In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

3/17/65
b

A STUDY OF THE BEAT FREQUENCIES BETWEEN MODES OF A HELIUM-NEON LASER

A THESIS

Presented to

The Faculty of the Graduate Division

by

John Henry Bordelon

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Electrical Engineering

Georgia Institute of Technology

June, 1966

A STUDY OF THE BEAT FREQUENCIES

BETWEEN MODES OF A HELIUM-NEON LASER

Approved:

Chairman

Date approved by Chairman: 11/14/66

ACKNOWLEDGMENTS

The author wishes to thank his thesis advisor, Dr. William B. Jones, Jr., for suggesting the subject of this thesis and for his patient criticism and assistance during the two years required for its completion. Thanks are also due to the other members of the reading committee, Dr. Roger P. Webb and Dr. Howard D. Edwards.

The author also wishes to thank his friends and relatives who lent moral support to him through many days of effort, frustration, and reward.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

SUMMARY

The usual type of helium-neon laser utilizes an optical cavity with mirrors separated by a distance in the order of magnitude of 1 meter. The tube containing the gas mixture is usually closed at the ends by windows oriented at the Brewster angle. This feature minimizes reflections from the windows and thereby results in a cavity of higher Q. It also causes the output light to be polarized in the axial plane of the Brewster window normal. This effect is also helpful, because otherwise the polarization of the output light has been found by experiment to vary randomly with time.

Depending upon the conditions of operation, the laser may produce one or many discrete optical frequencies. The frequencies of these oscillations are predictable from the spacing of the mirrors and their radii of curvature.

In this research, a study of the beat frequencies between the oscillating modes has been made by generating the beat frequencies in a photomultiplier tube and examining them with a radio frequency spectrum analyzer. Differences between the theoretical and observed values of the beat frequencies have been found. The magnitudes of the differences depend principally, it appears, on which modes of oscillation are present in the laser and on how each mode is spatially oriented with respect to the Brewster windows. An attempt is made to show that the Brewster windows are the principal, if not the only, cause of the differences observed.

Beat frequency studies have been used to determine the modes of oscillation present in the laser. In many cases the analysis is made easier by the presence of the differences discussed above.

Some experimental techniques which are useful in beat frequency studies are described. Chief among these is the use of an iris external to the optical cavity to allow only certain small portions of the output light pattern to reach the photomultiplier tube.

CHAPTER I

INTRODUCTION

A gas laser in its basic form is a light-frequency oscillator. Like other oscillators designed to deliver a useful output, it is composed of a resonant system and a gain mechanism for restoring energy lost both in the resonant system and as output taken off. The gain mechanism is the stimulated emission of radiation from atoms in an excited state. The energy stored in the gas is supplied by electrical excitation. The gain mechanism is spatial, since light signals passing through different gas volumes do not affect one another. It is also frequency dependent, the available gain depending upon the frequency to be amplified. The gain versus frequency plot resembles a resonance curve.

The resonant system is an optical interferometer, composed of two mirrors placed near the ends of the tube. These mirrors are made reflective by multiple coatings of dielectric material. The small fraction of the light energy which is not reflected is transmitted through the mirror as useful output. This optical resonator has dimensions many orders of magnitude greater than the optical wavelengths being handled. As is shown in Chapter II, it has many resonant frequencies, or possible modes of oscillation. Depending upon the mirror adjustment and spacing, and the degree of excitation of the gas, one or more of these modes of oscillation may be in operation.

Since the width of the gas resonance for helium-neon operating at 6328 Å is about 1 GHz., and the optical frequencies involved are approximately 475 THz., conventional optical spectrographs cannot easily provide the resolution necessary to discern each mode. In order to study the modes of oscillation, then, some other method must be used. Radio frequency spectrum analyzers are superheterodyne receivers with a visual display. If another laser having a single mode of oscillation could be used as a local oscillator together with an optical mixer, an optical spectrum analyzer of the superheterodyne type could be constructed. This approach is quite possible. There is a somewhat simpler and less expensive approach, however; this is to examine the beat frequencies between the modes of oscillation of the laser. This approach is less direct but, in some respects, more satisfactory; it may be accomplished by allowing the beam to fall on the photocathode of a photomultiplier tube. Because the tube is a square-law device, the beat frequencies appear at the output and may be studied with a radio frequency spectrum analyzer. Much about the behavior of the laser may be deduced from such studies.

It was the purpose of this research to study the beat frequencies between modes of a helium-neon laser. The existence of beat frequencies predicted by theory has been confirmed and their properties studied to some degree. The usefulness of beat frequency investigations to identify the modes of oscillation present has been investigated and useful experimental procedures have been developed.

CHAPTER II

REVIEW OF THE PERTINENT LITERATURE

Prior to, and in the early stages of development of lasers, several papers appeared on the resonant modes of optical cavities of the type used in lasers. Papers were published on optical cavities using plane mirrors and on cavities using spherical mirrors. Because the laser used in this research used spherical mirrors, papers on this type are of chief interest. Of principal interest are those by Boyd and Gordon[4], and Boyd and Kogelnik[5]. Boyd and Gordon show results for the confocal resonator, one in which the two spherical mirrors have equal radii of curvature and a common focus. Boyd and Kogelnik show results for a general resonator having separation d and radii of curvature b_1 and b_2. The confocal case is thereby included. The case of plane, parallel mirrors is not included. The Boyd and Kogelnik formulas are

$$\nu = \left(\frac{c}{2d}\right)\left[q + (1+m+n)f\right] \quad , \tag{1}$$

$$f = \frac{1}{\pi} \arccos \left[\left(1 - \frac{d}{b_1}\right)\left(1 - \frac{d}{b_2}\right)\right]^{\frac{1}{2}} \quad , \tag{2}$$

where c is the free space velocity of light, and q, m and n are integers which define the various resonant modes of the cavity.

The splitting factor, f, can vary from 0 to 1. For the confocal case, where $b_1 = b_2 = d$, $f = \frac{1}{2}$.

Modes which have the same m and n but different values of q are said to be longitudinal modes. It can be seen that with m and n constant that longitudinal modes are separated by integer multiples of the quantity c/2d. This separation is referred to as the longitudinal frequency, or when observed as a beat frequency, as the longitudinal beat frequency. It is

$$\nu_{\circ} = \frac{c}{2d} \quad . \tag{3}$$

It is the same for all values of m and n.

A group of longitudinal modes having the same m and n constitute a transverse mode set. Beat frequencies arising between longitudinal modes belonging to different transverse mode sets are called transverse beat frequencies. This beat frequency is zero if the two longitudinal modes have the same q and if (m+n) is the same for both. Otherwise the transverse beat frequency is

$$\nu_t = \Delta(m+n)\, f\, \nu_{\circ} \quad . \tag{4}$$

Because the modes of the resonator contain essentially no axial components of field, a mode is usually called a TEM_{mnq} mode. In this paper a particular transverse mode set is identified as an mn mode set, as for example the 10 mode set.

The actual frequencies of the modes in a helium-neon laser do not occur exactly at the resonant frequencies of the optical cavity. For one thing, the frequency characteristic of the gas causes the actual modes to shift from the cavity resonant frequencies toward the center frequency of the gas resonance. As pointed out by Bennett[1], the "pulling" increases with increasing excitation of the gas and varies with the distance of the cavity resonance from the center frequency of the gas resonance. The pulling is greater for modes farther from the center. It would be expected then that the longitudinal beat frequency, ν_0, would be split into two or more different frequencies. Bennett also shows that depletion of energy states by an oscillating mode alters the shape of the gas resonance curve so as to cause adjacent modes to move apart. The various modes, in effect, adjust their frequencies so as to arrive at the peaks of the curve that is the result of the optical cavity resonances and the altered gas resonance curve. The degree to which one mode repels another depends on the amount of gas volume shared in common by the two modes.

The pulling of modes toward the center of the gas resonance can be expressed approximately. Lengyel[7] and Bennett[1] show that the corrected frequency of oscillation is

$$\nu = \nu_c + (\nu_m - \nu_c) \frac{\Delta \nu_c}{\Delta \nu_m} \quad , \tag{5}$$

where ν_c and ν_m are the cavity resonance and the gas resonance center frequencies, respectively. The $\Delta \nu_c$ and $\Delta \nu_m$ terms are the widths of the resonances. Bennett[2] gives the gas resonance width as about 1500 MHz. For the TEM_{q00} mode, Boyd and Gordon give the resonator Q as

$$Q = \frac{2\pi d}{\alpha \lambda} \quad , \tag{6}$$

where α is the fractional power loss per reflection from a mirror surface. The mirror separation is d. For a loss per reflection of 5 percent, which is typical when Brewster window losses are taken into account, and a d = 150 cm., $Q \approx 3 \times 10^8$ at 6328 Å. For a mode separated from ν_m by 100 MHz., the pulling is about 100 KHz. This increases proportionately as the separation $\nu_m - \nu_c$ increases.

The question as to whether or not the longitudinal beat frequency, ν_o, will be split can be answered to a first approximation by the derivation below, where three modes are considered to be oscillating. The derivation can be extended to as many modes as desired.

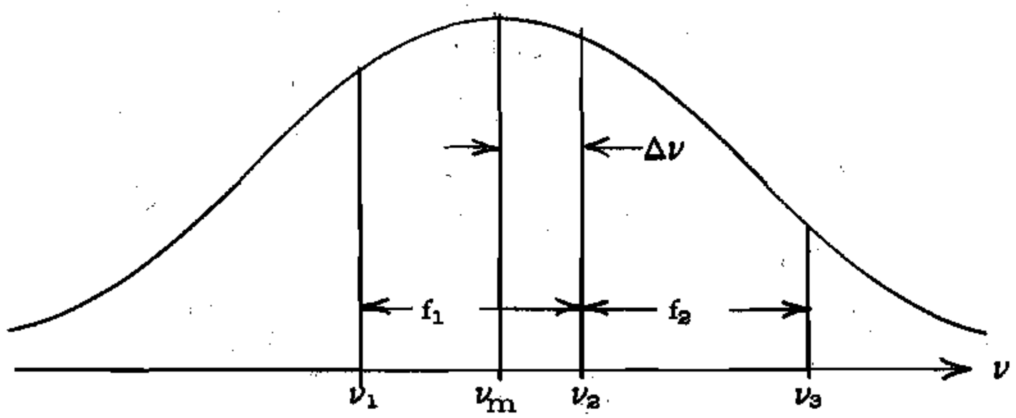


Figure 1. Pulling of Cavity Resonances Toward Line Center

The nomenclature used in the derivation is defined in Figure 1. Unprimed terms are used for frequencies found from cavity dimensions and primed terms are used for frequencies found when mode pulling is taken into account. The pulling toward the line center, ν_m, is $K(\nu_n - \nu_m)$ where ν_n is the cavity resonance ν_1, ν_2, ν_3.

$$\nu_2 - \nu_1 = \nu_0 \qquad \nu_3 - \nu_2 = \nu_0 \qquad f_1 = f_2 = \nu_0$$

$$\nu_1' = \nu_1 + K(\nu_0 - \Delta\nu) \qquad f_1' = \nu_2' - \nu_1'$$

$$\nu_2' = \nu_2 - K(\Delta\nu) \qquad f_2 = \nu_3' - \nu_2'$$

$$\nu_3' = \nu_3 - K(\nu_0 + \Delta\nu)$$

$$f_1' = \nu_2 - K(\Delta\nu) - \nu_1 - K(\nu_0 - \Delta\nu) = \nu_0 - K\nu_0$$

$$f_2' = \nu_3 - K(\nu_0 + \Delta\nu) - \nu_2 + K(\Delta\nu) = \nu_0 - K\nu_0$$

Therefore, to a first approximation, the longitudinal beat frequency should not be split.

The mode repulsion effect cannot be expressed in a mathematical form that lends itself to direct application except for one or two special cases. A few things may be said qualitatively however. When a mode oscillates at a particular frequency, a hole of the order of the natural line width is burned in the Doppler broadened line. Thus the shape of the gas resonance curve is modified, and nearby modes are repelled from the hole. The bottom of the hole is the threshold level for the particular system, so that the smaller the available gain above threshold, the less the distortion of the resonance curve and the smaller is the repulsion effect. The repulsion is greater for modes close together in frequency. The width and depth of the hole increase as the gain above threshold increases. Bennett observed repulsions of the order of 30 KHz. At the 6328 Å transition, gain above threshold is smaller and repulsion should be less. It should be possible, by using the laser with spherical reflectors, to move modes much closer together than the 160 MHz. used by Bennett, and to therefore observe much larger repulsion effects if the modes used occupy considerable common mode volume.

Many investigators have used beat frequency studies to learn of some particular characteristic of the gas laser. Bennett[1] utilized beat frequency studies in his work on mode pulling. Goldsborough[6] uses observation of the beat frequencies to identify the transverse mode sets present in complex output patterns. He deals only with the simplest cases, however. He also observes that the longitudinal beat frequency for different transverse mode sets is not the same. He concludes that this can be explained by Bennett's mode-repulsion theory. The splits he observes are of smaller magnitude than those observed by Bennett, but since the degree of repulsion is dependent upon the gain of the gas resonance, and the gain at 6328 Å is much smaller than at the wavelength used by Bennett, Goldsborough concludes that his results are reasonable.

Goldsborough's experimental technique has at least one fault. He shows that beat frequencies between modes of different transverse mode sets cannot be observed when the entire beam falls on the photocathode of the photomultiplier tube used to generate the beats. This phenomenon is due to the fact that the eigenfunctions describing each mode form an orthogonal set. Although the same beat frequency may arise out of the superposition of two modes at various points on the photomultiplier photocathode, the relative phases of the beat frequencies will be such that they will cancel out and no beat frequency will be observed. To avoid this cancelling effect, he merely blocks off large areas of the light pattern before it reaches the photomultiplier tube in the hope of destroying the symmetry of the pattern so that beat frequencies will be observed. This masking technique which he uses leaves a number of things to be desired. First, there is no assurance that the coarse masking technique as he uses it will yield the desired beat frequencies. Secondly, the method does not reveal from which areas of the light output pattern the beat frequencies originate. Also, if all the possible beat frequencies are displayed, the sensitivity of the photomultiplier tube is greatly reduced.

Subsequent to this research a paper was published by Uchida[13] which contains another point of interest. It is shown that spatially independent modes of the same transverse order, that is, ones having the same (m+n), do not oscillate at the frequencies predicted by theory. Splits of the order of 0 to 1.5 MHz. occurred.

CHAPTER III

INSTRUMENTATION AND EQUIPMENT

The laser tube used for these studies was a commercial helium-neon unit of standard design. Figure 2 shows in schematic form how the tube was arranged in relation to the other optical equipment employed and to the detection apparatus. Inside diameter of the quartz tube was 5 mm. The end bells and Brewster windows were 7056 glass. The tube was supported above an optical bench, on which rested the two spherical mirrors forming the resonant cavity. The mirrors each had a radius of curvature of one meter. The position of the mirrors was adjustable along the bench. A variable iris was available for use within the optical cavity for suppression of undesired modes. The laser was radio frequency excited. The excitation frequency was 28 MHz.

Essential to this research was the ability to use an external adjustable iris so as to allow only certain portions of the emerging light pattern to pass on to the beat frequency detecting apparatus. Two systems were tried. The detection apparatus was about ten feet from one end of the laser. At first the beam was enlarged with a diverging lens and the iris was placed at a point where the beam had become large enough that portions of the light pattern could be fairly easily discerned and masked with the iris. A converging lens system was then used to restore parallelism to the beam so that it would not diverge in travelling

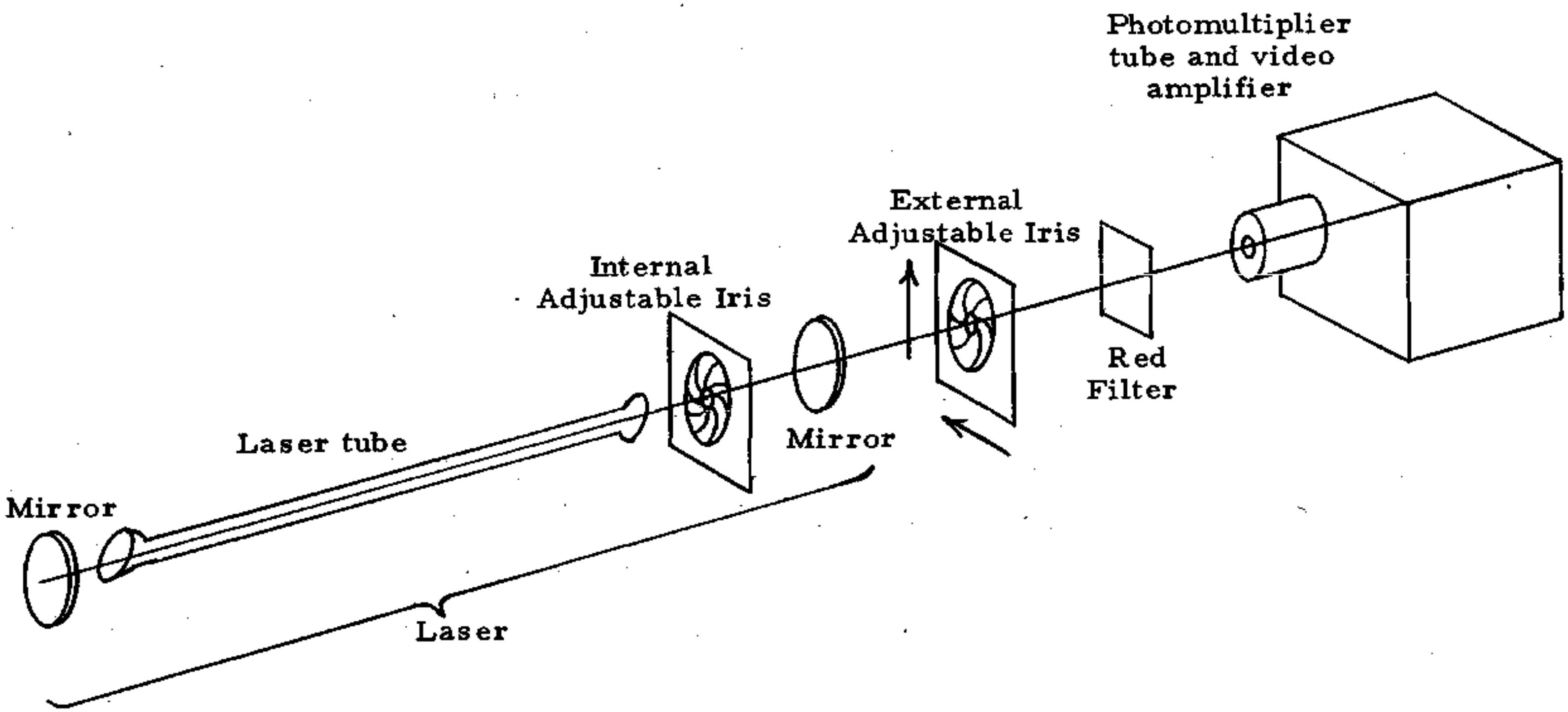


Figure 2. Physical Arrangement of Laser and Detection Apparatus

the remaining distance to the detection apparatus. Due to the poor quality of the lenses used, it became apparent that the experimental results were being rendered useless by distortion in the lenses. This system was abandoned. Instead, the iris was placed close to the detection equipment where the beam had enlarged sufficiently for details to be discernible in the pattern. The photomultiplier was operated inside a tubular light shield, with a 3/8" hole for entrance of the laser beam at one end. A red filter was placed over the hole to aid in excluding room light. A system which allowed the remaining light to be concentrated at a small spot near the center of the photomultiplier tube photocathode would be the most desirable. The frequency response of photomultiplier tubes is much better under such conditions.

The detection apparatus consisted of a photomultiplier tube coupled to a wideband video amplifier. The output of the video amplifier was examined on a spectrum analyzer. Figure 3 is a block diagram of the detection apparatus. The photomultiplier was an RCA type 7102. A variable voltage power supply was used to supply high voltage to the photomultiplier voltage divider string. An external vacuum tube voltmeter was used to monitor the photomultiplier anode current. The amplifier had a gain of 30 db. which was fairly uniform from 2 MHz. to 300 MHz. A detailed description of the amplifier may be found in the appendix. The output of the video amplifier was fed through 50 ohm coaxial cable to the spectrum analyzer. The spectrum analyzer was a Tektronix Model L20 plug-in unit for the Tektronix Model 545B oscilloscope. An accurate signal generator was available for frequency measurements by substitution.

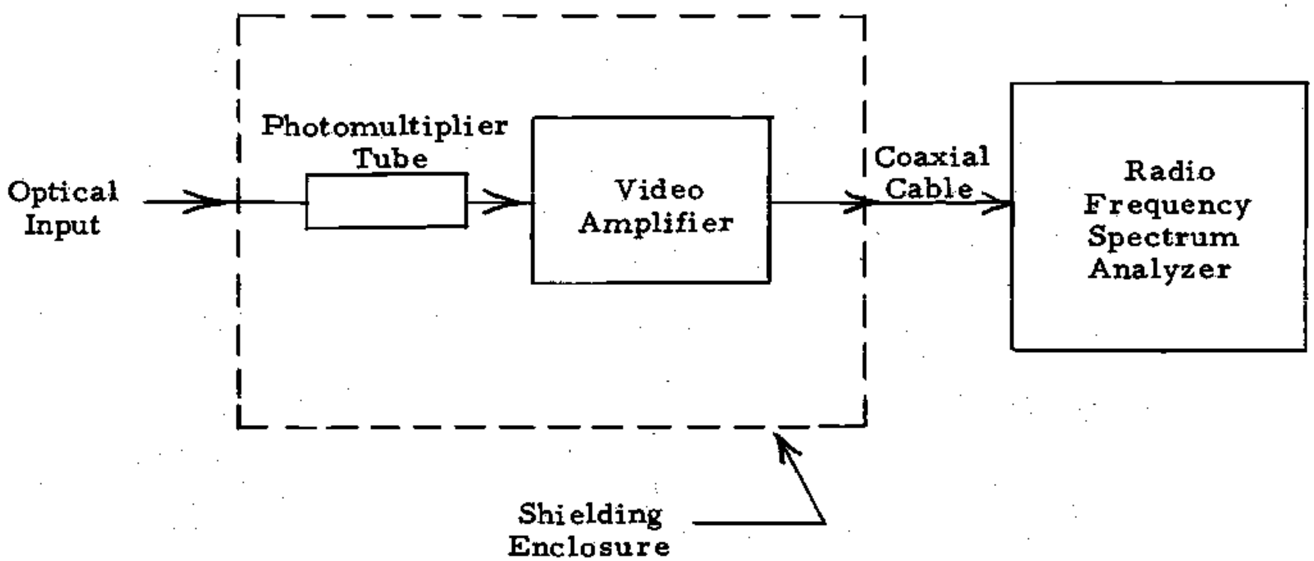


Figure 3. Block Diagram of Detection Apparatus

Because radio frequency excitation was used for the laser, interference from the source was a problem. This was eliminated as a problem by bonding the chassis of each piece of equipment to a large copper ground screen placed on the workbench. The photomultiplier tube circuits and the video amplifier were all built in a shielded aluminum chassis.

The characteristics of the photomultiplier tube are quite important to this research. In choosing a suitable tube, good sensitivity at 6328 Å was desired, along with good frequency response. The type 7102 seemed to have a satisfactory combination of these characteristics.

CHAPTER IV

PROCEDURE

To obtain a desired mode or modes in the laser, the mirrors were adjusted in their mounts. An iris placed in the cavity made it possible to discriminate against higher order modes by limiting the radial extent of the beam. Visual inspection was usually sufficient to determine what modes were present in simple cases.

To set the laser to a desired longitudinal beat frequency, the spectrum analyzer was set to the desired frequency by observing a signal of the desired frequency from an accurate signal generator. The mirrors were then moved until the longitudinal beat frequency appeared at the same point as the marker signal on the spectrum analyzer. Accuracy was better than ±50 KHz.

In order to measure a beat frequency, most tuning of the spectrum analyzer was done with a dispersion of ±500 KHz. After locating a beat frequency, the signal generator was substituted as a signal source and tuned to the same frequency as observed on the spectrum analyzer. The signal generator was then calibrated with its internal standard at the nearest 1 MHz. point and then tuned back to the beat frequency for measurement. Accuracy was ±100 KHz. or better.

Measurement of the length of the cavity was accomplished with a steel tape.

The determination of modes present in a pattern was dependent upon the iris external to the laser cavity. When examining a pattern for beat frequencies, the iris was first used to examine the center of the pattern only, then the outermost bright spots, to determine if they were parts of single modes or contained parts of more than one mode. Then, from the evidence gained in this manner, a reasonable assumption was made as to the modes present. Examination of other prominent portions of the pattern aided in confirming or modifying the original assumption.

CHAPTER V

DISCUSSION OF RESULTS

Beat Frequency Measurement

The first observations of interest pertained to beat frequencies which should exist for given transverse mode sets. The spacing of the mirrors was adjusted so that the longitudinal beat frequency, ν_0, existed at a desired frequency. The distance between the mirrors was measured and compared to the value obtained from equation (3). The data are summarized in Table 1.

Table 1. Measured and Calculated Values of Mirror Separation For Various Values of ν_0.

ν_0	d (measured)	d (calculated)	difference
115 MHz.	129.7 cm.	130.4 cm.	0.7 cm.
112	133.2	134.0	0.8
110	135.7	136.3	0.6
104	143.4	144.1	0.7
98	152.3	153.0	0.7

The average difference is 7 mm. Part of the error is due to the Brewster windows with their greater index of refraction as shown in the following derivation. Referring to Figure 4 and using Snell's law, it can be seen that

$$\sin\beta = \frac{\sin 56.6°}{1.48} = \frac{0.8348}{1.48} = 0.563 \ .$$

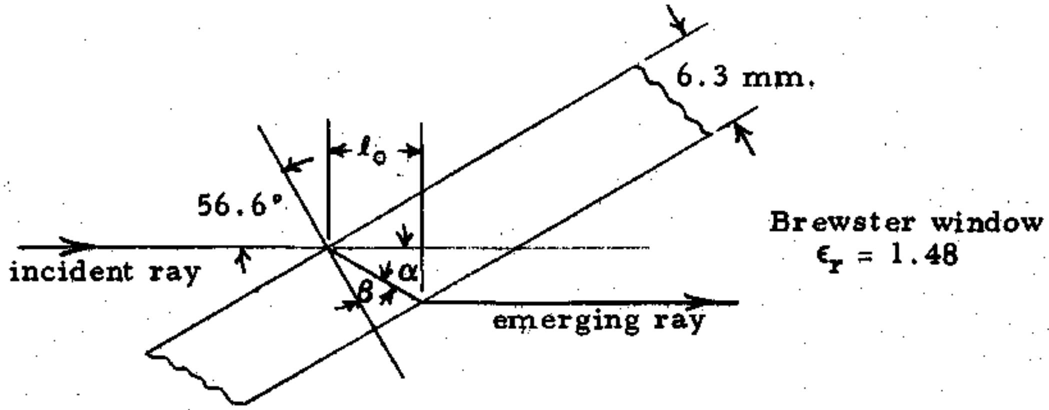


Figure 4. Apparent Increase in Path Length Through Brewster Window

$$\beta = 34.25°$$

The distance through the window is then

$$\frac{0.63}{\cos\beta} = \frac{0.63}{0.8265} = 0.762 \text{ cm.}$$

In order to find ℓ_0 the angle α must be known.

$$\alpha = 56.6° - 34.25° = 22.35°$$

Therefore

$$\ell_0 = 7.62 \cos 22.35° = 0.704 \text{ cm.}$$

The apparent increase in d is

$$\Delta\ell = 2[(0.762)(1.48) - 0.704] = 8.5 \text{ mm.}$$

Also to be considered is the fact that ν_0 will be somewhat lower than the expected value by the factor $K\nu_0$. With a ν_0 of 115 MHz., $K\nu_0 = 115$ KHz. Therefore, to establish a given ν_0, the cavity must be made somewhat shorter than the value predicted by equation (3). The amount may be found from

$$\frac{\partial \nu_0}{\partial d} = -\frac{c}{2d^2} = -\frac{3 \times 10^8}{2(1.304)^2} = -88 \text{ KHz./mm.}$$

The amount by which the spacing must be decreased is

$$\frac{115}{88} = 1.3 \text{ mm.}$$

Therefore the total difference between calculated and measured d should be 9.8 mm. but the observed value is 7 mm. The discrepancy has not been explained but is quite likely due to error in measurement of the distance between the mirrors.

The appearance of the longitudinal beat frequency on the spectrum analyzer depended on the adjustment of the laser. With only one mode set present, the beat frequency was sharp and clearly defined, but with more than one mode set available the beat frequency was not so sharp and was quite noisy in appearance. The presence of splittings between longitudinal beat frequencies of different mode sets and within mode sets is the probable cause. Uchida[13] has studied these in detail. Unfortunately, attempts to study the longitudinal beat frequency using greater resolution in the spectrum analyzer only yielded hints of splitting. The instability of the laser used tended to mask these effects.

A large amount of data was taken concerning beat frequencies between modes of different transverse order. The procedure again was to set ν_0 at a desired value and then to adjust the laser to produce certain desired modes. Visual inspection was used for mode determination, since only low order modes were used. This procedure entailed many hours of trial and error. An additional, though not serious difficulty, is the tendency for modes of lower transverse order to extinguish on the appearance of a higher order mode. Rigrod[10] has observed and discussed this phenomenon.

When the desired modes were obtained, the entire frequency range below ν_0 was covered and the observed beat frequencies recorded. In some cases, one or two modes more than the ones listed may have been present. The modes recorded are those between which the given beat frequencies were observed. The error is the difference between the frequencies observed and those computed from equations (2) and (3) and listed in Table 2. Blanks appear where data were not taken. The data appear in Table 3.

Table 2. Theoretical Values of d and ν_t and for Various Values of ν_0

ν_0	d	ν_t
83 MHz.	180.8 cm.	66.2 MHz.
94	159.6	66.1
98	153.0	66.4
104	144.1	67.1
110	136.3	68.0
112	134.0	68.3
115	130.4	68.8

Table 3. Observed Transverse Beat Frequencies

MHz.	Observed beat frequencies MHz.		Error MHz.	Modes used
83	65.1	17.8	1.1	00 & 01
	47.9	35.1	1.7	00 & 11
94	64.9	29.1	1.2	00 & 01
	36.5	57.5	1.7	00 & 11
104	66.8	37.2	0.3	00 & 10
	29.5	74.5	0.7	00 & 20
104	66.0	----	1.1	00 & 01
	----	75.6	2.2	00 & 02
115	67.5	47.5	1.3	00 & 01
	20.1	94.9	2.5	00 & 02

The consistency of these errors led to more experiments in which the purpose was to determine if the errors would change if the resonator axis were shifted with respect to the tube axis or if one of the mirrors were rotated about its axis. The data in Table 4 is the result.

For each set of modes, the sum of the observed beat frequencies is ν_0, as would be expected. The beat frequencies corresponding to $f\nu_0$ and $2f\nu_0$ are slightly lower than the theoretical value. The error is greater for modes distributed in the vertical direction than for modes distributed in the horizontal direction. Modes distributed vertically are called m0 modes. It should be noted that the axial plane of the Brewster window normal was also vertical. Since rotation of one mirror produces no change in these general observations, they are not the result of aberration of the mirror surface itself. Since there are no significant changes in the errors for different values of ν_0, frequency pushing or pulling effects are probably not the cause.

Table 4. Additional Transverse Beat Frequency Data

MHz.	Observed beat frequencies MHz.		Error MHz.	Conditions
115	68.3	46.7	0.5	m0 modes
	21.8	93.2	0.8	
115	67.6	47.4	1.2	0n modes
	20.5	94.5	2.1	
115	67.4	47.6	1.4	left mirror lowered
	----	----		slightly; 0n modes.
115	----	----		left mirror still
	21.9	93.1	0.7	lowered; m0 modes.
115	68.3	46.7	0.5	right mirror rotated
	21.6	93.4	1.0	1/4 turn; m0 modes.
115	67.7	47.3	1.1	right mirror still
	21.5	94.5	2.1	rotated; 0n modes.

Rigrod[9] et al., observed transverse beat frequencies but observed no differences with their experimental error. They did not give their experimental error, but it easily could have been large enough to mask the small differences observed here.

The Brewster windows are suspected as the cause of these errors. References 2, 6, 7, and 8 refer to the effects of Brewster windows on mode patterns. These references point toward the fact that the Brewster windows create some astigmatism, that is, the mirrors do not appear spherical and are not axially symmetric. Uchida[13] refers directly to the effect of this astigmatism on the frequency of modes of the same order but having different spatial distributions.

In order to study these errors, two cases must be considered. The first is the error between the observed and theoretical values of $f\nu_0$ for m0 modes. The second is the error for 0n modes.

A spherical mirror, excited by a plane wave reflects a spherical wave of radius half that of the mirror. What the mirror looks like, in a sense, through the Brewster window, may be evaluated by finding the shape of the wave front after it emerges from the window.

Only a small portion of the aperture of the mirror is involved. Boyd and Kogelnik show that for a nonconfocal resonator with mirrors of curvature b' and separation d that the confocal system which will generate the same phase fronts has separation and mirror radius of curvature b given by

$$b = \sqrt{2db' - d^2} \ .$$

In order to limit the calculations to be done, only the case of $b' = 1$ meter and $d = 130.4$ cm. will be considered, for which $\nu_0 = 115$ MHz. For this case $b = 0.95$ meter. For this equivalent confocal resonator the function describing the electric field in the horizontal direction for the 10 mode or in the vertical for the 01 mode is

$$g(x) = \sqrt{\frac{2}{b\lambda}} \ \pi x \, \epsilon^{-\frac{\pi x^2}{b\lambda}} \ .$$

This is the description for the pattern at the mirrors of the resonator. The point of greatest intensity of the light pattern can be found by differentiating g(x) and setting it equal to zero.

$$0 = 1 - \frac{2\pi x^2}{b\lambda} \ .$$

Therefore the distance from the axis of the resonator at which the pattern is of greatest intensity is

$$s = \sqrt{\frac{b\lambda}{2\pi}} \quad .$$

Therefore, using another Boyd and Kogelnik relation, the distance from the axis to the brightest point at any distance d/2 from the center is for the equivalent confocal or nonconfocal resonator

$$s = \frac{1}{2}\sqrt{\frac{b\lambda}{2\pi}} \sqrt{1 + \frac{d^2}{b^2}} \quad .$$

At the resonator center

$$s_0 = \frac{1}{2}\sqrt{\frac{b\lambda}{\pi}}$$

and for b = 0.95 meter

$$s_0 = 0.23 \text{ mm.}$$

At the mirrors s = 0.39 mm. The observed value was roughly 0.5 mm.

For m0 modes, a prediction of the errors may be made by considering a spherical wave front of radius 0.5 meter impinging on the Brewster window in the horizontal plane as shown in Figure 5. The approximation is made in Figure 5 that if α is very small, the distance T' is 7.62 mm., the distance through the window shown in Figure 4.

From Figure 6

$$l_1 = T' \tan\theta$$

and

$$l_1 + l_2 = T' \tan\alpha$$

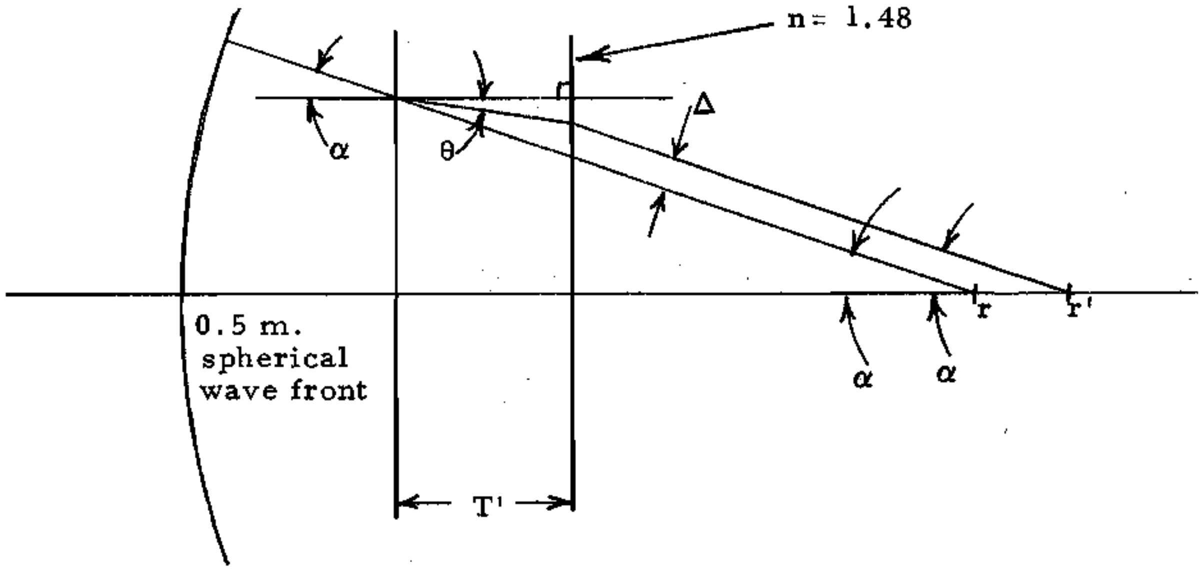


Figure 5. Spherical Wave Front Impinging On Brewster Window in Horizontal Plane

An enlarged view of Figure 5 is Figure 6.

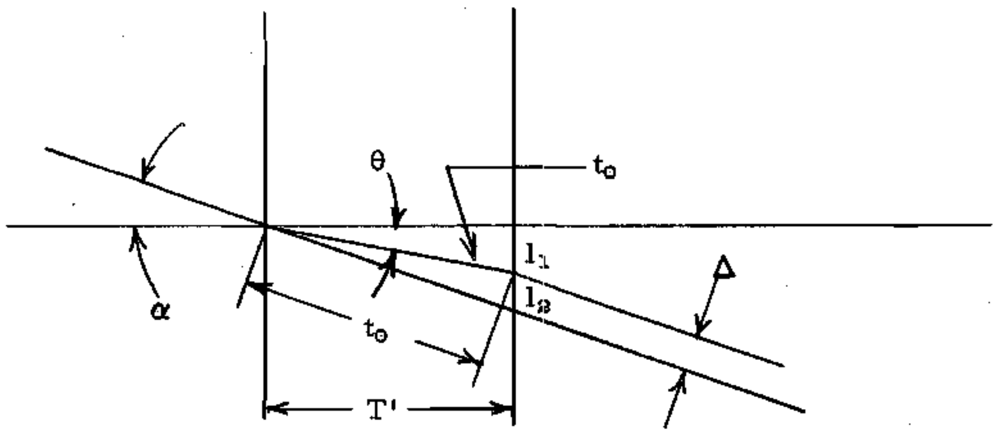


Figure 6. Detail of Figure 5

and

$$\Delta = l_2 \cos\alpha = T'(\tan\alpha - \tan\theta)\cos\alpha \quad .$$

Now from Figure 5

$$r' = r + \frac{\Delta}{\sin\alpha}$$

$$= r + \frac{T'(\tan\alpha - \tan\theta)}{\tan\alpha}$$

$$= r + T' - T'\frac{\tan\theta}{\tan\alpha} \quad .$$

But α is very small, being governed by the spot size s and the mirror focal length. It is about 0.001 radian, so

$$r' \approx r + T' - T'\frac{\theta}{\alpha} = r + T'(1 - \frac{\theta}{\alpha}) = r + T'(1 - \frac{1}{n})$$

$$= 0.5 + (0.00762)(0.325) = 0.5025 \text{ m.}$$

Therefore the apparent mirror b = 1.005 meters. Insertion of this value into equations (1) and (2) yields $f\nu_0$ = 68.5 MHz. The theoretical value without Brewster windows is 68.8 MHz. and the observed value is 68.3 MHz. Neglected in this calculation is the slight differences in distance traveled by rays through the window at different values of α. The result of such effects would be to modify the shape of the emerging wave front slightly. This effect is probably negligible as can be shown approximately by referring again to Figure 6. The distance $(t_1 - t_0)$ is the increase in path length. Again

$$\theta \approx \frac{\alpha}{n}$$

and therefore

$$t_1 = \frac{7.62}{\cos\theta}$$

and

$$t_o = t_1 \cos(\alpha - \theta) \quad .$$

Now

$$t_1 = \frac{7.62}{\sqrt{1 - \theta^2}}$$

or, since θ is very small,

$$t_1 \approx 7.62 \left[1 + \frac{\theta^2}{2}\right] \quad .$$

Similarly

$$t_o = \frac{7.62}{\cos\theta}\left[1 - \frac{(\alpha - \theta)^2}{2} \;\; = 7.62 \;\; 1 + \frac{\theta^2}{2} \;\;\; 1 - \frac{\alpha^2}{2} + \alpha\theta - \frac{\theta^2}{2}\right]$$

$$t_o = 7.62\left[1 - \frac{\alpha^2}{2} + \alpha\theta - \frac{\theta^2}{2} + \frac{\theta^2}{2} - \frac{(\alpha\theta)^2}{4} + \frac{\alpha\theta^3}{2} - \frac{\theta^4}{4}\right]$$

$$\approx 7.62\left[1 - \frac{\alpha^2}{2} + \alpha\theta\right] \quad .$$

With expressions for t_1 and t_o

$$t_1 - t_o = 7.62\left[1 + \frac{\theta^2}{2} - 1 + \frac{\alpha^2}{2} - \alpha\theta\right]$$

$$= 7.62\left[\frac{\theta^2}{2} + \frac{\alpha^2}{2} - \alpha\theta\right] \quad .$$

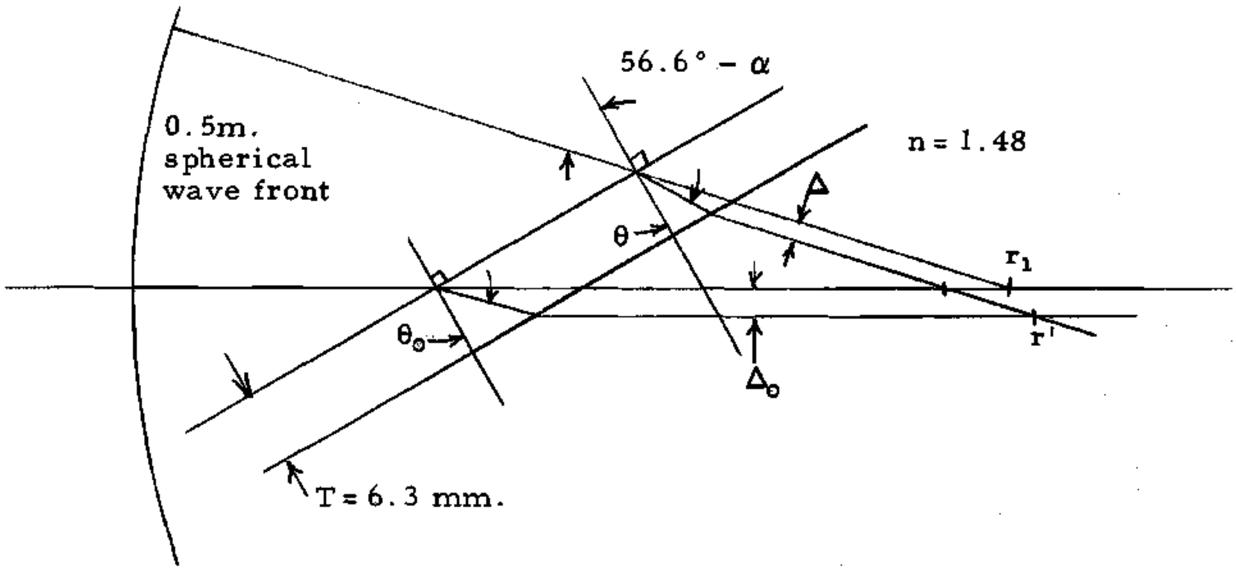


Figure 7. Spherical Wave Front Impinging On Brewster Window In Vertical Plane

Using an $\alpha = 0.001$

$$t_1 - t_0 = 2.14 \times 10^{6} \text{ mm.} = 21.4 \text{ Å} \quad .$$

This small difference in path length would cause a negligibly small frequency shift.

Figure 7 illustrates the geometry involved for the 0n modes. The point at which the apparent focus of the spherical wave lies is

$$r' = r - \frac{\Delta}{\sin\alpha} + \frac{\Delta_0}{\tan\alpha}$$

which for small α is

$$r' = r + \frac{(\Delta_0 - \Delta)}{\alpha} \quad .$$

Snell's refractive law gives

$$\sin\theta = \frac{\sin(56.6° - \alpha)}{1.48}$$

and

$$\sin\theta_0 = \frac{\sin 56.6°}{1.48} \quad .$$

And to complete the analysis

$$\Delta = \frac{T}{\cos\theta} \sin(56.6° - \alpha - \theta)$$

and

$$\Delta_0 = \frac{T}{\cos\theta_0} \sin(56.6° - \theta_0) \quad .$$

Numerical computation from these formulas for several values of α gives the results in Table 5.

Table 5. Effective Radius of Curvature for Various Values of α

α	b
0.0007 radian	100.98 cm.
0.0008	100.90
0.0009	100.96
0.0010	100.91
0.0011	100.96
0.0012	100.91

For a small range of α there is little change in b, much of the variation shown arising in computation. A value for b of 101 cm. yields

$f\nu_0$ = 68.3 MHz. The value without windows is again 68.8 MHz. and the observed value is 67.6 MHz. Again the right orders of magnitude are obtained but the exact experimental values are not obtained.

Although the computations given do not yield results that check exactly with experimental results, they seem to indicate that the Brewster windows are a contributing factor if not the primary cause of the errors observed.

Mode Analysis From Beat Frequency Studies

Early in this research, it was recognized that a better method for observing transverse beats was evident beyond Goldsborough's method.

It is necessary, in order to obtain beat frequencies, that the two light signals fall on the same area of the photomultiplier tube photocathode. Use of an iris to block off everything but a portion of the pattern where two modes overlap should be a better technique than mere blocking of large areas in Goldborough's manner. Since such a procedure also more precisely defines from what areas of the light pattern the beats originate, identification of the modes present in the laser is facilitated. Also, the sensitivity limitation imposed by the photomultiplier tube's maximum anode current rating is largely lifted. All these techniques have proved useful in the foregoing experiments and the ones following. Following are some experiments in which observations of the beat frequencies, using the iris, were used to aid in determining the transverse mode sets present in the laser under various conditions of adjustment. No effort was made to determine the number of longitudinal modes present in each transverse mode set because of equipment limitations and the great difficulty of the problem. The difficulty arises because in order to determine the number of longitudinal modes present in a single transverse mode set, the equipment must be capable of detecting the beat frequency $\nu_0(n-1)$, where n is the number of longitudinal modes present. The equipment used did not have the necessary bandwidth. Beat frequencies as high as $2\nu_0$ were observed quite often, however.

One of the more novel modes investigated is the annular pattern shown in Figure 8. This mode was studied by Goldsborough, who

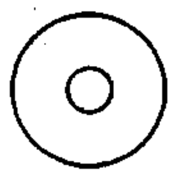


Figure 8. Annular Mode Pattern Resulting From Combination of 10 and 01 Modes

concludes that it is the combination of a 10 and a 01 mode. The laser was set into operation in this mode by careful adjustment. Using Goldsborough's technique of blocking large areas of the beam his observations and results were duplicated immediately. Briefly he observed that beat frequencies appeared at 700 KHz. above and below the longitudinal beat frequency and that these beat frequencies were weaker than the longitudinal one. Then an investigation was made using the iris.

With the beam unblocked, only the longitudinal beat frequency was seen. As explained by Goldsborough, beats between transverse modes can never be observed with an unblocked beam. This phenomenon is due to the fact that the eigenfunctions describing each mode form an orthogonal set. Therefore the same beat frequency may arise out of the superposition of two modes at various points on the photomultiplier photocathode. The relative phases of the beat frequencies will be such that they will cancel out and no beat frequency will be observed. With only the upper or lower or side edge exposed, the longitudinal beat frequency was observed with very weak beat frequencies 700 KHz. above and below the longitudinal beat frequency. This was due to imperfect coverage of the iris. Reference to Figure 9 aids in understanding these observations. When an area such as the upper right hand portion of the

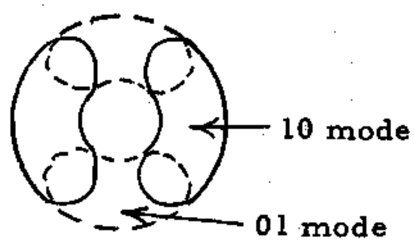


Figure 9. Superposition of 01 and 10 Modes To Form Annular Mode

pattern was observed the longitudinal beat frequency appeared and the weak sidebeats mentioned rose to an amplitude equal to that of the longitudinal beat frequency. The reason for this result is explained by Goldsborough as shown in Figure 10. The 700 KHz. split has previously

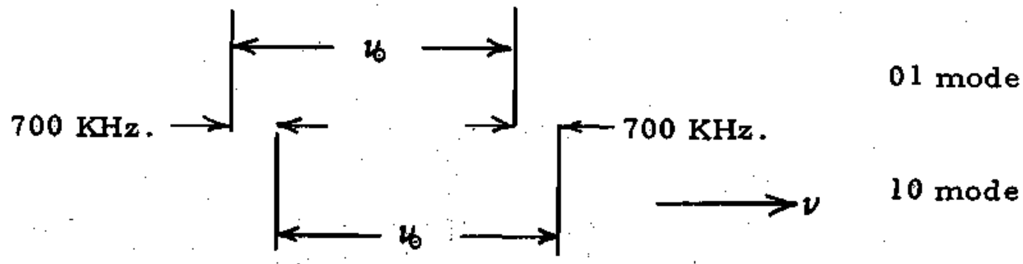


Figure 10. Spectral Diagram Explaining Presence of Sidebeats With Longitudinal Beat Frequency

been explained as due to the effects of the Brewster windows on the resonant frequencies of the cavity. It should be noted that use of the iris results in the sidebeats observed on either side of the longitudinal beat frequency being equal in amplitude to the longitudinal beat frequency. This would be expected with only one lobe of each mode exposed. Use of the blocking technique leaves portions of both lobes of each mode exposed and cancellation at the photomultiplier occurs as previously discussed. In this condition of laser adjustment there is usually a weak 00 mode present. This was evidenced in the last instance above by very weak beat frequencies at 37 and 67 MHz., which in this case were the transverse beat frequencies expected between the 00 mode set and either the 10 or 01 mode sets. Adjustment of the iris so as to allow more of the common area containing 10, 01, and 00 modes to fall on the photocathode of the photomultiplier tube increased the strength of these beat frequencies enormously. The beat frequencies occurred in pairs 700 KHz. apart at 37 and 67 MHz. Movement of the iris varied their relative amplitudes as would be expected. These observations are quite in agreement with those of Goldsborough, but use of the iris enhances the observations.

Figure 11 is a sketch of another pattern investigated. The shaded

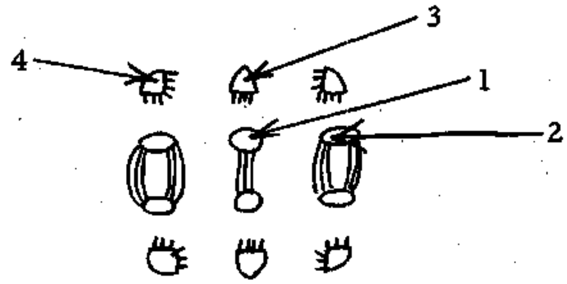


Figure 11. Mode Pattern Investigated

areas indicate fuzziness of the basic 23 mode pattern that indicates the presence of another mode. Visual inspection was not enough to determine what other mode or modes were present. Using the iris, the numbered areas shown were examined. In each case, the longitudinal beat frequency and a single strong clean beat frequency at 64 MHz. appeared. The longitudinal beat frequency was 94 MHz., and the transverse beat frequency at 64 MHz. indicated $\Delta(m+n)$ equal to 1. Therefore, the modes possible are those where $(m+n)$ is equal to 6 or 4. The possible modes are:

$m+n = 6$	$m+n = 4$
33 x	22 x
24 x	13 x
42	31 x
15 x	04 x
51 x	40
06 x	
60 x	

Those modes marked with an "x" are not compatible with the observed pattern because the unknown mode definitely appears to have m = 4. But since the unknown pattern extends up into the areas marked 4 and 3, the 40 mode is ruled out. Therefore 42 is the other transverse mode set present. It can be seen that much more complicated mode patterns would be difficult to identify.

Sketched in Figure 12 is another pattern investigated. In this case,

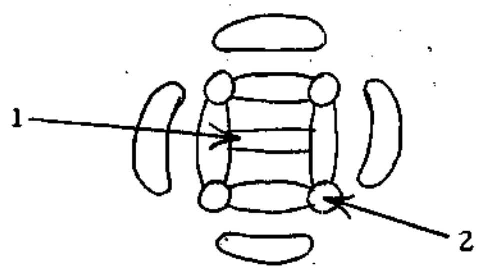


Figure 12. Another Mode Pattern Investigated.

the well defined brightness of the four spots surrounding the middle give definite indication of the 11 mode set. The actual pattern was nowhere so well defined as shown. The mode sets 04 and 30 are suspected however.

The very center of the pattern yielded the longitudinal beat frequency. There were very weak signals at frequencies indicating $\Delta(m+n) = 1$ and 2 but these were probably due to incomplete blocking by the iris so that light from unwanted areas of the pattern reached the photomultiplier tube. These were ignored.

Checking the four outermost lobes of the pattern revealed only the longitudinal beat frequency. It appeared, therefore, that each area investigated contained only one mode.

Checking the area marked 1 revealed the longitudinal beat frequency and a strong beat indicating $\Delta(m+n) = 1$. Such a beat frequency would occur with the suspected 04 and 30 modes present.

Area 2 revealed the longitudinal beat frequency and beat frequencies indicating $\Delta(m+n) = 1$ were also present but were double, the two beat frequencies being separated about 3 MHz. One of the beat frequencies in each pair was somewhat weaker and less stable in amplitude.

The evidence compiled does not lead to a clear cut solution. However, assumption of the presence of 11, 04, and 30 modes explains the results and is compatible with the observed pattern if the double beat frequencies in area 2 can be explained. The mode frequency diagram in Figure 13 is helpful. As shown in the diagram, the beat

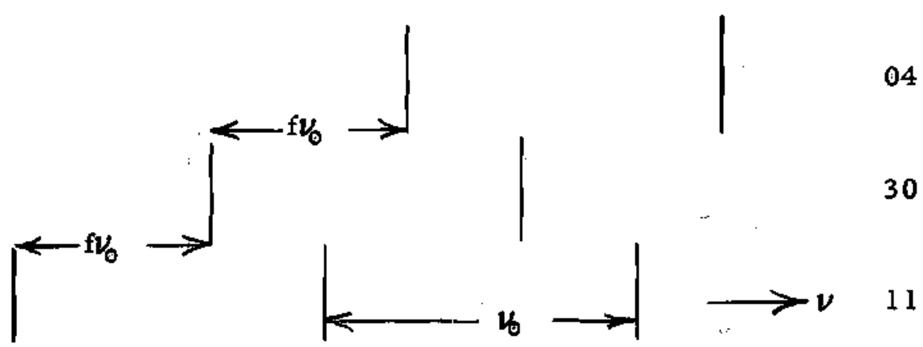


Figure 13. Mode Frequency Diagram Explaining Split In Transverse Beat Frequency

frequency indicating $\Delta(m+n) = 1$, may arise in two ways, first as a beat frequency between the 30 and 11 mode sets, and as a beat frequency between the 30 and 04 mode sets. The 04 and 11 mode sets, however, are very different in spatial distribution. As has already been shown, the resonant frequencies of modes of different spatial distribution are affected markedly by the Brewster windows. This explanation seems to be the most likely.

More complicated patterns become more a matter of guess-work unless other methods are used to supplement the information gained by observation of beat frequencies. For example, the pattern shown in Figure 14 was studied.

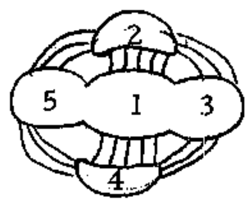


Figure 14. Another Mode Pattern Studied

This pattern appears to consist of 20, 02 and 10 modes and perhaps some other modes. With the iris adjusted to expose only the very center of the pattern, the longitudinal beat frequency was observed clearly, with beat frequencies of approximately the same amplitude as the longitudinal beat frequency 1.2 MHz. above and below. This result could be explained by the presence of 20 and 02 modes since they have a common area in the middle. The 1.2 MHz. split is also probably about the right magnitude for the split which will certainly exist between the two modes due to the Brewster windows. No beat frequencies indicating

$\Delta(m+n) = 1$ or 2 were found which means the 00 mode is evidently not present. As a check on these results, a knife edge was used inside the optical cavity to suppress some of the modes. This procedure is described by Goldsborough and is useful. Caution must be exercised in interpreting the results, however, for the suppression of some modes tends to cause the appearance of others not present before. Using this technique revealed the presence of the 02 mode set, a weaker 20 and the 10 mode set. The original conclusions seem justified.

Examination of the regions 5 and 3 revealed a beat frequency indicating $\Delta(m+n) = 1$ with a weaker beat frequency 1.2 MHz. lower, and a longitudinal beat frequency with beat frequencies 1.2 MHz. above and below and much smaller than before. Examination of areas 2 and 4 revealed only the longitudinal beat frequency with beat frequencies on either side as in regions 5 and 3. In each case the beat frequencies on each side of the longitudinal beat frequency probably result from overlap of the 02 and 20 modes. The $\Delta(m+n) = 1$ beat frequency most likely is indicative of the presence of the 10 mode previously discovered.

Examination of the areas between 5 and 1, and 3 and 1 reveal a longitudinal beat frequency with beat frequencies 1.2 MHz. above and below as large as the longitudinal beat frequency. This again is just the result of 02 and 20 modes beating together. A single clean beat frequency indicating $\Delta(m+n) = 2$ appears. Beat frequencies appear indicating $\Delta(m+n) = 1$ as shown in Figure 15.

The results seem to indicate that the modes present are 20, 02, 10, and 30. The strange appearance of the beat frequencies shown in Figure 15 can be explained by the mode diagram of Figure 16.

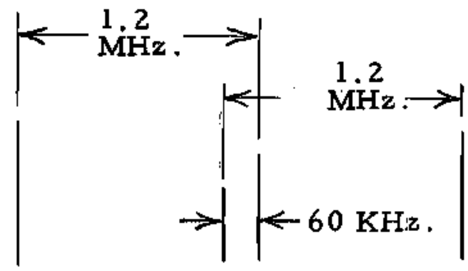


Figure 15. Beat Frequencies Appearing on Spectrum Analyzer

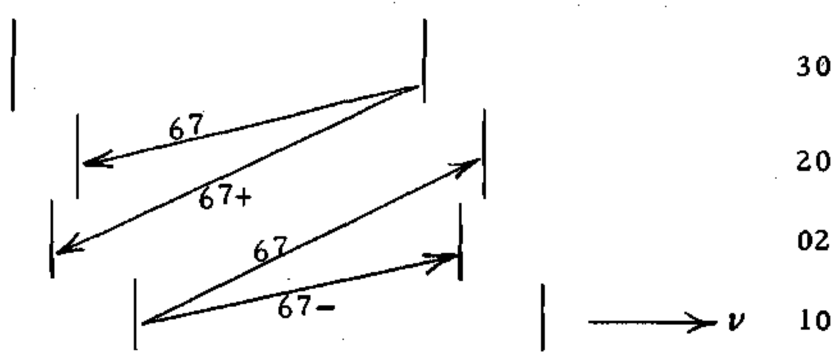


Figure 16. Mode Spectrum Explaining Beat Frequencies in Figure 15.

The 60 KHz. split shown in the beat frequencies could be explained as the result of a difference in the transverse beat frequency between mode sets differing by the same $\Delta(m+n)$.

It is seen that the 60 KHz. split is explained as the result of a phenomena which was also advanced as the cause of a 3 MHz. split in the case previous to this one. This can be explained because in the previous case one of the modes, 04, was at right angles to the 30 mode. It has previously been shown that vertically distributed modes, that is ones of 0n type, suffer marked frequency shifts as compared to m0 modes.

CHAPTER VI

CONCLUSIONS

The equations giving the beat frequencies in terms of the optical cavity dimensions do not include the effects of perturbations by the Brewster windows commonly used to terminate the gas tube. The Brewster windows have considerable effect on the frequencies of observed beat frequencies. The longitudinal beat frequency is lower than the value for given dimensions because of the greater path created by the Brewster windows. Modes of the same transverse order but having different spatial distributions oscillate at noticeably different frequencies chiefly as a result of Brewster window perturbing effects. The perturbation becomes greater as the mode order increases.

Splitting of the longitudinal beat frequency is small for the laser considered but the longitudinal beat frequency is lower than the predicted value also because of pulling effects which move each mode closer to the line center.

Beat frequencies occurring between pairs of modes where each pair differs by the same $\Delta(m+n)$ are not quite the same. The reason is the same as that in the first paragraph, the perturbing effect of the Brewster windows.

The technique of investigating only small areas of complex mode patterns for beats is more efficient than Goldsborough's method.

The video amplifier used in this research may not have been useful. Noise from the photomultiplier tube could not be seen above that of the amplifier. A better signal to noise ratio might be had without the video amplifier.

A more accurate investigation of beat frequencies will require that the laser be much more stable than at present.

An investigation of this type might make better use of some of the currently available photodiodes in place of the photomultiplier. These diodes have superior frequency response and require little space or auxiliary equipment.

BIBLIOGRAPHY

1. Bennett, W. R., Jr., "Hole Burning Effects in the He-Ne Optical Maser," Physical Review, 126, 580-593, 1962.

2. Bennett, W. R., Jr., "Gaseous Optical Masers," Applied Optics Supplement on Optical Masers, 1962, 24-61.

3. Boyd, G. D., A. G. Fox, H. Kogelnik, Tingye Li, and W. W. Rigrod, "Comment on 'Gaseous Optical Maser With External Mirrors,'" Journal of Applied Physics, Communications, 34, page 3412, 1963.

4. Boyd, G. D., and J. P. Gordon, "Confocal Multimode Resonator for Millimeter Through Optical Wavelength Masers," The Bell System Technical Journal, March 1961, 489-508.

5. Boyd, G. D., and H. Kogelnik, "Generalized Confocal Resonator Theory," The Bell System Technical Journal, July 1962, 1347-1369.

6. Goldsborough, J. P., "Beat Frequencies Between Modes of a Concave-Mirror Optical Resonator," Applied Optics, February 1964, 267-275.

7. Lengyel, Bela A., Lasers, John Wiley and Sons, Inc., New York, 1962.

8. Polanyi, Thomas G., and William R. Watson, "Gaseous Optical Maser with External Mirrors," Journal of Applied Physics, March 1963, 553-560.

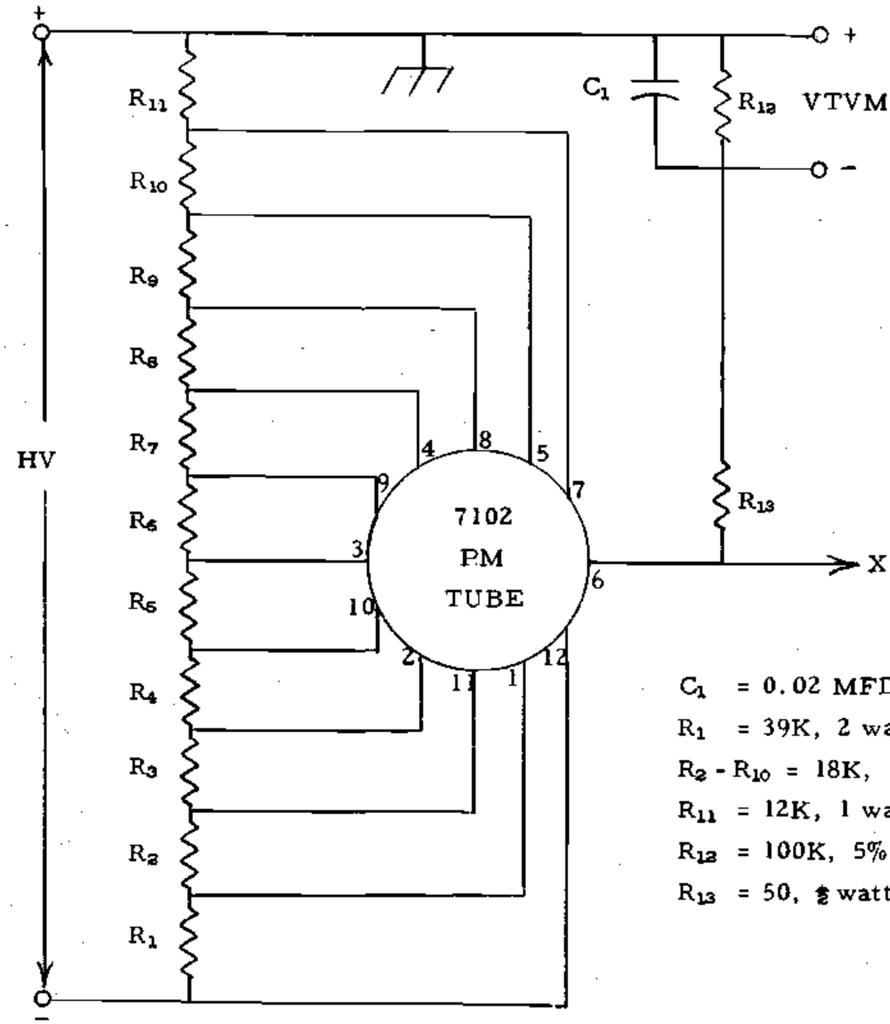
9. Polanyi, T. G., and W. R. Watson, "Reply to Comments on "Gaseous Optical Maser with External Mirrors,'" Journal of Applied Physics, Communications, 34, page 3412, 1963.

10. Rigrod, W. W., "Isolation of Axi-Symmetrical Optical Resonator Modes," Applied Physics Letters, February 1963, 51-53.

11. Rigrod, W. W., H. Kogelnik, D. J. Brangaccio, and D. R. Herriott, "Gaseous Optical Maser with External Concave Mirrors," Journal of Applied Physics, February 1962, 743-744.

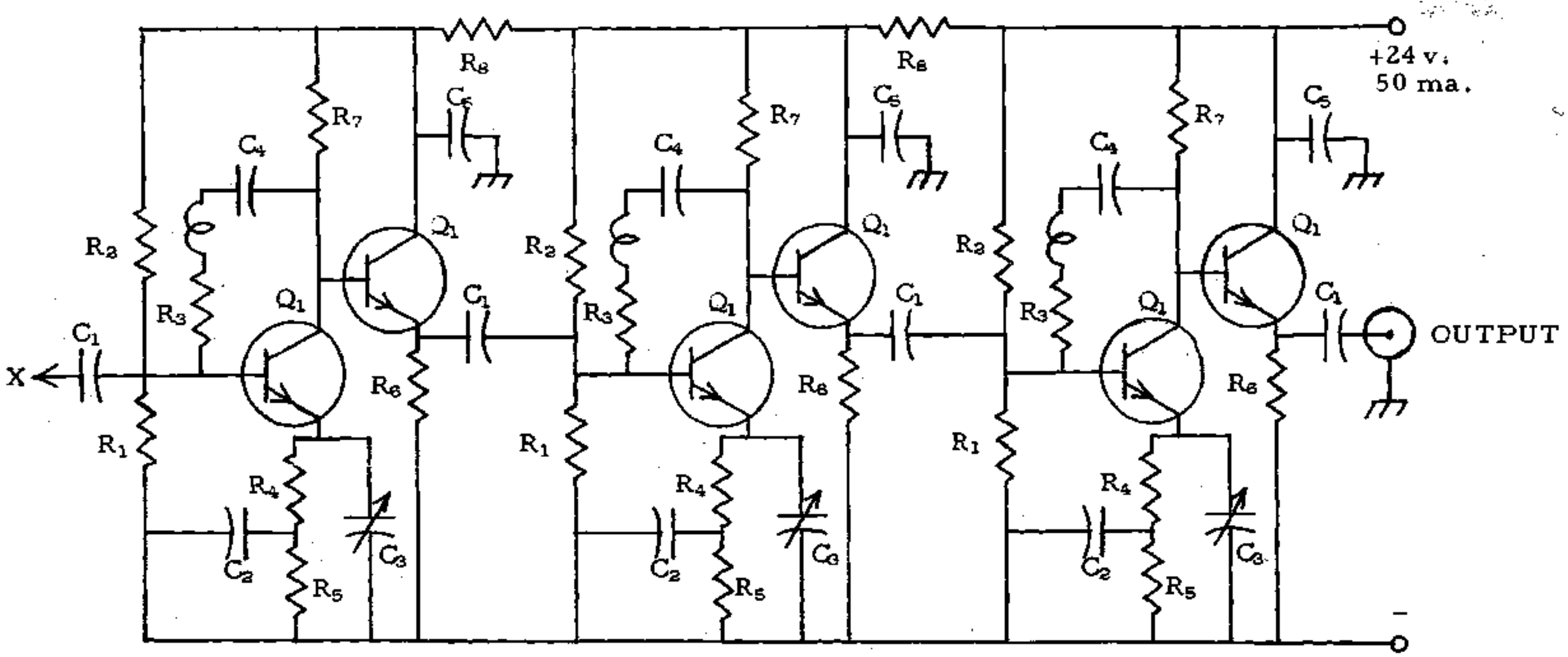
12. Rosenberger, D., "Mode Spectrum in the He-Ne Maser," Quantum Electronics, P. Grivet and N. Bleombergen, Columbia University Press, New York, 1964, 1301-1304.

13. Uchida, Teiji, "Frequency Spectra of He-Ne Optical Masers with External Concave Mirrors," Applied Optics, January 1965, 129-131.



C_1 = 0.02 MFD, 50V.
R_1 = 39K, 2 watts
R_2 - R_{10} = 18K, 1 watt
R_{11} = 12K, 1 watt
R_{12} = 100K, 5%, ½ watt
R_{13} = 50, ½ watt

Figure A-1. Phtomultiplier Portion of Detection Assembly



C_1 = 3000 PF	R_1 = 2.7K	R_6 = 2K
C_2 = 0.02 MF	R_2 = 18K	R_7 = 2K
C_3 = 7-45 PF	R_3 = 620	R_8 = 22
C_4 = 1000 PF	R_4 = 100	L_1 = 0.07 μH
C_5 = 0.1 MF	R_5 = 270	Q_1 = 2N918

Original Description Appeared In
The October 1964 Issue of
Solid State Design

All Resistors 1/4 Watt
Transistor Cases Grounded

Figure A-2. Video Amplifier Portion of Detection Assembly