality, continuity, and two non-negative abscissa intercepts), then the beta approximation to the distribution is at least correct with regard to its general shape. Different distributions, which possess these properties, however, could well have very different means and standard deviations; and hence--at least theoretically--an imprecise knowledge of the actual activity distribution could contribute significantly to any over-all error between the PERT calculated mean and standard deviation of an activity and its actual mean and standard deviation.

Moder and Phillips (9) suggest estimating the upper and lower ten percentage points rather than the end points of the hypothetical time distributions. They feel that it is unrealistic to ask a person, based on his experience, to estimate the end points (the ultimate limits) of the distributions since theoretically these times could never have been experienced. They also feel that between certain percentage points

of several distributions there would be approximately the same number of standard deviations, whereas between the end points of the same distributions the number of standard deviations would vary considerably.

Thus, there is criticism of

1. The assumption that the beta distribution approximates the true distribution of the activity times.
2. The subjective nature of the three PERT estimates.
3. The calculations used to estimate the moments of the activity time distributions.

These criticisms are the basis for this thesis.

CHAPTER III

PROCEDURE

The problem was approached in two steps. The first step challenged the assumption that the activity times are beta-distributed. The range in standard deviations of several distributions were noted at the zero and 100 percentiles and at various other percentiles. These ranges were calculated for various modal values from the left end point to the right end point. The results were compared to see if at some percentile the ranges for the distributions tested were approximately the same for varying modal values. If the ranges were the same, then at this percentile the assumption of the shape of the distribution is not important.

For example, suppose a triangular distribution is studied with a and b at the 25 percentile points (Figure 2). As m varies from left to right the range in standard deviations between the 25 and 75 percentile points changes. In Figure 2 the range decreases from 1.55σ with m at the left end point to 1.45σ with m at $2/5$ the distance from the left end point to the right end point. These calculations are from the Appendix A, Table 37.

In a similar manner the ranges for other distributions were calculated and compared. The distributions studied were the beta, negative exponential, normal, triangular, and uniform distributions.

The second step challenged the use of the end point estimates in the calculation of the mean and variance of the hypothetical performance time distributions.

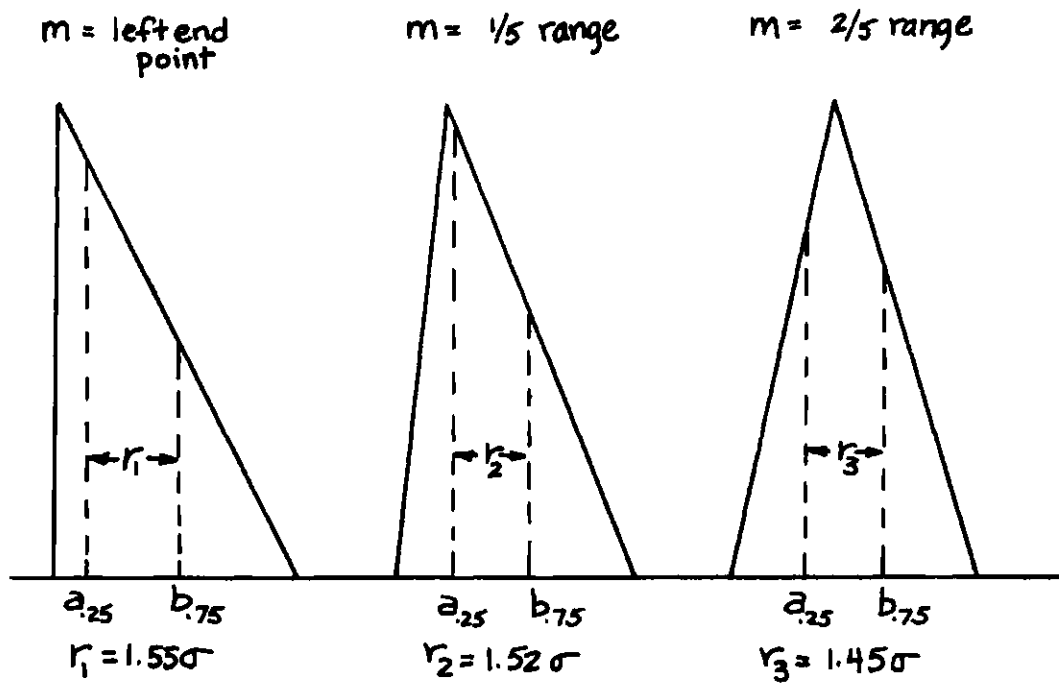


Figure 2. Range-Mode Relationship for a Triangular Distribution.

An experiment was used to test an individual's ability to estimate the mean, mode, end points, and certain percentiles of a hypothetical time distribution known only to the author. The subjects were given a random sample of points from the distribution to simulate their experience. The results of the experiment were analyzed to see if the subjects participating submitted estimates which yielded significantly different calculations of the moments of the distributions than with the usual PERT estimates.

The experiment was in the form of a test given to individuals with varying degrees of knowledge of PERT.

Description of the Experiment

The basic premise of the experiment was that estimation is based upon one's experience; that is, the estimator mentally divides an activity into sub-activities in which he has had experience.

For the experiment suppose the activity estimated was activity (A)-(B) in Figure 3.



Figure 3. Network Diagram for Experiment Activity.

Suppose further that activity (A)-(B) was broken down into sub-activities as follows (Figure 4):

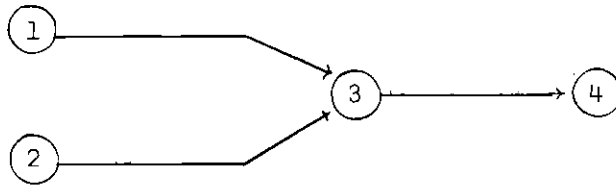


Figure 4. Network Diagram for Experiment Sub-Activities.

And suppose the past performance times for these sub-activities are shown in Table 1.

Table 1. Experienced Times for the Sub-Activities Which Constitute Activity (A) - (B)

Sub-Activity (1) - (3)	Sub-Activity (2) - (3)	Sub-Activity (3) - (4)
X_1	Y_1	Z_1
X_2	Y_2	Z_2
.	.	.
.	.	.
.	.	.
X_{n_1}	Y_{n_2}	Z_{n_3}

By referring to this "experience," the subject taking the test was asked to make the following four sets of estimates for the activity (A) - (B):

1. Estimate of the mean time for (A) - (B)
2. Estimate of a, m, and b (the usual PERT estimates)

3. Estimate of a , m and b with $a = \rho$ percentile and $b = (100 - \rho)$ percentile
4. Estimate of a , m , and b with $a = \rho'$ percentile and $b = (100 - \rho')$ percentile

where ρ and ρ' are the percentage points chosen in step one of the approach to the problem for which the distributions have almost identical ranges.

The subject was asked to make these estimates for three sets of "history" for the sub-activities of activity (A) - (B). The sets of history were as follows:

1. Set one consists of data which were the same for all subjects taking the test.
2. Set two consists of data which were unique for each subject taking the test.
3. Set three was identical to set one but each time differed by a constant and the order of the times was changed. This set was used to test the consistency of the subjects taking the test.

The history for the sub-activities was obtained by randomly sampling beta distributions with arbitrary parameters known only to the author.

The parameters for the distributions of the sub-activities were as follows (Table 2):

Table 2. Parameters for Sub-Activity Distributions

Sub-Activity	Mean	Standard Deviation
(1) - (3)	12	2
(2) - (3)	10	1
(3) - (4)	5	1

The beta distributions were made unique by requiring the following of each one (9):

1. The range of each distribution was six standard deviations.
2. The mode of each distribution was approximately one-third the distance from the lower boundary to the upper boundary. The exact location was chosen so that parameters α and β of the beta distribution were numbers for which the cumulative beta distribution have been tabled. This was necessary for convenience in sampling.

In order to determine the true mean and variance of the data in set two, which is different for each subject taking the test, Monte Carlo methods were applied directly to the data sheets used in the experiment.

For example, suppose one of the data sheets had the following sub-activity "history" for set two (Table 3).

Since activities (1) - (3) and (2) - (3) are performed simultaneously (see Figure 4), the shorter time was ignored in the calculation of the total time for activity (A) - (B).

Table 3. Example Sub-Activity Times for Set Two

Sub-Activity (1) - (3)	Sub-Activity (2) - (3)	Sub-Activity (3) - (4)
12	9	4
8	12	6
13	8	3
17	11	8
9	15	7
14	13	6
15	12	4
12		5
		5

The results of applying Monte Carlo methods to the times in Table 3 are shown in Table 4. The numbers in parentheses were considered in the calculation of the time for activity (A) - (B).

Thus, seven times for activity (A) - (B) were obtained from each data sheet used in the experiment. Since 100 subjects took the test, 700 times were used to calculate the mean and variance of (A) - (B). This should be a sufficiently accurate estimate of the true value of the mean and variance of the activity (A) - (B).

Table 4. Monte Carlo Methods Applied to Example
Sub-Activity Times for Set Two

Sub-Activity (1) - (3)	Sub-Activity (2) - (3)	Sub-Activity (3) - (4)	Total Time for (A) - (B)
(12)	9	(4)	16
8	(12)	(6)	18
(13)	8	(3)	16
(17)	11	(8)	25
9	(15)	(7)	22
(14)	13	(6)	20
(15)	12	(4)	19
12		5	
		5	

Procedure for Data Analysis

The data were analyzed to determine which of the four estimates yielded the best estimate of the true mean and variance of activity (A) - (B). The results were grouped as follows:

1. Subjects with PERT training
2. Subjects with technical background
3. First set of subjects with non-technical background
4. Second set of subjects with non-technical background

Within each of these classifications the results were grouped as follows:

1. Data which were the same for all subjects
2. Data which were unique for each subject
3. Data identical to 1, except different by a constant

Within each of these sets the results were grouped as follows:

1. For Mean
 - a. Estimate of the mean
 - b. t_e obtained by normal PERT estimates
 - c. t_e obtained by X percentile estimates
 - d. t_e obtained by Y percentile estimates
2. For Variance
 - a. V_e obtained by normal PERT estimates
 - b. V_e obtained by X percentile estimates
 - c. V_e obtained by Y percentile estimates

The analysis of variance technique was applied to determine if one method is significantly different from another. Interactions between methods were also studied.

CHAPTER IV

RESULTS

Comparison of Distributions

Step one of the procedure was to study the ranges of several distributions at various percentiles and at various positions of the mode. The distributions were studied at the following points:

1. Zero per cent points (end points): a, b
2. 0.1 per cent points: a_{.001}, b_{.999}
3. 1 Per cent points: a_{.01}, b_{.99}
4. 5 Per Cent points: a_{.05}, b_{.95}
5. 10 Per Cent points: a_{.1}, b_{.9}
6. 25 Per cent points: a_{.25}, b_{.75}

The ranges were calculated for the preceding points at the following positions of the mode for the following types of distributions:

1. Triangular distribution
 - a. m = left end point
 - b. m = 0.1 the range from the left end point
 - c. m = 0.2 the range from the left end point
 - d. m = 0.3 the range from the left end point
 - e. m = 0.4 the range from the left end point
 - f. m = 0.5 the range from the left end point
 - g. m = 0.6 the range from the left end point
 - h. m = 0.7 the range from the left end point

- i. $m = 0.8$ the range from the left end point
- j. $m = 0.9$ the range from the left end point
- k. $m =$ right end point

2. Beta distribution (these points were chosen for convenience in using tabled data).

- a. $m =$ left end point
- b. $m = 0.125$ the range from the left end point
- c. $m = 0.222$ the range from the left end point
- d. $m = 0.300$ the range from the left end point
- e. $m = 0.364$ the range from the left end point
- f. $m = 0.417$ the range from the left end point
- g. $m = 0.500$ the range from the left end point
- h. $m = 0.583$ the range from the left end point
- i. $m = 0.636$ the range from the left end point
- j. $m = 0.700$ the range from the left end point
- k. $m = 0.778$ the range from the left end point
- l. $m = 0.875$ the range from the left end point
- m. $m =$ right end point

3. Normal distribution.

The mode is at the midpoint.

4. Negative exponential distribution.

The mode is at the left end point.

5. Uniform distribution.

There is no mode for this distribution.

The calculations of the ranges for each of the distributions at the preceding percentiles and positions of the mode are found in Appendix A. Graphs of the results are shown in Figures 5 through 8 on the following pages.

One can see the reason for the criticism of the use of the end points in estimating the moments of the activity times by examining Figure 5, the comparison of ranges estimated from the end points of the distributions. The ranges for the distributions vary from 3.46σ for the uniform distribution, to 6.00σ for the beta distribution, not counting the fact that it would be infinite for the normal distribution. It is noted that the range for the triangular distribution varies from 4.24σ to 4.90σ as the mode changes. Thus, it is seen that at the end points the assumption of the shape of the distribution is very important.

The comparison of ranges estimated from the 0.1 per cent points is shown in Figure 6. The ranges for the distributions vary from 3.46σ for the uniform distribution to 6.20σ for the normal distribution. The range for the triangular distribution varies from 4.10σ to 4.68σ as the mode changes. There is still quite a difference in the ranges among the various distributions.

The comparison of ranges at the 1 per cent points are shown in Figure 7. These ranges vary from 3.39σ for the uniform distribution to 4.64σ for the normal distribution. The variation in the range decreases for the triangular distribution and increases for the beta distribution. The range for the triangular distribution varies from 3.79σ to 4.21σ , and the range for the beta distribution varies from 4.17σ to 4.30σ .

The comparison of ranges at the 5 per cent points is shown in

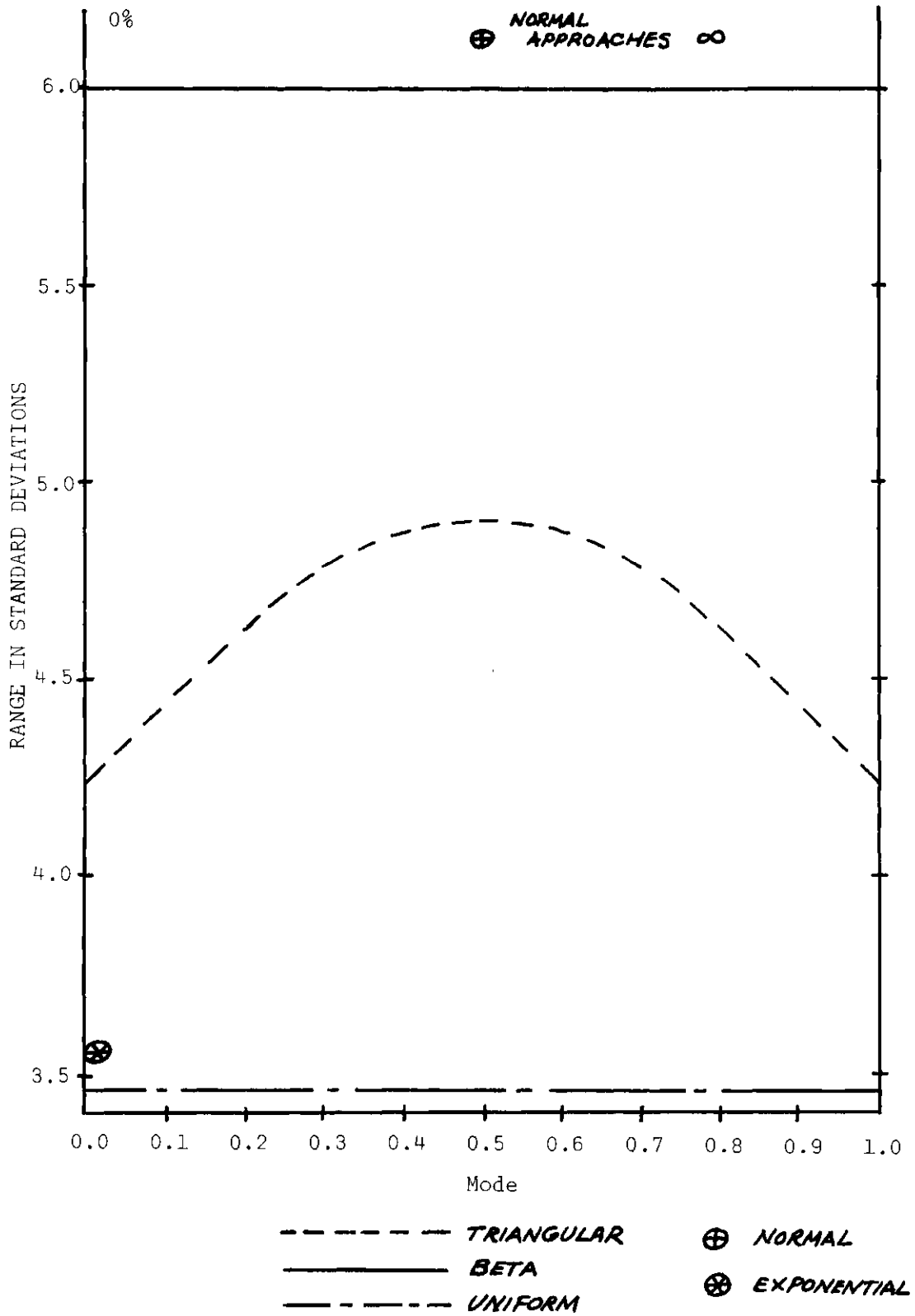


Figure 5. Range in Standard Deviations vs. Modal Positions for Zero Percentage Points of Five Distributions.

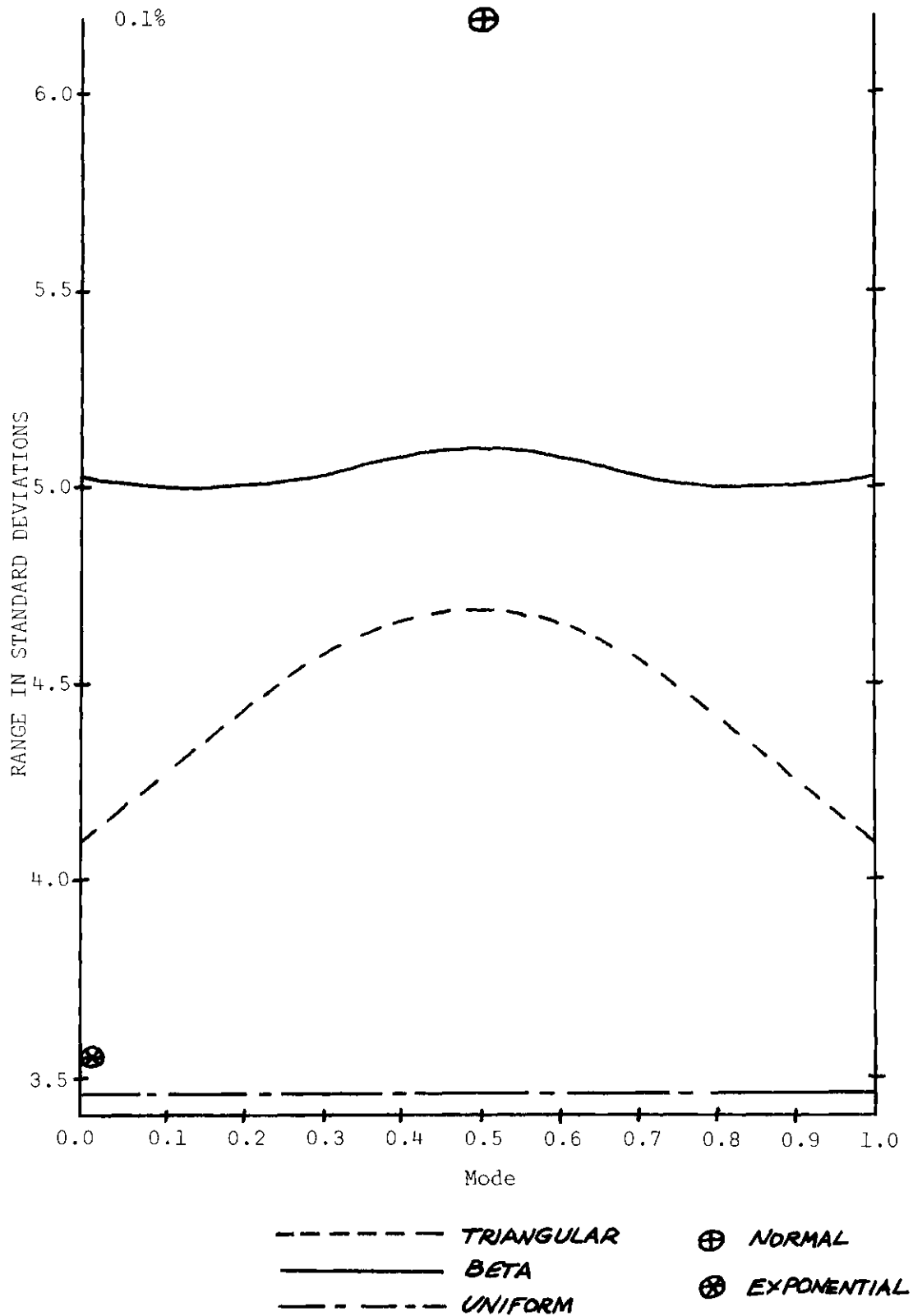


Figure 6. Range in Standard Deviations vs. Modal Positions for 0.1 Percentage Points of Five Distributions.

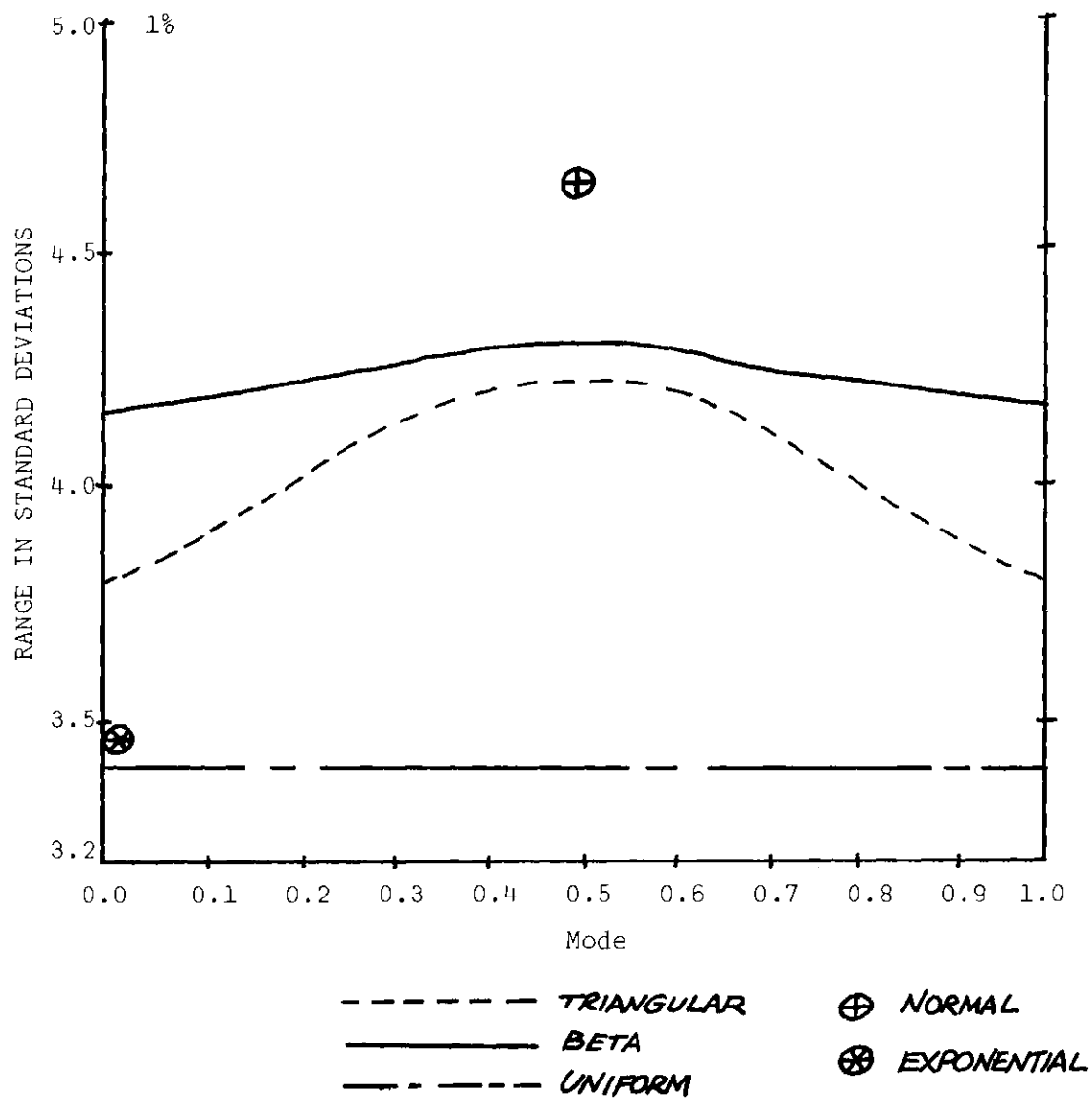


Figure 7. Range in Standard Deviations vs. Modal Positions for One Percentage Points of Five Distributions.

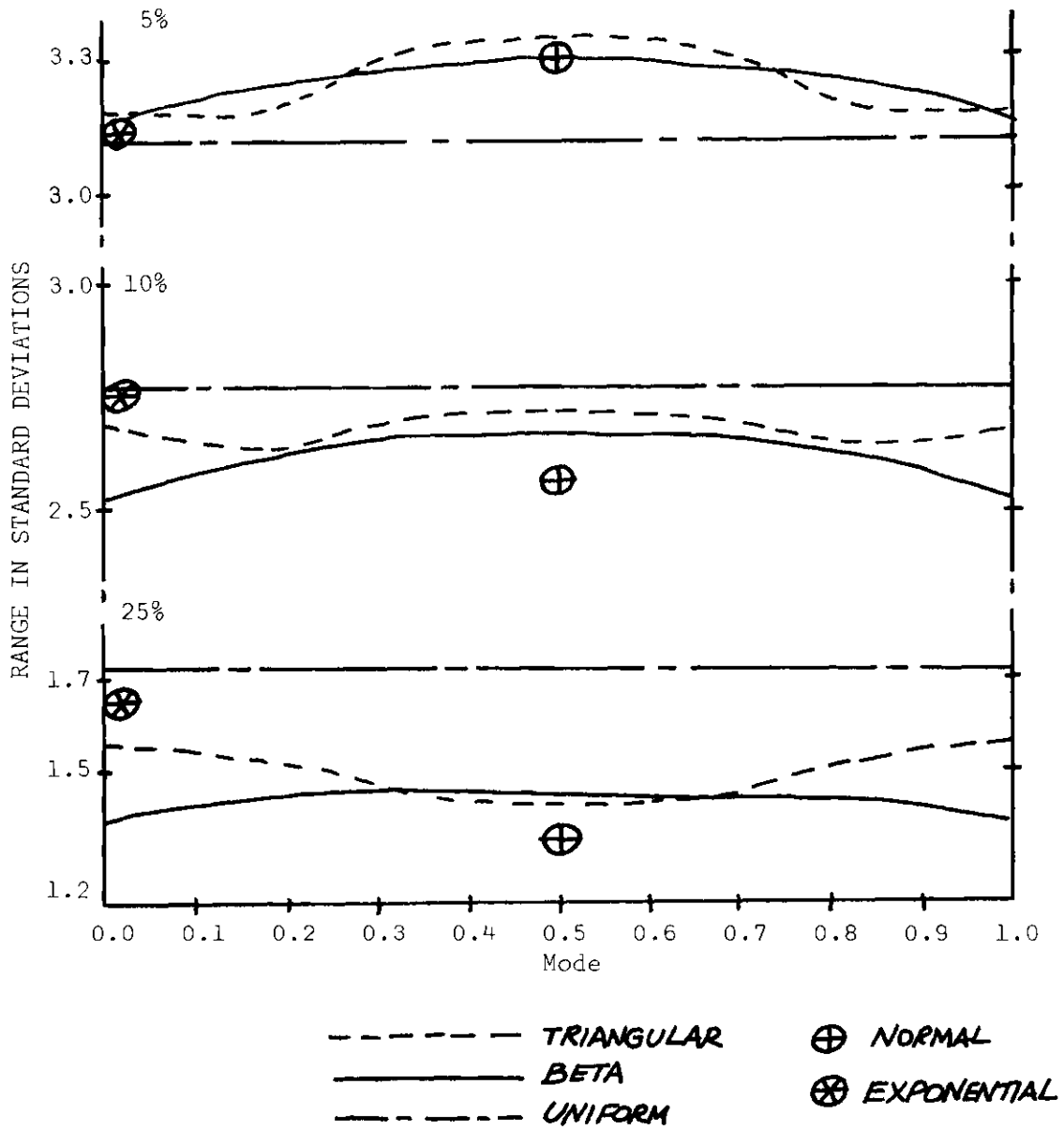


Figure 8. Range in Standard Deviations vs. Modal Positions for Various Percentage Points of Five Distributions.

Figure 8. The ranges for each of the distributions are almost identical. The ranges vary, from 3.12σ for the uniform distribution to 3.35σ for the triangular distribution. The range for the triangular distribution varies from 3.18σ to 3.35σ , and the range for the beta distribution varies from 3.17σ to 3.30σ . A range of 3.20σ is the average for the ranges for the observed distributions at the 5 per cent points.

The ranges of the distributions at the 10 per cent points (Figure 8) diverge slightly from the 5 per cent ranges. The ranges vary from 2.52σ for the beta distribution to 2.77σ for the uniform distribution. The range for the triangular distribution varies from 2.63σ to 2.72σ ; and the range for the beta distribution varies from 2.52σ to 2.66σ . A range of 2.70σ is the average for the ranges for all the observed distributions at the 10 per cent points.

The ranges for the distributions at the 25 per cent points (Figure 8) have a greater difference, from 0.67σ for the normal distribution to 1.73σ for the uniform distribution. The range for the triangular distribution varies from 1.44σ to 1.55σ , and the range for the beta distribution varies from 1.39σ to 1.45σ .

Thus, the 5 per cent points yield the least amount of variation in the ranges for the distributions studied. The 10 per cent points also yield a small amount of variation in the ranges. Thus, the 5 per cent and the 10 per cent points were used in the experiment in step two of the procedure.

Results of the Experiment

Step two of the procedure was to challenge the use of the end

point estimates in the calculation of the mean and variance of the hypothetical performance time distribution in PERT.

A test was devised to measure a person's ability to estimate the mean, mode, end points, 5 per cent points and 10 per cent points of a hypothetical performance time distribution. A sample of test is in Appendix B.

The subjects taking the test were divided into the following groups of 25 persons each:

1. PERT trained subjects--individuals who were experienced in the use of PERT.
2. Technically trained subjects--individuals who were not experienced in the use of PERT, but who had a sound background in statistics.
3. The first set of non-technically trained subjects--individuals with no background in statistics and no work experience.
4. The second set of non-technically trained subjects--individuals with no background in statistics, but with some work experience.

The source of subjects included employees at Management Science Atlanta, Atlanta, Georgia; employees at The General Tire and Rubber Company, Akron, Ohio; undergraduate and graduate students in the School of Industrial Engineering, Georgia Institute of Technology, Atlanta, Georgia; undergraduate students in the School of Business, University of Akron, Akron, Ohio; and other friends and associates of the author in Akron, Ohio.

The development of the data used in the test is shown in the Appendices. This development yielded the following observed values for

the mean and variance of the hypothetical distribution used in the test (Table 5):

Table 5. Actual Means and Variances by Data Group

Group	Mean	Variance
I	17.00	5.00
II	17.49	5.87
III	17.00	5.00
All Groups	17.16	5.29

In evaluating the test, the estimate of the end points and mode were substituted in Equations (1) and (2), the PERT calculations for the mean and variance of the distribution.

The estimate of the 5 per cent points and mode were substituted in the following variations of Equations (1) and (2):

$$t_e = (a_{.05} + 4m + b_{.95})/6.0 \quad (5)$$

$$V_e = (b_{.95} - a_{.05})^2/10.2 \quad (6)$$

The value 10.2 in Equation (6) was obtained by squaring 3.2, the average range in standard deviations for the observed distributions at the 5 per cent points.

The estimate of the 10 per cent points and mode were substituted into the following equations, also variations of Equations (1) and (2):

$$t_e = (a_{.1} + 4m + b_{.9})/6.0 \quad (7)$$

$$V_e = (b_{.9} - a_{.1})^2/7.3 \quad (8)$$

The value 7.3 in Equation (8) was obtained by squaring 2.7, the average range in standard deviations for the observed distributions at the 10 per cent points.

The results of the calculations are shown in Appendix C.

Comparison of Histograms for the Mean Calculations

Results of the mean calculations are shown in Tables 32 through 36 in the Appendices. Histograms summarizing these results are shown in Figures 9 through 12 on the following pages.

The histograms of results from the CPM estimate of the mean is shown in Figure 9. The average of the estimated means appeared to be about 18.00, which is greater than the actual value of 17.16. The analysis of variance which follows tested the significance of this difference.

The CPM results shown in Table 36 in the Appendices is summarized in Figure 11. This graph of the cumulative frequency indicated that 80 per cent of the estimates submitted were between the values of 17.0 and 19.0.

The histograms of results from the normal PERT estimates are also

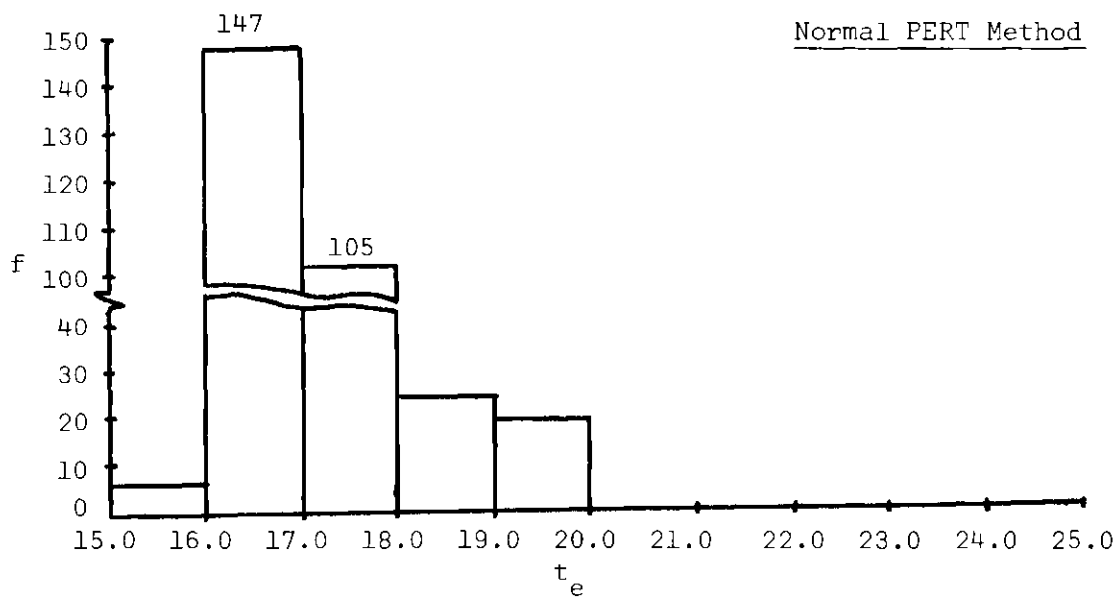
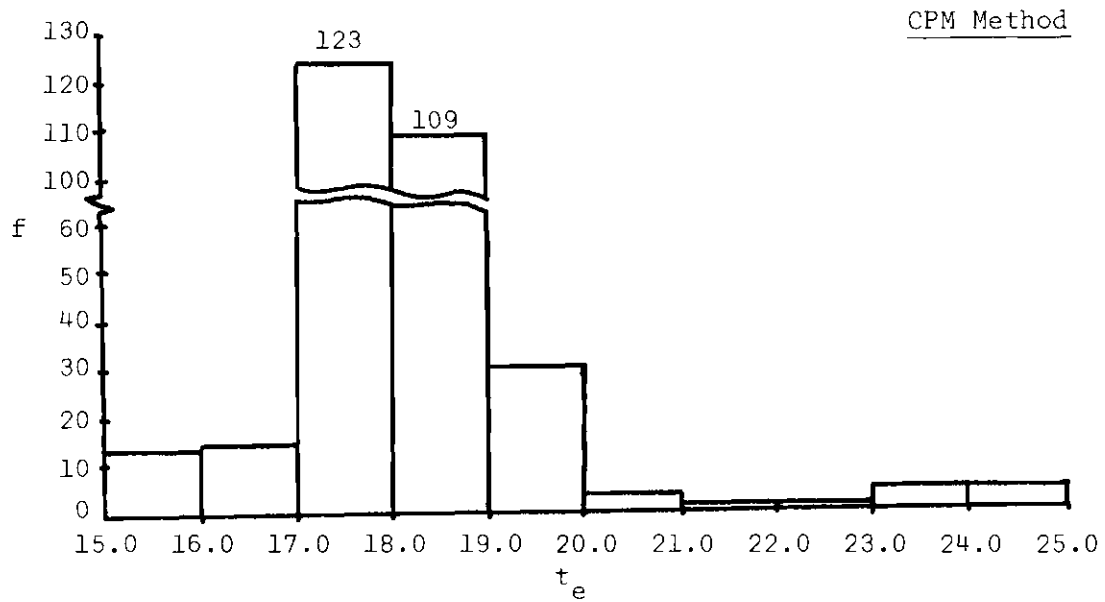


Figure 9. Estimated Mean Histograms--All Subjects and Groups--I.

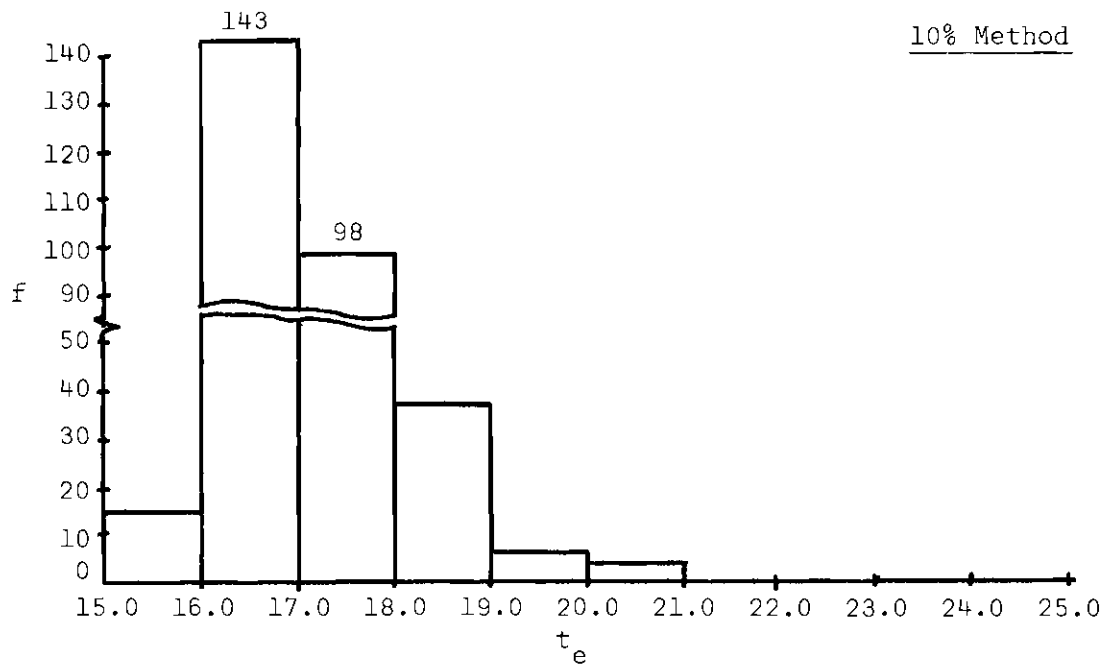
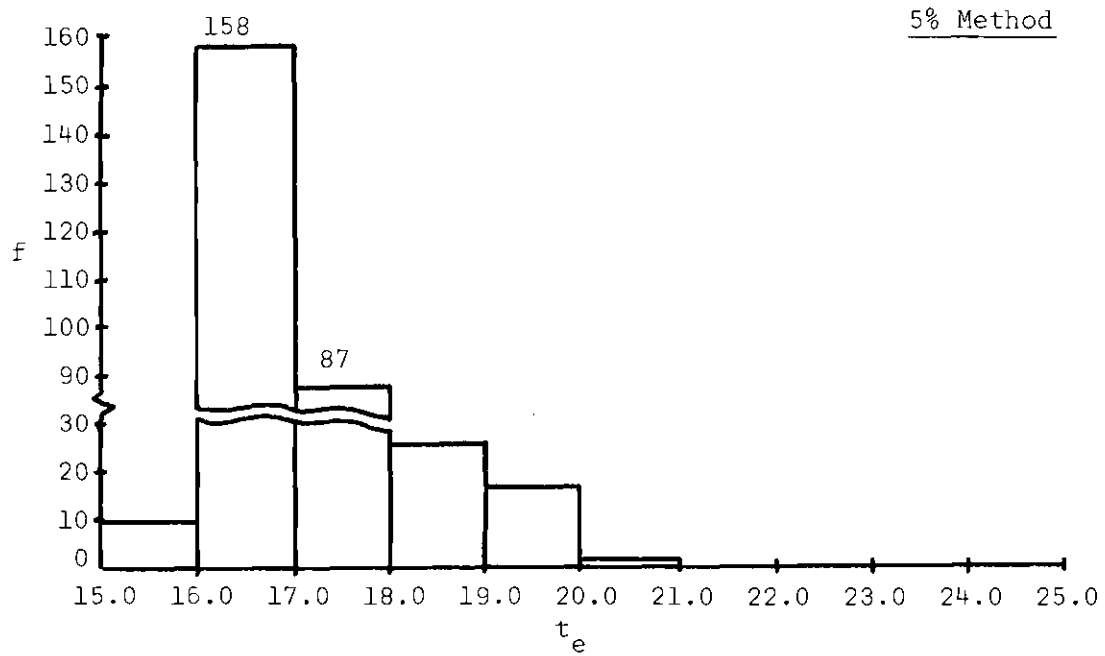


Figure 10. Estimated Mean Histograms--All Subjects and Groups--II.

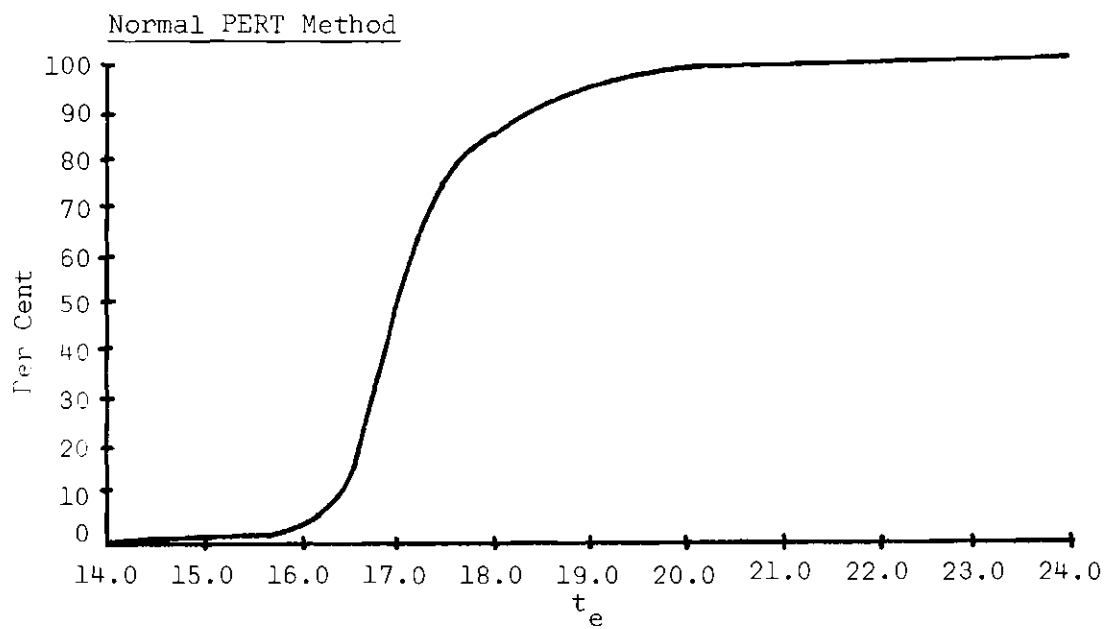
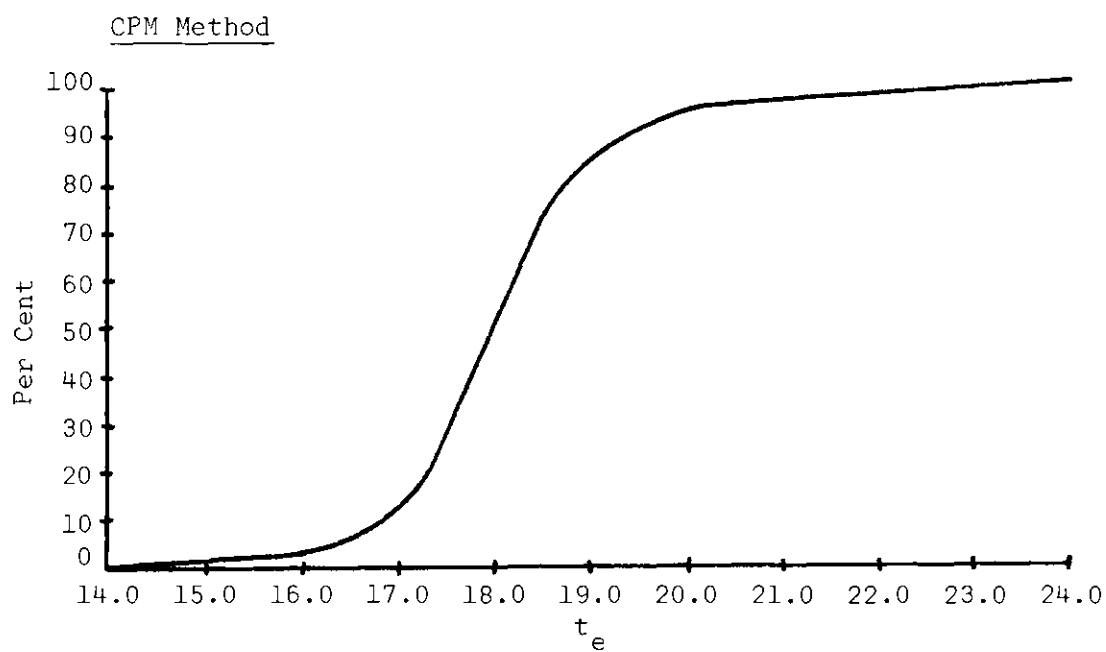


Figure 11. Cumulative Frequency Curves of Estimated Means--
All Subjects and Groups--I.

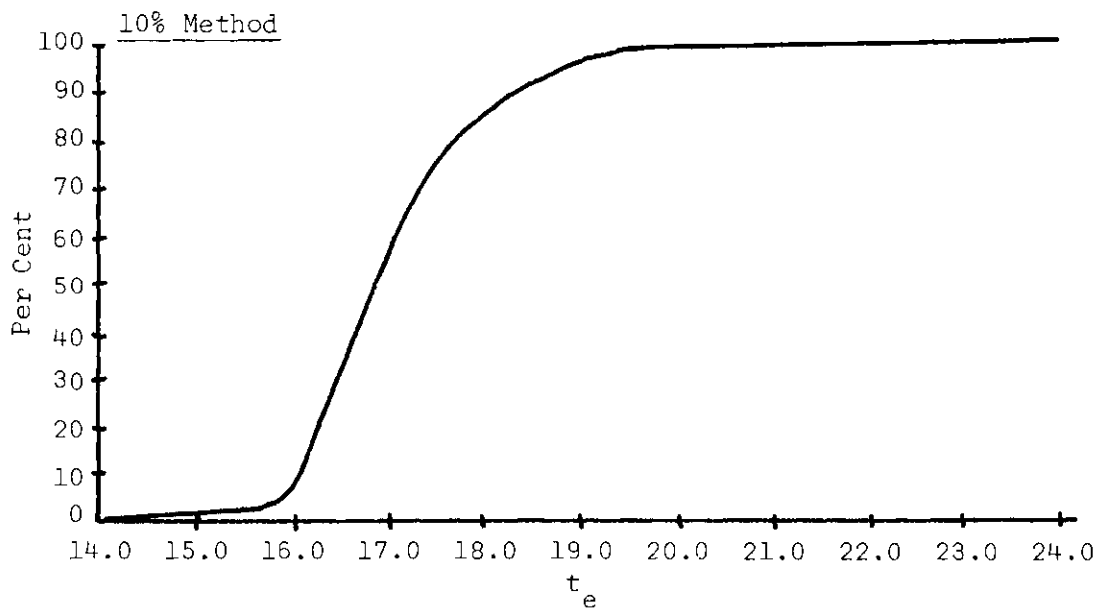
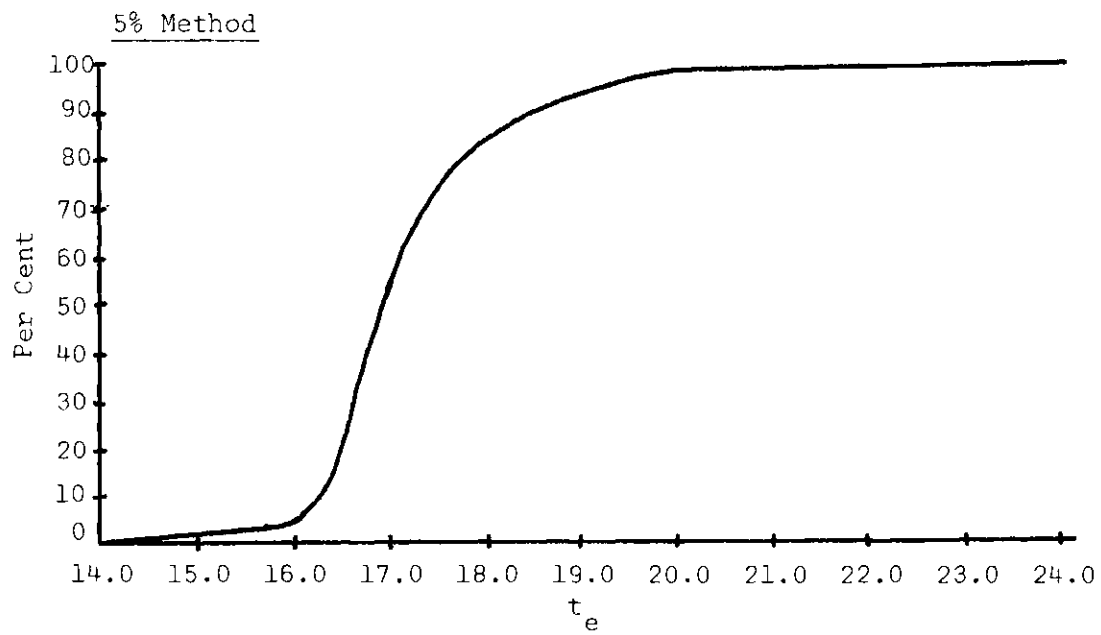


Figure 12. Cumulative Frequency Curves of Estimated Means--
All Subjects and Groups--II.

shown in Figure 9. Subjectively, the average appeared to be close to the true value of 17.16. The cumulative frequency graph in Figure 11 indicated that 80 per cent of the estimates calculated were between the values of 16.0 and 18.0.

The histograms of the results from the 5 per cent method is shown in Figure 10. Again subjectively, the average appeared to be about the same as the true value of 17.16. The cumulative frequency chart in Figure 12 indicated that 80 per cent of the estimates were between 16.0 and 18.0.

The histogram of the results from the 10 per cent method is also shown in Figure 10. The average also appeared to be about the same value as the actual mean. The cumulative frequency chart in Figure 12 also showed that 80 per cent of the estimates calculated were between 16.0 and 18.0.

Objective Analysis of the Mean Calculations--Estimated Means

The results of the analysis of variance test (10) on the estimated mean calculations are shown in Appendix D.

Table 6 on the following page summarizes the results.

Thus, there were no differences in the factors tested.

Comparison of Histograms for the Variance Calculations

Results of the variance calculations are shown in Tables 37 through 41 in the Appendices. Histograms summarizing these results are shown in Figures 13 through 22 on the following pages.

The histograms of results from the Present Method zero per cent, the normal PERT calculations, are shown in Figure 13. The average of the

Table 6. Results of the Analysis of Variance Test--
Means of Estimated Means

Source	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio	Significance Level	
					5%	1%
A	3	1.05	0.35	0.67	2.8	4.2
B	2	3.08	1.54	2.96	3.2	5.1
AB	6	1.77	0.29	0.55	2.3	3.2
C	3	3.62	1.21	2.33	2.8	4.2
AC	9	2.28	0.25	0.48	2.1	2.9
BC	6	0.55	0.09	0.02	2.3	3.2
ABC	18	4.51	0.25	0.48	1.9	2.4
Error	48	24.94	0.52			

estimated variances appeared to be about 2.50, which is less than the actual value of 5.29. The analysis of variance which follows tested the significance of this difference. The cumulative frequency curve for the present method 0% (Figure 14) indicated that 80 per cent of the calculated variances were between the values of 1.0 and 3.0.

The histogram of results from the 5 per cent method in Figure 13 indicated an average variance of about 4.5, still less than the true value of 5.29. The cumulative frequency curve in Figure 14 showed that 50 per cent of the calculated variances were between 1.0 and 3.0.

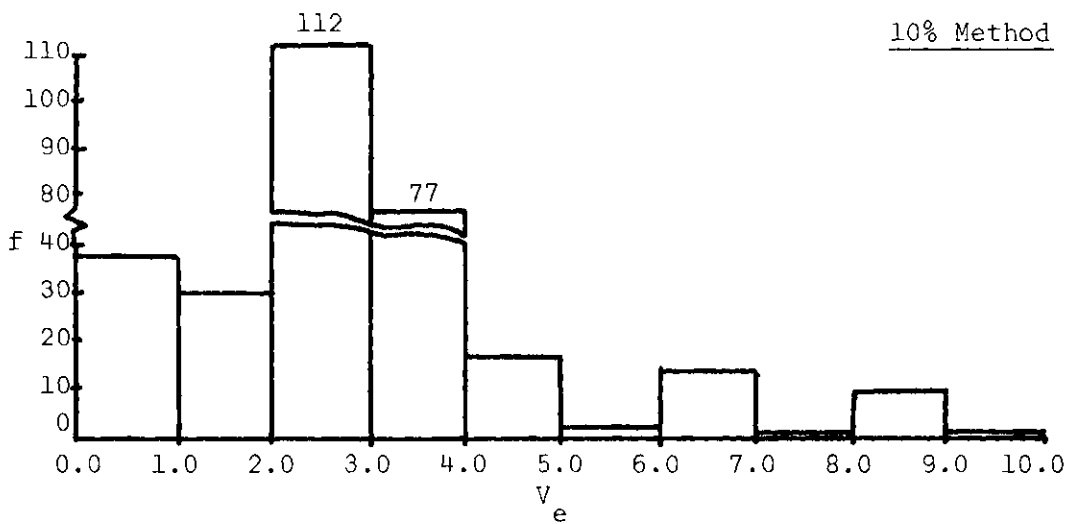
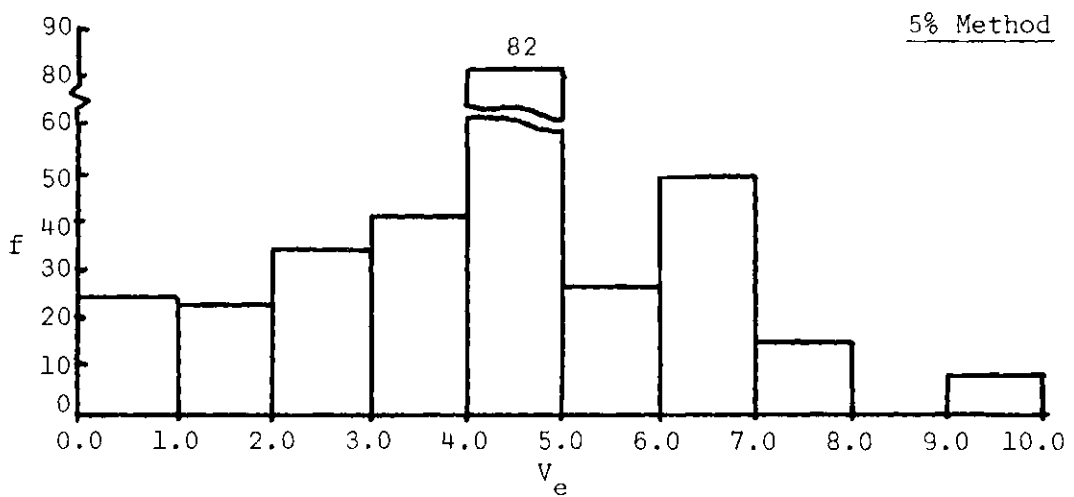
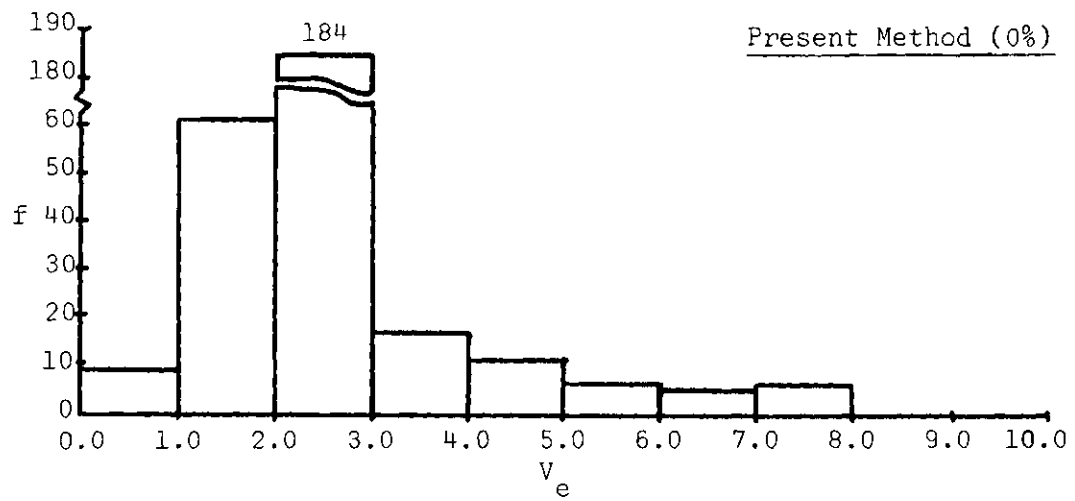


Figure 13. Estimated Variance Histograms--
All Subjects and Groups.

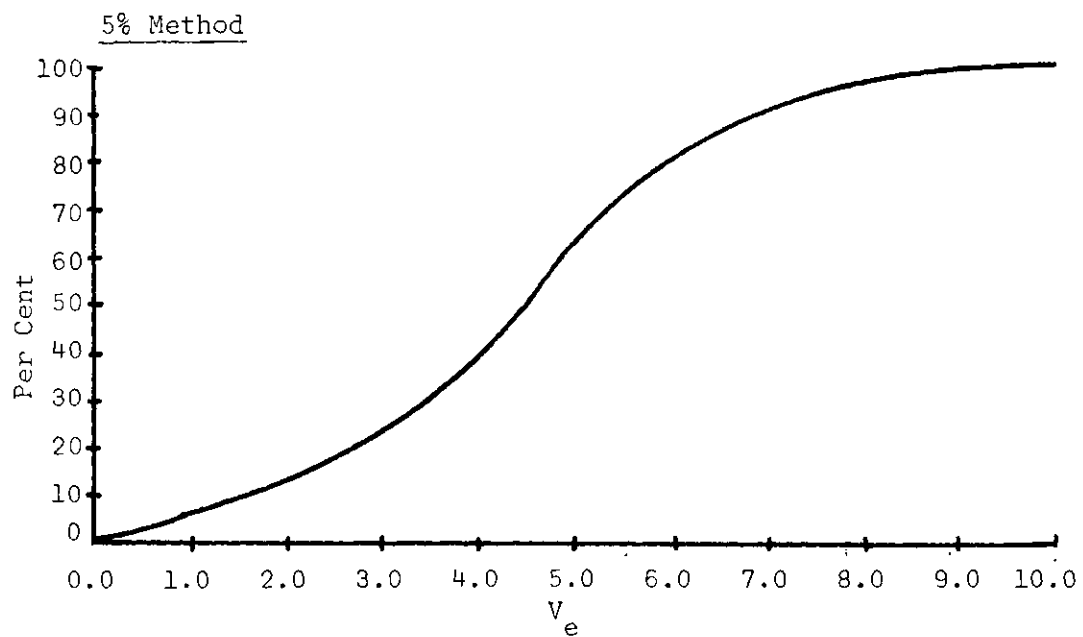
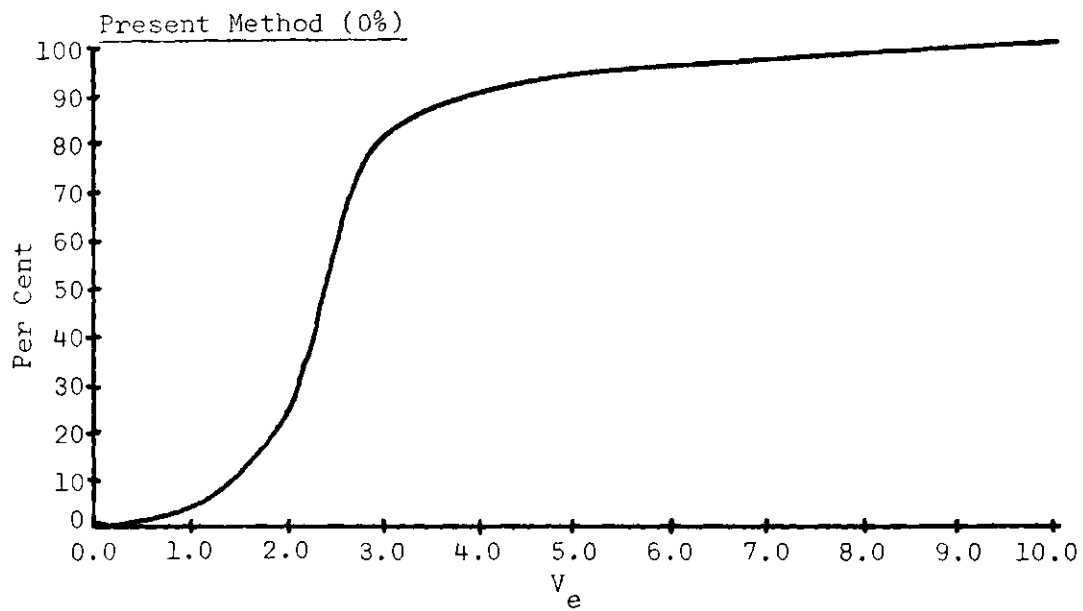


Figure 14. Cumulative Frequency Curves of Estimated Variances--
All Subjects and Groups--I.

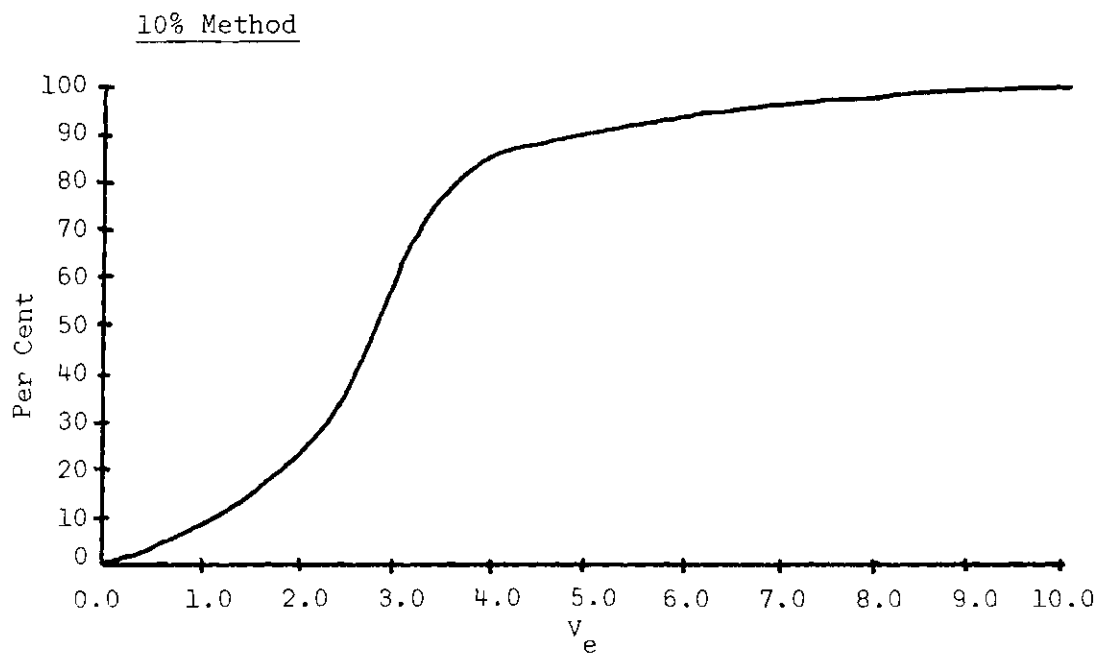


Figure 15. Cumulative Frequency Curves of Estimated Variances--
All Subjects and Groups--II.

The histogram of results from the 10 per cent method is shown in Figure 13. The average variance appeared to be 3.0, which is less than the actual value. The analysis of variance which follows tested this difference for significance. The cumulative frequency chart in Figure 15 showed that over 60 per cent of the calculated variances were between 2.0 and 4.0, and about 50 per cent were between 1.0 and 3.0.

The PERT trained subjects (Figure 16) were consistent using the zero per cent method, but the average variance was about 2.50 compared to the true value of 5.29. They were inconsistent using the 5 per cent method, but the average variance of 5.00 was close to the true value. In the 10 per cent method they were less consistent and the average variance was 3.00. These same results were found for the technically trained subjects (Figure 17).

Both groups of non-technically trained subjects (Figures 18 and 19) obtained an average variance of 2.0 for the zero per cent method. In the 5 per cent method they were inconsistent and the average variance was 3.5. They were also inconsistent using the 10 per cent method and the average variance was 2.5.

Subjectively each group appeared to be most accurate using the 5 per cent method, especially the PERT trained and technically trained subjects. The analysis of variance test that follows will statistically judge the apparent difference in methods.

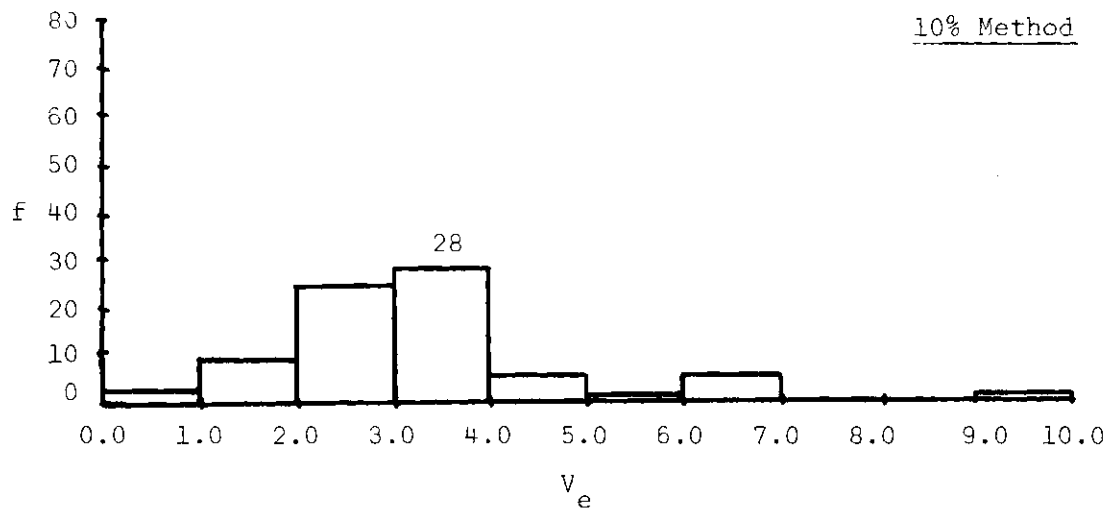
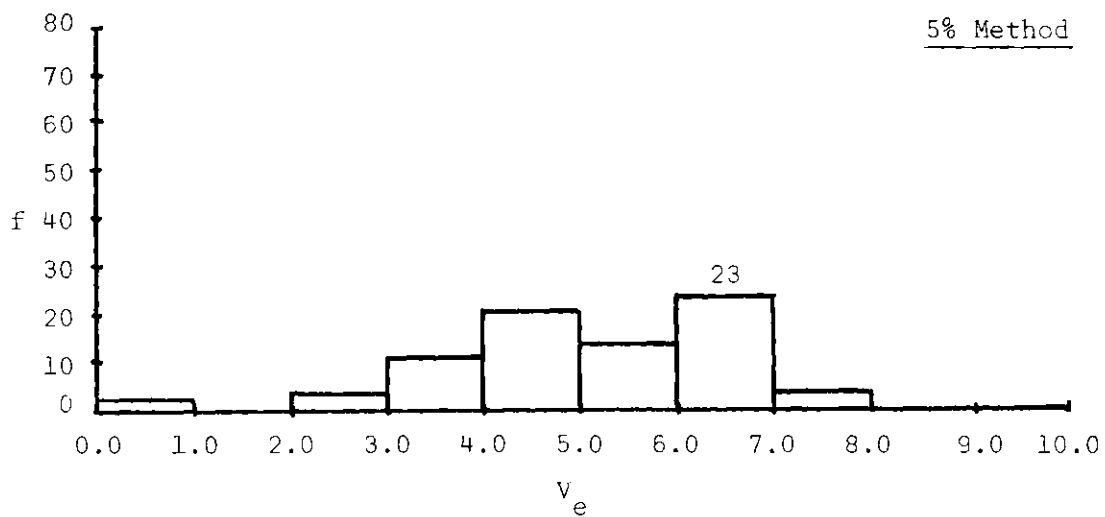
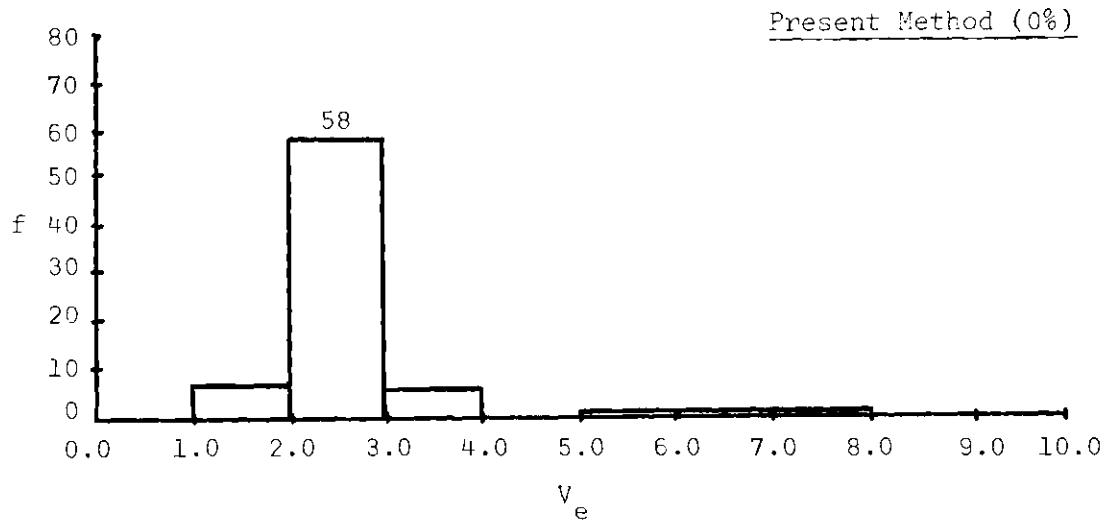


Figure 16. Estimated Variance Histograms--
PERT Trained Subjects

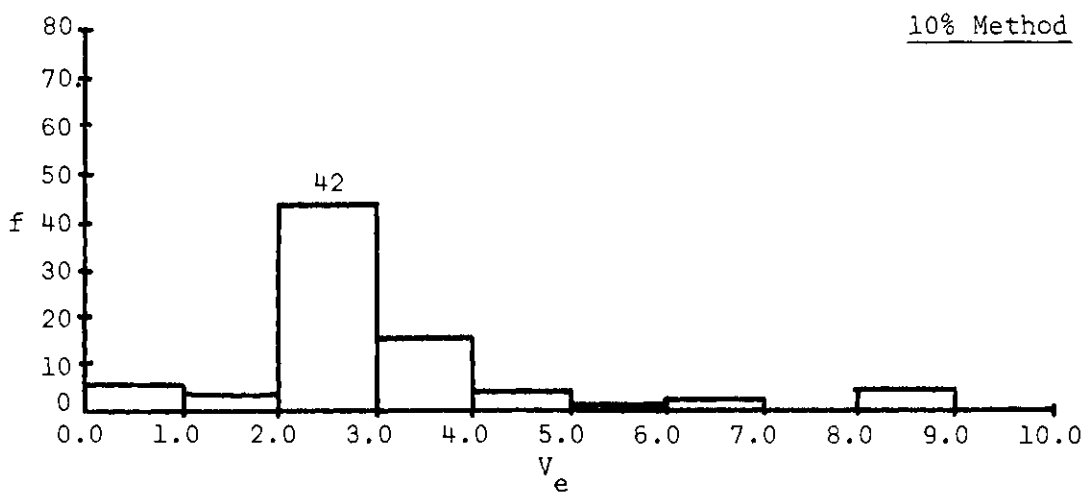
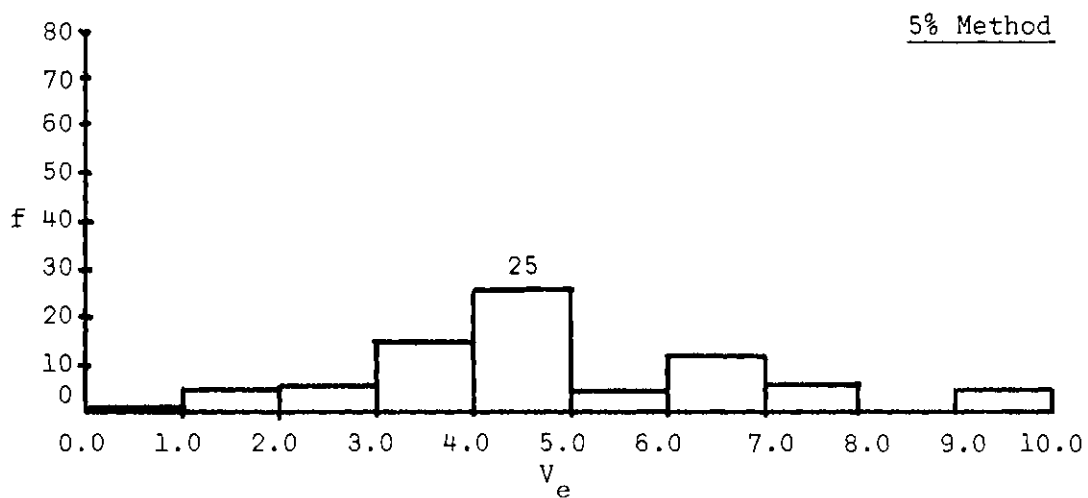
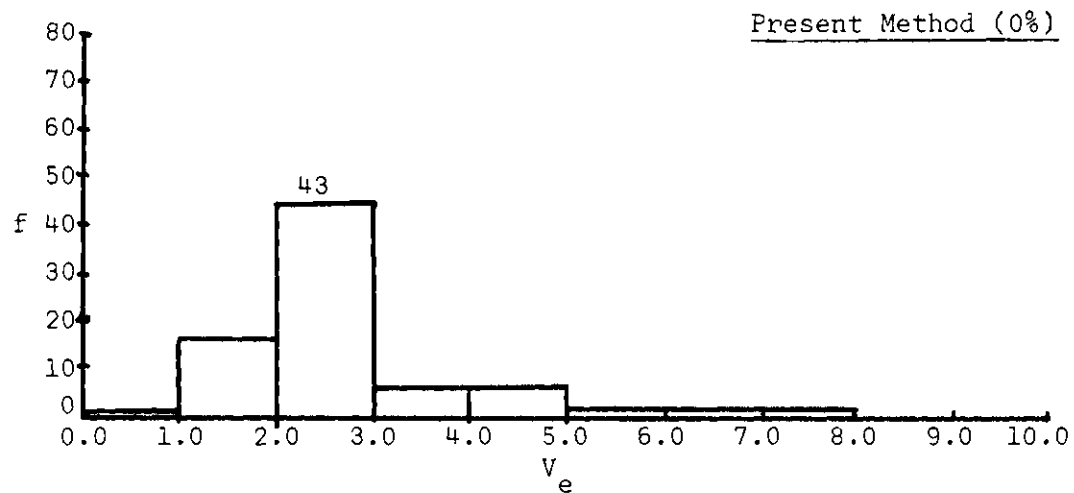


Figure 17. Estimated Variance Histograms--
Technically Trained Subjects.

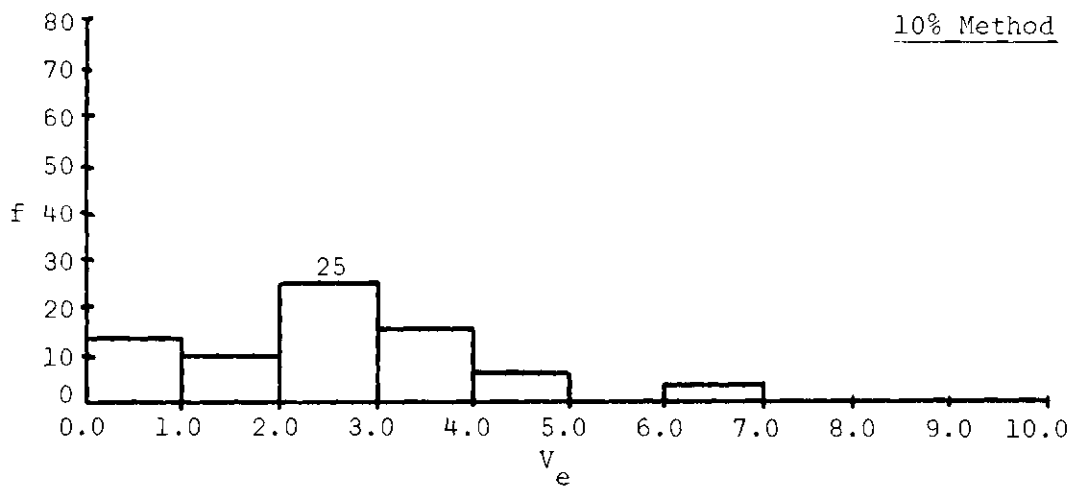
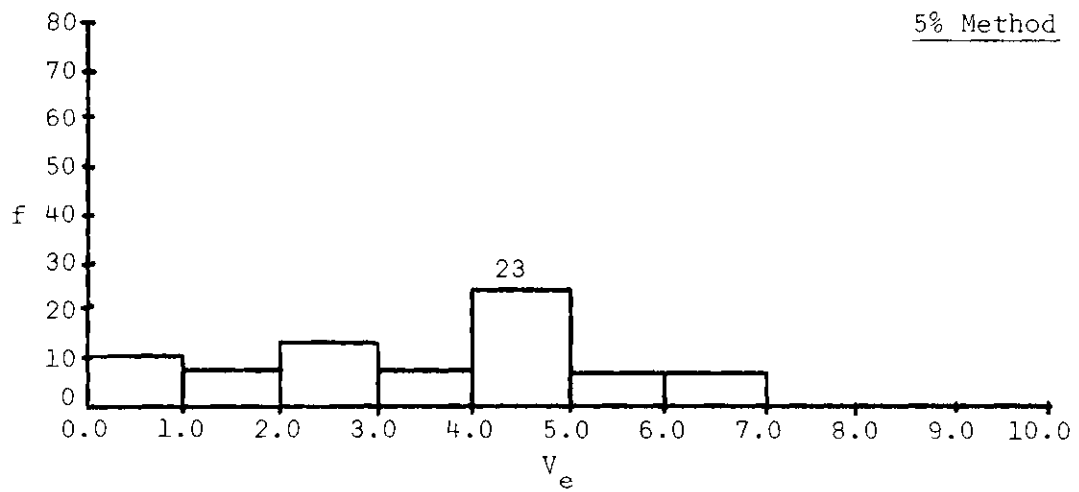
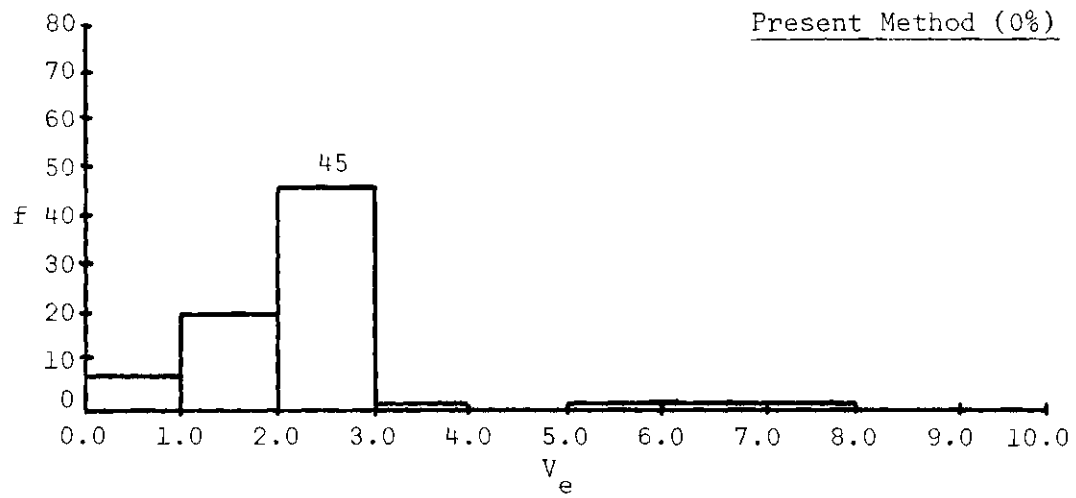


Figure 18. Estimated Variance Histograms--
Non-Technically Trained Subjects--A.

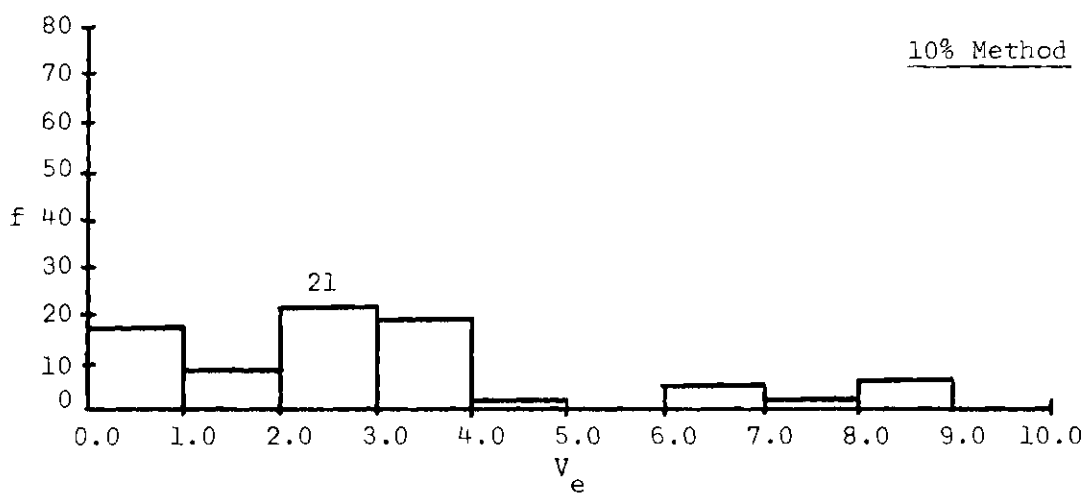
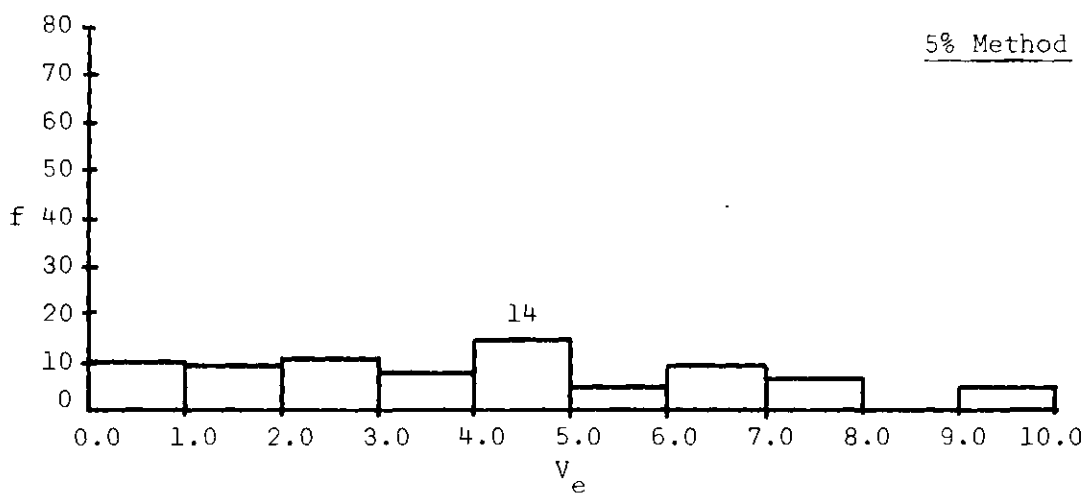
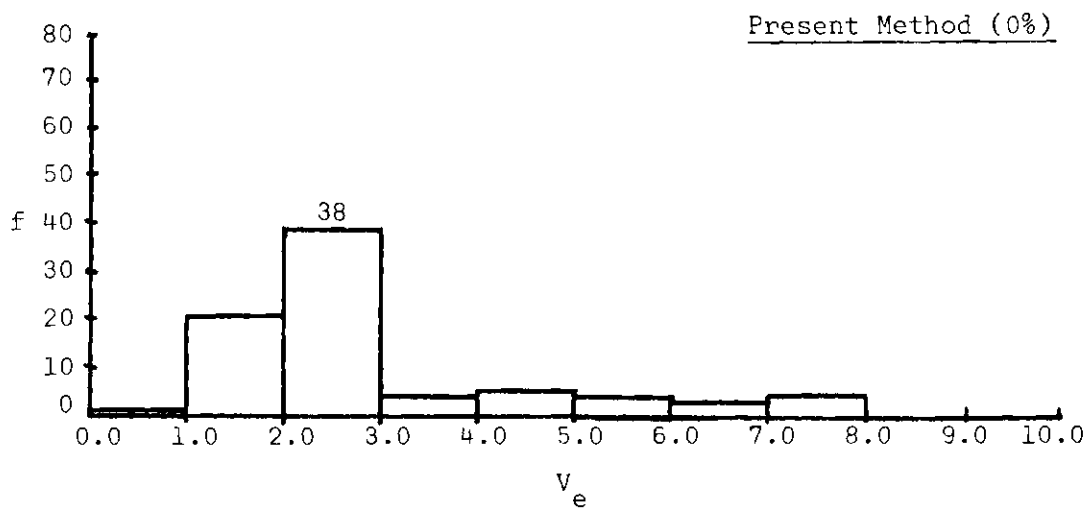


Figure 19. Estimated Variance Histograms--
Non-Technically Trained Subjects--B.

Objective Analysis of the Variance Calculations--

Estimated Variances

The results of the analysis of variance test on the means of the variance calculations are shown in the Appendices. Table 7 summarizes the results:

Table 7. Results of the Analysis of Variance Test--
Means of Estimated Variances

Source	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio	Level of Significance 5%	1%
A	3	9.27	3.09	12.36	2.9	4.4
B	2	1.52	0.76	3.04	3.3	5.3
AB	6	4.91	0.82	3.28	2.4	3.4
C	2	47.48	23.74	94.96	3.3	5.3
AC	6	3.02	0.50	2.00	2.4	3.4
BC	4	0.54	0.13	0.52	2.6	3.9
ABC	12	0.91	0.08	0.32	2.0	2.7
Error	36					

At the 1 per cent level of significance, source A and source C have significant F ratios.

Source C (Estimation Method)

To test for differences in source C, the standard error of the

mean (15) was calculated using Equation 9.

$$\sigma_{\text{mean}} = \sqrt{\frac{\text{Error Variance}}{\text{No. Observations}}} \quad (9)$$

From Table 7,

$$\sigma_{\text{mean}} = \sqrt{\frac{0.25}{24}} = 0.10$$

Then the standard error of the difference between two means is

$$0.10 \sqrt{2} = 0.14$$

From the t-Tables in (15) the 1 per cent level of significance at 36 degrees of freedom is 2.724.

Thus the significant difference between two means is

$$0.14(2.724) = 0.38$$

From the results of the analysis of variance the means for source C are as follows:

$$C_1 = 2.5$$

$$C_2 = 4.3$$

$$C_3 = 2.8$$

Using the significant difference, 0.38, C_2 is significantly different from both C_1 and C_3 .

Source A (Test Subjects)

$$\sigma_{\text{mean}} = \sqrt{\frac{0.25}{18}} = 0.12$$

The standard error of the difference between two means is

$$0.12 \sqrt{2} = 0.17$$

The significant difference between two means is

$$0.17(2.724) = 0.46$$

From the results of the analysis of variance the means for source A are as follows:

$$A_1 = 3.6$$

$$A_2 = 2.7$$

$$A_3 = 3.1$$

$$A_4 = 3.5$$

Using the significant difference, 0.46, A_1 is significantly different from A_2 and A_3 ; A_2 is significantly different from A_4 .

Thus, C_2 (5 per cent method) is significantly more accurate than

C_1 (usual PERT method) and C_3 (10 per cent method).

A_1 (PERT trained subjects) is significantly more accurate than A_2 (non-technical subjects--A) and A_3 (non-technical subjects--B). A_4 (technically trained subjects) is significantly more accurate than A_2 (non-technical subjects--A).

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

ConclusionsComparison of Distributions

1. At the end points the range in standard deviations varies greatly between distributions. Within some distributions the range also varies with the position of the mode. Thus, if the activity times in PERT are not beta-distributed, incorrect calculations of the moments of the distribution may be obtained by using Equations (1) and (2).

2. At the 5 per cent points and 10 per cent points the range in standard deviations for all the distributions is almost identical for all positions of the mode. Thus, if the activity times in PERT fit any of the distributions studied, accurate calculations of the moments of the distribution may be obtained by using Equations (5) and (6) or (7) and (8).

Results of the Experiment--Mean

1. There is no difference in means obtained from PERT trained subjects, technically trained subjects or non-technically trained subjects.

2. There is no difference in the mean values calculated from the normal PERT method, the 5 per cent method, or the 10 per cent method.

Results of the Experiment--Variance

1. There is no difference in the variances calculated from esti-

mates submitted by the PERT trained subjects and the technically trained subjects. However, estimates from the non-technically trained subjects are significantly lower.

2. There is no difference in the variances calculated from data which are identical for all test subjects and the variances calculated from data which are unique for each test subject. The data for each were obtained from the same distribution.

3. The calculation of the variance using the 5 per cent points in Equation (6) is more accurate than the normal PERT method or the 10 per cent method.

Recommendations

Since estimates of the 5 per cent points must be obtained to calculate the variance by Equation (6), it is suggested that the 5 per cent method, Equation (5), also be used to calculate the mean.

Thus, the author recommends the following equations for estimating the moments of the activity time distributions in PERT:

$$\text{Mean, } t_e = (a_{.05} + 4m + b_{.95})/6.0 \quad (5)$$

$$\text{Variance, } V_e = (b_{.95} - a_{.05})^2/10.2 \quad (6)$$

Recommendations for further study are as follows:

1. Other distributions be studied to see if the 5 per cent points again yield similar ranges for all positions of the mode.
2. The experiment be repeated using a larger sample of PERT

trained subjects to see if the 5 per cent method again yields the most accurate variance.

3. More studies be made to see if the normal PERT method yields consistently low values for the variance, as was indicated in this study.

4. The 5 per cent method be applied in parallel with the normal PERT method on a project to see what differences are encountered with its use.

APPENDICES

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APPENDIX A

Calculations of the Moments andRanges of the DistributionsRectangular Distribution

End points of zero and one are used to simplify the computations in all the distributions. An area of one is used for convenience in the rectangular distribution.

Mean

$$E(x) = \int_0^1 x f(x) dx$$

where $f(x) = 0$, $x < 0$

$f(x) = 1$, $0 \leq x \leq 1$

$f(x) = 0$, $1 < x$

$$E(x) = \int_0^1 x dx = x^2/2 \Big|_0^1 = 1/2$$

Variance

$$(E - \mu)^2 = \int_0^1 x^2 f(x) dx - \mu^2$$

where $\mu = \text{mean} = 1/2$

$$(E - \mu)^2 = \int_0^1 x^2 dx - 1/4$$

$$(E - \mu)^2 = x^3/3 \Big|_0^1 - 1/4 = 1/12$$

The standard deviation, σ , is the square root of the variance:

$$\sigma = \sqrt{1/12} = 1/3.464$$

Range

The range is computed between the end points and each of the percentage points as follows:

$$\text{Range} = F(x_2) - F(x_1)$$

where x_2 = the right percentage point

x_1 = the left percentage point

and $F(x) = 0, \quad x < 0$

$F(x) = x, \quad 0 \leq x \leq 1$

$F(x) = 0, \quad 1 < x$

Thus, the range in standard deviations between percentage points is as follows:

$$\text{Range} = (x_2 - x_1)/\sigma = 3.464(x_2 - x_1)$$

The ranges in standard deviations for the percentage points used in the comparisons are shown in Table 8.

Table 8. Ranges for the Rectangular Distribution

Per Cent	x_2	x_1	$(x_2 - x_1)$	$3.464(x_2 - x_1)$
End points (0%)	1.000	0.000	1.000	3.464
0.1%	0.999	0.001	0.998	3.457
1%	0.990	0.010	0.980	3.395
5%	0.950	0.050	0.900	3.118
10%	0.900	0.100	0.800	2.771
25%	0.750	0.250	0.500	1.732

Triangular Distribution

An area of one is assumed for convenience.

Mean

$$E(x) = \int_0^1 x f(x) dx$$

$$\text{where } f(x) = 0, \quad x < 0$$

$$f(x) = 2x/m, \quad 0 \leq x \leq m$$

$$f(x) = 2(1-x)/(1-m), \quad m \leq x \leq 1$$

$$f(x) = 0, \quad 1 < x$$

$$E(x) = \int_0^m (2x^2/m) dx + \int_m^1 (2x(1-x)/(1-m)) dx$$

$$E(x) = 1/m \int_0^m 2x^2 dx + 2/(1-m) \int_m^1 (x-x^2) dx$$

$$E(x) = 1/m(2x^3/3) \Big|_0^m + 2/(1-m)(x^2/2-x^3/3) \Big|_m^1$$

$$E(x) = (1+m)/3$$

Variance

$$E(x-\mu)^2 = \int_0^1 x^2 f(x) dx - \mu^2$$

where $\mu = (1+m)/3$

$$E(x-\mu)^2 = \int_0^m (2x^3/m) dx + \int_m^1 (2x^2(1-x)/(1-m)) dx - (1+m)^2/9$$

$$E(x-\mu)^2 = 2/m \int_0^m x^3 dx + 2/(1-m) \int_m^1 (x^2-x^3) dx - (1+m)^2/9$$

$$E(x-\mu)^2 = 2/m(x^4/4) \Big|_0^m + 2/(1-m)(x^3/3-x^4/4) \Big|_m^1 - (1+m)^2/9$$

$$E(x-\mu)^2 = (m^2-m+1)/18$$

$$\sigma = \sqrt{(m^2-m+1)/18}$$

Range

$$\begin{aligned}
 F(x) &= 0, & x < 0 \\
 F(x) &= x^2/m, & 0 \leq x \leq m \\
 F(x) &= 1 - ((1-x)^2/(1-m)), & m \leq x \leq 1 \\
 F(x) &= 0, & 1 < x
 \end{aligned}$$

The values of the standard deviation for several positions of the mode are shown in Table 9.

Table 9. Standard Deviations for
the Triangular Distribution

Mode	Standard Deviation
0.0	0.236
0.1	0.225
0.2	0.216
0.3	0.209
0.4	0.205
0.5	0.204
0.6	0.205
0.7	0.209
0.8	0.216
0.9	0.225
1.0	0.236

The ranges in standard deviations for the triangular distribution are shown in Tables 10 through 15.

Table 10. Ranges for the Triangular Distribution--Zero Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
0.0	1.000	0.000	1.000	4.237
0.1	1.000	0.000	1.000	4.444
0.2	1.000	0.000	1.000	4.630
0.3	1.000	0.000	1.000	4.785
0.4	1.000	0.000	1.000	4.878
0.5	1.000	0.000	1.000	4.902
0.6	1.000	0.000	1.000	4.878
0.7	1.000	0.000	1.000	4.785
0.8	1.000	0.000	1.000	4.630
0.9	1.000	0.000	1.000	4.444
1.0	1.000	0.000	1.000	4.237

Table 11. Ranges for the Triangular Distribution--0.1 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
0.0	0.968	0.001	0.967	4.097
0.1	0.970	0.010	0.960	4.266
0.2	0.971	0.015	0.956	4.426
0.3	0.973	0.017	0.956	4.574
0.4	0.975	0.020	0.955	4.658
0.5	0.977	0.023	0.954	4.677
0.6	0.980	0.025	0.955	4.658
0.7	0.983	0.027	0.956	4.574
0.8	0.985	0.029	0.956	4.426
0.9	0.990	0.030	0.956	4.266
1.0	0.999	0.032	0.960	4.097

Table 12. Ranges for the Triangular Distribution--1 Per Cent

Mode	x_2	x_1	(x_2-x_1)	$(x_2-x_1)/\sigma$
0.0	0.900	0.005	0.895	3.792
0.1	0.905	0.032	0.873	3.880
0.2	0.911	0.047	0.864	4.000
0.3	0.916	0.055	0.861	4.120
0.4	0.923	0.063	0.860	4.195
0.5	0.929	0.071	0.858	4.206
0.6	0.937	0.077	0.860	4.195
0.7	0.945	0.084	0.861	4.120
0.8	0.953	0.089	0.864	4.000
0.9	0.968	0.095	0.873	3.880
1.0	0.995	0.100	0.895	3.792

Table 13. Ranges for the Triangular Distribution--5 Per Cent

Mode	x_2	x_1	(x_2-x_1)	$(x_2-x_1)/\sigma$
0.0	0.776	0.025	0.751	3.182
0.1	0.787	0.071	0.716	3.182
0.2	0.800	0.106	0.694	3.213
0.3	0.813	0.123	0.690	3.302
0.4	0.826	0.141	0.685	3.341
0.5	0.842	0.158	0.684	3.353
0.6	0.859	0.174	0.685	3.341
0.7	0.877	0.187	0.690	3.302
0.8	0.894	0.200	0.694	3.213
0.9	0.929	0.213	0.716	3.182
1.0	0.975	0.224	0.751	3.182

Table 14. Ranges for the Triangular Distribution--10 Per Cent

Mode	x_2	x_1	(x_2-x_1)	$(x_2-x_1)/\sigma$
0.0	0.684	0.051	0.633	2.682
0.1	0.700	0.100	0.600	2.666
0.2	0.717	0.149	0.568	2.630
0.3	0.736	0.173	0.563	2.694
0.4	0.755	0.200	0.555	2.707
0.5	0.777	0.223	0.554	2.716
0.6	0.800	0.245	0.555	2.707
0.7	0.827	0.264	0.563	2.694
0.8	0.851	0.284	0.568	2.630
0.9	0.900	0.300	0.600	2.666
1.0	0.949	0.316	0.633	2.682

Table 15. Ranges for the Triangular Distribution--25 Per Cent

Mode	x_2	x_1	(x_2-x_1)	$(x_2-x_1)/\sigma$
0.0	0.500	0.134	0.366	1.551
0.1	0.525	0.178	0.347	1.542
0.2	0.553	0.225	0.328	1.519
0.3	0.581	0.274	0.307	1.469
0.4	0.613	0.316	0.297	1.449
0.5	0.647	0.353	0.294	1.441
0.6	0.684	0.387	0.297	1.449
0.7	0.726	0.419	0.307	1.469
0.8	0.775	0.447	0.328	1.519
0.9	0.822	0.475	0.347	1.542
1.0	0.866	0.500	0.366	1.551

Negative Exponential Distribution

Mean

$$E(x) = \int_0^1 x f(x) dx$$

where $f(x) = e^{-x} / \int_0^1 e^{-x} dx, 0 < x \leq 1$

$$f(x) = e^{-x} / (1 - e^{-1}) = (e/e - 1)e^{-x}$$

$$f(x) = 1.582e^{-x}$$

$$E(x) = 1.582 \int_0^1 x e^{-x} dx = -1.582(e^{-x}(x+1)) \Big|_0^1$$

$$E(x) = 0.4180$$

Variance

$$E(x-\mu)^2 = \int_0^1 x^2 f(x) dx - \mu^2$$

where $\mu = 0.4180$

$$E(x-\mu)^2 = \int_0^1 x^2 e^{-x} dx - 0.1747$$

$$E(x-\mu)^2 = -1.582(e^{-x}(x^2+2x+2)) \Big|_0^1 - 0.1747$$

$$E(x-\mu)^2 = 0.2539 - 0.1747 = 0.0792$$

$$\sigma = 0.2814$$

Range

$$F(x) = \int_0^x f(x) dx = \int_0^x (e/e-1)e^{-x} dx = (e/e-1)(1-e^{-x})$$

$$1 - e^{-x} = (e-1/e) F(x)$$

$$e^{-x} = 1 - (.6321F(x))$$

The ranges in standard deviations for the exponential distribution are shown in Table 16.

Table 16. Ranges for the Exponential Distribution

Per Cent	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/.2814$
End Points	1.000	0.0000	1.0000	3.55
0.1%	0.9983	0.0006	0.9977	3.55
1 %	0.9830	0.0063	0.9767	3.47
5 %	0.9175	0.0321	0.8854	3.15
10 %	0.8414	0.0653	0.7761	2.76
25 %	0.6427	0.1720	0.4707	1.67

Beta Distribution

The ranges and moments for the beta distribution are taken from Pearson's tables (13). A range of approximately six standard deviations is held constant over several values of the mode for which the results are tabled. The ranges in standard deviations by percentage point for the beta distribution are shown in Tables 18 through 22.

The range at the end points (0%) is assumed to be six standard deviations.

Table 17. Standard Deviations for the
Beta Distribution

Mode	Standard Deviation
1.000	0.1633
0.875	0.1636
0.778	0.1685
0.700	0.1694
0.636	0.1680
0.583	0.1653
0.500	0.1667

Table 18. Ranges for the Beta Distribution--0.1 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
1.000	0.9997	0.1777	0.8220	5.03
0.875	0.9987	0.1773	0.8214	5.02
0.778	0.9917	0.1517	0.8400	4.99
0.700	0.9800	0.1329	0.8471	5.00
0.636	0.9662	0.1186	0.8476	5.05
0.583	0.9488	0.1070	0.8418	5.09
0.500	0.9238	0.0762	0.8476	5.09

Table 19. Ranges for the Beta Distribution--1 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
1.000	0.9977	0.3161	0.6816	4.17
0.875	0.9882	0.3002	0.6880	4.21
0.778	0.9707	0.2593	0.7114	4.22
0.700	0.9484	0.2291	0.7193	4.25
0.636	0.9231	0.2053	0.7178	4.27
0.583	0.8966	0.1862	0.7104	4.30
0.500	0.8578	0.1421	0.7154	4.29

Table 20. Ranges for the Beta Distribution--5 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
1.000	0.9898	0.4725	0.5173	3.17
0.875	0.9637	0.4371	0.5266	3.22
0.778	0.9311	0.3824	0.5487	3.26
0.700	0.8958	0.3408	0.5550	3.28
0.636	0.8601	0.3077	0.5524	3.29
0.583	0.8255	0.2808	0.5447	3.29
0.500	0.7748	0.2252	0.5496	3.30

Table 21. Ranges for the Beta Distribution--10 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
1.000	0.9740	0.5623	0.4117	2.52
0.875	0.9402	0.5161	0.4241	2.59
0.778	0.8986	0.4551	0.4435	2.63
0.700	0.8566	0.4081	0.4485	2.65
0.636	0.8161	0.3703	0.4458	2.65
0.583	0.7780	0.3391	0.4389	2.65
0.500	0.7214	0.2785	0.4429	2.66

Table 22. Ranges for the Beta Distribution--25 Per Cent

Mode	x_2	x_1	$(x_2 - x_1)$	$(x_2 - x_1)/\sigma$
1.000	0.9306	0.7028	0.2278	1.39
0.875	0.8798	0.6477	0.2321	1.42
0.778	0.8241	0.5805	0.2436	1.45
0.700	0.7729	0.5267	0.2462	1.45
0.636	0.7268	0.4824	0.2444	1.45
0.583	0.6853	0.4452	0.2401	1.45
0.500	0.6212	0.3788	0.2424	1.45

Normal Distribution

The ranges in standard deviations for the normal distribution are taken from the table in reference (14). Since the range in the table is between the mean and the percentage point, it represents half the value of the range between percentage points.

The ranges for the normal distribution are shown in Table 23.

Table 23. Ranges for the Normal Distribution

Per Cent	Tabled Value	$(x_2 - x_1)/\sigma$
End Points		
0.1%	3.100	6.200
1 %	2.320	4.640
5 %	1.645	3.290
10 %	1.280	2.560
25 %	0.675	1.350

APPENDIX B

Development of the Test Data

The test data are obtained for each activity by randomly sampling a beta distribution transformed to meet the requirements for the moments of the activity.

The value of x for the beta distribution with range $(0,1)$ is transformed to x' with range (a,b) as follows:

$$x' = (b-a)x + a, \quad 0 \leq x \leq 1 \quad (10)$$

Then the transformed mean is as follows:

$$\mu_{x'} = (b-a)\mu_x + a \quad (11)$$

The transformed variance is as follows:

$$\sigma_{x'}^2 = (b-a)^2 \sigma_x^2 \quad (12)$$

The beta distribution used conforms to the PERT idea of a typical activity time distribution; that is, it has a range of about six standard deviations and a mode approximately one-third the range.

Pearson's tables (13) give values for distributions only where the mode is greater than half the range. Thus, a distribution with a mode of

approximately two-thirds the range is sampled, but the complement of the result is used in the transformation. This simulates sampling a distribution with a mode of one-third the range.

The distribution used has the following moments:

$$\mu_x = 3/8$$

$$\sigma_x^2 = 5/192$$

$$\text{mode} = 2/3$$

Each activity contains an arbitrary number of "performance times." If there are N times, N-2 of the times are determined by randomly sampling the beta distribution. The other two times are chosen to allow the activity sample to have approximately the same moments as the arbitrary moments.

Group I (Same Data for All Subjects)

Sub-Activity (1)-(3). The moments of the activity are arbitrarily assigned as follows:

$$\mu_{x'} = 12$$

$$\sigma_{x'}^2 = 4$$

From Equation (11):

$$12 = (b-a)(3/8) + a$$

let $R = (b-a)$ (13)

then $12 = (3/8)R + a$

$$a = 12 - (3/8)R \quad (14)$$

From Equation (12):

$$4 = (b-a)^2(5/192)$$

$$4 = (5/192)R^2$$

$$R^2 = 4(192/5)$$

$$R = 2\sqrt{(192/5)} = 12.39 \quad (15)$$

Substituting (15) into (14):

$$a = 12 - (3/8)(12.39) = 7.35 \quad (16)$$

Substituting (16) into (13):

$$12.39 = b - 7.35$$

$$b = 19.75 \quad (17)$$

The mode equals one-third the range:

$$m = (1/3)(b-a) + a$$

$$m = (1/3)(12.39) + 7.35 = 11.49 \quad (18)$$

Substituting (16) and (17) into (10), the transformation equation becomes:

$$x' = 12.39 x + 7.35 \quad (19)$$

Let $N = 8$ for activity (1)-(3). The transformations for the six random values of x are shown in Table 24.

Table 24. Times for Activity (1)-(3)--Group I

N	Random Number	x	(1-x)	x'	$(x' - \mu_{x'})^2$
1	3380547	.56	.44	13	1
2	6046608	.68	.32	11	1
3	6905733	.72	.28	11	1
4	9667988	.89	.11	10	4
5	2611701	.52	.48	13	1
6	6469981	.70	.30	11	1
	Sum			69	9

Since we desire $\mu_{x'} = 12$ and $\sigma_{x'}^2 = 4$, choose x'_7 and x'_8 as follows:

$$\mu_{x'} = \frac{\sum_{i=1}^N x'_i}{N} = 12$$

Thus from Table 24:

$$12 = (69 + x'_7 + x'_8)/8$$

$$x'_7 + x'_8 = 27 \quad (20)$$

$$\sigma_{x'}^2 = \frac{\sum_{i=1}^N (x'_i - \mu_{x'})^2}{(N-1)} = 4$$

Thus from Table 24:

$$4 = (9 + (x'_7 - 12)^2 + (x'_8 - 12)^2)/7$$

Thus

$$(x'_7 - 12)^2 + (x'_8 - 12)^2 = 19 \quad (21)$$

Since the sum of squares of the factors in (21) is 19, it may be assumed that the square root of one factor would have a maximum absolute value of five. Then x'_7 and x'_8 are found in the range:

$$(\mu_{x'} - 5) \leq x' \leq (\mu_{x'} + 5)$$

$$7 \leq x' \leq 17 \quad (22)$$

Since from Equation (20) we desire

$$x'_7 + x'_8 = 27$$

let

$$26 \leq (x'_7 + x'_8) \leq 28 \quad (23)$$

The possible values of x' in the range in (22) that yield the desired values in (23) are shown in Table 25.

Table 25. Possible Values of x'_7 and x'_8

26		27		28	
x'_7	x'_8	x'_7	x'_8	x'_7	x'_8
9	17	10	17	11	17
10	16	11	16	12	16
11	15	12	15	13	15
12	14	13	14	14	14
13	13				

The possible values in Table 25 are then evaluated by Equation (21) to see if they meet the requirements. The results are shown in Table 26.

Table 26. Evaluation of Possible Values of x_7' and x_8'

x_7'	x_8'	$(x_7'-12)^2$	$(x_8'-12)^2$	Sum
9	17	9	25	34
10	17	4	25	29
11	17	1	25	26
10	16	4	16	20
11	17	1	16	17
12	16	0	16	16
11	15	1	9	10
12	15	0	9	9
13	15	1	9	10
12	14	0	4	4
13	14	1	4	5
14	14	4	4	8
13	13	1	1	2

Table 26 shows that the values of x_7' and x_8' that yield the results closest to 19, the requirements of Equation (21), are as follows:

$$x_7' = 10$$

$$x_8' = 16$$

Thus, the performance times for activity (1)-(3) are the values of x' in Table 24 and the values of x'_7 and x'_8 .

Sub-Activity (2)-(3)

The moments of the activity are arbitrarily assigned as follows:

$$\mu_{x'} = 10$$

$$\sigma_{x'}^2 = 1$$

From Equation (11):

$$10 = (b-a)(3/8) + a$$

let

$$R = (b-a) \tag{24}$$

then

$$10 = (3/8)R + a$$

$$a = 10 - (3/8)R \tag{25}$$

From Equation (12):

$$1 = (b-a)^2(5/192)$$

$$1 = (5/192)R^2$$

$$R^2 = 192/5$$

$$R = \sqrt{192/5} = 6.20 \quad (26)$$

Substituting (26) into (25):

$$a = 10 - (3/8)(6.20) = 7.68 \quad (27)$$

Substituting (27) into (24):

$$6.20 = b - 7.68$$

$$b = 13.88 \quad (28)$$

The mode equals one-third the range, or

$$m = (1/3)(b-a) + a$$

$$m = (1/3)(13.88 - 7.68) + 7.68 = 9.74 \quad (29)$$

Substituting (27) and (28) into (10), the transformation becomes:

$$x' = 6.20x + 7.68 \quad (30)$$

Let $N = 8$ for activity (2)-(3). The transformations for the five random values of x are shown in Table 27.

Table 27. Times for Activity (2)-(3)--Group I

N	Random Number	x	(1-x)	x'	$(x' - \mu_{x'})^2$
1	3360772	.56	.44	10	0
2	5694308	.67	.33	10	0
3	9255602	.85	.15	9	1
4	4644300	.62	.38	10	0
5	6909122	.72	.28	9	1
Sum				48	2

Since we desire $\mu_{x'} = 10$ and $\sigma_{x'}^2 = 1$, choose x'_6 and x'_7 as follows:

$$\mu_{x'} = \frac{\sum_{i=1}^N x'_i}{N} = 10$$

Thus from Table 27:

$$10 = (48 + x'_6 + x'_7)/7$$

Thus

$$x'_6 + x'_7 = 22 \quad (31)$$

$$\sigma_{x'}^2 = \frac{N}{\sum_{i=1}^N} (x'_i - \mu_{x'})^2 / (N-1) = 1$$

Thus from Table 27:

$$1 = (2 + (x'_6 - 10)^2 + (x'_7 - 10)^2) / 6$$

Thus

$$(x'_6 - 10)^2 + (x'_7 - 10)^2 = 4 \quad (32)$$

Since the sum of the squares of the factors in (32) is four, it may be assumed that the square root of one factor would have a maximum absolute value of two. Then x'_6 and x'_7 are found in the range:

$$(\mu_{x'} - 2) \leq x' \leq (\mu_{x'} + 2)$$

$$8 \leq x' \leq 12 \quad (33)$$

Since from Equation (31) we desire

$$x'_6 + x'_7 = 22$$

Let

$$21 \leq (x'_6 + x'_7) \leq 23 \quad (34)$$

The possible values of x' in the range in (33) that yield the desired values in (34) are shown in Table 28.

Table 28. Possible Values of x'_6 and x'_7

21		22		23	
x'_6	x'_7	x'_6	x'_7	x'_6	x'_7
9	12	10	12	11	12
10	11	11	11		

The possible values in Table 28 are then evaluated by Equation (32) to see if they meet the requirements. The results are shown in Table 29.

Table 29. Evaluation of Possible Values of x'_6

x'_6	x'_7	$(x'_6-10)^2$	$(x'_7-10)^2$	Sum
9	12	1	4	5
10	12	0	4	4
11	12	1	4	5
10	11	0	1	1
11	11	1	1	2

Table 29 shows that the values of x'_6 and x'_7 that yield the results closest to four, the requirements of Equation (32), are as follows:

$$x'_6 = 10$$

$$x'_7 = 12$$

Thus the performance times for activity (2)-(3) are the values of x' in Table 27 and the values of x'_6 and x'_7 .

Sub-Activity (3)-(4)

The moments of the activity are arbitrarily assigned as follows:

$$\mu_{x'} = 5$$

$$\sigma_{x'}^2 = 1$$

From Equation (1):

$$5 = (b-a)(3/8) + a$$

Let

$$R = (b-a) \tag{35}$$

Then

$$5 = (3/8)R + a$$

$$a = 5 - (3/8)R \tag{36}$$

From Equation (12):

$$1 = (b-a)^2(5/192)$$

$$1 = (5/192)R^2$$

$$R^2 = 192/5$$

$$R = \sqrt{192/5} = 6.20 \quad (37)$$

Substituting (37) into (36):

$$a = 5 - (3/8)(6.20) = 2.68 \quad (38)$$

Substituting (38) into (35):

$$6.20 = b - 2.68$$

$$b = 8.88 \quad (39)$$

The mode equals one-third the range:

$$m = (1/3)(b-a) + a$$

$$m = (1/3)(8.88 - 2.68) + 2.68 = 4.74 \quad (40)$$

Substituting (38) and (39) into (10), the transformation becomes:

$$x' = 6.20x + 2.68 \quad (41)$$

Let $N = 9$ for activity (3)-(4). The transformation for the seven random values of x are shown in Table 30.

Table 30. Times for Activity (3)-(4)--Group I

N	Random Number	x	(1-x)	x'	$(x' - \mu_{x'})^2$
1	4857377	.63	.37	5	0
2	6011233	.68	.32	5	0
3	0963371	.40	.60	6	1
4	0152708	.26	.74	7	4
5	3770011	.58	.42	5	0
6	2613342	.52	.48	6	1
7	1527749	.45	.55	6	1
Sum				40	7

Since we desire $\mu_{x'} = 5$ and $\sigma_{x'}^2 = 1$, choose x'_8 and x'_9 as follows:

$$\mu_{x'} = \frac{\sum_{i=1}^N x'_i}{N} = 5$$

Thus from Table 30:

$$5 = (40 + x'_8 + x'_9)/9$$

Thus

$$x'_8 + x'_9 = 5 \quad (42)$$

$$\sigma_{x'}^2 = \frac{\sum_{i=1}^N (x'_i - \mu_{x'})^2}{(N-1)} = 1$$

Thus from Table 30:

$$1 = (7 + (x'_8 - 5)^2 + (x'_9 - 5)^2)/8$$

Thus

$$(x'_8 - 5)^2 + (x'_9 - 5)^2 = 1 \quad (43)$$

The only values of x' that would satisfy (43) are (4,5) or (5,6). The values (4,5) more closely satisfy the requirements of Equation (42). Thus, the following results are obtained:

$$x'_8 = 4$$

$$x'_9 = 5$$

Thus, the performance times for activity (3)-(4) are the values of x' in Table 30 and the values of x'_8 and x'_9 .

Activity (A)-(B)

The moments of activity (A)-(B) are the sums of the corresponding moments of the sub-activities.

Since sub-activities (1)-(2) and (2)-(3) are performed simultaneously, the one with the smaller mean would be ignored in calculating the mean of (A)-(B).

Thus, the moments of (A)-(B) are as follows:

$$\mu_{(A)-(B)} = \mu_{(1)-(3)} + \mu_{(3)-(4)}$$

$$\mu_{(A)-(B)} = 12 + 5 = 17$$

$$\sigma_{(A)-(B)}^2 = \sigma_{(1)-(3)}^2 + \sigma_{(3)-(4)}^2$$

$$\sigma_{(A)-(B)}^2 = 4 + 1 = 5$$

Group II (Different Data for Each Subject)

The same method is used as in Group I, except all the performance times are obtained by random sampling. The method for determining the moments for this group is shown in the Procedure.

The results shown in Table 31 and Figures 20 and 21 are as follows:

$$\mu_{(A)-(B)} = 17.49$$

$$\sigma_{(A)-(B)}^2 = 5.87$$

Mode = 17

a = 11

b = 26

a._{.05} = 13b._{.95} = 21a._{.10} = 14b._{.90} = 20

Table 31. Distribution of Activity Times for Group II

x_i	f	x_i^2	fx_i	fx_i^2	Cumulative f	Cumulative Per Cent
13	26	169	338	4394	26	3.3
14	62	196	868	12152	88	11.2
15	78	225	1170	17550	166	21.2
16	116	256	1856	29696	282	36.0
17	137	289	2329	39593	419	53.4
18	114	324	2052	36936	533	68.0
19	92	361	1748	33212	625	79.7
20	68	400	1360	27200	693	88.4
21	44	441	924	19404	737	94.0
22	17	484	374	8228	754	96.2
23	25	529	575	13225	779	99.4
24	4	576	96	2304	783	99.9
25	1	625	25	625	784	100.0

Group III (Same Data for All Subjects)

The results of Group I are used except that a constant of 3 is added to each time and the order of the data is changed. Thus, the moments obtained are as follows:

$$\mu_{(A)-(B)} = 23$$

$$\sigma_{(A)-(B)}^2 = 5$$

Before calculating the moments in Group III, the constant value of six is removed from the estimates submitted.

Sample Calculations

Test subject number one submitted the following estimates for Group I:

$$\text{Mean} = 18.0$$

$$\text{Mode} = 16.0$$

$$a = 14.0$$

$$b = 23.0$$

$$a_{.05} = 14.5$$

$$b_{.95} = 22.5$$

$$a_{.10} = 15.0$$

$$b_{.90} = 22.0$$

Calculations of the mean and variance are as follows:

FREQUENCY DISTRIBUTION OF SIMULATED ACTIVITY

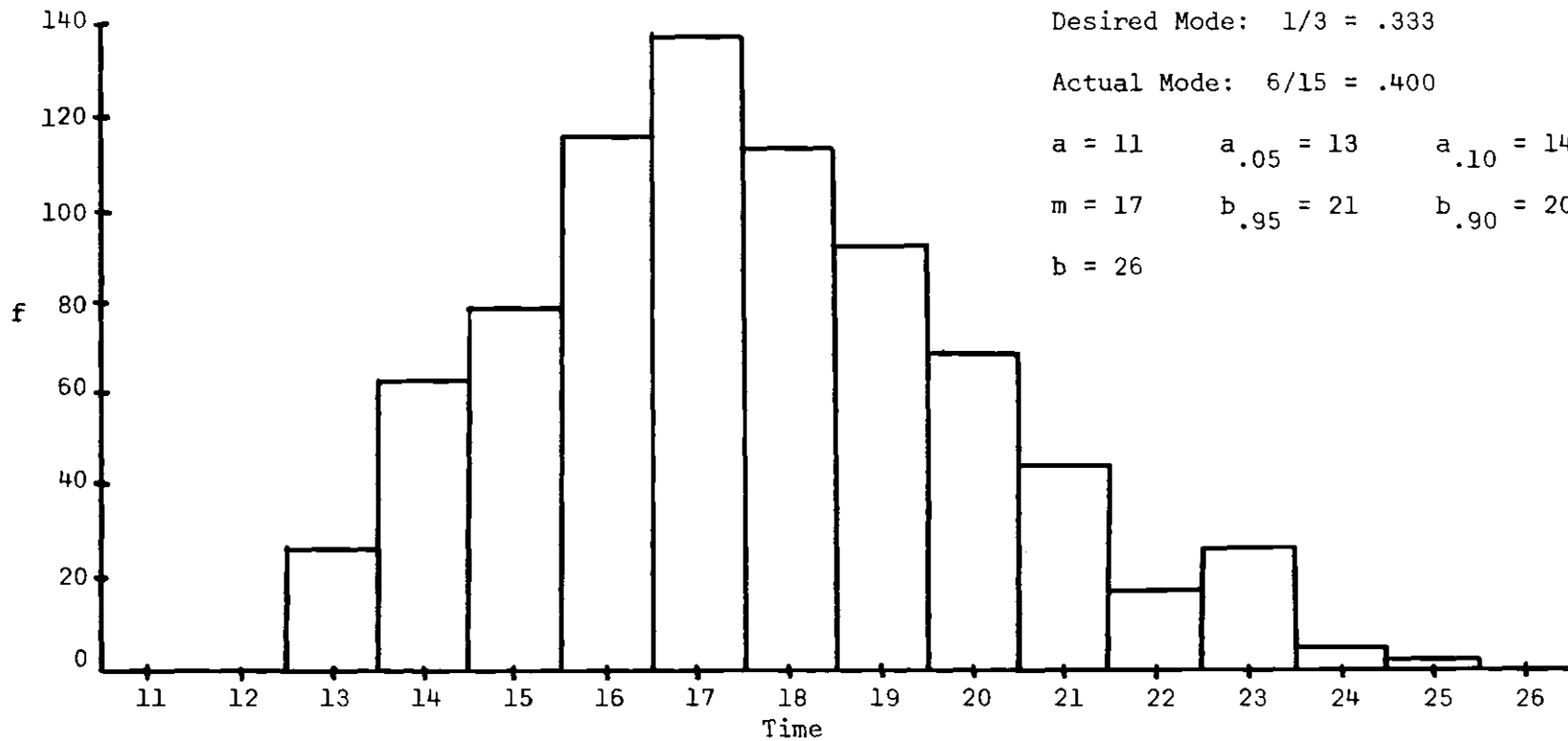


Figure 20. Distribution of Activity Times for Group II.

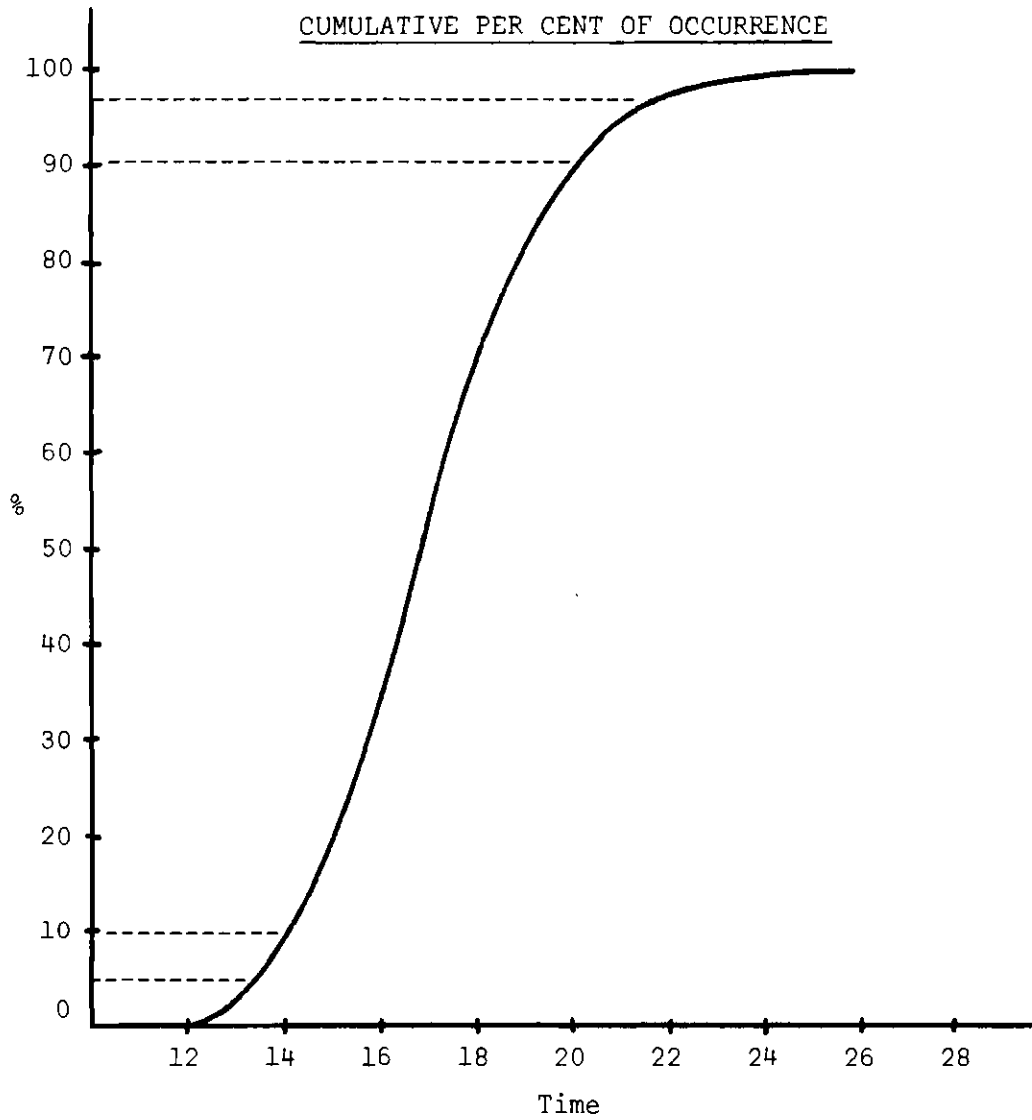


Figure 21. Cumulative Frequency of Activity Times for Group II.

THREE GROUPS OF "EXPERIENCE" DATA FOR ACTIVITY, "FINISH DESK"

	Activity 1-3 (Sand Desk)	Activity 2-3 (Mix Paint)	Activity 3-4 (Paint Desk)	Estimates
GROUP I	16	10	6	Mean _____
	10	10	5	Mode _____
	11	9	5	a _____
	13	10	4	b _____
	13	9	6	a (.05) _____
	10	12	6	b (.95) _____
	11	10	5	a (.10) _____
	11		7	b (.90) _____
			5	
GROUP II	11	10	6	Mean _____
	12	11	4	Mode _____
	15	11	6	a _____
	15	9	5	b _____
	13	8	4	a (.05) _____
	14	10	4	b (.95) _____
	12	12	6	a (.10) _____
	16		5	b (.90) _____
			5	
GROUP III	14	13	10	Mean _____
	19	13	8	Mode _____
	14	15	8	a _____
	13	13	9	b _____
	13	12	7	a (.05) _____
	14	12	8	b (.95) _____
	16	13	9	a (.10) _____
	16		8	b (.90) _____
			9	

Figure 22. Sample of the Test Used in the Experiment.

Normal PERT Method

From Equation (1):

$$t_e = (14.0 + 64.0 + 23.0)/6.0 = 16.9$$

From Equation (2):

$$V_e = (23.0 - 14.0)^2/36.0 = 2.3$$

Five Per Cent Method

From Equation (5):

$$t_e = (14.5 + 64.0 + 22.5)/6.0 = 16.9$$

From Equation (6):

$$V_e = (22.5 - 14.5)^2/10.2 = 6.3$$

Ten Per Cent Method

From Equation (7):

$$t_e = (15.0 + 64.0 + 22.0)/6.0 = 16.9$$

From Equation (8):

$$V_e = (22.0 - 15.0)^2 / 7.3 = 6.7$$

These same calculations are used for the Group II and Group III estimates.

Table 32. PERT Mean Computed from Experiment Data Sheets
(Test Subjects with PERT Training)

Test No.	GROUP I				GROUP II				GROUP III			
	CPM	0%	5%	10%	CPM	0%	5%	10%	CPM	0%	5%	10%
1	18.0	16.9	16.9	16.9	17.0	15.3	15.2	15.1	19.0	17.8	17.8	17.8
2	18.0	16.2	16.1	16.2	17.5	17.7	17.7	17.6	17.5	16.7	16.6	16.3
4	19.0	18.2	18.1	18.0	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
5	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
6	17.0	16.9	16.7	16.7	17.0	16.5	16.5	16.4	19.0	17.5	17.5	17.5
7	17.0	16.9	16.5	16.7	20.0	19.3	19.5	19.8	17.0	17.5	17.0	17.0
8	17.0	17.5	17.6	17.1	17.0	17.1	17.0	17.1	17.0	17.3	17.3	17.3
10	18.0	17.5	17.5	17.3	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
12	18.0	17.0	16.5	16.4	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
60	17.0	16.9	16.5	16.7	20.0	19.3	19.5	19.8	17.0	17.5	17.0	17.0
61	18.0	17.5	17.5	17.3	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
63	17.0	17.5	17.6	17.1	17.0	17.1	17.0	17.1	17.0	17.3	17.3	17.3
66	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
67	17.0	17.5	17.6	17.1	17.0	17.1	17.0	17.1	17.0	17.3	17.3	17.3
71	17.3	18.5	18.4	18.3	17.1	17.5	17.5	17.4	17.3	18.5	18.4	18.3
74	18.0	16.9	16.9	16.7	18.5	17.8	17.8	17.8	17.0	16.7	16.7	16.5
77	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
79	17.0	17.5	17.6	17.1	17.0	17.1	17.0	17.1	17.0	17.3	17.3	17.3
81	17.0	16.9	16.7	16.7	17.0	16.5	16.5	16.4	19.0	17.5	17.5	17.5
83	17.0	17.5	17.6	17.1	17.0	17.1	17.0	17.1	17.0	17.3	17.3	17.3
86	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
92	18.0	16.9	16.9	16.7	18.5	17.8	17.8	17.8	17.0	16.7	16.7	16.5
95	19.0	18.2	18.1	18.0	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
97	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
109	18.0	16.9	16.9	16.9	17.0	15.3	15.2	15.1	19.0	17.8	17.8	17.8
Sum	446.3	430.3	428.3	423.9	442.1	431.4	431.1	430.1	444.8	431.2	429.4	426.8
Mean	17.9	17.2	17.1	16.9	17.7	17.3	17.2	17.2	17.8	17.2	17.2	17.1

APPENDIX C

Table 33. PERT Mean Computed from Experiment Data Sheets
(Test Subjects with Technical Training)

Test No.	GROUP I				GROUP II				GROUP III			
	CPM	0%	5%	10%	CPM	0%	5%	10%	CPM	0%	5%	10%
14	17.0	16.0	15.7	15.5	18.0	15.1	15.0	14.8	17.0	15.8	15.7	15.7
15	18.0	16.9	16.9	16.7	18.5	17.8	17.8	17.8	17.0	16.7	16.7	16.5
16	18.0	16.9	16.7	16.5	18.0	16.0	16.0	16.0	17.0	16.9	16.6	16.4
18	17.0	16.7	17.5	16.5	16.0	15.7	15.5	15.2	21.0	18.0	18.0	18.0
19	18.0	16.7	16.7	16.5	22.0	19.8	18.9	20.0	19.0	17.5	17.8	17.8
22	18.5	16.9	16.6	16.5	16.5	16.4	16.4	16.4	18.5	16.4	16.4	16.3
25	17.0	16.9	16.0	16.2	18.0	18.0	18.0	18.0	17.0	16.9	16.4	16.4
72	18.0	17.5	17.5	17.3	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
76	19.0	18.2	18.1	18.0	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
94	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
98	18.0	16.9	16.9	16.9	18.6	19.1	19.1	18.6	15.0	16.9	16.9	16.5
102	18.0	17.0	16.5	16.4	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
104	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
105	18.0	16.9	16.7	16.5	18.0	16.0	16.0	16.0	17.0	16.9	16.6	16.4
106	18.0	17.5	17.5	17.3	18.3	17.3	17.3	17.3	18.0	16.9	16.7	16.7
108	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
110	18.0	16.9	16.7	16.5	18.0	16.0	16.0	16.0	17.0	16.9	16.6	16.4
111	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
113	18.0	16.9	16.7	16.5	18.0	16.0	16.0	16.0	17.0	16.9	16.6	16.4
114	17.0	16.9	16.0	16.2	18.0	18.0	18.0	18.0	17.0	16.9	16.4	16.4
115	18.0	17.5	17.5	17.3	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
116	17.0	16.9	16.7	16.7	17.0	16.5	16.5	16.4	19.0	17.5	17.5	17.5
117	18.0	16.9	16.9	16.7	18.5	17.8	17.8	17.8	17.0	16.7	16.7	16.5
118	18.5	16.9	16.6	16.5	16.5	16.4	16.4	16.4	18.5	16.4	16.4	16.3
119	19.0	18.2	18.1	18.0	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
Sum	449.0	425.7	421.0	418.4	448.0	431.1	430.8	428.0	436.0	423.8	421.3	417.8
Mean	18.0	17.0	16.8	16.7	17.9	17.2	17.2	17.1	17.4	17.0	16.9	16.7

Table 34. PERT Mean Computed from Experiment Data Sheets
(Test Subjects with Non-Technical Training--A)

Test No.	GROUP I				GROUP II				GROUP III			
	CPM	0%	5%	10%	CPM	0%	5%	10%	CPM	0%	5%	10%
29	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
30	19.0	16.0	16.9	16.9	17.5	16.4	16.2	16.2	18.0	16.9	16.8	16.8
31	17.3	18.5	18.4	18.3	17.1	17.5	17.5	17.4	17.3	18.5	18.4	18.3
32	17.0	16.9	16.4	16.7	17.4	19.8	20.2	20.3	17.2	17.0	16.7	16.7
33	17.3	16.9	16.4	16.6	16.4	16.2	16.3	16.4	17.3	16.9	17.0	16.7
34	16.0	16.9	16.2	16.0	24.0	19.0	19.3	19.5	17.0	16.9	16.5	16.5
35	24.0	16.9	16.2	16.2	19.0	17.3	16.5	17.3	19.0	17.0	15.9	15.7
37	17.5	16.7	16.9	17.2	23.2	17.4	17.2	17.7	17.4	16.4	15.9	16.3
38	16.9	16.9	16.5	16.5	16.8	17.0	17.3	17.3	17.3	16.9	16.9	17.0
39	16.5	16.7	16.7	16.6	18.0	19.8	19.8	19.8	17.5	16.7	16.7	16.8
40	17.1	16.9	16.9	16.4	15.3	15.3	15.5	15.4	17.1	17.0	16.6	16.2
64	17.0	16.9	16.9	16.7	17.0	16.5	16.5	16.4	19.0	17.5	17.5	17.5
69	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
70	17.5	16.7	16.7	16.9	23.2	17.4	17.2	17.7	17.4	16.4	15.9	16.3
73	18.0	17.0	17.0	16.5	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
75	19.0	18.2	18.2	18.1	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
78	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
80	17.3	18.5	18.5	18.4	17.1	17.5	17.5	17.4	17.3	18.5	18.4	18.3
82	18.0	17.5	17.5	17.5	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
84	16.0	16.9	16.9	16.2	24.0	19.0	19.3	19.5	17.0	16.9	16.5	16.5
88	17.0	16.9	16.9	16.7	17.0	16.5	16.5	16.4	19.0	17.5	17.5	17.5
89	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
90	18.0	17.5	17.5	17.5	18.0	17.3	17.3	17.3	18.0	16.9	16.7	16.7
96	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
99	19.0	18.2	18.2	18.1	17.0	17.0	17.0	17.0	18.0	17.5	17.5	17.5
Sum	445.4	429.0	424.1	423.8	462.5	442.7	442.9	442.1	429.8	427.5	423.5	421.2
Mean	17.8	17.2	17.0	17.0	18.5	17.7	17.7	17.7	17.2	17.1	16.9	16.8

Table 35. PERT Mean Computed from Experiment Data Sheets
(Test Subjects with Non-Technical Training--B)

Test No.	GROUP I				GROUP II				GROUP III			
	CPM	0%	5%	10%	CPM	0%	5%	10%	CPM	0%	5%	10%
41	17.4	16.9	16.7	16.5	17.6	17.3	17.3	17.6	17.4	17.0	16.9	17.0
43	23.5	17.0	16.0	16.8	15.9	17.6	17.3	17.6	17.3	16.9	16.7	16.9
45	18.8	17.3	17.1	17.0	17.8	18.5	18.5	18.5	17.1	18.5	18.0	18.0
46	17.2	16.9	16.9	17.0	18.1	18.2	18.2	18.2	17.2	16.4	16.4	16.5
47	17.0	16.9	16.9	16.2	16.5	16.7	16.7	17.0	17.0	17.0	16.5	16.7
48	17.3	17.5	18.0	18.0	18.0	17.5	17.5	17.3	17.0	16.4	16.4	16.4
52	17.4	16.9	16.5	16.9	18.5	17.6	17.4	17.9	17.5	16.4	16.4	16.4
54	17.3	16.7	16.8	16.7	16.1	16.5	16.6	16.5	17.0	16.9	16.7	16.7
56	17.3	16.9	16.9	16.5	16.1	16.2	16.2	16.0	17.3	16.9	16.2	16.7
57	17.3	16.9	16.3	16.4	20.1	19.8	20.2	20.4	16.2	16.9	16.8	16.7
58	17.5	17.5	17.5	17.5	19.0	18.3	18.5	18.5	17.5	16.9	16.5	16.5
59	18.4	16.8	16.9	17.0	17.5	17.9	18.0	18.1	18.2	16.9	16.2	16.6
62	18.0	17.0	16.5	16.4	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
65	18.0	16.9	16.9	16.7	18.5	17.8	17.8	17.8	17.0	16.7	16.7	16.5
68	19.0	16.9	16.8	16.5	18.0	17.3	17.3	17.1	19.0	16.9	16.8	16.5
85	17.3	18.5	18.4	18.3	17.1	17.5	17.5	17.4	17.3	18.5	18.4	18.3
87	18.0	16.9	16.9	16.9	18.5	19.1	19.1	18.6	15.0	16.9	16.9	16.5
91	17.3	17.5	18.0	18.0	18.0	17.5	17.5	17.3	17.0	16.4	16.4	16.4
93	18.0	17.0	16.5	16.4	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
100	18.0	16.9	16.7	16.5	18.0	16.0	16.0	16.0	17.0	16.9	16.6	16.4
101	17.4	16.9	16.7	16.5	17.6	17.3	17.3	17.6	17.4	17.0	16.9	17.0
103	18.0	17.0	16.5	16.4	17.0	16.0	16.0	15.8	18.0	17.2	17.4	17.2
107	18.8	17.3	17.1	17.0	17.8	18.5	18.5	18.5	17.1	18.5	18.0	18.0
112	17.0	16.9	16.9	16.2	16.5	16.7	16.7	17.0	17.0	17.0	16.5	16.7
120	18.0	16.9	16.7	16.5	17.6	17.3	17.3	17.6	17.4	17.0	16.9	17.0
Sum	449.2	426.8	423.1	420.8	439.8	435.1	435.4	435.9	431.9	426.5	422.0	450.3
Mean	18.0	17.1	16.9	16.8	17.6	17.4	17.4	17.4	17.3	17.1	16.9	16.2

Table 36. Frequency Distributions of Estimated Means
(All Subjects and Groups)

Mean	Frequency	Cumulative Frequency	Per Cent of Total Frequency
<u>CPM Method</u>			
15.0 - 15.9	13	13	4.33
16.0 - 16.9	14	27	9.00
17.0 - 17.9	123	150	50.00
18.0 - 18.9	109	259	86.33
19.0 - 19.9	30	289	96.33
20.0 - 20.9	3	292	97.33
21.0 - 21.9	1	293	97.67
22.0 - 22.9	1	294	98.00
23.0 - 23.9	6	300	100.00
<u>Normal PERT Method</u>			
15.0 - 15.9	6	6	2.00
16.0 - 16.9	147	153	51.00
17.0 - 17.9	105	258	86.00
18.0 - 18.9	23	281	93.67
19.0 - 19.9	19	300	100.00
<u>5% Method</u>			
15.0 - 15.9	10	10	3.33
16.0 - 16.9	158	168	56.00
17.0 - 17.9	87	255	85.00
18.0 - 18.9	26	281	93.67
19.0 - 19.9	17	298	99.33
20.0 - 20.9	2	300	100.00
<u>10% Method</u>			
14.0 - 14.9	1	1	0.33
15.0 - 15.9	13	14	4.67
16.0 - 16.9	143	157	52.33
17.0 - 17.9	98	255	85.00
18.0 - 18.9	37	292	97.33
19.0 - 19.9	5	297	99.00
20.0 - 20.9	3	300	100.00

Table 37. PERT Variance Computed from Experiment Data Sheets
(Test Subjects with PERT Training)

Test No.	GROUP I			GROUP II			GROUP III		
	0%	5%	10%	0%	5%	10%	0%	5%	10%
1	2.3	6.3	6.7	1.8	4.1	3.4	2.3	4.8	3.4
2	2.3	5.8	9.2	2.8	6.3	6.0	2.8	7.6	5.6
4	2.3	5.5	4.9	1.0	2.5	2.2	2.3	6.3	6.7
5	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
6	2.3	3.5	2.2	2.3	4.8	2.2	2.3	4.8	1.2
7	2.3	4.8	2.2	2.8	4.8	1.2	2.3	0.4	0.0
8	2.3	3.5	1.2	3.4	6.3	3.4	2.8	6.3	2.2
10	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
12	5.4	4.8	2.2	7.1	6.3	3.4	6.3	6.3	3.4
60	2.3	4.8	2.2	2.8	4.8	1.2	2.3	0.4	0.0
61	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
63	2.3	3.5	1.2	3.4	6.3	3.4	2.8	6.3	2.2
66	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
67	2.3	3.5	1.2	3.4	6.3	3.4	2.8	6.3	2.2
71	2.3	5.1	3.8	2.3	5.1	4.1	2.3	4.3	2.3
74	2.3	4.8	2.2	3.4	7.9	3.4	2.8	6.3	4.9
77	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	4.5
79	2.3	3.5	1.2	3.4	6.3	3.4	2.8	6.3	2.2
81	2.3	3.5	2.2	2.3	4.8	2.2	2.3	4.8	1.2
83	2.3	3.5	1.2	3.4	6.3	3.4	2.8	6.3	2.2
86	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
92	2.3	4.8	2.2	3.4	7.9	3.4	2.8	6.3	3.4
95	2.3	5.5	4.9	1.0	2.5	2.2	2.3	6.3	6.7
97	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
109	2.3	6.3	6.7	1.8	4.1	3.4	2.3	4.8	3.4
Sum	60.6	119.4	79.0	68.0	132.1	75.5	65.5	128.7	75.7
Mean	2.4	4.8	3.2	2.7	5.3	3.0	2.6	5.1	3.0

Table 38. PERT Variance Computed from Experiment Data Sheets
(Test Subjects with Non-Technical Training--A)

Test No.	GROUP I			GROUP II			GROUP III		
	0%	5%	10%	0%	5%	10%	0%	5%	10%
29	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
30	2.3	6.3	6.7	1.0	2.2	2.0	2.3	4.8	4.5
31	2.3	5.1	3.8	2.3	5.1	4.1	2.3	4.3	2.3
32	1.4	1.6	2.2	3.4	2.1	4.9	1.8	1.6	0.5
33	2.2	1.6	0.3	1.4	3.0	1.7	2.3	4.8	2.2
34	2.3	0.9	0.0	1.4	1.6	1.2	2.3	0.9	0.1
35	2.3	7.9	1.2	1.8	2.5	2.2	1.8	0.9	4.9
37	0.4	1.2	1.2	0.3	0.2	0.0	0.4	0.1	0.9
38	2.3	2.5	1.2	1.0	1.6	0.5	2.3	2.5	0.5
39	1.8	3.5	2.8	2.3	6.3	6.7	2.3	4.8	4.1
40	2.3	2.3	2.2	0.4	0.9	0.9	1.8	0.6	0.1
64	2.3	3.5	2.2	2.3	4.8	2.2	2.3	4.8	3.4
69	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
70	0.4	1.2	1.2	0.3	0.2	0.0	0.4	0.1	0.9
73	5.4	4.8	2.2	7.1	6.3	3.4	6.3	6.3	3.4
75	2.3	5.5	3.4	2.8	6.3	3.4	2.3	4.8	1.2
78	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
80	2.3	5.1	3.8	2.3	5.1	4.1	2.3	4.3	2.3
82	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
84	2.3	0.9	0.0	1.4	1.6	1.2	2.3	0.9	0.1
88	2.3	3.5	2.2	2.3	4.8	2.2	2.3	4.8	1.2
89	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
90	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
96	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
99	2.3	5.5	4.9	1.0	2.5	2.2	2.3	6.3	6.7
Sum	55.3	94.9	62.9	45.4	76.6	58.3	56.2	88.6	58.5
Mean	2.2	3.8	2.5	1.8	3.1	2.3	2.2	3.5	2.3

Table 39. PERT Variance Computed from Experiment Data Sheets
(Test Subjects with Non-Technical Training--B)

Test No.	GROUP I			GROUP II			GROUP III		
	0%	5%	10%	0%	5%	10%	0%	5%	10%
41	2.3	3.5	1.2	2.8	1.6	2.2	1.8	0.9	0.0
43	0.4	3.5	1.2	4.0	6.3	2.2	2.3	3.5	6.7
45	1.8	0.9	0.0	1.4	0.9	2.2	2.3	1.6	0.3
46	3.1	7.9	7.9	2.9	7.9	8.1	1.7	3.5	2.1
47	2.3	7.9	3.4	4.0	9.8	2.2	1.8	2.5	0.5
48	2.3	1.6	0.5	1.4	2.5	2.2	1.0	1.6	0.5
52	2.3	0.9	6.7	2.8	4.8	8.5	1.8	3.5	2.2
54	1.8	4.8	6.7	2.3	6.3	8.7	2.3	1.6	0.5
56	2.3	2.5	3.4	1.4	2.5	2.2	2.3	0.9	0.5
57	2.3	1.0	0.9	1.4	0.7	0.4	2.3	3.4	3.2
58	2.3	2.5	1.2	1.0	0.9	0.1	2.3	2.5	1.2
59	2.9	7.9	2.8	2.0	3.5	1.4	3.0	2.5	2.8
62	5.4	4.8	2.2	7.1	6.3	3.4	6.3	6.3	3.4
65	2.3	4.8	2.2	3.4	7.9	3.4	2.8	6.3	3.4
68	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
85	2.3	5.1	3.8	2.3	5.1	4.1	2.3	4.3	2.3
87	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
91	2.3	1.6	0.5	1.4	2.5	2.2	1.0	1.6	0.5
93	5.4	4.8	2.2	7.1	6.3	3.4	2.3	4.8	1.2
100	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.1	2.2
101	2.3	3.5	1.2	2.8	1.6	2.2	1.8	0.9	0.0
103	5.4	4.8	2.2	7.1	6.3	3.4	6.3	6.3	3.4
107	1.8	0.9	0.0	1.4	0.9	2.2	1.8	2.5	0.5
112	2.3	7.9	3.4	4.0	9.8	2.2	1.8	2.5	0.5
120	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.8	1.2
Sum	64.8	103.0	67.2	76.2	122.8	89.9	60.5	82.7	45.9
Mean	2.6	4.1	2.7	3.0	4.9	3.6	2.4	3.3	1.8

Table 40. PERT Variance Computed from Experiment Data Sheets
(Test Subjects with Technical Training)

Test No.	GROUP I			GROUP II			GROUP III		
	0%	5%	10%	0%	5%	10%	0%	5%	10%
14	4.0	6.3	3.4	3.4	6.3	2.2	3.4	6.3	4.9
15	2.3	4.8	2.2	3.4	7.9	3.4	2.8	6.3	3.4
16	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.1	2.2
18	2.8	4.8	3.4	0.7	0.9	1.2	2.8	6.3	4.9
19	1.8	3.5	1.2	4.7	7.9	4.9	2.3	1.2	0.5
22	3.4	7.1	6.7	1.8	3.5	2.8	1.6	3.5	2.8
25	2.3	3.5	1.2	1.8	1.6	0.5	2.3	1.6	0.5
72	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
76	2.3	5.5	4.9	1.0	2.5	2.2	2.3	6.3	6.7
94	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
98	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
102	5.4	4.8	2.2	7.1	6.3	3.4	6.3	6.3	3.4
104	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
105	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.1	2.2
106	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
108	2.3	4.8	3.4	1.4	2.5	2.2	2.3	4.8	3.4
110	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.1	2.2
111	2.3	5.5	3.4	2.8	6.3	3.4	2.3	5.5	3.4
113	2.3	4.8	3.4	4.0	9.8	8.7	2.3	4.1	2.2
114	2.3	3.5	1.2	1.8	1.6	0.5	2.3	1.6	0.5
115	2.3	4.8	2.2	1.8	3.5	2.2	2.3	3.5	2.2
116	2.3	3.5	2.2	2.3	4.8	2.2	2.3	4.8	1.2
117	2.3	4.8	2.2	3.4	7.9	3.4	2.8	6.3	3.4
118	3.4	7.1	6.7	1.8	3.5	2.8	1.6	3.5	2.8
119	2.3	5.5	4.9	1.0	2.5	2.2	2.3	6.3	6.7
Sum	64.5	123.0	79.6	64.0	123.2	85.3	62.7	111.9	74.1
Mean	2.6	4.9	3.2	2.6	4.9	3.4	2.5	4.5	3.0

Table 41. Frequency Distribution of Estimated Variances
(All Subjects and Groups)

Variance	Frequency	Cumulative Frequency	Per Cent of Total Frequency
<u>Normal PERT Method</u>			
0.0 - 0.9	9	9	3.00
1.0 - 1.9	62	71	23.67
2.0 - 2.9	184	255	85.00
3.0 - 3.9	17	272	90.67
4.0 - 4.9	11	283	94.33
5.0 - 5.9	6	289	96.33
6.0 - 6.9	5	294	98.00
7.0 - 7.9	6	300	100.00
<u>5% Method</u>			
0.0 - 0.9	24	24	8.00
1.0 - 1.9	22	46	15.33
2.0 - 2.9	33	79	26.33
3.0 - 3.9	41	120	40.00
4.0 - 4.9	82	202	67.33
5.0 - 5.9	27	229	76.33
6.0 - 6.9	49	278	92.67
7.0 - 7.9	14	292	97.33
8.0 - 8.9	0	292	97.33
9.0 - 9.9	8	300	100.00
<u>10% Method</u>			
0.0 - 0.9	38	38	12.67
1.0 - 1.9	30	68	22.67
2.0 - 2.9	112	180	60.00
3.0 - 3.9	77	257	85.67
4.0 - 4.9	17	274	91.33
5.0 - 5.9	2	276	92.00
6.0 - 6.9	13	289	96.33
7.0 - 7.9	1	290	96.67
8.0 - 8.9	9	299	99.67
9.0 - 9.9	1	300	100.00

APPENDIX D

Analysis of Variance

The analysis of variance test was made on the following sources from the experiment:

A. Test Subjects

1. PERT trained
2. Non-technical--A
3. Non-technical--B
4. Technical

B. Data Groups

1. Same for all subjects
2. Different for each subject
3. Same for all subjects

C. Estimation Method (Means of Estimated Variances)

1. Present method (0%)
2. 5% method
3. 10% method

D. Estimation Method (Means of Estimated Means)

1. CPM method
2. Present method (0%)
3. 5% method
4. 10% method

Means of Estimated Means

Table 42. Means of Estimated Means--ANOVA Input

		B ₁				B ₂				B ₃			
		C ₁	C ₂	C ₃	C ₄	C ₁	C ₂	C ₃	C ₄	C ₁	C ₂	C ₃	C ₄
A ₁	r ₁	18.1	17.1	17.0	16.8	17.6	17.2	17.1	17.0	17.9	17.1	17.1	16.9
	r ₂	17.5	17.3	17.2	17.0	17.7	17.3	17.3	17.3	17.6	17.3	17.2	17.2
A ₂	r ₁	17.8	17.2	17.1	17.1	18.2	17.9	17.9	17.8	17.0	17.0	17.0	16.9
	r ₂	17.7	17.0	16.7	16.7	18.8	17.4	17.4	17.5	17.4	17.1	16.8	16.7
A ₃	r ₁	18.2	17.0	16.8	16.8	17.2	17.2	17.2	17.3	17.0	16.9	16.9	18.1
	r ₂	17.6	17.1	17.0	16.8	18.0	17.6	17.7	17.2	17.0	16.8	15.4	17.7
A ₄	r ₁	17.2	17.0	16.9	17.6	16.9	16.9	16.8	17.9	17.1	17.1	17.0	16.9
	r ₂	16.8	16.6	16.5	18.3	17.5	17.5	17.4	16.9	16.9	16.7	16.6	16.5

ANOVA Results

C	= 28459.15
C_a	= 28460.20
$C_a - C$	= 1.05
C_b	= 28462.23
$C_b - C$	= 3.08
C_{ab}	= 28465.05
$C_{ab} - C_a - C_b + C$	= 1.77
C_c	= 28462.77
$C_c - C$	= 3.62
C_{ac}	= 28466.10
$C_{ac} - C_a - C_c + C$	= 2.28
C_{bc}	= 28466.40
$C_{bc} - C_b - C_c + C$	= 0.55
C_{abc}	= 28476.01
$C_{abc} - C_{ab} - C_{ac} - C_{bc} + C_a + C_b + C_c - C$	= 4.51
C_{abcr}	= 28500.95
$C_{abcr} - C_{abc}$	= 24.94

Table 43. Means of Estimated Variances--ANOVA Input

Means of Estimated Variances

		B ₁			B ₂			B ₃		
		C ₁	C ₂	C ₃	C ₁	C ₂	C ₃	C ₁	C ₂	C ₃
A ₁	r ₁	2.5	4.9	3.5	2.8	5.3	3.2	2.8	5.7	3.5
	r ₂	2.3	4.6	2.8	2.6	5.2	2.8	2.5	4.6	2.5
A ₂	r ₁	2.1	4.2	3.1	1.6	2.7	2.2	2.1	3.9	2.5
	r ₂	2.3	3.4	1.9	2.1	3.4	2.4	2.4	3.2	2.2
A ₃	r ₁	2.9	4.7	3.0	3.5	5.5	3.9	2.7	3.5	2.1
	r ₂	2.2	3.5	2.3	2.5	4.3	3.3	2.1	3.1	1.6
A ₄	r ₁	2.7	5.0	3.4	2.7	5.3	3.5	2.7	5.2	3.5
	r ₂	2.5	4.8	2.9	2.4	4.6	3.3	2.3	3.6	2.3

ANOVA Results

C	= 742.41
C_a	= 751.68
$C_a - C$	= 9.27
C_b	= 743.93
$C_b - C$	= 1.52
C_{ab}	= 758.11
$C_{ab} - C_a - C_b + C$	= 4.91
C_c	= 789.89
$C_c - C$	= 47.48
C_{ac}	= 802.18
$C_{ac} - C_a - C_c + C$	= 3.02
C_{bc}	= 791.95
$C_{bc} - C_b - C_c + C$	= 0.54
C_{abc}	= 810.06
$C_{abc} - C_{ab} - C_{ac} - C_{bc} + C_a + C_b + C_c - C$	= 0.91
C_{abcr}	= 818.96
$C_{abcr} - C_{abc}$	= 8.90

The summary of the analysis of variance results are shown in the body of the thesis.

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