

COMPLEX LOAD SIMULATOR DESIGN

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

By

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

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COMPLEX LOAD SIMULATOR DESIGN

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## ACKNOWLEDGMENT

Sincere appreciation is extended to members of this committee, Dr. David M. Sanborn and Dr. Wendell M. Williams, and especially to the committee chairman and thesis advisor, Dr. Harold L. Johnson, all of the School of Mechanical Engineering of the Georgia Institute of Technology for their generous cooperation that made possible the undertaking of this Complex Load Simulator Design as a thesis.

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CHAPTER I  
INTRODUCTION

Most machine and structural elements and components encountered in practical applications are usually acted upon by complex loads, forces, in a manner that produces a resultant maximum stress in these elements or components that has a time varying complex direction and magnitude. Testing of these actual elements and components under simulated actual load conditions is usually difficult if not impossible. Therefore, a small specimen of the material, that represents the element or component under consideration, usually is tested under a much simpler load than the actual complex load. An attempt usually is made with this simpler load to approximate the maximum stress in the test specimen that would be induced by the actual complex load.

If the actual complex load could be simulated in a test specimen, a more authentic test would result. This more authentic test would permit more accurate laboratory tests of materials under complex load conditions that are close to the actual application conditions. Also, this more authentic test would permit research to be conducted readily into the inter-relationship of the applied complex loads, and into the resultant stress magnitudes and directions produced by these loads in the test specimen.

### OBJECTIVE

The detailed design of a Complex Load Simulator that will permit complex loads, that simulate actual loads encountered in practical applications of machine and structural elements and components, to be applied to small test specimen is the objective of this thesis for the fulfillment of the thesis requirement for the degree of Master of Science Mechanical Engineering at the Georgia Institute of Technology, Atlanta, Georgia.

### CRITERIA

General limits and conditions that the Complex Load Simulator will be designed to satisfy, the criteria, are primarily in the areas of economy, performance, manufacturing and safety.

Economic criteria encompass costs, useful life, capacity, and productivity. The initial and operating costs of the Complex Load Simulator shall be kept as low as possible while attempting to obtain a maximum useful life. Its capacity shall be established as accomodating one sample of the smallest specimen that can be tested satisfactorily with a reasonable degree of accuracy under stress magnitudes of 100,000 psi or less for all load types and modes. Only the initial installation and the final removal of the test specimen shall be required manually, thereby maximizing the productivity of the complex load simulator.

Performance criteria involve the functionability, versatility, flexibility, reliability, maintainability, durability, and serviceability. Functionability requires that the desired loads shall be applied to the test specimen by the Complex Load Simulator in a manner that insures the maintenance of the loads' types, magnitudes, and variations throughout the duration of the tests. Versatility shall include the application of axial, bending, and torsional loads to the test specimen that have both constant and reversing components. Flexibility of the Complex Load Simulator shall provide that loads may be applied in any mode or in any combination of modes, including all three at the same time, with any or all of the loading modes having either constant and/or reversing components.

Anytime a test specimen is installed in the Complex Load Simulator the reliability of this apparatus shall insure that this apparatus will function properly for the entire duration of the test. Construction and components incorporated shall possess a high degree of maintainability for keeping this apparatus as close to its original condition as reasonably possible. Normal wear and deterioration shall be reasonably resisted by the durability of this apparatus. Serviceability shall provide that the test specimen be readily and easily installed, the desired loads readily applied, and maintenance and repairs readily performed.

Manufacturing criteria for the Complex Load Simulator involves its producibility, quality, simplicity, size and weight. The producibility of this apparatus shall permit only the utilization of standard manufactured components and materials in conjunction with the common machine tools found in the ordinary machine shop. Quality of the Complex Load Simulator shall be the highest degree of precision that may be attained in order that undesirable loads and errors in timing of applied load application shall not be induced. Simplicity in construction, function, and operation of this apparatus shall be required. Size of the Complex Load Simulator and its components shall be the minimum consistent with its required function, strength, and rigidity. Weight shall be limited to the minimum that is consistent with maintaining the necessary strength, rigidity, wear provisions, etc. Detachable fasteners shall be the method of assembly with no welding employed in any fabrications.

Safety criteria for the Complex Load Simulator shall provide reasonable and adequate protection, against injury and harm, to all persons

coming into contact with it or in its general vicinity during its operation, while also complying with all national, state, and local laws, regulations, and codes pertaining to safety.

## SUMMARY

Three principle mechanisms are required to form a machine that will function as a Complex Load Simulator, applying actual loads encountered in practical applications of machine and structural elements and components to a small test specimen. These three mechanisms consist of one each for bending, axial, and torsional loads with constant and cyclic loading provisions in each one. A Constant and Cyclic Bending Load Application Mechanism holds the test specimen while a Constant and Cyclic Axial Load Application Mechanism and a Constant and Cyclic Torsion Load Application Mechanism apply their loads to the bending mechanism through a special Thrust-Type Constant Angular Velocity Universal Joint.

Bending moments of 153.4 lb. in produce stresses of 100,000 psi through either the constant or cyclic load application components of the Bending Load Application Mechanism. Axial loads of 4000 lbs and torques of 310 in. lb similarly produce stresses of 100,000 psi through either the constant or cyclic load application components of their respective application mechanisms.

Each of these three load application mechanisms and the universal joint are shown in separate drawings, a total of four drawings, with each drawing having its own exclusive item numbering system that begins with an item number one.

## CHAPTER II

## OPERATING PROCEDURE

A specimen of material to be tested, that has been machined to the proper length dimension specifications, is installed into the Constant and Cyclic Bending Load Application Mechanism. This installation is accomplished by turning the threaded ends of the test specimen into the threaded sockets in the upper and lower universal joint extension shaft assemblies. When the test specimen is seated firmly into these threaded sockets, the adjacent set screws in both assemblies are tightened in order to secure the specimen and prevent rotational slippage under torsional loads. If the upper drive chain between the torsional and this bending load application mechanism is connected, disconnect it while making this installation.

Connect the upper drive chain between the torsional and bending load application mechanism after the test specimen has been installed. Place a cut-off switch on an appropriate load application cylinder so that the simulator's drive motor will be stopped when this cylinder's piston rod moves to its outer limit at the fracturing of the test specimen. Care must be taken by the operator to avoid placing this cut-off switch at a location that will permit it to be activated by the yielding rather than the fracturing of the test specimen.

Pneumatic pressure is adjusted in the appropriate load application cylinders to produce the desired loads on the test specimen after the simulator's drive motor has been started and has reached its operating speed. Bending moments of 153.36 lb. in., that needed to produce a maximum

bending stress of 100,000 psi in the test specimen, are produced by the bending load application mechanism when the cyclic load pneumatic cylinders exert a thrust of 51.12 lb. that acts through a three inch lever arm and when the tension load in the cable (constant bending load) is 38.35 lb. Axial loads of 4909 lb., which will produce a maximum axial stress of 100,000 psi in the test specimen, are produced by the axial load application mechanism when the constant load pneumatic cylinder exerts this amount of thrust and when the tension in the cable (cyclic load) is this same amount. Torques of 306.80 in. lbs., those which produce shear stresses of 100,000 psi in the test specimen, are produced by the cyclic torque pneumatic cylinder when it exerts a thrust of 87.66 lbs. that acts through a 3.5 inch lever and when the cable tension (constant torsional load) is 885.77 lbs.

Components of the Complex Load Simulator that must oscillate during the application of a cyclic torsional load to the test specimen have a theoretical critical speed of 884.48 rpms. Therefore, this apparatus should be operated at approximately 3600 rpm.

## THE SPECIMEN

A minimum size test specimen is used in the Complex Load Simulator in order to minimize the size of the three principle mechanisms and the intensity of the loads they apply. The specimen, item 1 in figure 1, is machined from a five and one-half inch ( $5\frac{1}{2}$ " ) length of one-half inch ( $\frac{1}{2}$ " ) diameter of test material. Each end of this specimen has a three-eights inch ( $3/8$ " ) diameter fine thread by one-half inch ( $\frac{1}{2}$ " ) long machined on it. The center of this specimen is machined to a diameter of 0.250 inch through the use of a ten inch (10" ) swing radius, as shown in figure 1. Heat treating, as required, is applied to the test specimen after it is machined.

## CONSTANT AND CYCLIC BENDING LOAD APPLICATION MECHANISM

The test specimen (item 1) is mounted between the two major components of the Constant and Cyclic Bending Load Application Mechanism as shown on sheet one of four on page 21. These two major components are constructed on this mechanism's two main shafts (item 2), the extended self-aligning output shafts of the constant-velocity-thrust universal joints (item 42), and they are identical. Each of these major components applies both constant and cyclic bending load components to the end of the test specimen held by it.

Close sliding fit socket inserts (item 3), utilizing set screws (item 39), have smaller threaded sockets at their ends and these socket inserts are located in the ends of the main shafts. This arrangement securely holds the test specimen without slipping while bending, transverse, axial, and torsional loads are applied to it by the Constant and Cyclic Bending Load Application Mechanism.

Cyclic bending loads are applied identically to the test specimen by these two major components. Each major component has a bearing mounted (item 23) rotating collar (item 4), located on its main shaft, a specific distance from the center of the spherical element in the universal joint. When forces transverse to each main shaft are applied to each of these two collars, a bending moment is produced between these two collars if a test specimen is being held in place by the two major components. Rotation of these two major components, with the test specimen being held between them, causes the applied transverse loads to produce a cyclic bending moment, in

the test specimen, that complete a cycle with every revolution.

Constant bending loads are applied to the test specimen with the aid of a radial arm (item 6) mounted on each main shaft. A cable (item 41) is attached to the upper arm and extends vertically downward to the lower arm, around a cable pulley (item 24), into the hollow lower main shaft, around a cable pulley inside this shaft (item 24), and then downward through the hollow shafts of the lower universal joint to the load application. When tension loads are applied to this cable the two radial arms are drawn together, at their outboard limits, producing a bending moment in the test specimen that is a product of the cable tension and the total lever arm length. This total lever arm length is the combined distance from the main shaft center line to the application points of the cables on the radial arms plus the diameter of the cable pulleys. This cable loading arrangement rotates with the test specimen and produces a constant bending moment in the test specimen. Axial loads that tend to be induced in the test specimen by these cable loads are prevented by the pulley arrangement at the lower radial arm that balances the vertical forces produced in this region by the cable; in addition, the lower universal joint assembly is free to move vertically.

The lower radial arm is securely attached to the lower main shaft, rotating together with no relative rotation between them, while the upper radial arm is free to rotate relative to the upper main shaft. This relative rotational freedom between the upper radial arm and upper main shaft is necessary in order that torsional loads being transmitted to the test specimen will not produce angular deflections in the specimen that will interfere with the proper functioning of radial arms in applying the

constant bending load component. An alignment rod (item 17) and tube (item 18) assembly maintain alignment of the upper radial arm with the lower one at all times.

Axial and torsional loads are transmitted to the test specimen as it is held and rotated by this bending load application mechanism. The lower universal joint assembly (figure 2) is vertically movable (figure 3) and it transmits the axial loads to the test specimen. Torsional loads are transmitted to the test specimen from torsional load application mechanism (figure 4) by chains that drive the input shafts of the two universal joint assemblies. The upper universal joint input shaft is connected directly to the top of the torsional load application mechanism while the lower universal joint input shaft is connected to both, the bottom of the torsional load application mechanism and the complex load simulator input drive assembly, by the same chain. Pneumatic cylinders are utilized in applying both the cyclic and constant bending load components in order to minimize unwanted loads that may be induced by movements in the various mechanisms as a result of deflections in the test specimen.

## CONSTANT VELOCITY THRUST TYPE UNIVERSAL JOINT

An essential part of the Complex Load Simulator is the special universal joint, shown on sheet two of four on page 22, to each end of the test specimen. The function of each of these two universal joints is to transmit torsional, transverse, and axial loads at constant angular velocity to the test specimen as it deflects to the degree necessary under the given loading conditions.

The primary element of this constant angular-velocity, thrust-transmitting type universal joint is a pair of floating pins with one pin (item 11) placed through a hole in the other pin (item 10). Torsional loads transmitted by this universal joint are conveyed from the input shaft (item 2) to the output shaft (item 1) by these floating pins. These floating pins are constrained to remain in the homokinetic plane as the universal joint assumes its various possible angles for the transmission of rotation. Angled slots in the outer drive sleeve (item 11) and in the spherical element (item 8) attached to the end of the self-aligning output shaft accomplishes the necessary constraining alignment of the pins. Slots in the outer drive sleeve are angled in a direction opposite to those in the spherical element in order to align the pins into a plane, i.e. the homokinetic plane, that bisects the angle formed by the centerlines of the two shafts coupled by this universal joint. Also, the slots for one pin are angled in an opposite direction from those for the other pin in both the outer drive sleeve and the spherical element in order to prevent the pins from traversing axially relative to the centerlines of the coupled

shafts; illustrated in figure 2. This slot arrangement applies equal enter line thrust loads in one direction to one pin and in the opposite direction to the other pin for any possible phase this universal joint may assume during the transmission of forces and torques. Also, this balanced force system prevents the input and output shafts of this universal joint from "winding-up" in relation to each other.

Angular deflection capacity between the input and output shafts of this universal joint is provided by the spherical element, attached to the end of the output shaft, that seats in a pair of matching spherical-surfaced thrust rings (item 9) located inside the slotted outer drive sleeve. These thrust rings are positioned firmly into proper contact with the spherical element by the two thrust ring retainer plates (item 4 and 5). One of these two thrust ring retainer plates (item 4) has a cone shaped opening machined into it in order to permit the output shaft to deflect fifteen degrees ( $15^{\circ}$ ) in any direction relative to the center line of this universal joint's main assembly. Also thrust and transverse loads are transmitted through this universal joint by this special spherical element and these thrust rings.

Remaining elements and features of this universal joint consist of: 1) the self-aligning, double thrust radial bearing (item 12) that surrounds and contains the functional elements of the universal joint, 2) the bearing housing, and 3) the general method of assembly. Threaded fasteners are used exclusively to assemble the universal joint's components, which are readily purchaseable or are easily machined from standard material stock shapes. The two thrust ring retainer plates are drawn together by long studs (item 14) that pass completely through the outer drive sleeve, lengthwise, in a manner

similar to the two cover plates being drawn together by long studs passing completely through the cylindrical body sections (item 7) and support plates (item 6). The long studs are used in order to accomplish the flexible fastener-rigid joint concept of design for purposes of lessening the effect of fatigue loading.

Torsional loads are transmitted through this universal joint from the input shaft, where they are applied initially, to the thrust ring retainer plates with the aid of the connecting studs. These connecting studs transmit the torsional loads to the slotted outer drive sleeve which applies forces to the floating pins. Reaction forces are applied to these floating pins as they apply the transmitted torsional load to the output shaft through the slotted spherical element.

Thrust loads are transmitted through this universal joint from the mounting plate (item 18) through the cylindrical body sections and the connecting studs into the cover plates. These cover plates transmit the thrust load to the self-aligning, double thrust radial bearing that transmits the thrust load to the thrust ring retainer plates, with the aid of their connecting studs. Thrust loads from the thrust ring retainer plates are applied to the thrust rings which applies these loads directly to the output shaft through the spherical element. If necessary, thrust loads could be transmitted to the input shaft, via the attached thrust ring retainer plate, from the mounting plate, or thrust loads may be transmitted through this universal joint from shaft to shaft.

Transverse loads are transmitted from the mounting plate to the bearing, outer drive sleeve, thrust rings, and finally to the output shaft through the spherical element. Also, transverse loads may be transmitted

from the mounting plate to the input shaft through the thrust ring retainer plate. This retainer plate is attached to the outer drive sleeve, with the aid of long studs, and to the input shaft. Transverse loads may be transmitted through this universal joint from shaft to shaft, if necessary.

Provision has been made in the design and construction of this universal joint to permit a cable or cables to pass completely through this universal joint for the purpose of applying loads to other mechanisms in the Complex Load Simulator. These cables pass longitudinally through the hollow output shaft, by the floating pins, and longitudinally through the input shaft.

## CONSTANT AND CYCLIC AXIAL LOAD APPLICATION MECHANISM

Constant and cyclic axial loads are applied to the lower universal joint assembly, which applies these loads to the test specimen, by the Constant and Cyclic Axial Load Application Mechanism on sheet three of four on page 23. Cables (item 26) apply these axial loads to two load pins (item 24), one each on opposite sides of the universal joint assembly. This universal joint is constrained to move vertically by ball bushings (item 33) operating on four alignment pins (item 20). Anchor blocks (item 5), two mounted on each of two wing plates (item 2) that are located on opposite sides of this universal joint assembly, apply loads horizontally to each cable that extends around pulleys (item 26) in order to apply axial loads vertically to the load pins. These cable pulleys are mounted on the wing plate support brackets (item 6), on each side of the universal joint assembly, and mate with the grooved wing plate support bars (item 4), providing support and alignment for the wing plates. Provisions are included for adjusting the cable tension and reversing their anchoring in order to reverse the direction of the axial loads.

An attached load plate (item 1) spans the two wing plates and receives constant horizontal loads from a directly attached pneumatic cylinder (item 41). Cyclic loads are applied horizontally to this same load plate through a ring cam (item 3) attached to the under side of the load plate. Loads which are applied to this load plate are transmitted directly to the axial load application cables through the two wing plates.

A cam follower (item 30) rotates around the ring cam at the same

angular velocity that the test specimen rotates. Rotation of the horizontal cam follower is produced by the load simulator's drive assembly that is located directly below the horizontal load plate and cam. This drive assembly consists of an input belt pulley (item 9) attached to the input shaft (item 10) that has an attached chain sprocket (item 40) and is supported by a tapered roller bearing (item 35). Ball bushings (item 31) mounted on the input pulley support the horizontal cam follower and permit it to reciprocate as necessary. Horizontal constant load is applied to this cam follower by a cable (item 27) that extends over a cable pulley (item 29), mounted on brackets (item 11) on the input belt pulley, and vertically downward to a pneumatic load application cylinder. Cyclic loads are generated by this constantly loaded cam follower as it rotates around the cam loading the load plate in first, one direction and then, the other, i.e. sinusoidal. This cyclic load is in the direction the load plate and the wing plates are free to move.

Pneumatic cylinders are used to apply both the cyclic and constant axial loads in order to minimize unwanted loads that may be induced by movements in the various mechanisms as a result of specimen deflections.

The lower universal joint assembly and the lower end of the torsional load application mechanism is driven directly by a chain (item 42) from the sprocket (item 40) mounted on the lower end of this input drive assembly for the entire load simulator. Drive to the lower universal joint is provided by pins (item 52) that extend through a chain sprocket mounted on a bearing-supported (item 36) sleeve (item 22), into a slotted input shaft (item 21) that reciprocates within this same sleeve. This reciprocating shaft arrangement permits axial loads to be applied to the test specimen by the lower universal joint assembly.

## CONSTANT AND CYCLIC TORSIONAL LOAD APPLICATION MECHANISM

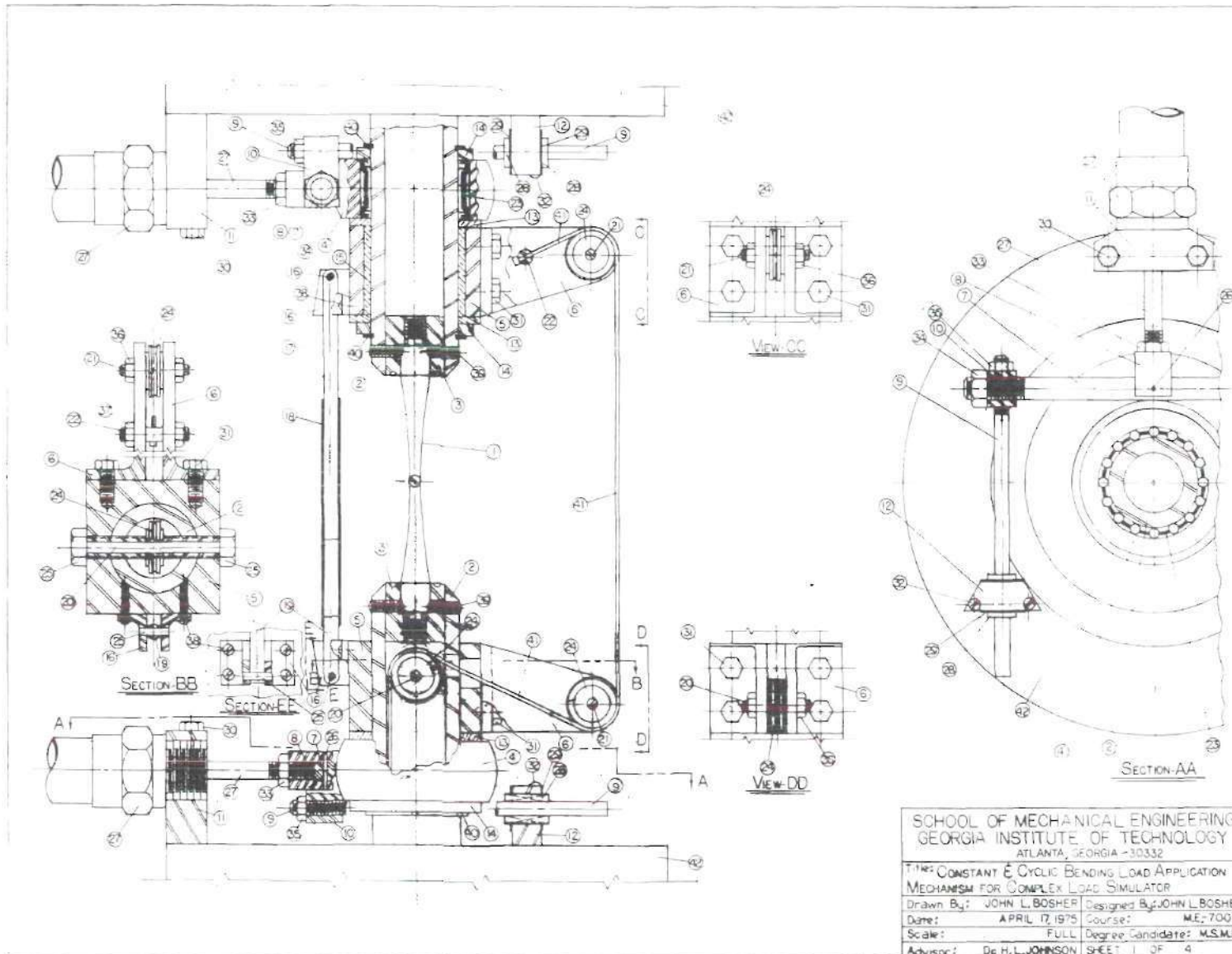
Torsional loads are applied to the test specimen, as it rotates with the Constant and Cyclic Bending Load Application Mechanism, by another mechanism rotating about an axis parallel to that of the bending load mechanism. This Constant and Cyclic Torsional Load Application Mechanism, sheet four of four on page 24, is connected to the mechanism constraining the test specimen by a chain (item 36) at each end of both mechanisms. This arrangement induces twist into the specimen while both of these mechanisms rotate at the same angular velocity. One section of this two-section torsional load mechanism produces a constant torsional load on the test specimen while the other section produces a cyclic torsional load on the test specimen.

Constant torsional loads are produced by the lower section of this torsional load mechanism. These loads are induced in the test specimen when the upper and lower shafts (item 2 and 3) of this torsional load mechanism are forced to rotate relative to each other during the rotation of both the torsional and bending load mechanisms. Since the top end of the upper shaft is connected to the top of the mechanism containing the test specimen by a chain and the lower shaft is connected to the bottom of the mechanism containing the test specimen also by a chain, a constant torsional load is applied to the test specimen. Relative rotation of these two shafts is produced by a floating load pin (item 6) that passes through slots in both shafts in the area where the upper shaft fits inside the lower shaft. These slots are inclined to the center line in one direction in the upper shaft and in the other direction in the lower shaft. This arrangement

permits the floating load pin to produce relative rotation between the two shafts when this pin is forced downward by a cable (item 37) that is under a constant tensile force. A pneumatic cylinder is used to apply this constant tensile force to this cable in order to minimize unwanted variations in load that may be caused by cyclic deflection from cyclic loads.

Cyclic torsional loads are produced by the upper section of this torsional load mechanism. A radial arm (item 7) attached to the upper shaft applies a cyclic torque to this shaft as it rotates due to a load of constant magnitude and direction being applied to this arm by the floating plate (item 8) that rotates with this arm. This floating plate is equipped with cam followers (item 21) that rotate within the internal ring of the cyclic load plate (item 9). Guide pins (item 11) operating in ball bushings (item 22) constrain this cyclic load plate to transmit load in one direction only to the floating plate. A pneumatic cylinder (item 24) is used to apply load to the cyclic load plate in order to minimize unwanted variations in load that may be induced by cyclic deflections from cyclic loads.

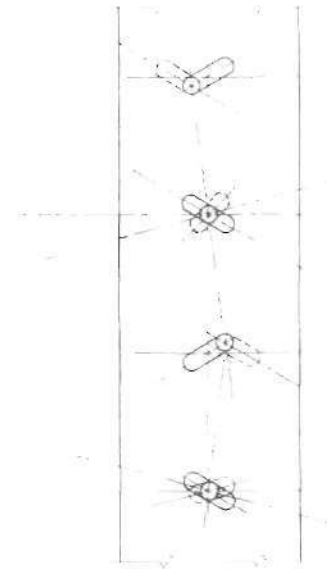
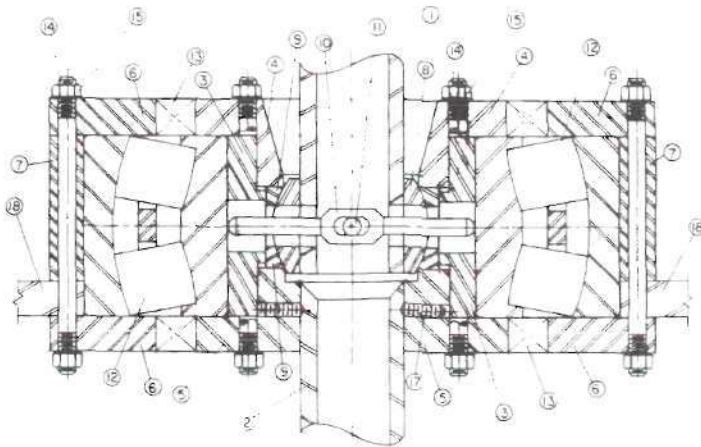
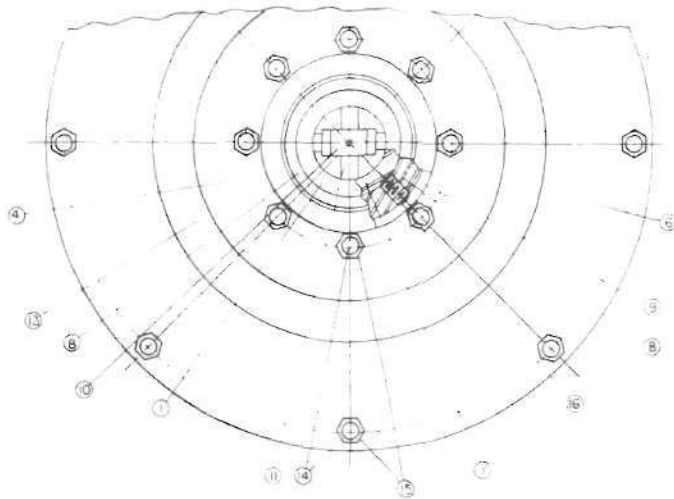
The radial arm induces cyclic torque in the upper shaft when it forces this shaft to rotate relatively and clockwise during one-half of its revolution and relatively and counterclockwise during the remaining half of its revolution. The effective lever arm length of radial arm through which the cyclic load plate's constant load is applied is a function of the length of this arm and the cosine of the angle between it and the direction of this applied load; i.e. sinusoidal or simple harmonic motion.



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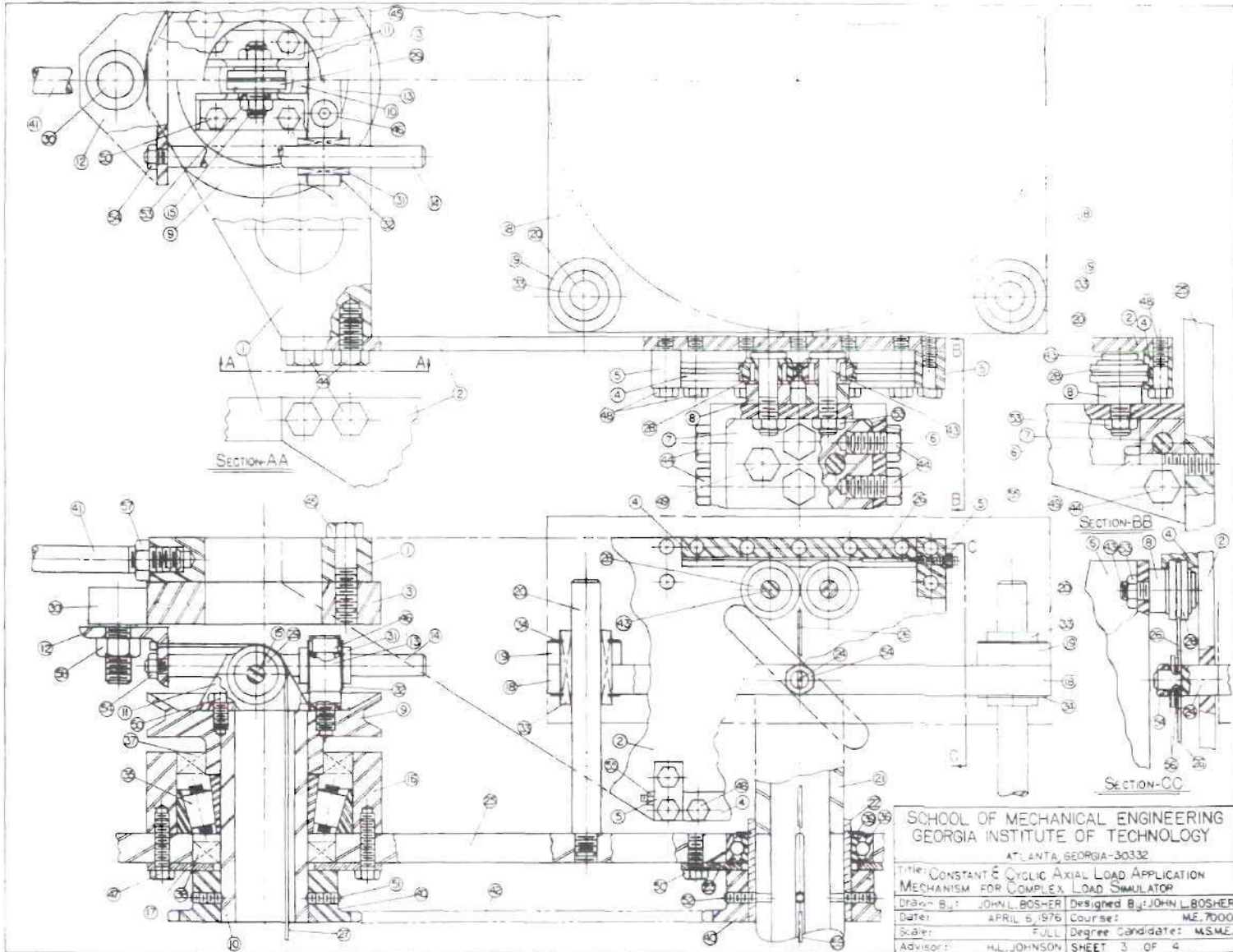
TITLE: CONSTANT & CYCLIC BENDING LOAD APPLICATION  
 MECHANISM FOR COMPLEX LOAD SIMULATOR

Drawn By: JOHN L. BOSHER    Designed By: JOHN L. BOSHER  
 Date: APRIL 17, 1975    Course: M.E. 7000  
 Scale: FULL    Degree Candidate: M.S.M.E.  
 Advisor: Dr. H.L. JOHNSON    SHEET 1 OF 4

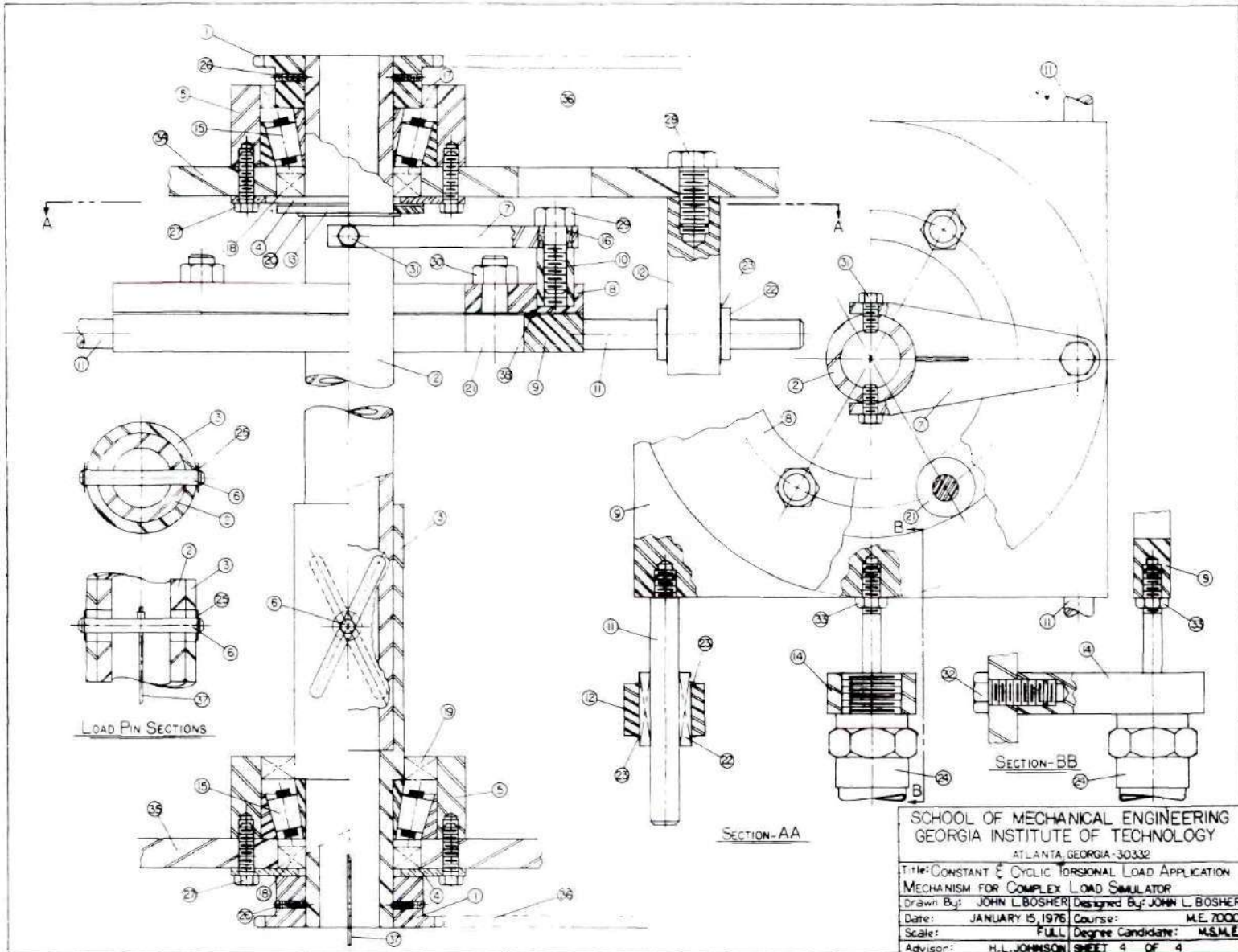


SLOTS ARRANGEMENT AT MAXIMUM ANGLE

SCHOOL OF MECHANICAL ENGINEERING	
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TITLE: THRUST BEARING & UNIVERSAL JOINT DESIGN FOR COMPLEX LOAD SIMULATOR	
Drawn By: JOHN L. BOSHER	Designed By: JOHN L. BOSHER
Date: JANUARY 14, 1975	Course: M.E. 7000
Scale: FULL	Degree Candidate: M.S.M.E.
Advisor: Dr. H.L. JOHNSON	SHEET 2 OF 4



SCHOOL OF MECHANICAL ENGINEERING  
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 Title: CONSTANT & CYCLIC AXIAL LOAD APPLICATION  
 MECHANISM FOR COMPLEX LOAD SIMULATOR  
 Drawn By: JOHN L. BOSHER | Designed By: JOHN L. BOSHER  
 Date: APRIL 5, 1976 | Course: ME 7000  
 Scale: FULL | Degree Candidate: MSME  
 Advisor: H.L. JOHNSON | SHEET 3 OF 4



SCHOOL OF MECHANICAL ENGINEERING  
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 TITLE: CONSTANT & CYCLIC TORSIONAL LOAD APPLICATION  
 MECHANISM FOR COMPLEX LOAD SIMULATOR  
 Drawn By: JOHN L. BOSHER    Designed By: JOHN L. BOSHER  
 Date: JANUARY 15, 1976    Course: M.E. 7000  
 Scale: FULL    Degree Candidate: M.S.M.E.  
 Advisor: H.L. JOHNSON    SHEET 4 OF 4

## APPENDIX A

## PARTS LIST

## CONSTANT AND CYCLIC BENDING LOAD APPLICATION MECHANISM

ITEM	QUANTITY	DESCRIPTION	$I_1$ (lb.-in-sec. <sup>2</sup> )
1	1	Test Specimen	
2	2	Self-Aligning Universal Joint	
		Output Shaft	.0293
3	2	Socket Insert	
4	2	Rotating Collar	
5	2	Arm-Mounting Base	
6	6	Radial Arm	
7	2	Load Rod	
8	2	Clevis	
9	4	Guide Rod	
10	4	Connector	
11	2	Cylinder Mounting Block	
12	4	Ball Bushing Mounting Block	
13	3	Bronze Washer	
14	3	Collar	
15	1	Bronze Bushing	
16	4	Alignment Rod Bracket	
17	1	Alignment Rod, Long	
18	1	Alignment Tube	
19	1	Alignment Rod, Short	
20	1	Cable Pulley Shaft, Long	
21	2	Cable Pulley Shaft, Short	
22	1	Cable Anchor Pin	
23	2	Needle Bearing Assembly	
24	3	Cable Pulley, Aircraft	
25	2	Cap Screw, Modified	
26	2	Rolled Pin	
27	2	Cyclic Load Pneumatic Cylinder	
28	2	Ball Bushing	
29	4	Retainer Ring	
30	4	Cap Screw	
31	8	Cap Screw	
32	4	Machine Screw	
33	2	Lock Nut	
34	2	Lock Nut	
35	2	Lock Nut	
36	4	Lock Nut	
37	2	Lock Nut	
38	8	Machine Screw	
39	8	Set Screw	
40	3	Retainer Ring	
41	1	Aircraft Cable, Constant Bending	
42	2	Universal Joint Assembly	

## PARTS LIST

## THRUST TYPE CONSTANT VELOCITY UNIVERSAL JOINT

ITEM	QUANTITY	DESCRIPTION	$I_1$ (lb.-in.-sec <sup>2</sup> )
1	1	Self-Aligning Output Shaft	
2	2	Input Shaft	
3	1	Slotted Drive Cylinder	0.0167
4	1	Output Thrust Ring Retainer Plate	0.0130
5	1	Input Thrust Ring Retainer Plate	0.0203
6	2	Cover Plate	
7	1	Cylindrical Body	
8	1	Slotted Drive Sphere	0.0014
9	2	Spherical Thrust Rings	0.0083
10	1	Slotted Drive Pin	
11	1	Straight Drive Pin	
12	1	Thrust Bearing, Self-Aligning	0.0599
13	2	Seals	
14	16	Studs	
15	32	Nuts	
16	4	Set Screws (Sphere)	
17	4	Set Screws (Input)	
18	1	Mounting Plate	

Total  $I_1 = 0.1196$

## PARTS LIST

## CONSTANT AND CYCLIC AXIAL LOAD APPLICATION MECHANISM

ITEM	QUANTITY	DESCRIPTION
1	1	Load Plate
2	2	Wing Plate
3	3	Ring Cam
4	4	Wing Plate Support
5	4	Anchor Block
6	2	Wing Plate
7	2	Mounting Block
8	8	Spacer
9	1	Belt Pulley
10	1	Input Shaft
11	2	Cable Pulley Support Bracket
12	1	Cam Follower Bracket
13	2	Bushing Support Block
14	2	Alignment Pin
15	1	Cable Pulley Shaft
16	1	Bearing Housing
17	1	Bronze Cover Plate
18	1	Load Plate
19	4	Spacer
20	4	Guide Pin
21	1	Universal Joint Fixed Shaft
22	1	Bronze Sleeve
23	1	Bronze Cover Plate
24	2	Load Pin
25	1	Bottom Plate
26	2	Aircraft Cable, Axial Load
27	1	Aircraft Cable, Axial Load
28	8	Aircraft Cable Pulley
29	1	Aircraft Cable Pulley
30	1	Cam Follower
31	2	Ball Bushing
32	4	Retainer Ring
33	4	Ball Bushing
34	8	Retainer Ring
35	1	Tapered Roller Bearing
36	1	Ball Bearing, Sealed
37	1	Seal
38	1	Seal
39	1	Retainer Ring
40	2	Sprocket
41	1	Pneumatic Cylinder Rod, Constant Load

ITEM	QUANTITY	DESCRIPTION
42	1	Chain
43	8	Modified Steel Rivets, Bottom Head
44	16	Cap Screw
45	4	Cap Screw
46	2	Cap Screw
47	4	Cap Screw
48	28	Cap Screw
49	8	Cap Screw
50	4	Cap Screw
51	4	Set Screw
52	4	Modified Set Screw
53	10	Lock Nut
54	2	Lock Nut
55	4	Adjusting Nut
56	4	Clamping Washer
57	1	Lock Nut
58	1	Lock Nut
59	1	Aircraft Cable, Axial

## PARTS LIST

## CONSTANT AND CYCLIC TORSION LOAD APPLICATION MECHANISM

ITEM	QUANTITY	DESCRIPTION	I. (lb.-in.-sec. <sup>2</sup> )
1	2	Sprocket	0.0269
2	1	Upper Shaft	0.0544
3	1	Lower Shaft	
4	2	Bronze Cover Plate	
5	2	Bearing Housing	
6	1	Load Pin	
7	1	Radial Arm	0.0054
8	1	Floating Plate	0.1382
9	1	Cyclic Load Plate	
10	1	Connector	0.0028
11	4	Guide Pin	
12	2	Bushing Support Block	
13	1	Collar	
14	1	Cylinder Support Block	
15	2	Tapered Roller Bearing	
16	1	Needle Bearing	
17	1	Seal	
18	2	Seal	
19	1	Seal	
20	1	Retainer Ring	
21	6	Cam Follower	0.0914
22	4	Ball Bushing	
23	8	Retainer Ring	
24	1	Pneumatic Cylinder, Cyclic Load	
25	2	Retainer Clip	
26	8	Set Screw	
27	8	Cap Screw	
28	4	Cap Screw	
29	1	Cap Screw	
30	6	Lock Nut	
31	2	Cap Screw	
32	1	Cap Screw	
33	1	Lock Nut	
34	1	Top Plate	
35	1	Bottom Plate	
36	2	Chain	0.0026
37	1	Aircraft Cable, Constant Load	
38	180	Steel Ball	

Total  $I_1 = 0.3486$

## APPENDIX B

## CALCULATIONS: CYCLIC AND CONSTANT BENDING

## 1. Test Specimen Cross Section

a) Minimum Diameter (d) = 0.250 in.

b) Minimum Moment of Inertia (I)

$$I = \frac{\pi d^4}{64}$$

$$I = \frac{\pi (0.250)^4}{64}$$

$$I = 1.917 \times 10^{-4} \text{ in.}^4$$

## 2. Bending Moments (M) and Stresses (s)

$$s = \frac{Mc}{I}$$

and

$$M = \frac{sI}{c} \quad \text{where} \quad c = \frac{d}{2}$$

$$\text{If } s = 100,000 \text{ lb./in.}^2$$

and

$$c = \frac{0.250 \text{ in.}}{2}$$

$$M = \frac{(100,000 \text{ lb./in.}^2)(1.917 \times 10^{-4})}{\frac{(0.250 \text{ in.})}{2}}$$

$$M = 153.35 \text{ in. lb.}$$

## 3. Cable Tension (T) For Constant Bending Moment (M)

$$M = lT$$

and

$$T = \frac{M}{l} \quad \text{where} \quad l = 4 \text{ in. (the total lever arm)}$$

$$T = \frac{153.36 \text{ in. lb.}}{4 \text{ in.}}$$

$$T = 38.35 \text{ lb.}$$

## 4. Cylinder Rod Thrust (F) For Cyclic Moment (M)

$$M = lF$$

and

$$F = \frac{M}{l} \quad \text{where} \quad l = 3 \text{ in. (the lever arm)}$$

$$F = \frac{153.36 \text{ in. lb.}}{3 \text{ in.}}$$

$$F = 51.12 \text{ lb.}$$

5. Moment Inertia ( $I_1$ ) For Members Isolating During the Application of Cyclic Torsional Load.

For upper extension shaft: (Item 2)

$$I_1 = \frac{\pi w l (R^4 - r^4)}{2g}$$

Where:  $w = 0.283 \text{ lb./in.}^3$   
 $l = 6.25$   
 $g = 386 \text{ in./sec.}^2$   
 $R = 1.5 \text{ in.}$   
 $r = 1.0 \text{ in.}$

$$I_1 = \frac{\pi (.283 \text{ lb./in.}^3)(6.25 \text{ in.}) [(1.5 \text{ in.})^4 - (1.0 \text{ in.})^4]}{2(386 \text{ in./sec.}^2)}$$

$$I_1 = .0293 \text{ lb.-in.-sec.}^2$$

## CALCULATIONS: UNIVERSAL JOINT

## 1. Bearing Selection For Torrington Type SD (Item 12)

$$a) \text{ Load Factor} = \text{Constant} \times \frac{\text{Thrust Load}^n}{\text{Radial Load}}$$

$$F = 5.5 \left( \frac{5,000 \text{ lb.}}{50 \text{ lb.}} \right)^{0.86}$$

$$F = 288.6 \text{ lb.}$$

$$b) \text{ Equivalent Radial Load } (P_e) = \text{Actual Radial Load} \times \text{Load Factor}$$

$$P_e = (50 \text{ lb.})(288.6)$$

$$P_e = 14,430 \text{ lb.}$$

$$c) \text{ Basic Dynamic Capacity} = \frac{P_e \times AF \times LF}{SF \times RF}$$

Where:

AF = Application Factor; 2.0 for Load Reversal

LF = Life Factor; 0.76 for 200 Hrs.

SF = Speed Factor; 0.245 @ 3600 rpms

RF = Ring Rotation Factor; 1.0 for Inter Ring Rotation

$$BDC = \frac{(14,430 \text{ lb.})(2.0)(0.76)}{(0.245)(1.0)}$$

$$BDC = 89,524.90 \text{ lb.}$$

Use Bearing 90SD23 (BDC = 89,500 lb.)

Note: Bearing design conditions are for extremely severe loading conditions which will seldom be encountered therefore, making a B-10 life (less than 10% failures) of 200 hours satisfactory since this is sufficient time to reach  $10^7$  cycles eight times at 3600 rpms.

## 2. Drive Pins (Item 10 and 11)

$$T = nA s_s l$$

and

$$s_s = \frac{4T}{n \pi d^2 l}$$

Where: T = Torque Transmitted  
 n = Number of Pin Shear Surfaces  
 A = Cross Section of Pin,  $\frac{\pi d^2}{4}$   
 $s_s$  = Shear Stress  
 l = Lever Arm that Produces Torque

If: d = 0.250 in.  
 n = 4  
 l = 1.250 in.

$$s_s = \frac{4(306.8 \text{ in./lb.})}{4 \pi (0.250 \text{ in.})^2 (1.250 \text{ in.})}$$

$$s_s = 1250.02 \text{ lb./in.}^2$$

## 3. Moment of Inertia of Members Oscillating During the Application of Cyclic Torsional Load

$$I_1 = \frac{\pi w l}{2g} (R^4 - r^4)$$

Where: w = 0.283 lb./in.<sup>3</sup>  
 g = 386 in./sec.<sup>2</sup>

## a) Slotted Drive Cylinder (Item 3)

R = 1.750 in.; r = 1.375 in.; l = 2.500 in.

$$I_1 = \frac{\pi (0.283 \text{ lb./in.}^3) (2.500 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times$$

$$\left[ (1.750 \text{ in.})^4 - (1.375 \text{ in.})^4 \right]$$

$$I_1 = 0.0167 \text{ lb.-in.-sec.}^2$$

b) Output Thrust Ring Retainer Plate (Item 4)

$$R = 2.188 \text{ in.}; r = 0.751 \text{ in.}; l = 0.500 \text{ in.}$$

$$I_1 = \frac{\pi(0.283 \text{ lb./in.}^3)(0.500 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times \left[ (2.188 \text{ in.})^4 - (0.750 \text{ in.})^4 \right]$$

$$I_1 = 0.0130 \text{ lb.-in.-sec.}^2$$

c) Input Thrust Ring Retainer Plate (Item 5)

$$R = 2.188 \text{ in.}; r = 0.750 \text{ in.}; l = 0.750 \text{ in.}$$

$$I_1 = \frac{\pi(0.283 \text{ lb./in.}^3)(0.750 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times \left[ (2.188 \text{ in.})^4 - (0.750 \text{ in.})^4 \right]$$

$$I_1 = 0.0203 \text{ lb.-in.-sec.}^2$$

d) Slotted Drive Sphere (Item 8)

$$R = 1.062 \text{ in.}; r = 0.750 \text{ in.}; l = 1.312 \text{ in.}$$

$$I_1 = \frac{\pi(0.283 \text{ lb./in.}^3)(1.312 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times \left[ (1.062 \text{ in.})^4 - (0.750 \text{ in.})^4 \right]$$

$$I_1 = 0.0014 \text{ lb.-in.-sec.}^2$$

e) Spherical Thrust Ring (Item 9)

$$R = 1.250 \text{ in.}; r = 1.000 \text{ in.}; l = 0.500 \text{ in.}$$

$$I_1 = \frac{\pi(0.283 \text{ lb./in.}^3)(0.500 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times \left[ (1.250 \text{ in.})^4 - (1.000 \text{ in.})^4 \right]$$

$$I_1 = 0.0083 \text{ lb.-in.-sec.}^2$$

f) Thrust Bearing - Inter Race Only (Item 12)

$$R = 2.312 \text{ in.}; r = 1.750 \text{ in.}; l = 2.500 \text{ in.}$$

$$I_1 = \frac{\pi(0.283 \text{ lb./in.}^3)(2.500 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times$$

$$\left[ (2.312 \text{ in.})^4 - (1.750 \text{ in.})^4 \right]$$

$$I_1 = .0599 \text{ lb.-in.-sec.}^2$$

$$\text{Total } I_1 = 0.1196 \text{ lb.-in.-sec.}^2$$

## CALCULATIONS: CYCLIC AND CONSTANT AXIAL LOADING

## I. Test Specimen Cross Section

a) Minimum Diameter (d) = 0.250 in.

b) Minimum Cross Sectional Area (A)

$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi(0.250 \text{ in.})^2}{4}$$

$$A = 4.910 \times 10^{-2} \text{ in.}^2$$

2. Axial Loads ( $P_a$ ) and Stresses ( $s_a$ )

$$s_a = \frac{P_a}{A}$$

and

$$P_a = s_a A$$

$$\text{If } s_a = 100,000 \text{ lb./in.}^2$$

$$P_a = (100,000 \text{ lb./in.}^2)(4.910 \times 10^{-2} \text{ in.}^2)$$

$$P_a = 4,910 \text{ lb.}$$

3. Pin Loads (V) and Pin Shear Stress ( $s_s$ )

a) Load Per Pin

$$V = \frac{P_a}{N} \quad \text{where } N \text{ is the number of pins:}$$

$$\text{If } N = 2$$

$$V = \frac{4910 \text{ lb.}}{2}$$

$$V = 2455 \text{ lb.}$$

Note: Aircraft Cable Tensile Strength is 5,000 lb.

b) Pin Area ( $A_p$ )

$$A_p = \frac{\pi d^2}{4}$$

$$\text{If } d = 0.500 \text{ in.}$$

$$A_p = \frac{\pi(0.500 \text{ in.})^2}{4}$$

$$A_p = 0.196 \text{ in.}^2$$

c) Pin Shear Stress ( $s_s$ )

$$s_s = \frac{4V}{3A_p}$$

$$s_s = \frac{4(2455 \text{ lb.})}{3(0.196 \text{ in.}^2)}$$

$$s_s = 16,700 \text{ lb./in.}^2$$

4. Required Pin Yield Strength ( $s$ )

$$\text{Factor of Safety (f.s.)} = 2.0$$

$$\text{Fatigue Factor (f.f.)} = 2.0$$

$$\text{Shear Factor (s.f.)} = 2.0$$

$$s = (\text{f.s.})(\text{f.f.})(\text{s.f.})s_s$$

$$s = (2.0)(2.0)(2.0)(16,770 \text{ lb./in.}^2)$$

$$s = 133,600 \text{ lb./in.}^2$$

Note: Pins must be heat treated.

## CALCULATIONS: CYCLIC AND CONSTANT TORQUE

## 1. Test Specimen Cross Section

a) Minimum Diameter (d) = 0.250 in.

b) Minimum Polar Moment of Inertia (J)

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi (0.250 \text{ in.})^4}{32}$$

$$J = 3.835 \times 10^{-4} \text{ in.}^4$$

2. Torques (T) and Shear Stresses ( $s_s$ )

$$s_s = \frac{Tc}{J}$$

and

$$s_s = \frac{TJ}{c}$$

$$\text{If } s_s = 100,000 \text{ lb./in.}^2$$

and

$$c = \frac{0.250 \text{ in.}}{2}$$

$$T = \frac{(100,000 \text{ lb./in.}^2) (3.835 \times 10^{-4} \text{ in.}^4)}{\frac{(0.250 \text{ in.})}{2}}$$

$$T = 306.80 \text{ in.-lb.}$$

## 3. Load Pin Forces

## a) Torque Development

## 1) Upper Shaft

$$T = 2 F_i \times D_i$$

and

$$F_i = \frac{T}{2D_i}$$

If  $D_i$  is 1.000 in. (Inside Diameter)

$$F_i = \frac{306.8 \text{ in. lb.}}{2(1 \text{ in.})}$$

$$F_i = 153.4 \text{ in. lb.}$$

## 2) Lower Shaft

$$T = 2 F_o \times D_o$$

and

$$F_o = \frac{T}{2D_o}$$

If  $D_o = 1.500$  in. (Outside Diameter of Upper Shaft)

$$F_o = \frac{306.8 \text{ in.-lb.}}{2(1.500 \text{ in.})}$$

$$F_o = 102.3$$

## b) Vertical Pin Load

$$P_t = 2(F_i + F_o) \tan \theta$$

$$\text{If } \theta = 60^\circ$$

$$P_t = 2(153.4 \text{ lb} + 102.3 \text{ lb.}) \tan 60^\circ$$

$$P_t = 885.8 \text{ lb.}$$

4. Moment of Inertia of Members Oscillating During the Application of Cyclic Torsional Load

$$I_1 = \frac{iiwl(R^4 - r^4)}{2g}$$

$$\text{Where: } w = 0.283 \text{ lb./in.}^3 \\ g = 386 \text{ in./sec.}^2$$

a) Sprocket (Item 1)

$$R = 2.812 \text{ in.}; r = 1.500 \text{ in.}; l = 0.875 \text{ in.}$$

$$I_1 = \frac{ii(0.283 \text{ lb./in.}^3)(0.875 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times$$

$$\left[ (2.375 \text{ in.})^4 - (1.500 \text{ in.})^4 \right]$$

$$I_1 = .0269 \text{ lb.-in.-sec.}^2$$

b) Upper Shaft (Item 2)

$$R = 1.500 \text{ in.}; r = 1.000 \text{ in.}; l = 11.625 \text{ in.}$$

$$I_1 = \frac{ii(0.283 \text{ lb./in.}^3)(0.875 \text{ in.})(11.625 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times$$

$$\left[ (1.500 \text{ in.})^4 - (1.000 \text{ in.})^4 \right]$$

$$I_1 = .0544 \text{ lb.-in.-sec.}^2$$

c) Radial Arm (Item 7)

$$I_1 = I_o + Md^2$$

$$\text{If } I_o = M \left[ \frac{a^2 + b^2}{12} \right]$$

and

$$M = \frac{wabt}{g}$$

$$I_1 = \frac{wabt}{g} \left[ \frac{a^2 + b^2 + d^2}{12} \right]$$

Where: a = 1.000 in.  
 b = 3.250 in.  
 d = 2.250 in.  
 t = 0.375 in.

$$I_1 = \frac{(0.283 \text{ lb./in.}^3)(1.000 \text{ in.})(3.250 \text{ in.})(0.375 \text{ in.})}{386 \text{ in./sec.}^2} \times \left[ \frac{(1.000 \text{ in.})^2 + (3.250 \text{ in.})^2 + (2.250 \text{ in.})^2}{12} \right]$$

$$I_1 = .0054 \text{ lb.-in.-sec.}^2$$

d) Floating Plate (Item 8)

R = 4.000 in.; r = 2.000 in.; l = 0.500 in.

$$I_1 = \frac{wl(0.283 \text{ lb./in.}^3)(0.500 \text{ in.})}{2(386 \text{ in./sec.}^2)} \times \left[ (4.000 \text{ in.})^4 - (2.000 \text{ in.})^4 \right]$$

$$I_1 = 0.1382 \text{ lb.-in.-sec.}^2$$

e) Connector (Item 10)

$$I_1 = \frac{wr^2}{g}$$

Where: w = .087 lb.

r = 3.500 in.

$$I_1 = \frac{.087 \text{ lb.} (3.5 \text{ in.})^2}{386 \text{ in./sec.}^2}$$

$$I_1 = .0028 \text{ lb.-in.-sec.}^2$$

f) Cam Follower (Item 21)

$$I_1 = \frac{Nw}{g} r^2$$

Where:

$$w = 0.200 \text{ lb.}$$

$$r = 2.500 \text{ in.}$$

$$N = 6 \text{ cam followers}$$

$$I_1 = \frac{6(0.200 \text{ lb.})(2.500 \text{ in.})^2}{386 \text{ in./sec.}^2}$$

$$I_1 = .0914 \text{ lb.-in.-sec.}^2$$

g) Chain (Item 36)

$$I_1 = \frac{wd^2}{g}$$

$$\text{If } w = (.21 \text{ lb./ft.})(2 \text{ ft.})$$

and

$$r = 1.562 \text{ in.}$$

$$I_1 = \frac{(.21 \text{ lb./ft.})(2 \text{ ft.})(1.562 \text{ in.})^2}{386 \text{ in./sec.}^2}$$

$$I_1 = .0026 \text{ lb.-in.-sec.}^2$$

$$\text{Total } I_1 \text{ (Including two item 1's)} = 0.3486 \text{ lb.-in.-sec.}^2$$

5. Critical Speed of Members Oscillating During the Application of Cyclic Torsional Load

$$W_n = k \sqrt{\frac{I_1 + I_2}{I_1 I_2}}$$

Where:

$$k = \frac{GJ}{l}$$

$$G = 11.4 \times 10^6 \text{ lb./in.}^2$$

$$J = \frac{\pi d^4}{32} = (.000936 \text{ in.})^4 \quad d = 0.3125 \text{ in.}$$

$$l = 2.500 \text{ in.}$$

$$\text{Therefore: } k = \frac{(11.4 \times 10^6 \text{ lb./in.}^2)(.000936 \text{ in.})^4}{2.500 \text{ in.}}$$

$$k = 4268.16 \text{ lb./in.}$$

$$I_1 = I_1 \text{ Bending} + I_1 \text{ Universal Joint} + I_1 \text{ Torsion}$$

$$I_1 = 0.0293 \text{ lb.-in.-sec.}^2 + 0.1196 \text{ lb.-in.-sec.}^2 + 0.3486 \text{ lb.-in.-sec.}^2$$

$$I_1 = 0.4975 \text{ lb.-in.-sec.}^2$$

$$I_2 = \infty$$

$$W_n = \sqrt{\frac{k(I_1/I_2 + 1)}{I_1}}$$

$$W_n = \sqrt{\frac{k}{I_1}}$$

$$W_n = \sqrt{\frac{4268 \text{ lb.-in.}}{0.4975 \text{ lb.-in.-sec.}^2}}$$

$$W_n = \left(\frac{92.6 \text{ rad.}}{\text{sec.}}\right) \left(\frac{\text{rev.}}{2\pi \text{ rad.}}\right) \left(\frac{60 \text{ sec.}}{\text{min.}}\right)$$

$$W_n = 884.48 \text{ rpm}$$

## 6. Amplification Ratio

$$\frac{\theta}{(T_r/h)} = \frac{-k}{I_1 W^2 - k}$$

Where:  $T_r$  = Applied cyclic torque

$$\text{When: } W = (3600 \text{ rpm}) \left(\frac{2\pi \text{ rad.}}{\text{rev.}}\right) \left(\frac{\text{min.}}{60 \text{ sec.}}\right)$$

$$W = 376.99 \frac{\text{rad.}}{\text{sec.}}$$

$$\frac{\theta}{(T_r/k)} = \frac{-4268.16 \text{ lb.-in.}}{(0.4975 \text{ lb.-in.-sec.}^2)(376.99 \frac{\text{rad.}}{\text{sec.}})^2 - 4268.16 \text{ lb.-in.}}$$

$$\frac{\theta}{(T_r/h)} =$$