The Relationship Between Mathematical Performance and GDP per Capita

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Abstract

It is a commonly held belief that a country's scholastic performance in STEM related subjects is indicative of a country's future economic standing. This paper is intended to present a quantitative analysis on the subject matter, testing our hypothesis that there is indeed a positive correlation between academic performance in STEM subjects and a country's economic strength. The PISA 2012 math scores and GDP per Capita were chosen as indicators of academic performance and economic strength respectively. Other indicators such as Literacy Rate and Educational Spending (indicators that are commonly associated with economic standing), are also included in the analysis. The findings from our study confirm our hypothesis and suggest that there is a positive correlation between academic performance in STEM subjects (in particular math) and economic strength.

1. Introduction

Education is vital to any nation's economic strength, long term survival, and prosperity. Specifically, investments in STEM education (especially in the areas of mathematics and science) can lead to economic growth in that citizens who become better educated in STEM are likely to get higher paying technological jobs that support innovation. In the United States, it is important for students to have quality education and demonstrate strong academic performance in order for the U.S. to maintain its long term economic productivity and dominance. For developing nations, early science and math education can either make or break efforts to become an economic powerhouse. Although it is clear that there is value in STEM education, opinions differ on the level of involvement the government should take in funding education, the effectiveness of educational spending, and the correlation between academic performance and economic welfare. Thus, we performed our own cross-country analysis to formulate an unbiased conclusion on the correlation between STEM academic performance and economic indicators such as Gross Domestic Product per-Capita.

As technology continues to expand into every corner of society and the global economy, understanding the relationship between economic growth and STEM education is a topic of importance in the 21st century. In order to compete on a global scale, nations need to understand the importance of investing in STEM education and technology. Countries that fail to adequately educate their populations in STEM related fields will not only suffer from the long term consequences of stagnating economies but also suffer from a "talent vacuum"; a phenomenon in which talented workers migrate to more promising areas which can hurt the existing economy. The United States in particular has the very real danger of losing its place among the top tier of the world's economy if it continues to reduce its investment in STEM education. If the United States does not invest in quality STEM education, the U.S. economy will lose its rank as being one of the strongest in the world to nations that are investing heavily in STEM education .

For our project we are examining the relationship between a country's economic strength and the academic performance of its students. Our hypothesis is that countries with higher GDP per capita will have higher mean math test scores. When looking at economic strength, generally three factors are considered: labor productivity, human capital, physical capital, and technology. Although each factor holds equal importance in determining economic strength of different countries, our paper in particular will focus on the impact that human capital (the amount of knowledge or skill a person has) has on GDP per capita. Smarter, more skilled workers are able to produce more. Therefore, increasing the human capital within a country increases the country's productivity and economic strength.

In order to analyze the correlation between academic performance and general wealth, we will first model the relationship between mean math scores and GDP per Capita for each country. Second, we will add GDP growth rate to the model in order to determine the variable's correlation with GDP per Capita and to also see if GDP growth rate affects mean math test scores. Finally, we will add non economic factors (education expenditures and literacy rates) to the model in order to achieve a holistic representation of the factors that have a partial effect on mean math scores. For our study, we will look at twenty-seven countries that vary in different levels of political regimes. For our final analysis, we will categorize each country as either being a democracy or non-democracy in order to determine if a country's' regime type ,as well as economic strength, affects mean math test scores of each country.

Our topic is an important area of research because education policy and corresponding systems and institutions can have a dramatic effect on a nation's economic performance and the quality of life of its citizenry. Because of this, differences in educational performance between countries can dramatically change the outcome of a country's future.

2. Literature Review

2.1 Examining The Traditional Methods for Analyzing Education and Economic Strength

The main idea of this article (Benos & Zotou 2014) is that traditional measures, such taking into account the number of years citizens have been in school, have been used to analyze and asses international education levels. Although using these measure have been the norm, because of the more readily available cross-sectional test score data samples from international mathematics and science exams, other methods of analysis can be used. By using these new analytical methods , it has been observed that a one standard deviation increase in educational test scores in a particular nation is estimated to increase that nation's per capita income growth by roughly 1.4 percent per year. Although groundbreaking, there are some limitations to this new research, since the tests that are used to conduct such research are given infrequently, which can lead to frequent errors in test administration. The cause and effect relationship is also difficult to discern, since many tests are conducted before income data is collected while others are conducted after economic data is collected.

The authors of this paper used a cross-country regression model with test scores as the independent variable and annualized GDP growth as the dependent variable. Additionally control variables, such as population growth and labor force participation are also included in the model. From the cross-sectional regressions, it can be observed that improving performance on international tests correlates to an increase in the country's income. This paper is different than ours in terms of the controlled variables. For our paper, we included literacy rates,GDP per Capita, GDP growth rate, and educational expenditure as control variables.

2.2 The Relationship Between Socioeconomic Background and Mathematical Performance

For our paper, we used the math scores of PISA 2012 in order to model the dependent variable *MeanMath* , which are the same math scores used for this paper. Unlike our paper, the author's analysis primarily centers its independent variables around the socio-economic characteristics attributed to the lives of individuals. However, the authors of the article, as well as ourselves, see a direct correlation between individual "wealth" and a country's economy.

For our analysis, we will use *GDP per Capita, GDP Growth Rate, Literacy Rate*, and *Educational Spending as independent variables..* Unlike Bailey's analysis, we will not include qualitative variables, however, we do believe that there will be a positive correlation between test scores and country's wealth that is similar to the correlation between test scores and socioeconomic status.

2.3. Non Economic Factors Relation to the Differentiation of Academic Performance Across Countries.

Woessmann (2001) looks at how institutions(and not education expenditure per student) within a particular country affect student performance on math and science test scores. For this study, the author uses data from the Third International Mathematics and Science Study (TIMSS) to analyze the test scores from 39 countries, as well as data from the Organisation for Economic Co-operation and Development (OECD) to study the education systems of those countries. The author uses a robust linear regression model, with countries as strata and schools as the primary sampling unit. Within the sampling unit, the author studies the effects of five institutional features of a nation's educational system, and how these factors affect student achieve on the TIMSS. The five institutional features tested were: 1) centralized exams; 2) the distribution of decision-making power between schools and their governing bodies; 3) the level of influence that teachers and teacher unions have on school policy; 4) the distribution of decision-making power among levels of government and 5) the extent of competition from the private-school sector. The effects of family background and the level of resources devoted to education were controlled during the experiment.

The results of the experiment showed that there was no strong positive relationship between spending and student performance. The correlation coefficient between spending per student and average TIMSS test scores is 0.13 in primary school and 0.16 in middle school (1.0 denotes a positive correlation). However, the effects of all five institutional variables affected student performance on the TIMSS significantly; math scores increase by more than 210 points and science scores increased by 150 points. A student who experienced institutions that were all conducive to student performance would have scored more than 200 points higher in math than a student who hasn't experienced such institutions.

This paper is different than ours as it primarily attempts to study the effects that a country's institutions has on math and science test scores. However, for our final analysis, we will test whether the distribution of decision-making power among levels of government (i.e. the domestic affairs within the countries tested) has either a strong or weak correlation to mean math PISA scores.

3. Data

The specific variables that we chose to examine include macroeconomic indicators such as a country's GDP per capita and corresponding GDP growth rate. We are also examining indicators of educational development, such as the literacy rate of citizens and educational expenditure. Lastly, we will study indicators relating to the government structure of each country such as the democracy index. This mix of economic and non-economic variables gives us a good mix of data to regress upon and make our determinations about the relationship between educational development and economic strength.

In the simple regression model, we study how the GDP per capita of countries affect the PISA math scores of the countries tested. GDP per capita is the independent variable of the model and the dependent variable is mean math test scores. For the first multiple regression model, GDP per capita and GDP growth are independent variables (GDP per capita is the control variable). In the second regression model, GDP per capita, GDP growth rate, and literacy rate per country are the independent variables (GDP per capita and GDP growth rate are the control variables). The third, and last multiple regression model includes GDP per capita, GDP growth rate, literacy rates, and educational expenditure as independent variables (GDP per capita, GDP growth rate, and literacy rates are control variables). Like in the simple regression, the dependent variable is test scores in all three multiple regression models.

3.1 Descriptive statistics

Table 1. shows the summary of all the data used in this paper. Our data was sourced from The World Bank, International PISA test results (from 15 year olds) and The Economist data sets from 2012. Each variable is recorded from the year 2012 from twenty-seven countries around the world. This data is consistent and represents a good range of values with few outliers. The range for some of the values is quite large . For example, the range in GDP growth rates goes from -2.8% to +6%. The Mean Math scores had lower variability compared to values of GDP per capita, which varied, significantly across countries.

Table 1: Summary Statistics- Summary of all data input into STATA for regression analysis

DATA SUMMARY

Variable	Observations	Mean	Std. Dev.	Min	Max
MeanMath	27	457.963	60.84816	368	573
LiteracyRate	27	95.81481	4.596713	79.1	100
GDPperCapita	27	32003.78	18047.5	11341	83066
GDPgrowthr	27	2.655556	2.232683	-2.8	6
EdSpending	27	17.69931	4.015386	10.40832	26.74356

3.2 Gauss Markov Assumptions

Our data does not appear to violate any Gauss Markov assumptions. As the Table 2. shows, there is some correlation between variables but none are perfectly correlated.

Table 2. Statistical Correlation between variables

CORRELATION TABLE					
	MeanMath	LiteracyRate	GDPperCapita	GDPgrowthr	EdSpending
MeanMath	1.0000				
LiteracyRate	0.4855	1.0000			
GDPperCapita	0.7469	0.2536	1.0000		
GDPgrowthr	-0.4109	-0.4467	-0.2007	1.0000	
EdSpending	0.3665	0.2000	0.1421	-0.3425	1.0000

4. Results

The following results were obtained by using STATA to perform statistical analysis on our simple linear regression as well as our multiple linear regressions.

4.1 Simple Linear Regression

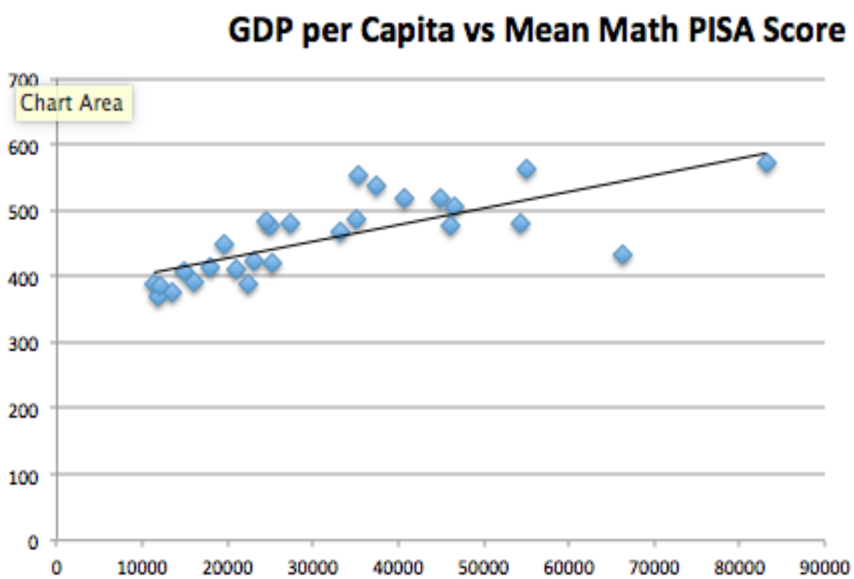
Figure 1. shows that the Mean Math Pisa Score is indeed correlated with a country's GDP per Capita. The t-value of t=5.62 and p=0.000 shows that GDP per Capita is significant at all levels. The R-squared value, 0.5579, shows that GDP per Capita has a moderate positive correlation with mean math PISA scores. It is however not as strong as we had predicted that it would be given the expected relationship between quality of education and a country's economic strength. GDP per Capita has a low standard error, which indicates that the sample mean is a more accurate reflection of the actual population mean. To further study how other indicators affect math performance, we created additional regression models that contain relevant variables, such as literacy rate and education spending as a percent of GDP.

Equation 1: GDP per Capita vs. Mean Math PISA Score

Math = 377.368 + 0.0025183gdppercapita

(16.3997) (0.0004484)

Figure 2: Simple Linear Regression Graph- GDP per Capita vs Mean Math PISA Score



4.2 Multiple Regression

Equation 2. looks at the partial effect that GDP growth rate has on mean math PISA scores for each country (GDP per capita is controlled for).

Equation 2: GDP per Capita and GDP Growth Rate vs. Mean Math PISA Score

Math = 377.368 + 0.0025183gdppercapita + -7.412818gdpgrowthrate

(19.43181) (0.000428) (3.459296)

The STATA results from Figure 3. demonstrates a more significant R-Squared value than what our simple regression model. showed. The R-squared value has increased from 0.5579 to

0.6289, which exemplifies a stronger correlation than the first simple regression model. GDP per Capita is still significant at all levels (t- statistic = 5.45 and the p value= 0.000) and still has a low standard error (the standard error has decreased by 2.04E-5). However, GDP growth rate is insignificant at the 99% level (the t-statistics for GDP growth rate is -2.14 and the p- value = 0.042) when conducting a 2-tailed hypothesis test. The standard error for GDP growth rate is also relatively high, which signifies that the sample mean for GDP growth rate is an inaccurate reflection of the actual population mean and that the estimate of the sample mean is not reliable. The high standard error of GDP growth rate is also an indication of the possibility that the coefficient of GDP growth rate is zero. An F-test will be applied to test the significance of GDP growth rate on mean math PISA score.

<u>Equation 3: GDP per Capita,GDP Growth Rate, and Literacy vs. Mean Math PISA Score</u>

$$Math = 90.88668 + 0.0021938gdppercapita + -4.671959gdpgrowthrate + 3.227821literacy$$

$$(174.1328) \quad (0.0004166) \quad (3.640636) \quad (1.790899)$$

The STATA results in Figure 4. demonstrates a slightly more significant R-Squared value than shown in the previous regression model. The R-squared value has increased from 0.6289 to 0.6748 , which exemplifies a stronger correlation . GDP per Capita is still significant at all levels (t- statistic = 5.27 and the p value= 0.000) and has a low standard error estimate. However, literacy has a t-value of 1.80 and a p-value of 0.085, which causes this variable to be insignificant at the 95% and 99% levels when conducting a two-tailed hypothesis test. Also, the standard error for literacy is relatively high, which suggests that the sample population for literacy deviates from the actual population. GDP growth rate is insignificant at all levels in this regression model, with a t-value of -1.28 and a p-value of 0.212. Also, GDP growth rate has increased by 0.18134, which further shows that the estimate for GDP growth rate is not reliable. Because literacy has significance at the 90% level, an F- test will not be conducted to determine its significance.

<u>Equation 4: GDP per Capita,GDP Growth Rate, Literacy, and Education Spending vs. Mean Math PISA Score</u>

$$Math = 73.53536 + 0.0021843gdppercapita + -4.405049gdpgrowthrate + 3.349879literacy + 0.3563159educationspending$$

$$(183.0978)\ (0.0004252)\ (3.774255)\ (1.852368)\ (0.9258267)$$

The STATA results in Figure 5. shows that regression model shown in Equation 4. has a R-Squared value that is slightly more significant than the R-Squared value from the previous regression model. The R-squared value has increased from 0.6748 to 0.6770 , which exemplifies a stronger correlation . GDP per Capita is still significant at all levels (t- statistic = 5.14 and the p value= 0.000) and its standard error has decreased by 2.8E-6. Literacy is only significant at the 90% significance level and thus insignificant at the 95% and 99% percent levels. The standard of error for literacy is still high, and has increased by 0.061469 . Education spending has a t-statistic 0.38 and a p-value of 0.704, which signifies that education spending is insignificant at all levels when conducting a two-tailed hypothesis test. Like literacy, education spending also has a relatively high standard error, which shows that the sample mean is varied in comparison with the actual population. Growth rate is also insignificant at all levels in this regression model, with a t-value of -1.17 and a p-value of 0.256, and still has a high standard error (the standard error has increased by 0.133619). An F- test will be conducted in order to verify the insignificance of education spending as well as GDP growth rate.

5. Robustness Tests: F-Tests

To test if there is joint significance between GDP growth rate and education spending, we create a restricted model excluding these variables and conducted an F-test. The results from the restricted model is shown in Figure 6.

Equation 5: The Restricted Model-GDP per Capita and Literacy vs. Mean MAth PISA Score

$$Math = -15.22748 + 0.0022478gdppercapita + 4.187805literacy$$

$$(155.2983)\ (1.64896)\ (0.00042)$$

Equation 6: F-test Formula

$$F= ((SSRR\ - SSR_{UR}\ )/q)/(\ SSR_{UR}/n\text{-}k\text{-}1)$$

q= 2: number of restrictions

n= 27: number of observations

k= 4: number of independent variables

The restricted model shows that GDP per capita is highly significant at the 90%, 95%, and 99% levels whereas literacy shows high significance at the 90% and 95% levels. We used Equation 5 to obtain an F-test value of about 0.8670. At the 95% significance level, we obtain a critical value of 3.44. Therefore, we fail to reject our null hypothesis and can conclude that there is no joint significance between GDP growth rate and education spending.

For our final analysis, we studied the effects that government regime type had on mean math PISA test score. We separated the twenty-seven countries used in our previous regression models into two groups (more democratic and less democratic) and performed regressions on each group. Table 3. shows the grouping of all twenty-seven countries. Since GDP growth rate and education spending are both jointly insignificant, these two variables were excluded from our regression model when studying the more democratic and less democratic groups. Below are the regression models used .

Equation 7: Regression Models used for More Democratic Regimes

$$Math = -178.2805 + 0.0023547gdppercapita + 5.848108literacy$$

$$(485.5346) \qquad (0.0010172) \qquad (5.15334)$$

Equation 8: Regression Model used for Less Democratic Regimes

$$Math = 79.17753 + 0.0021588gdppercapita + 3.1679literacy$$

$$(218.9335) \qquad (0.0005376) \qquad (2.35582)$$

Equation 9: Regression Model used for Pooled Data

$$Math = -15.22748 + 0.0022478gdppercapita + 4.187805literacy$$

$$(155.2983) \qquad (1.64896) \qquad (0.00042)$$

Table 3: More Democratic vs. Less Democratic Grouping of 27 Countries

More Democratic			
Australia **Costa Rica** **Japan** **United States**	**Brazil** **Finland** **Lithuania** **Uruguay**	**Canada** **Israel** **South Korea**	**Chile** **Italy** **Sweden**
Less Democratic			
Argentina **Russia**	**Columbia** **Malaysia**	**Hong Kong** **Mexico**	**Hungary** **Peru**

Jordan **UAE**	**Singapore**	**Tunisia**	**Turkey**

With our breakdown of the data into two distinct subgroups, More Democratic and Less Democratic based on the political and economic structure of each nation, we ran our regression models again to see if there were any marked differences between the two subgroups. Indeed we found that the overall R-squared of the regression model increases from 0.5763 to 0.6635. This may be due to less democratic nations having a more effective means of executing education policy and becoming efficient at making improvements whereas more democratic nations may be slower to adopt new initiatives.

Equation 10: Chow Test Formula

$$F = ((SSRp - (SSR_{Dem} + SSR_{NonDem}))/q)/((SSR_{Dem} + SSR_{NonDem})/n-2(k+1))$$

q= 2: number of restrictions

n= 27: number of observations

k+1= 2: number of independent variables

After conducting the Chow Test, are F statistic is about 0.3753. At the 95% significance level, the critical value for the F-test is 3.42. Therefore, we fail to reject the null hypothesis and conclude that more democratic regimes and less democratic regimes (as it pertains to math performance) are not jointly significant.

IV. Conclusion

Our worldwide analysis of the relationship between STEM educational performance and economic strength among 27 different nations yielded some significant insights into the complex relationship between education and economic development. As we had initially hypothesized, nations with stronger performance on STEM related PISA tests were in fact more likely to show greater economic strength as measured by each nation's GDP per capita. Overall this is a satisfactory result that shows our initial hypothesis was not unfounded.

However, the relationship between STEM educational performance and economic strength is very complex, as we discovered from further statistical analysis. Our results were based on an analysis of 27 individual nations, and having a relatively small sample size can cause results to be biased. This is an example of one of the complications that stem from micronumerosity, and further research should be conducted using a wider range of nations in order to determine if micronumerosity was indeed skewing our results in a particular direction.

We also conducted an extensive analysis on each variable to understand its statistical significance in relation to our regression models. We wanted to test if a nation's political structure played a role in determining the education and economic performance that we measured. To conduct such an analysis we broke our 27 countries down into two groups based on the level of democratic freedom that exists in that particular nation. From conducting this analysis, we obtained that countries who exhibit more authoritarian political structures have slightly better STEM test scores that countries that are more democratic. This could be due to more rigid political structures being able to become more efficient at adjusting to nationwide educational initiatives than more democratic and bureaucratic countries. We also conducted a Chow test to test this empirically, but we found that given the results of the Chow test the democracy ratings were not statistically significant. This result may be influenced by the methodology that we used to separate the countries into groups. A further analysis with a more rigid definition of what makes a nation democratic may yield different results. Overall we learned that placing an emphasis on STEM education is vital to a nation's long term economic strength and survival, just as we had hypothesized.

References

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Appendix (STATA Output)

Figure 1: Simple linear regression model- STATA output table of GDP per Capitavs Mean Math PISA Score

Source	SS	df	MS			
Model	53705.2322	1	53705.2322	Number of obs	=	27
Residual	42559.7308	25	1702.38923	F(1, 25)	=	31.55
				Prob > F	=	0.0000
				R-squared	=	0.5579
				Adj R-squared	=	0.5402
Total	96264.963	26	3702.49858	Root MSE	=	41.26

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0025183	.0004484	5.62	0.000	.0015949	.0034417
_cons	377.3684	16.3997	23.01	0.000	343.5926	411.1442

Figure 3. Multiple Linear Regression Model- STATA output table of GDP per Capita and GDP Growth Rate vs. Mean Math PISA Test Score

Source	SS	df	MS			
Model	60540.3693	2	30270.1846	Number of obs	=	27
Residual	35724.5937	24	1488.52474	F(2, 24)	=	20.34
				Prob > F	=	0.0000
				R-squared	=	0.6289
				Adj R-squared	=	0.5980
Total	96264.963	26	3702.49858	Root MSE	=	38.581

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0023343	.000428	5.45	0.000	.001451	.0032175
gdpgrowthrate	-7.412818	3.459296	-2.14	0.042	-14.55245	-.2731815
_cons	402.9425	19.43181	20.74	0.000	362.8372	443.0478

Figure 4. Multiple Linear Regression Model- STATA output table of GDP per Capita, GDP Growth Rate, and Literacy Rate vs. Mean Math PISA Test Score

Source	SS	df	MS
Model	64961.5691	3	21653.8564
Residual	31303.3939	23	1361.01712
Total	96264.963	26	3702.49858

Number of obs	=	27
F(3, 23)	=	15.91
Prob > F	=	0.0000
R-squared	=	0.6748
Adj R-squared	=	0.6324
Root MSE	=	36.892

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0021938	.0004166	5.27	0.000	.0013321	.0030555
gdpgrowthrate	-4.671959	3.640636	-1.28	0.212	-12.20319	2.859271
literacy	3.227821	1.790899	1.80	0.085	-.4769364	6.932578
_cons	90.88668	174.1328	0.52	0.607	-269.3345	451.1078

Figure 5. Multiple Linear Regression Model- STATA output table of GDP per Capita, GDP Growth Rate, Literacy Rate vs. Mean Math PISA Test Score

Source	SS	df	MS
Model	65170.9156	4	16292.7289
Residual	31094.0474	22	1413.36579
Total	96264.963	26	3702.49858

Number of obs	=	27
F(4, 22)	=	11.53
Prob > F	=	0.0000
R-squared	=	0.6770
Adj R-squared	=	0.6183
Root MSE	=	37.595

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0021843	.0004252	5.14	0.000	.0013025	.0030662
gdpgrowthrate	-4.405049	3.774255	-1.17	0.256	-12.23238	3.422276
literacy	3.349879	1.852368	1.81	0.084	-.4916961	7.191454
educationspending	.3563159	.9258267	0.38	0.704	-1.563731	2.276363
_cons	73.53536	183.0878	0.40	0.692	-306.1654	453.2361

Figure 6: Restricted Linear Regression Model

Source	SS	df	MS
Model	62720.2326	2	31360.1163
Residual	33544.7303	24	1397.6971
Total	96264.963	26	3702.49858

Number of obs	=	27
F(2, 24)	=	22.44
Prob > F	=	0.0000
R-squared	=	0.6515
Adj R-squared	=	0.6225
Root MSE	=	37.386

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0022478	.00042	5.35	0.000	.0013809	.0031146
literacy	4.187805	1.64896	2.54	0.018	.7845189	7.591091
_cons	-15.22748	155.2983	-0.10	0.923	-335.7474	305.2925

Figure 7: More Democratic Nations Regression Results

Source	SS	df	MS
Model	19344.733	2	9672.36652
Residual	14225.267	11	1293.20609
Total	33570	13	2582.30769

Number of obs	=	14
F(2, 11)	=	7.48
Prob > F	=	0.0089
R-squared	=	0.5763
Adj R-squared	=	0.4992
Root MSE	=	35.961

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0023547	.0010172	2.31	0.041	.0001159	.0045936
literacy	5.848108	5.15334	1.13	0.281	-5.494317	17.19053
_cons	-178.2805	485.5346	-0.37	0.720	-1246.935	890.3739

Figure 8: Less Democratic Nations Regression Results

Source	SS	df	MS
Model	35995.8112	2	17997.9056
Residual	18259.2657	10	1825.92657
Total	54255.0769	12	4521.25641

Number of obs	=	13
F(2, 10)	=	9.86
Prob > F	=	0.0043
R-squared	=	0.6635
Adj R-squared	=	0.5961
Root MSE	=	42.731

math	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gdppercapita	.0021588	.0005376	4.02	0.002	.000961	.0033566
literacy	3.1679	2.35582	1.34	0.208	-2.081195	8.416994
_cons	79.17753	218.9335	0.36	0.725	-408.6368	566.9919