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SYMBOLIC MODELING OF FLEXIBLE ROBOTIC MANIPULATORS

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ABSTRACT

This paper presents a new systematic algorithm to symbolically derive the full nonlinear dynamic equations of motion of multi-link flexible manipulators. Lagrange's-assumed modes method is the basis of the new algorithm and adapted in a way suitable for symbolic manipulation by digital computers. The advantages of obtaining dynamic equations in symbolic form and of the presented algorithm are discussed. Application of the algorithm to a two-link flexible arm example via a commercially available symbolic manipulation program is presented. Simulation results are given and discussed.

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I. Introduction :

Dynamics of a typical industrial manipulator, with six degrees of freedom, is governed by coupled highly nonlinear ordinary differential equations. These equations present a very complicated problem in control system design, mainly because the knowledge in nonlinear control system theory is very limited. Traditionally independent servo controllers are designed based on the assumption that nonlinear coupling terms are negligible. However, this assumption is reasonable and the control system performance may be satisfactory only if the speed of manipulator is "relatively slow". Increasing demand for higher industrial productivity requires manipulators that move faster and more accurately. As a result, the speed of manipulators must increase and the independent linear servo controllers, designed based on the slow motion dynamics, will perform unsatisfactorily.

In recent years there has been considerable progress in the adaptive control of robotic manipulators. Computed torque based methods are aimed at better performance by designing controllers based on more accurate models. Ultimately the performance and the capabilities of a system, i.e. maximum speeds etc., are limited by the initial design of the overall system. A control system at best can utilize these capabilities in an optimum manner. In other words no control law can make the system move at speed which can not be afforded by the existing actuators. Apparently one way of designing manipulators that can move faster is to increase the actuator sizes. However, since actuators themselves are carried by the other actuators, increasing size also increases the effective inertia resulting in a very massive structure. Thus this approach can be quickly self defeating and is not the ultimate answer.

The next option is to design light weight systems. Lightweight systems could have the following advantages: higher speed of operation, less overall cost, less energy consumption, smaller actuator sizes, higher productivity. The drawback of such systems is the structural flexibility which deteriorates the accuracy and repeatability. Rigid body dynamic analysis will no longer be accurate and controllers based on this will not perform satisfactorily. Flexibility has to be included in the analysis.

Background:

Modeling and control of a single link flexible arm [Fig.1] has been investigated by many authors [1,2,3,4]. The system is essentially modeled as Bernoulli-Euler beam and vibration coordinates are approximated by a finite number of assumed mode shapes. This allows the application of the whole finite dimensional linear control theory to the problem.

The multi-link flexible manipulator [Fig.2, and 3] modeling and control problem has not been researched as much as single -link case. First of all, the modeling problem is not a trivial one. Due to coupling between links, large configuration changes, and high speeds, the system can no longer be accurately represented by simple beam equations. An accurate dynamic model of a light weight arm involves highly complicated algebraic manipulations and can become impossible to deal with by hand. Moreover, the possiblity of making errors along the way is very high. Making some changes in an existing model also requires long algebraic manipulations. There are two basic methods used in the modeling : 1. Lagrange's-Finite Element based methods, 2. Lagrange's- assumed mode based methods. The end result of these methods are essentially the same. Many of the finite element based works on the analysis of closed chain mechanisms can be applied to the dynamic modeling of multi-link flexible arms [5,6].

In [7,8] the nominal joint variable time histories are assumed to be known and the small vibration dynamic model of the manipulators and mechanisms about nominal motions are developed. In [9] this assumption is removed and full dynamic model is derived. The main advantages of this method are : a) very systematic, b) Can be applied to complex shaped systems, applicable to a very wide class of problems. The disadvatages are: a) requires a substantial amount of software organization, b) results in constrained model, c) does not give much insight to the dynamic structure of the system. Static deflection modes are included in the modes to improve the accuracy of models with limited number of mode shapes [6] . Usuro et.al. investigated the performance of LQR with prescribed degree of stability on a two-link planar arm by digital simulations [10].

The Lagrangian - assumed modes method is used in the modeling of a two-link robotic manipulator in [11]. Distributed frequency domain analysis of non-planar manipulators using transfer-matrices has been developed in [12]. A recursive method using homogeneous transformation matrices to generate full coupled nonlinear dynamics of multi-link flexible manipulators is presented in [13].

It was experienced by the authors that the application of this technique to multi-link manipulators works well, but with an important drawback: <u>Algebraic complexity of intermediate steps</u>. When carried out by hand the length of expressions becomes very large and very time consuming. In addition, the possibility of making algebraic errors was quite high. On the other hand, the modeling method is easy to understand, recursive, does not require any dedicated special software and derives the full nonlinear dynamic model.

The symbolic manipulation programs eliminate the major drawback of the method. Symbolic modeling allows one to model systems with large order in a very short time, check the elements of the dynamic equations in explicit form and manipulate them very conveniently. Leu and co-workers developed programs to obtain dynamic equations of rigid robotic manipulators symbolically using commercially available symbolic manipulation programs [15, 16]. The method presented here is more general in the sense that it can handle structural flexibilities and rigid manipulator modeling problem is a special case of it.

The remaining part of this paper is organized as follows:
Section II summarizes the Lagrangian - assumed Modes method. Section III presents a new algorithm which adapts this method to a form suitable for symbolic manipulation by digital computer. In section IV, the algorithm is applied to a two-link flexible arm example. Application details and simulation results are discussed.

II. Lagrangian - Assumed Modes Method :

Kinematics: The first step in dynamic modeling of any mechanical system is to establish the kinematical relationships and be able to define fundamental vector quantities: position, velocity and acceleration. Consider the kinematic structure shown in [Fig.2] representing a manipulator with serial links and joints. Let the coordinate systems used for kinematics of the system be;

O_0XYZ - Fixed to base (Global Coordinate Frame)

O_ixyz - Fixed to the base of the link i

O'_ixyz - Fixed to the end of link i

If arms are rigid then $O_i'xyz$ coordinates are not needed. The position vector of any point on link i can be expressed with respect to O_ixyz as

$$^ih(x_i) = [x_i,0,0,1]^T + [w_x(x_i,t),w_y(x_i,t),w_z(x_i,t),0]^T \quad (2.1)$$

where; $w_x(x_i,t)$, $w_y(x_i,t)$, $w_z(x_i,t)$ are displacements of the flexible arm due to flexibility in respective directions. The dependence of w's on the spatial coordinates makes the system infinite dimensional, leading to coupled ordinary and partial differential equations of motion. In general these are approximated by finite series consisting of spatial variable dependent functions multiplied by time-dependent generalized coordinates. Once the number of generalized coordinates to be used to represent the distributed flexibility of each link has been decided on, w's can be approximated as;

$$w_\beta(x_i,t) = \sum_{j=1}^{n_i} \phi_{\beta j}(x_i)\, \delta_j(t) \quad ; \beta : x,y,z \quad (2.2)$$

where n_i is the number of assumed mode shapes used for link i for the w_β, $\phi_{\beta j}(x_i)$ are assumed mode shape functions from an admissable class, $\delta_j(t)$ are the generalized coordinates of approximation, $^ih(x_i)$ is uniquely defined. Next we need to be able to transfer this position vector with respect to global coordinate frame to obtain absolute position vector. Let ${}^{o}W_i$ be the homogeneous matrix transformation from moving coordinate frame $O_i xyz$ to fixed inertial frame $O_0 XYZ$. Then the absolute position vector, [Fig.3]

$$ {}^{o}h(x_i) = {}^{o}W_i \cdot {}^{i}h(x_i) \tag{2.3}$$

It is clear that the transformation ${}^{o}W_i$ consists of two parts: joint variables and flexible deflections. More clearly, [Fig.2]

$$ {}^{o}W_i = {}^{o}W_{i-1} \cdot E_{i-1} \cdot A_i \tag{2.4}$$

where

A_i - the transformation between $O_i xyz$ and $O'_{i-1} xyz$ - joint transformation

E_{i-1} - the transformation from the end of the link coordinates to link base coordinates.

${}^{o}W_{i-1}$ - the total transformation to the base cordinates from the link base coordinates.

The form of these transformation matrices are ;

$$ {}^{j}W_i = \left[\begin{array}{c|l} {}^{j}R_i & x_j \text{ component of } O_i \\ & y_j \text{ component of } O_i \\ & z_j \text{ component of } O_i \\ \hline 0^T & 1 \end{array} \right] ; \tag{2.5}$$

${}^{j}R_i$ is (3x3) matrix of direction cosines, 0^T (1x3);

$$E_i = \begin{bmatrix} 1 & 0 & 0 & l_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \sum_{j=1}^{n_i} \delta_{ij}(t) \begin{bmatrix} 0 & -\theta_{zij} & \theta_{yij} & x_{ij} \\ \theta_{zij} & 0 & -\theta_{xij} & y_{ij} \\ -\theta_{yij} & \theta_{xij} & 0 & z_{ij} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.6}$$

where $\theta_{\beta ij}$'s are rotation components of link i due to mode j, assuming small rotations due to flexible deflections, and l_i is the length of the link i.

Once the kinematic description of the system is set up, the process of obtaining the equations of motion is as follows:

1. Pick generalized coordinates (natural choices are joint variables and a finite number of an assumed modes series approximation for every flexible element).

2. Form the kinetic, and potential energy, and virtual work for the system.

3. Take the necessary derivatives of the Lagrangian Equations and assemble the equations.

If the system has N_j number of joints with single degree of freedom and N_l number of flexible elements with n_i modal coordinates for each element, the dynamic model of the system will be governed by a set of

$$N_j + \sum_{i=1}^{N_l} n_i \tag{2.7}$$

coupled second order ordinary differential equations.

III. Symbolic Implementation of Lagrangian - Assumed Modes Method:

Although the Lagrangian - assumed modes method is theoretically very well understood and documented [13], it is not quite in a form suitable for symbolic implementation on a digital computer, i.e. insufficient memory problems are likely to occur. Let us first specify some desired features of a modeling algorithm.

First, the mode shapes and the mode shape dependent parameters should be easily varied by the analyst. The selection of "appropriate" or "best" mode shapes for a given flexible system is not a clearly answered problem [12]. One should be able to easily simulate the effect of different mode shapes on the system behavior. For the case of a simple beam under bending vibrations the mode shapes effectively determine the natural frequencies of the system. Effective mass and spring matrix elements are functions of mode shapes as; (with simple boundary conditions)

$$m_{ij} = \int_0^{l_i} \rho \, A(x) \, \phi_i(x) \, \phi_j(x) \, dx \tag{3.1}$$

$$k_{ij} = \int_0^{l_i} E \, I(x) \, \phi_i''(x) \, \phi_j''(x) \, dx \tag{3.2}$$

If mode shapes are orthonormalized such that $m_{ij} = 1$ for i=j and 0 , for $i \neq j$, then $k_{ij} = w^2{}_i$ for i=j, 0 for $i \neq j$. The most accurate way is to update the mode shapes as the boundary conditions of the links vary as function of controller impedance.

Second, a recursive algorithm would be very desirable. For instance, when the number of modal coordinates increased or additional links included, the dynamic modeling process should not be repeated all over.

Third, method should eliminate any unnecessary algebraic operations so that it would be more efficient and require less memory.

The equations governing the dynamics of the system are given by;

$$\frac{d}{dt}\left(\frac{\partial \Sigma KE}{\partial \dot{q}_i}\right) - \frac{(\partial \Sigma KE)}{\partial q_i} + \frac{(\partial \Sigma PE)}{\partial q_i} = Q_i \tag{3.4}$$

where;

$\Sigma KE = \sum_{i=1}^{N} (KE)_i$; N: total number of discrete elements in the system (joints, links, payload).

$$\Sigma PE = \sum_{i=1}^{N} (PE)_i \text{ gravitational} + (PE)_i \text{ elastic} \tag{3.5}$$

q_i's are the generalized coordinates which are joint variables and flexible generalized coordinates of flexible elements.

Kinetic energies for rotary joints, if considered as mass with rotary inertia about the axis of rotation

$$(KE)_{joint\ i} = 1/2\ m_i V_{gi}^2 + 1/2\ \underset{\sim}{H}_{gi} \cdot \underset{\sim}{w}_i \tag{3.6}$$

where m_i is the mass of joint i, V_{gi} is the speed of joint i mass center, H_{gi} is angular momentum vector of joint with respect to its center of mass, w_i is the total angular velocity vector of the joint.

Kinetic energy of the flexible links;

$$(KE)_i = 1/2 \int_0^{l_i} \rho_i(x) \left(\dot{\underset{\sim}{r}}_i \cdot \dot{\underset{\sim}{r}}_i \right) dx \tag{3.7}$$

If all the modal coordinates and associated mode shapes were given, then the integration over the spatial variable could be evaluated. However since the mode shapes and dependent parameters are desired to be entered later by the user for analysis purposes, we identify all possible elements that are functions of spatial variables of link i and assign them parametric names. KE_i is spatially dependent only because of link i flexibility. The effect of previous element flexibilities on KE_i are reflected in W terms which depend only on resulting end point motions, and thus have no spatial variable dependence. From (2.3)

$$ {}^{o}\dot{h}_i(x) = {}^{o}\dot{W}_i \; {}^{i}h_i(x) + {}^{o}W_i \; {}^{i}\dot{h}_i(x) \tag{3.8}$$

$$\dot{\underset{\sim}{r}}_i \cdot \dot{\underset{\sim}{r}}_i = {}^{o}\dot{h}_i{}^{T}(x) \cdot {}^{o}\dot{h}_i(x)$$

$$= {}^{i}h_i{}^{T}(x)\; {}^{o}\dot{W}_i{}^{T}\; {}^{o}\dot{W}_i\; {}^{i}h_i(x) + {}^{i}\dot{h}_i{}^{T}(x)\; {}^{o}W_i{}^{T}\; {}^{o}\dot{W}_i\; {}^{i}h_i(x) +$$
$$ {}^{i}h_i{}^{T}(x)\; {}^{o}\dot{W}_i{}^{T}\; {}^{o}W_i\; {}^{i}\dot{h}_i(x) + {}^{i}\dot{h}_i{}^{T}(x)\; {}^{o}W_i{}^{T}\; {}^{o}W_i\; {}^{i}\dot{h}_i(x) \tag{3.9}$$

where;

$${}^{i}h_i{}^{T}(x) = [x + \Sigma\phi_{xij}(x)\;\delta_{xij}(t),\; \Sigma\phi_{yij}(x)\;\delta_{yij}(t),\; \Sigma\phi_{zij}(x)\;\delta_{zij}(t),\; 1]$$

$${}^{i}\dot{h}_i{}^{T}(x) = [\; \Sigma\phi_{xij}(x)\;\dot{\delta}_{xij}(t),\; \Sigma\phi_{yij}(x)\;\dot{\delta}_{yij}(t),\; \Sigma\phi_{zij}(x)\;\dot{\delta}_{zij}(t),\; 0\;]$$

$$\tag{3.10}$$

Elements of the transformations ${}^{o}W_i$ and ${}^{o}\dot{W}_i$ are functions of the generalized coordinates and parameters of the links $k < i$, such as $\{ \theta_i$, $\theta_i \theta_k(t)$, $\phi_{\beta kj}(l_k)$, $\delta_{\beta kj}(t)$, $\theta_k(t)$, where $k=1,..i-1$, $\beta{:}x,y,z\}$, l_k is the length of link k.

In general for serial link robotic manipulators, the kinetic energy of link i will have the following form ; (.) is used to indicate the possible existence of terms that are independent of spatial variable x . At this point, from a symbolic modeling point of view it is not important what these (.) terms are. But what is important is to extract all the possible combination of spatial-variable dependent terms and replace them with symbolic names so that the first objective of the modeling is accomplished.

$$(K.E)_i=(.)\int \rho(x)\, dx+(.)\int \rho(x)\, x\, dx+(.) \int \rho(x)\, x^2\, dx$$
$$+ \Sigma\, \Sigma \int \rho(x)\, \phi_{\beta ij}(x)\, \phi_{\xi ik}(x)\, dx\, .$$
$$[(.)\, \dot{\delta}_{\beta ij}\, \dot{\delta}_{\xi ik} +(.)\, \dot{\delta}_{\beta ij}\, \delta_{\xi ik} +(.)\, \delta_{\beta ij}\, \delta_{\xi ik}\ ] +$$
$$\Sigma\, \Sigma \int \rho(x)\, \phi_{\beta ij}(x)\, x\, dx\, [(.)\, \delta_{\beta ij} +(.)\, \dot{\delta}_{\beta ij}\ ] +$$
$$\Sigma\, \Sigma \int \rho(x)\, \phi_{\beta ij}(x)\, dx\, [(.)\, \delta_{\beta ij} +(.)\, \dot{\delta}_{\beta ij}\ ] \qquad (3.11)$$

where; β and ξ : x,y,z , $j=1,\ldots m_i$. At the calculation of absolute velocity of differential element of a flexible member, the parameters which are functions of the spatial variable can be extracted and be given symbolic names by the symbolic manipulation program very easily. These parameters represent the elements in the dynamic model which are functions of mode shapes, link length, and mass distribution of the flexible element.

It is possible to anticipate the forms resulting from Lagrangian equations and never explicity evaluate the Kinetic energy. This equivalent to substituting (3.12) into (3.9), but replace the spatially dependent terms with numerical values obtained by multiplying with the density and integrating over the associated link length as shown in (3.13).

Replace in (3.9) the following equations (3.12):

$$nm\beta\xi ijk \leftarrow \phi_{\beta ij}(x)\ \phi_{\xi ik}(x)\ ,\quad nw\beta ij \leftarrow \phi_{\beta ij}(x)\ x\ ,$$
$$nq\beta ij \leftarrow \phi_{\beta ij}(x)$$
$$m_i \leftarrow 1.,\ m_i\ l_i/2 \leftarrow x\ ,\quad J_{oi} \leftarrow x^2 \tag{3.12}$$

and in the simulation level evaluate these terms by multiplying with $\rho(x)$ and integrating over the link length.

$$nm\beta\xi ijk = \int \rho(x)\ \phi_{\beta ij}(x)\ \phi_{\xi ik}(x)\ dx\ ,$$
$$nw\beta ij = \int \rho(x)\ \phi_{\beta ij}(x)\ x\ dx\ ,\ nq\beta ij = \int \rho(x)\ \phi_{\beta ij}\ dx$$
$$m_i = \int \rho(x)\ dx\ ,\quad m_i\ l_i/2 = \int \rho(x)\ x\ dx\ ,$$
$$J_{oi} = \int \rho(x)\ x^2\ dx\ ; \tag{3.13}$$

There are six basic parameters related to the inertia properties of the flexible element and with their use there is no longer spatial dependence in the kinetic energy expressions. With this approach one can see more explicitly the effect of mode shapes and system parameters on the dynamic model, leading to a better understanding of the dynamics, which is not offered by numerical or other modeling methods. Notice that if the mode shapes associated with a coordinate (i.e. y) are chosen to be orthonormal with respect to distributed mass and flexibility many of the above terms will be zero, such as $nm\beta\xi ijk = 1$ if $j=k$, 0 if $j\neq k$.

Similiarly for the elastic potential energy of the link i (gravitational potential energy is omited here to save space)

$$(P.E.)_i=1/2\ \Sigma\int\ (EI_y(\phi''_{yij}(x)\ \phi''_{yik}(x)\ \delta_{yij}(t)\ \delta_{yik}(t))$$
$$EI_z\ (\ \phi''_{zij}(x)\ \phi''_{zik}(x)\ \delta_{zij}(t)\ \delta_{zik}(t)\ ) +$$
$$EA(x)\ (\ \phi'_{xij}(x)\ \phi'_{xik}(x)\ \delta_{xij}(t)\ \delta_{xik}(t)\ ))dx\ ; \tag{3.14}$$

Similiarly

$$k\beta ijk = \int_0^{li} EI_\beta(x)\, \phi''_{\beta ij}(x)\, \phi''_{\beta ik}(x)\, dx \;; \qquad \beta : y,z \text{ and } j,k = 1, n_i$$

$$kxijk = \int_0^{li} EA(x)\, \phi'_{xij}(x)\, \phi'_{xik}(x)\, dx \;;$$

$$(P.E.)_i = 1/2 \Sigma \Sigma [\, k\beta ijk\, \delta_{\beta ij}(t)\, \delta_{\beta ik}(t) + kxijk\, \delta_{xij}(t)\, \delta_{xik}(t) \,] \;; \qquad (3.15)$$

Now the next important topic is the development of a recursive method which will not run into memory problems as the system dimension gets large as well as eliminating unnecessary algebraic operations. Moreover once a model is developed, some variations of the model should be possible without repeating the whole modeling process. As the system dimension gets larger, carrying out the derivations using total energy expressions can easily run into memory problems. Thus

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}(\Sigma KE_j) - \frac{\partial}{\partial q_i}(\Sigma KE_j) + \frac{\partial}{\partial q_i}(\Sigma PE_j) = Q_i \qquad (3.16)$$

$$\Sigma \left(\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}(KE_j) - \frac{\partial}{\partial q_i}(KE_j) + \frac{\partial}{\partial q_i}(PE_j)\right) = Q_i \qquad (3.17)$$

Due to serial nature of manipulator arm ;

$$\frac{\partial}{\partial \dot{q}_i}(KE_j) = \frac{\partial}{\partial q_i}(KE_j) = \frac{\partial}{\partial q_i}(PE_j) = 0. \quad \text{for } i > j \qquad (3.18)$$

The equations of motion of the system are found to be;

$$\sum_{j=1}^{N} \left(\frac{d}{dt} \frac{\partial}{\partial \dot{q}_i} (KE_j) - \frac{\partial}{\partial q_i} (KE_j) + \frac{\partial}{\partial q_i} (PE_j) \right) = Q_i \quad ; \quad i = 1,j \qquad (3.19)$$

The following algorithm, in combination with equation (3.19), can be effectively programmed in any commercially available general purpuse symbolic manipulation program to obtain dynamic model equations of multi-link flexible robotic manipulators symbolically.

Algorithm :

For j = 1 to N

For i=1, to j

Find and store KE_j , PE_j (3.11) and (3.15)

$$\frac{d}{dt} \left(\frac{\partial}{\partial \dot{q}_i} (KE_j) \right) , \quad \frac{\partial}{\partial q_i} (KE_j) , \quad \frac{\partial}{\partial q_i} (PE_j)$$

Next i

Next j

Given the results of the algorithm, substitute these to equation (3.19) and assemble the equations in a convenient form for simulations and analysis purposes. After the equations are assembled, it is very easy to program them in one of the standard scientific programming languages using the capabilities of the commercial symbolic manipulation packages.

Let us assume that after modeling a manipulator , it is desired to add another link to the model with n_i degrees of freedom. Based on the above algorithm one must evaluate ;

For i=1, to $N + n_i$

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_i}(KE_{N+1}),\ \frac{\partial}{\partial q_i}(KE_{N+1}),\ \frac{\partial}{\partial q_i}(PE_{N+1}) \qquad (3.20)$$

Next i

Let us assume previous model was assembled in the form:

$$[\,M\,]\ \ddot{q} + f = Q \qquad (3.21)$$

where the inertia matrix dimension is (NxN) , q , f, Q vector dimensions are (Nx1), N is the total number of generalized coordinates up to that point.

The additional link contribution is of the form:

$$\left[\begin{array}{c|c} m_{nn} & m_{nn+1} \\ \hline m_{nn+1} & m_{n+1} \end{array}\right] \left[\begin{array}{c} q_{nn+1} \\ \hline q_{n+1} \end{array}\right] + \left[\begin{array}{c} f_{nn+1} \\ \hline f_{n+1} \end{array}\right] = \left[\begin{array}{c} Q_{nn+1} \\ \hline Q_{n+1} \end{array}\right] \qquad (3.22)$$

where the inertia matrix is of dimension $(N+n_i) \times (N+n_i)$ and the vector quantities are of $(N+n_i) \times 1$ dimension.
Partition of the equation (3.22) is made such that it would clearly reflect the increase in the dimension of the system compared to (3.21). The complete equations of motion are obtained by the addition of (3.22) to (3.21), where (3.21) is extended to (3.22) dimensions with additional zeros corresponding to the new generalized coordinates q_{n+1} introduced by the new element.

The implementation adapted here has the following advantages: a) mode shape and dependent parameters can be easily varied, b) all unnecessary derivatives avoided, c) it is recursive, and d) memory problems are not likely to occur.

IV.Applications and Discussion of Simulation Results:

Here the described modeling method is applied to a two-link planar flexible arm, with rotary joints and payload. Two mode shapes for each link are considered to represent the structural flexibilities. As noted earlier, mode shapes can be input into the simulation program and the effect of different mode shapes on the dynamic response and the accuracy of modes can be checked. Joints and payload are considered as mass with rotary inertia. These inertial parameters can be set to zero as well [Fig.4]

System input parameters for simulation are as follows:

Joint 1 mass and rotary inertia about its center of mass ; mj_1 , jj_1
and similiarly for joint 2 ; mj_2, jj_2 , and for payload ; m_p , j_p
For link 1 and 2 ; mass per unit length, link lengths, flexural rigidity constants .

ρA_1 , ρA_2 , l_1, l_2 , EI_1 , EI_2

Assumed mode shapes and gravity vector ;

$\phi_{11}(x)$, $\phi_{12}(x)$, $\phi_{21}(x)$, $\phi_{22}(x)$; g_x , g_y , g_z

Initialization procedures

Time independent parameters are calculated at the initialization of the program only once per session. If mode shapes are up dated as function of changing boundary conditions, then these parameters need to be reevaluated. These parameters are:

nm11, nm12, nm21, nm22, nw11, nw12, nw21, nw22,
nq11, nq12, nq21, nq22, kw11, kw12, kw21, kw22

$\phi_{11}(l_1)$, $\phi_{12}(l_1)$, $\phi_{21}(l_1)$, $\phi_{22}(l_2)$

$$\frac{\partial}{\partial x}(\phi_{11})\Big| , \frac{\partial}{\partial x}(\phi_{12})\Big|_{x=l1} , \frac{\partial}{\partial x}(\phi_{21})\Big| , \frac{\partial}{\partial x}(\phi_{22})\Big|_{x=l2}$$

Here, the objectives of digital simulations are as follows :

1. Verify that the model generated by the above algorithm is correct.
2. Demonstrate the ease of changing mode shapes and the resulting change in the dynamic response due to different mode shapes used in the model.

1. Model verification will be done by comparing the response of the flexible arm model with that of a rigid arm, which has the same corresponding parameters.

a) Cleary as the flexural rigidity, EI(x), of the links increases, joint angle response of flexible model should converge to that of rigid model response. Figures (5) and (6a-b) clearly show that joint angle responses converge to those of rigid arm case, as flexural rigidity, EI, of links is increased.

b) The same test simulation is done with clamped-clamped mode shapes for the first link. For this case, when EI is set to 100 Nt m^2, the joint angle responses were almost the same as the rigid case (See Fig.5 and 7a-b). The reason for faster convergence for the clamped-clamped case than the clamped-free case is that clamped-clamped mode shapes result in a stiffer system. However, clamped-free case is a more accurate prediction of the system response than the clamped-clamped case, as discussed below.

c) As EI(x) increases, the frequencies associated with structural flexiblity should increase, for the simple beam case natural frequencies are fuctions of EI as ;

$$w_i = (\gamma_i /l)^2 \quad (EI/\rho A)^{1/2} \tag{4.1}$$

where; γ_i is the characteristic value of the simple beam eigenvalue problem. Even though in two link arm case we are considering here (4.1) does not hold exactly, it is still valid in principle and gives a quantitative idea about what to expect. Rayleigh's energy principle also supports this expectation. Figures (8a and 8b) confirm these expectations.

2. modeling method cleary reveals that mode shapes are important parameters of the system dynamics (e.g. Eqn (3.12)). What assumed mode shapes should be used? Would different shapes make an important difference in the system dynamic characteristics? Theoretically, the only constraint on the assumed mode shapes is that they must satisfy the geometric boundary conditions, but not neccessarily the natural boundary conditions nor the governing differential equations. The governing differential equations and natural boundary conditions are results of the functional variation of the Hamiltonian and are approximately satisfied in any case. The controlled end of each link, driven by a high gain feedback controller, behaves more like a clamped end [1]. The other end condition of the intermediate links should be approximated by a mass with rotary inertia due to other links of the serial structure and payload. However, for different structures and even for different payloads the resultant simple beam analysis will give different mode shapes. Given the fact that these are natural boundary conditions and will be approximately satisfied even if assumed mode shapes do not satisfy them, a clamped-free simple beam mode shape would be an appropriate choice for the assumed modes used in the model. The clamped-clamped case results in a stiffer system. As a result, joint variable

response converges to rigid arm response much faster than clamped-free case as function of flexural rigidity (See Fig. 5,6,7). Frequency of flexible vibrations are significantly higher than those of clamped-free case for the same parameters and conditions (See Fig. 8). This analysis further reveals the importance of mode shapes in the dynamic behavior of the system, hence the importance of keeping the mode shapes as parameters in general at modeling level.

V.Conclusion :

From the modeling technique point of view, it has been shown that Lagrangian - assumed modes method can be effectively used for multi-link flexible arms. The availability of general purpose symbolic manipulation programs overcomes the algebraic complexity of derivation steps, and allows the researchers to obtain more complete models in very short time, in spite of their complexity. A new systematic algorithm based on Lagrangian-assumed mode method is presented suitable for symbolic manipulation by digital computers. The algorithm results in scalar dynamic equations of motion of the system in explict form. There is one scalar diferential equation for each generalized force. This is very usefull in the parallel computation of control torques based on inverse dynamics (computed-torque) since the computation task of each of the scalar equations can be assigned to a single processor which are totaly independent of each other from computations point of view. The algorithm is applied to a two link flexible arm. Simulation results are discussed and shown that the method worked very well for this example case.

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Figure Captions

Fig. 1. One Link Flexible Arm

Fig. 2. A Flexible Serial Manipulator

Fig. 3. (4x4) Homogeneous Coordinate Transformations

Fig. 4. Two Link Flexible Arm Example.

Fig. 5. Two Link Rigid Model Joint Angles

Fig. 6. Two Link Flexible Model Joint Angles, Clamped-Free Mode shapes.

a) EI_i = 10. Nt-m^2 , b) EI_i = 100. Nt-m^2. i=1,2

Fig. 7. Two Link Flexible Model Joint Angles, Clamped-Clamped Mode Shapes for link 1.

a) EI_i = 10. Nt-m^2. , b) EI_i = 100. Nt-m^2. i=1,2

Fig. 8. Comparisions of Flexible vibration coordinate responses (Clamped-Free Mode shapes)

a) EI_i = 10. Nt-m^2. , b) EI_i = 100. Nt-m^2 , i=1,2

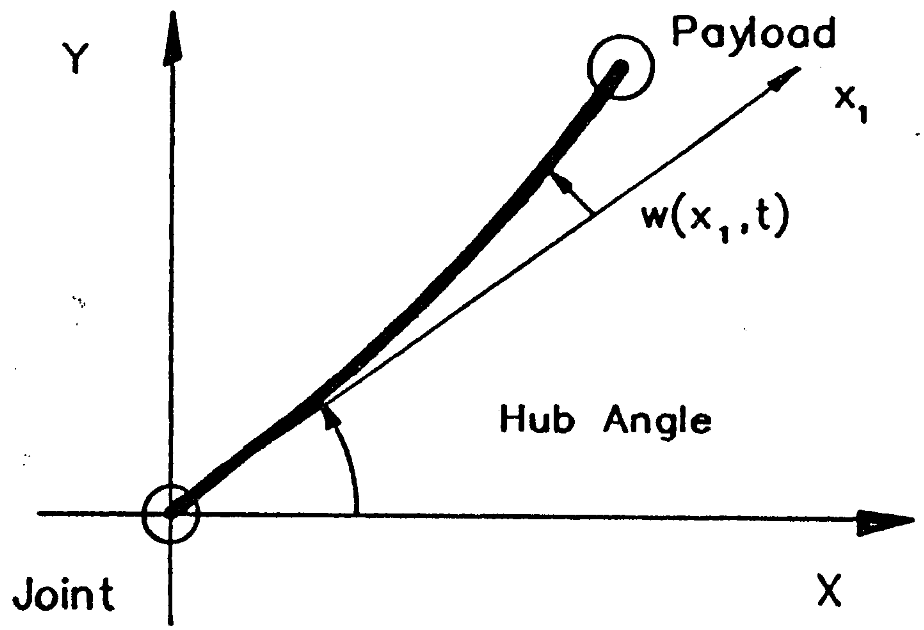

Fig. 1 Cetinkunt and Book

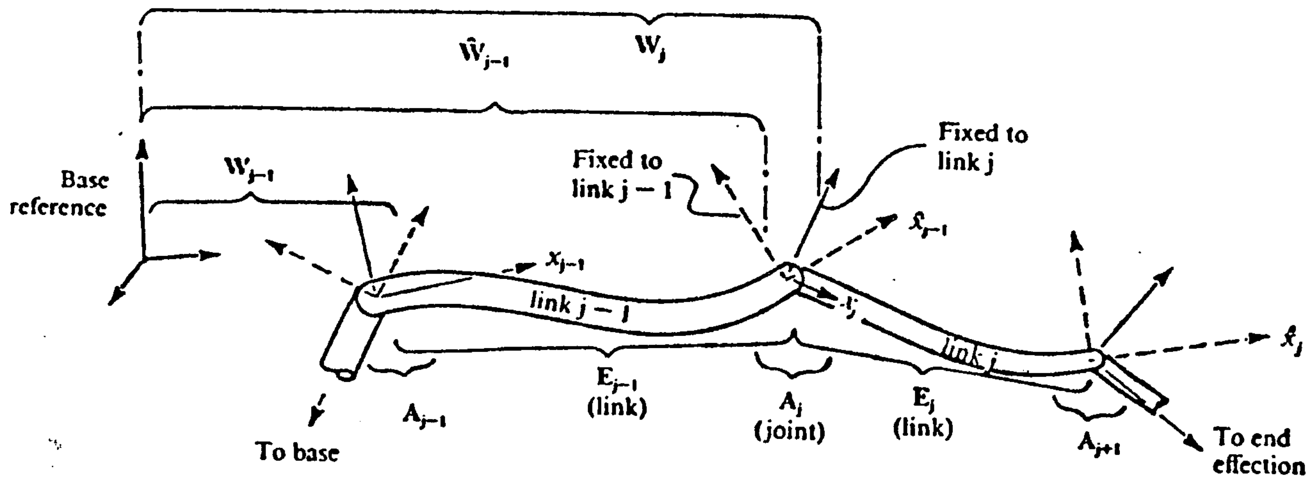

Fig. 2.

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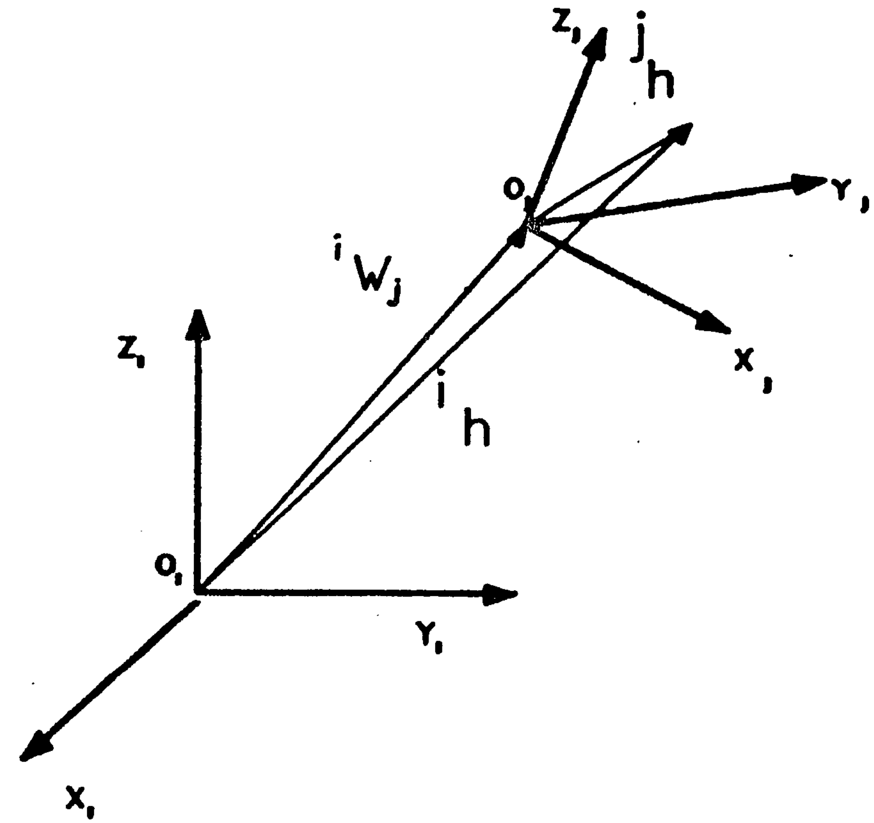

Fig. 3.

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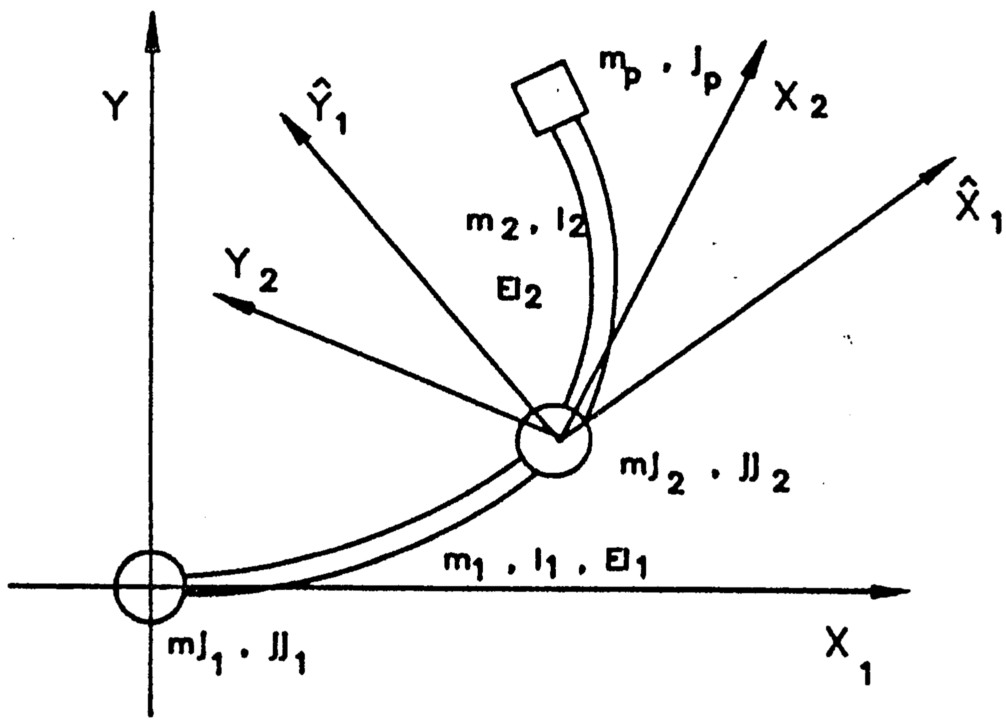

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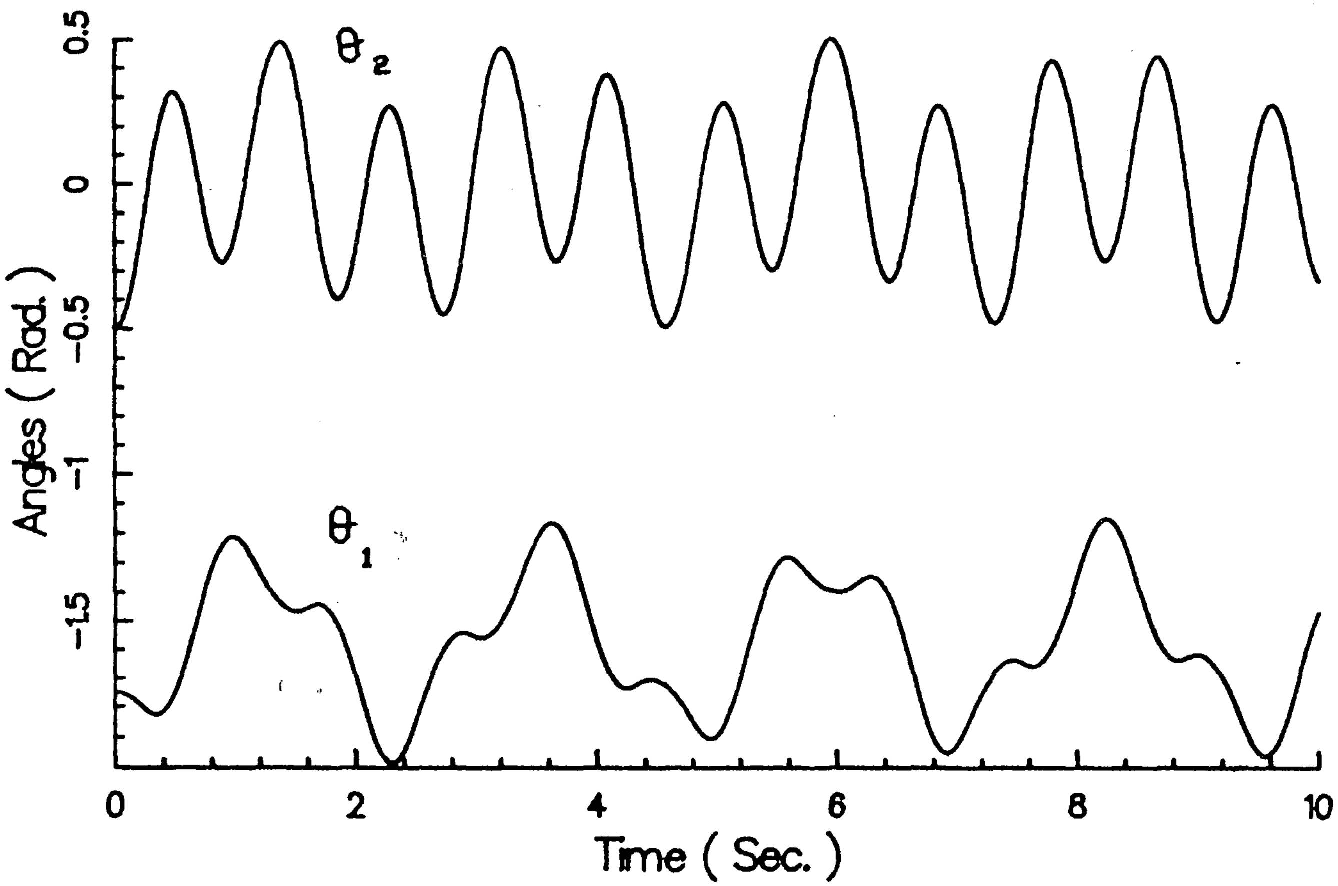
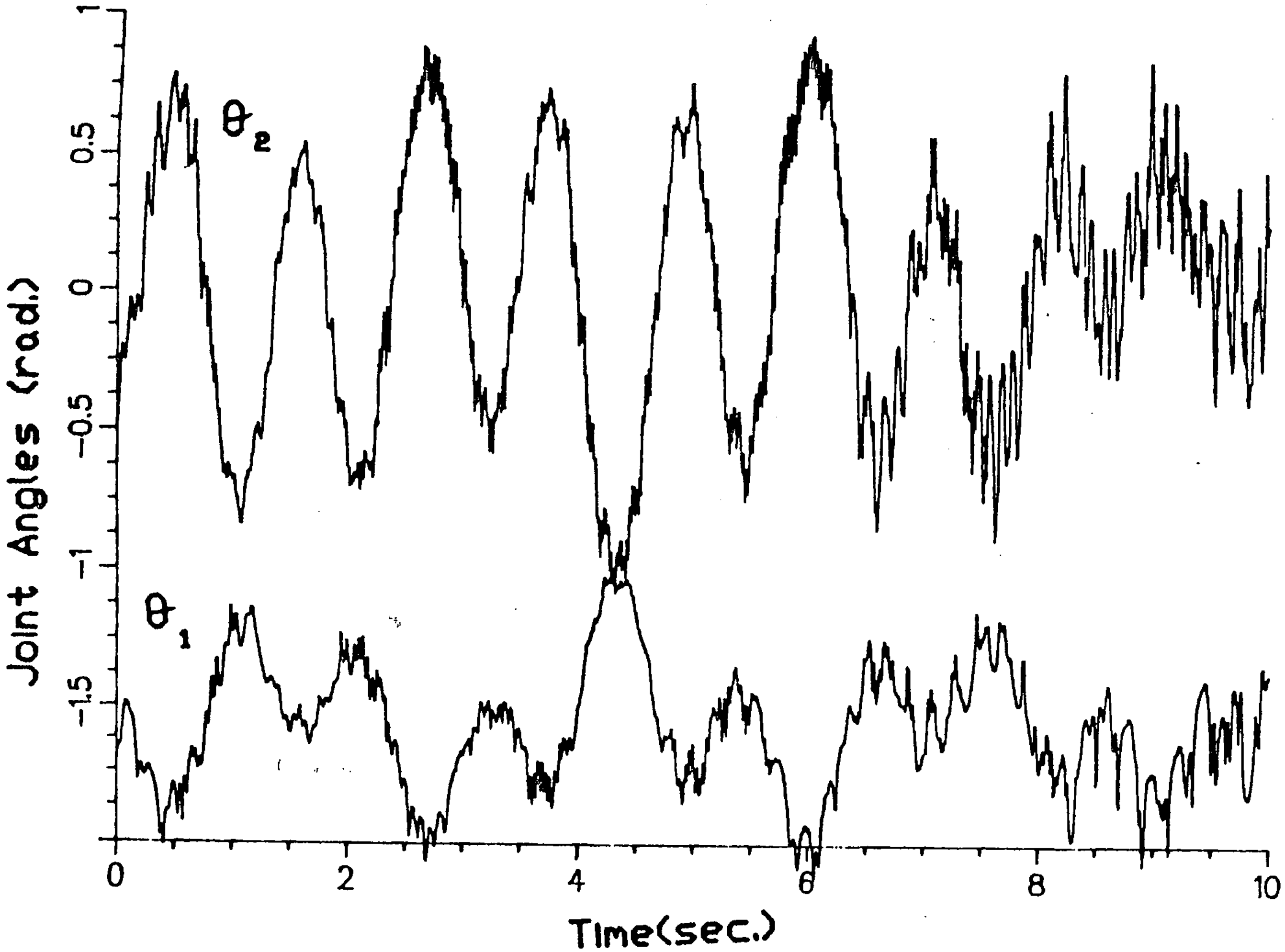


Fig. 5 Cetinkunt and Book



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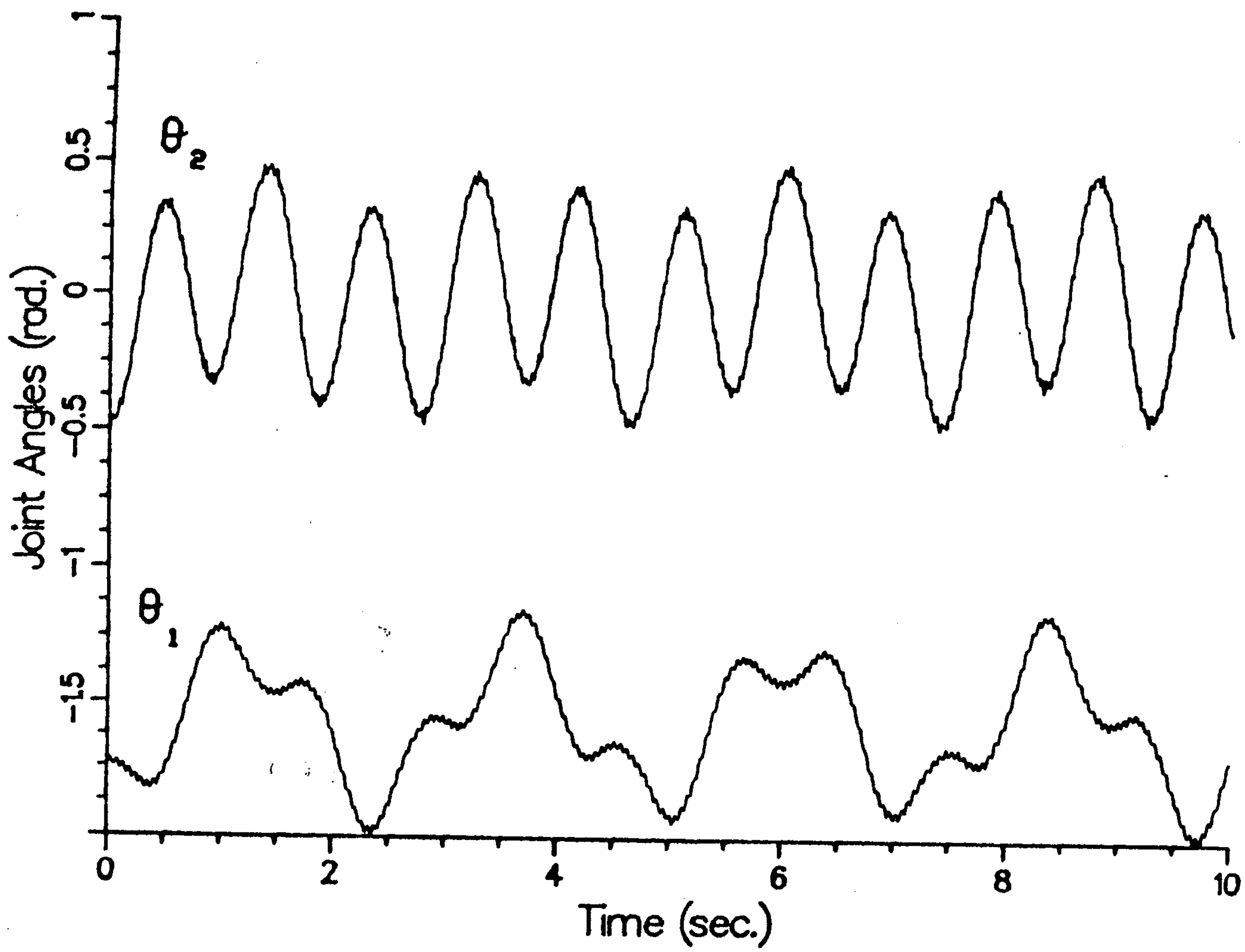


Fig. 6b Cetinkunt and Book

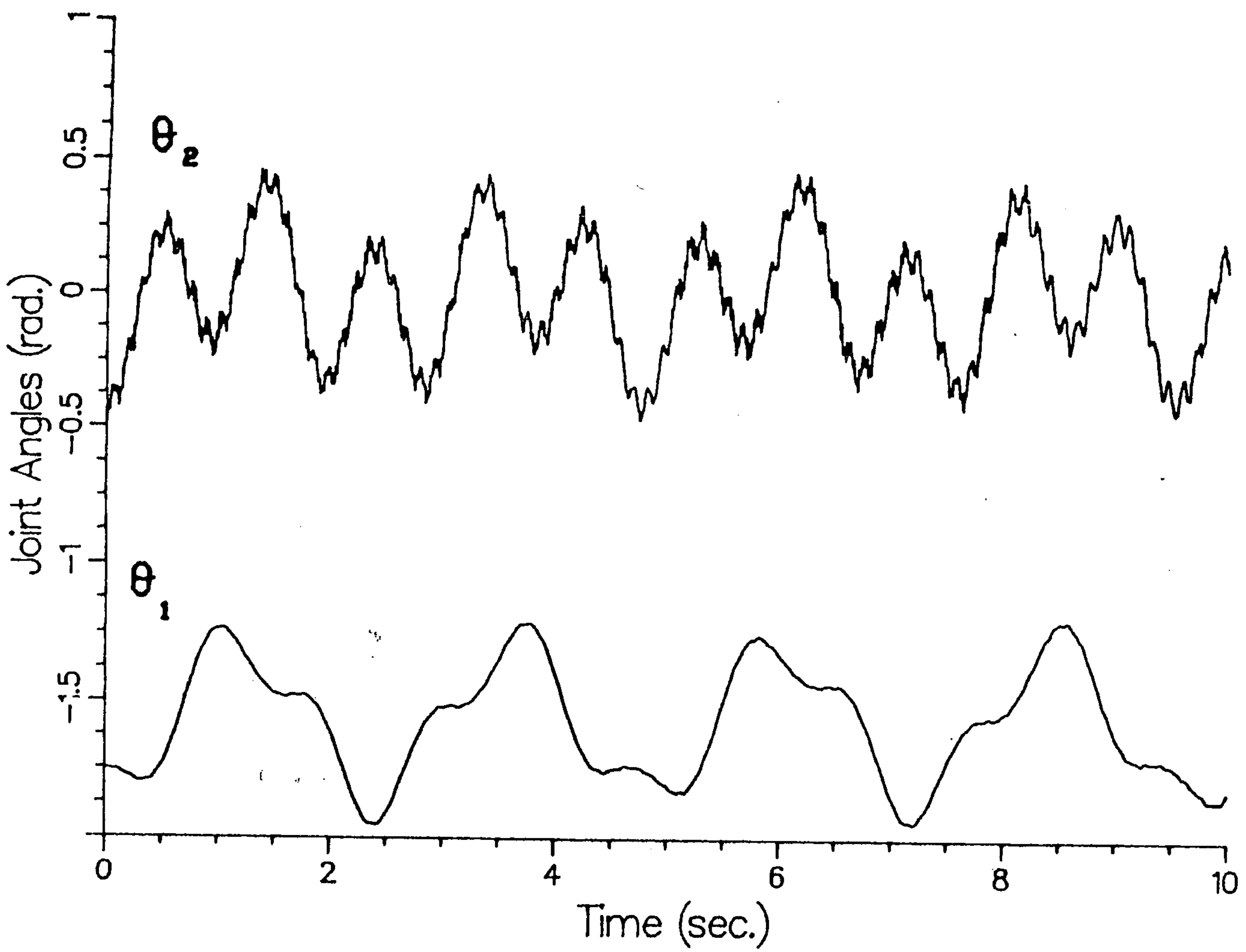
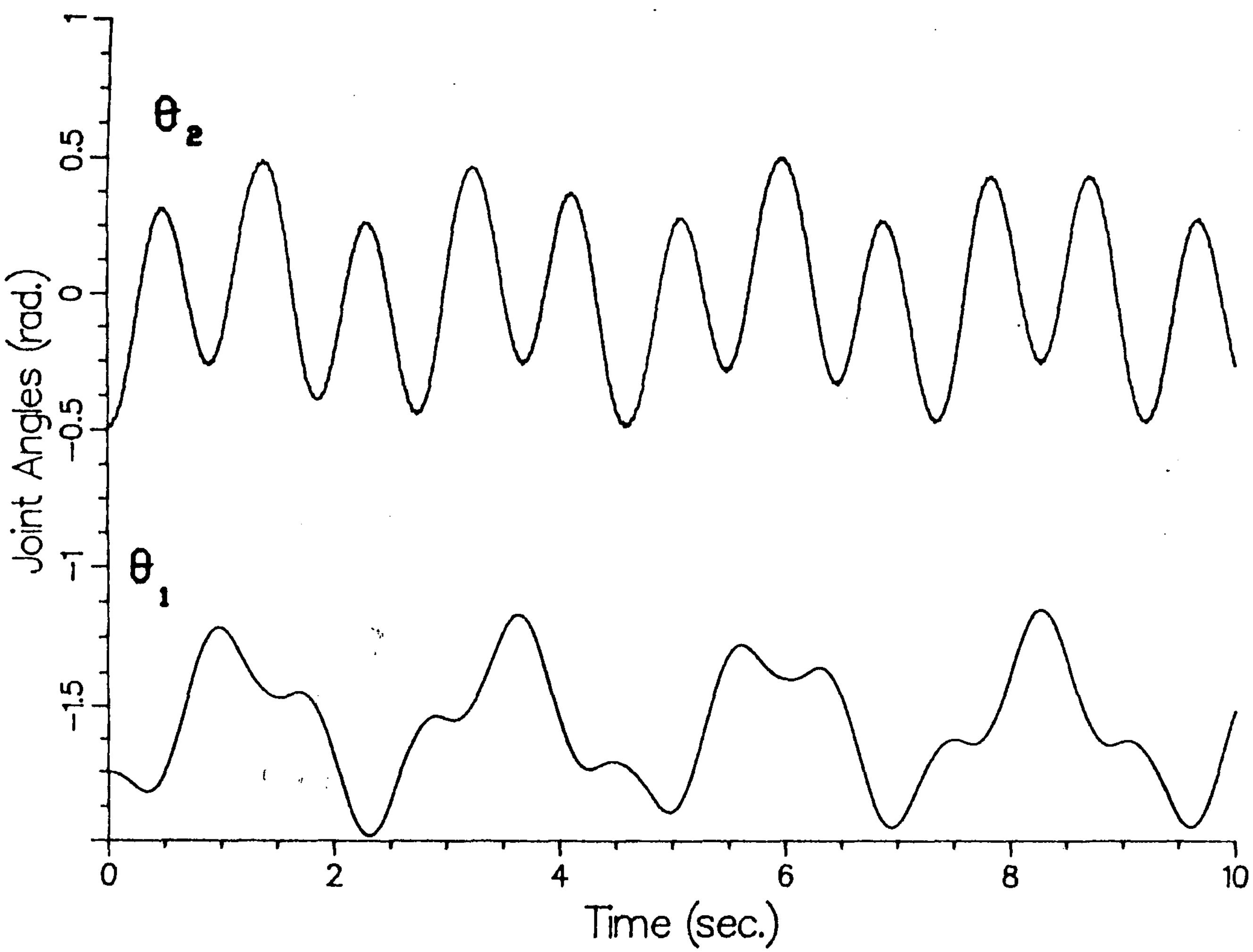


Fig. 7a Cetinkunt and Bool



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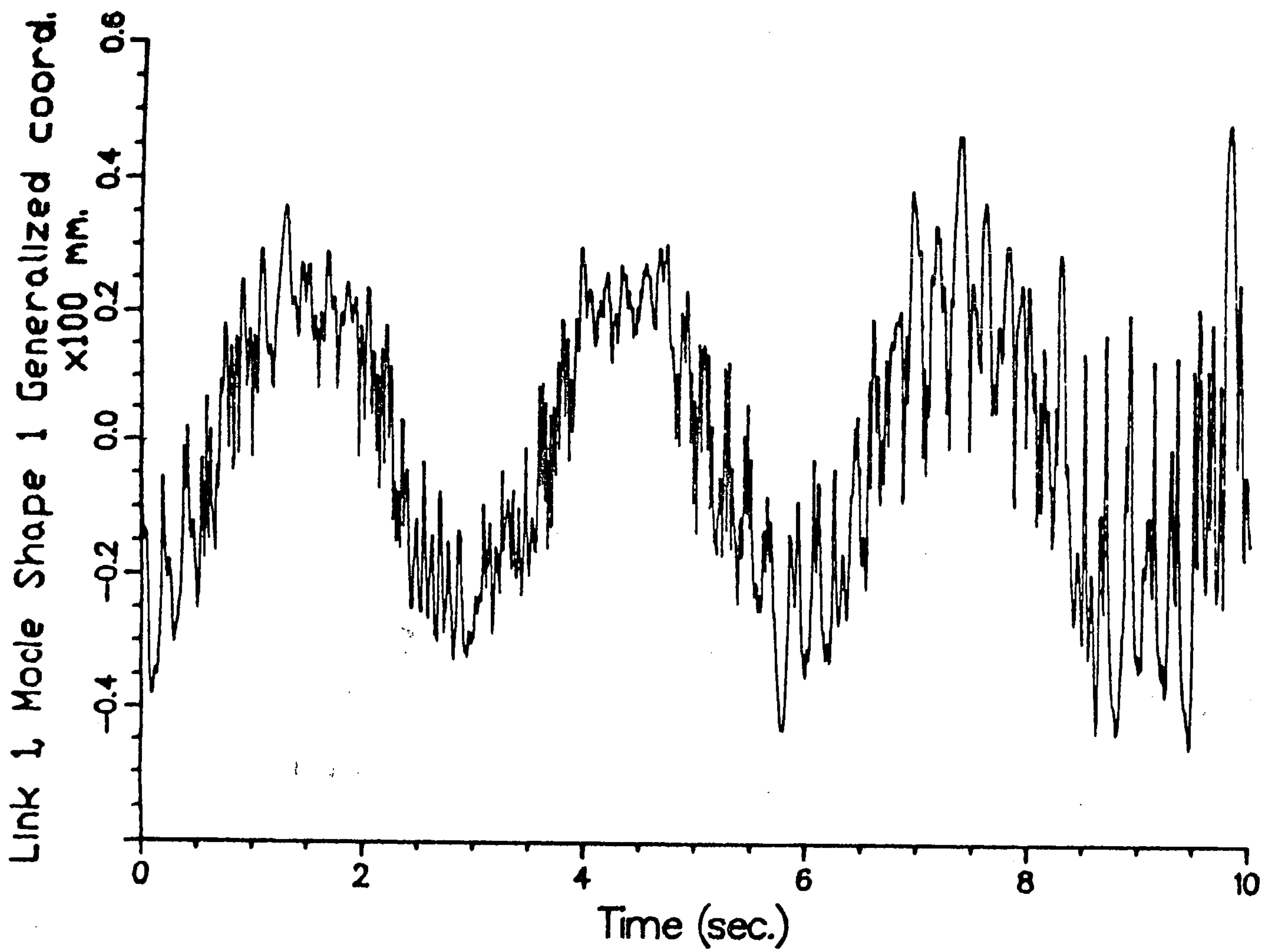

Fig. 8a Cetinkunt and Book

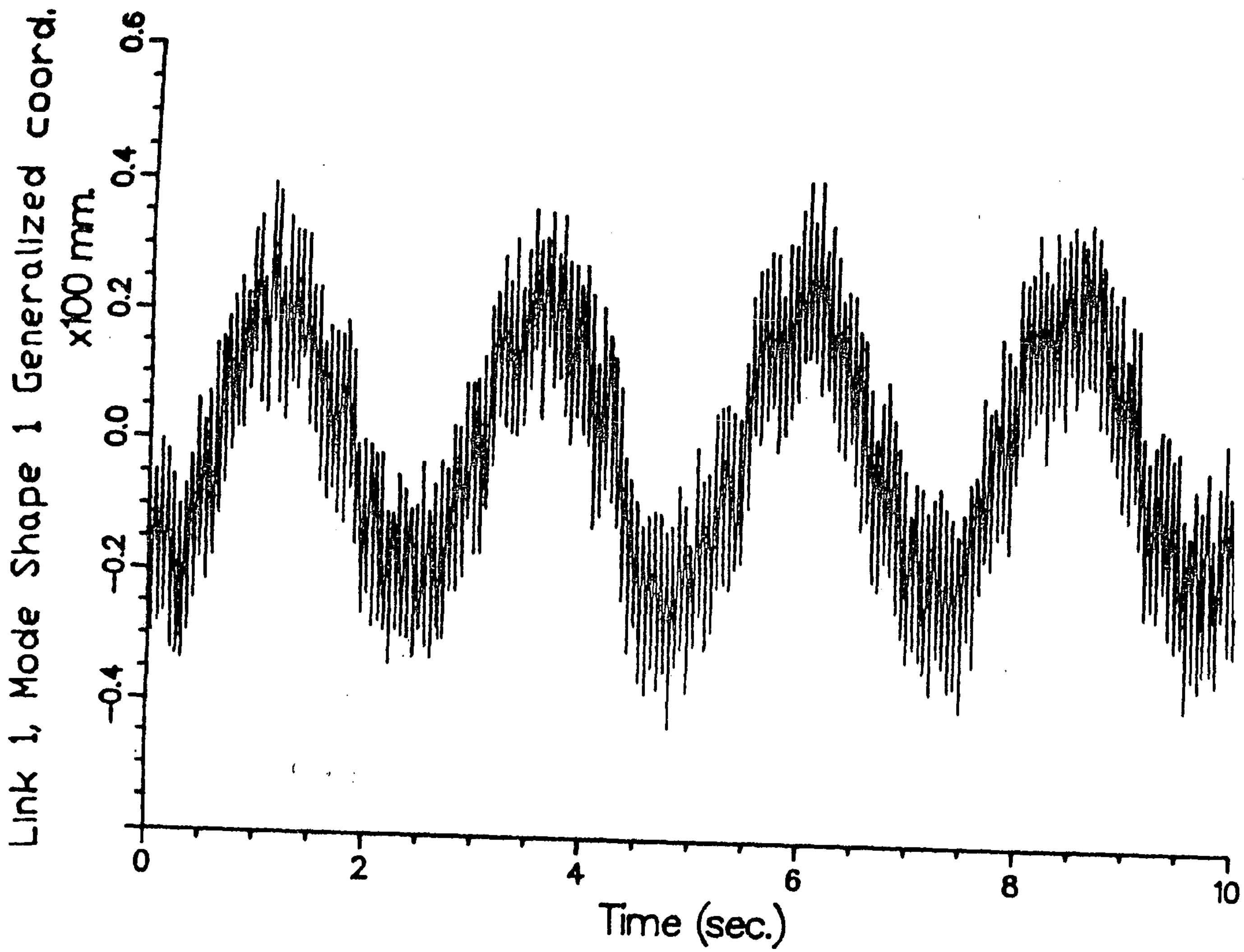

Fig. 8b. Cetinkunt and Book