E-25-605

FINAL REPORT

FLOW-INDUCED VIBRATIONS IN FLEXIBLE COMPONENTS OF SPACE SHUTTLE FEED SYSTEMS

By

P. V. Desai

Under

Contract No. NAS10-10685

Georgia Tech Project No. E-25-605

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GEORGIA INSTITUTE OF TECHNOLOGY

A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA

SCHOOL OF MECHANICAL ENGINEERING

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ABSTRACT

This report presents the development of a methodology to estimate life time of a bellow or a flexhose with a single- or multi-ply wall construction. In particular, it examines the fatigue life of a corrugated flexible line, such as the one used in feedlines of external tanks of the space shuttle, when subjected to vibrations and noise accentuated by the fluid flowing through it. Both facets of the problem, the calculation of the natural frequency and modes of vibration as well as the prediction of fatigue strength and failure, are addressed in the report. Sample calculations, together with a computer program, are presented to illustrate the methodology.

I. INTRODUCTION

I.1 Project Background and Overview

This report describes all the work performed by Georgia Institute of Technology, College of Engineering, School of Mechanical Engineering under Contract number NAS10-10685, "Flow-Induced Vibrations in Flexible Components of Space Shuttle Feed Systems". This study was performed for the Kennedy Space Center of the National Aeronautics and Space Administration.

The general objective of this work was to investigate the physical phenomena associated with flow-induced vibrations of a flexible line and to develop a methodology to examine its fatigue life. In response to NASA needs, much emphasis was placed on developing an illustrative procedure for calculating the time to failure of the assumed vibration model.

Flexible expansion joints are currently being used in a variety of applications. These include the nuclear reactor cooling systems, supply and return loading lines for submarines from the mother ship on the base, liquid cryogenic fuel lines for the space shuttle external tanks and so forth. There is, in fact, a manufacturer's association for expansion joints which publishes its recommended practices for their use.

Serious difficulties are encountered when practical solutions are not obtained to control the so-called "self-generated" sound or vibrations in flow systems which use such corrugated lines. For the special case of the internally corrugated flexible hoses, segements of which are utilized in the feed lines of Space Shuttle external tanks, it has been possible to identify the fluid flow related parameters governing the physical phenomena. Unshrouded hose vibration has been attributed [1] to an approximate matching between the longitudiual structural frequency and the frequency of vortex shedding at a critical flow velocity over the first few corrugations in the hose. This has led to the computation of the modal frequencies of vibration by representing the hose with a series of lumped spring-mass elements and relating the amplitude of vibrations to the damping characteristics of the system. The postulated model was supplemented by a large number of empirical correlations obtained from a series of experiments to relate the system geometry, the mean flow rate and the fluid properties with the modal frequencies and amplitudes of vibration, the damping characteristics, the maximum alternating stress and the expected fatigue life of the hose material [2].

Experimental observations under simulated field conditions have proved the predicted failure results of the two cited works to be very conservative. Furthermore, an overwhelming reliance on empirically obtained functions to predict the fagitue life of the hose in these studies has made it cumbersome to directly utilize the suggested failure prediction procedure.

In the context of flow past cavities, there are three types of self generated oscillations. These are the fluid-dynamic, the fluid-resonant and the fluid elastic types of oscillations [3,4]. The fluid dynamic oscillations arise from some type of flow instability inherent to the flow configuration. The fluid-resonant oscillations occur due to standing wave resonance effects which are particularly important for compressible fluid flow. The fluid-elastic oscillations involve a coupling of the solid boundary vibrations with the fluid flow perturbations causing those. The fluid-dynamic and the fluid-elastic oscillation flow field in bellows are considered in examining the longitudinal vibration modes in this work.

I.2 Statement of Research Problem

This research concerns the development of a methodology to estimate life time of a bellow or a flexhose with a single- or multi-ply wall construction. In particular, it examines the fatigue life of a corrugated flexible line, such as the one used in feedlines or external tanks of the Space Shuttle, when subjected to vibrations and noise accentuated by the fluid flowing through it. Both facets of the problem, the calculation of the natural frequency and modes of vibration as well as the prediction of fatigue strength and failure, are addressed in this work.

I.3 Summary Statement of Results

This report furnishes

1. a mechanical structural vibration model which characterizes the flexhose or bellows flow-induced vibration, with the added mass due to the fluid dynamic coupling and material damping effects being accounted for in the model
2. a set of general expressions for calculating the natural frequencies of the flexhose and the forced vibration responses induced by the fluid flowing through the flexhose and
3. a methodology to predict the flexhose fatigue life time under different flow velocities.

II. LONGITUDINAL VIBRATION MODES ANALYSIS

II.1 Development of the Mathematical Model

The structural vibrations of flexible pipes, with an incompressible fluid flow through it are coupled to the motion of the fluid. The complexity of the problem is greatly increased when the viscous damping and vortex shedding effects of the internal fluid are taken into account. The structural vibration model of the system may be represented by a series of limped masses representing the convolution, with springs and dashpots connecting these as shown in Figure 1. This system has $(2N_c - 1)$ degrees of freedom, N_c being the total number of convolutions. In order to appreciate the physical basis for such a model, at first a structure with only one convolution is considered.

The equation of motion is given by

$$m_m \ddot{x} + c\dot{x} + kx = F, \tag{1}$$

where m_m is the mass of the convolution, c is the internal material damping coefficient, k is the stiffness coefficient of the flexible pipe, and F is the force exerted on the system by the flowing fluid.

If, under the stead-state, the structure oscillates harmonically with amplitude x_0 and angular frequency ω , the fluid force is mainly due to two components: one acts in phase with the acceleration of the structure, and the other acts in phase with the velocity of the system. The former is the fluid force applied to the structure due to the inertia of the fluid entrained by the moving convolution, which is known as the added mass. This added mass inertia acts with the same sign, frequency, and phase as the inertia of the structural mass and always decreases the natural frequencies of the structure. The latter is the fluid force caused by viscous damping and vortex shedding effets of the fluid. It is a nonlinear term and could be so complicated a function in most flow induced vibration problems that the apparant way to specify it is through experiments. A combination of the inertia effects due to the structure as well as the fluid entrained in the convolution of the flexible pipe gives (with $m = m_m + m_a$)

$$(m_m + m_a)\ \ddot{x} + c\dot{x} + kx = \frac{C_D \rho_f V^2 A}{2}\ , \tag{2}$$

where m_a is the added mass, ρ_f is the density of the fluid, V is the relative velocity of flow, A is the cross-sectional area on which the fluid force acts and C_D is the coefficient of drag of the fluid force. In general, C_D is a function of the fluid flow Reynolds number, Re, the Struhal number of the vibrating system, S and the geometry of the convolutions; that is

$$C_D = f(\mathrm{Re},\ S,\ \mathrm{geometry})\ . \tag{3}$$

The value of C_D must be determined by experiments. Once the mathematical model for a single convolution is established, Equation (2) may be generalized to obtain a model for the whole flexible line. It is noted from Figure 1 that if the material of the flexible pipe is homogeneous, the value of K for each spring should be equal. This approximation greatly simplifies the problem since the overall stiffness coefficient then equals that for an element divided by the total number of springs connected in series. The overall stiffness of the flexible pipe, k_a, is obtained as

$$k = 2N_c k_a\ . \tag{4}$$

If it is further assumed that each convolution of the flexible pipe be equally split into two parts shown in Figure 1, all masses including the added masses will be the same. Although this is not entirely valid at the two ends, it nevertheless represents a good practical approximation that substantially simplifies the calculations. For the same reason, all material damping coefficients are also assumed to be identical. With these simplifications, the governing equation of motion for the flexible pipe may be written as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}, \tag{5}$$

where [M], [C], and [K] are $(2N_c - 1) \times (2N_c - 1)$ mass, damping, and stiffness matrices, respectively. In other words,

$$[M] = \begin{bmatrix} m & o & \cdots\cdots & o \\ o & m & \cdots\cdots & o \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ o & o & \cdots\cdots & m \end{bmatrix}, \tag{6a}$$

$$[C] = \begin{bmatrix} 2c & -c & o & \cdots & o \\ -c & 2c & -c & \cdots & o \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & -c \\ o & o & & \cdots\cdots -c & 2c \end{bmatrix} \tag{6b}$$

and

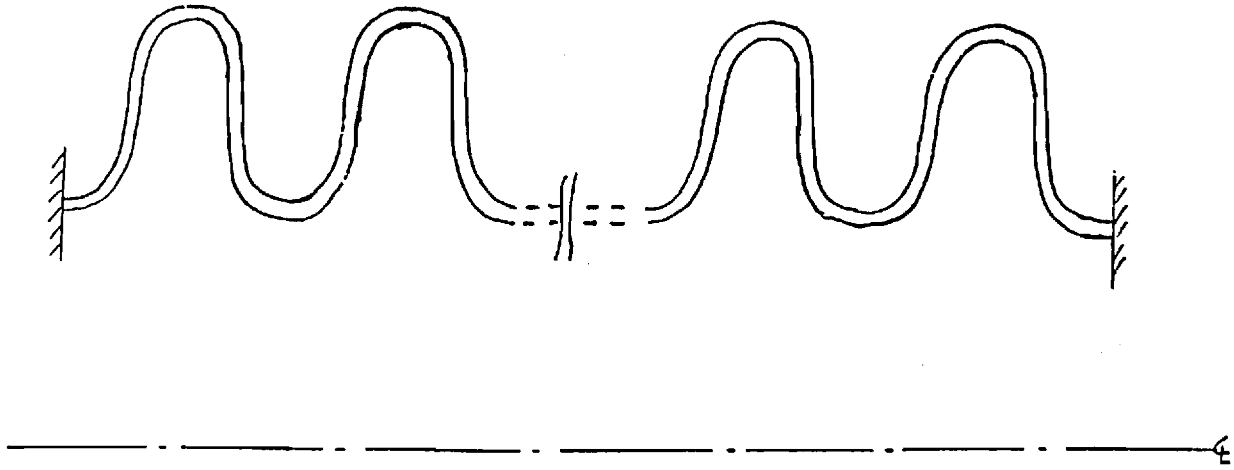


Figure 1a. Schematic of Flexhose Convolutions.

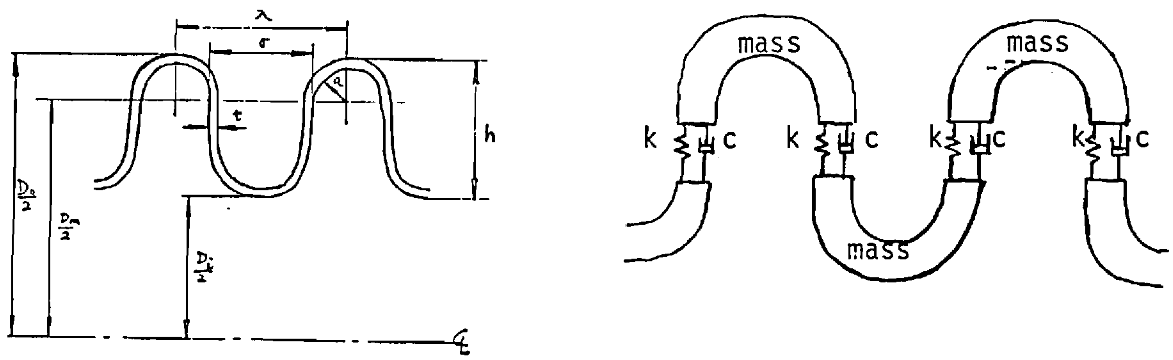


Figure 1b. Mechanical Model of Convolution Vibrations.

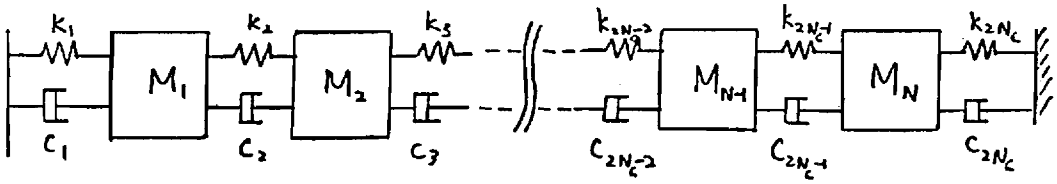


Figure 1c. Geometry of Convolutions.

$$
[K] = \begin{bmatrix} 2k & -k & o & \cdot & \cdot & \cdot & o \\ -k & 2k & -k & \cdot & \cdot & \cdot & o \\ o & \cdot & & & & & \\ \cdot & \cdot & & & & & \\ \cdot & \cdot & & & & & -k \\ o & o & \cdot & \cdot & \cdot & -k & 2k \end{bmatrix} . \tag{6c}
$$

To find the natural frequencies and natural modes of vibration of the system, one first solves the corresponding homogeneous part of Equation (5). Since under the steady-state, the flexible pipe is assumed to oscillate harmonically with amplitude x_o and angular frequency ω_o, one can assume a solution to Equation (5) to be of the form

$$
x_i = x_o e^{i\omega_n t} , \tag{7}
$$

substitute it into the governing equation of motion and subsequently set

$$
D = |[K] - \omega_n^2 [M]| = 0 . \tag{8}
$$

All eigenvalues or natural frequencies can be obtained from this characteristic equation. Substituting those eigenvalues back into the corresponding homogeneous equation then yields the corresponding eigenvectors or natural modes. It is understood here that the harmonic motion is defined by the real part of Equation (7).

The force $\{F\}$ should also be harmonic and have the same general form as that of the response of the system, but with different amplitude and angular frequency. Therefore, using the determinants D and Cramer's rule, the magnitude of the motion for the ith element is found to be

$$x_i = \frac{\begin{bmatrix} (2k-\omega_e^2 m)+i2c\omega_e & -k-ic\omega_e & 0 & \frac{C_D\rho_f V^2 A}{2} & 0 \\ -k-ic\omega_e & (2k-\omega_e^2 m)+i2c\omega_e & & \frac{C_D\rho_f V^2 A}{2} & 0 \\ & & & & \\ 0 & & & & \\ \vdots & & & & \\ 0 & 0 & 0 & \frac{C_D\rho_f V^2 A}{2} & (2k-\omega_e^2 m)+i2c\omega \end{bmatrix}}{|[K] - \omega_e^2[M] + i\omega_e[C]|} , \quad (9)$$

where ω_e represents the excitation frequency.

The quantity c in the foregoing expressions is replaced by the damping ratio, ζ , defined as

$$\zeta = \frac{c}{2\sqrt{km}} , \quad (10)$$

which can be obtained by using the bandwidth technique [1].

II. 2 Calculation of Structural Vibration Modes

As an application of the method outlined in section II.1, natural frequencies are calculated for a flexhose with two convolutions. A similar procedure can also be applied to a flexhose with higher number of convolutions. Previously it has been established [1,2] that the values of m and k are given by

$$m = \pi\rho_m t N_p D_m[\pi a t(h-2a)] + \frac{\pi}{2} \rho_f D_m h(2a-tN_p) \quad (11)$$

and

$$k = 2D_m E N_p \left(\frac{t}{h} \right)^2 . \tag{12}$$

In Equations (11) and (12), ρ_m and ρ_f are the densities of the flexhose wall material and the fluid, respectively, N_p refers to the number of plys in the construction of the wall, D_m is the mean diameter of the flexhose, t is the wall thickness, a is the mean forming radius, h is the mean disc height and E is the Young's modulus of the material. Table I [1,2] provides specific data pertaining to P/N - 08047, #106 as

$D_m = 2.16$ (in)	$a = 0.0445$ (in)
$N_p = 1$	$E = 2.8 \times 10^7$ (lb/in^2)
$h = 0.16$ (in)	$\rho_m = 0.28$ (lb/in^3)
$\sigma = 0.095$ (in)	$\rho_f = 0.036042$ (lb/in^3)
$t = 0.006$ (in)	

Substitution of these numbers into Equations (11) and (12) gives

$$m = 0.1296744 \text{ (lb sec}^2\text{/in)}$$

and

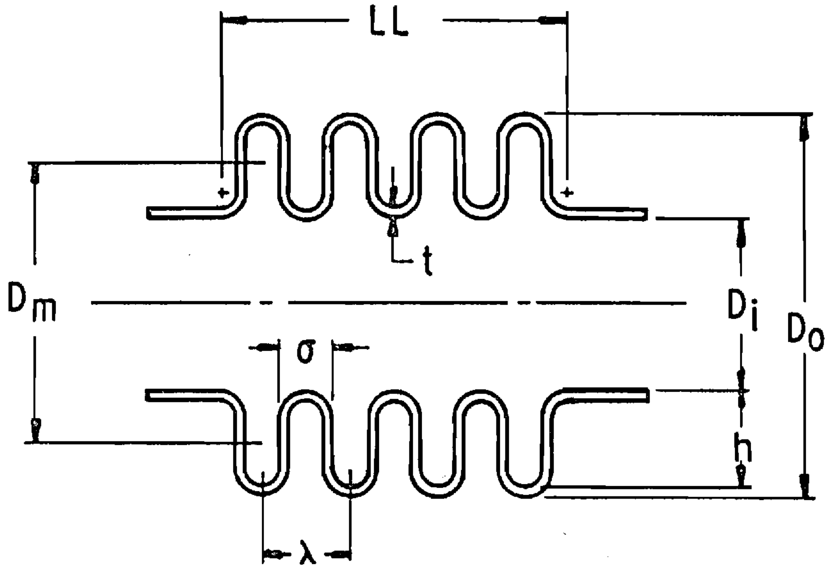
$$k = 1.7 \times 10^5 \text{(lb/in)} \quad .$$

Available experiments [1,2] yield the value of damping as

$$\zeta = 0.0029 .$$

Consequently, for one convolution the natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.7 \times 10^5}{0.1296744}} = 1145 \text{ (rad/sec)} ,$$



N_c = Number of convolutions counted from the outside.

N_p = Number of plys

Bellows Number	D_i	D_o	D_m	LL	N_c	N_p	h	λ	σ	t
#101, 102, 105, 112	1.49	2.22	1.85	1.80	7	1	.35	.26	.125	.013
#103, 104, 110, 114	1.46	2.03	1.74	1.77	8	1	.27	.22	.144	.013
P/N-08047, #106	1.98	2.32	2.16	2.15	9	1	.16	.24	.095	.006
P/N-08049, #107	1.97	2.33	2.15	2.15	9	2	.17	.24	.140	.006
P/N-08051, #108	2.00	2.34	2.17	2.15	9	3	.15	.24	.140	.006
P/N-08052, #113	1.99	2.45	2.22	2.41	14	1	.22	.17	.090	.013

Note: These dimensions are for geometric description only. Actual dimensions for a particular bellows may vary somewhat from these representative dimensions.

TABLE I. DIMENSIONAL DATA FOR TEST BELLOWS CITED IN REF. [1,2].

or

$$f_n = \frac{\omega_n}{2\pi} = 182 \text{ (Hz)} \ .$$

For two convolutions, there are three ($2N_c$-1) natural frequencies. The characteristic equation from Equation (8) is

$$(2k - \omega^2 m) \ [m^2 \ \omega_n^{\ 4} - 4mk(1 + 2\zeta)^2 \omega_n^{\ 2} + 2k^2] = 0 \ ,$$

This results in

$$\omega_{n1} = 876.3 \text{ (rad/sec)}; \qquad f_{n1} = 139.5 \text{ (Hz)}$$

$$\omega_{n2} = 1145 \text{ (rad/sec)}; \qquad f_{n2} = 182 \text{ (Hz)}$$

$$\omega_{n3} = 2115.7 \text{ (rad/sec)}; \qquad f_{n3} = 336.7 \text{ (Hz)}$$

II.3 A Comment on Model Stiffness

The fundamental natural frequency of the model decreases as the number of convolutions increases. This is because the overall stiffness of the flexible pipe decreases as N_c increases. A comparison with the numerical results reported in the literuatre [1,2] indicates that the natural frequencies obtained in the present work are all much lower than those obtained previously. This is attributed to the fact that the previous mathematical model has imposed too many constraints which inevitably increase the overlll stiffness of the sytem. This results in raising the overall natural frequencies calculated. This in turn results in a prediction of the fatigue life of the flexible pipe which is too conservative. It is claimed that the model and its calculation procedure adopted here are fundamental, explicit and more likely to lead to experimentally observed fatigue life of flexhoses than those reported in the literature cited previously.

III. STRAIN AND FATIGUE LIFE ANALYSIS

III.1 Fatigue Behavior by Strain-Life Approach

Modern approach to characterize the fatigue behavior of materials is to focus on the cyclic strain-life [5]. The total strain, ε , is considered as having an elastic ε_e , and a plastic, ε_p , components. Expressed as strain amplitudes, this implies

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} . \tag{13}$$

The elastic strain-life can be expressed as

$$\frac{\Delta\varepsilon_e}{2} = \frac{\sigma_a}{E} = \frac{1}{E}\,\sigma_f{}'\,(2N_f)^b \quad , \tag{14}$$

where σ_a and $\sigma_f{}'$, respectively, represent the true stress amplitude and the fatigue strength coefficient, E is the elastic modulus, b is the fatigue strength exponent (typically, b varies between -0.05 and -0.12) and N_f is the number of cycles to failure. The plastic strain-life is related by a power function as

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f{}'(2N_f)^d \quad , \tag{15}$$

where $\varepsilon_f{}'$ is the fatigue ductility coefficient and d is the fatigue ductility exponent (typically, d varies betwen -0.5 and -0.7). The total strain amplitude may thus be expressed as

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f{}'}{E}\,(2N_f)^b + \varepsilon_f'(2N_f)^d \tag{16}$$

Equation (16) is called the strain-life relation and forms the basis for the strain-life approach to predicting fatigue behavior of such material as wrought metals. For thin-walled shells, such as the flexhose or bellows in question, N_f is typically replaced by cycles to initiate the crack, N_I. For steel bellows

$$b \approx -0.1,$$

$$\sigma_f' \approx 150{,}000 \text{ psi},$$

$$0.5 \leqslant \varepsilon_f' \leqslant 1 ,$$

and $$d \approx -0.7.$$

III.2 Estimation of Fatigue Life

The flexhose with N_c convolutions has $(2N_c - 1)$ degrees of freedom, with the corresponding $(2N_c - 1)$ responses. Each degree of freedom vibration leads to N_I number of cycles to initiate the crack. The operational value from safe design point of view is the smallest of the calculated N_I values.

As an example of calculation of the fatigue life, consider a flexhose with two convolutions. The properties of the hose, Bellows #105, are listed in Table I. The natural frequencies and forced vibration responses for each convolution are calculated by the procedure discussed under section II of this report and a computer program called "FLEXHOS" listed in the Appendix. Several flow conditions are examined in the calculations. The flow velocity, V is related to the Struhal number, S, of the flow and can be used to estimate the frequency of forced vibrations, f_e, by using the relation

$$f_e = \frac{VS}{\sigma}, \tag{17}$$

where σ is the convolution width. When f_e coinsides with any of the natural frequencies, there occurs a resonance of the system in that mode, and the fatigue life is greatly reduced.

If it is assumed that the deformation of each lumped mass approximately equals the vibration response induced by the flowing fluid, the effective strain is given by

$$\Delta\varepsilon_i = \frac{|x_i|}{h + (\pi - 2)a} \quad , \tag{18}$$

where $|x_i|$ is the amplitude of deformation for the ith lumped mass. Use of the calculated $\Delta\varepsilon$ from Equation (18) into the strain-life relation (16) gives the corresponding number of cycles to failure, N_{Ii}. The actual time to failure τ_{fi} is then obtained as

$$\tau_{fi} = \frac{N_{Ii}}{f_e} \quad . \tag{19}$$

Although the program "FLEXHOS" is designed only for this example, it may be expanded to the general case of N_c convolutions.

The input to the program is the frequency of forced vibration that corresponds to a specific flow velocity related by the Struhal number. The output is the fatigue life time. Table II shows a summary of calculation of fatigue life estimation in seconds for three different natural frequencies [because there are three degrees of freedom for a two convolution system]. The calculations are performed for flow velocities ranging from 2.4 ft/sec to 14 ft/sec. The table also shows the excitation frequency and the number of cycles to failure for each case. Figure 2 shows this data on a time-to-fail vs input-velocity diagram. It is seen that the fatigue life is surprisingly short near the natural frequencies. In other words, the effect of resonance is very critical. In order for the fatigue life to be longer than sixty (60) seconds, it is necessary to avoid the flow velocity between 3.05 to 3.78 ft/sec and between 8.04 and 8.46 ft/sec for a two convolution model.

III.3 Remarks on the Results of Fatigue Life Analysis

From Figure 2 and Table II it is seen that when the excitation frequency approaches the natural frequency, the expected resonance occurs and the fatigue life time is substantially reduced. The absence of resonance at an excitation frequency of 129 Hz needs to be examined further before making more definite statements.

It should be noted that the present mathematical model includes many simplifications. As a structure vibrates in a fluid, its motion is retarded by fluid drag, composed of viscous and pressure drags. Viscous drag is produced by shearing of fluid between the free stream and the surface of the structure, and the pressure drag by the flow separating from the structure and forming a turbulent wake. The drag force acts in line with the relative flow velocity and opposes the relative motion. In the cited literature [1,2], the drag coefficient, C_D, is obtained only in terms of system geometry. One way to find C_D analytically is to examine the vortex shedding effects and pressure drop within each convolution of the flexible pipe at moderate to high flow velocities. A more rigorous mathematical model would then lead to an accurate prediction of fatigue life for the flexible pipe.

TABLE II. Numerical Results of Estimated Fatigue Life.

Flow Velocity V (ft/s)	Excitation frequency, f_e	Natural frequency, f	Number of cycles allowed, N_I	Estimation of fatigue life, t (Seconds)
2.4225	50 (Hz)		5.5×10^7	1.1×10^6
2.9070	60		2.9×10^4	490
3.3915	70	69.83 (Hz)	7.1	0.102
3.6337	75		1.6×10^3	21.488
3.8760	80		7.3×10^3	90.98
4.1182	85		2.3×10^4	268.74
4.3605	90		6.2×10^4	693.08
4.450	100		3.7×10^5	3671.49
5.3295	110		1.7×10^6	1.5×10^4
5.8140	120		6.4×10^6	5.3×10^4
6.2985	130	129.03	2.5×10^7	1.9×10^5
6.7830	140		1.7×10^8	1.2×10^6
7.2674	150		1.1×10^{10}	7.5×10^7
7.7519	160		9.6×10^{12}	5.9×10^{10}
8.2364	170	168.58	655	3.85
8.7209	180		4.7×10^4	262.97
9.2054	190		2.5×10^5	1.3×10^3
9.6899	200		6.4×10^5	3.2×10^3
10.1744	210		1.2×10^6	5.7×10^3
10.6589	220		1.9×10^6	8.4×10^3
11.1434	230		2.6×10^6	1.1×10^4
11.6279	240		3.3×10^6	1.4×10^4
12.1124	250		4.1×10^6	1.6×10^4
13.0814	270		5.6×10^6	2.1×10^4
14.0503	290		7.1×10^6	2.4×10^4

Example: The natural frequencies for bellows #105 with two convolutions are found to be 69.8 Hz and 168.6 Hz.

$$f_e = \frac{VS}{\sigma},$$

where V is flow velocity;
S is Strouhal number;
σ is convolute width.

In order for the fatigue life to be longer than 60(s), it is necessary to avoid the ranges of flow velocity between (3.05 ∿ 3.78 ft/s) and (8.04 ∿ 8.46 ft/s).

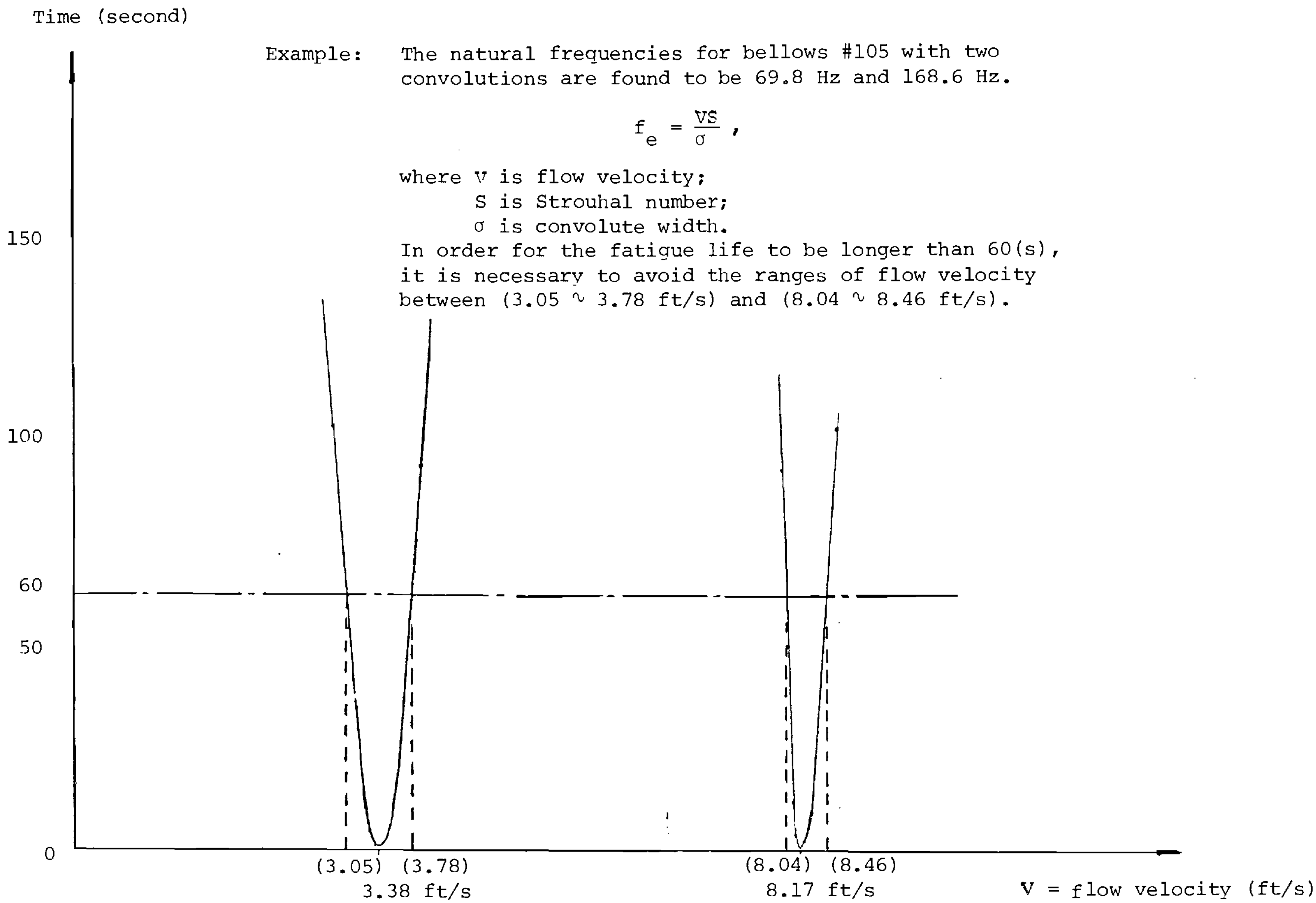


Figure 2. Calculated Fatigue Life Variation with Flow Velocity.

IV. RECOMMENDATIONS FOR FUTURE WORK

A method to calculate the longitudinal structural vibration modes of a flexhose with a fluid flowing through it, together with a strain-life analysis to estimate its fatigue life has been established in this research. The method has been applied to a model with two convolutions to predict ranges of flow velocity for which resonance occurs and fatigue life is drastically reduced. It is expected that the method may also be applied to a flexhose with a large number of convolutions. However, it is first necessary to further examine the model with a view toward modifying some of the underlying assumptions.

As regards the longitudinal vibration modes, it is recognized that the flexhose is a continuous system possessing an infinite number of natural vibration modes. The coarse assumption of $(2N_C - 1)$ lumped masses may be refined by increasing their number in a finite element computation scheme. The model also needs to include end corrections for spring stiffness, since the flexhose is fixed at both ends. Modeling of the forcing function due to fluid dynamic coupling is a crucial issue that needs a more comprehensive attention. This force plays a dominant role in determining the structural vibration response. It is quite essential to adequately identify the roles of the Reynolds and Struhal numbers and the system geometry in determining the forcing function. This latter task cannot be accomplished by using a single drag coefficient.

An issue that is not included in this report concerns the accoustic vibration modes. When the excitation frequencies are high, the acoustic modes provide the primary mechanism for failure. A mathematical model that can predict acoustic resonance frequencies should be developed to estimate the fatigue life of flexhoses at high frequencies.

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APPENDIX

COMPUTER PROGRAM FOR FATIGUE LIFE ESTIMATION

Start

Enter dimensions of particular bellows

Enter the excitation frequency, f_e, corresponding to the given flow velocity, V.

Calculate the spring constant, k, for each element

$$k = 2N_c k_a$$

Calculate the total mass, m

$$m = m_m + m_a$$

Determine the natural frequencies of the particular bellows

$$|[K] - \omega_n^2 [M]| = 0 \ , \ n = 1,2,\ldots$$

Solve for the forced vibration responses and choose the largest value See Eq. (9)

Calculate the strain

$$\Delta\varepsilon = \frac{|x_i|}{h + (\pi-2)a}$$

Find the allowable number of cycles N_I

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_I)^b + \varepsilon_f'(2N_I)^d$$

Give the estimation of fatigue life

$$t = \frac{N_I}{f_e}$$

Print f_e, V, N_I, t

Stop

Program Symbol	Definition
A	$a = \frac{\lambda - \sigma}{2}$, approximate radius of convolution
PAI	π
RK	Spring constant, k
RM1	Material mass, m_m
RM2	Added mass, m_a
RM	Total mass, $m = m_m + m_a$
C	Damping coefficient, c
FRE1,FRE2,FRE3	Natural frequencies, f_{n1}, f_{n2}, f_{n3}, (Hz)
V	Flow velocity, V(ft/s)
OMEGA	Angular velocity $\omega_e = 2\pi f_e$
D	Determinant $D = \|[K]-\omega_e^2[M] + i2\omega_e[C]\|$
F	External fluid force
X1	The ith lumped mass' response, $\|x_i\|$
DE	Strain, $\Delta\varepsilon$
RNI	Allowable number of cycles, N_I
TIME	Fatigue life time, t

```
      PROGRAM FLEXHOS(INPUT,ANSWER,TAPE5=INPUT,TAPE6=ANSWER)
C
C
C     ENTER THE DIMENSIONS OF BELLOWS #105
C
C
      DATA DM/1.85/,DO/2.22/,DI/1.49/,RH/.35/,T/.013/,RNP/1.0/,
     *     RLAM/.26/,SIGMA/.125/,ZETA/.0029/,S/.215/,CD/.0325/,
     *     EPS/1.0/,SIG/150000.0/,E/2.8E+07/,ROM/.28/,ROF/.036042/
      READ(5,*) FRE
      A=0.5*(RLAM-SIGMA)
      PAI=4.0*ATAN(1.0)
      RK=2.0*DM*E*RNP*(T/RH)**2
      RM1=PAI*ROM*T*RNP*DM*(PAI*A+(RH-2.0*A))
      RM2=0.5*PAI*ROF*DM*RH*(2.0*A-T*RNP)
      RM=32.2*(RM1+RM2)
      C=2.0*ZETA*(RK*RM)**0.5
C
C
C     CALCULATE THE NATRUAL FREQUENCIES
C
C
      FRE1=(1.0/(2.0*PAI))*((2.0-2.0**0.5)*RK/RM)**0.5
      FRE2=(1.0/(2.0*PAI))*(2.0*RK/RM)**0.5
      FRE3=(1.0/(2.0*PAI))*((2.0+2.0**0.5)*RK/RM)**0.5
      V=FRE*SIGMA/S
      OMIGA=2.0*PAI*FRE
      A11=2.0*RK**2-2.0*OMIGA**2*(2.0*RK*RM+C**2)+OMIGA**4*RM**2
      A22=RK-RM*OMIGA**2
      A1=(2.0*RK-OMIGA**2*RM)*A11-8.0*(C*OMIGA)**2*A22
      D=ABS(A1)
      F=4.025*PAI*(DO**2-DI**2)*CD*ROF*V**2
      A3=F*((2.0*RK-OMIGA**2*RM)*(3.0*RK-OMIGA**2*RM)-6.0*(C*OMIGA)**2)
      A4=F*((2.0*RK-OMIGA**2*RM)*(4.0*RK-OMIGA**2*RM)-8.0*(C*OMIGA)**2)
      IF(ABS(A3).LT.ABS(A4)) GO TO 50
      X1=ABS(A3)
      GO TO 55
   50 X1=ABS(A4)
   55 DE=X1/(D*(RH-2.0*A+PAI*A))
      X1=(A3**2)**0.5
      DE=X1/(D*(RH-2.0*A+PAI*A))
      Y1=(2.0*SIG)/(DE*E)
      Y2=(2.0*EPS)/DE
      Y3=0.0
  100 W1=Y3**6*(Y3-Y1)
      IF(Y2.LT.W1) GO TO 200
      Y3=Y3+0.001
      GO TO 100
C
C
C     DETERMINE THE NUMBER OF CYCLES ALLOWED AND THE FATIGUE LIFE
C
C
  200 RNI=(Y3**10)/2.0
```

```
      TIME=RNI/FRE
      V=V/12.0
      WRITE(6,*) A1,D,A3,A4,X1,DE
      WRITE(6,400)FRE1,FRE2,FRE3
      WRITE(6,500)V,FRE
      WRITE(6,600)RNI,TIME
400   FORMAT(5X,F10.5,'(HZ)',5X,F10.5,'(HZ)',5X,F10.5,'(HZ)')
500   FORMAT(2X,'V=',F10.5,'(FT/S)',5X,'FRE=',F10.5,'(HZ)'//)
600   FORMAT(2X,'NUMBER OF CYCLES, RNI=',F20.5,5X,'TIME=',F20.5,'(S)')
      STOP
      END
EOI. 0 FILES. 1 RECS. 237 WORDS.
/
```