OPTIMUM ASSIGNMENT AND SCHEDULING OF ARTILLERY UNITS TO TARGETS

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ARTILLERY UNITS TO TARGETS

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SUMMARY

This thesis is concerned with a particular type of assignment and scheduling problem, one involving optimizing the assignment of artillery units, by their capabilities and location relative to a particular target, to fire on certain enemy targets according to a schedule which orders the firing in priority of the importance of the targets. The method assumes that the target analyst will be able to determine the type of artillery unit to fire on a given target, thus enabling him to determine the number of units and number of volleys required to achieve the desired results. Also, using a given weighting scheme, the analyst can derive comparative weights for the targets which, when inserted into the assignment problem would insure optimum allocation of artillery units. This allows the scheduling of those same targets in a manner such that they would be scheduled at the earliest possible time.

Although developed to schedule 155 mm and 8" artillery units, the problem formulation lends itself to change to allow for other types of field artillery firing units. Further, the formulation is such that it can simply be converted to a computerized format capably handled by most computers now in use in the U. S. Army. This computerization would greatly enhance the target analysis requirement in a combat situation. Its applicability is not limited to the scheduling phase, but for all phases of fire planning.

CHAPTER I

INTRODUCTION

1.1 Description of the Problem

Throughout history artillery has played an important role in military engagements. Its first use, due to range limitations, was directed against enemy forces as they approached a defender's position. Little prior thought was given to organizing all firing pieces or coordinating these inaccurate, random firings. When a target appeared, one merely aimed the artillery weapon in the general direction of the acquired target and fired it.

As range, accuracy, and effectiveness of artillery improved, more consideration was given to organization and coordination. Plans were developed to better utilize this formidable weapon. For such applications as the seige and reduction of a fortified enemy position, coordination of efforts was found to be quite effective. The artillery was organized to fire on pre-designated targets, such as a strong point or a wall which must be breached. All pieces were fired in unison to achieve an initial surprise and were concentrated on one point to maximize the inherent destructive effect on a single point. Once a hole was punched in the wall and ground troops were advancing, all fires would be shifted to another area on a prearranged signal to support other aspects of the attack.

To be most effective, the earliest artillery fire had to be "observed fire." An observer had to physically see the rounds hitting, such as the wall in the above example, and make appropriate adjustments. During the American Civil War, balloon observers improved artillery accuracy where distant firing was concerned. Aircraft with radios further increased this capability. Now targets could be engaged at maximum ranges of weapons. The study of projectile trajectories was another great advance. Once a target was located and fires adjusted to strike the target, fires could be brought later on that same point without someone observing the fires. Ballistic tests had shown that projectiles of the same weight shot by a fixed amount of propellent in a given direction with the tube elevated the same amount achieved roughly the same range each time.

This concept of accurate, unobserved fires plays an important role in today's military actions. Specifically, most offensive actions are now preceded not just by artillery, but by naval gunfire or air attack or a combination of the three. These attacks, called "preparations," are by definition, "intense prearranged fires delivered in accordance with a time schedule in support of an attack, to disrupt the enemy's command and communications, disorganize his defenses, and neutralize his fire support means," Field Artillery School (24). There may be requirements for two, three, or even more preparations in a given day, depending upon the situation. Herein lies the problem, for one must consider at least the following:

a. Length of time for the preparation

b. Units available to fire

c. Type and capabilities of the units available

d. Ammunition available

e. Location of friendly artillery units

f. Plan of attack

g. Information about the enemy targets to include

  1. location and proximity of friendly forces

  2. size, composition, vulnerability, mobility and recuperability

  3. terrain in which the target is located

h. Commander's guidance

All of these factors must be taken into consideration when an analyst is trying to decide which unit(s) i ($i = 1, 2, \ldots, m$), depending upon their capabilities and expected results, should fire on which target(s) j ($j = 1, 2, \ldots, n$), in what order these targets should be fired on, and at what time t ($t = 0, 1, 2, \ldots, T - 1$, where T represents the total time in minutes that the preparation will last and $t = 0$ is the time the preparation is to start).

If one were to consider, say $m = 6$, $n = 10$, and $T = 10$, and one unit firing only one volley on a given target, this would result in $m \times n \times T$ or $6 \times 10 \times 10 = 600$ possible unit-target-time combination possibilities. Which of those 600 would be the most effective and yield the best results? Using the present methods of unit-target assignment, this decision would be a very difficult task. Certainly one can easily obtain a feasible arrangement, but only by sheer chance might it be the best, or optimal solution.

1.2 Purpose of the Research

The purpose of this research is to determine a more timely and effective means of attacking enemy targets in a manner which will insure optimal utilization of all means available. By "more timely" it is desired that a method be introduced which would greatly reduce the lengthy, time-consuming process presently used to assign units to fire on targets; "effective means" implies utilization of a unit or units located such that their firing on a given target would result in the smallest possible error in the target area; and by "optimal utilization" it is meant that a combination of effects and priorities of all given targets be considered.

1.3 Scope of the Research

The problem formulation can be used by the divisional artillery level of command. The problem formulation also considers 155 mm and 8" howitzer units, and looks at preparation fires. This is a reasonable representation of what might be expected. With a little manipulation, the problem formulation could readily be adjusted to handle different units of different weapon calibers (105 mm, 175 mm, etc.), varying types of preplanned fires (counter-battery, groups or series of targets, final protective fires, etc.), and handle problems for higher levels of command than division artillery level.

1.4 Thesis Layout

Chapter II summarizes the literature search for this thesis. Chapter III contains the model assumptions, symbols and definitions, formulations, an example problem, and partial problem solution. The assumptions at the start of Chapter III establish the overall extent of the problem. In Chapter IV the objective function is developed. In Chapter V, the problem as formulated in Chapter III is divided into two parts, a unit-target assignment problem and a scheduling problem. A complete example problem containing all aspects of the model appears in Chapter VI and is followed by conclusions and recommendations in Chapter VII.

CHAPTER II

LITERATURE REVIEW

A review of available literature indicates that the related area of missile/bomber-assignment problems has received considerable attention. This area is concerned with expensive, highly complex, strategically important weapon systems for which there is a one time deployment prospect. Matlin (16) in his 1970 article reviewed 36 missile-assignment related articles. Various approaches to the artillery-assignment problem were found.

In 1957 Manne (15) looked at this problem in a similar fashion as the personnel-assignment problem. Although the model was probabilistic and nonlinear in nature, he discovered it was possible to devise a linear programming formulation which provided a close approximation to the original problem.

Two 1958 articles are of interest. In the first, denBroeder, et al. (6) approached the assignment problem in a manner which yields an optimal solution by maximizing the expected value of targets destroyed. To accomplish this he assumed that these target values were known. Dobbie's (7) article was similar as he discussed a way to best allocate deterrent systems. For both articles, these systems were missile related.

Lemus and David (14) in 1963 considered a similar problem, that of optimizing strategies of defense. Their efforts were devoted towards obtaining an analytical solution to the problem where one has more than one type of weapon available for assignment to an undefended target complex. Day's (5) 1965 article also considered the nonlinear weapon (missiles/bombers) allocation problem. He developed a target weighting scheme and approached the problem by breaking it down into simpler subproblems.

Wollmer (18) in 1969 developed two algorithms to treat aircraft attacks in a lines-of-communication network. Wollmer's approaches used network flow with arc costs being linear functions of flow or piecewise linear functions of flow.

In 1970 two articles addressed the problem. Passy (17) developed and applied to an example an algorithm, again initially nonlinear, which enabled the problem to be reformulated as a geometric problem or to be transformed into a complementary geometric program. Bracken and McGill (1) formulated a convex programming model for allocating submarine-launched ballistic missiles. Their work developed a nonseparable concave objective function.

Lastly, in 1972 Furman and Greenberg (9) presented a strategy for finding an optimal allocation of weapons to maximize the expected damage of a given collection of targets. Their approach utilized a generalized Lagrange-multiplier method.

All of the above topic areas are related items of theoretical interest for the artilleryman, but do not totally solve his problem. Although similarities exist, none formulated the battlefield unit-target assignment problem; none could assist in setting up a target weighting scheme; and none offered a rapid means of solving the problem.

A second related problem area is in the determining an order of precedence for the targets to be fired. Guidance from U. S. Army Field Manuals and Field Artillery School reference notes are so vague and generalized as to be of little help. How will a target, if untouched or insufficiently fired upon, effect the outcome of a specific engagement? To answer this, one would need to know the target's characteristics (composition, size and shape, vulnerability, mobility, and recuperability), its location and proximity to friendly forces, and a combination of terrain and weather factors. These coupled with the priority of the target are necessary to be able to determine the proper ordering of targets within an array.

To make an algorithm work, therefore, an analyst must be able to determine values for w_j, a weighting factor for target j based upon its relative importance. In review of the literature, target weighting schemes were mentioned by Day (5) and Bracken and McGill (1), but did not go into depth on how to get a military target weight. Literature on utility and measurement theory hold some useful information in solving this problem. Churchman, et al. (3) have an excellent discussion of how to obtain comparative weights for an objective function. They discuss two methods which might be used and through examples show the application of these methods. Although comparisons were made in dollar figures, they explain how one might also work with other scales.

In 1965 Eckenrode (8) compared six methods which might be applied to obtain a relative value of sets of criteria. Those methods were ranking, rating, three versions of paired comparisons, and the method of successive comparisons as previously suggested by Churchman et al. (3). He concluded that ranking is "increasingly more efficient than paired comparisons as the number of items to be judged increases from six to 30."

Lastly, Hull (13) in 1973, considering Eckenrode's and others' findings, explicitly considered those factors which would influence the measurement method chosen in a decision-making situation.

In this thesis a combination of the above measurement theory approaches will be used to obtain comparable objective function, target weights.

CHAPTER III

MATHEMATICAL FORMULATION OF THE PROBLEM

This research is based on a situation which regularly occurs in tactical military operations. The model, based on the normal composition of U. S. mechanized or armored division artillery, was formulated to handle 155 mm and 8" howitzer batteries, the type units regularly assigned. The mission which each battalion will perform will be known. Therefore, it is reasonable to assume that the person or persons responsible for setting up a preparation fire table will know exactly how many units the model will contain.

3.1 Basic Model Assumptions

Inherent in the problem are a number of situational variables which may change with the time or which the individual analyst may change. To constrain the problem to a workable size, the following assumptions were made:

a. The function f_{ij} (effectiveness factor of unit i firing on j, see Section 4 in Chapter IV) can be determined and will be known $\forall$ i, j.

b. Given a specific target j, the individual assessing the situation will be able to assign j a priority weight w_j which conforms to the necessary required standards set forth in Section 3, Chapter IV.

c. The values for U_j (U_j = 1, 2, 3, ..., represents the number of units required to fire on j) and V_j (V_j = 1, 2, 3, ..., is the number of volleys each unit will fire on j) will be known. This information can be obtained from Department of the Army Publication (22), classified CONFIDENTIAL, or from other unclassified documents. For example, a target such as an enemy platoon in the open, designated target number 5, may require that three units fire two volleys each. Thus $U_5 = 3$ and $V_5 = 2$.

d. Units of only one type (155 mm or 8") will be predetermined to fire on a given j. No mix of types will be allowed by the model.

e. The total duration of time T of the preparation will be established.

3.2 Symbols Used and Definitions

Many of the following symbols have already been mentioned. However they are repeated here for convenience in following the problem model formulation.

$i = 1, 2, \ldots, m$	represents the ordered indices of all battery size artillery units considered.
$j = 1, 2, \ldots, n$	represents the indices assigned to the targets.
$t = 0, 1, \ldots, T - 1$	represents the time progression increments in seconds from the time of the

first volley (at t = 0 until one minute before the time T when the last round must be fired.

$d_{ij} = f_{ij}w_j$ where

f_{ij} represents the effectiveness function of unit i firing on target j.

w_j represents the comparative weight or priority of target j.

U_j represents the number of units needed to fire on target j.

V_j represents the number of volleys that each of the U_j units will fire on target j.

$$C_j = \begin{cases} 0 & \text{if 155 mm's are to fire on target j.} \\ 1 & \text{if 8'' units are to fire on target j.} \end{cases}$$

$$x_{ij}^t = \begin{cases} 1 & \text{if unit i fires on target j for the first time at time t.} \\ 0 & \text{otherwise.} \end{cases}$$

It is important to know when i commences firing on j so that it can be determined when i is available to commence firing on other j's.

$$I_j = \begin{cases} \text{set of i's which are permitted to fire on the given} \\ \text{target j.} \end{cases}$$

Note that each I_j will contain either all 155 mm units or all 8" units.

3.3 Problem Formulation

The objective is to match the most appropriate firing units with the highest priority targets first, and to do this in the allotted time for firing the prepoint in. The objective function, then, would be:

$$\text{Max} \sum_i \sum_j \sum_t d_{ij} \; x_{ij}^t \tag{1}$$

(d_{ij} for a given j is greatest when the distance from i to j is least and when that j is the highest priority target, thus a maximization problem is applicable. See Chapter III for development and further explanation of d_{ij}.)

There are eight limiting constraints on the above objective function. Initially, we must insure that unit i can be assigned to fire on target j for the first time no more than once. This is accomplished by:

$$\sum_t x_{ij}^t \leq 1 \quad \forall \; i, j \tag{2}$$

Secondly, once a unit i is chosen to fire on a specific target, that i cannot be again assigned to fire on any other target for that given t. This is accomplished with the following:

$$\sum_j x_{ij}^t \leq 1 \quad \forall \; i, t \tag{3}$$

Next a constraint is needed which will insure that U_j units will fire on target j. The equation

$$\sum_{i \varepsilon I_j} \sum_t x_{ij}^t = U_j \quad \forall\ j \tag{4}$$

will do this.

The results of equations (2), (3), and (4) must be combined in such a manner as to insure that all first fires on target j occur simultaneously. This is accomplished by:

$$\sum_{h \varepsilon I_j} \sum_t x_{hj}^t \left[\sum_{i \varepsilon I_j} x_{ij}^t - U_j \right] = 0 \ \forall\ j \tag{5}$$

The next restriction is to insure that once a unit i has been assigned to fire on target j at time t, no other targets will be assigned to be fired on by unit i until all required volleys, V_j, have been fired. For the 155 mm battery, firing one volley per minute, Department of the Army (20), this is straight forward. If, for example, $V_j = 3$ and $x_{ij}^t = 1$ for a specific i,j, and t, then i cannot be assigned to fire on another target until t + 3. In other words, unit i must not receive another target on which to fire from t to $t + \gamma$ where in this example equals 1 and 2. Generally, $\gamma = 1, 2, \ldots, V_j - 1$. For an 8" battery firing one round every two minutes, Department of the Army (21), $2(V_j - 1)$ time periods are needed plus one extra minute after the last round is fired. Again, if $V_j = 3$ and $x_{ij}^t = 1$, that 8" unit cannot be assigned to fire on an additional target over the period $\gamma = 1$ through $\gamma = 2\ (3 - 1) + 1 = 5$. Thus, for the given i and j, x_{ij}^{t+1} through x_{ij}^{t+5} cannot equal 1. Recalling that $C_j = 0$ for 155 mm units and $C_j = 1$ for 8" units, this constraint takes the form:

$$\sum_{k=1}^{n} x_{ik}^{t} \sum_{j} \sum_{\gamma=1}^{(C_k+1)(V_k-1)\,+\,C_k} x_{ij}^{t+\gamma} \leq 1 - \sum_{j} x_{ij}^{t} \quad \forall i, t \tag{6}$$

Here the $\sum_{k=1}^{n} x_{ik}^{t}$ serves a purpose similar to $\sum_{t} x_{hj}^{t}$ in equation (5). That is, it searches over all j's to find where $x_{ij}^{t} = 1$, and once that value of k is found, it is fixed for the upper limits of γ.

The seventh constraint serves to impose a different, but important, purpose of insuring that the last volley is fired not later than time T - 1. For this constraint we have:

$$\sum_{j} \sum_{\gamma=1}^{(C_j+1)(V_j-1)} x_{ij}^{T-\gamma} \leq 0 \quad \forall \text{ i and } V_j > 1 \tag{7}$$

If $V_j = 1$, that mission could be fired at time T - 1 without violating the time constraint.

The last constraint restricts the problem to integer values.

$$x_{ij}^{t} = 0 \text{ or } 1 \; \forall \; i, j, t \tag{8}$$

The equations (1) through (8) comprise the model, a nonlinear integer programming problem because of equations (5) and (6). The model is:

$$\text{MAX} \quad \sum_i \sum_j \sum_t d_{ij}\, x_{ij}^t \quad \forall\, i,\ j,\ t \tag{1}$$

$$\text{S.T.} \quad \sum_t x_{ij}^t \leq 1 \qquad \forall\ i,\ j \tag{2}$$

$$\sum_j x_{ij}^t \leq 1 \qquad \forall\ i,\ t \tag{3}$$

$$\sum_{i \varepsilon I_j} \sum_t x_{ij}^t = U_j \quad \forall\ j \tag{4}$$

$$\sum_{h \varepsilon I_j} \sum_t x_{hj}^t \left[\sum_{i \varepsilon I_j} x_{ij}^t - U_j \right] = 0 \quad \forall\ j \tag{5}$$

$$\sum_{k=1}^{n} x_{ik}^t \sum_j \sum_{\gamma=1}^{(C_k+1)(V_k-1)+C_k} x_{ij}^{t+\gamma} \leq 1 - \sum_j x_{ij}^t \quad \forall\ i,\ t \tag{6}$$

$$\sum_j \sum_{\gamma=1}^{(C_j+1)(V_j-1)} x_{ij}^{t-\gamma} \leq 0 \quad \forall\, i \text{ and } V_j > 1 \tag{7}$$

$$x_{ij}^t = 0 \text{ or } 1 \quad \forall\ i,\ j,\ t \tag{8}$$

3.4 Example Problem Formulation

To better understand the above model, consider an example problem.

Suppose there are four 155 mm units, two targets, T = 5, V_1 and $V_2 = 3$, U_1 and $U_2 = 2$ (for now no specific values for d_{ij} will be used). The model thus becomes:

$$\text{MAX} \quad d_{11}\, x_{11}^0 + d_{11}\, x_{11}^1 + \dots + d_{42}\, x_{42}^4 \tag{1}$$

$$\text{S.T.} \quad \left.\begin{array}{l} x_{11}^0 + \dots + x_{11}^4 \leq 1 \\ x_{42}^0 + \dots + x_{42}^4 \leq 1 \end{array}\right\} \tag{2}$$

$$\left.\begin{array}{l} x_{11}^{0} + x_{12}^{0} \leq 1 \\ x_{41}^{4} + x_{42}^{4} \leq 1 \end{array}\right\} \quad (3)$$

$$\left.\begin{array}{l} x_{11}^{0} + \ldots + x_{41}^{4} = 2 \\ x_{12}^{0} + \ldots + x_{42}^{4} = 2 \end{array}\right\} \quad (4)$$

$$\left.\begin{array}{l} x_{11}^{0}[x_{11}^{0}+\ldots+x_{41}^{0}-2]+\ldots+x_{41}^{4}[x_{11}^{4}+\ldots+x_{41}^{4}-2]=0 \\ x_{12}^{0}[x_{12}^{0}+\ldots+x_{42}^{0}-2]+\ldots+x_{42}^{4}[x_{12}^{4}+\ldots+x_{42}^{4}-2]=0 \end{array}\right\} \quad (5)$$

$$\left.\begin{array}{l} x_{11}^{0}(x_{11}^{0+1}+\ldots+x_{12}^{0+2})+\ldots+x_{12}^{0}(x_{12}^{0+1}+\ldots+x_{2}^{0+2})\leq 1-(x_{11}^{0}+x_{12}^{0}) \\ x_{41}^{4}(x_{41}^{4+1}+\ldots+x_{42}^{4+2})+\ldots+x_{42}^{4}(x_{42}^{4+1}+\ldots+x_{42}^{4+2})\leq 1-(x_{41}^{4}+x_{42}^{4}) \end{array}\right\} \quad (6)$$

$$\left.\begin{array}{l} x_{11}^{3} + \ldots + x_{12}^{2} \leq 0 \\ x_{41}^{3} + \ldots + x_{42}^{2} \leq 0 \end{array}\right\} \quad (7)$$

$$x_{ij}^{t} = 0 \text{ or } 1 \quad \text{For } i=1, \ldots, 4 \quad j=1, 2 \quad t=0, \ldots, 4 \quad (8)$$

3.5 Example Problem Solution

Using "branch and bound" techniques a solution can be obtained to the above problem formulation. For a review of Branch and Bound methods see Garfinkel and Nemhauser (10).

Recall that there were four 155 mm units, $i = 1, 2, 3, 4$; two targets, $j = 1, 2$; total allowable preparation time $T = 5$; $V_1 = 3$; $V_2 = 3$; $U_1 = 2$; $U_2 = 2$. Consider $W_1 > W_2$; $F_{11} > F_{21} > F_{31} > F_{41}$; and $F_{12} > F_{22} > F_{32} > F_{42}$. From the definition, $x_{ij}^{t} = 0$ otherwise. With these two possible outcomes, one would have $m \times n \times t = 4 \times 2 \times 5 = 40$ levels of our Branch and Bound tree with a maximum of 2^{40} possible combinations. Some of these branches can be ignored as infeasible.

For example, if $x_{11}^{0} = 1$, then $x_{11}^{1} = 1$ is infeasible from equation (3) and $V_1 = 3$. Nevertheless, many other branches remain. Thus, obtaining a solution is a tedious and time consuming, if not virtually impossible, task. Figure 1 shows a partially completed branch and bound tree.

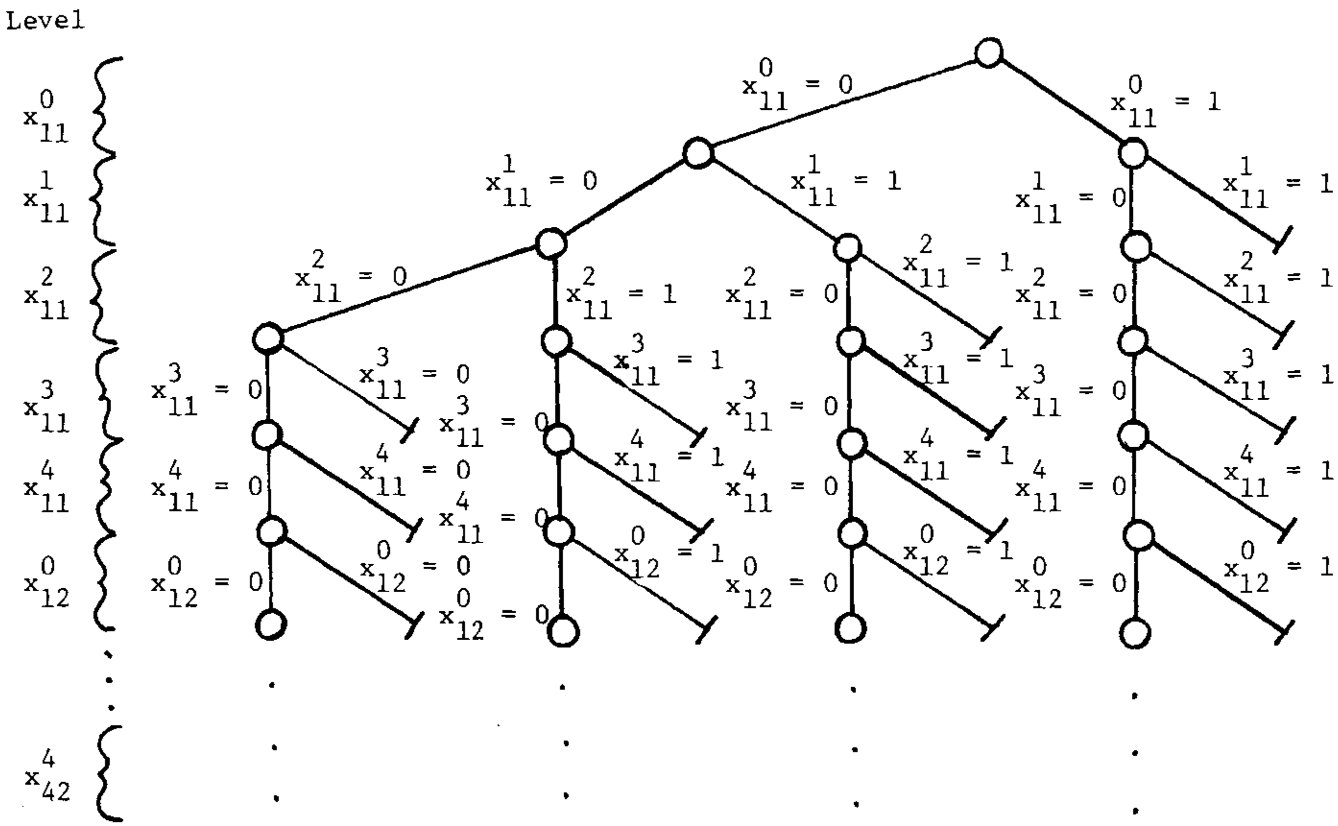


Figure 1. Partial Branch and Bound Solution to the Example Problem

CHAPTER IV

DETERMINATION OF THE OBJECTIVE FUNCTION

As seen in Chapter III, d_{ij}, the coefficient of x^t_{ij}, is actually the product of two factors, w_j or target weight, and f_{ij}, the effectiveness corresponding to the range from unit i to target j. Of the two, w_j is considerably more important. Its value will dictate which target(s) will be considered first. In addition, w_j will dictate, somewhat, the order in which the targets are attacked by fires. The f_{ij} is of a magnitude which will not significantly reduce the importance placed upon the assigned weight. If a target is within range, any unit firing on it can be expected to achieve some degree of results. However, as the range increases, accuracy decreases, thereby reducing overall effectiveness. Therefore, this f_{ij} function was introduced to help differentiate between units within whose range the particular target falls. The development of w_j and f_{ij} follows.

4.1 Problems with a Weighting Scheme

Determination of the target weights, w_j, is critical to the overall problem formulation. As this is a maximization problem, targets with higher priorities would require higher weights than those of lesser priorities.

Consider the size and complexity of the problem. The problem model was formulated to consider 155 mm and 8" artillery batteries and was focused on a division artillery level. Typically in a mechanized or armored division we find nine 155 mm and three 8" batteries of which perhaps only three 155 mm and three 8" batteries would have their fires planned at Division Artillery level. (This assumes that two 155 mm battalions would be in direct support of the two front line infantry brigades, and would therefore plan their own fires, while the third 155 mm and the 8" battalions with their three organic batteries might have either a general support or general support reinforcing mission requiring Division Artillery firing planning.)

The 155 mm howitzer has a sustained rate of fire of one round per minute (FM 6-88) while the 8" howitzer can fire one round every two minutes (FM 6-94). Assuming an average of two volleys required per target, a preparation to last not more than 10 minutes would allow approximately 20 targets for the fire plan.

To assist with assigning of weights to the targets, consider first the four priorities delineated in Department of the Army (19):

1. <u>Priority I</u>. Targets immediately capable of preventing the execution of the plan of action. . .

2. <u>Priority II</u>. Targets capable of immediate serious interference with the execution of the plan of action. . .

3. <u>Priority III</u>. Targets capable of ultimate serious interference with the execution of the plan of action. . .

4. <u>Priority IV</u>. Targets capable of limited interference with the execution of the plan of action. . .

This guidance has been intentionally left vague so as to allow greater flexibility and the interjection of subjective military judgement on the part of the fire planner. Generally speaking, Priority I includes artillery/mortar targets plus forward observers and other means of artillery adjustment, Priority II contains enemy command and control (communications) elements, III consists of the enemy fighting forces, both front line and reserve, and IV contains all of the remaining targets such as truck parks, re-supply points, avenues of approach, etc. However, this additional information alone does not suffice for problem formulation. A literature search yielded little in the way of assistance from the military aspect. Therefore, a combination of measurement theory methods is required. The following represents a suggested approach to this problem.

The very nature of utility or measurement theory is such that assigned weights will vary from one person to the next. For example, a possible target might be listed as a machinegun emplacement. To the platoon leader whose platoon's mission is to secure the hill on which that machinegun is located, the priority rating for this weapon would be very high. However, to the platoon's battalion commander, that machinegun represents a target of much less significance. The battalion commander is much more concerned with the entire battalion's mission, and such things as where the enemy's reserve is located, which for the battalion poses a greater threat.

One must also consider branch differences. To the armor company commander, this machinegun would pose little threat to his 18 tanks when compared with what a machinegun might do to the 100 men in an infantry company. If, however, this machinegun were to be replaced by an anti-tank weapon the target assumes greater priority.

This is a weapon capable of knocking out a tank, but one which is relatively ineffective against spread-out troops. With these two weapons one might expect to find exactly opposite priority ratings when assessed by these two different branch individuals.

These are but two of the many differences which might occur for the analyst. There are target-related differences which must also be considered. Such items of importance are target characteristics (composition, size and shape, vulnerability, mobility and recuperability), location (in regards to proximity of targets to friendly troops where their safety or the enemy threat is concerned), terrain (some weapon systems, for example, tanks in a thickly forrested area, pose less of a threat than if located on a gently rolling plain), and weather (poor visibility might preclude a target from being hit with a preferred airstrike, thus it may become extremely important that it be engaged by artillery fire).

4.2 Determination of Comparable Target Weights

The following is an approach to the problem of determining target weights. It is not proposed that this is the only method which could be used. However, if these procedures are followed, they should yield desirable and usable results to be applied to the objective function. It need only be done once with occasional re-evaluation as experience is gained through usage.

4.2.1 Step 1 - Complete Expected Target List

Draw up as complete a listing of expected targets as possible. This list can be generalized, that is, designate a unit size, (though not specifically by name; i.e., a company, not B Company, 2nd Bn 23rd Infantry). The S2/G2 Intelligence/Order of Battle personnel should be able to provide this information.

Example:

1. Troop units: squad, platoon, mech rifle company, tank platoon, tank company
2. Headquarters elements: battalion, regiment, or divisional level
3. Crew served weapons: machineguns, anti-tank weapons
4. Artillery/mortars: 82 mm, 120 mm, 160 mm mortars, 122 mm, 152 mm howitzers, free flight rockets, 140 mm, 200 mm multiple rocket launchers, forward observers
5. Miscellaneous: supply point, truck park, POL dump

4.2.2 Step 2 - Group the Targets

The above way the targets are listed, Step 2 follows quite easily. For the priorities mentioned earlier, Priority I equals to Number 4 above, II is Number 2, II contains Numbers 1 and 3, and IV, Number 5.

4.2.3 Step 3 - Rank Order the Targets

Within each group, the importance of each type of target must be considered. With pair-wise comparisons, ask which target, if untouched or insufficiently fired upon, would have the greatest effect on friendly forces achieving their mission. Some are rather straight forward. Obviously, a company could do more harm to friendly forces than a platoon. But, when considering a machinegun and a squad, for example, there may be no clear cut differentiation. Neither would have priority over the other. Thus, as a possible result, one might have the following partially depicted order:

Free flight Rocket $\supseteq$ 152 mm howitzer battery $\supseteq$. . . $\supseteq$ Regimental Headquarters $\supseteq$. . . $\supseteq$ Tank Company $\supseteq$ Mechanized rifle company $\supseteq$. . . Rifle squad $\supseteq$. . . $\supseteq$ Truck park where the $\supseteq$ means "of higher or equal priority to."

4.2.4 Deriving Comparable Weights

The determination of the weights for every target is more involved and not easily developed. The following represents a scenario of the approach to be used. As an analogy, one might ask, "Which is preferred, a dish of ice cream or a piece of cake?" This may be difficult to answer. But one might come up with an answer if asked, "Which is preferred, two dishes of ice cream or a piece of cake?" or "three dishes of ice cream?" At some point the feeling of indifference will be replaced with one of preference. This is the general idea of the approach we will take here.

When considering target comparisons, many similarities exist which will make starting comparisons easier. For example, a platoon is typically composed of a fixed number of squads, or a tank company contains a number of tank platoons. Consider, then, as a basic building block, arbitrarily assigning a weight of a target described as a squad of infantry equal to 1. Then, each target can be compared to that squad taking into consideration previously mentioned significant target features, plus the commander's guidance.

For example, if one is assessing the weight of a platoon containing three squads, the question asked would be, "Which target, if unfired upon or insufficiently hit, would create a greater obstacle to the advancing friendly forces--three separate, independently acting squads or one coordinated, singly functioning platoon?" "Four separate squads or one platoon?" "Five similar squads or one platoon?" At some point we would find a situation for which we are indifferent between the two choices. If the analyst was rather indifferent between four squads and one platoon, but preferred one platoon to three squads and preferred five squads to one platoon, then the weight of that platoon has been determined to be four (or equal to four squads whose weight was assigned as one). The weight of this platoon may now be used to determine weights of larger size units such as a company in much the same manner.

When considering targets such as a headquarters or a communications center, an important factor to remember is that one is actually weighting the importance placed upon the command and control element of many smaller units. For example, if a battalion headquarters is knocked out, three subordinate companies suddenly find themselves left to act without coordination or control thereby reducing their effectiveness.

Once weights have been assigned, return to the ordered sequence. If an order as listed in Step 3 has been determined where the target is listed as another target, but its resulting comparative weight is less than the other, then consider either making appropriate weight adjustments or perhaps change the Step 3 order. Does the new order make sense? Are the four priority groups concept still being adhered to?

4.3 Determination of w_j

Once comparable target weights have been established, we are now ready to apply them to actual targets as this information is received. This will be a relatively simple, quick process. The following represents the procedure one could follow. Upon receipt of a target, such as target BZ 8012 (according to the target numbering system set forth in Department of the Army (19), the analyst would assign the target the first free j number. For this example, let j = 8. An 8000 target is an artillery type. One would then look in the comparative weighting list for the weight of that particular type of unit.

Two things might happen. First of all, perhaps this target was not listed. Then, by simply locating where this target would fit into the ordered listing, compare it to the target considered to be more critical and less critical. This new target weight should then fall somewhere in between or equal to one of the other two targets which bracket it. This target can then be added to the list for future reference.

A second problem which might arise is when one feels a particular weight does not necessarily show the true value of the target. Perhaps a weight of six has been given to a rifle platoon, but this specific platoon is well dug-in on a principal objective. This would elevate the importance of firing on that target and it should be adjusted accordingly using the steps outlined earlier. For another example, a tank company may have been assessed a value of 26. However, this particular company's location places it on a critical flank thereby jeopardizing a friendly force's penetration. An upward adjustment could therefore be utilized to increase the likelihood of his target's being scheduled for firing early in the schedule. A third example considers the conventional vs. nuclear situation. If the threat of nuclear escalation is imminent, highest priority must be given to all enemy nuclear-capable units or means. One might introduce a fifth priority group, Priority 0, signifying top priority. These units should be reassessed using the steps previously mentioned. Included in this grouping would be all nuclear delivery means (rockets and artillery), their fire control elements, and the nuclear weapons themselves (identified in convoys or in special ammunition supply points).

Once w_j, or w_8 in this example, is assigned, obtain the values for U_8 and V_8, record the pertinent target information such as location and altitude, and the process is completed, ready to handle an additional target.

4.4 Determination of f_{ij}

As mentioned in Chapter III, f_{ij} represents a function which will help decide which unit i is best, in terms of location, to fire on target j. As a basic consideration, as range from i to j increases, so does range probable error, PE_R. PE_R is defined as "a value which, when added to and subtracted from the expected range, will produce an interval, along the line of fire, that should contain 50% of the rounds fired. Variations in muzzle velocity, in angle of departure, and in total drag during flight all contribute to the probable error in range to impact, Department of the Army (23).

To be able to get comparable figures, first consider Table 1. Represented are the preferred propellant charge or charges (charges with the smallest PE_R and thus least range) and the corresponding PE_R for the 155 mm and 8" howitzer in increments of one kilometer.

The values from Table 1 were plotted (see Figure 2) and a linear curve approximating the data was drawn. The equations for the lines yielded the following:

$$PE_R = \begin{cases} 2 + 2.5R_{ij} & \text{for 155 mm and } R \leq 14.5k \\ 3 + 3R_{ij} & \text{for 155 mm and } 14.5 < R \leq 18k \\ 2 + 2R_{ij} & \text{for 8' and } R \leq 16.5k \\ \infty & \text{otherwise} \end{cases}$$

Note: R is in kilometers and measures the range from unit i to target j.

It is expected that the greatest value for w_j might be between 100 and 200. It is not the intent for f_{ij} to be such that a target with a very high w_j would be reduced to such a degree that one of considerably lesser value would be preferred.

Therefore, the following f_{ij} values are suggested. These values will enable the analyst to determine which unit can be expected to achieve the best results on the target. However, the total effect on the reduction of w_j will not exceed 10%, that is, $f_{ij}w_j > .9w_j \ \forall$ i, j, or $.9 < f_{ij} < 1$. Letting $f_{ij} = 1 - \frac{PE_{Rij}}{600}$ (where PE_{Rij} is the specific PE_R for i firing on j) will satisfy this requirement. Thus, for example, if $PE_{Rij} = 58$, or its absolute maximum (for 155 mm and a range of 18k's), if $f_{ij} = 1 - \frac{58}{600} > .9$.

PE_{Rij} can be readily found from an applicable firing table, or from Figure 2. Should computerization of the problem be desired, let

$$S_{ij} = \begin{cases} 0 \text{ if } R_{ij} \le 14.5k \\ 1 \text{ if } R_{ij} > 14.5k \end{cases} .$$

(Note: this only applies where $c_j = 0$.) Then

$$PE_{Rij} = \begin{cases} (1-c_j)[(2+2.5R_{ij})(1-S_{ij}) + (3+3R_{ij})S_{ij}] + c_j(2+2R_{ij}) \\ \text{if the target is in range and some large number} \\ (\gg 600) \text{ if the target is out of range} \end{cases}$$

Table 1. PE_R for 155 mm and 8" Howitzers

Range in kilometers	155 mm Preferred charge/PE_R	8" Preferred charge/PE_R
1	3G or 4G/ 4	6 or 7/ 3
2	4G/ 6	6 or 7/ 6
3	4G or 5G/ 9	6/ 8
4	5G/10	6/10
5	5G/12	4 or 5/12
6	5G/15	4 or 5/14
7	5G/18	5 or 6/16
8	5G/21	5/17
9	6W/24	6/19
10	6W or 7W/26	6/20
11	6W or 7W/28	6/21
12	7W/30	6/22
13	7W/32	6/24
14	7W/34	7/26
15	8 /49	7/27
16	8 /51	7/27
17	8 /54	
18	8 /58	

Note: G is for green bag propellent and W is for white bag.

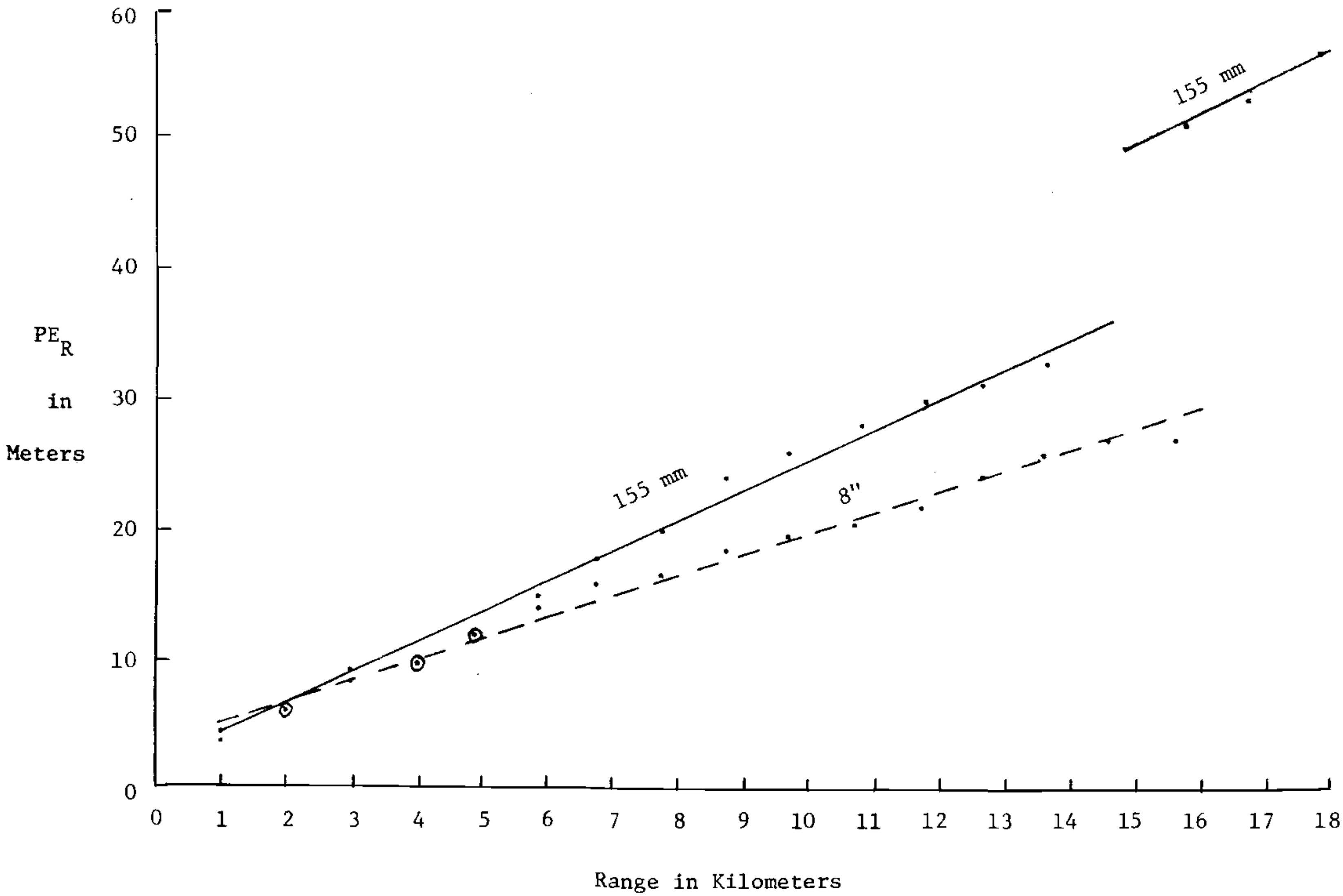


Figure 2. Plot of PE_R for 155 mm and 8" Howitzers

CHAPTER V

PROBLEM SOLUTION

In Chapter III it was indicated that obtaining a solution to the nonlinear integer problem would be difficult. With a better idea of what the objective function represents we are now ready to look for a method of solution.

The problem appears to be one which could be separated into two parts. One part would be concerned with determining which unit or units should fire on which target or targets. Once it is known which units are firing on the various targets, all that remains is to schedule these units.

5.1 Unit-Target Assignment Problem

To transform the original problem formulation into a form easier to work with, it is necessary to change the form of a few of the equations. Consider equation (4). If we multiply both sides by V_j the result is

$$V_j \sum_{i \varepsilon I_j} \sum_t x_{ij}^t = V_j U_j$$

and rearranging some terms on the left we get

$$\sum_{i\epsilon I_j} (\sum_t V_j \; x_{ij}^t) = V_j U_j .$$

Substituting

$$y_{ij} = \sum_t V_j \; x_{ij}^t$$

yields

$$\sum_{i\epsilon I_j} y_{ij} = V_j U_j \quad \forall \; j \tag{9}$$

Here y_{ij} represents the number of volleys that unit i fires on target j. By summing over all i's it is assured that target j will be fired on by the desired number of units, U_j.

We still need to insure that each unit is not assigned to fire on more targets than it can handle between time t = 0 and t = T - 1. From equation (2) we know that $\sum_t x_{ij}^t = 1$ if unit i ever fires on target j. Thus $\sum_j (C_j + 1) \; V_j \sum_t x_{ij}^t - C_j$ will be the total number of time periods unit i will be firing on all targets. Since these cannot overlap we must have

$$\sum_j \sum_t (C_j + 1) \; V_j \; x_{ij}^t - C_j \leq T$$

Note: the "$-C_j$" in this equation takes into account the fact that the 8" would fire its last volley at T - 1 if T is an odd number and at T - 2 if T is even.)

Again, making the substitution of y_{ij} and rearranging the terms we get:

$$\sum_j y_{ij} \leq \frac{T+C_j}{C_j+1}$$

Since y_{ij} must be an integer this may be rewritten as

$$\sum_j y_{ij} \leq R_i \quad \forall\ i \tag{10}$$

where

$$R_i = \begin{cases} T \\ \frac{T+1}{2} \end{cases}$$

for 155 mm units integer part for 8" units. Also, since

$$\sum_t V_j \ x_{ij}^t = \begin{cases} 0 \\ V_j \end{cases}$$

then

$$0 \leq y_{ij} \leq V_j \quad \forall\ i,\ j \tag{11}$$

and

$$y_{ij} = 0 \text{ or } V_j \quad \forall \; i, j \tag{12}$$

Thus, the necessary constraint equations are developed. Making the y_{ij} substitution into the objective function completes the transformation with

$$\text{MAX} \sum_i \sum_j \frac{d_{ij}}{V_j} y_{ij} \quad \forall \; i, j \tag{13}$$

A check for feasibility can be accomplished rather easily. If one 155 mm unit can fire T volleys at a maximum, then M_j 155 mm units can fire $M_j T$ volleys. If we sum over every j the number of required units times the number of required volleys, the summed product cannot exceed $M_j T$ to be feasible. To include our use of C_j to be able to consider 8" firing one volley every two minutes the question of feasibility can be represented by the question: if

$$M_j \; (T + C_j) > \sum_j \; (C_j + 1) \; V_j U_j \quad \forall \, i \, \varepsilon \; I_j \tag{14}$$

then, our solution will be feasible.

To summarize, equations (9) through (12) are as follows:

$$\text{MAX} \quad \sum_i \sum_j \frac{d_{ij}}{V_j} y_{ij} \quad \forall \; i, j \tag{13}$$

$$\text{S.T.} \quad \sum_{i \varepsilon I_j} y_{ij} = V_j U_j \quad \forall \; j \tag{9}$$

$$\sum_j y_{ij} \leq R_i \quad \forall\, i \tag{10}$$

$$0 \leq y_{ij} \leq V_j \quad \forall\, i, j \tag{11}$$

$$y_{ij} = 0 \text{ or } V_j \quad \forall\, i, j \tag{12}$$

5.2 Example of the Unit-Target Assignment Formulation

Let us see how equations (9) to (13) have simplified original example problem. Recall from Section 4 of Chapter II we had four units, two targets, $T = 5$, V_1 and $V_2 = 3$, U_1 and $U_2 = 2$ (again we will not consider values for α_{ij}).

We first check for feasibility. Substituting into equation (13) we get

$$4(5) > (3)(2) + (3)(2)$$

Yes, $20 > 12$ and therefore we can get a feasible solution.

Substituting our values into equations (9) to (13) and we get:

$$\text{MAX } \frac{d_{11}}{3} y_{11} + \frac{d_{12}}{3} y_{12} + \cdots + \frac{d_{42}}{3} y_{42}$$

$$\text{S.T.} \quad \left.\begin{aligned} y_{11} + y_{21} + y_{31} + y_{41} &= (3)(2) \\ y_{12} + y_{22} + y_{32} + y_{42} &= (3)(2) \end{aligned}\right\} \text{from equation (9)}$$

$$\left.\begin{aligned} y_{11} + y_{12} &\leq 5 \\ y_{21} + y_{22} &\leq 5 \\ y_{31} + y_{32} &\leq 5 \\ y_{41} + y_{42} &\leq 5 \end{aligned}\right\} \text{from equation (10)}$$

$$\left.\begin{array}{l} 0 \leq y_{i1} \leq 3 \quad \forall\, i \\ 0 \leq y_{i2} \leq 3 \quad \forall\, i \end{array}\right\} \text{from equation (11)}$$

$$\left.\begin{array}{l} y_{i1} = 0 \text{ or } 3 \\ y_{i2} = 0 \text{ or } 3 \end{array}\right\} \text{from equation (12)}$$

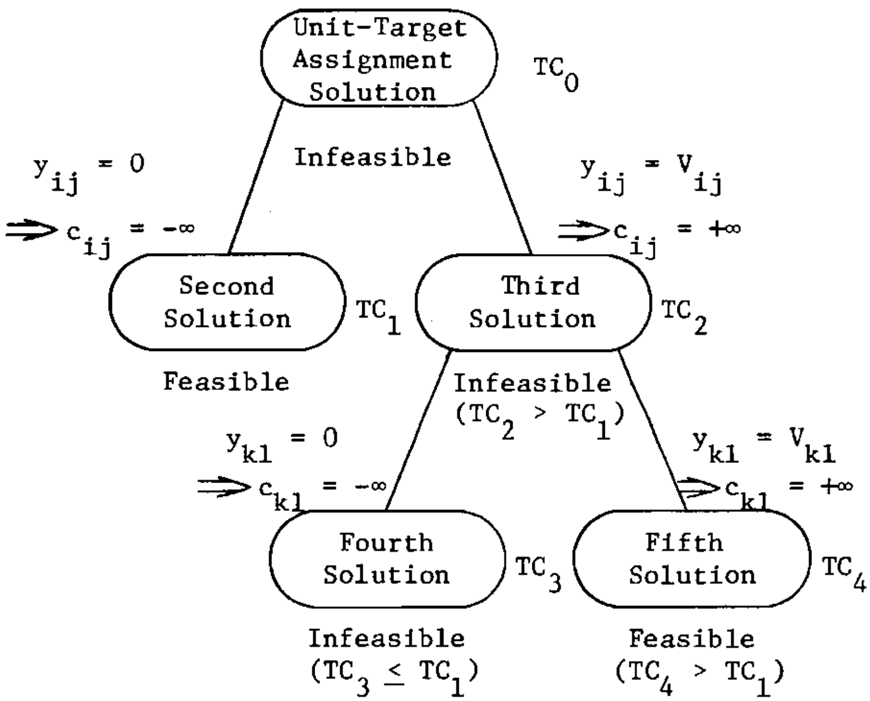
Ignoring constraints (12) for the moment, the above problem can be solved in a number of ways. One way is to solve the problem as a linear program.

Another approach to solving the problem is to use a variation of a transportation problem which allows for upper limits on the variables. Such a method is called a "capacitated transportation" problem. Ways of solving this type of problem are explained in Hadley (11), and are summarized in Appendix A. The example used in Chapter V was solved using this method. Then if $y_{ij} \neq 0$ or V_j Branch and Bound could be applied to satisfy (12).

As the Branch and Bound technique is an important concept in this problem, it is important to understand its application. Suppose that the capacitated transportation method yielded an optimum solution and y_{ij} is known for every i and j. The transportation method will insure that these values for y_{ij} satisfy equation (11), that is, $0 \leq y_{ij} \leq V_j \;\forall\; i,j$. However, it may not be the case that $y_{ij} = 0$ or V_{ij} as required by equation (12) of the model. If y_{ij} equals 0 or V_j, the "best" unit-target solution may be used directly in the scheduling process. If, however, one or more y_{ij}'s $\neq 0$ or V_j, the Branch and Bound approach can be used to obtain this required result.

It is this situation which needs further explanation.

In general terms, then, associated with the optimal y_{ij}'s would be a total cost TC_0. Since this is a maximization problem, TC_0 represents the upper bound on the objective function. To handle the situation where some $y_{ij} \neq 0$ or V_{ij} the unit-target assignment must be tightened for the specific y_{ij}. We accomplish this by separating the original problem into two problems, one with $y_{ij} = 0$ and one with $y_{ij} = V_j$. Schematically this might appear as follows:



In this case the fifth solution obtained in the branch and bound method is the "best solution" and would be utilized in the scheduling phase. If we were unable to schedule the fifth unit-target assignment solution we would then return to the branch and bound method to obtain the second "best solution" for scheduling purposes. Note that this may require further branching in the fifth solution. Further discussion of the branch and bound process for this problem can be found in Appendix A.

5.3 Scheduling Problem with Example

From the unit-target assignment problem we will have an optimum solution indicating which targets unit i will fire on and the number of volleys unit i will fire. Given a set of y_{ij}'s which solve the assignment problem, we must schedule the volleys on the targets within the time limits $t = 0, 1, \ldots, T - 1$ in such a way that:

a. fires on a given j commence simultaneously when more than one i is involved

b. fires are continuous (uninterrupted) on a given j

c. fires on highest priority targets are started as early as possible

d. as many units as possible should start firing at $t = 0$ to capitalize on the initial shock and surprise effect

e. as many units as possible fire their last volley at $t = T - 1$ to allow for a smoother transition between the cessation of artillery fires and commencement of infantry, armor, or air delivered fires, or a combination of the three fires.

To accomplish the above, consider the following as a suggestion, but certainly not an only approach to handling this problem. From the unit-target assignment problem result, make two groups of all j's to be fired on. One group will be composed of 155 mm targets, the other of 8". Arrange these in decreasing priority order according to their assigned weights in both groups. For example, we might get the result for 155 mm units:

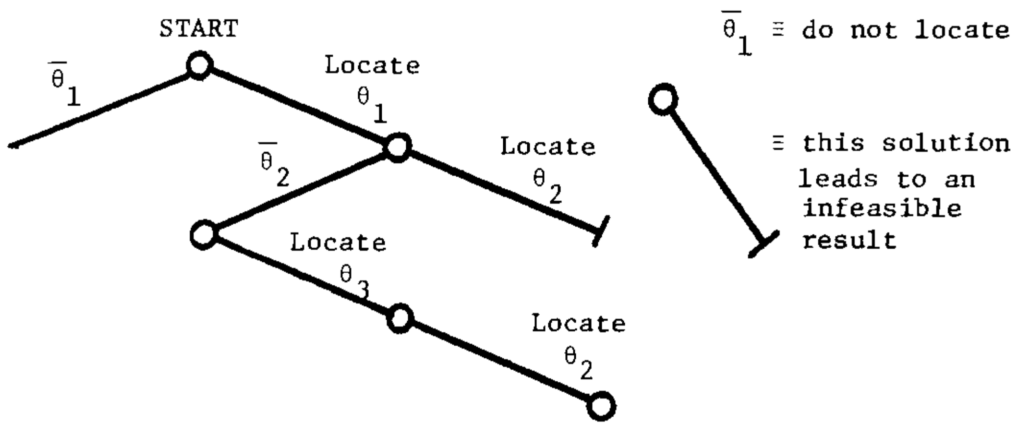
$$w_8 \geq w_4 \geq w_{12} \geq w_2 \geq w_3 \geq w_7$$

(Note, the total number of j's to be fired on as a result of the unit-target assignment problem not necessarily equals n, the total number of targets identified, but rather equals r where $r \leq n$.)

Next, define θ_1 to be the set of i's (initially consider only the 155 mm units, later repeated for 8" units), which must fire on the target with the highest priority (largest w_j), θ_2 the next highest, and so forth through θ_r, the units to fire on the lowest priority target. As in the above example, θ_1 would represent the set of units to fire on target 8 (since $w_8 \geq$ all other w's) and might be $\theta_1 = \{1, 4, 5\}$, determined from the result of the unit-target assignment problem which, for this example, must have been:

$$y_{18} = y_{48} = y_{58} = 1$$

We now have an order in which to begin scheduling the units and targets. Starting at t = 0 we locate θ_1 units which will occupy the spaces for those units through $C_j V_{\theta_1}$ spaces where V_{θ_1} represents the number of volleys associated with the θ_1 units. We next locate θ_2, and so on through θ_r. If at this time we find that a volley is to be fired beyond time T - 1, we must consider relaxing our "highest weighted target fired first" consideration by changing the scheduling order. Suppose, for example, we had three targets and found that the order $\theta_1\ \theta_2\ \theta_3$ did not work. When we tried $\theta_1\ \theta_3\ \theta_2$ order our time limit was not exceeded, thus we had our solution. This process would appear graphically as a tree of solutions. A tree of the above example would appear as follows:

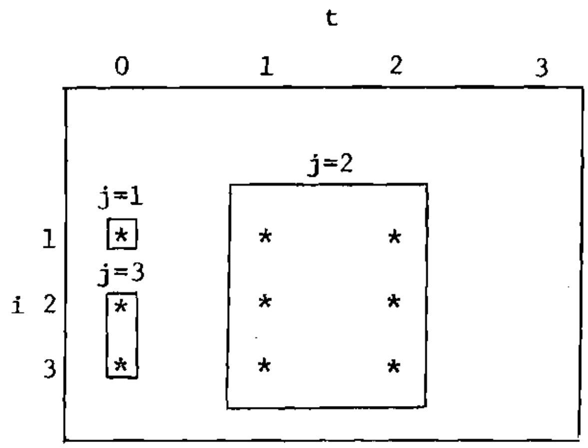


Numerically the above might appear as follows:

$\theta_1 = \{1\}$ $\quad \theta_2 = \{1, 2, 3\}$ $\quad \theta_3 = \{2, 3\}$ $\quad T = 3$

$V_{\theta_1} = 1$ $\quad V_{\theta_2} = 2$ $\quad V_{\theta_3} = 1$

Using the $\theta_1\ \theta_2\ \theta_3$ order we get the following schedule of fires



* represents a volley to be fired.

$\boxed{*}$ encloses a complete fire mission.

If one started sliding θ_3 from the left one would find that it can fit. Thus we have a feasible solution.

5.4 Second Best Solution

A question is raised, "What does one do if the optimal y_{ij}'s from the unit-target assignment problem cannot be scheduled such that all firing is completed by time T - 1?" Consider the following example of this:

$i = 1, 2, 3, 4 \qquad j = 1, 2, 3, 4 \qquad T = 3$

$U_j = 2, 1, 2, 2 \qquad V_j = 2, 2, 1, 1 \qquad d_{i1} > d_{i2} > d_{i3} > d_{i4} \; \forall \; i$

The optimal solution to the unit-target assignment problem using the capacitated transportation problem solution yielded:

		j					
		1	2	3	4	D	
i	1	* $\boxed{2}$	$\boxed{2}$	0 $\boxed{1}$	1 $\boxed{1}$	$\boxed{2}$	3
	2	$\boxed{2}$	$\boxed{2}$	* $\boxed{1}$	1 $\boxed{1}$	1 $\boxed{2}$	3
	3	2 $\boxed{2}$	$\boxed{2}$	1 $\boxed{1}$	$\boxed{1}$	$\boxed{2}$	3
	4	$\boxed{2}$	2 $\boxed{2}$	$\boxed{-1}$ $\boxed{1}$	$\boxed{-3}$ $\boxed{1}$	1 $\boxed{2}$	3
		4	2	2	2	2	

Thus:

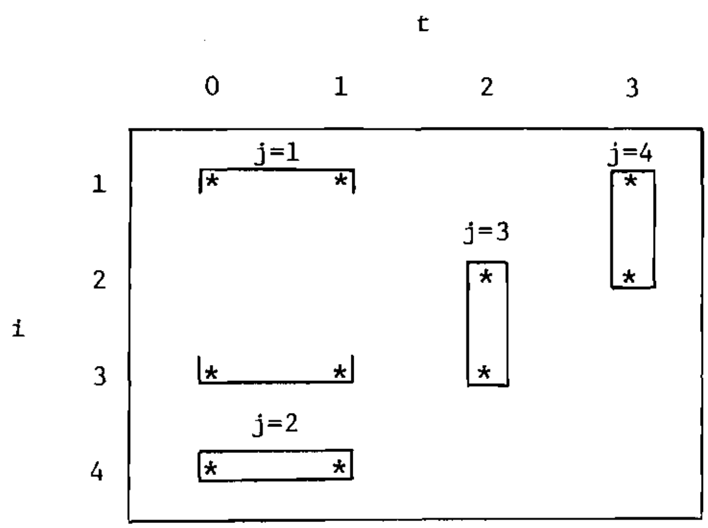
$\theta_1 = \{1,3\}$

$\theta_2 = \{4\}$

$\theta_3 = \{2,3\}$

$\theta_4 = \{1,2\}$

Scheduling these θ's in numerical order yields:



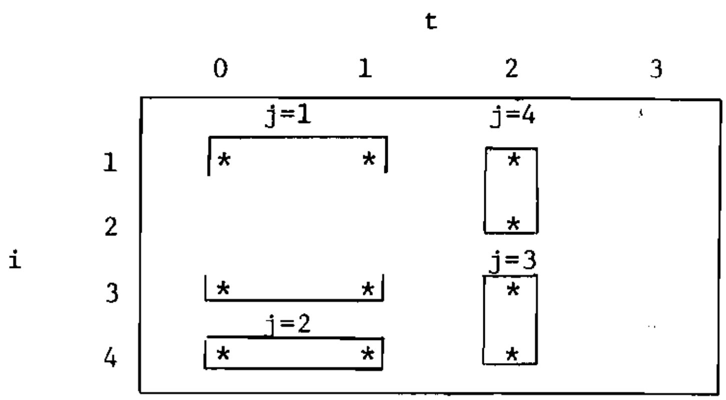
This solution and every other combination yields infeasible solutions, i.e., no schedule containing the optimal y_{ij}'s allows completion of firing by $T - 1 = 2$. A second best solution, therefore, is required.

In the above schedule, the problem obviously stems from the fact that unit 2 is to fire on targets 3 and 4 with two different units. A simple solution to this would be either to change y_{23} to 0 (instead of 1) and set $y_{43} = 1$ or let $y_{24} = 0$ and $y_{44} = 1$. But how can this be done algorithmically?

The answer is discussed by Hadley (11). With the final tableau of the capacitated transportation problem we have $C_j - Z_j$ values for all non-basic cells. Look at the value in cell (4,3) and (4,4). As neither of these are at their upper limit, the cell with the least negative value of the two should be selected to enter the basis.

For this example, since $C_{43} - Z_{43} = -1$ is the least negative, we can achieve this desired result by changing d_{43} such that $d_{43} > d_{13}$ or d_{23} or d_{33}. This change would result in $C_{43} - Z_{43} > 0$, thus being a candidate to enter the basis. Now we go through the usual process to obtain a new, next best optimal solution which should enable one to get a feasible schedule. If not, repeat the above procedure until a feasible schedule is obtained.

In our example, the above change would yield the following:



which is now the optimal, feasible solution. See also section 3 in Appendix A for further discussion of this procedure.

5.5 Feasibility and Optimality of the Final Solution

The final question one might ask is, since we manipulated the equations of Chapter II to reach those shown in this chapter, are these results also feasible and optimal to the original problem?

First consider feasibility. Clearly we have:

Lemma 1

For any feasible solution to the unit-target assignment model which can be scheduled, there is a corresponding feasible solution to the original problem with an equal objective function value.

Proof

We have a feasible solution to the unit-target assignment and scheduling problems.

$\Rightarrow$Target j will be fired upon with the proper number of volleys coming from the desired number of simultaneously firing units. This satisfied equations 6, 5, and 4.

In addition, the scheduling requirements for feasibility $\Rightarrow$no firing will be interrupted until completed nor will firing extend beyond the time limit thereby satisfying equations 3 and 7.

$\Rightarrow$Target j will be fired upon for the first time once and only once by U_j i's during the given time frame, thus $x_{ij}^t = 1$ for some value of i, j and t.

Now, from the schedule, if $y_{ij} = 0 \Rightarrow x_{ij}^t = 0 \quad \forall t$.

However, if $y_{ij} = V_{ij}$, then the schedule must be checked to determine that time t_0 when unit i fires on target j for the first time. Given t_0 we let $x_{ij}^{t_0} = 1$. This satisfies equations 2 and 8.

Finally, the decomposition model resulted from the substitution of y_{ij} for $\sum_t V_j\ x_{ij}^t$. For some value of i and j such that $\frac{d_{ij}}{V_j} y_{ij} \neq 0$,

$\Rightarrow y_{ij} \neq 0, \therefore y_{ij} = V_{ij}$

$\Rightarrow x_{ij}^t \neq 0, \therefore$ for some time t_0, $x_{ij}^{t_0} = 1$

$$\Rightarrow \frac{d_{ij}}{V_j} y_{ij} = \frac{d_{ij}}{V_j} \sum_t V_j\ x_{ij}^t = d_{ij} \sum_t x_{ij}^t$$

$$\Rightarrow \frac{d_{ij}}{V_j} y_{ij} = \frac{d_{ij}}{V_j} \sum_t V_j\ x_{ij}^t = d_{ij} \sum_t x_{ij}^t$$

But $\sum_t x_{ij}^t = 1$ for only one value of t, namely t_0, the time at which unit i fires on target j for the first time. This holds true $\forall$ i, j, therefore the value of the objective function for both the original model and the decomposition model are equal.

Conversely, we have:

Lemma 2

For any feasible solution to the original problem there is a corresponding feasible solution to the unit-target assignment model which can be scheduled and has an equal objective function value.

Proof

i) If $\sum_t \overline{x}_{ij}^t = 1$, then let $\overline{y}_{ij} = V_j$.

If $\sum_t \overline{x}_{ij}^t = 0$, then let $\overline{y}_{ij} = 0$.

This is feasible to the unit-target assignment problem.

$$\sum_i \sum_j \frac{d_{ij}}{V_j} \bar{y}_{ij} = \sum_i \sum_j \frac{d_{ij}}{V_j} (\sum_t V_j \bar{x}^t_{ij}) = \sum_i \sum_j \sum_t d_{ij} \bar{x}^t_{ij}$$

Thus, the objective values of the two are the same.

Theorem

The final solution produced by the above decomposition procedure is optimal for the original problem.

Proof

By Lemmas 1 and 2 the feasible sets of the decomposition procedure and the original problem coincide. Moreover, objective function values for corresponding solutions are equal.

The simple example shown in Section 4 of this chapter can be used to illustrate the above Theorem and Lemmas. In that example the best results of the transportation problem solution lead to an infeasible scheduling result. These results also violate equation (7) of the original model since volleys must be fired by two units beyond time $t = T - 1$. When the second best solution was obtained, its scheduling was now found to be feasible.

From this solution, then, $y_{11} = 2 = y_{31}$; $y_{14} = 1 = y_{24}$; $y_{33} = 1 = y_{43}$; and $y_{42} = 2$. All other $y_{ij} = 0$.

The objective function is maximized with

$$\frac{d_{11}}{2} y_{11} + \frac{d_{14}}{1} y_{14} + \frac{d_{24}}{1} y_{24} + \frac{d_{31}}{2} y_{31} + \frac{d_{33}}{1} y_{33} + \frac{d_{42}}{2} y_{42} + \frac{d_{43}}{1} y_{43}$$

$$= d_{11} + d_{14} + d_{24} + d_{31} + d_{33} + d_{42} + d_{43}$$

When these same results are translated into the original model variables, one obtains

$$x_{11}^{0} = x_{14}^{2} = x_{31}^{0} = x_{33}^{2} = x_{42}^{0} = x_{43}^{2} = 1$$

and $x_{ij}^{t} = 0$ for all other values of i, j, t. Thus, equation 8 is satisfied. The remaining constraint equations would then be:

$$\left.\sum_t x_{11}^t \le 1,\ \sum_t x_{12}^t \le 1,\ \ldots,\ \sum_t x_{44}^t \le 1 \right\}\text{Equation 2}$$

$$\left.\sum_j x_{1j}^0 \le 1,\ \sum_j x_{1j}^1 \le 1,\ \ldots,\ \sum_j x_{4j}^3 \le 1 \right\}\text{Equation 3}$$

$$\left.\begin{array}{l} x_{11}^0 + x_{31}^0 = 1 + 1 = U_1 = 2 \\ x_{42}^0 = 1 = U_2 = 1 \\ x_{33}^2 + x_{43}^2 = 1 + 1 = U_3 = 2 \\ x_{14}^2 + x_{24}^2 = 1 + 1 = U_4 = 2 \end{array}\right\}\text{Equation 4}$$

$$\left.\begin{array}{l} x_{11}^0[x_{11}^0+x_{31}^0-U_1]+x_{31}^0[x_{11}^0+x_{31}^0-U_1]=1[1+1-2]+1[1+1-2]=0 \\ x_{42}^0[x_{42}^0-U_2] = 1[1-1]=0 \\ x_{33}^2[x_{33}^2+x_{43}^2-U_3]+x_{43}^2[x_{33}^2+x_{43}^2-U_3]=1[1+1-2]+1[1+1-2]=0 \\ x_{14}^2[x_{14}^2+x_{24}^2-U_4]+x_{24}^2[x_{14}^2+x_{24}^2-U_4]=1[1+1-2]+1[1+1-2]=0 \end{array}\right\}\text{Equation 5}$$

$$\left.\begin{array}{l} x_{11}^0(x_{11}^1)+\ldots+x_{14}^0(x_{14}^1)=0+\ldots+0\le 1-(x_{11}^0+\ldots+x_{14}^0)=1-1=0 \\ \vdots \\ x_{41}^2(x_{41}^3)+\ldots+x_{44}^2(x_{44}^3)=0+\ldots+0\le 1-(x_{41}^2+\ldots+x_{44}^2)=1-1=0 \end{array}\right\}\text{Equation 6}$$

$$\left.\begin{array}{l} x_{11}^{2} + x_{11}^{1} \leq 0 \\ x_{31}^{2} + x_{31}^{1} \leq 0 \\ x_{42}^{2} + x_{42}^{1} \leq 0 \end{array}\right\} \text{Equation 7}$$

Since all of the above constraint equations are satisfied, only the objective function

$$d_{11}x^{0}_{11} + d_{31}x_{31}^{0} + d_{42}x_{42}^{0} + d_{33}x_{33}^{2} + d_{43}x_{43}^{2} + d_{14}x_{14}^{2} + d_{24}x_{24}^{2}$$
$$= d_{11} + d_{31} + d_{42} + d_{33} + d_{43} + d_{14} + d_{24}$$

remains to be considered. This value is exactly the same value as that which resulted from the decomposition procedure.

CHAPTER VI

COMPLETE EXAMPLE PROBLEM

To get a better idea how all of the pieces fit together, an example problem is in order. Due to time and space limitations, not every combination of possible situations can be considered. Only a representative sampling is presented here.

There are basically three components of this example problem. The first is a running scenario of the situation. This scenario is greatly simplified and does not contain the usual details which would normally be contained in an operation order, but is rather reduced for simplicity. The second component describes the actions or required activities of the individual required to develop the division artillery level fire support plan. The third component consists of additional information used to assist in following the activities. They include tables and other data which normally would be used only internally by a computerized system.

6.1 The Situation

We are with the S-3 (Operations Section) of the 8th Infantry Division Artillery somewhere in Germany. Our Division, along with the 2nd Armored Division on our left and the 4th Infantry Division on our right will commence the attack four hours from now to secure certain designated objectives. The 8th Infantry Division Commanding

General has decided that the attack will be with two brigades, the 1st Brigade on the left, 2nd Brigade making the main attack on the right, and the 3rd Brigade kept in reserve.

6.2 Division Artillery Activities

With this information, the divisional Artillery units were assigned the following missions:

1-2nd FA (155 mm) Direct Support of the 1st Brigade

5-31st FA (155 mm) Direct Support of the 2nd Brigade

5-83rd FA (155 mm) General Support

7-16th FA (8") General Support

Through the above assigned missions, the 5-83rd and 7-16th Battalions would be positioned and have fires planned by Division Artillery where they can best approach the attack. (See Figure 2.) These locations and other pertinent information are entered into a computer. Also, as target information is received, that data is also entered in the computer. This data would include the target number j, the target weight w_j, (taken from the previously developed comparative target weighting scheme), the C_j designation for whether 155 mm or 8" units are desired to fire on a particular target, plus the corresponding number of units, U_j, and volleys, V_j, required to be fired. (See Table 2 for the target list and Figure 3 for relative target locations.)

Once the requirement for an artillery preparation is presented, the fire planners need only enter the value for T, or preparation duration time. For this problem T = 11. All other information has

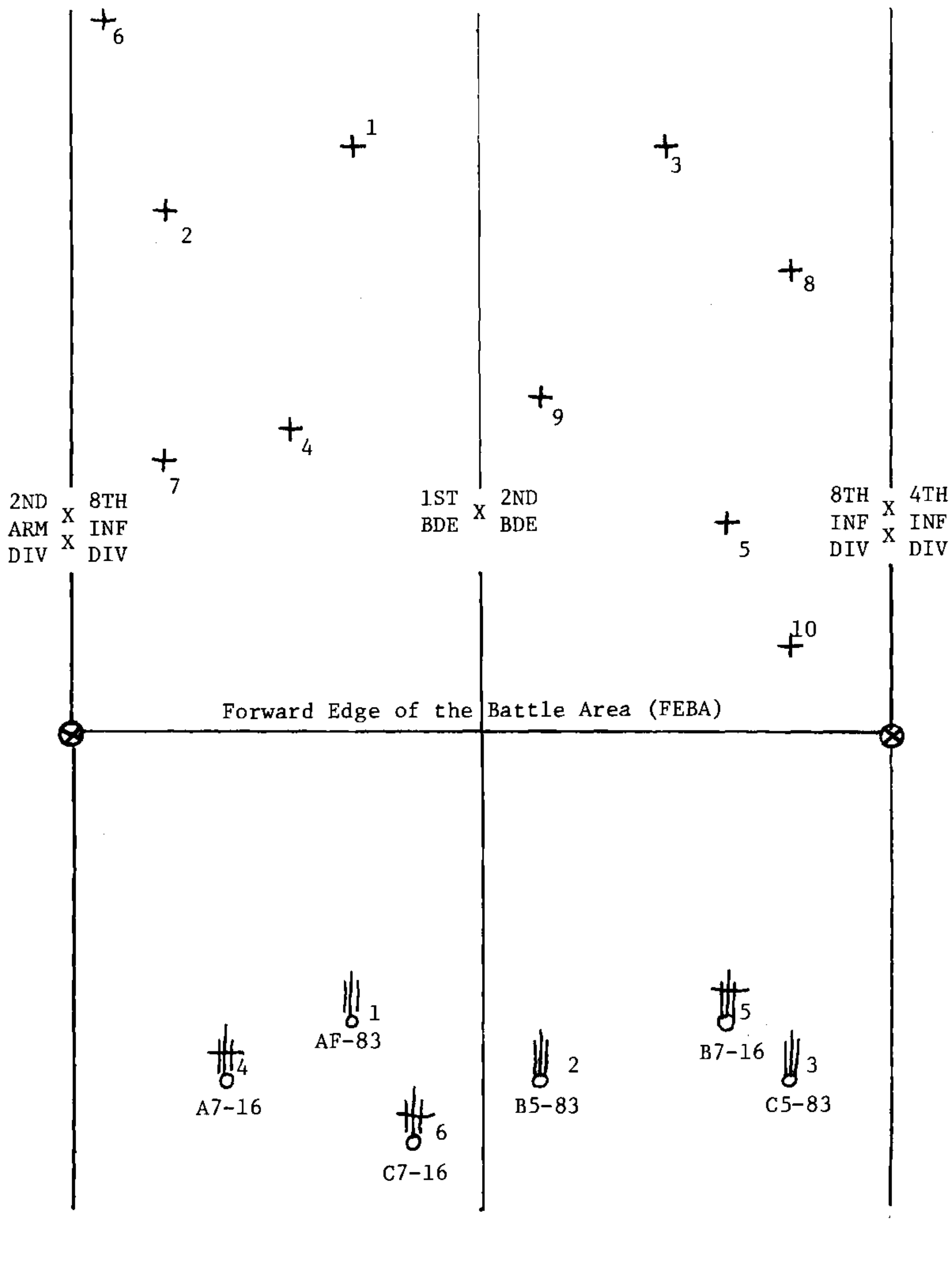


Figure 3. Unit and Target Situation for Example Problem

already been entered. For this problem, since the problem formulation has not been computerized, the data was hand computed and values for d_{ij}, R_{ij}, and $PE_{R_{ij}}$, and f_{ij}, are displayed on Tables 2, 3, 4, and 5 respectively. The results of the problem formulation follow:

Table 2. Target List

Target Number j	Description	C_j	W_j	d_{ij} 1	2	3	4	5	6	U_j	V_j	Notes
1	Mech. Regt. Hq.	1	45	42.3	41.4	44.1	42.8	42.8	42.8	3	2	
2	Resupply Point (Ammo & Equipment)	0	2	1.9	1.84	1.82	1.92	1.9	1.9	1	3	
3	152 mm Gun How. Btry. (Nuclear Capable)	1	100	92	92	92	95	96	95	3	4	W_j=200 in a nuclear situation
4	Mech. Inf. Comp. Res.	0	12	11.5	11.4	11.4	11.6	11.5	11.5	3	3	
5	Plt. Size Unit Dug in on an Objective	1	4	3.8	3.84	3.84	3.84	3.88	3.84	2	2	Change C_5 to 0*
6	Rocket Launcher (Nuclear Capable)	-	130	118	0.0	0.0	0.0	0.0	0.0	2	2	Request Air Strike
7	Truck Park	0	1	.96	.95	.94	.97	.96	.96	2	1	
8	120 mm Mortar Btry.	0	65	61.1	61.1	61.8	61.8	62.4	61.8	2	2	
9	Tank Comp. Res. Assembly	1	15	14.4	14.2	14.2	14.4	14.6	14.4	3	3	
10	Observation Post	0	50	47.5	48.0	48.5	48.0	48.5	48.0	1	1	

*These changes resulted when feasibility check found the 8" units were designated to fire on too many targets.

Table 3. Range from Unit i to Target j

R_{ij} (i \ j)	1	2	3	4	5	6	7	8	9	10
1	14.0	13.2	14.7	9.6	9.9	16.3	9.3	13.6	10.3	9.2
2	15.1	15.0	14.9	11.0	9.3	18.1	11.5	13.4	11.0	8.0
3	16.3	16.8	14.9	13.0	8.8	19.9	13.9	12.8	11.5	7.0
4	14.9	13.7	16.3	10.4	11.9	16.8	9.9	15.6	11.9	11.3
5	15.0	15.6	13.8	11.6	8.0	18.7	12.6	11.8	10.3	6.0
6	15.8	15.3	16.3	11.5	11.0	18.4	11.5	15.0	12.0	9.9

Table 4. Probable Error in Range for Unit i Firing on Target j

$PE_{R_{ij}}$ (i \ j)	1	2	3	4	5	6	7	8	9	10
1	34	32	48	25	26	52	25	33	27	25
2	49	49	49	28	25	∞	29	33	28	22
3	15	54	49	32	24	∞	34	32	29	20
4	27	25	28	20	22	∞	20	27	22	25
5	27	27	26	22	17	∞	23	22	20	14
6	27	27	28	22	21	∞	22	27	22	22

Table 5. Objective Function Values for Unit i Firing on Target j

		j=1	2	3	4	5	6	7	8	9	10
	F_{ij}	1	2	3	4	5	6	7	8	9	10
155 mm	1	.94	.95	.92	.96	.95	.91	.96	.94	.96	.95
155 mm	2	.92	.92	.92	.95	.96	.00	.95	.94	.95	.96
155 mm	3	.98	.91	.92	.95	.96	.00	.94	.95	.95	.97
8"	4	.95	.96	.95	.97	.96	.00	.97	.95	.96	.96
8"	5	.95	.95	.96	.96	.97	.00	.96	.96	.97	.97
8"	6	.95	.95	.95	.96	.96	.00	.96	.95	.96	.96

W_j	45	2	100	12	4	130	1	65	15	50

V_j	2	3	4	3	2	2	1	2	3	1

6.3 Test for Feasibility

Initially, it was desired that 155 units fire on targets 2, 4, 7, 8, and 10. From equation (13) we find:

$$3(11) = 33 \geq 3(1) + (3)(3) + (1)(2) + (2)(2) + (1)(1) = 19$$

Therefore, we can get a feasible solution. Targets 1, 3, 5, and 9 were to be fired on by 8" units. Checking for feasibility we find:

$$3(11 + 1) = 36 \not\geq 2[(2)(3) + (4)(3) + (2)(2) + (3)(3)] = 62$$

Therefore, it is infeasible. Checking the right hand side for a sum as close to the left hand side without exceeding it we find assigning targets 1 and 3 best satisfies the feasibility check, that is,

$$36 \geq 2[(2)(3) + (4)(3)] = 36$$

Now check to see if 155's can attack targets 5 and 9.

$$33 > 19 + (2)(2) + (3)(3) = 32$$

Therefore, this is still feasible and we may proceed to the unit-target assignment problem.

6.4 Unit-Target Assignment Initial Solution

For the 155 mm unit-target problem, our formulation is:

$$\text{MAX} \quad \frac{1.9}{3} y_{12} + \frac{1.84}{3} y_{22} + \ldots + \frac{48.0}{3} y_{210} + \frac{48.5}{3} y_{310}$$

$$\text{S.T.} \quad y_{12} + y_{22} + y_{32} = (3)(1)$$

$$y_{14} + y_{24} + y_{34} = (3)(3)$$

$$y_{15} + y_{25} + y_{35} = (2)(2)$$

$$y_{17} + y_{27} + y_{37} = (1)(2)$$

$$y_{18} + y_{28} + y_{38} = (2)(2)$$

$$y_{19} + y_{29} + y_{39} = (3)(3)$$

$$y_{110} + y_{210} + y_{310} = (1)(1)$$

$$y_{12} + y_{14} + y_{15} + y_{17} + y_{18} + y_{19} + y_{110} \leq 11$$

$$y_{22} + y_{24} + y_{25} + y_{27} + y_{28} + y_{29} + y_{210} \leq 11$$

$$y_{32} + y_{34} + y_{35} + y_{37} + y_{38} + y_{39} + y_{310} \leq 11$$

$$0 \leq y_{ij} \text{ for } i = 1, 2, 3 \text{ and } \forall\, j$$

$$y_{ij} \leq 1 \text{ for } i = 1, 2, 3 \text{ and for } j = 7, 10$$

$$y_{ij} \leq 2 \text{ for } i = 1, 2, 3 \text{ and for } j = 5, 8$$

$$y_{ij} \leq 3 \text{ for } i = 1, 2, 3 \text{ and for } j = 2, 4, 9$$

To solve, we first determine the optimal solution using the capacitated transportation model method. Note that targets 4 and 9 require three volleys from all three units. Thus this solution is trivial and we may omit them for the time being as we come up with our transportation model solution. The initial tableau using the

North-West corner method yields the following:

		2	5	7	8	10	Dummy	
	1	1.9 3 3	3.8 2 2	.96 1 $\boxed{+.76}$	61.1 2 $\boxed{+.04}$	47.5 1 $\boxed{-.26}$	0 1 $\boxed{+.74}$	5
i	2	1.84 3 $\boxed{-.1}$	3.84 2 2	.95 1* 1 $\boxed{+.71}$	61.1 2 2	48.0 1 $\boxed{+.2}$	0 1 $\boxed{+.7}$	5
	3	1.82 3 $\boxed{-.82}$	3.84 2 $\boxed{-.7}$	.94 1 1	61.8 2 2	48.5 1 1	0 1 1	5
		3	4	2	4	1	1	

(See Appendix A for explanation of solution.)

The final tableau is:

	2	5	7	8	10	D	
1	3	$\boxed{-.04}$	$\boxed{+.01}$ *	1	$\boxed{-1.0}$	$\boxed{0}$	0
2	$\boxed{-.06}$	$\boxed{0}$ *	1	1	$\boxed{-.5}$	1	0
3	$\boxed{-.08}$	2	$\boxed{-.01}$	$\boxed{+.7}$ *	1	0	0
	1.9	3.84	.95	61.1	48.5	0	

This solution, then, is optimal. It would mean the following:

$y_{12} = 3$

$y_{25} = y_{35} = 2$

$y_{17} = y_{27} = 1$

$y_{18} = y_{28} = 1$

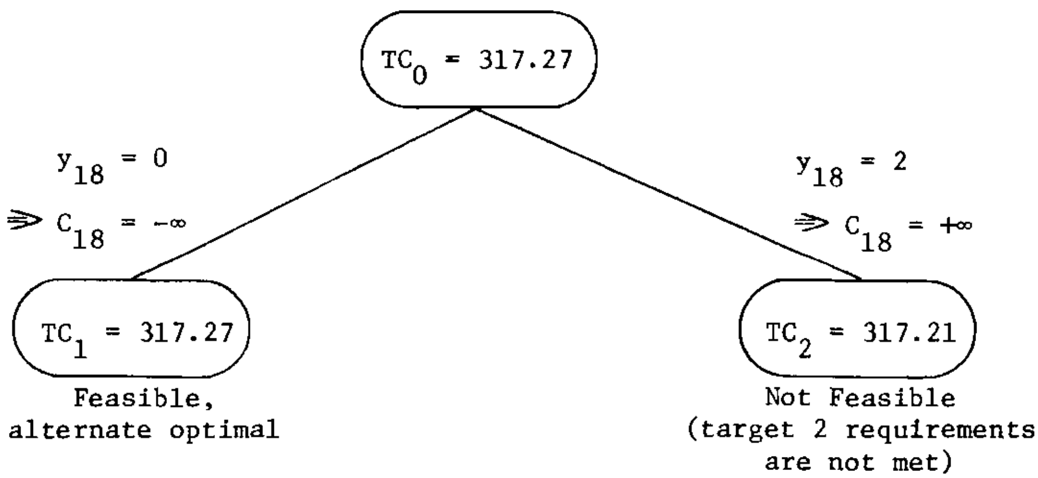
$y_{38} = 2$

$y_{310} = 1$

$y_{2D} = 1$ or, unit 2 has one free slack period

The problem, then, lies with target 8 where we desired two units, two volleys each. Which unit, 1 or 2, should pick up the additional volley?

To answer this question recall the branch and bound problem discussion in Chapter V. In this example, either $y_{18} = 0$ or $y_{18} = 2$. This is portrayed with the corresponding total costs as follows:



The above result came from the following final optimal tableaus letting $C_{18} = +\infty$ and $C_{18} = -\infty$.

$C_{18} = +\infty$

	2	5	7	8	10	D
1	2	$\boxed{-.06}$	1	$\boxed{+\infty}^{*}$	$\boxed{-1.02}$	$\boxed{-.06}$
2	1	$\boxed{+.04}^{*}$	$\boxed{+.05}^{*}$	$\boxed{-.66}$	$\boxed{-.46}$	1
3	$\boxed{-.06}$	2	0	2	1	$\boxed{-1.04}$

TC = 317.21

Optimal

$C_{18} = -\infty$

	2	5	7	8	10	D
1	3	$\boxed{-.04}$	$\boxed{+.01}^{*}$	$\boxed{-\infty}$	$\boxed{-1.0}$	1
2	$\boxed{-.06}$	$\boxed{0}^{*}$	1	2	$\boxed{-.5}$	0
3	$\boxed{-.08}$	2	$\boxed{-.01}$	$\boxed{+.7}$	1	0

TC = 317.21

Optimal

Note that the optimal solution to the unit-target assignment problem was obtained on the first branching. If, however, the branch with $y_{18} = 2$ had resulted in, say, $TC_2 = 317.40$, then further branching on this node would be required before the "best" unit-target assignment would be obtained.

If it is determined, during the scheduling phase, that the "best" unit-target assignment cannot be scheduled then we must return to the branch and bound tree for the unit-target assignment problem and select the "second best" feasible assignment for scheduling. Selecting the "second best" assignment may require further branching depending on the bounds. Further discussion of the branch and bound method applied to this problem is presented in Appendix A. The solution to the 8" unit-target problem is determined the same way, but, with only two targets concerned, its solution is rather trivial. This set-up and solution is as follows:

$$\text{Max} \quad \frac{42.8}{2} y_{41} + \frac{42.8}{2} y_{51} + \frac{42.8}{2} y_{61} + \frac{95}{4} y_{43} + \frac{96}{4} y_{53} + \frac{95}{4} y_{63}$$

$$\text{S.T.} \quad y_{41} + y_{51} + y_{61} = (2)(3)$$

$$y_{43} + y_{53} + y_{63} = (4)(3)$$

$$y_{41} + y_{43} \le \frac{11 + 1}{2}$$

$$y_{51} + y_{53} \le \frac{11 + 1}{2}$$

$$y_{61} + y_{63} \le \frac{11 + 1}{2}$$

$$0 \le y_{ij} \quad \text{for } i = 4, 5, 6 \text{ and } j = 1, 3$$

$$y_{i1} \le 2 \quad \text{for } i = 4, 5, 6$$

$$y_{i3} \le 4 \quad \text{for } i = 4, 5, 6$$

Again, using the capacitated transportation problem method of solution we get the following optimal tableau:

		j		
		1	3	
		21.4 2*	23.75 4	
	4	2	4	0
		21.4 2*	24 4	
i	5	2	4	0
		21.4 2	23.75 4	
	6	2	4	0
		0	0	

The meaning of this result is

$$y_{41} = y_{51} = y_{61} = 2$$

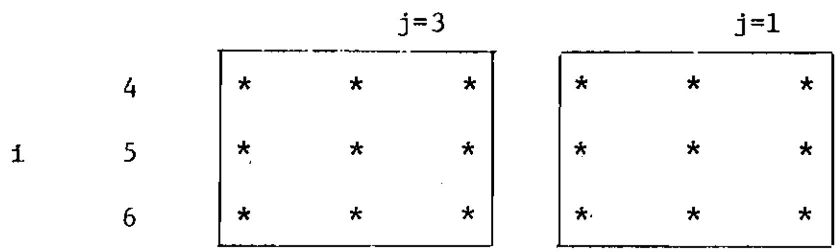
$$y_{43} = y_{53} = y_{63} = 4$$

We are now ready to see if we can schedule the unit-target assignments in accordance with schedule requirements.

6.5 Scheduling the Unit-Target Assignment Result

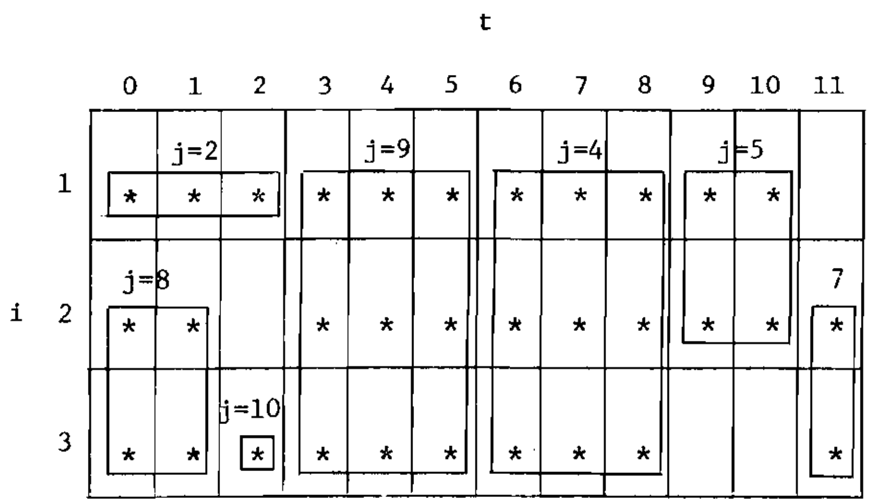
Let us look at the 8" unit schedule first as it again has a rather trivial solution. Now, $w_3 \geq w_1$ thus $\theta_1 = \{4, 5, 6\}$ and $\theta_2 = \{4, 5, 6\}$. We first locate θ_1 at time $t = 0$. This mission occupies all three units from time $t = 0$ through $t = (C_{j1} + 1)\, V_j - 1 = (1 + 1)\ (4) - 1 = 7$. Thus we cannot schedule θ_2 until time $t = 8$.

This schedule would appear as

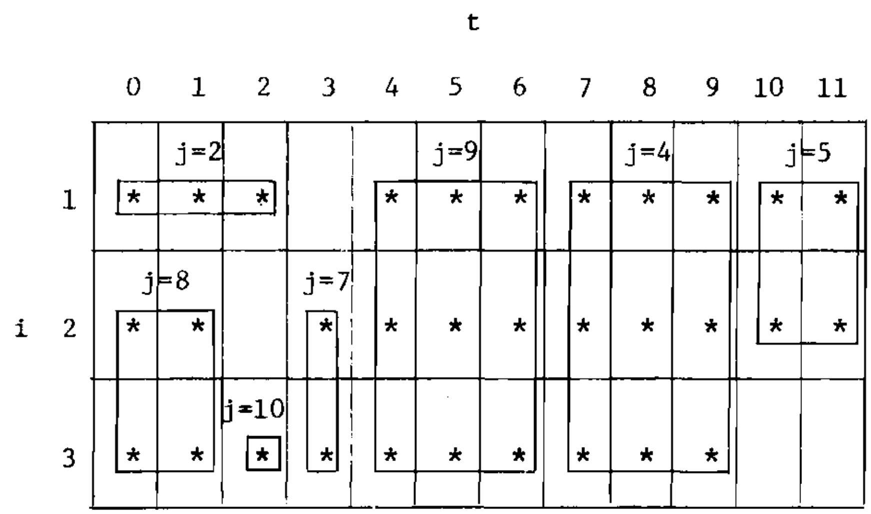


The 155 mm unit scheduling problem is not as simple. As above, we first order the w_j's: $w_8 \geq w_{10} \geq w_9 \geq w_2 \geq w_4 \geq w_5 \geq w_7$. The corresponding θ's would be $\theta_1 = \{2, 3\}$, $\theta_2 = \{3\}$, $\theta_3 = \{1, 2, 3\}$, $\theta_4 = \{1\}$, $\theta_5 = \{1, 2, 3\}$, $\theta_6 = \{2, 3\}$, and $\theta_7 = \{1, 2\}$.

Now we are ready to follow the branch and bound technique to schedule these fires. We start out by trying to schedule the targets in the given ordered weight sequence. Locating these related θ's in this order would yield the following schedule:

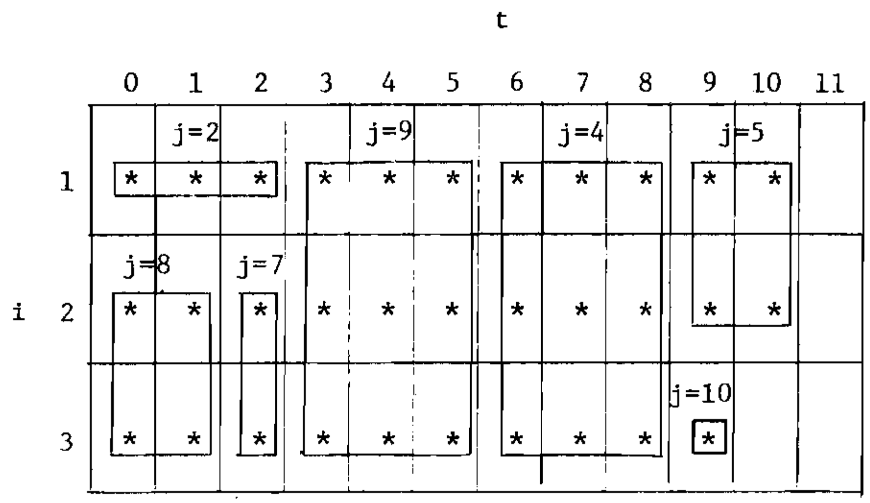


This schedule is infeasible as it requires firing on target 7 after time T - 1. Thus, we go back to the branch and bound technique to look at the other possible ways of ordering the targets. As mentioned earlier, to do this we are relaxing the requirement that the highest weighted target be fired first. For example, we may have considered the combination 8, 10, 7, 9, 2, 4, and 5. This would graphically appear as follows:



which again is infeasible as it too exceeds the T - 1 time restriction.

Finally, we would try the combination 8, 7, 9, 2, 4, 10, 5 portrayed as follows:



This is a feasible solution and our problem is finished. Had we exhausted every possible ordering combination and were still not able to find a feasible solution we would have been forced to look for a second best solution, determine new values for the θ's and go through the above scheduling procedure again.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The artillery's presently used, outdated system of assigning units to fire on given targets can effectively be replaced by a time-saving procedure. This procedure will insure that the units with the most advantageous location relative to a particular target will fire on that target where possible. Also, highest priority targets will be fired on as early as possible.

In addition, a means of comparing the relative importance of targets was introduced, an item which at best is only mentioned within by artillery literature in the most general form. This research presents a means whereby one can differentiate priorities between targets. This system could conceivably be applied to bomber-missle-strategic type targets as well.

Obviously, the first extension of this problem would be to convert the model to a computer formulation. Considering the magnitude of possible problems, computerization would allow one to get a solution in seconds.

Next, the model could be improved to allow for it (rather than target analyst) to decide what type of unit would be best for firing on a given target. This would involve changing V_j and U_j to V_{jk} and U_{jk} thereby increasing the number of constraint equations. Also, the objective function would require some adjustments to enable a decision between unit types on a given target mode.

One might also consider the accuracy of the target location. A target whose location is not precisely known would require more volleys to be fired to achieve the same level of assurance of desired results compared to a similar target with a more accurate location. Thus, the more accurate target would be more desirable for firing than a less accurate plot.

There are many other considerations one might want to include to improve the model. For example, where would be the best location to move a unit to to enable it to bring more effective fires on a given target or a group of targets? (This is critical when considering a single gun which might be separated from its parent unit to fire a nuclear mission.) Extending this idea, knowing where the targets are located and what the offensive missions of supported units are, where can all artillery units be located to best support the attack. One might also consider integrating air-delivered munitions into the model to handle targets, such as tanks, where artillery is not as effective.

The limits on how far this problem can be extended cannot be seen. With time and effort many related areas can be effectively handled by applicable extensions of this thesis.

APPENDIX A

SOLVING THE CAPACITATED TRANSPORTATION PROBLEM

Hadley (11) discussed how to solve capacitated transportation problems. This, in part, is iterated here with additional notes to assist in solving problems related to this thesis.

A.1 Generalized Notation

The following tableau represents a generalized layout for the unit-target j assignment problem:

j

i		1	2		n	D	
	1	d_{11} | V_1	d_{12} | V_2	....	d_{1n} | V_n	0 | ?	T
	2	d_{21} | V_1	d_{22} | V_2	....	d_{2n} | V_n	0 | ?	T
	⋮	⋮	⋮		⋮	⋮	
	m	d_{m1} | V_1	d_{m2} | V_2	....	d_{mn} | V_n	0 | ?	T
		U_1V_1	U_2V_2		U_nV_n	D_T	

A circled number in a cell, i.e., ① or ③, represents a cell value which is in the basis.

A * represents a cell's value which is at that cell's upper limit.

A number in a rectangle, i.e., $\boxed{-1.6}$ or $\boxed{+.7}$, represents the value of $C_j - Z_j$.

The d_{ij} represents the cost coefficient as presented in the objective function.

The V_j, the same value as the number of volleys target j requires, represents the upper limit of the particular cell. The lower limit of every cell is zero.

Column totals, U_jV_j, are the total number of volleys all units will fire on target j.

The total of each row, T, is actually equal to T for 155 mm units and the integer part of $\frac{T+1}{2}$ for 8" units.

The D column is the dummy, or slack column. Its cost is zero for each cell. The upper limit of each cell = $M_j(T+C_j) - \sum_j (C_j+1)V_jU_j$

This value also equals the column total D_T.

A.2 Chapter VI Example Returned

Consider the Chapter VI example problem. As mentioned there, to get a starting solution, the northwest corner method will suffice. Cells are filled to their upper limit where possibly everywhere except the D column where this is not a requirement. The starting tableau is repeated here for convenience.

		j						
		2	5	7	8	10	D	
	1	1.9 [3] 3	3.8 [2] −Δ 2	.96 [1] +Δ $\boxed{+.76}$	61.1 [2] $\boxed{+.04}$	47.5 [1] $\boxed{-.26}$	0 [1] $\boxed{+.74}$	5
i	2	1.84 [3] $\boxed{-.1}$	3.84 [2] +Δ 2	.95 [1] * $\boxed{+.71}$	61.1 [2] −Δ 2	48.0 [1] $\boxed{+.2}$	0 [1] $\boxed{+.7}$	5
	3	1.82 [3] $\boxed{-.82}$	3.84 [2] $\boxed{-.7}$	.94 [1] −Δ 1	61.8 [2] +Δ 2	48.5 [1] 1	0 [1] 1	5
		3	4	2	4	1	1	

Since $m + n - 1 = 3 + 6 - 1 = 8$, we need eight cell values in the basis. These are circled. To get to the optimal solution, the usual transportation methods are used. $C_{ij} - Z_{ij}$ values are obtained (in $\square$'s above). Candidates to enter the basis are the most positive $C_{ij} - Z_{ij}$ values for cells at the zero level or the most negative $C_{ij} - Z_{ij}$ for cells at their upper limit. (Recall that we are maximizing the objective function). In this case we want cell (1, 7) to enter. The $\pm$ Δ's are indicated on the initial tableau. Since cell (2, 5) cannot be increased, it drops out of the basis at the zero level, or 0. New $C_{ij} - Z_{ij}$ values are computed. This is made easier by the use of the following:

$C_{ij} - Z_{ij} = C_{ij} - (W_i + W_{m+j})$ where W_i and W_{m+J} are defined by the relationship:

$C_{ij} - Z_{ij} + C_{ij} - (W_i + W_{m+j}) = 0$ for basic cells (i, j) and, say, $w_1 = 0$.

Those values are indicated on the following tableau:

	2	5	7	8	10	D	W_i
1	3	2	0	-.62	-1.02	-.02	0
2	+.66	* +.76	* +.71	2	+.2	+.7	-.72
3	-.06	+.06	1	2	1	1	-.02
W_{m+j}	1.9	3.8	.96	61.82	48.52	.02	

Next to enter is cell (2, D). But, cell (3, 5) can also come in as it is not associated with the basic elements cell (2, D) is. This results in:

	2	5	7	8	10	D	
1	3	1	1	+.14	-.96	+.04	0
2	-.1	.0 *	-.05 *	2	-.5	0	+.04
3	-.11	1	-.06	+.7 *	1	1	+.04
	1.9	3.8	.96	61.06	48.46	-.04	

Next to enter is cell (1, 8) yielding:

	2	5	7	8	10	D	
1	3	$\boxed{-.06}$	1	1	$\boxed{-1.0}$	$\boxed{-.1}$	0
2	$\boxed{+.04}$	* $\boxed{+.12}$	* $\boxed{-.09}$	1	$\boxed{-.5}$	1	-.1
3	$\boxed{+.02}$	2	$\boxed{+.08}$	$\boxed{+.06}$	1	0	-.1
	1.9	3.86	.96	61.2	48.6	+.1	

Finally after entering cell (2, 7) we get the optimal tableau:

	2	5	7	8	10	D	
1	3	$\boxed{-.04}$	* $\boxed{+.01}$	1	$\boxed{-1.0}$	0	0
2	$\boxed{-.06}$	* 0	1	1	$\boxed{-.5}$	1	0
3	$\boxed{-.08}$	2	$\boxed{-.01}$	* $\boxed{+.7}$	1	0	0
	1.9	3.84	.95	61.1	48.5	0	

As noted in Chapter VI, target 8 results are infeasible. Letting cell (1, 8) have a cost of $-\infty$ in the above tableau led to the optimal solution shown in Chapter VI in one step, thus, that part is omitted. Letting $C_{18} = +\infty$, however, did not. Therefore, those results are included here for illustrative purposes. Because of its positive $C_{ij} - Z_{ij}$ value, cell (1, 8) enters the basis and yields

	2	5	7	8	10	D
1	0	$\boxed{-\infty}$	$\boxed{-\infty}$ *	0	$\boxed{-\infty}$	$\boxed{-\infty}$
2	$\boxed{+\infty}$	UC *	0	0	UC	0
3	$\boxed{+\infty}$	0	$\boxed{\text{UC}}$	$\boxed{\text{UC}}$ *	0	0

$\boxed{\text{UC}}$ means $C_{ij}-Z_{ij}$ is unchanged

Now enter cell (1, 7) and (2, 2) yields

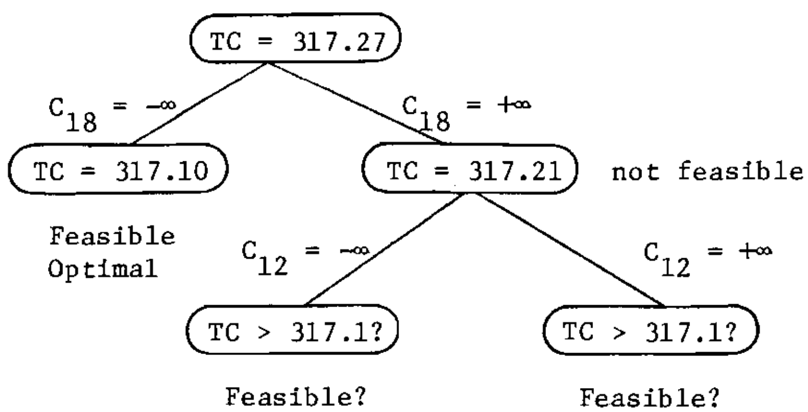
	2	5	7	8	10	D
1	2	$\boxed{-.1}$	1	2	$\boxed{-1.06}$	$\boxed{-.06}$
2	1	$\boxed{0}$ *	$\boxed{+.05}$ *	$\boxed{-\infty}$	$\boxed{-.5}$	1
3	$\boxed{-.02}$	2	$\boxed{+.04}$	$\boxed{-\infty}$ *	1	0

Enter next (3, 8)

	2	5	7	8	10	D
1	2	$\boxed{-.1}$	1	$\boxed{+\infty}$ *	$\boxed{-1.06}$	$\boxed{-.06}$
2	1	0 *	$\boxed{+.05}$ *	$\boxed{-.7}$	$\boxed{-.5}$	1
3	$\boxed{-.02}$	2	$\boxed{+.04}$	2	1	0

Lastly, enter (3, 7) to yield the already shown final tableau. Again one can immediately note on the final tableau (as above) that now target 2 poses a problem. Thus, once this point is reached, we are not finished. Since we already had an optimal feasible result when we let $C_{18} = -\infty$ and this tableau had an associated total cost (317.27) which was greater than the one from letting $C_{18} = +\infty$ (TC = 317.21), we were able to stop right there. Had that not been the case, one would have to continue, this time trying $C_{12} = +\infty$ and $C_{12} = -\infty$. Once the optimal is reached, again check for feasibility and if necessary repeat the above until the optimal, feasible point is reached.

The branch and bound portrayal of this was shown in Chapter V. Let us just say that where $C_{18} = -\infty$ the TC were 317.10. In this case, we would have to follow out the $C_{18} = +\infty$ course. This would appear as follows:

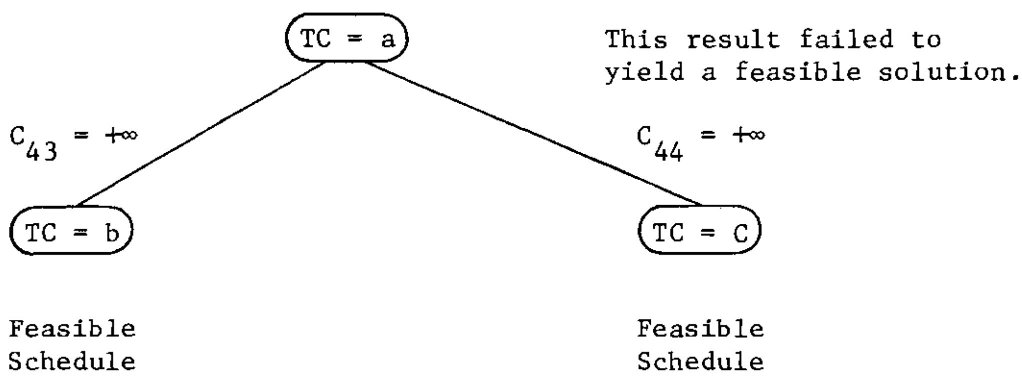


If either branch from $C_{18} = +\infty$ were feasible and had a TC > 317.1, it would be more desirable that the $C_{18} = -\infty$ branch and would be the approved solution.

A.3 Next Best Solution

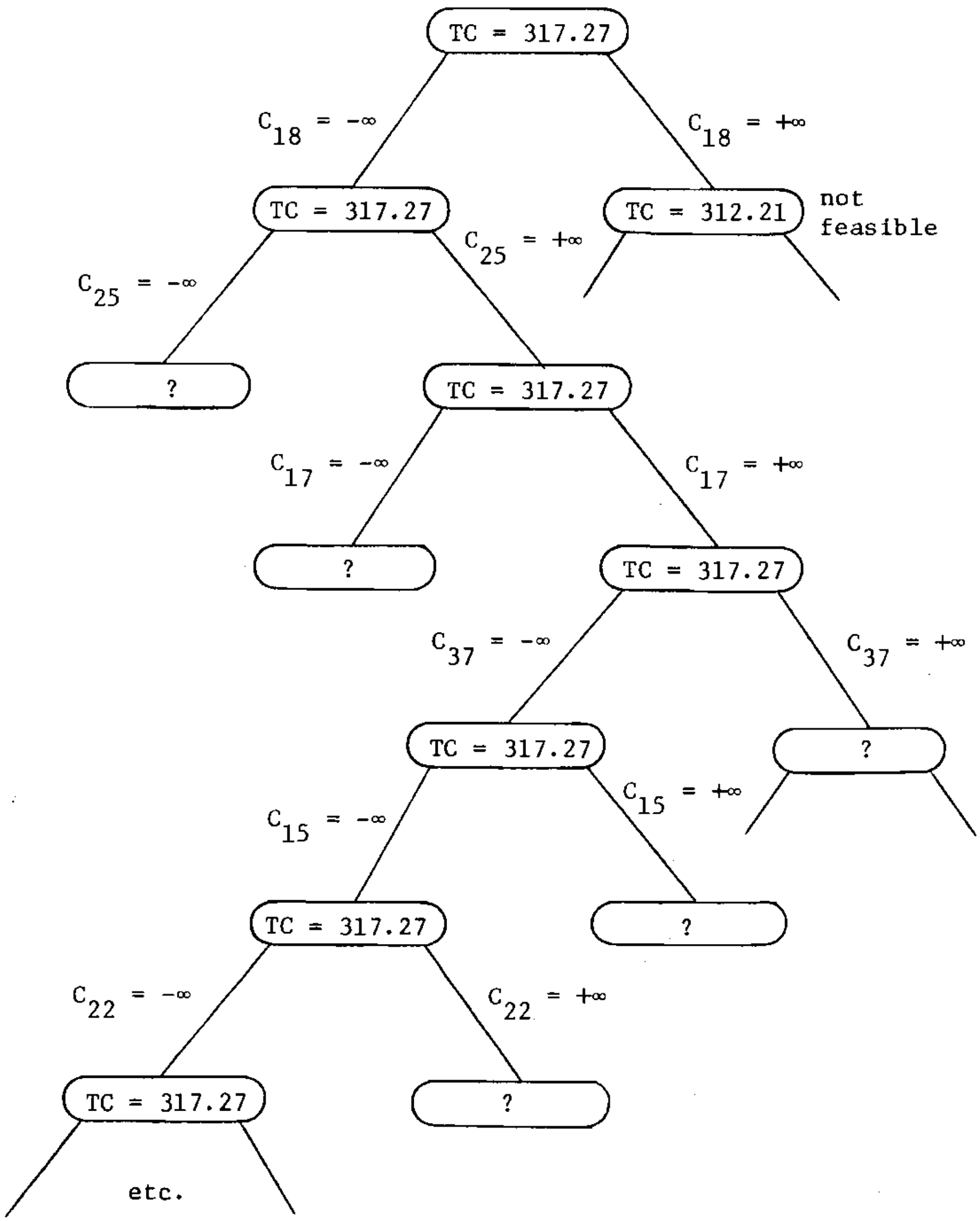
The next best (or second best) solution was addressed in Chapter V, Section 4. For clarity purposes, it is reiterated here.

For the Chapter VI example, the optimal, feasible result could be scheduled (after letting $C_{18} = -\infty$) from this unit-target assignment problem solution. Suppose this had not been possible. This would call for obtaining a second best, or third best, etc., solution. This is accomplished in much the same manner as mentioned in Section 2 above and in Chapter V. The branch and bound portrayal of the Chapter V example would be:



where $a > b > c$. $\therefore C_{43} = +\infty$ was preferred to $C_{44} = +\infty$.

For the Chapter VI example, if a next best solution was needed then consider the following branch and bound tree:


TC = 317.27
C18 = -∞
C18 = +∞
TC = 317.27
TC = 312.21
not feasible
C25 = -∞
C25 = +∞
?
TC = 317.27
C17 = -∞
C17 = +∞
?
TC = 317.27
C37 = -∞
C37 = +∞
TC = 317.27
?
C15 = -∞
C15 = +∞
TC = 317.27
?
C22 = -∞
C22 = +∞
TC = 317.27
?
etc.

The above would be continued for every non-basic cell to find the total cost which is the second best. If no "?" is greater than TC = 317.21 (for $C_{18} = +\infty$), that branch must be continued. Of course, each branch end must meet feasibility requirements.

As is readily apparent, the above process is an extremely tedious, time consuming one. Fortunately, the need for its use is scarce indeed. The greater T, the less likely a scheduling problem will result.

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