Option Paper

The Impact of Traffic Density on Lane-Changing Frequency

Qian Jiang

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Abstract

Fluctuations on roadways are widely considered as an effect of lane-changing activity. Lane-changing has been recognized as microscopic behaviors and elements in lane-changing models are considered to be mostly dynamic. But lane-changing decisions can still be influenced by some traffic conditions reflected as macroscopic factors. This paper attempts to correlate microscopic models with macroscopic models by exploring the relationship between lane-changing frequency and density. A descriptive analysis is generalized to explain lane-changing behaviors as a reaction to traffic density. It is observed that the lane-changing frequency increases in the low-density region and reaches a peak around a certain density. Five simple regression models are constructed to fit the NGSIM (Next Generation SIMulation) data. Based on three statistical indicators, the cubic model is selected as the best fit for the relationship between lane-changing frequency and density.

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1. Problem Statement

Congestion is a recurrent phenomenon on all road types around the world. Fluctuations on roadways appear as an effect of lane changing instead of car following, for the periods of heavy traffic conditions. Lane-changing maneuvers have received increased attention in a number of scientific studies. If we can increase the understanding of lane-changing behaviors it will contribute significantly to the advancement of traffic management strategies and the autonomous vehicle development. Compared to other traffic flow models, lane-changing models have not been studied enough. Most lane-changing models only consider the kinematic factors such as gap distances, velocities, and relative speed, but miss ambient traffic conditions such as traffic density (Rahman, 2005). While traffic density has been proven to have an impact on lane-changing behavior (Huang, 2002; Laval, 2006; Yang et al. 2018). Traffic flow theories, since initiated, are mainly put forward along two different schools of thought microscopic models, and macroscopic models. Traffic density is a significant indicator in the macroscopic models. But most macroscopic models do not incorporate lane-changing behaviors as variables. A gap exists between the two model types, while the lane-changing model is in essence a hybrid model concerning both macroscopic and microscopic features. This paper attempts to fill the gap between two traffic flow models by figuring out the relationship between a macroscopic factor, density, and a microscopic factor, lane-changing frequency based on a frequently used dataset, NGSIM (Next Generation SIMulation).

Objectives

- Define proper density and lane-changing indicators.
- Identify how traffic density affects lane-changing behaviors.

- Validate the descriptive relationship between lane-changing frequency and density.
- Find a best model to fit the lane-changing frequency and density correlation.

2. Literature Review

Traffic flow is a system consisting of strongly interacting travelers with transportation infrastructure. infrastructure. The theory of traffic flow includes s models and methodologies to better understand the spatiotemporal patterns of traffic flow. In 1935, Greenshields started scientific studies for traffic problems and that is commonly considered as the beginning of traffic flow theory (Greenshields 1935). At that time, due to the small size of the of vehicle population, every vehicle was treated individually. Kinzer (1934) and Adams (1936) pointed out the arrival of a vehicle is a stochastic behavior and that could be explained or predicted by the Poisson distribution (Kinzer 1934, Adams 1936).

In the following decades, with the development of the automobile industry, the vehicle population saw a great increase. Inevitably, traffic jams and crashes increased and interactions between vehicles began to play a major role in traffic studies. Since then, the activity of a single vehicle has become hard to observe and the original probability theory began to be unsuitable for most situations, where the number of vehicles was too big to be analyzed. Scientific traffic planning and control need the support of updated traffic flow theory. Since the 1950s, many branches of traffic flow theory have appeared one after another, including the car-following theory, the kinematic wave theory, etc. In 1959, the first international conference on traffic flow theory was held in Detroit, Michigan, which became an important symbol of the birth of traffic flow theory. Since then, the study of traffic flow theory has entered a period of rapid development (Gerlough and Huber 1976). These traffic theories are mainly put forward along two different lines, namely, microscopic, and macroscopic. In the microscopic models, the traffic is treated as a system of interacting particles driven far from equilibrium. In contrast, in the so-called macroscopic models, the traffic is viewed as a compressible fluid.

2.1 Macroscopic traffic models

Macroscopic models treat vehicular traffic as a compressible fluid. Compared with microscopic models, the continuum models have a better understanding of the collective behaviors of traffic flow and analyze the traffic flow status dynamically. The aim is to study the service level of intersections and roads on the basis of three fundamental variables: velocity, flow, and density. If properly defined, the three variables could also be used to depict the service level of transportation networks.

Lighthill and Whitham proposed a simple hydrodynamic model, that was commonly accepted as the oldest and most popular macroscopic traffic model based on the fluid-dynamic theory (Lighthill and Whitham, 1955; Nagel 1992; Papageorgiou, 1998; Helbing, 2001; Nagatani, 2002). They studied traffic jams as a shock wave based on two assumptions. The first is that the number of vehicles within a certain time and space is balanced and the second is that there exists a simple equation to describe the relationship between traffic flow and density. The equation is expressed as (Lighthill and Whitham, 1955):

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = s(x, t) \tag{1}$$

where x is the vehicle position. t represents time. k and q describe traffic density and flow respectively. $s(x, t)$ stands for the traffic flow generation rate. In the ideal model where a lane has no exits and entrances, the value of $s(x, t)$ is 0. The model includes a few simplifications and fails to reproduce some real dynamic phenomena observed on freeways (Payne, 1971; Phillips, 1979; Kuhne, 1984; Rathi et al., 1987; Ross, 1988; Papageorgiou, 1998). It can explain the traffic shockwaves well over a long time and space span but shows a poor performance when the traffic wave is near a bottleneck. Also, failure to consider the effects of acceleration and inertia prevents the model from accurately reflecting the dynamic characteristics of traffic flow under a steady state. To circumvent the deficiencies mentioned above, some efforts were made to improve the first-order model to a higher-order one by taking reaction time into account (Payne 1971). The equation is expressed as (Payne, 1979):

$$\frac{\partial k}{\partial t} + \frac{\partial (kv)}{\partial x} = 0 \tag{2}$$

$$k\frac{\partial v}{\partial t} + kv\frac{\partial v}{\partial x} = \frac{k}{\tau}[V(k) - v] - c_0^2\frac{\partial k}{\partial x} + \mu\frac{\partial^2 v}{\partial x^2} \tag{3}$$

Where τ is the reaction time constant. c_0^2 and μ are phenomenological constants. *V(k)* is the phenomenological function representing the desired velocity at density *k*. Phillips (1979) improved the Payne model by incorporating a traffic pressure function affected by velocity and density. The Phillips model held that vehicle interactions could be assumed as intermolecular interactions in gas dynamics and this assumption allowed the model to simulate maneuvers in roadways with more than one lane. But the constant in the traffic pressure function was complicated, making the Phillips model difficult to be widely used. The Phillips model and the Payne model present similar results in low-density regimes, while perform very differently when density goes up. In the following years, more critics pointed out when road geometry and traffic volume change rapidly in a short time, the model may not capture the real traffic dynamic characteristics because of its slow adjustment to the optimal speed (Ross, 1988; Papageorgiou *et al.*, 1989). After conducting a linear stability analysis, Castillo (1994) claimed that vehicles in the Payne continuum model were compelled to travel in a steady state and that was not in line with reality obviously. Papageorgiou (1983) expanded the Payne model and introduced ramp flow as a variable, improving the performance in highway simulation, but the stability at high density was not greatly improved. Ross (1988) proposed a simplified second-order continuum model skipping over the traditional velocity-density relationship in a steady flow and therefore avoiding the deficiencies of the Payne model. The equation is given by (Ross, 1988):

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{v - v_f}{\tau} \tag{4}$$

Where v_f is the optimal velocity in the free flow regime.and τ is the delay time constant. However, it assumed that with an incompressible traffic fluid the propagation of shockwave could be theoretically infinite if the leading vehicle takes a stop-and-go action. Kerner and Konhauser (1993) have investigated the continuum model and have shown that the jamming transition occurs at high density and the stationary cluster moves with a constant velocity and has an invariable form.

So far, the macroscopic models have focused on traffic density in great depth, mostly focused on the velocity-density relationship. Although there is a general tendency for speed to decrease with increasing traffic density, Ross (1988) has found that a deterministic velocity-density relationship is not possible and each velocity-density curve can be applied in a particular situation. To simulate more realistic traffic dynamics, the macroscopic models have been continuously expanded and improved to reproduce different traffic scenarios. But it is difficult to tell which model is better as each has a unique applicable scenario. Even though one could claim that higher-order models improve the accuracy level provided by first-order models and simultaneously circumvent the qualitative deficiencies, the higher complexity models often result in analytically difficult outcomes virtually impossible (Papageorgiou, 1998). Despite the diversity, the applicable scope of macroscopic models is still limited. Most macroscopic models are suitable for uniform, dense, and steady traffic flow rather than light traffic regimes, or unstable situations such as the go-and-stop, or congested scenarios. Besides, macroscopic models fail to consider driving behaviors as factors. Even though some models incorporated microscopic elements, e.g. reaction time, they do not consider lane-changing behaviors as variables. Consequently, to some extent, macroscopic models are out of line with microscopic models.

2.2 Microscopic traffic models

In the microscopic models, the traffic is treated as a system of interacting particles. Microscopic models analyze the interactions between a driver and another driver or concentrate on a single driver on the different features of a road. The main purpose of microscopic models is to describe the individual driver behaviors for two driving situations: car-following and lane-changing.

2.2.1 Car-following models

The car-following model is one of the most important microscopic models, depicting the process in which the rear vehicle follows a front vehicle when driving in a single lane and no overtaking is allowed. Reuschel (1950) and Pipes (1951) created differential equations to describe car-following phenomena. The basic assumption is that the velocity of the car behind is a linear function of the distance from the car in front. Chandler (1958) incorporated acceleration into the model and found that the follower's acceleration is proportional to the difference in velocity of the vehicle pair. The model is known as the General Motors (GM) model and it is given by (Chandler, 1958):

$$a_n\,(t + T) = C \cdot \Delta v \tag{5}$$

Where $a_n(t)$ is the acceleration of vehicle n at time t. Δv is the difference in velocity of vehicle n

and the one in front. T is the constant time lag. C is the sensitivity constant. Gazis et al. (1959) developed a non-linear "reciprocal-spacing" car-following model based on which a tentative discussion of the stability of traffic flow was expanded and a critical velocity below which the flow becomes unstable was derived. Surprisingly, the model drew a macroscopic conclusion. They found that drivers become more concentrated and faster to react when the traffic volume gets increased. To incorporate the reaction factor and make the conclusion compatible with other macroscopic theories, Gazis *et al* modified the Chandler model and changed the sensitivity function to C/ Δx. The equation is expressed as (Gazis et al. 1959):

$$a_n\,(t+\ T) = \frac{C}{\Delta x}\cdot \Delta v_n \qquad (6)$$

Where $a_n(t)$ is the acceleration of vehicle n at time t. Δv is the difference in velocity of vehicle n and the one in front. Δx is the distance between vehicle n and its leading vehicle. T is the constant time lag. C is the sensitivity constant. Herman and Potts (1959) validated the model through experiments in three main tunnels in New York. Edie (1961) compared the model with other macroscopic models and discovered the alteration of accelerations is relevant to vehicle velocity and hence the model was improved as (Edie, 1961):

$$a_n\,(t+\ T) = \frac{C\cdot v_n}{(\Delta x)^2}\cdot \Delta v_n \qquad (7)$$

Where $a_n(t)$ is the acceleration of vehicle n at time t. Δv is the difference in velocity of vehicle n and the one in front. Δx is the distance between vehicle n and its leading vehicle. T is the constant time lag. C is the sensitivity constant. The Edie model performs well in low-density regimes and it can be used to predict the free-flow speed. Nevertheless, it is not suitable for traffic congestion regimes. To overcome the defect, Gazis et al. (1961) studied two distinctive regimes, congested and uncongested, and a more general model was proposed (Gazis et al. 1961):

$$a_n\,(t+\,T) = \frac{C\cdot x_n^m}{(\Delta x)^l}\cdot \Delta v_n \tag{8}$$

Where $a_n(t)$ is the acceleration of vehicle *n* at time *t*. Δv is the difference of velocity of vehicle *n* and the one in front. Δx is the distance between vehicle *n* and its leading vehicle. *T* is the constant time lag. C is the sensitivity constant. *m* and *l* are constants used to depict traffic conditions. However, there is no consensus on the value of *m* and *l*. A great deal of work has been done to find suitable *m* and *l* values, to distinguish traffic regimes including congested and uncongested, accelerated, and decelerated, etc. (May and Keller, 1967; Hayes and Ashworth 1972; Treiterer and Myers 1974; Ceder 1976; Ceder *et al* 1976; Ozaki 1993).

Newell (1961) has proposed the optimal velocity (OV) model and that was the first time the concept of "velocity-headway function" was put forward (Newell, 1961):

$$v_n\,(t+\,T) = V\,(1-\exp\,(-\frac{C}{V}\cdot \Delta x - d) \tag{9}$$

Where $v_n(t)$ is the velocity of vehicle *n* at time *t*. Δx is the distance between vehicle *n* and its leading vehicle. *d* and *C* are constants. *V* is the optimal velocity. The idea is that a driver tends to adjust the vehicle velocity according to the observed headway. Bando *et al* (1995) updated the model by introducing an acceleration constant based on a "velocity-headway function".

Over the same period, another branch of car-following models, the Cellular Automata (CA) model, was put forward. Car-following models adhere to a rule, starting with obtaining information from the leading vehicle and ending with reacting to the stimulus. The CA model concentrates on avoiding collisions. In 1959, Kometani and Sasaki (1959) put forward a CA model adhering to collision avoidance. The equation is expressed as (Kometani and Sasaki, 1959):

$$\Delta x\ (t-0.5)=\alpha v_{n-1}{}^{2}(t-0.5)+\beta v_{n}{}^{2}(t)+\gamma v_{n}(t)+d \quad (10)$$

Where $v_n(t)$ is the velocity of vehicle *n* at time *t*. Δx is the distance between vehicle *n* and its leading vehicle. α, β, γ, and *d* are constants. Gipps (1984) also contributed to the CA model by noticing that every driver has a subconscious mind to keep a safe distance in order to avoid crashes and the corresponding reaction function was modified.

Car-following models have been modified and improved over a long time. Nevertheless, further studies found that this branch focused on traffic activities on a single lane and did not address most traffic issues since lane changes were common and overtaking behaviors could hardly be explained by car-following models.

2.2.2 Lane-changing models

The earliest lane-changing model was introduced in 1986. Since then, lane-changing models have seen great improvements. The initial lane-changing model was a traditional rule-based model, containing limited factors and the output was binary with certain results, change or not change. Further efforts found that lane change is a probabilistic event and developed discrete-choice models to explain the results. In discrete-choice models, the drivers' heterogeneity is considered and more complex factors are incorporated. Over the same period, some studies claimed that lane-changing, involving ambient vehicles, is not only an individual behavior and these merging and weaving maneuvers cover dynamic interactions between multiple vehicles. The game theory-based lane-changing decision-making model was developed to interpret the phenomena. The processing of data and the estimation of parameters were found to be tedious using the models mentioned above. So, the artificial intelligence (AI) model was put forward to address the problems.

The first lane-changing model, applied in microscopic fields, was proposed in 1986 by Gipps. The model was designed to deal with urban traffic problems. When driving on arterial roads, drivers are expected to encounter various conditions and this model simulated the lane-changing process as a decision tree to cover complex urban driving situations, including traffic signals, obstructions, and heavy vehicles. The lane-changing decision process was considered to be determined by three factors: possibility, necessity, and desirability. Based on the three factors, driving conditions were expanded to a series of conflicting goals drivers may face and a binary decision result (change or not change) could be generated through the decision tree. However, the model was designed to be used in conjunction with a car-following model (Gipps, 1981). So, a driver tends to stay in the current lane and maintain the desired speed if no surrounding traffic or road environment affects his driving intention, and lane-changing decisions are forced by the introduction of obstructions or intersections. As a result, the Gipps Model is robust in simulating this type of coercive lane-changing behaviors but is poor in explaining casual lane-changing phenomena. Yang and Koutsopoulos (1996) classified these two types of lane-changing as mandatory lane-changing (MLC) and discretionary lane-changing (DLC). Mandatory lane-changing occurs when drivers have to change lanes due to some obstructions or restrictions. Discretionary lane-changing refers to cases in which drivers change lanes in order to get better driving conditions, but the lane change is not required. This updated model was implemented in MIcroscopic Traffic SIMulation Laboratory (MITSIMLab) simulator platform, starting from defining the type of change, selecting the target lane, and ending with executing the desired lane change if gap distances are acceptable. The MITSIMLab model introduces impatient factors and speed indifference factors to check traffic conditions of relevant lanes and determines whether to consider a discretionary lane change. When lead and lag gaps of the target lane are acceptable, a merging maneuver may be executed. Another significant difference from the Gipps model is that the output of the MITSIMLab model is probabilistic. When encountering conflicting goals in the decision tree, drivers' lane-changing decisions are probabilistic instead of binary, which is closer to reality. Apart from velocity and gap distance, more variables, such as acceleration, are considered as elements in lane-changing models. Kesting el at. (2007) proposed a MOBIL model (Minimizing Overall Braking Induced by Lane change) to derive lane-changing rules for MLC and DLC. The MOBIL model captured the attractiveness and risks associated with lane-changing behaviors and these two indicators were expressed by a function of acceleration.

The second type of lane-changing models is the discrete-choice model and it has developed as the most commonly used tool for studying individual behaviors. Ahmed (1996) modeled lane-changing motivation with a utility function and drivers tend to maximize their benefits in a lane-changing maneuver in terms of a series of explanatory factors and individual variables. The equation is given by (Ahmed, 1996):

$$U_{tn} = X_{tn}\gamma + v_n + \varepsilon_{tn} \tag{11}$$

Where U_{tn} is the utility for individual n at time t. X_{tn} is the vector of explanatory variables. γ is the vector of unknown parameters. v_n is the individual specific random term and ε_{tn} stands for the drivers' inherent heterogeneity, and also for a given induvial across different time periods. In theory, MLC and DLC could be explicitly distinguished since proposed. However, in the practical application, the boundary between the two lane-changing types is fuzzy and one can hardly identify the subjectivity of each lane-changing maneuver according to objective conditions. Toledo (2003) captured the trade-offs between MLC and DLC and combined these two lane-changing types into a single probabilistic lane-changing decision model through a utility function. The integrated equation is expressed as (Toledo, 2003):

$$U_n^d(t) = X_n^d(t)\beta^d + \gamma^d EMU_n^d(t) + \alpha^d v_n + \varepsilon_n^d(t) \tag{12}$$

Where $U_n^d(t)$ depicts the utility of lane-changing decision. $X_n^d(t)$ is the vector of explanatory variables. $EMU_n^d(t)$ is the expected maximum utility of lower level choices that are available it the decision is made. v_n is the individual specific random term. ε_{tn} is a random term that varies across different time periods for a given individual, as well as across individuals. β^d, γ^d, and α^d are vectors of parameters. The Gap acceptance model is a significant component of the utility function and a lane change is considered only if both the lead gap and the lag gap of the target lane are adequate. However, gap distances are hardly acceptable in congestion regimes, causing the inefficiency of the model in the simulation of congested traffic situations. Toledo (2005) and Choudhury et al. (2007) developed a latent lane-changing model with the explicit choice of target lanes to overcome the defects and the drivers were considered to have latent plans before acting. The latent plan model was validated in various scenarios and proved to improve previous models in different traffic regimes, especially in extreme conditions, e.g. severe congestion, or roadways where the level of service (LOS) of lanes varies significantly.

The third type of lane-changing models are game theory-based models. Traditionally, lane-changing models were used to deal with the subject maneuvers in sections where lane-changing behaviors happen. Further efforts revealed that surrounding vehicles also play a big role in the merging and weaving decision process. Game theory-based models were developed to capture the interactions between two vehicles or more in lane-changing maneuvers. In the merging-giveway scenario, both the merging vehicle and through vehicles are facing conflicting goals in the merging section and the subjects involved would evaluate the trade-offs based on instantaneous knowledge derived from the traffic environment. Kita (1999) proposed a likelihood function to simulate this scenario of merging-giveway conflicts with the purpose of avoiding collision and modeled the behaviors of the merging vehicle and the closest through vehicle as a game. However, this short-term knowledge acquisition process was based on an assumed perfect condition and, in reality, drivers may take longer to observe the traffic environment, receive knowledge, and act. Kita (2002) improved the model to incorporate the lead vehicle as a factor. However, for the purpose of simplifying the model, Kita also failed to consider other detailed elements, making the model unrealistic. Hidas (2005) classified lane-changing as three types: free lane change, forced lane change, and cooperative lane change, and the latter two types captured the interactions between the subject vehicle and follower vehicles in the target lane. Merging vehicles and follower vehicles were considered to interact and cooperate with each other to solve conflicting goals. Based on the Kita model, Hidas incorporated more factors such as gaps and differential speed and made it more realistic. However, the model assumed that involved drivers have adequate information about plans of other vehicles and they can cooperate perfectly, while drivers, in reality, may take seconds or longer to take actions or reactions. To better understand merging behaviors, Liu (2007) improved the Kita model by separating conflicting vehicles and modeling merging vehicles and lag vehicles respectively through payoff functions. Some unique behavior rules, assuming that the merging vehicle from the on-ramp attempts to join mainline traffic as quick as possible and the target lane vehicle aims at maintaining original speed, were considered to make the model more realistic. But it still did not address the problem of processing time. Kang (2018) put forward a repeated game approach to overcome the defects of instantaneous fluctuations in a short-time. With the iteration of lane-changing decisions, the simulation accuracy was improved for most situations. However, this method is limited by the data acquisition and computation process, preventing it from simulating scenarios containing more than four vehicles.

The fourth type of lane-changing models is the artificial intelligence (AI) model. Since proposed, the AI model has attracted the attention of researchers because it can simulate the uncertain characteristics as well as fuzzy intentions of drivers and does not have to understand lane-changing maneuvers inherently. Technically, all models can be discussed at a fuzzy level, although it is difficult to explain them in explicit physical terms (Moridpour, 2009). Das et al. (1999), on the basis of implementing fuzzy rules, divided lane change strategies into mandatory MLC and DLC for analysis but did not consider the type of vehicles in lane-changing decisions. To solve this problem, Moridpour et al. (2012) developed lane-changing models for different car types using fuzzy logic. Fuzzy logic models show better accuracy in simulating the driver's actual decision-making process by incorporating more variables. But the complexity of fuzzy rules prevents it from been validated efficiently and widely applied (Ma, 2004). Apart from fuzzy models, the artificial neural network (ANN) model is another type of the AI model. Hunt et al. (2004) used the ANN model to predict drivers' lane-changing decisions on dual carriageways. A major drawback of the ANN model is that the model is highly data-driven and relies too much on field-collected traffic data (Dumbuya et al. 2006).

2.3 Macro-micro link

It can be observed from the reviews above that macroscopic models are partly disjointed with microscopic models. Most macroscopic models do not contain kinematic variables in the equations, especially those related to lane-changing behaviors. While most microscopic models focus on dynamic variables, such as gap distances, velocities, and relative speed to determine subsequent behaviors, but do not give much consideration to traffic conditions. Rahman (2005) claimed that existing lane-changing models only considered the dynamics of the front and lag vehicles, while traffic density and other ambient traffic conditions were largely ignored. However, studies have shown that drivers' lane-changing decisions are affected by surrounding traffic conditions (Huang, 2002; Chen, 2015; Li and Sun, 2015; Yang et al., 2018). Yang et al. (2018) conducted a simulator experiment and found that the drivers' intention for lane change and overtaking is enhanced as traffic density increases. It was also found that the lane-changing rate increases in the low-density region, reaches the maximum around a certain density, and then decreases in the high-density region (Huang, 2002). But previous studies only analyzed the patterns descriptively. Further research into the mathematical relationship between lane-changing behaviors and traffic density is few supported by large-scale data collections (Ma, 2004).

The other significant constraint is the computational amount. The traditional traffic flow theory is based on the conventional mathematical and physical methods that could be physically explained explicitly, with harsh constraints and rigorous derivations. Although straightforward and robust in simple scenarios, the traditional models need lots of efforts to estimate parameters, and this process is tedious when the number of scenarios increases. With the development of the automobile industry, traffic conditions have become increasingly complicated, and such complex situations are more difficult to be explained by a uniform model. The modern traffic flow theory creates a solution to the problem by taking advantage of traffic simulators to reproduce realistic traffic conditions virtually. The best prediction results can be obtained through traffic simulators, given all features and models related to traffic conditions as input. One of the most attractive remedial measures for developing traffic simulators is the deployment of the Intelligent Transportation System (ITS). The ITS focuses on the application of current and evolving technologies to transportation systems and the careful integration of system functions to provide more efficient and effective solutions to multimodal transportation problems (Boxill et al., 2000). A lot of effort has been made to integrate different types of models into one scheme. But it is impossible to adopt a general model to all situations. ITS has developed a number of traffic simulators, and each software has its unique application scope. Due to more reliable data collection methods and more diverse traffic models, the traffic simulators have higher accuracy in reproducing real-life traffic scenarios.

This analysis, on the one hand, is expanded with the purpose of validating the descriptive results observed by other studies. On the other hand, it aims at statistically examining the effect of traffic density on drivers' lane-changing intentions. The developed equation can improve the prediction accuracy of existing traffic simulators if properly used

2.4 Summary

A summary of the findings from the literature review is presented below.

- The primary attention of macroscopic models focuses on three basic traffic factors: density, flow, and velocity. Factors related to lane-changing behaviors are rarely considered.

- Most microscopic models focus on dynamic variables but do not consider traffic conditions such as density.
- Car-following models focus on micro elements but fail to address lane-changing maneuvers.
- Lane-changing models contain explanatory variables as well as unknown variables, and this allows macroscopic independents to be incorporated.
- Traffic simulators serve to integrate different types of models and highly intricate analyses of traffic activities can be obtained.
- Figuring out statistical relationships between lane-changing behaviors and traffic density contributes to improving the accuracy of traffic simulators.

3. Methodologies

3.1 Data source

The vehicle trajectory data was collected by the program called Next Generation SIMulation (NGSIM) launched by Federal Highway Administration (FHWA). The NGSIM datasets include four sites and the data gathered on a section of southbound U.S. Highway 101 (US-101) in Los Angeles, CA. This site is chosen in order to examine the lane-changing behaviors. Figure 1 shows the location of the detection zone.

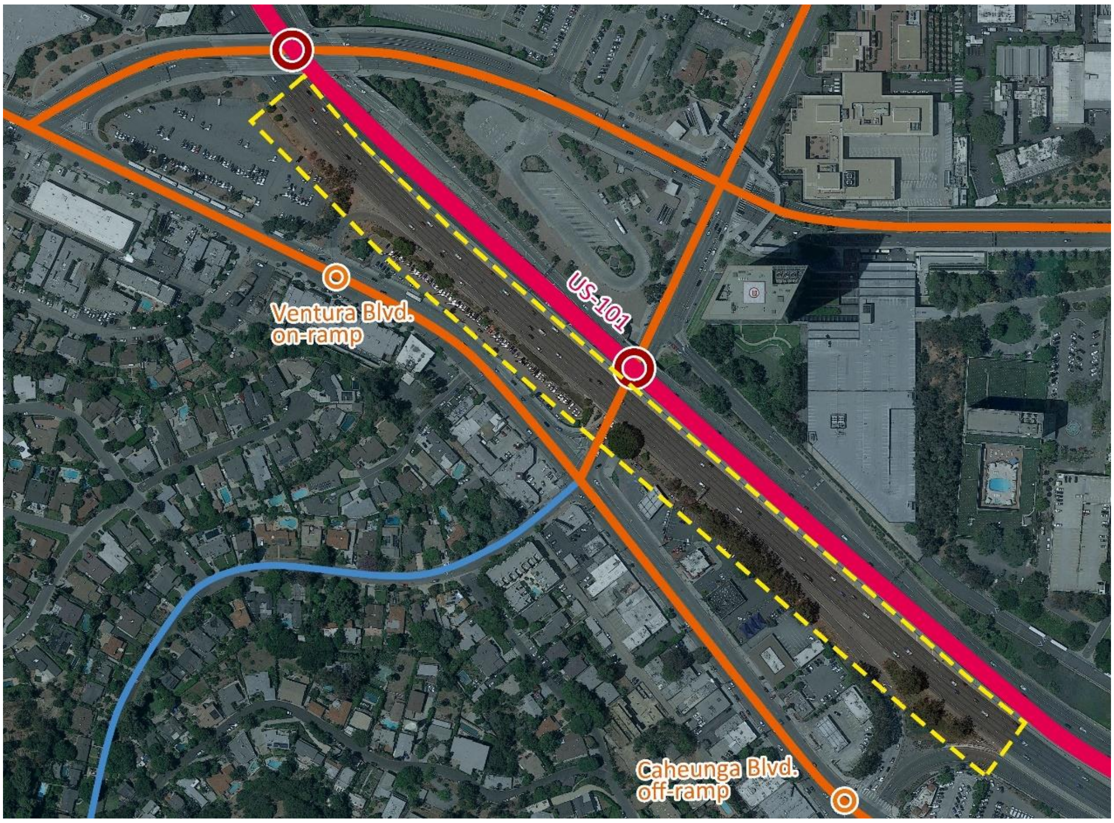

Figure 1. U.S. Highway 101 research segment (Source: Google Maps, 2020)

The US-101 dataset was collected on June 2005 by FHWA on a segment of US-101. The road section is approximately 2200 feet in length, with five mainline lanes and one auxiliary lane connecting to the Ventura Blvd on-ramp and the Cahuenga Blvd off-ramp. Figure 2 shows the configuration of the road section.

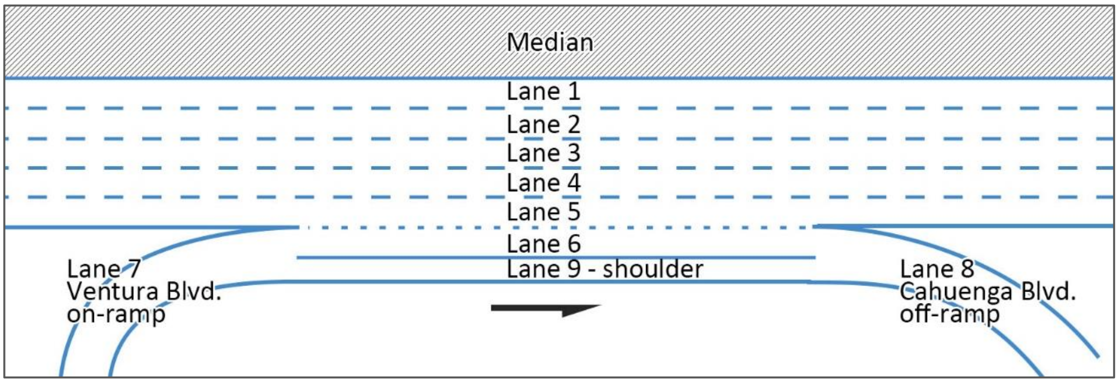

Figure 2. U.S. Highway 101 structural layout (Source: self-depicted)

Eight video cameras are installed on a 36-story building right next to US-101. The eight cameras take pictures every one-tenth of a second. A customized software application was developed to extract detailed vehicle positions and covert the pictures to usable data. The US-101 trajectory dataset has three subsets during morning rush hours, segmented into three 15-minute periods (7:50 a.m. to 8:05 a.m., 8:05 a.m. to 8:20 a.m., and 8:20 a.m. to 8:35 a.m.). These three periods represent congested regimes as well as the transition between congested and uncongested conditions.

3.2 Data processing

Traffic density is a commonly used indicator of traffic condition. Lane-changing behaviors could be described in terms of frequency, implying the chance of taking a lane-changing maneuver, or pattern, clarifying the distribution of lane changes between specific lane-lane pairs. In this paper, lane-changing frequency is selected as the indicator of lane-changing behaviors because the frequency may vary as a response to different densities. Traffic density is a variable related to the spatial extent while estimating lane-changing frequency requires both spatial and temporal coverage (Coifman, 2003).

Traffic velocity, flow, and density are three fundamental variables in the traffic analysis and the relationships between them are major focuses of the traffic flow theory. Traditionally, velocity and flow can be easily derived through a single detector for the reason that the values could be measured at a point. On the contrary, the measurement of density has been somewhat controversial because of its spatial nature. Greenshields (1934) first calibrated density from photogrammetric tools, but this method is only for measuring the instantaneous density and the estimation can be highly complicated with the data from a point detector. Some efforts developed various density estimation methods to deal with graph-constrained situations. However, one can calibrate the instantaneous density precisely through a photograph, and with the development of traffic data collection technologies, the photogrammetric tools make data more accessible. The equation is given by:

$$k_j = \frac{n_j / n_t}{\Delta l \cdot \alpha} \tag{13}$$

Where

k_j = the average density extrapolated to one mile on a particular detection section and during a particular observation period of the jth sample.

n_j = the number of records in the jth sample

n_t = the number of 0.1 seconds in the temporal interval.

Δl= the length of the detection zone.

α = the number of lanes

The lane-changing frequency is an important indicator of lane-changing behaviors. It is defined as the number of lane changes occurring among all lanes along a given length of road and over a given time span (Worral and Bullen, 1970). Consequently, quantifying lane change maneuvers requires both spatial and temporal coverage. Pahl (1972) defined the total number of lane changes as well as lane-changing frequencies. The lane-changing frequency was expressed as the total lane-changing number divided by the number of through-vehicle trajectories that were available in the section. With the summary of previous studies, the equation is given by:

$$r_j = \frac{\sum_{i=1}^{n_j} b_i}{q_j \cdot \Delta l} \tag{14}$$

Where

r_j = the expected number of lane-changing maneuvers of one vehicle extrapolated to one mile on a particular detection section based on the jth sample.

b_i = whether the ith recorded vehicle takes a lane-changing maneuver. 0 represents staying in the current lane and 1 represents successfully changing a lane.

n_j = the number of records in the jth sample.

q_j = the traffic flow extrapolated to one hour on a particular detection section and during a particular observation period of the jth sample.

Δt = the temporal interval.

Δl = the spatial interval.

To conduct a numerical analysis, the values of parameters should be estimated, namely, the temporal interval Δt, and the spatial interval Δl. With respect to the temporal interval, Worral and Bullen (1970) expressed the lane-changing frequency as the lane-changing maneuvers per 500 ft of roadway per minute. Munjal and Hsu (1973) applied 3 minutes as the spatial coverage. Chang and Kao (1991) estimated lane-changing frequency with a 5-min interval. Kesting (2007) used 1 minute as the temporal interval. For the purpose of improving the modeling accuracy, some simulators attempted to reduce the coverage to 0.5 seconds. However, further efforts claimed this change increased computational efforts significantly (Kesting, 2007). Lane change maneuvers are microscopic so that it is hard to say a lane-changing behavior one minute in advance will influence maneuvers in the same section a minute later. Although one could claim that shockwaves caused by a lane-changing behavior would propagate along the road and the impact would last longer than a single lane-changing maneuver, shortening the time interval has a positive effect on this analysis in terms of the fixed detection road section. Consequently, we want the time interval to be as short as possible on the premise that a lane change can be completed. Most lane changes are done in less than 10 seconds, and considering the sample size as well as the calculation accuracy, the temporal interval Δt is set as 10 seconds.

The value of the spatial interval basically hinges on objective conditions, usually dependent on the length of the detection section or the distance between two point-detectors. Gazis and Knapp (1971) studied a road section in the Lincoln Tunnel and estimated traffic density from the sequential speed and the flow data in time series. The maximum length of the tunnel section was 2833ft. Pahl (1972) did not define specific temporal intervals and the spatial distance was 600ft. Gazis and Szeto (1974) improved the measurement accuracy through the analysis of aerial photographs of expressway sections taken every 2 seconds and the spatial interval of the expressway section was about 3/4 mile. Coifman (2002) provided a methodology to estimate the density in a single lane between two loop detectors, the length of which was about 1500ft, based on the assumption that no vehicles change lanes between the two stations. The study site of this analysis is the section of southbound US-101 and the detection area is approximately 2200 feet in length. Similar to the temporal interval, we want the spatial interval to be short and simultaneously a complete lane change maneuver can be performed. Considering the temporal interval is 10s, and the average speed of the dataset is approximately 50ft/s, the spatial interval is considered around 500ft. Considering the total length of the detection area could be equally divided into four sections, the spatial interval Δl is set as 550ft.

So, equation (13) and (14) is transformed to:

$$k_j = \frac{n_j/100}{550ft \cdot 6\ lanes} \tag{15}$$

Where

k_j = the average density extrapolated to one mile on a particular detection section and during a particular observation period of the jth sample.

n_j = the number of records in the jth sample.

and

$$r_j = \frac{\sum_{i=1}^{n_j} b_i}{q_j \cdot 550ft} \tag{16}$$

Where

r_j = the lane-changing frequency of one vehicle extrapolated to one mile on a particular detection section based on the jth sample.

b_i = whether the recorded vehicle makes a lane-changing maneuver. 0 represents staying in the same lane and 1 represents changing a lane.

n_j = number of records in the jth sample.

q_j = the traffic flow extrapolated to one hour on a particular detection section and during a particular observation period of the jth sample.

4. Results

The proposed descriptive analysis includes the plotting of the estimated density and lane-changing frequency data. This allows the unveiling of several interesting relationships between the state of traffic and lane-changing activities. Figure 3 shows the scatter plot of empirical data and each point represents the lane-changing frequency and the average density per lane of a scenario within 550ft in length and 10 seconds in time. It's noteworthy that the lane-changing frequency values of some data points are 0, which means no lane-changing maneuvers happen in the record. It is also found that the values of density of most data points are between 20 and 40. According to Table 1, most of the plotted data are in LOS C, D, and E. This is consistent with the objective condition of the dataset that the data was collected during morning peak hours. The dataset indicates that 30 out of 45 minutes of the collected data were in congestion and the other 15 minutes were in transition.

Table 1. LOS criteria for freeway facilities

Level of Service (LOS)	**Density (pc/mi/ln)**
A	≤11
B	>11-18
C	>18-26
D	>26-35
E	>35-45
F	>45 or any component v_d/c ratio > 1.00

Source: Highway Capacity Manual (HCM), 2010.

According to the LOS criteria, if we separate the density by 18 and 45, traffic conditions could be categorized as three regimes. When the density is below 18, the free flow regime is detected from a small number of plots. Data above the density of 45 exhibits congestion conditions during the observation period. Most data records are observed between 18 and 45, and this is considered to be in the transition of traffic regimes.

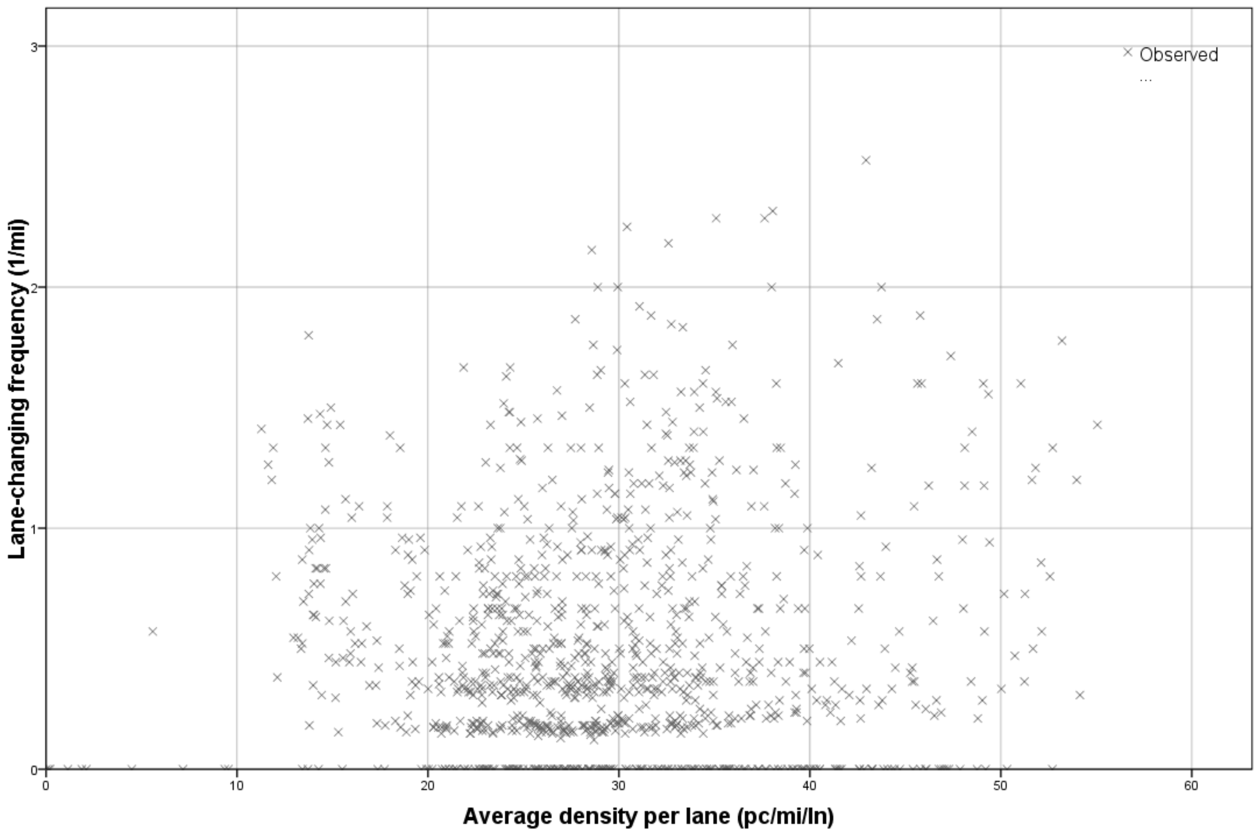

Figure 3. Lane-changing frequency (r) versus average density per lane (k) (Source: self-depicted)

In the free flow regime, drivers feel safe and comfortable. They have adequate space to make decisions and they have lots of options so most lane changes are discretionary. As a result, lane-changing frequencies fluctuate widely in this regime. When the density goes up, the number of vehicles increases, and some impatient drivers need to overtake. Hence the lane-changing frequencies are expected to increase. But it is also found that the highest-peaking of lane-changing frequency happens around the density of 30 to 35. The number of vehicles continues to go up and gaps between vehicles begin to decrease as a result. When the density reaches a bottleneck that gaps are not adequate for most drivers to complete a comfortable lane change. The drivers' lane changing maneuvers might be over-intervened and restrained by the congested traffic flow. Some discretionary lane changes might be rejected and the lane-changing frequencies are expected to decrease. Another bottleneck appears around the density of 40 to 45 where the lane-changing space is further reduced until congestion.

Some numerical analyses were conducted on the platform of SPSS (Statistical Package for the Social Sciences) to validate the findings above. A simple regression procedure was applied to the data, using five statistical models to fit the scatter points, namely, linear, logarithmic, inverse, quadratic, and cubic. R-square, statistical significance and root-mean-squared error (RMSE) were quantified to evaluate the model fit. Figure 4 shows the results of regression fit lines and Table 2 summarizes the statistics of the models.

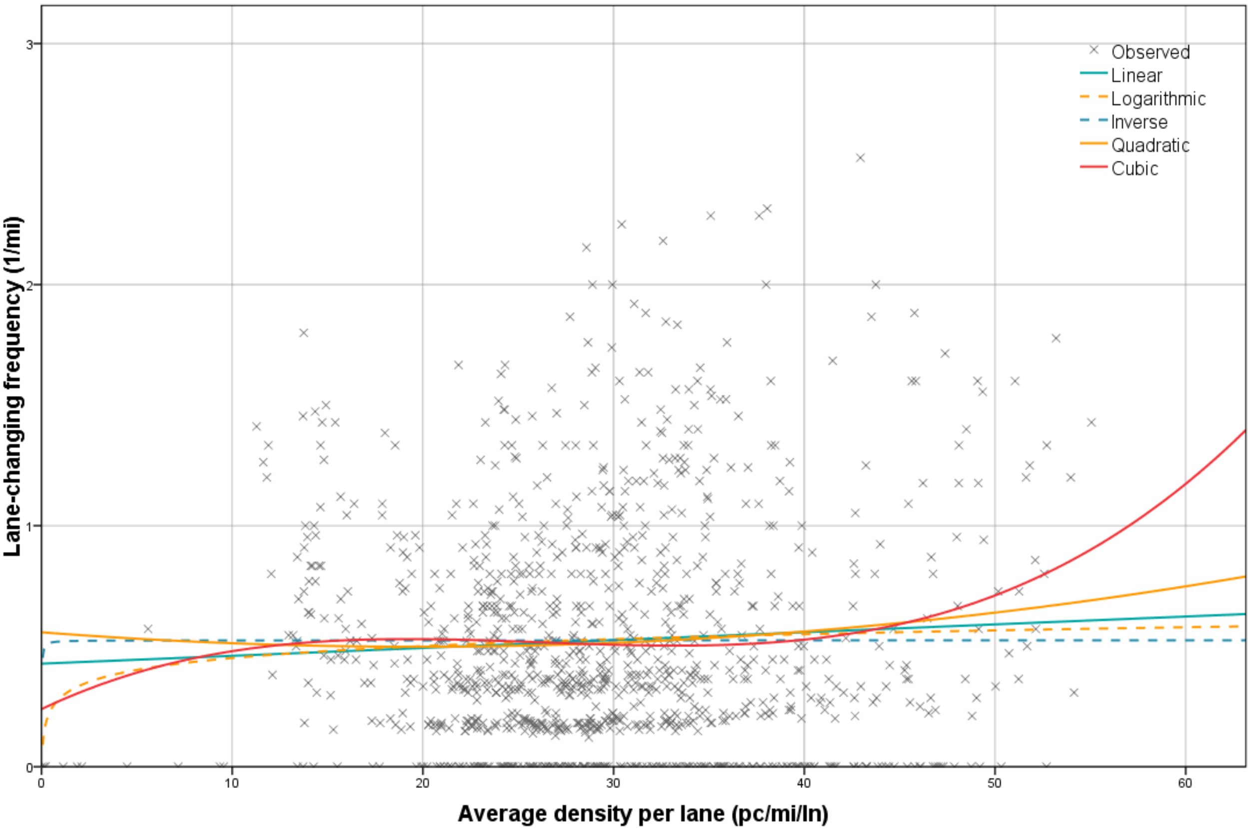

Figure 4. Regression lines to fit the scatter plot (Source: self-depicted)

Table 2. Statistical summary of regression models

Model	Summary		
	R Square	Sig.	RMSE
Linear	.003	.053	0.475
Logarithmic	.005	.020	0.475
Inverse	.001	.222	0.476
Quadratic	.005	.069	0.475
Cubic	.010	.010	0.474

In terms of r-square, the cubic model has a greater r-square than the other models, suggesting that the cubic model is more explanatory than the other four models. The r-square of the cubic model is 0.01, indicating that the density could only explain 1% of the variation of lane-changing frequencies using this model. The r-square values of the other four models are below 0.005, implying that less than 0.5% of the lane-changing frequency variation can be explained by density if using any of these four models. For the sake of stochasticity and complexity, lane-changing decisions are random and hard to be predicted by regression models with only one independent, the small values of r-square are understandable. Comparisons between the five models are more indicative than the absolute values. In terms of statistical significance, the cubic model is statistically significant in a 99 percent confidence interval. The logarithmic model is statistically significant in a 95 percent confidence interval. The linear model and the quadratic model are statistically significant in a 90 percent confidence interval and the inverse model is statistically insignificant. From the perspective of significance, the cubic model is more robust than the other four models, and the inverse model is unreliable. With respect to the RMSE, all the five models have similar values, indicating that these five fit lines have similar errors. It can be concluded from the statistical summary that the cubic model is suggested as the best fit. According to the software output, the equation is expressed as:

$$r = 0.239 + 3.745 \cdot 10^{-2} \cdot k - 1.54 \cdot 10^{-3} \cdot k^2 + 1.9587 \cdot 10^{-5} \cdot k^3 \quad (17)$$

If we differentiate equation (17) with respect to k, the local maximum of on the density of 19.2 and the local minimum on the density of 33.2 can be obtained. The results suggest that in the low-density region from 0 to 19 (LOS A to LOS C), the lane-changing frequency increases significantly with traffic density and reaches the maximum around the density of 19. After that, a slight decrease can be observed. But generally, the lane-changing frequency remains steady between the density from 20 to 40 (LOS C to LOS E). In the high-density region, as the density goes up, the lane-changing frequency starts to increase and approach infinity as a limit, which is contrary to previous studies and common sense. This is the limitation of the cubic model that the prediction results are unrealistic in high-density regions.

5. Conclusions and outlook

The fluctuation on roadways is widely considered as an effect of the lane-changing activity. Capturing lane-changing behaviors explicitly contributes a lot to many aspects including traffic management, autonomous vehicles, etc. Lane-changing maneuvers have been recognized as microscopic behaviors, and the factors of relevant models are mostly dynamic. However, in fact, macroscopic features also play an important role in lane-changing decision processes. It has been proven by previous studies that traffic density, as a traffic condition factor, has an impact on lane-changing behaviors. The analysis presented in this paper attempts to validate the descriptive findings. In order to model these qualitative observations, a simple regression approach is used. This enables to define analytically the relationship between lane-changing behaviors and traffic density.

The results indicate that the cubic model can depict the relationship between lane-changing behaviors and traffic density the best. The r-square of the regression is 0.01, implying that traffic density can explain 1% of the variation of lane-changing behaviors using the cubic model. The result is not plausible but indicative because the r-square value of the cubic model is greater than other models, and it is statistically significant. The fit line suggests that in the low-density region, as traffic density increases, drivers' intention enhances significantly and reaches a highest-peaking around the density of 20. Then the correlation turns negative and the lane-changing frequency goes down as the density goes up, but the fluctuation is not obvious. As for the high-density region, the pattern displayed by the cubic model is contrary to common sense and previous findings. The cubic model needs lots of fine-tuning for the parameters to reproduce high-density scenarios. Above all, the equation obtained in this analysis, if put in traffic simulators, can improve the prediction accuracy by 1% in low-density regimes.

There are several deficiencies of this analysis and future work should focus on the drawbacks. First is that the r-square value of the cubic model is too small that the variation of lane-changing frequencies can hardly be explained by a single independent. Incorporating more variables would improve the interpretability of the model. But considering more independents implies the expected model should be more complex than a simple regression process. Second, the cubic model is poor in the high-density region. The same analysis procedure should be conducted with different datasets to validate the conclusions achieved in this research and this could expand the applicable scope to different types of traffic regimes. Note that the qualitative observations show different patterns in the three distinct regimes. Separating the three traffic conditions and modeling the relationship between lane-changing frequencies and density independently for each regime is another solution. Last but not least, this analysis did not separate MLC and DLC from lane-changing maneuvers. Detailed research into the two lane-changing types would help to comprehensively understand the inherent logic of the relationship.

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