HYBRID CONTROL OF FLEXIBLE MANIPULATORS WITH MULTIPLE CONTACT

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This paper proposes a hybrid position/force controller for flexible link manipulators that make contact with the environment at more than one point. The manipulator maintains contact at the bracing point to reduce its structural vibrations, and at the same time, the end effector may make contact with the workpiece to perform the task. This approach requires hybrid control not only at the end effector but also at the bracing point. In this paper, the dynamic equations of the motion with multiple contact constraints are derived. The dynamic equations are transformed into two subspaces, the constrained and constraint-free subspace, using the singular value decomposition of the constraint equations. Each force and position controller are developed based on the orthogonality of these two subspaces. A hybrid controller is proposed from the new coordinates, and its stability is proved analytically under the quasi-static condition. Finally, an experimental study is carried out to justify the feasibility and application of proposed ideas.

Introduction

It is known that light weight manipulators can provide distinct advantages over conventional ones. The reduction of the component weight allows the actuators to move faster and carry heavier loads with longer links. However, in exchange for lighter weight, one must accept an increase in system flexibility. The inherent flexibility might result in undesirable structural vibration, which is a major concern for most applications.

Active vibration feedback control by the joint actuator may reduce structural vibration. However, its performance is limited by the actuator bandwidth, and it could excite high modes in a complex multiple link system. Bracing a flexible manipulator may be one effective method to reduce the flexibility or damp out its structural vibration. The manipulator braces against a stationary frame while the end effector performs the fine motion control just as the human arm braces at the wrist for accurate writing. Bracing will secure the end point positioning by forming a closed kinematic chain. [Kwon,88] and [West,85] proved that bracing reduces the positioning uncertainty and increases the stiffness of the manipulator.

This paper provides a hybrid position/force controller for a flexible manipulator that makes multiple contacts with the environment. The basic frame work of the proposed ideas is developed from the following previous literature. The author will briefly review them. [Book,84] introduced the idea of a bracing strategy for flexible link manipulators, and various bracing devices were compared. [West,85] showed advantages of bracing and designed a hybrid control for bracing manipulators. [Mason,81] presented a guideline for understanding constrained task at the end effector. The task broke down into sub tasks that are defined as the natural and artificial constraints. The position and force controllers are designed in each sub task by the use of a selection matrix. Hybrid control based on Mason's work was proposed and demonstrated by [Raibert,81]. Some researchers such as [Yoshikawa,87] improved the hybrid control by considering the dynamics of the manipulator. [Fisher,92] considered the kinematic stability condition of a hybrid control. However, most of the previous work on the hybrid control considers only the contact at the end effector. [McClamroch,86] took a different approach to control a constrained manipulator system. First, he modeled the constrained manipulator with differential equations and algebraic equations. He formulated this set of equations as a singular system. He proposed a systematic way to reduce system order and applied a nonlinear feedback controller to control the constrained systems. However, the application is limited to a rigid link manipulator with a single contact. [Mills,91] recognized the similarity between the approaches like [Mason,81] and [McClamroch,86]. [Singh,85] used the singular value decomposition method to reduce the equations of motion for a class of constrained dynamic systems. However, the effects of constrained force dynamics are not formulated in the problem.

Problem Statement

The bracing arm control problem is generalized to a hybrid control of flexible manipulators with multiple contacts with the environment. Figure 1 shows a flexible

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manipulator with n joints, all active, constrained by m contacts with the environment.

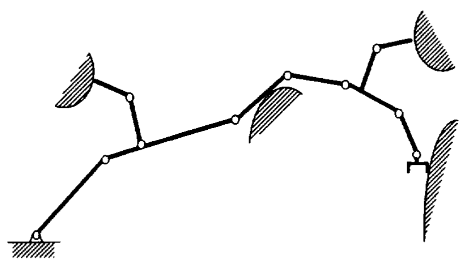

Figure 1 A Flexible Manipulator with Multiple Contact with Environment

Due to the contacts with the environment, the motion of the end effector becomes constrained, and the system order (degree of the freedom) is reduced. As a result, the constraint forces become new variables to be controlled. Some researchers define the constraint forces as internal forces. Each contact point needs to be controlled in position/force depending on the constraint conditions.

The objective of the controller is the control of the position/force of the end effector while satisfying all the constraints and maintaining desired contact forces at the bracing points. Thus, a bracing manipulator, which is a redundant manipulator, executes the necessary task at the end effector and uses the internal(null) motion to brace at the bracing points. Thus, the task of the manipulators can be represented as an augmented vector of main task and sub task.

$$Task = \begin{bmatrix} task_t \\ task_{b1} \\ \vdots \\ task_{bm} \end{bmatrix} \tag{1}$$

Each $(task)_i$ can be position or force variables that need to be controlled at each contact point. For example, the $(task)_t$ is for the end effector to perform the required task, and the $(task)_{bi}$ is the augmented task for maintaining bracing forces and positions by internal (null) motion. However, the selection of the $(task)_i$ requires the following consideration. There should exist a nonsingular Jacobian matrix J, which is the mapping from the active joint coordinates to the task coordinates defined in equation (1). This condition is very important because it guarantees that the constraints from contact are mutually independent, and each task can be controlled independently by the active joints. The condition will be examined later in this paper.

Further assumptions are made to formulate the problem:

1. The locations and geometry of constraint surfaces are known in advance. Thus, we can express the constraints with algebraic equations, the so called configuration constraint equations:

$$\left.\begin{aligned} \phi'_1(x_1) &= \phi_1(q) = 0 \\ \phi'_2(x_2) &= \phi_2(q) = 0 \\ &\vdots \\ \phi'_m(x_m) &= \phi_m(q) = 0 \end{aligned}\right\} \text{ m algebraic equations} \tag{2}$$

where x_i is the position of each contact point in Cartesian coordinates, and q is in the manipulator generalized coordinate.

2. The constraint equations can be written as a set of m constraint surfaces, each of which is assumed to be mutually independent.
3. The manipulator always maintains contacts with the environment while it is in motion.
4. Constraint surfaces are very rigid compared to the manipulator and do not deform due to contact.
5. All joints are rotational joints.

Modeling of Constrained Flexible Manipulators

The dynamics of open chain flexible manipulators can be derived using the Lagrangian formulation with the assumed modes method. Details can be found in [Lew,92]. When a flexible manipulator makes contact with the environment, one can introduce unknown reaction forces at the contacts between the manipulator and the environment (carrying this out with Lagrange multipliers), and then these reaction forces can be included in the equations of motion as generalized forces. Later, one may solve the dynamic equations simultaneously with the constraint equations to determine the constraint forces as well as the reduced order dynamic equations. The equations of the motion of multiple constrained flexible manipulators can be represented as

$$\begin{bmatrix} M_{rr} & M_{rf} \\ M_{fr} & M_{ff} \end{bmatrix} \begin{pmatrix} \ddot{q}_r \\ \ddot{q}_f \end{pmatrix} + \begin{bmatrix} C_{rr} & C_{rf} \\ C_{fr} & C_{ff} \end{bmatrix} \begin{pmatrix} \dot{q}_r \\ \dot{q}_f \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} I \\ b \end{bmatrix} \tau + \Phi_1^T \lambda_1 + \Phi_2^T \lambda_2 + \cdots + \Phi_m^T \lambda_m \tag{3}$$

where q_r = rigid coordinate such as joint angles for rotational joints, $q_r \in R^n$

q_f = flexible mode amplitude coordinates for each link, $q_f \in R^{N-n}$

M_{ij} = a partition of the inertia matrix of the manipulator. The subscript r denotes rigid, and the subscript f denotes flexible

C_{ij} = a partition of Coriolis and centrifugal matrix

k = link stiffness matrix

τ = torque from each joint actuator
I = nxn identity matrix
b = a part of input matrix that relates between input torque and flexible mode coordinate. It is determined by the boundary conditions for mode shape functions.
λ_i = Lagrange's multiplier which is the reaction force magnitude at each contact point

Also $\Phi_1 = \frac{\partial \phi_1}{\partial q}$, $\Phi_2 = \frac{\partial \phi_2}{\partial q}$, ..., $\Phi_m = \frac{\partial \phi_m}{\partial q}$

where $q = \begin{bmatrix} q_r^T & q_f^T \end{bmatrix}^T$ and also satisfies the constraint equation (2).

If one combines all of the reaction forces into one matrix and rewrites equation (3) in a simpler form,

$$M\ddot{q} + C\dot{q} + Kq = B\tau + \Phi^T\lambda \tag{4}$$

where

$$\Phi^T = \begin{bmatrix} \Phi_1^T & \Phi_2^T & \cdots & \Phi_m^T \end{bmatrix} = \begin{bmatrix} \Phi_r^T \\ \Phi_f^T \end{bmatrix}$$

$$\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_m \end{bmatrix}^T$$

with the configuration constraint equations which are defined in equation (2). Also, we may equivalently replace the configuration constraint by a velocity constraint, which is a restriction on the velocity when it is in a specified position. The time derivative of the configuration constraint equation, which is the velocity constraint, is

$$\Phi \dot{q} = \begin{bmatrix} \Phi_r & \Phi_f \end{bmatrix} \begin{pmatrix} \dot{q}_r \\ \dot{q}_f \end{pmatrix} = 0 \tag{5}$$

Recall that $\lambda \in R^m$ and $\Phi^T \in R^{N\times m}$ and $rank(\Phi^T) = m$ since each constraint is assumed to be independent. Now, we have N (= rigid+flexible) differential equations with m unknowns, and m velocity constraint equations exist.

Elimination of Constraint Force (λ)

In the previous section, one obtained the dynamic equations of a constrained flexible manipulator with multiple constraints. To design a controller for constrained manipulators, the equation of motion should be represented as a standard form without constraint forces. The constraint forces can be found using Singular Value Decomposition (SVD). The rigid part of the Jacobian constraint matrix, Φ_r, with rank m, can be decomposed into the following form,

$$\Phi_r = \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} \Sigma & 0 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix} \text{ and } \Phi_r^T = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} \begin{bmatrix} u^T \end{bmatrix} \tag{6}$$

where u and v_i are orthornormal bases for four fundamental subspaces as shown in Figure 2. The columns of u are the normalized eigenvectors of the matrix $\Phi_r\Phi_r^T$. The columns of v_i are the normalized eigenvectors of the matrix $\Phi_r^T\Phi_r$ where $v_1 \in R^{n\times m}$ and $v_2 \in R^{n\times(n-m)}$. Σ is a diagonalized matrix with the square roots of non-zero eigenvalues of $\Phi_r\Phi_r^T$. i.e., $\Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_m)$ with $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m$. Notice that v_2 is the null space of Φ_r which satisfies the following relationship.

$$\Phi_r v_2 = 0$$

The reason for taking the SVD of only the rigid part of the constraint matrix is that only rigid generalized coordinates have actuators at the same physical locations. The actuators generate direct control of the rigid joint angle so that any arbitrary motion can be realized. On the other hand, the flexible generalized coordinates do not have independent actuators in the coordinates. Thus, the flexible motion cannot be controlled independently. The flexible motion is indirectly influenced by the motion of joint angles and the constraint forces. Therefore, it is impossible to generate an orthogonal actuation to all the generalized coordinates.

Now, one transforms the original equations to a new set of differential equations. Let

$$\bar{v} = \begin{bmatrix} v & 0 \\ 0 & I \end{bmatrix} \quad \text{where } v = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$$

If we pre-multiply by $\bar{v}^T$, then the equation of motion becomes

$$\bar{v}^T M\ddot{q} + \bar{v}^T C\dot{q} + \bar{v}^T Kq = \bar{v}^T B\tau + \bar{v}^T \Phi^T \lambda$$

$$= \bar{v}^T B\tau + \begin{bmatrix} \Sigma u^T \lambda \\ 0 \\ \Phi_f^T \lambda \end{bmatrix} \tag{7}$$

One can notice that the constraint force is eliminated in the second equation corresponding to the zero in the last term of equation (7). The first m differential equations include the constraint forces and will be used to compute the constraint force. The next n-m differential equations are constraint-free rigid generalized coordinate dynamics. The last N-n differential equations represent the flexible generalized coordinates.

System Order Reductions

Each configuration constraint equation may be solved for one of the generalized coordinates. Substitution of that result into the equations of motion, and the other constraint equations will remove the selected generalized coordinate from the formulation. The result will be a reduction in the order of the system equations. However, such an approach is effective only for a special form of the algebraic constraints. In this section, the use of the pseudo inverse achieves a systematic reduction of the system order of constrained flexible manipulators .

From constraint equations, we have

$$\Phi \dot{q} = \Phi_r \dot{q}_r + \Phi_f \dot{q}_f = 0 \quad (8)$$

If one takes the pseudo inverse of the rigid part of the constraint matrix which gives minimum norm and represents the null space projection matrix with v_2 from equation (6), one can obtain an allowable rigid joint motion from the constraint.

$$\dot{q}_r = -\Phi_r^+ \Phi_f \dot{q}_f + v_2 \dot{z} \quad (9)$$

where $\Phi^+ = \Phi^T(\Phi\Phi^T)^{-1}$. The second term of equation (9) gives the null solution for the constraint and shows the allowable joint motion which is free from the constraints. z is any arbitrary vector ($z \in R^{n-m}$). Eventually z becomes the new reduced order coordinates. It is difficult to interpret the physical meaning of z for the general case of multiple contact. However, when the end of the manipulator is constrained, z measures contact point motion which is tangential to the constraint surfaces.

If we take the time derivative of equation (8) one more time, we can get an acceleration relationship as

$$\ddot{q}_r = -\Phi_r^+ \Phi_f \ddot{q}_f + v_2 \ddot{z} \quad (10)$$

The integration of equation (9) gives a position relationship

$$q_r = -\Phi_r^+ \Phi_f q_f + v_2 z + C \quad (11)$$

Assume that the initial conditions are zero. Thus, $C = 0$.

Now, the generalized coordinate q_r ($q_r \in R^n$) may reduce to *new* coordinates $z(z \in R^{n-m})$. Substitute equations (9), (10) and (11) into (7) to reduce the order of the system as follows.

$$v_1^T M_{rr} v_2 \ddot{z} + v_1^T (M_{rf} - M_{rr}\Phi_r^+ \Phi_f)\ddot{q}_f + v_1^T C_{rr} v_2 \dot{z} + v_1^T (C_{rf}) - C_{rr}\Phi_r^+ \Phi_f)\dot{q}_f = v_1^T \tau + \Sigma u^T \lambda \quad (12)$$

$$v_2^T M_{rr} v_2 \ddot{z} + v_2^T (M_{rf} - M_{rr}\Phi_r^+ \Phi_f)\ddot{q}_f + v_2^T C_{rr} v_2 \dot{z} + v_2^T (C_{rf} - C_{rr}\Phi_r^+ \Phi_f)\dot{q}_f = v_2^T \tau \quad (13)$$

$$M_{fr} v_2 \ddot{z} + (M_{ff} - M_{fr}\Phi_r^+ \Phi_f)\ddot{q}_f + C_{fr} v_2 \dot{z} + (C_{ff} - C_{fr}\Phi_r^+ \Phi_f)\dot{q}_f + K q_f = \Phi_f^T \lambda \quad (14)$$

These three sets of differential equations represent the constrained flexible arm dynamics without constraint equations. The first set of equations shows the relationship between constraint forces and arm dynamics. The second set of equations shows the arm's dynamics in the constraint-free space. The third set of equations shows the flexible mode behaviors.

Note: If the manipulator is over-constrained physically, the manipulator can not accomplish the desired motion. It is important to check whether the constraint-free coordinate z can realize the desired motion in the task coordinates of the manipulator. One needs to examine the mapping relationship between the constraint-free coordinates and task coordinates. Since only the joint coordinate is directly controlled by actuators and the flexible mode coordinate is indirectly influenced by the rigid joint motion, we will consider the rigid joint motion only, i.e., ignore the flexible mode coordinates for the inspection. The possible rigid joint motion from the constraint equation is expressed in equation (9). Thus, there should exist a mathematical relationship as

$$\dot{X}_{motion} = J_r \dot{q}_r = J_r v_2 \dot{z}$$

where $\dot{X}_{motion}$ represents motion in the task coordinate for the end effector and the bracing points. Based on the above transformation relationship, we can conclude the following: (1) If $Rank[J_r v_2]$ is larger than $Dim[z]$, then it is impossible to perform the desired task. This is the situation when the manipulator is in a singular configuration or over-constrained. One needs to change the configuration of the manipulator. (2) If $Rank[J_r v_2]$ is equal to $Dim[z]$, the manipulator can execute the desired task under the given constraint. (3) If $Rank[J_r v_2]$ is smaller than $Dim[z]$, then there exists redundancy in z for the desired motion. We may utilize the redundancy to optimize criteria such as the energy consumption of the system or obstacle avoidance [Yoshikawa,90].

Quasi-Static Assumptions

Assume that the flexible mode becomes static after bracing although the joint angles have dynamic motions. Thus, we may assume that terms in equation (12,13,14) involving $\ddot{q}_f$ and $\dot{q}_f$ are zero. The justifications of this quasi-static assumption are:

1. Bracing forms kinematic closed loops, consequently, the kinematic structure of the manipulator becomes rigid.
2. After bracing, the manipulator moves in a relatively slow motion. Therefore, the structural vibration is not excited by the rigid motion of joint angles.

The constrained system dynamic equations (12) and (13), and (14) become

$$v_1^T M_{rr} v_2 \ddot{z} + v_1^T C_{rr} v_2 \dot{z} = v_1^T \tau + \Sigma u^T \lambda \quad (15)$$

$$v_2^T M_{rr} v_2 \ddot{z} + v_2^T C_{rr} v_2 \dot{z} = v_2^T \tau \quad (16)$$

$$M_{fr} v_2 \ddot{z} + C_{fr} v_2 \dot{z} + K q_f = \Phi_f^T \lambda \quad (17)$$

Based on these quasi-static assumptions, a hybrid controller is proposed and the closed loop system stability will be investigated.

Proposed Feedback Controller

The control objective is to make $z \to z_d$ and $\lambda \to \lambda_d$. Each contact point should be able to follow the desired trajectory and to maintain the desired contact force. In this work, we consider only a regulator problem. (The trajectory tracking control can be developed too.) Let us design a controller as

$$\tau = v_1\tau_f + v_2\tau_p \tag{18}$$

Recall that v_1 and v_2 are orthonormal. Thus the position control input τ_p does not affect the constrained force dynamics, which is represented by equation (15) . On the other hand, the force control input τ_f does not effect the constraint-free space motion, which is shown in equation (16). The matrices v_1 and v_2 work as a kinematic filter to separate the control input into the force controlling input and the position controlling input. The hybrid controller proposed by [Raibert,81] is similar to the proposed controller, and actually it is a special case of the proposed controller.

Let the position controller input be

$$\tau_p = K_p(z_d - z) - K_d\dot{z} \tag{19}$$

where K_p and K_d are the proportional and derivative controller gain matrices. If uncertainty of the system exists, an extra robust controller can be added to guarantee the stability. The detail description is found later in [Lew,93] and [Chen,89]. If the proposed position controller is applied to equation (16), it becomes

$$v_2^T M_{rr} v_2 \ddot{z} + v_2^T C_{rr} v_2 \dot{z} = K_p(z_d - z) - K_d\dot{z} \tag{20}$$

The solution of the state z is asymptotically stable as long as K_p and K_d are positive definite. In other words, z converges to a constant desired state z_d. This can be proved by using the Lyapunov analysis and the Invariant Set Theorem since the matrix $\dot{M}_{rr} - 2C_{rr}$ is skew symmetric. Details can be found in {Lew,93}

Recall that the response of the system differential equations has to satisfy equations (15),(16), and (17). If the state z is controlled to be the desired state z_d by the position controller, $\dot{z}$ and $\ddot{z}$ become zero at steady state. Then equation (15) becomes an algebraic equation as

$$\tau_f + \Sigma u^T\lambda = 0 \tag{21}$$

Let the force controller input be

$$\tau_f = -\Sigma u\lambda + K_f(\lambda - \lambda_d) \tag{22}$$

where K_f is the force controller gain. Thus, the position controller τ_p should be able to make $z \rightarrow z_d$ and at the same time, the force controller will make $\lambda \rightarrow \lambda_d$ with the proposed force control input. On the other hand, the magnitude of the flexible mode becomes

$$q_f = K^{-1}\Phi_f^T\lambda_d \tag{23}$$

To represent the control input in terms of joint angles and flexible modes which are measurable, multiply the equations (4-9) and (4-11) with v_2^T. We can obtain

$$z = v_2^T q_r - v_2^T \Phi_r^+ \Phi_f q_f \tag{24}$$

$$z = v_2^T q_r - v_2^T \Phi_r^+ \Phi_f q_f \tag{25}$$

Since the bracing arm dynamics is assumed to be quasi-static, $\dot{q}_f$ is zero. Thus the control input is

$$\tau = v_1\{-\Sigma u^T\lambda + K_f(\lambda_d - \lambda)\} + v_2[-K_d v_2^T \dot{q}_r + K_p\{z_d - v_2^T(q_r - \Phi_r^+\Phi_f q_f)\}] \tag{26}$$

Experimental Case Study

A large experimental arm designated RALF (Robotic Arm, Large and Flexible) has been constructed and is under computer control. RALF has two degrees of freedom in the vertical plane. The length of each link is about 10 feet. At the tip of RALF, SAM (Small Articulated Arm) is mounted as shown in Figure 2. SAM also has two degrees of freedom in the vertical motion. The dominant structural natural frequency of RALF with SAM is observed around 2 Hz. Both the arms are controlled by a PC 486-33 with two A/D boards

The program is written in the C language, and 10 msec is used as a sampling time for experiments. Two "custom-made" force sensors are used to measure the contact forces, and the strain gages at each link of RALF give the information of how much the link deflects.

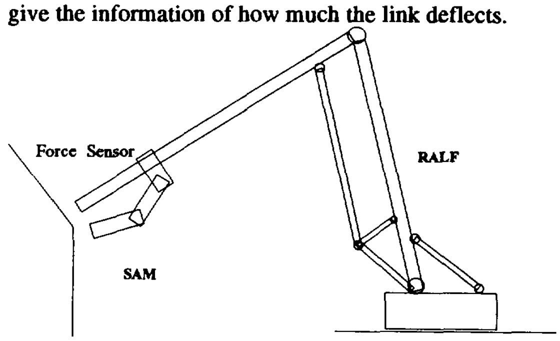

Figure 2. Experimental Setup of RALF and SAM for Hybrid Control

Experiment 1: The proposed hybrid control is applied to a one point contact case. The experiment is carried out with only RALF. The constraint surface is located 16 ft way from the base of RALF and has a slope of 125 degrees. The tip of RALF moves very close to the constraint surface by the PD position control. Then RALF performs a force control normal to the surface and follows a desired trajectory along the surface. The desired force is 1.5 lbf, and the trajectory is given as a cycloidal motion for 1 ft travel distance.

Figure 3(a) and (b) show the experimental results. Figure 3(a) shows the good tracking motion of the tip of RALF. The plot of the tip position is obtained from the measured joint angles. The switch from the PD position control mode to the hybrid control mode causes the initial jump in the motion as shown in Figure 3(a). Figure 3(b) shows the contact force measured by the force sensor at the tip of RALF. The excitation of the measured force at the start of the motion is due to structural vibration of the force sensor. After the contact at 3.7 sec, the contact force followed the desired force as we expected.

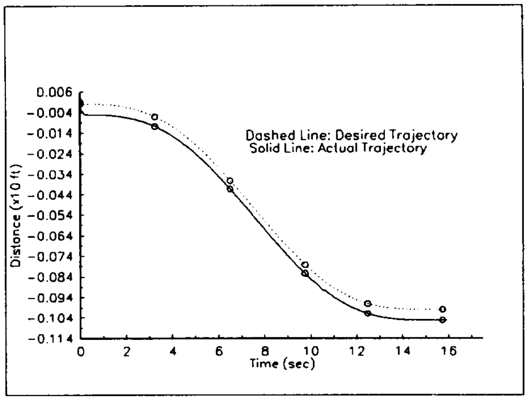

Figure 3(a) RALF Position Control along the Constraint Surface (Experiment 1)

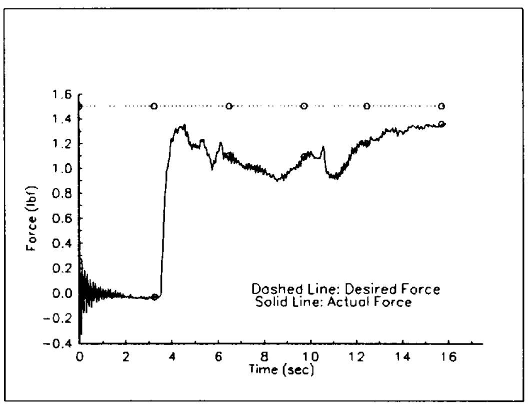

Figure 3(b) RALF Force Control Normal to the Constraint Surface (Experiment 1)

Experiment 2: The second experiment will be performed to show the two points hybrid control. The tip of RALF is going to brace against the same constraint surface as Experiment 1, and SAM will also make a contact with a vertical constraint surface as shown in Figure 3. Each arm will carry out the hybrid control against two different surfaces. Its results will be published in [Lew,93]

Conclusion

This paper deals with a constrained flexible manipulator that makes contact with the environment at more than one point . A hybrid controller is derived from the constrained dynamics of the flexible manipulator. Its stability is proved analytically. Experimental study is being carried out to show the feasibility of the proposed controller. As a case study, Experiment 1 accomplished the tip point hybrid control of a flexible manipulator. Each position and force control motion is achieved in a reasonable manner. The on-going Experiment 2 should be able to achieve the hybrid control at both the tip and bracing points.

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