Controlled Motion in an Elastic World

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The flexibility of the drives and structures of controlled motion systems are presented as an obstacle to be overcome in the design of high performance motion systems, particularly manipulator arms. The task and the measure of performance to be applied determine the technology appropriate to overcome this obstacle. Included in the technologies proposed are control algorithms (feedback and feedforward), passive damping enhancement, operational strategies, and structural design. Modeling of the distributed, nonlinear system is difficult, and alternative approaches are discussed. The author presents personal perspectives on the history, status and future directions in this area.

1 Introduction

Engineering consists largely of tradeoff decisions and constraint observance resulting from key physical limitations. A list of the most important such limitations would include friction, strength of materials, and stiffness of materials. This paper will address the design, dynamics and control of motion systems in a world of finite stiffness. Even as pervasive as compliance (flexibility) is, mechanical engineers of motion systems have skillfully avoided dealing directly with its good and its bad aspects in large part up to the present. High performance systems of the future will gain advantage over their competition by ceasing to view stiffness as an inequality constraint and confronting the tradeoffs that are possible with novel designs.

One of the most challenging motion control problems for engineers is the robot or manipulator arm. It is a multi-input, multi-output, nonlinear system with many constraints on power and geometric form. Perhaps more difficult is the fact that it is expected to be "general purpose." This translates to ambiguous constraints and performance measures, and sometimes unrealistic expectations. Much of the following discussion will be couched in terms of robotics, but the conclusions are relevant to a much broader range of motion and force control problems.

2 Motivation and Perspective

When optimizing the design of motion systems, or any system, one will look at (1) the task to be done, (2) how system performance is to be measured, and (3) what technology is to be employed in an optimum design. As problem solvers, we will start with the task. What is to be done?

2.1 The Need for Expanded Capabilities. Applications are difficult to characterize, but we should include free movement, achieving contact, application of force after contact, grasping, transporting, and releasing payloads. If all technical specifications are within easy reach, cost may be the only relevant performance measure. More likely there are other performance measures including, perhaps, speed or cycle time, range, accuracy or repeatability, and payload mass. What technology can be used to enhance performance and obey any inequality constraints? These technologies need to include feedback control algorithms implemented on suitable processors. Technologies also need to include improved material stiffness and damping characteristics, more efficient structural designs, placement of actuators, and structures comprised of actuators. We must consider new strategies for using motion systems.

The application that has lead to careful examination of previous solutions to the stiffness constraint is the space manipulator. With the extreme penalty on weight carried into orbit, NASA commissioned research into the behavior of lightweight, flexible manipulator arms. Long arms were eventually built for the Space Shuttle by the Canadian company SPAR with close involvement by U.S. firms and researchers. The design studies illuminated the difficulty of the problem and the inadequacies of existing tools, structural dynamics and multibody dynamics, to study the issues. Structural dynamics was oriented to studying linear systems vibrating about a nominal point, and multibody dynamics was limited to rigid bodies joined by simple kinematic pairs. The technique for space manipulators was, and continues to be, to move the joints slowly and wait for the tip of the arm to settle to equilibrium. Buffeted by public and congressional ambivalence and constrained by the primacy of manned space flight, the U.S. space program may loose its lead in lightweight motion systems to the Japanese and European space programs (Whittaker et al., 1991).

Industrial tasks require productivity and cost effectiveness. Industrial solutions also tend to be very conservative in production concepts and machinery. The slow incorporation of numerically controlled machine tools is an excellent example. Gradually, however, inroads are being made in the conservative mentality, largely by virtue of the need for higher speeds and larger work spaces at a reasonable cost.

Service robots are the current driving force for innovative approaches to the stiffness limitations of motion systems. En-

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vironmental restoration and waste management needs demand slender arms for access to areas of high radiation where no human laborers will be permitted (Krieg et al., 1990). In the nuclear industry, one finds a workspace designed 50 years ago that cannot now be made friendly to robots of a more traditional design. The magnitude of the problem demands an approach that is at least time efficient and preferably economical. Also critical to the advancement of the technology is a well-organized research funding source, which in this case is the U.S. Federal Government. The U.S. Government has a recognized responsibility and a commitment to (eventually) restoring the environment and managing nuclear waste. The problem is multinational, and every industrialized nation, certainly every nuclear power, has need for the capability of these robots. Military robots have some similar specification (large workspace, compact design), but contractors have eschewed novel approaches for conservative, rigidized designs that marginally meet specifications (Orbach and Ball, 1991).

Commercial service robots may hold the ultimate hope for breakthroughs due to the competitive nature of the business, an environment that will not be designed for robots exclusively (humans must at least share the workspace), and a production volume that will justify a substantial engineering effort.

2.2 The Engineering Tradeoffs. Flexibility becomes important as one reduces the structural material available to support the payload or when demanding a faster response of a given structure. By normalizing the motion bandwidth by the lowest natural frequency of the structure with the joint motion eliminated, one could readily predict the effect of flexibility. As shown in Fig. 1 (Book et al., 1975), the effect of striving for higher bandwidth with conventional control was that the damping ratio was not sufficient for desirable motion control. The solid lines in Fig. 1 are the loci of dominant roots as one changes joint velocity feedback gains to increase the damping ratio. For a rigid model the solid lines would be circular arcs with the radii of the arcs equal to the closed-loop natural frequency and approximately the system bandwidth. The bandwidth that could be achieved by a simple P-D control was limited to approximately 1/3 of the clamped joint natural frequency. This result was initially found for one specific configuration: two links of equal length separated by a P-D controlled joint. As other configurations were tested, the limitations were similarly stated. This became a useful rule of thumb for design. Bandwidth exceeding the clamped joint natural frequency temporarily became a worthy goal.

Low damping of the dominant mode is certainly unacceptable for most motion control systems. Space systems did not need to support their own weight, and it was clearly possible to deploy a structure so light that the rule of thumb was violated. For systems in earth's gravity, it is possible that other constraints would prevent the flexibility from ever becoming so great. The other constraint on structural cross section is strength. The arm must support its weight (if any) and it must not fail when the joints are torqued. If you *want* to study flexible arms, you must only show that one realistic case is constrained by flexibility (I did this for NASA in 1973). If you don't want to study flexible arms you can look at numerous existing physical examples, such as industrial arms for sale, and show that the links are rigid (Good et al., 1984).

Short of completely designing many arms with incremental changes in the relevant parameters (length, payload, etc.) how can one characterize the conditions under which flexibility is important? A study published by Book (1974) and refined by Book and Majette (1983) normalized relevant parameters and generated a family of optimized skeleton designs by computer, incrementing the nondimensional length and payload mass. To my relief, I found that flexibility can cut in half the angle moved in a motion cycle of fixed time. The solid lines in Fig. 2 show the best performance possible if a rigid design strategy is used. The dashed line shows the best performance if the flexibility constraint could be eliminated. The separation between the dashed line and the solid lines shows the penalty suffered due to flexibility. The strong effect of arm length is apparent in these results.

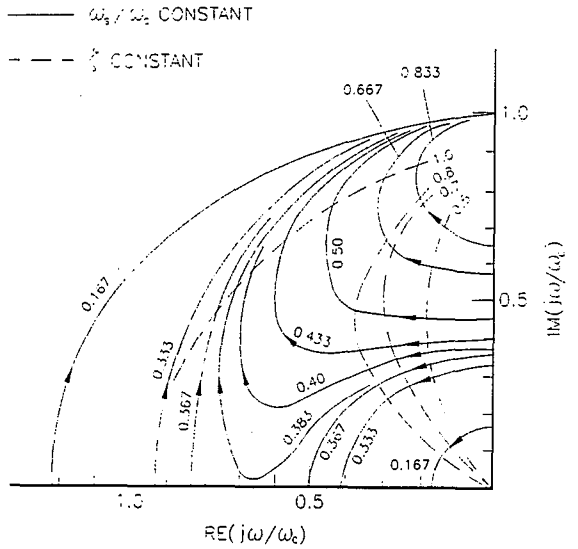

Fig. 1 Root loci of a flexible arm. (Joint feedback P-D gains are varied to achieve the given damping ratio (ζ) and natural frequency ω_s/ω_c for a rigid arm of fixed inertia. First clamped joint natural frequency ω_c normalizes the servo frequency ω_s.)

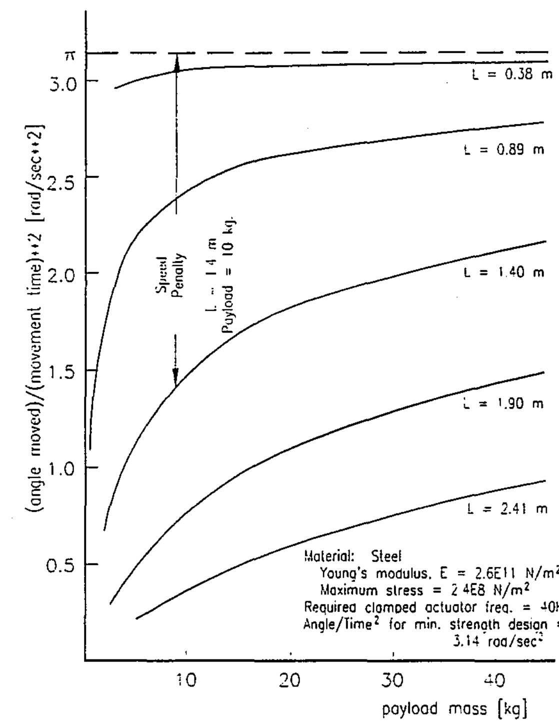

Fig. 2 Speed penalty of the rigidity constraint

A new view of the tradeoff problem evolved from these studies. The ratio of torque to inertia and the joint velocity limits determine the speed to make a gross (finite) motion of the arm. Reducing arm structure reduces the inertia and improves this measure of performance. Reducing arm structure also reduces the natural frequency of the arm and hence the bandwidth of the controlled motion system, descriptive of the small motion of the arm. It is thus reasonable and useful to

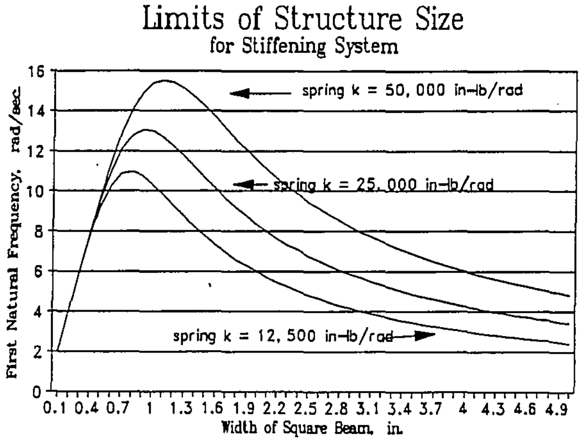

Fig. 3 Variation of fundamental frequency of a pinned-spring-beam system as the square beam is widened. (Beam length = 100 in., material: aluminum, springs: 12,500; 25,000; 50,000 in·lb/rad, bottom to top.)

characterize the selection of structure cross section as a tradeoff between fine motion and gross motion. As long as one maintains the same motion system for large and small gross motion, this tradeoff must be made.

A word is in order about the relative importance of drive compliance and structural (link) compliance. Good et al. (1984) found link compliance to be almost insignificant in a standard GE P50 robot, yet Book and Majette (1983) showed that, with an optimized drive, the link compliance was quite significant (see also Alberts et al., 1992). In fact, existing arms are not designed with an optimal distribution of structural mass between links and drives. As shown in Fig. 3, reducing the link cross section can improve the arm's natural frequency. The additional inertia does more damage than the additional stiffness does good.

3 Modeling

Dynamic models are used for design, simulation and control of flexible motion systems. The difficulty perceived in modeling leads many designers to avoid solutions that include some link flexibility. Because arm dynamics may be both flexible and nonlinear, it is necessary to make simplifying assumptions regarding either the flexibility or the nonlinearity. It thus is helpful to have several model types that serve as cross checks on the overall system.

3.1 Representing Flexible Motion With Transformation Matrices. The resistance to including flexibility in the dynamic characteristics of motion systems stems from the complexity of the dynamic models that incorporate flexibility. The perception of complexity is as important as complexity itself. This perception was changed for rigid robots by the use of 4 × 4 transformation matrices to describe kinematics, originally by Denavit and Hartenberg (1955) and later by Paul (1983) in his popular early textbook. It seemed possible to incorporate this approach into flexible arm kinematics and dynamics which tended to involve confusing vector notation and be performed by mechanicians and not roboticists. During a leave of absence at Carnegie Mellon, I was finally able to work through the details of this approach (Book, 1984).

The transformation matrix is a capable bookkeeping tool. It makes the problem easy to think about. Just as with rigid link dynamics, the implementation of the dynamics on computer was straight forward from the 4 × 4 matrix notation.

To use transformation matrices, attach a coordinate system on one end of a system element of known (at least by assumption) position and another coordinate system on the other end of unknown position. Then analyze from first principles how the element may deform. Simple kinematic pairs like translating or rotating joints have simple motion descriptions involving a joint variable (call it q). The transformation matrix involves only one variable.

$$p_i = T_i(q_i)p_{i+1}$$

$$\text{where } p_i = [x\ y\ z\ 1]^T$$

$$= \text{position of point in frame } i$$

$$T_i = \text{a } 4 \times 4 \text{ transformation matrix}$$

$$q_i = \text{the joint } i \text{ variable.}$$

Serial complexity is added by multiplication of successive transformations

$$p_i = T_i T_{i+1} p_{i+2}.$$

For distributed parameter links modeled by an assumed modes method, as described below, the addition of successive matrices for each additional mode of vibration assumed is required.

$$p_i = T_i[q_i(t, x)]p_{i+1}$$

$$= T_i\left[\sum_{j=1}^{\infty} q_{ij}(t)\delta_{ij}(x)\right]p_{i+1}$$

$$T_i = X_i + \sum_{j=1}^{\infty} q_{ij}(t)E[\delta_{ij}(x_i)]$$

$$\text{where } X_i = \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$E = \begin{bmatrix} 0 & -\theta_{zij} & \theta_{yij} & \delta_{xij} \\ \theta_{zij} & 0 & -\theta_{xij} & \delta_{yij} \\ -\theta_{yij} & \theta_{xij} & 0 & \delta_{zij} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

δ_{yij} = *displacement of function j in the* y_i *direction*

$\theta_{zij} = \partial\delta_{yij}/\partial x_i$

$= \angle$ *of the tangent to the beam w.r.t. the* z_i *axis*

$\theta_{yij} = -\partial\delta_{zij}/\partial x_i$

θ_{xij} = *torsion about* x_i *axis*

Flexible drives or joints with concentrated flexibility are important but require only a slight complication of the modeling procedure. For distributed flexible links, the variables are less intuitive and somewhat arbitrary. Every point on the link can be moved to a great variety of locations without violating the boundary conditions or the differential equations that relate it to other points. All possible positions must be described for representing the complete dynamics. The spatial variable (x) joins the time variable t as an independent variable. When separation of variables can be assumed, a basis set of shape functions $\delta_{ij}(x)$ can represent the possible shapes of the link when they are multiplied by their time varying amplitude and summed.

When the summation is truncated to a finite number of terms, a practical way of dealing with the otherwise infinite number of variables results. Justification for the truncation is based on the high frequency and low amplitude of the terms that are dropped. Unfortunately, it is somewhat an art to figure out which terms should be dropped for complex systems (such as space structures) with many vibration modes of nearly the same frequency. For the clean designs built and analyzed in our laboratory, a relatively small number of modes (two or three) suffices to represent flexible dynamics (Tsujisawa and Book, 1989). The mode shapes may be based on analytical results for simple shapes (e.g., uniform Bernoulli-Euler beams) or finite element models of links with more complex geometry can be solved for eigenvalues (frequencies) and eigenfunctions (shapes) (Huggins et al., 1987). Boundary conditions used for these shape determinations depend on the joint variables to be used and affect the accuracy of the truncated model and its suitability for certain kinds of further applications, such as inverse dynamic analysis.

After the kinematics of a flexible system is known by whatever method, one can mechanically progress to the equations of motion using Lagrange's approach. The kinetic and potential energy (gravity and strain) are formulated and the work terms involving each variable are established. The energy terms for the distributed elements are obtained by integration over the spatial variable(s). After moving the time domain variable outside the integral, modal masses and modal stiffnesses with definitions analogous to the conventional masses and stiffnesses of rigid bodies can be defined. The choice of mode shape ultimately determines these coefficients and they can be determined off-line. It is common to insert modal damping to represent in a linear way the dissipation occurring in the material and structure independent of joint motion. Enhancement of this modal damping is discussed elsewhere in this paper. The determination of appropriate modal damping is typically based on experiment and sometimes extreme accuracy is not required. Exceptions are when the joint is not moving (due to joint stiction or brakes) and when extremely light damping is present with a sampled data controller (Alberts, 1986). Friction at actuated joints is typically added as if it were a control torque but the control law is replaced by a friction law.

MACSYMA was in its infancy when symbolic equation generation by computer was first attempted for this problem by Maizza-Neto (1974) and the large storage requirements prevented useful simplification of the results. Now this and similar programs (Wolfram, 1991) are indispensable tools to producing error-free simulation codes from the Lagrangian approach or other approaches (Cetinkunt and Book, 1987).

3.2 Formulation for Inverse Dynamics. It is possible to take advantage of variations in the coordinate systems to solve some problems more easily. A high performance servo might be most efficiently (in terms of number of variables for a given accuracy) represented by a joint angle between lines tangent to the flexible links on either side. The assumed mode shapes must reflect this choice of joint variables and be tangent to or clamped to the joint angle. Hence, a clamped boundary condition is essential for any assumed mode, as shown in Fig. 4, case (*a*), using the joint angle θ. This makes it very difficult to solve for the torque that will generate a given tip motion since the tip motion depends on the joint variables and all flexible variables.

A somewhat less efficient set of variables uses a joint variable that goes directly to the point of interest, say the link tip. The joint angle is measured up to a line connecting the axis of rotation to the link tip as shown in Fig. 4, case (*c*). Now the tip position can be described in terms of the joint variables q_r only. As a result, the desired output can be related directly to a subset of states: the joint variables. Now the joint torques can be solved for and eliminated from the remaining flexible equations. The steps to do this are simple matrix manipulation as shown schematically in the following steps from Book and Kwon (1992). The equations presented here are for the linearized single link case, but the nonlinear case presents no difficulties, since the differences are that the mass matrix and input matrix are a function of state, and that the velocity terms

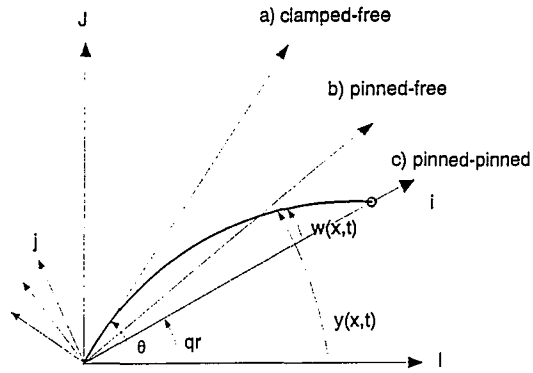

Fig. 4 The relationship between joint variable used and the permissible boundary condition on the shape function

include squares of velocities. One begins with the dynamic equations:

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{rf}^T & M_{ff} \end{bmatrix} \begin{Bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{Bmatrix} + \begin{bmatrix} D_{rr} & D_{rf} \\ D_{rf}^T & D_{ff} \end{bmatrix} \begin{Bmatrix} \dot{q}_r \\ \dot{q}_f \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{ff} \end{bmatrix} \begin{Bmatrix} q_r \\ q_f \end{Bmatrix} = \begin{bmatrix} B_r \\ B_f \end{bmatrix} \tau$$

where M_{ij} = *the mass matrices,*

D_{ij} = *the velocity coefficient matrices*

B_i = *the input matrices*

$i, j = r$ *(for rigid)*, f *(for flexible)*

$q_r = q_0$; *rigid body coordinates,*

$$q_f = \begin{Bmatrix} q_1 \\ \vdots \\ q_n \end{Bmatrix}, \text{ flexible mode coordinates}$$

Separating the partitioned equations and solving for the joint torque in terms of the known rigid coordinates which depend only on prescribed tip position:

$$[M_{rr}]\ddot{q}_r + [M_{rf}]\ddot{q}_f + [D_{rr}]\dot{q}_r + [D_{rf}]\dot{q}_f = [B_r]\tau$$

$$[M_{rf}]^T\ddot{q}_r + [M_{ff}]\ddot{q}_f + [D_{rf}]^T\dot{q}_r + [D_{ff}]\dot{q}_f + [K_{ff}]q_f = [B_f]\tau$$

$$\tau = [B_r]^{-1}\{[M_{rr}]\ddot{q}_r + [M_{rf}]\ddot{q}_f + [D_{rr}]\dot{q}_r + [D_{rf}]\dot{q}_f\}.$$

Substituting for the joint torque into the flexible equations:

$$[M_i]\ddot{q}_f + [D_i]\dot{q}_f + [K_i]q_f = [B_{i1}]\dot{q}_r + [B_{i2}]\ddot{q}_r$$

where $[M_i] = \{[M_{ff}] - [B_f][B_r]^{-1}[M_{rf}]\}$

$$[D_i] = \{[D_{ff}] - [B_f][B_r]^{-1}[D_{rf}]\}$$

$$[K_i] = [K_{ff}]$$

$$[B_{i1}] = \{[B_f][B_r]^{-1}[D_{rr}] - [D_{rf}]^T\}$$

$$[B_{i2}] = \{[B_f][B_r]^{-1}[M_{rr}] - [M_{rf}]^T\}.$$

Thus, one can solve for the flexible coordinates as well:

$$\ddot{q}_f = -[M_{ff}]^{-1}[M_{rf}]^T\ddot{q}_r - [M_{ff}]^{-1}[D_{rf}]^T\dot{q}_r - [M_{ff}]^{-1}[D_{ff}]\dot{q}_f - [M_{ff}]^{-1}[K_{ff}]q_f + [M_{ff}]^{-1}[B_f]\tau.$$

While the application of this approach for systems with one flexible link is straightforward and effective, it is not well-verified for multiple flexible links. Clearly it is possible to specify positions and angles that cannot be reached by an arbitrary arm configuration. This is a topic of current research.

3.3 Closed Kinematic Chain Dynamics. The above discussion of models applies cleanly to serial chains of links and joints. Closed kinematic chains or parallel structural elements with flexibility introduce constraint equations that are difficult to include. The problem is practically important, since motion

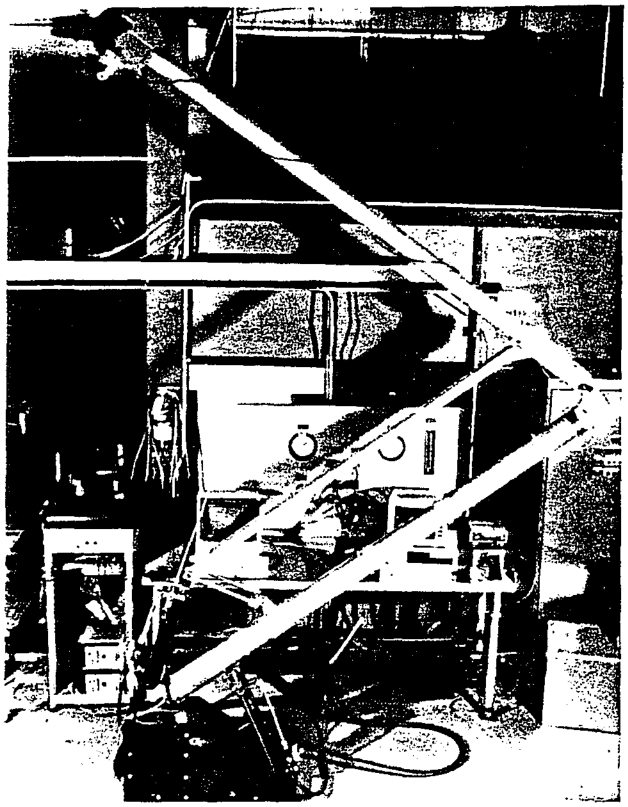

Fig. 5 The experimental arm RALF (Robot Arm Large and Flexible)

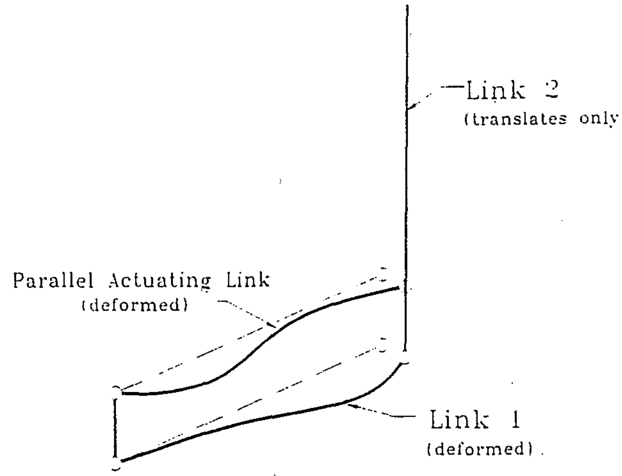

Fig. 6 Approximate kinematics of the flexible parallel link mechanism assuming constant pin to pin distance

systems often involve these structural elements to enhance stiffness and/or to locate the actuator remotely from the joint. We confronted this problem head on several years ago when we were creating a test bed, the Robotic Arm Large and Flexible (RALF) in our laboratory. As shown in Fig. 5, RALF has two degrees of freedom, with the second actuated through a parallelogram mechanism. Several methods have proven effective in modeling RALF. The straightforward approach is to establish algebraic constraint equations to describe the closure conditions as one moves around the parallelogram along parallel paths to arrive at the same point. These constraints must be enforced at the same time that the differential equations are simulated. The differential equations are derived based on multiple open chain topologies and consequently have joint variables that are not all independent. The simplistic approach would be to eliminate some of the excess variables. This is not possible due to the complex relationships of the equations and coupling between the algebraic and differential equations. An alternative is to insert constraint forces to enforce the chain closure. Another alternative (Lee, 1990) that is more computationally tractable is a change in variables that allows the solution to proceed tangent to the constraint surface as it is described in these variables. The simulation may proceed for many iterations, gradually moving to violate the constraint. When sufficient constraint error is observed, the change of variables is recomputed and the simulation proceeds again.

The parallelogram has certain characteristics that permit a totally different approach (Yuan, 1989). The major effect of the parallelogram when its long links (link one and the drive link) are deflected is to maintain link two parallel during the deflected motion, as shown in Fig. 6. This is true to the extent that the two ends of the deflected beams remain the same distance from the each other. This is easily enforced in the kinematic description of the arm, and in fact is simpler than the serial link arm. The drive link is much lighter than link one and its effect is considered as a massless compliance and lumped into the hydraulic actuator compliance. This technique is justified by the experimental data.

3.4 Premature Linearization. Several inaccuracies in conventional modeling have been noted (Kane et al., 1987). One is the failure to account for the stiffening of a rotating link. This effect is termed "centrifugal stiffening." The other inaccuracy is the failure to account for the difference in the length along a deflected beam and its projection onto its original undeformed axis. This is termed "the foreshortening effect." Kane has shown that this results from the premature linearization of the dynamic equations. Basically, we have substituted the assumed linearized shape function information into the nonlinear equations of motion before the equations had an opportunity to qualify the admissible shapes. Kane has shown that this can result in pessimistic predictions in some cases, where clearly passive, stable systems are predicted to go unstable. These effects are significant at large deflections and high rates of angular rotations. In some motion systems these effects are important, but in robotic systems deflections and rotational velocities are generally lower.

3.5 Frequency Domain Models. With the truncation of the series representing the deformed shape of a link proposed in Section 3.1, we must ask how accurate the approximation for distributed flexible behavior will be. No conclusive answer is available. However, when the distributed flexible behavior dominates the nonlinear aspects of the motion system, an alternative linear modeling technique is effective and avoids some of the concern for accuracy. The transfer matrix approach works with arbitrary boundary conditions applied to a linearized model of the elastic continuum in one dimension, i.e., the distributed flexible link. When the model is transformed to the frequency domain the two boundary conditions on the link are perfectly represented in the frequency domain. Upon multiplication by a transfer function and adding together the effects of all relevant boundary conditions one obtains an exact frequency domain representation of the system behavior. The algebra can be simplified by collecting the transfer functions into a transfer matrix, and collecting the boundary conditions into a state vector, as shown in the equation below. The transfer matrices multiply vectors of boundary conditions to get the boundary conditions at the other end of the element. Additional serial elements are included by multiplying transfer matrices, thus eliminating the intermediate "boundary condition" vectors. Transfer matrices representing flexible or rigid links, joints, payloads and drives have been constructed and used for modeling complex motion systems (Book and Majette, 1983).

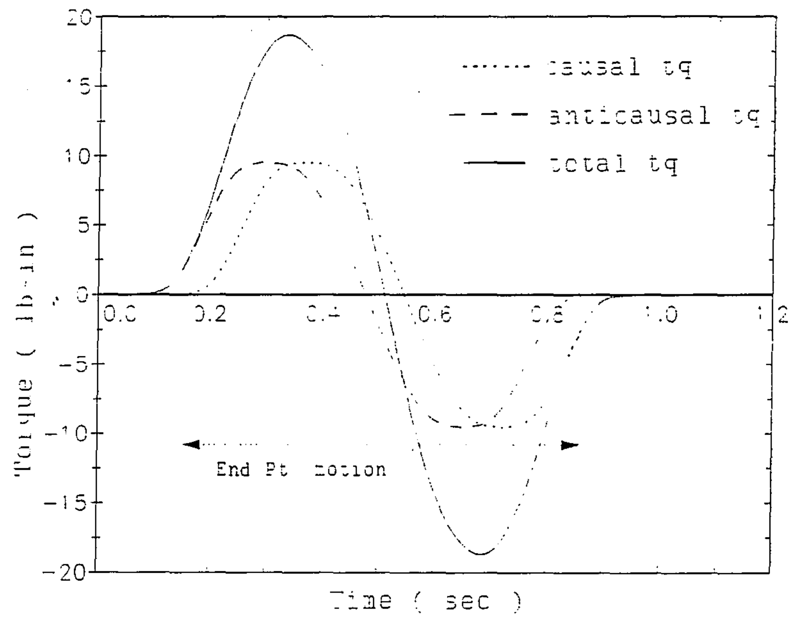

Fig. 7 Calculated torque by the inverse dynamic method using causal (with initial conditions) and anticausal (with final conditions) terms

For example, two beams separated by a controlled joint can be conveniently represented by a transfer matrix product:

$$z_L = B(\omega)C(\omega)B(\omega)z_R$$
$$= T(\omega)z_R$$

where z_L = *the state vector on the left boundary of the system*
z_R = *the state vector on the right boundary of the system*
B = *beam transfer matrix*
C = *joint control transfer matrix*

$$z_i = \begin{bmatrix} -\text{deflection } (-w) \\ \text{beam slope } (\psi) \\ \text{moment } (M) \\ \text{shear force } (V) \end{bmatrix}$$

A wide variety of results can be computed from the transfer matrix model, including frequency response, its inverse transform (impulse response), natural frequencies and mode shapes.

In addition to the restriction to linear or linearized motion, the transfer matrix approach does not readily allow some of the analysis techniques permitted by a state space model. The attempt to have the best of both worlds lead Book and Majette (1983) to alternate between the two when applying pole placement techniques to the dominant modes. From the transfer matrix model, closed-loop poles and the corresponding second-order state space model were determined. The low-order model with the linear control was exact in the pole positions, but the poles were not at the desired positions. From the state space model one can readily compute an approximate modified feedback control law that would properly place the reduced-order model poles. When these gains were used in the frequency domain model, the true pole positions could again be determined and the error in pole position indicated. This cycle was repeated with convergence to the desired pole positions in those cases where it was feasible, typically in three or four iterations.

The frequency domain techniques require much more user interaction but are crucial for understanding the true nature of distributed parameter systems (Spector and Flashner, 1990). The nature of the approximations we make with other methods, including finite element and assumed modes, can be understood with an alternate frequency domain model. Especially tenuous is our understanding of the zero dynamics (zeros of the transfer functions), that is critical to the use of inverse dynamics control techniques.

3.6 Nonminimum Phase and Inverse Dynamics. When a distributed parameter system is forced at one point in its spatial domain and its response is measured at another point, the system is said to be noncollocated. Since the system is theoretically infinite dimensional, a transfer function would contain an infinite number of terms with various time constants or periods and amplitudes. It can be shown for beam-like systems that nonminimum phase dynamics will result if the finite dimensional model retains terms of sufficiently high frequency (Schmitz, 1985). Nonminimum phase produces various symptoms: reverse initial action, zeros in the right-half plane, delay due to wave propagation and phase in the frequency response that is not "minimum" for the order of the system. Many theoretical results are complicated by or even totally voided by a system of nonminimum phase.

Inverse plant controls are viewed in the linear case as the inverse of a transfer function. Zeros of the transfer function become poles of the inverse plant and a transfer function with right-half plane poles is representative of instability. An unstable response results from a contour of integration enclosing the left-half plane. Alternatively, a contour enclosing the right-half plane results in an acausal response, that is, a system with response that occurs before the input. While a real time system must be causal, acausal systems can be used to calculate a torque history that will be nonzero before the prescribed motion begins. The input is the desired trajectory and the response of the inverse dynamics is the torque to be applied. This acausal inverse dynamics can be implemented if the trajectory is known in advance (Book and Kwon, 1992; Moulin and Bayo, 1991). While in theory the input to the noncollocated system must be applied infinitely far in advance, in practice anticipation of one or two time constants is needed. Figure 7 shows the torque resulting from the inverse plant calculation for a single pinned beam to achieve a piece-wise cubic tip acceleration. The system has been decoupled into causal and anticausal subsystems, the responses of which sum to give the complete response. The causal response entirely follows the contributing inputs and is solved from the input and the specified initial conditions. Anticausal is the condition when the response entirely precedes the contributing inputs. The anticausal response is solved backwards in time from the known final conditions and results in the portion of the response preceding the initial tip motion.

It should be emphasized that inverse dynamics as a total solution to open-loop control is not practical in most cases. The variability of the parameters demands corrections be provided by feedback controls. It is also imperative that the computational demands be kept in check. For a single link, linearization and time domain solution procedures have made on-line trajectory calculations feasible (Kwon, 1991). For multiple links, the techniques have demanded super computer effort in the past (Bayo, 1987). It appears likely that improved calculation schemes will evolve for multiple links as well, though perhaps involving simplifications and assumptions not applicable in all cases.

Inverse dynamic calculations can give desired values for all the system states that are consistent with the specified tip conditions. These trajectories are needed if certain tracking feedback controls are to be applied (Siciliano et al., 1986). Other feedback controls show almost equivalent performance using only a subset of the states (Book and Kwon, 1992). These flexible states also provide a convenient way to unite position and force control, since force will always result in deflection of a flexible arm. Hence, state trajectories for a tip force history are predictable from a static model of arm bending.

4 Feedback Control Algorithms

Feedback must be included to respond to unmodeled dynamics and disturbances. It is not possible to directly extend the rigid arm approach of linearizing feedback to flexible link systems since joint actuators are not available to cancel the nonlinear dynamic terms in the flexible equations. On the other hand, the goal of tracking the desired endpoint has been achieved for flexible link arms.

Many of the advanced modern control algorithms have been applied to flexible arm control, but in my opinion, no results crown any algorithm as superior in all cases. Our experience and observations to date show the following:

- Linear state feedback is effective at controlling multilink flexible structure dynamics (Cannon and Schmitz, 1984; Henrichfreise et al., 1987), but it may be too sensitive to variations of the dynamics during operation.
- Strain rate or some equivalent is important to feed back to damp vibrations (Yuan et al., 1990).
- Decoupled control of some arms is effective at controlling joint and corresponding flexible link motion (Yuan et al., 1990).
- Adaptive algorithms that ignore flexibility do not eliminate or greatly improve the vibration problem (Cetinkunt and Book, 1990).
- Simple adaptive gain scheduling is very effective in extending the local advantages of decoupled strain rate feedback to the overall work space.
- Multiple time scale composite controls, e.g., based on the singular perturbation method, are effective simplifications for dealing with the complex problem (Fraser and Daniel, 1991; Ghorbel and Spong, 1992; Siciliano and Book, 1988). One will be locked into a lower range of performance, however, in order to achieve the required separation of time scales between the rigid and flexible subsystems.
- Robust control techniques based on bounded uncertainty estimates can be extended to flexible link arms. While stability proofs are reassuring, they are not very helpful in obtaining a system of high performance (Lee, 1992).
- End point measurements can be used productively in servoing multilink arms with flexible links (Oakley and Cannon, 1990).

5 Command Shaping Techniques

While inverse dynamic equations allow the advance prescription of the entire tip trajectory, the demands on calculation time are substantial. Prescribing the complete trajectory may not be desirable, or even possible. Teleoperated arm motion is an example where this is true. One might still hope to modify the input specification of joint angles to result in tip motion without exciting the dominant modes of vibration. One thinks of smoothing or shaping the inputs to be better suited to the vibratory nature of the system. Notch filters can be applied, but while the frequency domain specification looks promising, the transient response of these filters is not very good.

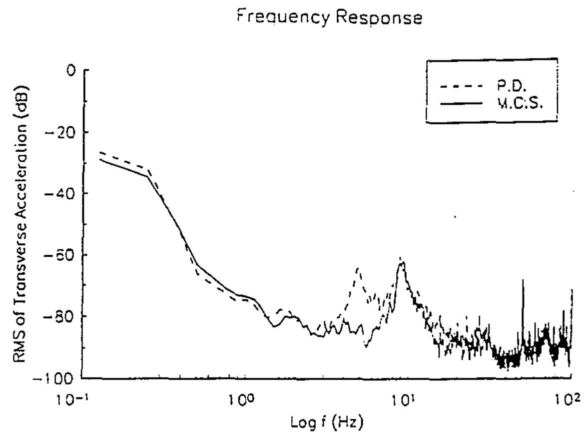

Fig. 8 Experimental frequency response of RALF for a circular trajectory using only PD and using Magee's modified command shaping

Singer and Seering (1990) proposed an alternative for linear systems they referred to as input shaping. In the simplest form of input shaping, an impulse input would be shaped into two impulses, with the second delayed by one-half period of the vibration frequency to be avoided. Singer showed that by further shaping, i.e., generating more pulses, the undesirable sensitivity to errors in the frequency could be reduced. Singer's work also showed that multiple modes of vibration could be handled as well.

For some flexible motion systems the natural frequencies change dramatically. An example is the experimental arm shown in Fig. 5 or any multilink arm with variable inertia outboard of flexibility. The variable frequency was treated by Magee and Book (1992) with an extension of input shaping he called command shaping. Magee first measured the frequency and damping ratio throughout the system's workspace. A variable delay between successive pulses in the shaped response was based on these measurements with care to avoid transient effects. Consider experiments performed on the arm in Fig. 5. A circular trajectory of diameter equal to about one-sixth of the maximum reach was commanded. Figure 8 shows the acceleration spectrum with command shaping inside a P-D feedback loop and the unshaped P-D response. All frequencies are passed except the frequencies of the resonance. Notice that a slight error results from the fact that the arm period is not a perfect multiple of the sampling rate. Note that the second mode at about 10 hz was not cancelled in this preliminary study. Higher sampling rates have been used in more recent experiments to avoid exciting multiple modes.

While Singer placed the input shaping outside the feedback loop, we chose to place the input shaping inside the loop, with the shaping filter acting on the error signal produced by the joint controllers. In this way, the commanded and actual values of the joints can be directly compared. This does raise concern about stability, since the modified command shaping law introduces time delays into the feedback loop. A step response does show additional overshoot with modified command shaping. Zuo and Wang (1992) have analyzed the destabilizing effect of this delay.

6 Passive Damping Enhancement

Active damping alone of an infinite number of undamped vibrational modes with bandwidth limited actuators is doomed to fail. The high frequency modes will be outside the bandwidth of the actuators. Fortunately, damping is always present in elastic systems, although perhaps not to the degree necessary for good performance. This section will discuss the need and sources of passive damping and one effective method, the constrained layer damping treatment, for enhancing passive damping.

While low frequency modes may be readily influenced by the active control being designed, unmodeled (low or high frequency) modes can cause nasty surprises. First, controllability and observability are influenced by the placement of actuators and sensors, respectively. It is possible to excite modes that are not observed from a given sensor set and to observe modes which are not excited by a given actuator set. Furthermore, controllers are designed based on a given model order, always lower than the physical system. When excitation of the unmodeled modes occur, it is referred to as "spillover." Balas (1978) receives credit for exposing this problem, particularly as oriented to large space structures. He showed that "observer spillover" could result in system instability.

The situation is complicated further by digital control of the distributed parameter system. By sampling the high order system below the Nyquist rate, "wrap around" of the high frequency poles occurs as explained by a Z-transformation of a linearized system transfer function. The Z-plane poles with angles greater than π radians (s-plane imaginary part greater than π divided by the sample rate) will begin to interact with lower frequency poles. This can drive branches of the Z-plane roof locus unstable when the s-plane root locus would show stable behavior (Alberts, 1986).

The dissipation mechanism to enhance damping used by Alberts (1986) for beam bending was the shearing of a thin viscoelastic layer, such as is commercially available for sound dampening, applied to the surface of the beam. A constraining layer of high extensional modulus should be applied on top of the viscoelastic layer. When the beam bends, its outer fibers stretch on one side and contract on the other, requiring corresponding motions in the viscoelastic material where it contacts the beam. The constraining layer ensures that the outer surface of the viscoelastic layer does not move as much; hence, shearing and energy dissipation results. If the constraining layer is long, shear forces will build up and cause it to stretch. This slightly increases the beam's stiffness, but shearing of the viscoelastic layer is reduced, defeating our effort to improve damping. By sectioning the constraining layer, the damping effect can be optimized for a given mode shape. The resulting damping has been shown to stabilize the controlled motion of a bending beam (Book et al., 1985).

Damping both torsion and bending in two axes is more difficult. Spiraled wraps to a constraining layer tape have been shown to be effective. Damping the torsion of a link directly may not be effective or necessary if the mode to be damped included bending of another link.

Sattinger and Sanjana (1991) have shown that fiber composites can be used in conjunction with the constrained layer damping treatment. A weight penalty is incurred by this approach that may be important. The viscoelastic material is also very temperature sensitive and subject to other environmental effects that may limit its use under severe conditions.

7 Operational Strategies

Since the technology limitations depend on the way a motion system is used, a modification of the way the system is used should be considered along with improvement of the technology. This alternative may be ruled out in some cases if compatibility with other systems or processes is required. In other cases a change in operational strategy is intertwined with a change in technology.

For example, as stated earlier, one must trade off large motion speed and work-space size against small motion speed (bandwidth) or accuracy when sizing a structure. This is true if the same structure is used for both motions. The bracing strategy (Book et al., 1984) seeks to do otherwise by using redundant degrees of freedom. A large motion subsystem carries a small motion subsystem. After the small motion system is properly placed, the inherent effects of its long extension can be eliminated by bracing it against a passive support, much as humans do with their arm when they write. The passive support allows the small motion system to operate from a new fixed base. The small motion system as proposed by Asada and West (1984) did not require additional actuators but reduced motion options.

As a generalization of the bracing strategy, Kwon and Book (1988) analyzed the concept of staged positioning systems. Positioning often occurs in stages. Ships, trains, trucks, forklifts, conveyors, and robots might all contribute to the repositioning of a part between factories. Movement to the other side of the world and positioning to an accuracy of 0.7 in. means a position accuracy of one part in 10^{10}! This figure of merit is only possible with staged positioning systems.

The use of staged positioning is found in practice. Some disk heads use a coarse and fine positioning system. IBM researchers (Karidis et al., 1992) have employed this approach for very high-speed probing of IC test points. Chiang et al. (1991) has employed a "fast wrist" to quickly position at the end of a flexible arm. NASA anticipates a small dexterous manipulator at the end of a space crane on the space station Freedom and DOE envisions a similar arrangement for waste removal from underground storage tanks (Krieg et al., 1990). In these cases bracing is not used, however. In an attempt to appreciate the tradeoffs, Book and Wang (1989) optimized a skeleton design to establish when staged positioning was effective. As expected, more small motion favors bracing.

In more imaginative designs, the small motion system can deform the large motion system, analogous to the deflection of a crane's cable by workmen engaged in precisely positioning a heavy load (Lew and Book, 1990).

The small arm can supplement the shortcomings of a large arm in other ways. Stiction and low actuator bandwidth in the large joints may render them unusable for active damping. Inertial forces generated by moving the small arm have been shown to be effective in damping a large arm's vibration (Lee, 1992; Lee and Book, 1990). The use of inertial forces forms an unconventional actuation approach and others have been proposed and deserve mention. Zalucky and Hardt (1984) proposed a stiffening actuator to straighten the deflected link. Asada et al. (1991) have refined the concept with tendon actuation oriented to producing a minimum phase flexible system. Piezoelectric films and ceramics have been used to deform structures for shape control and active damping (Tzou, 1989).

8 Conclusions

We understand more about the dynamics and control of flexible motion systems than 20, or even five years ago. Our database shows over 60 publications in 1990 alone and only about 20 from 1980 through 1984, so researchers clearly recognize the problem. Researchers regularly model flexible arm dynamics, and some verify their results with experimental results. When experiments are part of a paper, the test systems usually lack the complexity of real world applications as we would expect for focused test beds. Truckenbrodt (1983) and Cannon and Schmitz (1984) have controlled the linear behavior of a flexible arm system at bandwidths well above the clamped joint natural frequency, but the sensitivity to parameter variations is too great for most practical arm applications. Robustness of the controlled system must now be stressed. We are passing through the phase of the area where every control approach from neural nets and fuzzy control to H_∞, is applied to "the" flexible arm problem. "The" flexible arm problem does not exist, but all the available tools should be applied to "a" flexible arm problem to assess its suitability for problems of this type. The flexible arm has become one test case for the evaluation of control and dynamics algorithms. The rapid improvement of control computers makes many of these algorithms feasible.

In the near future, research will move from writing the appropriate dynamic equations efficiently to understanding the equations in terms of relevant dynamic characteristics: nonminimum phase, time delays, skew symmetric terms in the dynamics, for example, that enable control development to move forward. This must be coupled with physical experimentation to avoid irrelevant solutions. The research needs to be evaluated in the context of a real application. Three years ago I would have hesitantly guessed that space would provide the first opportunity for this evaluation. It now appears that environmental restoration and waste management efforts of the U.S. Department of Energy will more likely provide this opportunity. The crystal ball seldom works when, predicting the technology of the future, but adaptive or learning "feedforward" approaches are likely to contribute to future advances, and intelligent tip position sensors that will provide velocity and position of the tip relative to a predefined work piece. Our best hope to expand the envelope of feasible performance is a confluence of open- and closed-loop controls, design innovations, material improvements and sensor and actuator developments. These advances are underway, but will they be integrated into a viable system design with radical new features and wide applicability? This might happen if the same technology that controls flexible arms also allows tip-oriented control that is easily programmed off-line without the need for precise fixturing of the part relative to the work piece. But even short of these fondest desires one can expect an evolutionary change that allows future generations of engineers to design controlled motion systems to be lighter, faster, and more accurate.

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