THE STUDY OF CHARGE-RETENTION CURRENTS
IN WEAK INTERACTIONS

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Dedication to my parents

whose present location makes them

unaware of this work

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CHAPTER I

THE PRESENT STATE OF WEAK INTERACTIONS

Classification

As is well known, all slow processes can be divided into three classes:

1. Leptonic processes with $\Delta S = 0$

   $\mu \longrightarrow e + \nu + \bar{\nu}$

   $n \longrightarrow p + e + \nu$

   $\pi \longrightarrow \mu + \nu, \ e + \nu$

   $\mu + p \longrightarrow n + \nu$

   $\Sigma \longrightarrow \Lambda + e^- + \nu$

2. Leptonic processes with $|\Delta S| = 1$ e.g.,

   $K \longrightarrow \mu + \nu$

   $K \longrightarrow \pi + \mu + \nu, \ \pi + e + \nu$

   $\Lambda \longrightarrow p + \mu + \nu, \ p + e + \nu$

3. Non leptonic processes with $|\Delta S| = 1$ e.g.,

   $K \longrightarrow 2\pi, \ 3\pi$

   $\Lambda(\Sigma) \longrightarrow N + \pi$ (N = nucleon)

   $\Xi \longrightarrow \Lambda + \pi$

These processes are characterized by comparable coupling constants, G, of the interactions responsible for their occurrence. The values of G^2 range from $10^{-12} - 10^{-14}$ in units of $\hbar = c = m_\pi = 1$. Moreover there is parity violation in all of these processes, except in two cases:

$\Sigma^+ \rightarrow n + \pi^+$, $\Sigma^- \rightarrow n + \pi^-$ which will be discussed later (p. 13). Also CP-invariance and T-invariance seem to hold.

Therefore, one is tempted to assume only one basic mechanism to be responsible for all these slow processes.

By far the most detailed experimental information exists for class I; next comes class III, while the class II is still largely unchartered.

Since all leptonic processes involve neutrinos, it is worthwhile to review the theory of the neutrino before discussing each class in detail.

Theory of the Neutrino

The presence of the neutrino was first assumed from the energy-momentum balance in β-decays by Pauli in 1933.

It has zero mass zero electric charge and spin $\frac{1}{2}$. A theory for such a particle had been proposed by Weyl (1929) but remained rather obscure until 1957. The reason is that in Weyl's concept, the neutrino is described by a two-component wave function φ which satisfies

$$\vec{\sigma} \cdot \vec{p} \; \varphi = -i \frac{\partial \varphi}{\partial t}$$

Due to the zero mass, we can rewrite for a particular energy:

$$\frac{\vec{\sigma} \cdot \vec{p}}{p} \varphi = - \varphi$$

Then the Weyl neutrino has a definite handedness or helicity $h = -1$. But the helicity changes its sign by inversion of the coordinates, therefore in the period when there was no doubt about parity conservation, this theory was physically not taken seriously.

The Weyl equation can be understood by the following simple argument: for the description of a zero mass, zero charge, spin $\frac{1}{2}$ particle, it is natural to use a 2-component object, since there are 2 spin states. Also its dynamical properties are made up of the spin $\frac{\vec{\sigma}}{2}$, the momentum $\vec{p}$ and the energy E. Then the only Lorentz covariant equations which contain all dynamical properties of the particle are:

$$\vec{\sigma} \cdot \vec{p}\,\varphi = -i\,\frac{\partial \varphi}{\partial t} \tag{1}$$

$$\vec{\sigma} \cdot \vec{p}\,\chi = +\,i\,\frac{\partial \chi}{\partial t} \tag{2}$$

Note, that χ transforms as $\sigma_2\varphi^*$. From a pair of φ or χ alone, one can only form vectors under proper Lorentz transformations namely $(\varphi^+\varphi, -\varphi^+\vec{\sigma}\varphi)$ and $(\chi^+\chi, \chi^+\vec{\sigma}\chi)$. From a pair of φ and χ one can form scalars $\varphi^+\chi$ and $\chi^+\varphi$, tensors $\varphi^+\sigma^\mu\sigma^\nu\chi$ where

$$\sigma^\mu = \left|\begin{array}{ll} +1 & \text{for } \mu = 0 \\ -\vec{\sigma} & \text{for } \mu = 1,2,3. \end{array}\right.$$

From (1) and (2) we see immediately that the particle associated with φ is left-handed, i.e., the helicity $h = -1$ while χ corresponds to a right-handed particle, i.e., $h = +1$.

If we define the antiparticle in the Dirac sense, then we can consider χ as anti-particle of φ or vice versa. We can also consider χ and φ as 2 Weyl particles: two particles left and right handed and correspondingly two antiparticles right and left handed. The question of which possibility is realized in nature is left for experiments to decide. Since the discovery of parity non-conservation in β decay it has been suggested that the electron neutrino, i.e., the neutrino associated with β decay, be described by the Weyl equations.

The expressions electron neutrino, and later on μ-meson neutrino, are used generically to include neutrino and antineutrino.

Various experiments on the polarization of $e^{\pm}$ from $\beta^{\pm}$ decays yielded $h(e^{\pm}) = \pm 1$ (h = helicity) and thereby helped to prove the definite handedness of the electron neutrino.

A direct test of the completely polarized nature of the neutrino from β decay came also from the Cowan Reines experiment (1959).

From now on we shall consider the two component theory as a working hypothesis for the electron neutrino since all the present evidence is in accord with this assumption. As discussed above one now has to find out how many Weyl neutrinos there are in β-decay phenomena.

The absence of a definite observation of double β-decay, the outcome of the Davis experiment (1955), and the polarization experiments on β decays may be interpreted as evidence for the existence of two kinds of electron neutrinos and that β^+ decays yield left handed neutrino (ν_L^e) whereas β^- decays yield right handed neutrino (ν_R^e)

$$\beta^- \text{ decay}: \quad n \rightarrow P + e^- + \nu_R^e$$

$$\beta^+ \text{ decay}: \quad p \rightarrow n + e^+ + \nu_L^e$$

Let us take (e^-, ν_L) as leptons and (e^+, ν_R) as anti-leptons. Then one has a lepton conservation law in which electrons participate together with their neutrinos.

Fermions are composed of leptons, i.e., light fermions, e, ν, and μ, and baryons, i.e., heavy fermions e.g., p, n and Λ.

Experiments show that in all observed processes there is baryon conservation. The stability of nuclei proves this law for nucleons. One then is tempted to believe in a lepton conservation which includes μ-mesons. Let us look at the observed $\pi - \mu$ and $K - \mu$ decays:

$$\pi \rightarrow \mu + \nu^{\mu}$$

$$K \rightarrow \mu + \nu'^{\mu}$$

where ν^{μ}, ν'^{μ} are neutral particles with masses very small compared to the μ-mass. For instance, the upper limit of the ν^{μ} mass is 3 Mev (Yang, 1962).

Since the π-spin is zero, the polarization measurements on μ from π decay would yield the ν^{μ}-helicity. The experimental result is $h(\mu^{\pm}) \simeq \mp 1$ (Bardon, 1961) it means

$$\pi^{+} \rightarrow \mu^{+} + \nu_{L}^{\mu}$$

$$\pi^{-} \rightarrow \mu^{-} + \nu_{R}^{\mu}$$

Again the subscript L and R stand for left handed and right handed.

Similar polarization measurements were done for the $K - \mu$ decay. One obtained the same result: $h(\mu^{\pm}) \simeq \mp 1$ (Coombes, 1957).

However one cannot draw any conclusion as to the ν'^{μ} helicity because the K spin is experimentally not definitely known. For the sake of simplicity, we shall assume $h(\nu'^{\mu}) = h(\nu^{\mu})$, it then follows that the K spin is zero. Naturally one faces a new problem: the identity of ν^e and ν^μ. An experiment done at Brookhaven exhibited a difference between ν^μ from π and K decays and ν_e from β decays (Danby, 1962).

In order to maintain a single law of lepton conservation one can choose:

$$\text{leptons}: \ \nu_L^e \ \nu_R^\mu, \ e^-, \ \mu^+$$

$$\text{antileptons}: \ \bar{\nu}_R^e \ \bar{\nu}_L^\mu, \ e^+, \ \mu^-$$

Then the μ-decay reaction becomes $\mu^+ \rightarrow e^+ + \nu_L^e + \nu_R^\mu$ and the neutrinoless $\mu \rightarrow e + \ldots$ transition would be ruled out since μ and e have opposite lepton number.

However it is known that e and μ behave identically in electromagnetic interactions. There are also supports for $e - \mu$ universality in weak interactions. Indeed, the $(\mu\nu)$ coupling to nucleons in μ-meson capture is of roughly the same strength as the $(e\nu)$ coupling in β decay. Moreover, the hypothesis of a universal coupling for leptons would predict for the branching ratio

$$\frac{\pi \rightarrow e + \nu}{\pi \rightarrow \mu + \nu} \simeq 1.3 \times 10^{-4}$$

which is in striking agreement with experimental results.

Hence one is more inclined to assign the same lepton number to $e^\pm$ and $\mu^\pm$. In order to rule out neutrinoless transitions $\mu \rightarrow e + \ldots$ one assumes two separate conservation laws: the e-number conservation (e, ν^e) and the μ-number conservation (μ, ν^μ).

In reaction producing two neutrinos such as the μ-decay we then have: $\mu^+ \rightarrow e^+ + \nu_L^e + \bar{\nu}_R^\mu$.

The fact that we have to separate e-number and μ-number conservations is not too surprising.

Indeed we encounter a similar situation in electromagnetic interactions: we do not have an electric current

$$j_\lambda^{e.m.} = \bar{\Psi}_e \gamma_\lambda \Psi_\mu \quad \text{i.e.} \quad \mu \not\rightarrow e + \gamma$$

Although we have $e \rightarrow e + \gamma$ or $\mu \rightarrow \mu + \gamma$.

This fact may be understood through its connection with the gauge invariance of the theory. Because if there exists an electromagnetic interaction of the form:

$$\bar{\Psi}_e \gamma_\lambda \Psi_\mu A_\lambda$$

then under the transformation $A_\lambda \rightarrow A_\lambda + \partial_\lambda \chi$ the action integral would suffer the following variation:

$$\delta S = \int d^4x \bar{\Psi}_e \gamma_\lambda \psi_\mu \partial_\lambda \chi$$

$$= \int d^4x \, \partial_\lambda (\bar{\Psi}_e \gamma_\lambda \psi_\mu \chi) - \int d^4x \, \partial_\lambda (\bar{\Psi}_e \gamma_\lambda \psi_\mu)$$

$$= - \int d^4x \, \chi \, \partial_\lambda (\bar{\Psi}_e \gamma_\lambda \psi_\mu)$$

Since $m_e \neq m_\mu$ so $\partial_\lambda (\bar{\Psi}_e \gamma_\lambda \psi_\mu) \neq 0$ and since χ is arbitrary, so $\delta S \neq 0$. There is no gauge invariance.

We now have two different lepton assignments. It seems to us that in a (V - A) theory of interaction the first assignment i.e. a single lepton conservation, cannot be distinguished from the second assignment i.e. two separate e and μ number conservation laws. Let us bombard protons with neutrinos from π decay.

In the second assignment we have:

$$\pi^+ \rightarrow \mu^+ + \nu_L^\mu$$

$$\nu_L^\mu + p \not\rightarrow n + e^+$$

The second reaction does not go because there is no e-number (or μ-number) conservation.

In the first assignment we have:

$$\pi^+ \rightarrow \mu^+ + \bar{\nu}_L^\mu$$

because μ^+ is now a lepton.

$$\bar{\nu}_L^\mu + p \not\rightarrow n + e^+$$

the second reaction satisfies lepton conservation law but it cannot occur because such a reaction would be tantamount to: $n \rightarrow p + e^- + \bar{\nu}_L^\mu$ which is experimentally disproven.

From now on we shall use for convenience sake one convention, the separate e-number and μ-number conservation laws.

Extension of Theory to Finite Mass Fermions

Weak interactions involve, beside neutrinos, many finite mass fermions - even ν^μ might have a finite mass. So, before discussing the structure of weak interactions, we shall review briefly the theory of fermions with mass.

To us, the natural way is to start with zero mass fermions, i.e., with the Weyl equations:

$$(E + \vec{\sigma} \cdot \vec{p})\varphi = 0$$

$$(E - \vec{\sigma} \cdot \vec{p})\chi = 0$$

In order to arrive at dynamical equation for finite mass fermions, we need to write a Lorentz covariant relation manifesting all dynamical quantities: mass m, spin $\frac{\vec{\sigma}}{2}$, energy E and momentum $\vec{p}$.

We notice that the $(E + \vec{\sigma} \cdot \vec{p})\varphi$ transform as χ under proper Lorentz transformation. Indeed:

$$\varphi^+ \sigma_\lambda \varphi \equiv (\varphi^+\varphi, -\varphi^+ \vec{\sigma}\varphi)$$

is a 4-vector. So

$$\varphi^+ \sigma_\lambda p_\lambda \varphi \equiv \varphi^+ (E + \vec{\sigma} \cdot \vec{p})\varphi$$

is a scalar.

Since $\varphi^+\chi$ is a scalar, $(E + \vec{\sigma} \cdot \vec{p})\varphi$ transforms like χ. It is now clear that the simplest Lorentz covariant equation with a finite mass is:

$$(E + \vec{\sigma} \cdot \vec{p})\varphi = m\chi \qquad (1)$$

and similarly for $(E - \vec{\sigma} \cdot \vec{p})\chi$, we arrive at:

$$(E - \vec{\sigma} \cdot \vec{p})\chi = m\varphi \ . \qquad (2)$$

We see that a finite mass fermion can be described by two first order 2-component equations (1) and (2) where the mass plays the role of mixing

φ and χ so that parity is conserved. Indeed from (1) and (2) we get:

$$(E + \vec{\sigma} \cdot \vec{p})(E - \vec{\sigma} \cdot \vec{p})\chi = m(E + \vec{\sigma} \cdot \vec{p})\varphi = m^2\chi$$

Or

$$(E + \vec{\sigma} \cdot \vec{p})(E - \vec{\sigma} \cdot \vec{p})\chi = m^2\chi$$

The last equation containing only χ, is invariant under inversion. Note that this is a second order equation for χ.

We can go over to the Dirac theory, also invariant under inversion, by defining:

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \qquad \gamma_t = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma_k = \begin{pmatrix} 0 & +\sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

and writing (1) and (2) together as:

$$(\gamma_\lambda p_\lambda - m)\psi = 0 \quad \text{(Dirac equation)}$$

The operators $\vec{\alpha}$ and γ_5 which are defined respectively by:

$$H = \vec{\alpha} \cdot \vec{p} + \gamma_t m$$

$$\gamma_5 \psi_\nu = \psi_\nu$$

$$\text{where } \psi_\nu = \begin{pmatrix} \varphi \\ 0 \end{pmatrix} \text{ and } \psi_{\bar{\nu}} = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

$$\gamma_5 \psi_{\bar{\nu}} = -\psi_{\bar{\nu}}$$

would be given in this representation as:

$$\vec{\alpha} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

One notes here the following relations:

$$\gamma_t \gamma_k = \alpha_k \qquad \sigma_k = -\gamma_5 \alpha_k$$

Before discussing the structure of weak interactions, it is worthwhile to remark that, for zero mass fermions, φ and χ which describe two different particles, form a complete orthonormal basis in the two dimensional space. They are eigenvectors u_+ and u_- of the unitary and hermitian operator $\vec{\sigma} \cdot \hat{p} = \vec{\sigma} \cdot \frac{\vec{p}}{p}$.

Consequently we obtain the closure equation:

$$\varphi\varphi^+ + \chi\chi^+ = I \equiv \text{unitary operator.} \quad \text{(zero mass)}$$

For finite mass fermions, φ and χ are two linearly dependent vectors, they describe one particle, hence $\varphi\varphi^+ + \chi\chi^+ \neq I$

Nevertheless, we can arrive at some closure equation. Let us consider a particular case when:

$$\varphi = \lambda \; u_+ \qquad (\lambda = \text{arbitrary constant})$$

Then

$$\chi = \frac{E + \vec{\sigma} \cdot \vec{p}}{m} \varphi = \frac{E + p}{m} \lambda \; u_+$$

$$= \lambda' \; u_+$$

where $\lambda' = \frac{E + p}{m} \lambda$ and $\varphi^+\chi = \lambda\lambda'$ which is Lorentz invariant. The choice $\varphi^+\chi = 1$ yields $\lambda\lambda' = 1$ hence: $\varphi \chi^+ = u_+ u_+^+$.

The completeness of the set (u_+, u_-) leads to the following closure equation:

$$\sum_s \varphi_s \chi_s^+ = \sum_s \chi_s \varphi_s^+ = I \qquad \text{(finite mass)}$$

where s denotes the spin state of the particle. These closure equations will be useful in the calculations of lepton covariants in leptonic weak processes.

Class I. Weak Interactions

It is now well established that the effective weak interaction density at low energy can be written:

$$\mu \text{ decay : } \frac{G^\mu}{\sqrt{2}} \bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu \bar{\Psi}_e \gamma_\lambda (1 + \gamma_5) \Psi_\nu$$

$$\beta \text{ decay : } \frac{G^\beta}{\sqrt{2}} \bar{\Psi}_p \gamma_\lambda (1 + x \gamma_5) \Psi_n \bar{\Psi}_e \gamma_\lambda (1 + \gamma_5) \Psi_\nu$$

*

$$\mu \text{ capture: } \frac{G^\beta}{\sqrt{2}} \bar{\Psi}_p \gamma_\lambda (1 + x \gamma_5) \Psi_n \bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu$$

where $G^\mu \simeq G^\beta$ with an error of less than 3%

$$\simeq (1.01 \pm 0.01) \frac{10^{-5}}{M_p^2} \quad M_p = \text{proton mass}$$

$$x \simeq 1.20$$

The striking similarity between μ and β decay theories -- apart from $x \neq 1$ in β decays -- and also the study of μ capture* have led to the conjecture of an unrenormalized universal 4-fermion interaction in the class of strangeness conserving leptonic processes, namely:

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*For μ-capture, agreement with experiment is poorer (Telegdi, 1962).

$$\mathcal{H} = \frac{G}{\sqrt{2}} \bar{\Psi}_1 \gamma_\lambda (1 + \gamma_5) \Psi_2 \bar{\Psi}_3 \gamma_\lambda (1 + \gamma_5) \Psi_4$$

Usually one refers to this theory as the V-A theory, i.e., the space-time structure is V-A with n, p, μ^- and ν^μ as particles; to be complete one needs to add that the helicities of e^- and ν^e are negative.

In two-component form, the interaction density becomes:

$$\mathcal{H} \propto (\varphi_1^+ \sigma_\lambda \sigma_2)(\varphi_3^+ \sigma_\lambda \varphi_4) = (\varphi_1^+ \varphi_2)(\varphi_3^+ \varphi_2) - (\varphi_1^+ \vec{\sigma}\varphi_2)(\varphi_3^+ \vec{\sigma} \varphi_4)$$

The weak interaction couples only the "left handed" part of the participating fermions.

The fact that $x \neq 1$ in β-decay may be interpreted as due to a strong interaction disturbance of the nucleons. This brings in an admixture $\frac{x-1}{x+1}$ of V + A which is:

$$\bar{\Psi} \gamma_\lambda (1 - \gamma_5) \Psi = \chi^+ \sigma_\lambda \chi$$

i.e. there is a small part of "right handed" nucleon interaction.

Perhaps by some dynamical accident, the mixture due to strong interactions may yield parity conserving processes as in $\Sigma^+ \rightarrow n + \pi^+$ and $\Sigma^- \rightarrow n + \pi^-$.

In the class I, we still have other processes like:

$$\pi \rightarrow \mu + \nu \qquad \pi \rightarrow e + \nu \qquad \Sigma^- \rightarrow \Lambda + e^- + \nu$$

Due to the meager statistics of $\Sigma^- \rightarrow \Lambda + e^- + \nu$ (1 event) we shall only consider the first two processes. Following the usual way of analogy with electromagnetic interactions we regard

$$\pi \rightarrow \begin{cases} \mu \\ e \end{cases} + \nu$$

as a result of two successive steps:

$$\pi \rightarrow p + \bar{n} \rightarrow \begin{matrix} \mu + \nu \\ e + \nu \end{matrix}$$

(strong) (weak)

Before proceeding further, we note here an interesting property of the (V - A) 4-fermion coupling on the reordering of the spinors, namely the P_{13} invariance, i.e. the permutation between particle 1 and particle 3 does not lead to any physical change.

We have:

$$\bar{\Psi}_1 \gamma_\lambda (1 + \gamma_5) \Psi_2 \bar{\Psi}_3 \gamma_\lambda (1 + \gamma_5) \Psi_4 = \pm \bar{\Psi}_3 \gamma_\lambda (1 + \gamma_5) \Psi_2 \bar{\Psi}_1 \gamma_\lambda (1 + \gamma_5) \Psi_4$$

The $\pm$ signs correspond respectively to the assumption that the different Ψ's anticommute or commute.

In the c-number theory where the ψ's commute we have:

$$\bar{\Psi}_1 \gamma_\lambda (1 + \gamma_5) \Psi_2 \bar{\Psi}_1 \gamma_\lambda (1 + \gamma_5) \Psi_2 = 0$$

Characteristics of the 4-fermion interaction

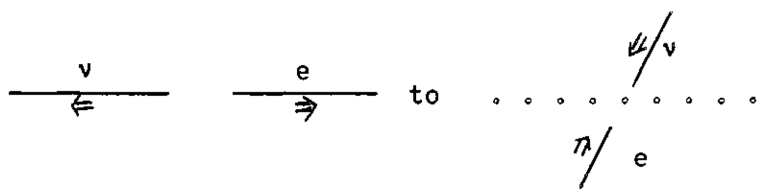
The dimension of the Dirac field is $\frac{1}{L^{3/2}}$ since $\int \psi^+ \psi \, dv = 1$. A product of 4-fermion fields has the dimension of $\frac{1}{L^6}$. The Hamiltonian density has the dimension of energy x L^{-3} or L^{-4} in units of $\hbar = c = 1$ where energy has the dimension L^{-1}.

Hence, in "natural units" the coupling constant G of four fermions interacting directly has the dimension of L^{-2}.

From experiment we know that $G \simeq \frac{10^{-5}}{M_p^2}$ where M_p is the proton mass. So the weak coupling constant is not a dimensionless constant and it is not so clear what we mean by "weak." For example, the cross section of $e - \nu$ scattering is given by $\sigma = \frac{4}{\pi}(G)^2 q^2$ where q is the incoming ν-energy and m_e is assumed to be zero (Bernstein, 1962).

For q small compared to M_p we can say the interaction is weak, and that is the case for β decay, μ decay, etc.

Since there are no spin changes possible in the e-ν scattering (see figure below) and the interaction is a contact coupling, there is only S-wave scattering.



In the center of mass frame.

The familiar partial wave analysis of scattering transitions yields:

$$\frac{d\sigma}{d\Omega} = \frac{\pi}{q^2} \sin^2 \delta_o \quad \text{or} \quad \sigma = \frac{4\pi^2}{q^2} \sin^2 \delta_o$$

where δ_o denotes the phase-shift of the S-wave. Since $\sin^2 \delta_o \leq 1$, the cross section will show a "unitary breakdown" at:

$$\frac{4\pi^2}{q^2} = \frac{4}{\pi}(G)^2 q_{max}^2 \longrightarrow \quad q_{max} \simeq 300 \text{ Bev}$$

which was to be expected since the unitarity condition of the scattering-matrix connects higher order processes with first order processes (Bernstein, 1962).

Unitarity however can be maintained in first order processes alone if higher order processes are negligible.

To have a good picture of the unitarity breakdown, let us calculate the $e - \nu$ cross-section at $q \simeq 300$ Bev. Then:

$$G\, q^2 \simeq \frac{10^{-5}}{M_p^2} (300 \text{ Bev})^2 \simeq 1 \quad \text{since} \quad M_p \simeq 1 \text{ Bev}$$

and

$$\sigma \;\alpha\; G^2 q^2 \simeq G = \frac{10^{-5}}{M_p^2} = \frac{10^{-5}}{(1 \text{ Bev})^2} .$$

But 1 Bev $\simeq 10^{14}$ cm^{-1} and thus $\sigma \simeq 10^{-33}$ cm^2. This means the weak interaction, to first order, becomes "strong."

At high energy, higher order weak processes might compete with first order processes because, as we just saw, the coupling constant G might become "strong."

<u>Effects of Strong Couplings on Weak Interactions</u>

So far we have considered the weak coupling constant G as being independent of the energy of the particles, or more precisely on the momentum transferred. Actually, as we shall see, the coupling constant G is energy dependent if we do not treat the particles as point particles. We know nucleons have an extension a bit smaller than 10^{-13} cm and the size corrections should in general be taken into account.

Since a reliable theory of the π-meson cloud surrounding a nucleon is not available, we must confine ourselves to a phenomenological description based on the condition of Lorentz covariance.

The weak vector nucleon current is:

$$V_\lambda = \langle P \mid \bar{\Psi}_p \gamma_\lambda \Psi_n \mid N \rangle$$

where $|P\rangle$ and $|N\rangle$ stand for physical proton and neutron. For free particles $V_\lambda = \bar{p}\gamma_\mu n$ where p, n stand for free Dirac spinors. In the most general case we have (Bernstein, 1962):

$$V_\lambda = \langle P \mid \bar{\Psi}_p \gamma_\lambda \Psi_n \mid N \rangle = \bar{p}\,[\gamma_\lambda F_1(q^2) + \sigma_{\lambda\alpha}\, q_\alpha\, F_2(q^2) + q_\lambda\, F_3\,(q^2)]\, n$$

and for the axial vector nucleon current:

$$A_\lambda = \langle P \mid \bar{\Psi}_p \gamma_\lambda \gamma_5 \Psi_n \mid N \rangle = \bar{p}[\gamma_\lambda \gamma_5 G_1(q^2) + \gamma_5 q_\lambda G_2(q^2) + \gamma_5\, \sigma_{\lambda\alpha}\, q_\alpha\, G_3(q^2)]\, n$$

q is the difference of the four-momenta of neutron and proton, F_L, G_L, $L = 1 - 3$ are the form factors. These expressions for V_λ and A_λ can be simplified. Without strong interactions we have:

$$V_\lambda \longrightarrow \bar{P}\,\gamma_\lambda\, n = \text{bare vector current}$$

$$A_\lambda \longrightarrow \bar{P}\,\gamma_\lambda \gamma_5 n = \text{bare axial vector current.}$$

Under the G conjugation which is defined as the product of charge-conjugation C and a rotation of 180° about the y-axis in isospace, i.e. $G = C\,e^{i\,\pi/2\,\tau_2}$ where $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, the bare vector current remains unchanged whereas the bare axial vector current changes its sign. These transformation properties cannot be varied as the strong interaction is gradually switched on, in order to establish the real vector V_λ and axial vector A_λ, because G commutes with the strong interaction.

Therefore we require: $V_\lambda \longrightarrow V_\lambda$

$$A_\lambda \longrightarrow -A_\lambda$$

under the G-conjugation. That leads to: $F_3(q^2) = G_3(q^2) = 0$. We are left with only four form factors in class I weak interactions. It is noted that $F_3(q^2) = 0$ is also a consequence of the CVC theory.

In the case of hyperon decays like $\Lambda \rightarrow p + e + \nu$; $\Sigma \rightarrow \Lambda + e + \nu$ the situation is much more complicated. Here there is no basis for reducing the number of form-factors.

The Conserved Vector Current in Weak Interactions

The remarkable result $G^\beta \simeq G^\mu$ led to the Feynman-Gell-Mann CVC hypothesis. To illustrate it, we first study the electromagnetic interaction of the proton.

The action integral of the physical system can be written:

$$S = \int \mathcal{L}\, d^4x$$

where $\mathcal{L}$ = total Lagrangian density

$$= \mathcal{L}_\gamma + \mathcal{L}_{\text{physical proton}} + \mathcal{L}_{\text{physical proton} - \gamma}$$

$\mathcal{L}_\gamma$ = free Lagrangian density of photon

$\mathcal{L}_{\text{physical proton - }\gamma}$ = interaction Lagrangian density of γ with the physical proton.

$\mathcal{L}_{\text{physical proton}} = \mathcal{L}_{\text{free proton}} + \mathcal{L}_{\text{free }\pi} + \mathcal{L}_{p-\pi}$

$\mathcal{L}_{p-\pi}$ = interaction Lagrangian density between proton and π meson.

The $\mathcal{L}_{\text{physical proton}-\gamma}$ is derived by applying the principle of minimum electromagnetic coupling: $\partial_\lambda \rightarrow \partial_\lambda + i\, e\, A_\lambda$. Let us assume there are no derivative terms in the strong interaction $\mathcal{L}_{p-\pi}$ we have:

$$\mathcal{L}_{\text{physical proton}-\gamma} = [\, j_\lambda\ (\text{proton}) + j_\lambda(\pi)] \circ A_\lambda$$

$$= J_\lambda A_\lambda$$

Where:

$$J_\lambda = j_\lambda(\text{proton}) + j_\lambda(\pi)$$

$$j_\lambda\ (\text{proton}) = i\, e\, \bar{\psi}_p \gamma_\lambda\, \psi_p$$

and to first order in e:

$$j_\lambda(\pi) = i\, e\, [\varphi^* \partial_\lambda \varphi - \partial_\lambda \varphi^*\, \varphi]$$

The electric charge conservation yields $\partial_\lambda J_\lambda = 0$,* and consequently:

physical proton electric charge = bare proton electric charge

(Berman, 1962). Indeed, the electric charge of the physical proton operator is by definition:

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*See Appendix A.

$$Q = \int J_4 \, d^3x = \int j_4 \text{ (proton) } d^3x + \int j_4(\pi) \, d^3x$$

Hence:

$$\langle p \mid Q \mid p \rangle = \langle p \mid \int j_4 \text{ (proton) } d^3x \mid p \rangle + \langle p \mid \int j_4(\pi) \, d^3x \mid p \rangle$$

where $|p\rangle$ denotes a state of the physical proton.

On the other hand equation $\partial_\lambda J_\lambda = 0$ yields $\frac{dQ}{dt} = 0$. Therefore $\langle p \mid Q \mid p \rangle$ = constant in time. Let us assume

$t \rightarrow -\infty$. Then

$|p\rangle \rightarrow$ free proton state

ψ_p obeys free Dirac equation

$\langle p \mid \int j_4 \text{ (proton) } d^3x \mid p \rangle \rightarrow$ bare proton electric charge by definition

$\langle p \mid \int j_4 \, (\pi) \, d^3x \mid p \rangle \rightarrow 0$ since a bare proton does not have any pion cloud around

Finally: physical proton electric charge = bare proton electric charge. In order to go easily to the case of weak interactions, we rewrite the electromagnetic current in such a way that the isospin dependence is explicitly shown.

$$J_\lambda = i\, e\, (\bar{\psi}_p \gamma_\lambda \psi_p) + (\varphi^k \partial_\lambda \varphi - \partial_\lambda \varphi^k \varphi)$$

$$= i\, e\, (\bar{\psi}_N \gamma_\lambda \frac{1 + \tau^3}{2} \quad {}_N + (\vec{\phi} \times \partial_\lambda \vec{\phi})^3$$

$$= i\, e\, \frac{1}{2} \; \bar{\psi}_N \gamma_\lambda \psi_N + \frac{1}{2} \bar{\psi}_N \tau^3 \psi_N + (\vec{\phi} \times \partial_\lambda \vec{\phi})^3$$

where

$$\psi_N = \begin{pmatrix} \psi_{proton} \\ \psi_{neutron} \end{pmatrix} = \text{nucleon field operator}$$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{\phi}$ = vector in isospace whose components ϕ_1, ϕ_2 are given by $\phi = \frac{1}{\sqrt{2}} (\phi_1 - i\,\phi_2)$

$$\phi^* = \frac{1}{\sqrt{2}} (\phi_1 + i\,\phi_2)$$

The fourth component of the first term in J_λ, i.e., $\bar{\psi}_N \gamma_4 \psi_N$ gives the number of nucleons.

We do have $\partial_\lambda (\bar{\psi}_N \gamma_\lambda \psi_N) = 0$ when ψ_N stands for the free nucleon. Because of baryon number conservation we still have $\partial_\lambda (\bar{\psi}_N \gamma_\lambda \psi_N) = 0$ when ψ_N stands for the physical nucleon.

From $\partial_\lambda J_\lambda = 0$ and $\partial_\lambda (\bar{\psi}_N \gamma_\lambda \psi_N) = 0$

We arrive at

$$\partial_\lambda \overset{V}{J}{}_\lambda^{3} = 0$$

where

$$\overset{V}{J}{}_\lambda^{3} = i\,e\,\bar{\psi} \gamma_\lambda \frac{\tau^3}{2} \psi + i\,e\,(\vec{\phi} \times \partial_\lambda \vec{\phi})^3$$

Since charge-independence holds within the domain of strong interactions, the equation $\partial_\lambda \overset{V}{J}{}_\lambda^{3} = 0$ leads to

$$\partial_\lambda J_\lambda^{V_\pm} = 0$$

where

$$J_\lambda^{V_\pm} = i\ e\,\bar{\Psi}\gamma_\lambda \frac{\tau^\pm}{2}\Psi + i\ e\ (\vec{\phi} \times \partial_\lambda\ \vec{\phi})^\pm$$

here

$$J_\lambda^{V_\pm} = J_\lambda^{V_1} \pm J_\lambda^{V_2}$$

Inspection of the first term in the expression of $J_\lambda^{V_\pm}$ shows that it is proportional to the vector part of the β-decay. Hence the so-called CVC theory in weak interactions was born in order to explain the unrenormalized* weak vector coupling constant in β decays. The CVC theory states that the vector part in β decays is actually given by the following conserved vector:

$$\frac{G}{\sqrt{2}}\ \bar{\Psi}\gamma_\lambda \frac{\tau^\pm}{2}\Psi + (\vec{\phi} \times \partial_\lambda\ \vec{\phi})^\pm$$

One usually calls it the "weak vector current" to stress the analogy with the electromagnetic vector current. The faith in the CVC theory has now substantially risen due to experimental studies. The measurement of the decay constants $\pi^+ \rightarrow \pi^o + e^+ \nu$ (Depommier et al., 1962) and the measurement of weak magnetism (Mayer-Kuckuck, 1961; Y, K. Lee, et al., 1963) in the decays of N^{12} and B^{12} produced results which are in good agreement with the CVC theory.

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*i.e., unrenormalized as far as strong interactions go.

It is noted that the theory is based on charge independence, therefore the weakly interacting vector current is only conserved neglecting electromagnetic effects.

Finally, it is natural to inquire whether the axial vector current in β decay is conserved.

We know that a bare axial vector $\bar{\psi}_1 \gamma_\lambda \gamma_5 \psi_2$ can only be divergence-free if the masses of the fields ψ_1 and ψ_2 are zero.

The fact that $x \neq 1$ in β-decays and that decays $\pi \rightarrow \mu + \nu$; $K \rightarrow \mu + \nu$ are observed although they are forbidden by $\partial_\lambda J_\lambda$ (axial) = 0 (Taylor, J. C. 1958) are conclusive enough to prove the lack of conservation of the axial vector current in β-decay theory.

The Intermediate Boson

The success of the CVC theory has drawn a deep analogy between weak interactions and electromagnetic interactions. It is natural then to suppose that weak interactions are mediated by a boson field, W. On this assumption the primary interaction is between the weakly interacting current J and the W field:

$$\mathcal{H}_{\text{primary}} = g\, J_\lambda\, W_\lambda$$

Here the coupling constant g is dimensionless. Observed slow processes are then of second order in the theory -- that is, two successive steps, creation and destruction of a W.

Due to the observed electric charge-exchange of the weakly interacting currents, there must at least exist a W^+ and a W^- boson. They should be massive because of the short range nature of the weak-interaction and in order to explain the absence of $K^+ \rightarrow W^+ + \gamma$ which would occur more rapidly than the observed $K^+ \rightarrow \pi^o + e^+ + \nu^e$ we should have $(\text{mass})_W \geq (\text{mass})_K$.

The intermediate boson hypothesis becomes more attractive if we recall that the contact 4-fermion coupling results in a breakdown of unitarity at high energy (page 15). The e-ν scattering cross section increases proportionally with the square of the bombarding ν-energy. Hence, as photons in electrodynamics, the assumption of intermediate bosons would introduce a damping propagator $D_{\alpha\beta}$ given by:

$$D_{\alpha\beta} = \frac{\delta_{\alpha\beta} + \frac{q_\alpha q_\beta}{m_w^2}}{q^2 + m_w^2}$$

where q_α = 4-momentum of the vector boson.

m_w = mass of W-boson

Subscripts α, β denote respectively the initial and final polarizations. At low energy

$$q^2 \ll m_w^2, \quad \text{then} \quad D_{\alpha\beta} \rightarrow \frac{\delta_{\alpha\beta}}{m_w^2}.$$

However, the introduction of these intermediaries does not make the lowest-order theory unitary although it does extend the unitary limit to a limit higher than the contact 4-fermion interaction.

Usually, one refers to the primary processes as semi-weak processes. The reason is as follows: taking into account the universality of class I interactions, the primary coupling constant g would be related to the

β-coupling constant G through:

$$\frac{G}{\sqrt{2}}_{\text{at low energy}} = \frac{g^2}{m_w^2}$$

Thus

$$g^2 = \frac{1}{\sqrt{2}} m_w^2 G = \frac{1}{\sqrt{2}} \frac{m_w^2}{M_p} \circ 10^{-5}$$

Assume $m_w \simeq M_p$ then $g = \sqrt{5} \circ 10^{-2}$ is semi-weak.

We make here one final remark about the intermediate boson theory $\mathcal{L}_{int} = -g J_\lambda W_\lambda$. The weakly interacting current - $\bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5)\psi_\nu$ for example - is not conserved, thus the theory is not gauge invariant of the second kind.* Both spin 1 and spin o intermediate boson participate in the weak interaction.

<u>The Universal Current x Current Theory</u>

The universality of Class I weak interactions led Feynman-Gell-Mann to writing the weak interaction as the product of a current and its adjoint:*

$$\mathcal{H} = \frac{G}{\sqrt{2}} J_\lambda J_\lambda^+$$

where

$$J_\lambda = B_\lambda + L_\lambda^{(e)} + L_\lambda^{(\mu)}$$

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*See Appendix A.

**At low energies we are not concerned whether $J_\lambda J_\lambda^+$ is a direct four spinor interaction or the result of two elementary processes involving intermediate bosons.

$$B_\lambda = \bar{p}\, n\, ;\; L_\lambda^{(e)} = \bar{\nu}\, e \quad L_\lambda^{(\mu)} = \bar{\nu}\, \mu \, .$$

We do not write the Dirac matrices for the sake of clarity. Then in addition to the experimentally observed processes, we get three new ones: $(\bar{p}\ n)(\bar{n}\ p)$; $(\bar{\mu}\ \nu)(\bar{\nu}\ \mu)$; $(\bar{e}\ \nu)(\bar{\nu}\ e)$. The first one gives a small correction to the n-p scattering due to strong interaction and the two last ones can be of astrophysical consequence (Pontecorvo, 1959).

These newly added interactions can be distinguished from the observed ones by the following property:

$$(\bar{p}\ n)(\bar{n}\ p) = (\bar{p}\ n)(\bar{p}\ n)^+ \quad \text{and} \quad (\bar{p}\ n)(\bar{p}\ n) = 0^*$$

due to P_{13} invariance (page 14). But:

$$(\bar{p}\ n)(\bar{e}\ \nu) = (\bar{p}\ n)(\bar{\nu}\ e)^+ \quad \text{and} \quad (\bar{p}\ n)(\bar{\nu}\ e) \neq 0^*$$

Also it is noted here that the weakly interacting currents J_λ were formed of baryon fields or lepton fields separately and that they are charge-exchange currents.

However these forms may not be necessary or the only ones due to the P_{13} invariance. Indeed, let us take three examples:

1. μ-decay: the interaction density is:

$$\frac{G}{\sqrt{2}} (\bar{\nu}^\mu\, \mu)(\bar{e}\ \nu^e) = \pm \frac{G}{\sqrt{2}} \quad (\bar{e}\ \mu)(\bar{\nu}^\mu\, \nu^e)$$

= product of charge-retention currents

$(\bar{e}\ \mu)$ and $(\bar{\nu}^\mu\ \nu^e)$

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* $(\bar{p}\ n)(\bar{p}\ n)$ and $(\bar{p}\ n)(\bar{\nu}\ e)$ cannot represent any physical processes because there would be no charge conservation.

2. The β-decay interaction density is given by:

$$\frac{G}{\sqrt{2}}(\bar{p}\,n)(e^{-}\,\nu^{e}) = \pm\frac{G}{\sqrt{2}}(\bar{p}\,\nu^{e})(\bar{e}\,n)$$

= product of charge-exchange currents where each current - say $\bar{e}\,n$ - is formed of a lepton field and a baryon field.

3. The n-p weak interaction is:

$$\frac{G}{\sqrt{2}}(\bar{p}\,n)(\bar{n}\,p) = \pm\frac{G}{\sqrt{2}}(\bar{n}\,n)(\bar{p}\,p)$$

= product of charge retention currents $(\bar{n}\,n)$; $(\bar{p}\,p)$

However, one can do away with such currents like $(\bar{e}\,\mu)$, and $(\bar{e}\,n)$ if one assumes weak interactions to be mediated by vector bosons of zero baryon number. Then Semi-Weak processes like $n \rightarrow e + W$; $\mu \rightarrow e + W$ would be ruled out because of baryon conservation and e-number and μ-number conservations. The intermediate boson hypothesis in general cannot eliminate charge-retention currents like $\bar{n}\,n$, $\bar{p}\,p$, $\bar{\nu}^{e}\,\nu^{e}$, $\bar{\nu}^{\mu}\,\nu^{\mu}$.

The question of their existence naturally arises. Actually there is not a single piece of evidence against their presence in this class. These currents would have many physical consequences; for instance, the electron proton scattering, the neutrino nucleon scattering, or the emission of $\bar{\nu}\,\nu$ pairs from exited nuclei.

Class II. Weak Interactions

The first question is: does the V-A theory apply to this class? Let us look at the ratio

$$R = \frac{K^+ \rightarrow e^+ + \nu}{K^+ \rightarrow \mu^+ + \nu}$$

The V-A theory predicts $r \simeq 2.5 \times 10^{-4}$. No $K \rightarrow e + \nu$ events have been reported. Of course, if the K meson is pseudoscalar then only the axial vector part is operative for these decays.

Considerable experimental information is beginning to be accumulated on the decay spectra of the process K_{e3} and $K_{\mu 3}$.

The present conclusion is that the V-A theory works here. Due to the meager statistics of $\Lambda \rightarrow p + \ell^- + \nu$, where ℓ stands for e or μ, nothing can be said about the space-time structure of leptonic hyperon decays, although we know that their rates are down by roughly a factor of 10 from the prediction by universality using the β-coupling constant.

Apart from the last fact, one is inclined to incorporate the class II into the class I; i.e. one adds a strangeness-changing current S_λ to the weakly interacting currents, for instance $S_\lambda = \bar{p}\,\Lambda$. One then interprets the relative smallness of the coupling constant of class II as being due to strong interaction effects.

The enlarged Hamiltonian becomes:

$$\mathcal{H} = \frac{G}{\sqrt{2}} \; J_\lambda J_\lambda^+$$

where $J_\lambda = S_\lambda + B_\lambda + L_\lambda^{(e)} + L_\lambda^{(\mu)}$.

Then Class II interactions would result from the coupling between S_λ and L_λ.

We observe the following rules in Class II:

+ $|\Delta S| = 1$

where Δ denotes the change of some quantum number in the current S_λ, e.g., ΔS = change in strangeness, ΔQ = change in charge. No decays $\Xi \rightarrow N + \ell + \nu$ - $\Delta S = 2$ - have been reported, although they are energetically favored over $\Lambda \rightarrow N + \ell + \nu$. Here N stands for nucleons.

+ $\Delta Q \neq 0$. No slow processes with $\Delta Q = 0$ have been observed, such as:

$$K_2^o \rightarrow \mu^+ + \mu^-, \quad e^+ + e^{-\,*} \quad \text{(Anikina, 1962)}$$

$$K^+ \rightarrow \pi^+ + \mu^+ + \mu^-, \quad \pi^+ + e^+ + e^-, \ldots$$

Hence in class II, we are left with two possibilities:

$$\Delta S = \Delta Q \quad \text{and} \quad \Delta S = -\Delta Q$$

The $\Delta S = \Delta Q$ rule does exist because of the observed processes:

$$\Lambda \rightarrow p + \ell^- + \nu, \quad K^\pm \rightarrow \pi + \ell^\pm + \nu; \quad K^\pm \rightarrow \mu^\pm + \nu, \ldots$$

There were some experimental results which indicate the existence of decays:

$$\Sigma^+ \rightarrow \mu^+ + \nu + n \quad \text{(one event reported, Barkas, 1962)}$$

$$K^o \rightarrow e^-(\mu^-) + \bar{\nu} + \pi^+ \quad \text{(Ely, 1962; Alexander 1962)}$$

which obey $\Delta Q = -\Delta S$.

The existence of the $(\Delta Q = -\Delta S)$ currents would imply two important consequences:

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*These reactions are allowed as far as CP invariance is concerned. They would be forbidden in a V-A theory if lepton masses are taken to be zero.

1. The universal current x current hypothesis would be excluded because the product of a $\Delta Q = \Delta S$ current S_λ with a $\Delta Q = -\Delta S$ current S_λ^+ will yield transitions with $|\Delta S| = 2$. These transitions belong to class III. So far no $\Delta S = 2$ class III events have been reported.

2. A strange current with $\Delta Q = -\Delta S$ does not satisfy $|\Delta I| = \frac{1}{2}$.* Indeed, from the very definition of strangeness $Q = I_3 + \frac{B + S}{2}$ it follows $\Delta Q = \Delta I_3 + \frac{\Delta S}{2}$

$$\Delta Q = \Delta S \rightarrow |\Delta I_3| = \frac{1}{2} \rightarrow |\Delta I| \geq \frac{1}{2}$$

$$\Delta Q = -\Delta S \rightarrow |\Delta I_3| = \frac{3}{2} \rightarrow |\Delta I| \geq \frac{3}{2}$$

The existence of $\Delta Q = \Delta S$ does not necessitate $|\Delta I| = \frac{1}{2}$ although $|\Delta I| = \frac{1}{2}$ necessitates $\Delta Q = \Delta S$. The existence of $\Delta Q = -\Delta S$ demands $|\Delta I| \geq \frac{3}{2}$.

In the case of K^0 leptonic decay, previous evidence (Ely, 1962) for large $\Delta S \neq \Delta Q$ contributions has not been confirmed in recent experiments (Kirsch, 1964).

Also, the experiments on K_{e4}^+ decay (Birge, 1963) and Σ-leptonic decay (Willis, 1964) seem to have confirmed the fact that no appreciable $\Delta S = -\Delta Q$ currents exist for these decays.

In support of the point that the strange currents satisfy $|\Delta I| = \frac{1}{2}$ was the fact that, in experiments by Neagu (1961) and Luers (1961), the probabilities of decays $K_2^0 \rightarrow e^+ + \nu + \pi^-$ and $K^+ \rightarrow e^+ + \nu + \pi^0$ were

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*Here $|\Delta I| = \frac{1}{2}$ means the change of isospin of the current S_λ.

roughly equal.* Recent results by Kirsch (1964) confirm this equality.

Class III. Weak Interactions

In the general current x current picture, these processes might result from the coupling $S_\lambda B_\lambda^+$ (their adjoints) and $S_\lambda S_\lambda^+$. The latter $S_\lambda S_\lambda^+$ would yield reactions with either $|\Delta S| = 2$ or $|\Delta S| = 0$. Note here that ΔS denotes the change of strangeness in the reaction. The case $|\Delta S| = 0$ belongs to Class I interactions. The case $|\Delta S| = 2$ is ruled out experimentally because of the unobserved reaction $\Xi \to N + \pi$ and the small mass difference between K_1^o and K_2^o. Hence observed non leptonic hyperon decays result from the coupling between S_λ and B_λ.

For these reactions where all of the participating particles are also engaged in strong interactions, one faces dynamical problems from the beginning, whereas in leptonic reactions, say the neutron decay, one can factor out the leptonic part and evaluate it, as long as electromagnetic effects are neglected.

Nevertheless one has had some success in investigating the space-time and isospin properties of these processes.

1. Parity non conservation

The historical $\tau - \theta$ puzzle is a good example for the parity non-conservation of the non-leptonic K-meson decays.

The tests for parity conservations in class III decays are not difficult because these processes lead a spin $\frac{1}{2}$ particle to a spin $\frac{1}{2}$ and a spin 0 particle. Therefore the angular distribution of the decay products can be used as a test for parity conservation without knowing the details

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*Alexander (1962) did not get the same result.

of the interaction. Consider for example the reaction $\Lambda \to p + \pi^-$

We have: $\frac{d\sigma}{d\Omega} \alpha \ (1 + \alpha \langle \vec{\sigma} \rangle \cdot \hat{p})$

where $\langle \vec{\sigma} \rangle$ is the polarization of the decay hyperon and $\hat{p}$ the unit vector in the direction of the proton momentum.

The measurements of the asymmetry parameter α can be summarized as follows:

$\alpha(\Lambda \to p + \pi^-) \simeq \alpha(\Lambda \to n + \pi^0) \simeq -0.61 \pm 0.07$ (Cork, 1960)

$\alpha(\Sigma^+ \to p + \pi^0) = 0.73 \begin{matrix} +.16 \\ -.11 \end{matrix}$ (Beall, 1962)

$\alpha(\Sigma^+ \to n + \pi^+) = 0.03 \pm 0.08$ (Cork, 1960)

$\alpha(\Sigma^- \to n + \pi^-) = 0.16 \pm 0.21$ (Tripp, 1962)

$\alpha(\Xi^- \to \Lambda + \pi^-) = 1 \begin{matrix} +0 \\ -0.35 \end{matrix}$ (Crawford, 1962)

There are two interesting remarks:

1. The $\alpha(\Lambda \to p + \pi^-)$ due to the V-A theory has the opposite sign of the experimental α. The enlarged current x current theory (to include S_λ) fails badly here.

2. Two final states of $\Sigma^\pm$ decay reactions have essentially definite parity.

2. $|\Delta I| = \frac{1}{2}$ rule*

So far there are two known violations of this rule. The larger violation comes from the observed decay $K^+ \to \pi^+ + \pi^0$. Recent experiments on the decay mode $K^+ \to \pi^+ + \pi^0 + \gamma$ (Fry, 1964) seem to indicate

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*Here ΔI stands for the change of isospin in the reaction.

that the reaction $K^+ \rightarrow \pi^+ + \pi^o$ occurs because of virtual electromagnetic effects. As to Σ decay, more experiments with greater accuracy are needed to test the $|\Delta I| = \frac{1}{2}$ rule which is desirable because it would predict the "dynamical accident" $\alpha(\Sigma^- \rightarrow n + \pi^-) \_ 0$ if

$$\frac{\alpha(\Sigma^+ \rightarrow n + \pi^+)}{\alpha(\Sigma^+ \rightarrow p + \pi^o)} << 1 \text{ as experimentally found.}$$

We shall summarize the experimental situation in two charts. The first chart corresponds to experiments agreeing with $|\Delta I| = \frac{1}{2}$ and the second chart contradicts the $|\Delta I| = \frac{1}{2}$ rule. Each chart has two columns. In the left column, we list theoretical predictions from $|\Delta I| = \frac{1}{2}$ and correspondingly in the right, experimental results.

Chart 1

Theory	Experiments
$\dfrac{K_1^o \rightarrow 2\pi^o}{K_1^o \rightarrow \text{total}} = \dfrac{1}{3}$	0.329 ± 0.013 (Brown, 1962)
$\dfrac{K_2^o \rightarrow \pi^+\pi^-\pi^o}{K^+ \rightarrow \pi^+\pi^o\pi^o} = 2 \times (1.032)$	$(1.39 \pm 0.11) \times 2 \simeq 2.90 \pm 0.72$ where: $(1.39 \pm 0.11) \times 10^6 \sec^{-1}$ = decay rate of $K^+ \rightarrow \pi^+\pi^o\pi^o$ $(2.90 \pm 0.72) \times 10^6 \sec^{-1}$ = decay rate of $K_2^o \rightarrow \pi^+\pi^-\pi^o$ (Stern, 1964)
$\dfrac{\Lambda \rightarrow p\,\pi^-}{\Lambda \rightarrow \text{total}} = \dfrac{2}{3}$	0.685 ± 0.04 Anderson (See Crawford, 1962)

Chart 2

$\sqrt{2}\ A^o + A^+ = A^-$

where $A^{\pm}_{o}$ stand for

transition amplitudes of

$\Sigma^+ \to n\pi^+$

$\Sigma^+ \to p\pi^o$

$\Sigma^- \to n\pi^-$

The inconsistency is between 2 and 3 standard deviation

(Tripp, 1962)

$K^+ \to \pi^+\pi^o$

$$\frac{K^+ \to \pi^+\pi^o}{K_1^o \to 2\pi} \simeq \frac{1}{500}$$ (Adair 1963)

The Validity of the Universal Current x Current Weak Interaction

The extension of the current x current formalism to explain all weak interactions encounters two main theoretical difficulties: the low rates of strangeness changing leptonic decays* and the $|\Delta I| = \frac{1}{2}$ rule in strangeness non conserving processes.

The latter difficulty would cause unwanted processes. Indeed, let us consider the decay $\Lambda \to p + \pi^-$. In the overall current x current formalism, this reaction results from the coupling $S_\lambda^+ B_\lambda = (\bar{p}\ \Lambda)(\bar{n}\ p)$. Since the Λ is an isoscalar and the nucleon is an isospinor, this coupling would transform in isospace as:

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*Recently, Cabibbo (1963) introduces a theory of weak interactions based on the eightfold representation of SU_3. The universal four-fermion interaction is modified such that the current $B_\lambda(\Delta S = 0,\ \Delta I = 1) \sim \cos\theta$ and the current $S_\lambda(\Delta S = \Delta Q,\ \Delta I = \frac{1}{2}) \sim \sin\theta$. Then with hyperon leptonic decay data, he has shown that there exists an angle $\theta \simeq 0.26$ which fits roughly the $K^+ \to \mu^+ + \nu$, $K^+ \to \pi^o + e^+ + \nu$ and baryon leptonic decays.

$$(\bar{n}\, p)(\bar{p}\, \Lambda) = |I = 1; \ I_3 = -1 \quad |I = \tfrac{1}{2}; \ I_3 = \tfrac{1}{2}$$

which has both $I = \frac{1}{2}$ and $I = \frac{3}{2}$ parts. In order to have the $|\Delta I| = \frac{1}{2}$ rule, which has been experimentally verified (see p. 33), we have to eliminate the $I = \frac{3}{2}$ part. To do so, we have to add a term which transforms as $|I = 1; I_3 = 0\rangle \quad |I = \frac{1}{2}, \ -\frac{1}{2}\rangle$ so that we have:

$$\left| \tfrac{1}{2}, -\tfrac{1}{2} \right\rangle = |1, -1\rangle \quad \left| \tfrac{1}{2}, \tfrac{1}{2} \right\rangle - \frac{1}{\sqrt{2}} \, |1, 0\rangle \quad \left| \tfrac{1}{2}, -\tfrac{1}{2} \right\rangle$$

according to the theorem of addition of angular momenta. A coupling of the type $|1,0\rangle \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ is $\frac{\bar{p}p - \bar{n}n}{\sqrt{2}} \bar{n}\Lambda$. It is a product of charge-retention currents. Thus the weak interaction can be made to satisfy $|\Delta I| = \frac{1}{2}$ rule for class III processes only at the price of introducing charge-retention baryon currents like $(\bar{p}\, p)$, $(\bar{n}\, n)$, $(\bar{n}\, \Lambda)$,...

If we believe in a universal weak interaction then charge-retention baryon currents should have as their counterparts charged retention lepton currents which would lead to the unobserved decays:

$$K^+ \rightarrow \pi^+ + \mu^+ + \mu^-, \quad \pi^+ + e^+ + e^-$$

$$K_2^o \rightarrow \mu^+ + \mu^-, \quad e^+ + e^-$$

Even with the assumption of a boson mediating the weak interactions, one runs into the same trouble. Indeed, Lee and Yang (1960) by assuming:

1. No $|\Delta S| > 1$ in first order weak interactions.
2. $|\Delta I| = \frac{1}{2}$ for strangeness non conserving decays

needed to introduce a set of four "schizoid" bosons W^+, W^-, W^o, $\overline{W}^o$.*
The W's behave as an isovector $[W^+, -\frac{1}{\sqrt{2}}(W^o + \overline{W}^o), W^-]$ when coupled with B_λ and as isospinors when coupled with S_λ.

In order to prevent $|\Delta S| > 1$, they were led to assume $\Delta S = \Delta Q$ for S_λ.

In order to forbid $K_2^o \rightarrow \mu^+ + \mu^-$, $K^+ \rightarrow \pi^+ + e^+ + e^-, \ldots$ they had to assume a non symmetric condition on W's, namely: only charge intermediate bosons can couple with leptons.

With the possible existence of $\Delta S = -\Delta Q$ in class II processes, T. D. Lee (1962) introduced a set of 6 bosons: $W^{\pm o}$ and $\overline{W}^{\pm o}$ in order to produce the following pattern:

1. No $|\Delta S| > 1$
2. $|\Delta I| = \frac{1}{2}$ for class III processes only.
3. Both $\Delta Q = \Delta S$ and $\Delta Q = -\Delta S$ exist in class II events. Once again he had to assume that only the charged W's interact with leptons.

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*The set of three bosons (W^+, W^o, W^-) would cause $|\Delta S| > 1$. For instance, the coupling $\overline{\Lambda}\, n\, W^o$ implies $n \rightarrow \Lambda + W^o$, hence in second order we have $n + n \rightarrow n + \Lambda + W^o \rightarrow \Lambda + \Lambda$.

**The decay $\Lambda \rightarrow p + W^-$ leads to $\overline{K}^o \rightarrow \Lambda + \overline{n} \rightarrow \Lambda + \overline{p} + \pi^+ \rightarrow \pi^+ + W^-$ (here $\Delta Q = \Delta S$). The assumption $|\Delta S| < 2$ implies $\pi^+ + W^- \not\rightarrow K^o$ (i.e. no $\Delta Q = -\Delta S$).

CHAPTER II

CHARGE-RETENTION CURRENT HYPOTHESES IN WEAK INTERACTIONS

Whereas isospin properties of strong and electromagnetic interactions are well established, we do not face the same situation in weak interactions.

As already pointed out, charge retention baryon currents are introduced in the weak interaction in order to explain the $|\Delta I| = \frac{1}{2}$ rule. Therefore the study of the above-mentioned currents might shed some light on the isospin structure of weak interactions.

If the arrangement of nature is prescribed by simplicity we are inclined to believe that a good theory of weak interactions which includes charge-retention currents would have a similar isospin structure with strong and electromagnetic interactions.

Part 1. Charge-Independence Hypothesis of Class I Weak Interactions

The deep analogy between leptonic $\Delta S = 0$ weak processes and electromagnetic processes has been enhanced greatly by the success of the CVC theory. The question whether 4-fermion interactions are being mediated by vector bosons $W^{\pm}$ has been raised and briefly discussed (p. 23).

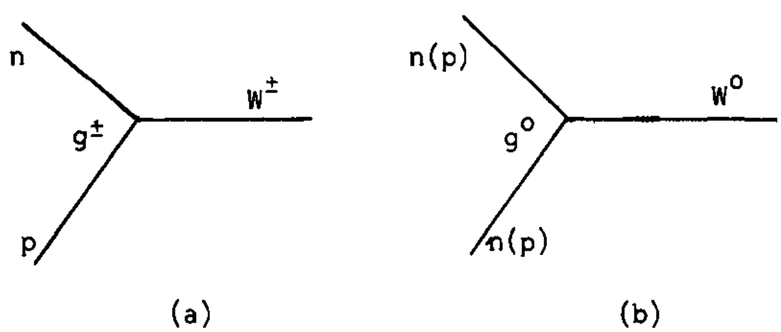
However there are difficulties in the way of constructing theories of Class I weak interactions by analogy with electrodynamics. One is that the axial vector currents are not conserved, actually even the conservation of the vector currents is broken by electromagnetism. The other difficulty is that whereas photons are massless and neutral, the vector particles W's are massive and must carry electric charge.

Probably, a better approach to the dynamical properties of W bosons is to look at the strong π-nucleon interaction because π-mesons appear also charged and have finite mass; although the π-nucleon coupling is supposedly pseudoscalar. At the present time, our theoretical knowledge regarding strong interactions is almost non-existent. We really know of separate P, C, T invariance - which is also true for electromagnetic interactions - and of charge-independence.

Naturally, one is inclined to ask: Would the semi-weak W-nucleon coupling and the strong π-nucleon coupling share some similar dynamical properties?

Since there is parity non-conservation in weak interactions, if there is anything they can share aside from T invariance, it should be charge-independence. In other words, they might share isospin properties.

For the present, we assume there is charge-independence in weak-interactions of class I; this means, we postulate the existence of a set of at least 3 bosons W^+, W^o, W^- such that the two diagrams (a) and (b) are



dynamically identical, except that $g^o = \frac{g^{\pm}}{\sqrt{2}}$, in analogy with charge-independence in strong interaction. With the hypothesis of charge-independence, the weak interaction implies:

1. The weakly interacting currents can be either charge-exchange - $\bar{p}\,n$, etc. - or charge-retention - $\bar{p}\,p$, etc....

2. Since the space-time structure of the charge-exchange current is V-A, so is the space-time structure of the charge-retention currents.

3. Since for low energies the coupling constant for the charge-exchange current x current interaction is:

$$G \simeq \frac{10^{-5}}{M_p^2}, \quad \text{where } M_p \text{ is the proton mass,}$$

Then at low energy the coupling constant for charge-retention current x current interaction would be:

$$G^o = \frac{G}{2} = \frac{1}{2}\,\frac{10^{-5}}{M_p^2}.$$

because we have assumed that the weak interaction density is:

$$\mathcal{H}_{\text{"charged"}} = \frac{G}{\sqrt{2}}\; J\, J^+$$

where $J = \bar{p}\,n + \bar{\nu}^e\,e + \bar{\nu}^\mu\,\mu$.

Together with nucleons which form a doublet $B = \binom{p}{n}$, leptons are assumed to be put into two doublets:

$$\ell^e = \binom{\nu^e}{e} \quad \ell^\mu = \binom{\nu^\mu}{\mu}$$

Then in the charge-independence hypothesis, the four-fermion weak interaction assumes the following form:

$$\mathcal{H}^{(C.I.)} = \frac{G}{\sqrt{2}} \left(\bar{B}\,\frac{\vec{\tau}}{2}\,B + \vec{\bar{\ell}^{e}}\,\frac{\vec{\tau}}{2}\,\ell^{e} + \vec{\bar{\ell}^{\mu}}\,\frac{\vec{\tau}}{2}\,\ell^{\mu} \right)^2$$

where $\vec{\tau}\,(\tau_x, \tau_y, \tau_z)$ are the Pauli spin matrices and we do not write the γ-matrices since they are not essential to this argument.

Developing we arrive at:

$$\mathcal{H}^{(C.I.)} = \frac{G}{\sqrt{2}} \left[(\bar{p}\ n)(\bar{e}\ \nu^{e}) + (\bar{n}\ p)(\bar{\nu}^{\mu}\ \mu) + (\bar{\nu}^{\mu}\ \mu)(\bar{e}\ \nu^{e}) + \text{Hermitian conjugate} \right]$$

$$+ \frac{1}{2}\frac{G}{\sqrt{2}} \left[(\bar{n}\ p)(\bar{p}\ n) + (\bar{e}\ \nu^{e})(\bar{\nu}^{e}\ e) + (\bar{\mu}\ \nu^{\mu})(\bar{\nu}^{\mu}\ \mu) \right]$$

$$+ \frac{1}{2}\frac{G}{\sqrt{2}} \left[(\bar{n}\ n)(\bar{e}\ e + \bar{\mu}\,\mu - \bar{\nu}^{e}\,\nu^{e} - \bar{\nu}^{\mu}\,\nu^{\mu}) + \bar{p}p(\nu^{e}\nu^{e} + \bar{\nu}^{\mu}\nu^{\mu} - \bar{e}e - \mu\mu) \right.$$
$$\left. + (\bar{e}e)(\bar{\mu}\mu) + (\bar{\nu}^{e}\nu^{e})(\bar{\nu}^{\mu}\nu^{\mu}) - (\bar{e}e)(\bar{\nu}^{\mu}\nu^{\mu}) - (\bar{\mu}\mu)(\bar{\nu}^{e}\nu^{e}) \right]$$

$$+ \frac{1}{4}\frac{G}{\sqrt{2}} \left[(\bar{p}p)(\bar{p}p) + (\bar{n}n)(\bar{n}n) + (\bar{e}e)(\bar{e}e) + (\bar{\mu}\mu)(\bar{\mu}\mu) + (\bar{\nu}^{e}\nu^{e})(\bar{\nu}^{e}\nu^{e}) \right.$$
$$\left. + (\bar{\nu}^{\mu}\nu^{\mu})(\bar{\nu}^{\mu}\nu^{\mu}) \right]$$

If $\mathcal{H}_i$ = interaction density of the i^{th} row of the above expression then we have

$$\mathcal{H}^{(C.I.)} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 \quad \text{(Charge-independence hypothesis)}$$

In the charge exchange current x current weak interaction, we have:

$$\mathcal{H}_{\text{"charged"}} = \mathcal{H}_1 + 2\,\mathcal{H}_2\ .$$

We see clearly that for all observed weak processes of class I - due to $\mathcal{H}_1$: β decay, μ-decay, μ capture - there is agreement between the charge-independence hypothesis and the Feynman-Gell-Mann current x current interaction.

So in order to test our hypothesis, we have to carry out three types of experiments:

1. Reactions allowed by $\mathcal{H}_2$, e.g.:

$$\nu^e + e \rightarrow e + \nu^e; \quad \nu^\mu + \mu \rightarrow \mu + \nu^\mu; \quad n + p \rightarrow n + p$$

The charge-independence hypothesis would give a cross section four times as small as the cross section given by the charge-exchange current x current hypothesis.

The weak n-p scattering is dominated by strong interactions.

The electron neutrino scattering has been studied. The main experimental difficulties are due to the presence of background. Available reactors provide high flux low energy antineutrinos. The average end point energy is a few Mev.

The μ - ν^μ scattering is out of the question . There is no μ-target.

2. Reactions allowed by $\mathcal{H}_3$: In our opinion, the best experiments involve neutrino-induced reactions because the only interaction known to be enjoyed by the neutrino is the weak interaction.

According to the expression $\mathcal{H}_3$, we can have three feasible targets: e, p, n.

We do not choose electron as targets because the only neutrino-induced reaction allowed by $\mathcal{H}_3$ is: $\nu^\mu + e \rightarrow e + \nu^\mu$ and one does not have any sufficiently high flux of μ-neutrinos yet, although one obtains a high energy μ-neutrino from K and π decays. So we are left with nucleons as targets. The simplest experiment is $\bar{\nu}^{e} + p \rightarrow p + \bar{\nu}^{e}$. In the center of mass system, and with $c = 1$, the proton recoil momentum is equal to the incoming antineutrino energy the average end-point of which is, for reactor neutrinos, a few Mev. The nuclear recoil energy is then $\frac{p^2}{2M} \lesssim \frac{1}{100}$ Mev.

Within present experimental possibilities, the signature of this reaction is insufficient.

Reactions of the type $\bar{\nu} + {}_{Z}X_{N} \rightarrow {}_{Z}X^{*}_{N} + \bar{\nu}$ have been suggested (see King, 1961).

We shall return to this point.

3. Reactions allowed by $\mathcal{H}_4$ which are all forbidden by the $\mathcal{H}_1 - \mathcal{H}_3$:

$$p + p \rightarrow p + p, \quad n + n \rightarrow n + n \quad \nu + \nu \rightarrow \nu + \nu$$

$$e + e \rightarrow e + e, \quad \mu + \mu \rightarrow \mu + \mu, \quad \ldots$$

Clearly, all of them are either dominated by strong and electromagnetic interactions (p-p scattering e-e scattering); or are experimentally impossible, $\nu + \nu \rightarrow \nu + \nu$.

The only way to detect - p-p scattering for example - is to find the effect of parity non-conservation.

<u>Conclusion</u>

From the above discussion, we think - within present experimental possibilities - the best way to test the charge independence hypothesis of the weak interaction is to study nuclear reactions induced by reactor antineutrinos.

Note that we do not have the vector interaction in allowed transitions of the type:

$$\bar{\nu} + {}_{Z}X_{N} \rightarrow {}_{Z}X_{N}^{*} + \bar{\nu}$$

$$\bar{\nu} + {}_{Z}X_{N} \rightarrow Z \text{ proton} + (N - Z) \text{ neutron} + \bar{\nu}$$

Indeed, the vector interaction nuclear matrix element in the allowed transition is:

$$< \text{final} \mid \sum_{j} \tau_{3j} \mid \text{initial} >$$

where j runs over all nucleons. This matrix element should be zero because the initial and the final states, determined by strong interactions and electromagnetic interactions, are eigenstates of $\sum_{j} \tau_{3j}$ and, on the other hand, they are in different energy levels.

Since we use antineutrinos, we want to go from neutrinos to antineutrinos. We transform as follows:

$$L_{\lambda} = \bar{\Psi}_{\nu} \gamma_{\lambda}(1 + \gamma_5)\Psi_{\nu} = 2\bar{\Psi}_{\nu} \gamma_{\lambda} \frac{1 + \gamma_5}{2} \Psi_{\nu}$$

$$= 2(\varphi^{+}\varphi, - \varphi^{+}\sigma_{k}\varphi)$$

Since

$$\varphi = \sigma_2 \chi^{*} \quad (* : \text{ complex conjugate})$$

$$\varphi^{+} = \chi^{T} \sigma_2 \quad (T = \text{transpose})$$

Then

$$L_t = \varphi_1^+ \varphi_2 = \chi_1^T \sigma_2 \sigma_2 \chi_2^* = \chi_1^T \chi_2$$

$$L_t = - \chi_2^+ \chi_1 {}^{(*)}$$

Also

$$\varphi_1^+ \sigma_k \varphi_2 = \chi_1^T \sigma_2 \sigma_k \sigma_2 \chi_2 = \chi_2^+ \sigma_2^T \sigma_k^T \sigma_2^T \chi_1$$

$$= + \chi_2^+ \sigma_k \chi_1 \quad (*)$$

because $\sigma_2 \vec{\sigma} \sigma_2 = - \vec{\sigma}^T$.

Part 2. The Electrically Neutral Current Hypothesis in Weak Interactions

If one assumes there is universality in all classes of weak interactions, then the charge-independence hypothesis has to be generalized. However the experimental $|\Delta I| = \frac{1}{2}$ rule in 2-body modes of Λ decay might be used as a counter example for a universal charge-independence theory.

The absence of such decays as $K_2^o \rightarrow \mu^+ + \mu^-$, $e^+ + e^-$, ... leads to the speculation that there are no electric charge-retention lepton currents in $|\Delta S| \neq 0$ leptonic processes. We note the following experimental upper limits on certain decay ratios:(**)

$$\frac{R(K^+ \rightarrow \pi^+ + \text{neutrinos})}{R(K^+ \rightarrow \pi^o + e^+ + \nu)} \leq \frac{1}{3}$$

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*We assume different fermion fields anticommute with each other.

**Lee, Yang, 1960.

$$\frac{R(K^+ \rightarrow \pi^+ + e^+ + e^-)}{R(K^+ \rightarrow \pi^o + e^+ + \nu)} \leq 2.5 \times 10^{-5} \; (*)$$

$$\frac{R(K^+ \rightarrow \pi^+ + \mu^+ + \mu^-)}{R(K^+ \rightarrow \pi^o + \mu^+ + \nu^\mu)} \leq 10^{-3}$$

$$\frac{R(K_2^o \rightarrow \mu^+ + \mu^-)}{R(K^+ \rightarrow \mu^+ + \nu^\mu)} \leq 10^{-3}$$

Whereas they might merely cast some doubt on the existence of a neutral $(\bar{\nu}\,\nu)$ current it appears that there are no $(\bar{\mu}\mu)$, $(\bar{e}e)$ currents.

We shall assume there exists a $(\bar{\nu}\nu)$ current. Then, as a consequence of universality, we also postulate the existence of other similar neutral currents, namely currents formed out of electrically neutral fields; for example $(\bar{n}n)$, $(\bar{\Lambda}\Lambda)$, $(\bar{\Sigma}^o\Sigma^o)$ $(\bar{\Xi}^o\,\Xi^o)$ $(\bar{\Lambda}n)$, $(\bar{\Sigma}^o\,\Lambda)$, ...

To make an acquaintance with this new physical picture easier we shall at first limit our discussion to class I weak interactions. In this class we assume as before three doublets

$$\binom{p}{n} ; \; \binom{e^+}{\nu^e} ; \; \binom{\mu^+}{\nu^\mu}$$

Our postulate is then equivalent to the assumption that the isospin structure of weakly interacting charge retention currents is $\frac{1 - \tau^3}{2}$. The 2 in the denominator will be explained.

Strong interactions are rotationally invariant in isospace. They allow several kinds of coupling among the components of the nucleon doublet.

The electromagnetic interaction does not have this invariance. It only allows coupling among the upper components of each doublet. Its

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*Camerini, 1964.

isospin structure is $\frac{1 + \tau^3}{2}$.

Now we already know about the existence of a coupling between the lower and the upper components of each doublet in weak interactions, e.g., $\bar{p}n$ in β decay. Then the assumption of an additional coupling between the lower components of each doublet in weak interactions, and no more, might be interpreted as an attempt by the weak interactions to restore all couplings allowed by strong interactions.

In line with the structure $\frac{1 + \tau^3}{2}$ in electromagnetic interaction, we shall postulate $\frac{1 - \tau^3}{2}$ as the isospin structure for the weakly interacting charge-retention currents.

The last question is the space-time structure of these neutral currents. Since our hypothesis stems originally from the perhaps possible observed decay: $K^+ \rightarrow \pi^+ + 2$ neutrinos - although it may be quite independent from it - the lepton current responsible for this decay should be a vector current because from one neutrino and one antineutrino in the final state, or equivalently from one neutrino in the initial state and one neutrino in the final state one can only form a vector $\varphi^+\sigma_\lambda\varphi$. In 4-component notation this vector becomes $\bar{\Psi}\gamma_\lambda(1 + \gamma_5)\Psi$ it means we have a V and/or A theory. In brief, we have, in addition to the electric charge exchange current interactions, the following interaction density:

$$\mathcal{H}^{(o)} = \frac{G^{(o)}}{\sqrt{2}} J_\lambda^o J_\lambda^{o+} \text{ if we use the current-current picture.}$$

Here

$$J_\lambda^{o} = S_\lambda^{o} + (\bar{p}\ \bar{n})\ \frac{1 - \tau_3}{2} \begin{pmatrix} p \\ n \end{pmatrix} + (\bar{e} \cdot \bar{\nu})\ \frac{1 - \tau_3}{2} \begin{pmatrix} e \\ \nu \end{pmatrix} + \ldots$$

$$= S_\lambda^{o} + (\bar{n}\ n)_\lambda + (\bar{\nu}\ \nu)_\lambda + \ldots$$

where a prototype of S_λ^{o} might be $\bar{\Lambda}\, n$.

The space time structure, V and/or A or their linear combination, and the neutral coupling constant $G^{(o)}$ would be left arbitrary if the charge-exchange and the charge-retention current interactions are independent from each other. Such a theory is not desirable in view of the $|\Delta I| = \frac{1}{2}$ rule.

The physical meaning of the neutral current hypothesis, when being incorporated into the charge-exchange current interaction, can be easily illustrated in the intermediate boson picture.

For the sake of simplicity, let us limit ourselves to class I weak interactions; also, the Dirac matrices are not written. The semi-weak interaction density can be written:

$$\mathcal{H}_{\text{"charge"}} = g\ (J^+\ W^- + J^-\ W^+)$$

where

$$J^{\pm} = \bar{\Psi}_1\ \frac{\tau_x \pm i\ \tau_y}{2}\ \Psi_2$$

The neutral current hypothesis, or in this class, the $\frac{1 - \tau^3}{2}$ hypothesis, can be interpreted as follows:

The current $J^3 = \bar{\Psi}_1\ \frac{\tau^3}{2}\ \Psi_2$ couples with a neutral vector boson W^3 to yield, in addition to $\mathcal{H}_{\text{"charge"}}$, the interaction $g^o\ J^3\ W^3$. If we take $g^o = g\sqrt{2}$, the total interaction would be an isoscalar, we go back to the charge-independence hypothesis. In order to have the $\frac{1 - \tau^3}{2}$ isospin structure, we assume there exists another weak coupling between the isoscalar current $J^o = \bar{\psi}_1 \frac{1}{2} \psi_2$ and W^3, according to $-g^o J^o W^3$. This interaction alone is not an isoscalar because W^3 is a neutral vector boson.

Finally we arrive at:

$$\mathcal{H}_{total} = g\ (\sqrt{2}\ J^o W^3 - \vec{J} \cdot \vec{W})$$

The arrow means: isovector.

In comparison with the Lee and Yang theory (1960), the newly added interaction $J^o W^3$ of this hypothesis does not result in any new isospin selection rules since J^o is an isoscalar. The neutral current hypothesis, on the one hand, introduces the neutral vector boson W^3 to explain the observed $|\Delta I| = \frac{1}{2}$ rule. On the other hand it reduces charge-retention currents to neutral currents.

Clearly the coupling constant $G^{(o)}$ is given by:

$$G^o = 2G \quad \text{where} \quad G = \beta\text{-coupling constant}$$

and the space-time structure is V-A.

The extension of the theory to strangeness changing slow processes can be made explicitly by using some model based on SU(3).

For instance, in the Sakata model, the neutral current hypothesis would yield the following isospin structures for neutral currents:

$$\frac{1 - \lambda_3}{2}\ ;\ \frac{\lambda_6 \pm i\lambda_7}{2}$$

$$\frac{1 - \lambda_3}{2} \text{ corresponds to: } \bar{n}\, n + \frac{\bar{\Lambda}\Lambda}{2}$$

$$\frac{\lambda_6 \pm i\lambda_7}{2} \text{ corresponds to: } \bar{\Lambda} n, \quad \text{or} \quad \bar{n}\Lambda$$

(for λ_i, see M. Gell-Mann, 1962).

The best test of the neutral current hypothesis is again the study of nuclear reactions induced by neutrinos. The coupling $(\bar{n}\,\Lambda)(\bar{n}\,n)$ which gives rise to:

$$\Lambda + n \rightarrow n + n$$

is purely dictated by weak interactions $(\Delta S \neq 0)$, however the meagre statistics of Λ and the fact that all electrically neutral particles (except γ, ν) are not stable, discourage experimentalists. They can be stabilized in nuclei (neutron) and hypernuclei (lambda) but other channels can then participate.

CHAPTER III

ANTINEUTRINO DISINTEGRATION OF THE DEUTERON

In the following, the cross section for the antineutrino disintegration of the deuteron is calculated in the allowed and first forbidden approximations following both the theory of charge-independence and the electrically neutral current theory.

We concern ourselves with the Deuteron for the following reasons:

1. The disintegration cross-section of the deuteron can be calculated close to exactly in terms of the pertinent coupling constants for bombarding energies below 10 Mev or so.

2. The binding energy of the deuteron is not excessive as far as reactor produced antineutrino energies are concerned.

3. As to the signature of the reaction, there is a superiority of $\bar{\nu} + D \rightarrow n + p + \bar{\nu}$ (1) over $\bar{\nu} + p \rightarrow p + \bar{\nu}$ (1').

The signature of the reaction (1') is the identification of the recoil proton, which is now impossible because of the small proton recoil energy. However the situation is different in the reaction (1). Its signature is the identification of neutron - by slowing down and capture - and proton. Although the neutrino momentum are small, and thus the momentum of the neutron-proton system, the individual nucleon momenta can be quite large - as large as energy conservation allows. Hence it is possible to detect the proton in the reaction (1).

4. The deuteron can exist in many well known compounds including scintillators.

We want to calculate two things:

1. The cross-section of the reaction $\bar{\nu} + D \rightarrow n + p + \bar{\nu}$ (1)

2. The nucleon spectrum.

Postulating sufficiently refined experimental methods, the latter would prove helpful in the identification of the reaction (1), particularly, in distinguishing it from $\gamma + D \rightarrow p + n$.

Since the reaction:

$$\bar{\nu} + D \rightarrow 2n + e^{+} \tag{2}$$

does occur, we also calculate its cross-section in order to compare with the cross-section of the reaction (1).

It is clear that the results of the subsequent investigation, once compared to experiment, will only illuminate the situation with charge retention currents in regards to processes without strange particles.

General Discussion of the Cross Section

We shall use the unit system where $\hbar = c = 1$ and neglect the proton-neutron mass difference. We normalize all interacting fields to unity $\int \psi^{+}\psi dv = 1$. In particular the incoming antineutrino flux is unity if we put V, the volume of normalization equal to 1. The cross-section is then given by:

$$\sigma = 2\pi \, S \, |H'_{if}|^{2} \, \rho_{f}$$

where ρ_f = density of final states

H' = weak interaction Hamiltonian responsible for the process i,f stand for the initial and final states. These states are eigenstates of the H'-free hamiltonian. In our problem they are determined by strong interactions

S means a summation over initial and final spin states and averaging over initial spin states.

We shall first calculate the density of final states, then the transition matrix elements in the allowed and first forbidden approximations.

The Density of Final States

Since the cross-section is a Lorentz invariant quantity, we shall limit our calculations to the zero-momentum frame. In this problem, the final state is composed of three particles: two are non-relativistic, the nucleons, and one is relativistic, the lepton.

The total energy, exclusive of the nucleon masses, is:

$$\begin{aligned} \varepsilon &= q - B \\ &= E_3 + \frac{p^2}{4M} + \frac{k^2}{M} \end{aligned}$$

where q is the incoming antineutrino energy,

B the binding energy of the deuteron,

E_3, p the energy and momentum of the final lepton,

M the nucleon mass,

and k the relative momentum between the two nucleons with momenta $\vec{p}_1$ and $\vec{p}_2$.

The density of final states is given by:

$$\rho_f = \frac{d}{d\varepsilon}\left[\frac{1}{(2\pi)^6}\, d^3p\; d^3k\right]$$

Since $\vec{k} = \vec{p}_1 - \frac{1}{2}\vec{p}$, once $\vec{p}$ and $\vec{k}$ are given, $\vec{p}_1$ and $\vec{p}_2$ are determined.

In order to calculate ρ_f, we first fix $\vec{p}$; then:

$$\frac{d}{d\varepsilon}(d^3p\ d^3k)\Bigg|_{\vec{p}} = d^3p\ k^2 \frac{dk}{d\varepsilon}\ d\Omega_k$$

$$= d^3p\ k^2 \frac{M}{2k}\ d\Omega_k$$

$$= \frac{M}{2} k\, d\Omega_k\ d^3p$$

For $\vec{p}$ in the range $(p, p + dp)$ and in the solid angle $d\Omega$, we have $d^3p = p^2 dp d\Omega$. Since we are dealing with $E_3 < q \leq 13.5$ Mev, we can make the following approximation:

$$\varepsilon = E_3 + \frac{p^2}{4M} + \frac{k^2}{M} = E_3\left(1 + \frac{p}{E_3} \cdot \frac{p}{4M}\right) + \frac{k^2}{M}$$

$$\simeq E_3 + \frac{k^2}{M}$$

The last equation yields:

$$dE_3 = \frac{p}{E_3}\, dp = - dE$$

$$p^2 = E_3^2 - m_3^2 = (\varepsilon - E)^2 - m_3^2$$

$$= (q - B - E)^2 - m_3^2$$

where $E = \frac{k^2}{M}$ is the relative kinetic energy of the two nucleons, and m_3 the mass of the final lepton.

Thus $$d^3p = p^2\, dp\, d\Omega$$

$$= \left[(q - B - E)^2 - m_3^2\right]\left(-\frac{E_3}{p}\right) dE\, d\Omega$$

$$k = (ME)^{1/2}$$

Finally:

$$\rho_f = \frac{+1}{(2\pi)^6}\frac{M^{3/2}}{2}\left[(q - B - E)^2 - m_3^2\right]^{1/2}(q - B - E)E^{1/2}\, dE d\Omega_k d\Omega \quad \text{(I-1)}$$

If the final lepton is the antineutrino we have:

$$\rho_f = \frac{1}{(2\pi)^6}\frac{M^{3/2}}{2}(q - B - E)^2\, E^{1/2}\, dE d\Omega_k d\Omega \qquad \text{(I-2)}$$

In terms of the relative momentum, the last equation becomes:

$$\rho_f = \frac{1}{(2\pi)^6} k^2 \left(q - B - \frac{k^2}{M}\right)^2 dk d\Omega_k d\Omega$$

<u>The Transition Matrix Element</u>

In the zero-momentum frame, the wave functions of the initial and final states are given by:

incoming antineutrino: $e^{i\bar{q} \cdot \vec{r}_\nu} \chi_i$

deuteron : $e^{-iq \cdot \frac{\vec{r}_1 + \vec{r}_2}{2}} \psi_D$

final lepton : $e^{i\vec{p} \cdot \vec{r}_\nu} \chi_f$ (*)

outgoing nucleons : $e^{-i\vec{p} \cdot \frac{\vec{r}_1 + \vec{r}_2}{2}} \psi_{1,2}$

*The leptons are assumed to couple locally with each other.

where χ_i, χ_f are the two-component spinors of the initial and final lepton,

Ψ_D is the deuteron wave function in the center of mass of the bound two-nucleon system

and $\Psi_{1,2}$ the wave function of the outgoing two-nucleon system in their center of mass.

The transition matrix element can be written:

$$\langle f \mid H' \mid i \rangle = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_{\bar{\nu}} \; e^{-i\vec{p}\cdot\left(\vec{r}_{\bar{\nu}} - \frac{\vec{r}_1+\vec{r}_2}{2}\right)}$$

$$\times \Psi^+_{1,2} \left[T'_\lambda \cdot (2\chi^+_f \sigma_\lambda \chi_i)\right] \Psi_D \, e^{i\vec{q}\cdot\left(\vec{r}_{\bar{\nu}} - \frac{\vec{r}_1+\vec{r}_2}{2}\right)}$$

where:

$$T'^{(o)}_\lambda = \frac{2G}{\sqrt{2}} \sum_{j=1,2} \left(\frac{1-\tau^3}{2}\Gamma_\lambda\right)_j \delta(\vec{r}_{\bar{\nu}} - \vec{r}_j)$$

$$T'^{(3)}_\lambda = \frac{1}{2}\frac{G}{\sqrt{2}} \sum_{j=1,2} (\tau^3\Gamma_\lambda)_j \, \delta(\vec{r}_{\bar{\nu}} - \vec{r}_j)$$

$$T'^{(-)}_\lambda = \frac{G}{\sqrt{2}} \sum_{j=1,2} \left(\frac{\tau^-}{2}\,\Gamma_\lambda\right)_j \delta(\vec{r}_{\bar{\nu}} - \vec{r}_j)$$

The Dirac delta function expresses the local coupling between weakly interacting currents.

τ^3, $\tau^- = \tau_1 - i\tau_2$ are the usual Pauli matrices in isospace, and Γ_λ is the non-relativistic approximation of $\gamma_t\gamma_\lambda\,(1 + x\gamma_5)$

The superscript (0) means that the interaction density used is given by the neutral current hypothesis. Similarly, the superscripts (3) and (-) correspond respectively to the components $\mathcal{H}_3$ and $\mathcal{H}_1$ of the charge-independence hypothesis. Clearly, $T'^{(o)}$ and $T'^{(3)}$ are responsible for: $\bar{\nu} + D \longrightarrow n + p + \bar{\nu}$ whereas the transition: $\bar{\nu} + D \longrightarrow 2n + e^+$ is realized by $T'^{(-)}$. By integrating over $d\vec{r}_{\bar{\nu}}$, we arrive at:

$$\langle f \mid H' \mid i \rangle = \int d\vec{r}_1 \, d\vec{r}_2 \, (\chi_f^+ \sigma_\lambda \chi_i) \psi^+_{1,2} \, h'_\lambda \, \psi_D \qquad \text{(II)}$$

where

$$h'^{(o)}_\lambda = 2\sqrt{2}\, G \left\{ e^{-i \frac{\vec{r}}{2} \cdot \vec{p}} \left(\frac{1-\tau^3}{2} \Gamma_\lambda \right)_1 e^{i \frac{\vec{r}}{2} \cdot \vec{q}} + \right.$$

$$\left. + \, e^{i \frac{\vec{r}}{2} \cdot \vec{p}} \left(\frac{1-\tau^3}{2} \Gamma_\lambda \right)_2 e^{-i \frac{\vec{r}}{2} \cdot \vec{q}} \right\}$$

$$h'^{(3)}_\lambda = \frac{G}{\sqrt{2}} \left\{ e^{-i \frac{\vec{r}}{2} \cdot \vec{p}} (\tau^3 \Gamma_\lambda)_1 \, e^{i \frac{\vec{r}}{2} \cdot \vec{q}} + \right.$$

$$\left. + \, e^{i \frac{\vec{r}}{2} \cdot \vec{p}} (\tau^3 \Gamma_\lambda)_2 \, e^{-i \frac{\vec{r}}{2} \cdot \vec{q}} \right\}$$

$$h'^{(-)}_\lambda = \sqrt{2}\, G \left\{ e^{-i \frac{\vec{r}}{2} \cdot \vec{p}} \left(\frac{\tau^-}{2} \Gamma_\lambda\right)_1 e^{i \frac{\vec{r}}{2} \cdot \vec{q}} + \right.$$

$$\left. + \, e^{i \frac{\vec{r}}{2} \cdot \vec{p}} \left(\frac{\tau^-}{2} \Gamma_\lambda\right)_2 e^{-i \frac{\vec{r}}{2} \cdot \vec{q}} \right\}$$

and $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Allowed Approximation

In the allowed approximation, all exponentials are equated to unity. Only the time-component of the vector interaction, i.e. $\Gamma_t^V = 1$, and space-components of the axial-vector interaction, i.e. $\Gamma_k^A = -x\sigma_k$, participate.

Since the deuteron ground state consists of mainly ^3S; the nucleons in the final state are in an S-state (^3S or ^1S).

As discussed, ^3S is excluded for the reaction

$$\bar{\nu} + D \rightarrow n + p + \bar{\nu} \tag{1}$$

and because of the Pauli principle, we also face the same case for the reaction:

$$\bar{\nu} + D \rightarrow 2n + e^+ \tag{2}$$

We shall, in the following, calculate the cross-sections of (1) and (2) separately.

<u>Cross-section of $\bar{\nu} + D \rightarrow n + p + \bar{\nu}$</u>

Since there is no contribution from the allowed vector interaction the allowed matrix elements become:

$$\langle f|H'^{(o)}|i\rangle = 2\sqrt{2}\; xG \int d\vec{r}_1\, d\vec{r}_2 (\chi_f^+ \vec{\sigma}\, \chi_i)\Psi_{1,2}^+ \left(\frac{1-\tau^3}{2}\vec{\sigma}\right)_1 + \tag{3}$$

$$+ \left(\frac{1-\tau^3}{2}\vec{\sigma}\right)_2 \Psi_D$$

$$\langle f|H'^{(3)}|i\rangle = \frac{xG}{2} \int d\bar{r}_1 d\bar{r}_2 (\chi_f^+\, \bar{\sigma}\, \chi_i)\Psi_{1,2}^+ \quad (\tau^3\, \bar{\sigma})_1 + (\tau^3\, \bar{\sigma})_2 \Psi_D \tag{3'}$$

The wave functions Ψ_D and $\Psi_{1,2}$ can be written:

$$\Psi_D = (^3S)\; \chi_1^m\, I_o$$
$$\Psi_{1,2} = (^1S)\; \chi_o\; I_{1,o}$$

where by convention, $({}^3S)$ denotes the $\vec{r}$-dependence of the deuteron wave function

X_1^m the spin state of the deuteron (triplet),

I_o the isospin state of the deuteron (singlet),

$({}^1S)$ the $\vec{r}$-dependence of the final two nucleon system

X_o the spin state (singlet),

and $I_{1,o}$ the isospin state (triplet) of the final two-nucleon system.

One can replace $\int d\vec{r}_1\, d\vec{r}_2\, ({}^1S)^+\, ({}^3S)$ by $\int d\bar{r}({}^1S)^+\, ({}^3S)$ because the integration over $\vec{r} = \vec{r}_1 - \vec{r}_2$ is equivalent to the integration over $\vec{r}_1$, keeping $\vec{r}_2$ fixed. Then the integration over $\vec{r}_2$ would yield the volume which was previously normalized to one.

Since:

$$\left(\frac{1-\tau^3}{2}\vec{\sigma}\right)_1 + \left(\frac{1-\tau^3}{2}\vec{\sigma}\right)_2 = \frac{1}{2}\,(\vec{\sigma}_1 + \vec{\sigma}_2) - \frac{1}{2}\,\left[(\tau^3\,\vec{\sigma})_1 + (\tau^3\,\vec{\sigma})_2\right]$$

and

$$X_o^+\,(\vec{\sigma}_1 + \vec{\sigma}_2)\,X_1^m = 0\,,$$

We have from (3), (3') and from the last equation:

$$\langle f\,|\,H'^{(0)}\,|\,i\rangle = -\,2\,\langle f\,|\,H'^{(3)}\,|\,i\rangle \tag{4}$$

The calculation of $\langle f\,|\,H'\,|\,i\rangle$ reduces to the calculation of

$\int ({}^1S)^+\,({}^3S)\,d\vec{r}$ and of:

$$(\chi_f^+ \vec{\sigma} \chi_i) \circ \quad I_{1,o}^+ \chi_o^+ (\tau_1^3 \vec{\sigma}_1 + \tau_2^3 \vec{\sigma}_2) \; \chi_1^m I_o \;=\; \chi_o^+ (\vec{\sigma}_1 - \vec{\sigma}_2) \chi_1^m \;\circ (\chi_f^+ \vec{\sigma} \chi_i)$$

Of course what we are really interested in is not $\langle f \mid H' \mid i \rangle$ but $\frac{1}{3} \sum_m |\langle f \mid H' \mid i \rangle|^2$ since the deuteron spin state is a triplet.

Let us calculate $\frac{1}{3} \sum_m |(\chi_o^+ (\vec{\sigma}_1 - \vec{\sigma}_2) \chi_1^m) \circ (\chi_f^+ \vec{\sigma} \chi_i)|^2 = D.$

We notice

$$\chi_o^+ (\vec{\sigma}_1 - \vec{\sigma}_2) \chi_1^m = \chi_o^+ (\vec{\sigma}_1 + \vec{\sigma}_2) \chi_1^m - 2\chi_o^+ \vec{\sigma}_2 \chi_1^m$$

$$= - 2\chi_o^+ \vec{\sigma}_2 \chi_1^m$$

Thus

$$D = \frac{4}{3} \sum_m |(\chi_o^+ \vec{\sigma} \chi_1^m)(\chi_f^+ \vec{\sigma} \chi_i)|^2$$

$$= \frac{4}{3} \chi_o^+ \sigma_{2j} (\sum_m \chi_1^m \chi_1^{m+}) \sigma_{2k} \chi_o \chi_f^+ \sigma_j \chi_i \chi_i^+ \sigma_k \chi_f$$

$$= \frac{4}{3} \sum_{j,k} \delta_{jk} \chi_f^+ \sigma_j \chi_i \chi_i^+ \sigma_k \chi_f$$

$$= \frac{4}{3} \sum_j \chi_f^+ \sigma_j \chi_i \chi_i^+ \sigma_j \chi_f$$

We choose the z-axis as the incident direction of antineutrino, then the $\chi_i = \binom{1}{0}$ since $\bar{\nu}$ is right handed, and:

$$\chi_i \chi_i^+ = \binom{1}{0} (1 \; 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} (\sigma_z + 1)$$

From $(p - \vec{\sigma} \circ \vec{p})\chi_f = 0$ follows:

$$\chi_f = \frac{1}{\sqrt{2p(p + p_z)}} \begin{pmatrix} p + p_z \\ p_x + ip_y \end{pmatrix}$$

and $$\sum_j \chi_f^+ \sigma_j \chi_i \chi_i^+ \sigma_j \chi_f = \frac{3}{2} \sum_j \chi_f^+ \sigma_j \sigma_z \sigma_j \chi_f$$

$$= \frac{3}{2} - \frac{1}{2} \chi_f^+ \sigma_z \chi_f$$

$$= \frac{3}{2} - \frac{1}{2} \frac{p_z}{p}$$

Finally

$$D = \frac{4}{3} \left(\frac{3}{2} - \frac{1}{2} \frac{p_z}{p}\right) = 2\left(1 - \frac{1}{3} \frac{p_z}{p}\right) \qquad (5)$$

Consider now $\int ({}^1S)^+ ({}^3S)\, d\vec{r}$. The radial dependence of the deuteron wave function can be approximated by:

$$({}^3S) = \sqrt{\frac{\gamma}{2\pi}} \frac{e^{-\gamma r}}{r}$$

where $B = \frac{\gamma^2}{M}$.

This implies the action of a δ-function potential between neutron and proton.

If we ignore the neutron-proton interaction, the final state of the two-nucleon system is given by:

$$({}^1S)_{free} = S_{wave} \text{ of } e^{i\vec{k} \cdot \vec{r}}$$

where $\vec{k}$ is the relative momentum between the nucleons.

Since $e^{i\vec{k}\cdot\vec{r}} = 4\pi \sum_\ell i^\ell j_\ell(kr) \sum_m Y_\ell^{m*}(k)\, Y_\ell^m(r)$. Then

$$({}^1S) = 4\pi j_0(kr) = Y_0^*(k)\, Y_0(r)$$

$$= j_0(kr) \qquad \left(\text{because } Y_0 = \frac{1}{\sqrt{4\pi}}\right)$$

$$= \frac{\sin kr}{kr}.$$

However if we take account of the strong interaction between neutron and proton, there is a phase-shift δ_o such that

$$({}^1S) = \frac{\sin(kr + \delta_o)}{kr} .$$

The integral $\int ({}^1S)^+ ({}^3S)\ d\vec{r}$ can be calculated by replacing

$$({}^1S) = \frac{\sin(kr + \delta_o)}{kr} \quad \text{and} \quad ({}^3S) = \sqrt{\frac{\gamma}{2\pi}}\ \frac{e^{-\gamma r}}{r}$$

We have:

$$\int ({}^1S)^+ ({}^3S)\ d\vec{r} = \int \frac{\sin(kr + \delta_o)}{kr} \sqrt{\frac{\gamma}{2\pi}}\ \frac{e^{-\gamma r}}{r}\ r^2\ dr d\Omega$$

$$= \sqrt{\frac{8\pi\gamma}{k^2}}\ \frac{k\cos\delta_o + \gamma\sin\delta_o}{k^2 + \gamma^2}$$

But $k\cot\delta_o = -\frac{1}{a}$ where a = singlet scattering length then

$$\left|\int ({}^1S)^+\ {}^3S\ d\vec{r}\right|^2 = \frac{8\pi\gamma}{k^2}\ \frac{(1 - a\gamma)^2\ k^2}{(1 + a^2k^2)(k^2 + \gamma^2)^2}$$

$$= 8\pi\gamma\ \frac{(1 - a\gamma)^2}{(1 + a^2k^2)(k^2 + \gamma^2)^2} \tag{6}$$

From (3), (4), (5) and (6), we arrive at:

$$\frac{1}{3}\sum_m |\langle f\,|\,H'^{(o)}\,|\,i\rangle|^2 = 4(xG)^2\left(1 - \frac{1}{3}\frac{p_z}{p}\right)\frac{8\pi\gamma(1 - a\gamma)^2}{(1+a^2k^2)(k^2+\gamma^2)^2}$$

$$\frac{1}{3}\sum_m |\langle f\,|\,H'^{(3)}\,|\,i\rangle|^2 = \frac{1}{4}\,\frac{1}{3}\sum_m |\langle f\,|\,H'^{(o)}\,|\,i\rangle|^2$$

Finally the cross-sections are given in the allowed approximation by:

$$\sigma = 2\pi \ S \ |H'_{if}|^2 \ \rho_f \tag{7}$$

$$d\sigma^{(o)} = 4 \ d\sigma^{(3)}$$

$$d\sigma^{(o)} = \frac{1}{(2\pi)^5} \frac{M^{3/2}}{2} 4(xG)^2 \frac{8\pi\gamma(1-a\gamma)^2}{(1+a^2k^2)(k^2+\gamma^2)^2} \left(1 - \frac{1}{3}\frac{p_z}{p}\right) \cdot$$

$$(q - B - E)^2 \ E^{1/2} \ dE d\Omega d\Omega_k$$

From now on, we shall only consider $d\sigma^{(o)}$; $d\sigma^{(3)}$ would be given by (7) once $d\sigma^{(o)}$ is calculated.

After integrating over $d\Omega$, the term $\frac{p_z}{p}$ vanishes, and:

$$d\sigma^{(o)} = \frac{8}{\pi^2} (xG)^2 \ B^{1/2} \ (1 - a\gamma)^2 \ \frac{E^{1/2}(q - B - E)^2}{(E+B)^2(1+Ma^2E)} dE \tag{8}$$

In our problem, the cross-section is evaluated for reactor antineutrinos and for three antineutrino energies: 5.5, 8.5, 13.5 Mev. The cross-sections $\sigma(q)$ for monoenergetic antineutrinos will be obtained by integrating $\int d\sigma$ for the range of E between 0 and $q - B$.

Let us denote $I = \frac{(q - B - E)^2}{(E + B)^2(1 + Ma^2E)}$, then:

$$d\sigma^{(o)} = \frac{8}{\pi^2} (xG)^2(1 - a\gamma)^2 \ (BE)^{1/2} \ I \ dE \ .$$

We rewrite:

$$I = \frac{1}{1 + Ma^2E} + \frac{q^2}{(E + B)^2(1 + Ma^2E)} - \frac{2q}{(1 + Ma^2E)(E + B)}$$

But:

$$\frac{1}{(1+Ma^2E)(E+B)} = \frac{1}{1-MBa^2}\left(\frac{1}{E+B} - \frac{Ma^2}{1+Ma^2E}\right)$$

hence:

$$I = \frac{1}{1+Ma^2E} + \frac{q}{E+B}\left(\frac{q}{(E+B)(1+Ma^2E)} - \frac{2}{(1+Ma^2E)}\right)$$

$$= \frac{1}{1+Ma^2E} + \frac{q}{(E+B)}\left[\frac{q}{1-MBa^2}\left(\frac{1}{(E+B)} - \frac{Ma^2}{1+Ma^2E}\right) - \frac{2}{1+Ma^2E}\right]$$

$$= \frac{1}{1+Ma^2E} + \frac{q}{E+B}\left[\frac{q}{1-MBa^2}\left(\frac{1}{E+B} - \frac{Ma^2q}{(1-MBa^2)(1+Ma^2E)}\right) - \frac{2}{1+Ma^2E}\right]$$

$$= \frac{1}{1+Ma^2E} + \frac{q^2}{1-MBa^2}\frac{1}{(B+E)^2} - \frac{1}{1+Ma^2E}\left(\frac{Ma^2q^2}{1-MBa^2} + 2q\right)\frac{1}{E+B}$$

$$= \left(1 + \frac{Ma^2q}{1-MBa^2}\right)^2 \frac{1}{1+Ma^2E} - \frac{1}{1-MBa^2}\left(\frac{Ma^2q^2}{1-MBa^2} + 2q\right)\frac{1}{B+E}$$

$$+ \frac{q^2}{1-MBa^2}\frac{1}{(B+E)^2}$$

The integration $\int d\sigma$ is now straightforward (Appendix B). We obtain:

$$\sigma^{(o)} = \int_o^{q-B} d\sigma^{(o)}$$

$$= \frac{8}{\pi^2}(xG)^2(1-a\gamma)^2 F \qquad (9)$$

where

$$F = \int_0^{q-B} (BE)^{1/2}\, I\, dE$$

$$= \sqrt{B(q-B)}\left(2E_S + \frac{q}{B/E_S - 1}\right) - 2\left(E_S - \frac{q}{B/E_S - 1}\right)^2 \sqrt{\frac{B}{E_S}}\, \text{arctg}\sqrt{\frac{q-B}{E_S}}$$

$$+ \frac{q}{B/E_S - 1}\left(q - 4B + 2\frac{q}{B/E_S - 1}\right) \text{arctg}\sqrt{\frac{q-B}{B}} \,.$$

($E_S = \frac{1}{Ma^2} \simeq 0.074$ Mev).

The total cross-section for reactor antineutrinos is obtained by integrating $\sigma(q)$ over the energy distribution of the incident antineutrinos.

By taking account of the finite range correction (Bethe, 1950) we arrive at the following results:

$$\sigma^{(o)}_{(\text{reactor antineutrinos})} = 2.4 \quad 10^{-44}\ \text{cm}^2$$

$$\sigma^{(o)}_{(5.5\ \text{Mev antineutrinos})} = 5.1 \quad 10^{-43}\ \text{cm}^2$$

$$\sigma^{(o)}_{(8.5\ \text{Mev antineutrinos})} = 2.2 \quad 10^{-42}\ \text{cm}^2$$

$$\sigma^{(o)}_{(13.5\ \text{Mev antineutrinos})} = 8.6 \quad 10^{-42}\ \text{cm}^2$$

<u>Relation of Differential Cross-section Maximum to Virtual Singlet Deuteron Level.</u>

From (8) we have:

$$\frac{d\sigma^{(o)}}{dE}\ \alpha\ \frac{E^{1/2}\,(q - B - E)^2}{(E+B)^2(1 + Ma^2E)} = f(E)$$

Or by denoting $E_S = \frac{1}{Ma^2} \simeq 0.07$ Mev and taking the logarithmic derivative:

$$\frac{f'(E)}{f(E)} = \frac{1}{2E} - \frac{2}{q-B-E} - \frac{1}{E_S(1+\frac{E}{E_S})} - \frac{2}{B+E}$$

$$= \frac{1}{2E}\left[1 - \frac{2\,E/E_S}{1+\frac{E}{E_S}} - 4E\left(\frac{1}{q-B-E} + \frac{1}{B+E}\right)\right]$$

$$= \frac{1}{2E}\left[\frac{1-E/E_S}{1+E/E_S} - \frac{4qE}{(q-B-E)(B+E)}\right]$$

$\frac{d\sigma^{(o)}}{dE}$ is maximum when $f'(E) = 0$ or when E satisfies the following equation:

$$G(E) \equiv (1 - \frac{E}{E_S})(q-B-E)(B+E) - 4qE(1+\frac{E}{E_S}) = 0$$

We can check easily that, for $q = 5.5, 8.5, 13.5$ Mev;

$$G(E_S) < 0$$

$$G(0.05\ \text{Mev}) > 0$$

In other words, the solution E_m of $G(E) = 0$ lies between 0.05 Mev and $E_S \simeq 0.07$ Mev.

We obtain the following results:

For $q = 13.5$ Mev $\quad E_m \simeq 0.058$ Mev

$q = 8.5$ Mev $\quad E_m \simeq 0.054$ Mev

$q = 5.5$ Mev $\quad E_m \simeq 0.054$ Mev.

Cross-section of $\bar{\nu} + D \rightarrow 2n + e^+$

The allowed matrix element is:

$$\langle f \,|\, H'^{(-)} \,|\, i \rangle = \sqrt{2}\; xG \int d\vec{r}_1 d\vec{r}_2\; (\chi_f^+ \vec{\sigma}\, \chi_i) \Psi_{1,2}^+ \left[\left(\frac{\tau^-}{2} \vec{\sigma}\right)_1 + \left(\frac{\tau^-}{2} \vec{\sigma}\right)_2 \right] \Psi_D$$

With the same notation used in the case of $\bar{\nu} + D \rightarrow n + p + \bar{\nu}$, the wave functions Ψ_D and $\Psi_{1,2}$ are given by:

$$\Psi_D = (^3S)\; \chi_1^m\; I_o$$

$$\Psi_{1,2} = (^1S)\; \chi_o\; I_{1,-1}$$

As before, one can replace $\int d\vec{r}_1 d\vec{r}_2\; (^1S)^+\; (^3S)$ by $\int d\vec{r}\; (^1S)^+\; (^3S)$, and since:

$$I_{1,-1}^+\; \chi_o^+ \left[\left(\frac{\tau^-}{2} \vec{\sigma}\right)_1 + \left(\frac{\tau^-}{2} \vec{\sigma}\right)_2 \right] \chi_1^m\; I_o = \chi_o^+ \frac{\vec{\sigma}_1 - \vec{\sigma}_2}{\sqrt{2}}\; \chi_1^m$$

$$= -\sqrt{2}\; \chi_o^+\, \vec{\sigma}_2\, \chi_1^m$$

the allowed matrix element can be rewritten:

$$\langle f \,|\, H'^{(-)} \,|\, i \rangle = -2\; (xG) \int d\vec{r}\; (^1S)^+\; (^3S)(\chi_o^+\, \vec{\sigma}_2\, \chi_1^m)\; (\chi_f \vec{\sigma} \chi_i)$$

We are led to calculate:

$$D' = \frac{1}{3} \sum_m \sum_s \left| (\chi_o^+\, \vec{\sigma}_2\, \chi_1^m)\; (\chi_f^+\, \vec{\sigma}\, \chi_i) \right|^2$$

where s denotes the spin state of the positron wave function χ_f.

We have:

$$D' = \frac{1}{3} \chi_o^+ \sigma_{2j} (\sum_m \chi_1^m \chi_1^{m+}) \sigma_{2k} \chi_o \sum_s \chi_f^+ \sigma_j \chi_i \chi_i^+ \sigma_k \chi_f$$

$$= \frac{1}{3} (\chi_o^+ \sigma_{2j} \sigma_{2k} \chi_o) \sum_s \chi_f^+ \sigma_j \frac{1}{2} (\sigma_z + 1) \sigma_k \chi_f$$

$$= \frac{1}{3} \sum_s \chi_f^+ \sum_j \sigma_j \frac{1}{2} (\sigma_z + 1) \sigma_j \chi_f$$

$$= \frac{1}{2} \sum_s \chi_f^+ (1 - \frac{\sigma_z}{3}) \chi_f$$

$$= \frac{1}{2} \sum_s \chi_f^+ (1 - \frac{\sigma_z}{3}) \frac{E_3 + \vec{\sigma} \cdot \vec{p}}{m_3} \varphi_f$$

where E_3, m_3 and $\vec{p}$ are respectively the energy, the mass and the momentum of the positron.

Since the 4-component spinor $\psi_f = \begin{pmatrix} \varphi_f \\ \chi_f \end{pmatrix}$ of the positron is normalized to unity, i.e.:

$$\psi_f^+ \psi_f = (\varphi_f^+ \varphi_f + \chi_f^+ \chi_f) = \frac{2E_3}{m_3} (\varphi_f^+ \chi_f) = 1$$

Or $\varphi_f^+ \chi_f = \dfrac{m_3}{2E_3}$,

the closure equation (p. 12) becomes:

$$\sum_s \varphi \chi_s^+ = \frac{m_3}{2E_3}$$

Consequently:

$$D' = \frac{1}{2} \sum_s \chi_f^+ (1 - \frac{\sigma_z}{3}) \frac{E_3 + \vec{\sigma} \cdot \vec{p}}{m_3} \varphi_f$$

$$= \frac{1}{2} \operatorname{Trace} \left[\left(1 - \frac{\sigma_z}{3}\right) \frac{m_3}{2E_3} \frac{E_3 + \vec{\sigma} \cdot \vec{p}}{m_3} \right]$$

$$= \frac{1}{2} (1 - \frac{1}{3} \frac{p_z}{E_3}) .$$

The cross-section is given in the allowed approximation by:

$$d\sigma^{(-)} = (2\pi)\ 4(xG)^2 \frac{8\pi\gamma(1-a\gamma)^2}{(1+a^2k^2)(k^2+\gamma^2)^2} \frac{1}{2} \left(1 - \frac{1}{3}\frac{p_z}{E_3}\right) \rho_f$$

where

$$\rho_f = \frac{1}{(2\pi)^6} \frac{M^{3/2}}{2} \left[(q-B-E)^2 - m_3^2\right]^{1/2} (q-B-E)E^{1/2} dE d\Omega_k d\Omega .$$

After integrating over $d\Omega$, $\frac{p_z}{E_3}$ vanishes; we finally have:

$$d\sigma^{(-)} = \frac{4}{\pi^2} (xG)^2 (1-a\gamma)^2 (BE)^{1/2} \frac{\left[(q-B-E)^2 - m_3^2\right]^{1/2} (q-B-E)}{(E+B)^2\ (1+Ma^2E)} dE \quad (10)$$

For monoenergetic antineutrinos, the cross-section will be obtained by integrating $\int d\sigma^{(-)}$ for the range of E between 0 and $q - B - m_3$.

We obtain the following results:

$$\sigma^{(-)}_{(\text{reactor antineutrino})} = 2.5 \quad 10^{-45}\ \text{cm}^2$$

$$\sigma^{(-)}_{(5.5\ \text{Mev antineutrino})} = 6.9 \quad 10^{-44}\ \text{cm}^2$$

$$\sigma^{(-)}_{(8.5\ \text{Mev antineutrino})} = 6.4 \quad 10^{-43}\ \text{cm}^2$$

$$\sigma^{(-)}_{(13.5\ \text{Mev antineutrino})} = 3.2 \quad 10^{-42}\ \text{cm}^2$$

The Proton Spectrum of $\bar{\nu} + D \rightarrow n + p + \bar{\nu}$.

In order to obtain the proton spectrum, we have to express the differential cross-section as a function of the proton momentum - which we denote as $\vec{p}_1$ – . The neutron momentum is denoted by $\vec{p}_n$ and the final antineutrino momentum by $\vec{p}$.

The density of final states is:

$$\rho_f = \frac{d}{d\varepsilon}\left[\frac{1}{(2\pi)^6} d^3p\, d^3p_1\right]$$

with ε the total energy excluding the nucleon rest masses.

We first fix the proton momentum $\vec{p}_1$ in the range $\vec{p}_1 + d\vec{p}_1$; this means the number of available states is proportional to $p_1^{\,2} dp_1 d\Omega_1$

Then:

$$\rho_f = \frac{1}{(2\pi)^6}\; p_1^{\,2} \circ dp_1 d\Omega_1 \left[\frac{d^3p}{d\varepsilon}\right]_{\vec{p}_1}$$

$$\left(\frac{d^3p}{d\varepsilon}\right)_{\vec{p}_1} = p^2 \;\frac{dp}{d\left(\varepsilon - \frac{p_1^{\,2}}{2M}\right)}\; d\Omega$$

Energy conservation yields: $\varepsilon = q - B = p + \frac{p_1^{\,2}}{2M} + \frac{p_n^{\,2}}{2M}$

Momentum conservation gives: $\vec{p}_1 + \vec{p}_n + \vec{p} = 0$. Then

$$\frac{d\left(\varepsilon - \frac{p_1^{\,2}}{2M}\right)}{dp} = 1 + \frac{p_n}{M}\frac{dp_n}{dp} \quad \text{(from energy conservation)}$$

From momentum conservation we get:

$$p_n^{\,2} = p^2 + p_1^{\,2} + 2\vec{p} \circ \vec{p}_1$$

$$\left(p_n \frac{dp_n}{dp}\right)_{\vec{p}_1} = p + \frac{\vec{p} \circ \vec{p}_1}{p}$$

Hence

$$\frac{d(\varepsilon - p_1^2/2m)}{dp} = 1 + \frac{p^2 + \vec{p} \cdot \vec{p}_1}{pM} = \frac{pM + p^2 + \vec{p} \cdot \vec{p}_1}{pM}$$

and

$$\left(\frac{d^3p}{d\varepsilon}\right)_{\vec{p}_1} = \frac{p^3M}{p^2 + pM + \vec{p} \cdot \vec{p}_1} \quad d\Omega$$

Finally:

$$\rho_f = \frac{1}{(2\ )^6} p_1^2 dp_1 d\Omega_1 \frac{p^3M}{p^2 + pM + \vec{p} \cdot \vec{p}_1} d\Omega$$

$$= \frac{1}{(2\pi)^6} p^2 p_1^2 \frac{1}{(1 + \frac{p}{M}) + \frac{\vec{p} \cdot \vec{p}_1}{pM}} dp_1 d\Omega_1 d\Omega$$

Since $\frac{p}{M} \lesssim \frac{q - B}{M} \lesssim \frac{1}{100}$, we can ignore $\frac{p}{M}$ and write:

$$\rho_f = \frac{1}{(2\pi)^6} \quad p^2 p_1^2 dp_1 \frac{1}{1 + \frac{\vec{p} \cdot \vec{p}_1}{pM}} d\Omega_1 d\Omega$$

The proton momentum is a maximum when $p = 0$ and $\vec{p}_1 = -\vec{p}_n$. So:

$$\left(\frac{p_1^2}{M}\right)_{max} = q - B \lesssim 10 \text{ Mev} .$$

Since $M \simeq 10^3$ Mev, hence:

$$(p_1)_{max} \lesssim 100 \text{ Mev}$$

and

$$\frac{p_1}{M} \lesssim \frac{1}{10}$$

To first order in $\frac{p_1}{M}$ we have:

$$\rho_f = \frac{1}{(2\pi)^6} \; p^2 p_1^{\,2} [1 - \frac{p_1}{M} \cos \theta] \; dp_1 d\Omega_1 d\Omega$$

where $\cos \theta = \cos (\vec{p} \circ \vec{p}_1)$.

We now want to express p^2 in terms of p_1. From

$$q - B = p + \frac{p_1^{\,2}}{2M} + \frac{p_n^{\,2}}{2M}$$

and

$$p_n^{\,2} = p^2 + p_1^{\,2} + 2\vec{p} \circ \vec{p}_1$$

we get:

$$q - B = p + \frac{p_1^{\,2}}{M} + \frac{p^2}{2M} + \frac{\vec{p} \circ \vec{p}_1}{M}$$

$$\simeq p[1 + \frac{p_1}{M} \cos \theta] + \frac{p_1^{\,2}}{M}$$

Or:

$$p \simeq (q - B - \frac{p_1^{\,2}}{M})(1 - \frac{p_1}{M} \cos \theta)$$

In terms of the proton momentum, the density of final states can be written:

$$\rho_f = \frac{1}{(2\pi)^6} \; p_1^{\,2} (q - B - \frac{p_1^{\,2}}{M})^2 \; (1 - 3 \frac{p_1}{M} \cos \theta) dp_1 d\Omega_1 d\Omega$$

As to the transition matrix element, we get from previous results:

$$\frac{1}{3} \sum_m |H'_{if}(o)|^2 = 4(xG)^2 (1 - \frac{1}{3} \frac{p_z}{p}) \frac{8\pi\gamma(1 - a\gamma)^2}{(1 + a^2 k^2)(k^2 + \gamma^2)^2}$$

We shall express the relative momentum $\vec{k}$ in terms of the proton momentum $\vec{p}_1$:

$$\vec{k} = \vec{p}_1 - \frac{1}{2}\vec{p}$$

$$k^2 = p_1^{\,2} + \frac{1}{4} p^2 - pp_1 \cos\theta$$

$$= p_1^{\,2}\left[1 - \frac{p}{p_1}\cos\theta + \frac{1}{4}\left(\frac{p}{p_1}\right)^2\right]$$

Let us assume a proton with energy $E_1 \geq 0.05$ Mev can be identified. Then:

$$E_1 = \frac{p_1^{\,2}}{2M} \geq 0.05 \text{ Mev} \longrightarrow p_1^{\,2} \geq \frac{M}{10} \simeq 100 \text{ (Mev)}^2$$

$$p_1 \gtrsim 10 \text{ Mev}.$$

and
$$\frac{p}{p_1} < \frac{q - B}{p_1} \leq \frac{q - B}{10}$$

$+$ For $q \leq 5.5$ Mev, we have:

$$\frac{p}{p_1} < \frac{q - B}{10} \lesssim \frac{3.3}{10}$$

$$\frac{1}{4}\left(\frac{p}{p_1}\right)^2 < \frac{1}{4}\ \frac{9.9}{100} < 0.03$$

We shall ignore the last term. Thus:

$$k^2 \simeq p_1^{\,2}\left(1 - \frac{p}{p_1}\cos\theta\right)$$

and

$$1 + a^2k^2 \simeq (1 + a^2p_1^{\,2})\left(1 - \frac{a^2pp_1\cos\theta}{1 + a^2p_1^{\,2}}\right)$$

$$(1 + a^2k^2)^{-1} \simeq (1 + a^2p_1{}^2)^{-1} \left[1 + \frac{a^2pp_1 \cos\theta}{1 + a^2p_1{}^2} + \right.$$

$$\left. + \frac{a^2pp_1 \cos\theta}{1 + a^2p_1{}^2} + \dots \right]$$

Also

$$\gamma^2 + k^2 \simeq (\gamma^2 + p_1{}^2)\left(1 - \frac{p_1p \cos\theta}{\gamma^2 + p_1{}^2}\right)$$

$$(\gamma^2 + k^2)^{-2} \simeq (\gamma^2 + p_1{}^2)^{-2} \left[1 + 2\,\frac{p_1p \cos\theta}{\gamma^2 + p_1{}^2} - \left(\frac{p_1p \cos\theta}{\gamma^2 + p_1{}^2}\right)^2 + \right.$$

$$\left. + 4\left(\frac{p_1p \cos\theta}{\gamma^2 + p_1{}^2}\right)^2 + \dots \right]$$

$$\simeq (\gamma^2 + p_1{}^2)^{-2} \left[1 + 2\,\frac{pp_1 \cos\theta}{\gamma^2 + p_1{}^2} + 3\left(\frac{pp_1 \cos\theta}{\gamma^2 + p_1{}^2}\right)^2 + \dots \right]$$

Hence $(1 + a^2k^2)^{-1}\,(\gamma^2 + k^2)^2 = (1 + a^2p_1{}^2)^{-1}\,(\gamma^2 + p_1{}^2)^{-2}\,K$

where:

$$K = 1 + pp_1 \cos\theta\left(\frac{2}{\gamma^2 + p_1{}^2} + \frac{a^2}{1 + a^2p_1{}^2}\right) + (pp_1 \cos\theta)^2 \left[\frac{3}{(\gamma^2 + a^2p_1{}^2)^2} + \right.$$

$$\left. + \frac{a^4}{(1 + a^2p_1{}^2)^2} + \frac{2a^2}{(\gamma^2 + p_1{}^2)(1 + a^2p_1{}^2)} \right]$$

Finally:

$$d\sigma^{(o)} = 2\pi \frac{1}{3} \sum_m |H'_{if}|^2 \rho_f$$

$$= \frac{4}{(2\pi)^5} (xG)^2 \frac{8\pi\gamma(1 - a\gamma)^2}{(1 + a^2p_1^2)(\gamma^2 + p_1^2)^2} (1 - \frac{1}{3} \frac{p_z}{p}) Kp_1^2 (q - B - \frac{p_1^2}{M})^2 \cdot$$

$$(1 - 3 \frac{p_1}{M} \cos \theta) \, dp_1 d\Omega_1 d\Omega$$

We first integrate over $d\Omega$, $\frac{p_z}{p}$ vanishes, so does $\cos \theta$, however $\int \cos^2 \theta \, d\Omega = \frac{4\pi}{3}$. We then get the differential cross-section for the proton, in the range $(p_1, p_1 + dp_1)$, after integrating over $d\Omega_1$:

$$\frac{d\sigma^{(o)}}{dp_1} = \frac{16}{\pi^2} (xG)^2 p_1^2 (q - B - \frac{p_1^2}{M})^2 \frac{\gamma(1 - a\gamma)^2}{(1 + a^2p_1^2)(\gamma^2 + p_1^2)^2} L$$

where:

$$L = \frac{1}{4\pi} \int (1 - 3 \frac{p_1}{M} \cos \theta) \, K \, d\Omega$$

$$= 1 + \frac{p^2p_1^2}{3} \left[\frac{3}{(\gamma^2 + p_1^2)^2} + \frac{a^4}{(1 + a^2p_1^2)^2} + \frac{2a^2}{(\gamma^2 + p_1^2)(1 + a^2p_1^2)} - \frac{3}{pM} \left(\frac{2}{\gamma^2 + p_1^2} + \frac{a^2}{1 + a^2p_1^2} \right) \right]$$

Since

$$\frac{p^2p_1^2}{3} \frac{3}{pM} \left(\frac{2}{\gamma^2 + p_1^2} + \frac{a^2}{1 + a^2p_1^2} \right) < \frac{p}{M} p_1^2 \left(\frac{2}{p_1^2} + \frac{1}{p_1^2} \right) \leq 3 \frac{q-B}{M} < 0.03 ,$$

we can ignore that term. We now have:

$$L \simeq 1 + \frac{p^2 p_1{}^2}{3} \left(\frac{3}{(\gamma^2 + p_1{}^2)^2} + \frac{a^4}{(1 + a^2 p_1{}^2)^2} + \frac{2a^2}{(\gamma^2 + p_1{}^2)(1 + a^2 p_1{}^2)} \right)$$

$$\simeq 1 + \frac{1}{3} \left(\frac{p}{p_1}\right)^2 N$$

where

$$N = \frac{3}{\left(1 + \frac{\gamma^2}{p_1{}^2}\right)^2} + \frac{1}{\left(1 + \frac{1}{a^2 p_1{}^2}\right)^2} + \frac{2}{\left(1 + \frac{\gamma^2}{p_1{}^2}\right)\left(1 + \frac{1}{a^2 p_1{}^2}\right)}$$

$$= \frac{3}{\left(1 + \frac{B}{p_1{}^2/M}\right)^2} + \frac{1}{\left(1 + \frac{E_S}{p_1{}^2/M}\right)^2} + \frac{2}{\left(1 + \frac{B}{p_1{}^2/M}\right)\left(1 + \frac{E_S}{p_1{}^2/M}\right)}$$

with $B = \frac{\gamma^2}{M}$, $E_S = \frac{1}{Ma^2}$.

Due to energy and momentum conservation, one has:

$$\left(\frac{p_1{}^2}{M}\right)_{max} = q - B, \quad \text{hence:}$$

$$N \leq \frac{3}{(1 + \frac{B}{q-B})^2} + \frac{1}{(1 + \frac{E_S}{q-B})^2} + \frac{2}{(1 + \frac{B}{q-B})(1 + \frac{E_S}{q-B})}$$

$$\leq 3 \left(\frac{q-B}{q}\right)^2 + \left(\frac{q-B}{q-B+E_S}\right)^2 + \frac{2(q - B)^2}{q(q - B + E_S)}$$

Since we are considering incoming antineutrinos with $q \leq 5.5$ Mev we have:

$$\frac{q - B}{q} \leq \frac{5.5 - 2.226}{5.5} \simeq 0.6$$

$$\frac{q - B}{q - B + E_S} \simeq 1 \quad \text{because} \quad E_S = \frac{1}{Ma^2} = 0.074 \text{ Mev}$$

and $$N \leq 3\ (0.6)^2 + 1 + 2(0.6) = 3.3$$

$$\frac{1}{3}\ (\frac{p}{p_1})^2\ N \leq (1.1)\ (\frac{p}{p_1})^2 \leq (1.1)\ (\frac{q-B}{10})^2 < \frac{1}{10}$$

because we are considering protons with $p_1 \geq 10$ Mev.

Finally, within an error less than 10%, we can write:

$$d\sigma^{(o)} = \frac{16}{\pi^2}\ (xG)^2\ p_1^2\,(q - B - \frac{p_1^2}{M})\ \frac{\gamma(1 - a\gamma)^2}{(1 + a^2p_1^2)\ (\gamma^2 + p_1^2)^2}\ dp_1$$

or in terms of $E_1 = \frac{p_1^2}{2M}$, $dE_1 = \frac{p_1}{M}\ dp_1 = (\frac{2E_1}{M})^{1/2}\ dp_1$;

$$d\sigma^{(o)} = \frac{8}{\pi^2}\ (xG)^2\ B^{1/2}\ (1 - a\gamma)^2\ \frac{(2E_1)^{1/2}\ (q - B - 2E_1)^2}{(2E_1 + B)^2\ (1 + Ma^2 2E_1)}\ d(2E_1) \qquad (11)$$

which is also the expression $\frac{d\sigma}{dE}$ (see equation (8)) if one replaces the relative kinetic energy E by $2E_1$.

By a similar analysis, we get the same result (equation (11)) for $q = 8.5$ and $q = 13.5$ provided we consider protons with energies respectively higher than 0.2 Mev and 0.5 Mev.

In brief, the expression (11) yields the proton energy spectrum under the following conditions:

For $q \leq 5.5 \longrightarrow E_1 \geq 0.05$ Mev

For $q \leq 8.5 \longrightarrow E_1 \geq 0.2$ Mev

For $q \leq 13.5 \longrightarrow E_1 \geq 0.5$ Mev

The differential cross-sections $\frac{d\sigma^{(o)}}{dE}$ are plotted as functions of E in Figure 1 and Figure 2 for reactor antineutrinos and for 13.5 Mev antineutrinos respectively.

First Forbidden Approximation

From the general expression (II) of the transition matrix element (p. 56) we can write down the first forbidden matrix element by recalling that in this approximation, we are dealing with two different types:

1. The exponentials are replaced by their second terms in the expansion, e.g. $e^{i\,\vec{r}/2\,\circ\,\vec{q}} \longrightarrow i\,\frac{\vec{r}}{2}\circ\vec{q}$.

2. The non-relativistic approximations of

$$\Gamma_k^V = \alpha_k \quad \text{and} \quad \Gamma_t^A = x\gamma_5$$

In this problem, we shall use the Ahrens' approximation (1952). It is noted here that the operator γ_5 in our representation (p. 10) is different from the conventional (Weyl) one by a factor (-1).*

We have for the component $\mathcal{H}_3$:

$$\langle f\,|H'^{(3)}|\,i\rangle = \int d\vec{r}_1 d\vec{r}_2\,(\chi_f^+\,\sigma_\lambda\chi_i)\psi_{1,2}^+\,h_\lambda'^{(3)}\,\psi_D$$

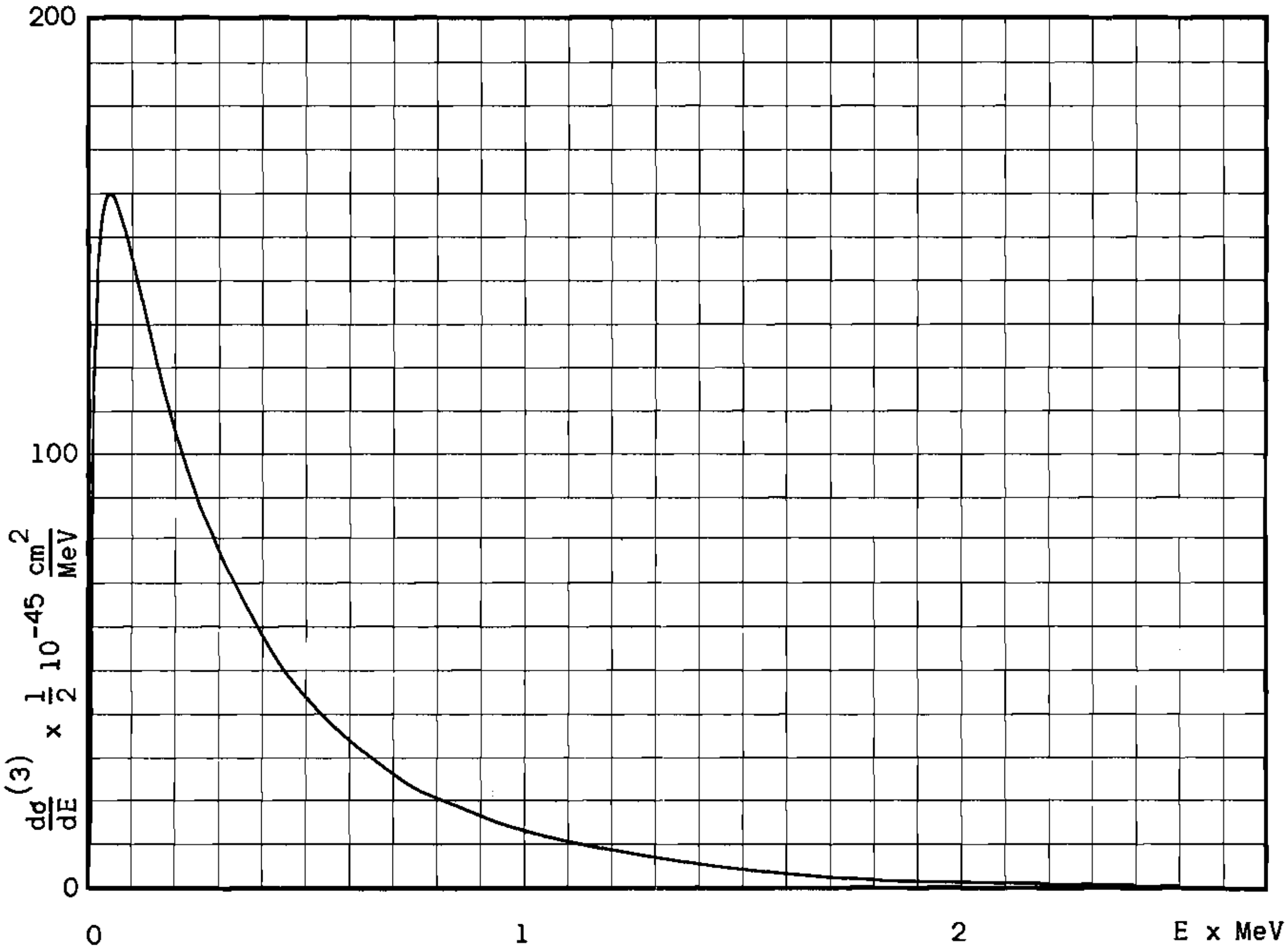
where

$$h_t'^{(3)} = \frac{G}{\sqrt{2}}\left\{\left[i\,(\vec{q}-\vec{p})\circ\frac{\vec{r}}{2}\right](\tau_1^3-\tau_2^3) + x\left[(\tau^3\gamma_5)_1 + (\tau^3\gamma_5)_2\right]\right\}$$

$$h_k'^{(3)} = \frac{G}{\sqrt{2}}\,[(\tau^3\alpha_k)_1 + (\tau^2\alpha_k)_2] - x[i(\vec{q}-\vec{p})\circ\frac{\vec{r}}{2}][(\tau^3\sigma_k)_1 - (\tau^3\sigma_k)_2]$$

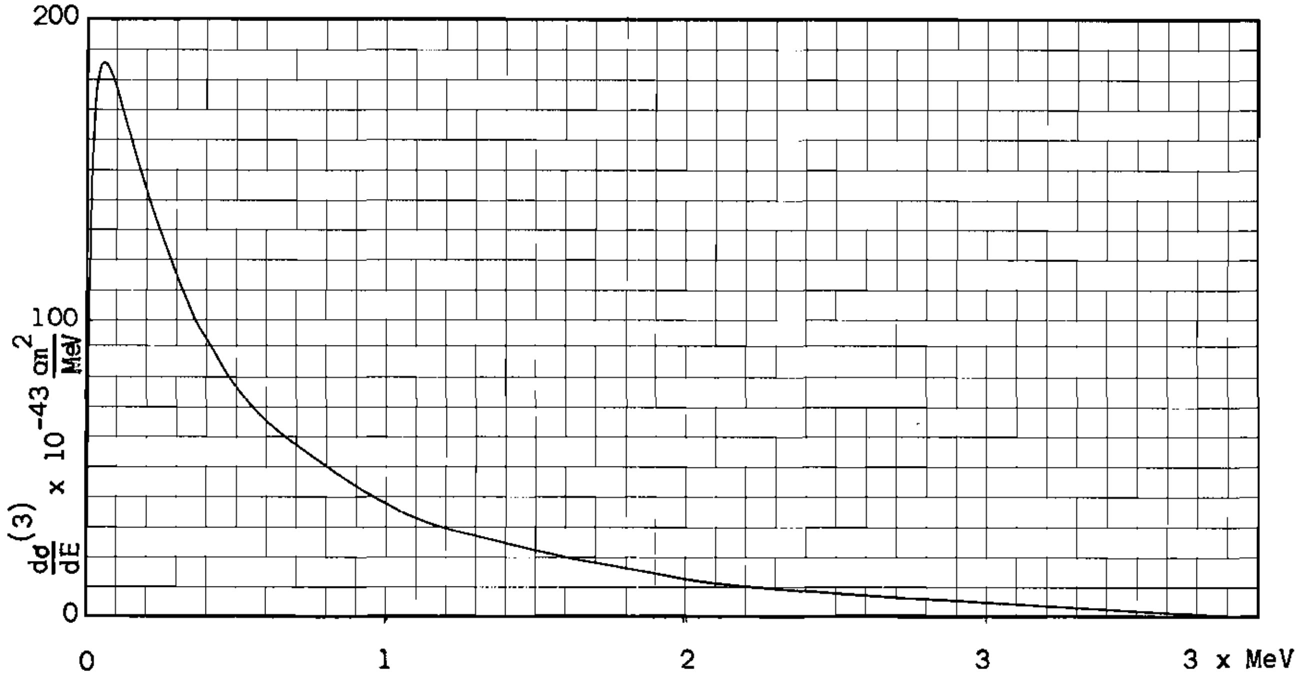
---

*Also our γ_k are different from the conventional ones by a factor (i) since we use the metric $p\circ x = Et - \vec{p}\circ\vec{x}$ (Sakurai, 1959).



E is the relative kinetic energy of the neutron-proton system.

Figure 1. The Proton Energy Spectrum for Reactor Antineutrino.



E is the relative kinetic energy of the neutron-proton system.

Figure 2. The Proton Energy Spectrum for 13.5 MeV Antineutrino.

Since the deuteron isospin state is a singlet, we have:

$$h_t^{\circ} = \frac{G}{\sqrt{2}}\left\{[i(\vec{q} - \vec{p}) \cdot \vec{r}] + x[(\gamma_5)_1 - (\gamma_5)_2]\right\}$$

$$h_k^{\circ} = \frac{G}{\sqrt{2}}\left\{[(\alpha_k)_1 - (\alpha_k)_2] - x[i(\vec{q} - \vec{p}) \cdot \frac{\vec{r}}{2}][(\sigma_k)_1 + (\sigma_k)_2]\right\}$$

The superscript (3), hereafter is not written. Also:

$$\langle (\alpha_k)_1 - (\alpha_k)_2 \rangle = \int d\vec{r}\, \Psi_{1,2}^{+} [\alpha_k)_1 - (\alpha_k)_2] \Psi_D$$

$$\simeq -i\,(W_f - W_i) \langle \vec{r}_1 - \vec{r}_2 \rangle = -i(W_f - W_i) \langle \vec{r} \rangle$$

$$\langle (\gamma_5)_1 - (\gamma_5)_2 \rangle \simeq -i\,(W_f - W_i) \langle \sigma_2 \vec{r}_2 - \sigma_1 \vec{r}_1 \rangle$$

$$\simeq -i\,\frac{W_f - W_i}{2} \left\langle (\vec{\sigma}_1 + \vec{\sigma}_2)(\vec{r}_2 - \vec{r}_1) + (\vec{\sigma}_2 - \vec{\sigma}_1)(\vec{r}_1 + \vec{r}_2) \right\rangle$$

$$\simeq i\,\frac{W_f - W_i}{2} \langle (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{r} \rangle$$

where W_f is the total energy of the neutron-proton system in the final state,

and W_i the total energy of the neutron-proton system in the initial state.

Thus

$$W_f = 2M + \frac{k^2}{M}$$

$$W_i = 2M - B$$

$$W_f - W_i = \frac{k^2}{M} + B = E + B$$

The first forbidden matrix element can be written:

$$\langle f \mid H' \mid i \rangle = -\frac{G}{\sqrt{2}} \Big\{ \Big[\langle i(\vec{q} - \vec{p}) \circ \vec{r} \rangle + ix(E+B) \Big\langle \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \circ \vec{r} \Big\rangle \Big] \chi_f^+ \chi_i$$

$$+ \Big[i(E+B) \langle \vec{r} \rangle + ix \Big\langle [(\vec{q} - \vec{p}) \circ \vec{r}] \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \Big\rangle \Big] \chi_f^+ \vec{\sigma} \chi_i \Big\}$$

Since the deuteron is in a ^3S state, clearly the final neutron-proton system is in a ^3P state:

$$\psi_D = (^3S)\ \chi_1^m$$
$$\psi_{1,2} = (^3P)\ \chi_1^{m'}$$

and by denoting:

$$\int \vec{r} = \int (^3P)^+ \ \vec{r}\ (^3S)\ d\vec{r}$$

$$\vec{S} = \chi_1^{m'+} \ \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \ \chi_1^m$$

$$I = (\vec{q} - \vec{p}) \circ \int \vec{r}$$

$$\vec{J} = (E + B) \int \vec{r}$$

we can rewrite the matrix element as follows:

$$\langle f \mid H' \mid i \rangle = -i \frac{G}{\sqrt{2}} \Big[(I\delta_{mm'} + x\, \vec{J} \circ \vec{S})\, \chi_f^+ \chi_i + (\vec{J}\delta_{mm'} + xI\vec{S})\chi_f^+ \vec{\sigma} \chi_i \Big]$$

We notice that the vector interaction selects $m = m'$, whereas the $m = m'$ contribution of the axial vector interaction is proportional to m because its spin operator is $(\vec{\sigma}_1 + \vec{\sigma}_2)/2$. Therefore, after summing over spin states, the interference between the vector and the axial vector interaction vanishes.

The calculation of $\frac{1}{3} \sum_{m,m'} | \langle f \mid H' \mid i \rangle |^2$ is straightforward.

We have:

$$|\chi_f^+ \chi_i|^2 = \chi_f^+ \chi_i \chi_i^+ \chi_f = \frac{1}{2} \chi_f^+ (\sigma_z + 1) \chi_f = \frac{1}{2} (1 + \frac{p_z}{p}) \quad \text{(p. 59)}$$

As we shall see, $\int r_i \int r_j = \frac{1}{3} | \int \vec{r} |^2 \delta_{ij}$, hence:

$$\frac{1}{3} \sum_{m, m'} |\vec{J} \cdot \vec{S}|^2 = \frac{1}{3} \sum_{m, m'} \sum_k J_k^2 S_k S_k^+$$

$$\frac{1}{3} \sum_{m, m'} S_k S_K^+ = \frac{1}{3} \sum_{m'} \chi_1^{m'} \left(\frac{\sigma_1 + \sigma_2}{2}\right)_k \sum_m \chi_1^m \chi_1^{m+} \left(\frac{\sigma_1 + \sigma_2}{2}\right)_k \chi_1^{m'}$$

$$= \frac{1}{3} \text{Trace } (1)$$

$$= \frac{2}{3}$$

and

$$\frac{1}{3} \sum_{m, m'} |\vec{J} \cdot \vec{S}|^2 = \frac{2}{3} \vec{J}^2$$

Note that $\frac{1}{3} \sum_{m, m'} \delta_{mm'} = 1$, thus:

$$\frac{1}{3} \sum_{m, m'} |\vec{J} \chi_f^+ \vec{\sigma} \chi_i|^2 \delta_{mm'} = \sum_k \chi_f^+ \sigma_k J_k \chi_i \chi_i^+ \sigma_K J_K \chi_f$$

$$= \frac{1}{2} \sum_k \chi_f^+ \sigma_k (\sigma_z + 1) \sigma_k \chi_f J_k^2$$

$$= \frac{1}{2} (1 - \frac{1}{3} \frac{p_z}{p}) \vec{J}^2 \qquad \text{(p. 60)}$$

$$\frac{1}{3} \sum_{m, m'} |I \vec{S} \chi_f^+ \vec{\sigma} \chi_i|^2 = I^2 \frac{1}{3} \sum_{m, m'} |\vec{S} \chi_f^+ \vec{\sigma} \chi_i|^2$$

$$= \frac{2}{3} I^2 \chi_f^+ \vec{\sigma} \chi_i \chi_i^+ \vec{\sigma} \chi_f$$

$$= I^2 (1 - \frac{1}{3} \frac{p_z}{p}) .$$

Let us denote:

$$H = I\vec{J}\,\chi_f^+\,\vec{\sigma}\,\chi_i\chi_i^+\,\chi_f \; + \; \text{its complex conjugate}$$

$$L = I\vec{S}\,\chi_f^+\,\vec{\sigma}\,\chi_i\chi_i^+\,\chi_f\;\vec{J}\circ\vec{S}^+ + \text{its complex conjugate}$$

we then have:

$$H = I\,\chi_f^+\,(\vec{\sigma}\circ\vec{J}\,\chi_i\chi_i^+ + \chi_i\chi_i^+\,\vec{\sigma}\circ\vec{J})\,\chi_f$$

$$= I\,\chi_f^+\,\sum_k J_k\,(\sigma_k\,\frac{\sigma_z + 1}{2} + \frac{\sigma_z + 1}{2}\,\sigma_k\,\chi_f$$

$$= I(J_z + \chi_f^+\,\vec{\sigma}\circ\vec{J}\,\chi_f)$$

Since $\hat{q} = \frac{\vec{q}}{q}$ is a unit vector along the z-axis, we can write: $J_z = \vec{J}\circ\hat{q}$.

Due to the closure relation $\varphi\varphi^+ + \chi\chi^+ = I$ and since:

$$\frac{1 + \vec{\sigma}\circ\hat{p}}{2}\,\chi = \chi\;;\quad \frac{1 + \vec{\sigma}\circ\hat{p}}{2}\,\varphi = 0$$

we have:

$$\chi_f^+\,\vec{\sigma}\circ\vec{J}\,\chi_f = \chi_f^+\,\vec{\sigma}\circ\vec{J}\;\frac{1 + \vec{\sigma}\circ\hat{p}}{2}\;\chi_f$$

$$= \text{Trace}\,(\vec{\sigma}\circ\vec{J}\,\frac{1 + \vec{\sigma}\circ\hat{p}}{2})$$

$$= \vec{J}\circ\hat{p}$$

Thus

$$H = I\,(J_z + \chi_f^+\,\vec{\sigma}\circ\vec{J}\,\chi_f)$$

$$= I\,(\hat{q} + \hat{p})\circ\vec{J}\,.$$

The calculation of $\frac{1}{3}\sum_{m,m'} L$ is actually the calculation of H. Indeed from:

$$L = I\,\chi_f^+ (\vec{\sigma}\circ\vec{S})(\vec{J}\circ\vec{S}^+)\chi_i\chi_i^+\chi_f + \text{ its complex conjugate}$$

and noticing that:

$$\frac{1}{3}\sum_{m,m'} (\vec{\sigma}\circ S)(\vec{J}\circ\vec{S}^+) = \frac{2}{3}\vec{\sigma}\circ\vec{J} \quad (\text{p. } 82)$$

we have:

$$\frac{1}{3}\sum_{m,m'} L = \frac{2}{3} H = \frac{2}{3} I(q+\hat{p})\circ\vec{J}.$$

Finally:

$$\frac{1}{3}\sum_{m,m'} |\langle f\,|H'|\,i\rangle|^2 = \frac{G^2}{2}\Big\{(I^2 + \frac{2x^2}{3}\vec{J}^2)\frac{1}{2}(1+\frac{p_z}{p})$$

$$+ (\vec{J}^2 + 2x^2 I^2)\,\frac{1}{2}\,(1 - \frac{1}{3}\frac{p_z}{p})$$

$$+ (\frac{2}{3}x^2 + 1)\, I\,\vec{J}.(\hat{q}+\hat{p})\Big\}$$

where

$$I^2 = [(\vec{q}-\vec{p})\circ\int\vec{r}]^2 = \frac{1}{3}(q^2+p^2-2\vec{p}\circ\vec{q})\,|\int\vec{r}\,|^2$$

$$\bar{J}^2 = (E+B)^2\,|\int\vec{r}\,|^2$$

$$I\,\vec{J}\circ(\hat{q}+\hat{p}) = (\vec{q}-\vec{p})\circ\int\vec{r}\,(E+B)\int\vec{r}\circ(\hat{q}+\hat{p})$$

$$= (E+B)\,\frac{1}{3}\,|\int\vec{r}\,|^2(q-p)$$

By integrating over $d\Omega_{\bar{\nu}}$, we get:

$$\int d\Omega_{\bar{\nu}} \ \frac{1}{3} \sum_{m,m'} | \ f \,|\, H' \,|\, i \ |^2 = 2\pi G^2 \, A \ | \int \vec{r} \,|^2$$

with:

$$A = \frac{q^2 + p^2}{6} - \frac{pq}{9} + x^2 \left(\frac{q^2 + p^2}{3} + \frac{2pq}{27}\right) + (E + B)^2 \left(\frac{1}{2} + \frac{x^2}{3}\right)$$

$$+ \frac{1}{3} (E + B)(q - p)\left(\frac{2}{3} x^2 + 1\right)$$

We shall express A in terms of $E = \frac{k^2}{M}$ by noting that:

$$q - B \simeq p + E$$

$$q - p \simeq B + E$$

$$q^2 + p^2 = (B + E) + 2pq = (B + E) + 2q(q - B - E).$$

Then:

$$A = \frac{1}{6} \left[(E + B)^2 + 2pq \right] - \frac{pq}{9} + (E + B)^2 \left(\frac{1}{2} + \frac{x^2}{3}\right)$$

$$+ x^2 \left[\frac{(E+B)^2 + 2pq}{3} + \frac{2pq}{27} \right] + \frac{1}{3} (E + B)^2 \left(\frac{2}{3} x^2 + 1\right)$$

$$= (E + B)^2 \left(1 + \frac{8x^2}{9}\right) + 2pq \left(\frac{1}{9} + \frac{10}{27} x^2\right)$$

$$= \frac{1}{9} \left[(9 + 8x^2)(E + B)^2 + 2\left(1 + \frac{10x^2}{3}\right) q(q - B - E) \right]$$

<u>Calculation of $| \int \vec{r} \,|^2 = | \int (^3P)^+ \, \vec{r} \, (^3S) \, d\vec{r} \,|^2$.</u>

Let us denote $r^\mu = \sqrt{\frac{4\pi}{3}} \, r \, Y_1^\mu$ where Y_1^μ are the conventional spherical harmonics. We can write:

$$| \int \vec{r} \,|^2 = \int r^\mu \cdot \int r^{*\mu}$$

$$= \frac{4\pi}{3} \int r Y_1^\mu \cdot \int r \, Y_1^{\mu *}$$

At low energy, the P-wave of the neutron-proton system is not disturbed by the short range strong interaction, therefore:

$$^3P = P_{wave} \text{ of } e^{i\vec{k}\cdot\vec{r}}$$

$$= 4\pi i\, j_1\,(kr) \sum_m Y_1^{m*}(\hat{k})\, Y_1^m(\hat{r})$$

and

$$\left|\int \vec{r}\right|^2 = (4\pi)^2 \frac{4\pi}{3} \left| \int j_1(kr)\; r(^3S)\; dv\right|^2$$

$$\times \int d\,\Omega_r \sum_m Y_1^{m*}(\hat{r})\; Y_1\; (\hat{r})\; Y_1^m(\hat{k})$$

$$\times \int d\,\Omega_r \sum_{m'} Y_1^{m'*}\; (\hat{r})\; Y_1^{\mu*}\; (\hat{r})\; Y_1^{m'*}(\hat{k})$$

$$= \frac{(4\pi)^3}{3} \left| \int j_1(kr)\; r\; (^3S)\; dv\right|^2 Y_1^{\mu}(k)\; Y_1^{\mu*}(k)$$

The last expression shows that: $\left|\int x\right|^2 = \left|\int y\right|^2 = \left|\int z\right|^2 = \frac{1}{3}\left|\int \vec{r}\right|^2$

Since

$$^3S = \sqrt{\frac{\gamma}{2\pi}}\; \frac{e^{-\gamma r}}{r}$$

$$j_1(kr) = \frac{1}{kr}\left(\frac{\sin kr}{kr} - \cos kr\right)$$

and since in the phase-space factor we have $d\Omega_k$, we can make the following integration:

$$\frac{1}{4\pi}\int d\Omega_k \left|\int \vec{r}\right|^2 = (4\pi)^2 \frac{\gamma}{2\pi}\left(\int j_1(kr)\; e^{-\gamma r}\; r^2 dr\right)^2$$

But

$$\int j_1(kr)\, e^{-\gamma r}\, r^2 dr = \frac{1}{k}\int \left(\frac{\sin kr}{kr} - \cos kr\right) e^{-\gamma r}\, r dr$$

$$= \frac{2k}{(k^2 + \gamma^2)^2}$$

Finally $\left| \int \vec{r} \right|^2 = 32\pi\gamma \dfrac{k^2}{(k^2 + \gamma^2)^2}$.

The cross-section for the first forbidden transition is given by:

$$\sigma = 2\pi \frac{1}{3} \sum_{m,m'} \left| \langle f \mid H' \mid i \rangle \right|^2 \rho_f$$

$$d\sigma = \frac{1}{4\pi^3} G^2 A \left| \int \vec{r} \right|^2 \frac{M^{3/2}}{2} (q - B - E)^2 E^{1/2}\, dE$$

But:

$$\left| \int \bar{r} \right|^2 = 32\pi\gamma \frac{k^2}{(k^2 + \gamma^2)^4} = 32\pi\gamma \frac{ME}{M^4 (B + E)^4}$$

$$d\sigma = \frac{4}{\pi^2} G^2 A \frac{\gamma\, E^{3/2}\, M^{3/2}}{M^3\, (B + E)^4} (q - B - E)^2\, dE$$

$$\frac{d\sigma}{dE} = \frac{4}{9\pi^2} \frac{B^{1/2}}{M} \left\{ (q + 8x^2) \frac{E^{3/2}\, (q - B - E)^2}{(E + B)^2} + 2q\left(1 + \frac{10x^2}{3}\right) \cdot \frac{E^{3/2}\, (q - B - E)^3}{(E + B)^4} \right\}$$

The cross section for monoenergetic antineutrinos would be obtained by integrating $\frac{d\sigma}{dE}$ over the range of E between 0 and $q - B$. The cross-section for reactor antineutrinos is then obtained by integrating $\sigma(q)$ over the energy distribution of the incident antineutrinos.

Taking account of the finite range correction (Bethe, 1950), we arrive at the following results:

First Forbidden Approximation

$$\sigma^{(3)}_{(\text{reactor antineutrinos})} = 4.6 \ 10^{-48} \ \text{cm}^2$$

$$\sigma^{(3)}_{(5.5 \text{ Mev antineutrinos})} = 8.5 \ 10^{-47} \ \text{cm}^2$$

$$\sigma^{(3)}_{(8.5 \text{ Mev antineutrinos})} = 1.4 \ 10^{-45} \ \text{cm}^2$$

$$\sigma^{(3)}_{(13.5 \text{ Mev antineutrinos})} = 1.8 \ 10^{-44} \ \text{cm}^2 .$$

A P P E N D I C E S

APPENDIX A

GAUGE INVARIANCE AND CONSERVATION LAWS

Gauge invariance of the first kind stems from the fact that all physical observables which are bilinear forms in non-hermitian fields ψ and ψ^+ are unaffected by a phase factor $e^{i\Lambda}$, where Λ is an arbitrary constant. The transformation

$$\psi \rightarrow e^{i\Lambda}\,\psi \tag{1}$$

is called the "gauge transformation of the first kind." Of course such a transformation cannot be applied to hermitian fields.

We shall show that "charge* conservation laws" are related to the invariance of the Lagrangian under (1). This transformation results in the following variation of the Lagrangian:

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\psi_\alpha}\,\delta\psi_\alpha + \frac{\partial\mathcal{L}}{\partial\partial_\lambda\psi_\alpha}\;\delta(\partial_\lambda\psi_\alpha)$$

Due to the Euler-Lagrange equation, and abbreviating $\partial_\lambda = \frac{\partial}{\partial x_\lambda}$

$$\delta\mathcal{L} = \partial_\lambda\;\frac{\partial\mathcal{L}}{\partial\partial_\lambda\psi_\alpha}\;\delta\psi_\alpha + \frac{\partial\mathcal{L}}{\partial\partial_\lambda\psi_\alpha}\;\delta(\partial_\lambda\psi_\alpha)$$

$$= \partial_\lambda\;\frac{\partial\mathcal{L}}{\partial\partial_\lambda\psi_\alpha}\;\delta\psi_\alpha$$

For an infinitesimal Λ:

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*Here the term "charge" stands for electric charge, baryonic charge, etc.

$$\delta \mathcal{L} = i\,\partial_\lambda\, j_\lambda$$

where $j_\lambda = -\,i\Lambda \dfrac{\partial \mathcal{L}}{\partial \partial_\lambda \Psi_\alpha}\Psi_\alpha$ is called the "charge current" as will be explained later.

The invariance of the Lagrangian yields:

$$\delta \mathcal{L} = i\,\partial_\lambda j_\lambda = 0$$

which would give rise to the charge conservation law. Indeed, let us consider a free baryon field Lagrangian:

$$\mathcal{L}_o = -\,\bar{\Psi}\,(\gamma_\lambda \partial_\lambda - m)\,\Psi$$

$$\frac{\partial \mathcal{L}_o}{\partial \partial_\lambda \Psi} = -\,\bar{\Psi}\,\gamma_\lambda$$

$$\frac{\partial \mathcal{L}_o}{\partial \partial_\lambda \Psi}\Psi = -\,\bar{\Psi}\,\gamma_\lambda\,\Psi$$

and $\partial_\lambda j_\lambda = \partial_\lambda (\Lambda\,\bar{\Psi}\,\gamma_\lambda \Psi) = 0$

We can write:

$$\partial_t j_t = -\,\partial_k (\Lambda\,\bar{\Psi}\,\gamma_k \Psi)$$

$$\int \partial_t j_t \, dV = -\int \partial_k\,(\Lambda\,\bar{\Psi}\,\gamma_k \Psi)$$

$$= 0$$

Hence $\quad \int j_t \, dV = \Lambda \int \bar{\Psi}\,\gamma_t \Psi \, dv = \text{constant}$

If we expand the field operator in terms of the creation and destruction operators, we get:

$$\int j_t \, dV = \Lambda \int \bar{\Psi}\,\gamma_t \Psi \, dV = \Lambda \sum_r \sum_k \,(n_{rk} - \bar{n}_{rk}) \qquad \text{(Marshak 1962)}$$

where n_{rk} and $\bar{n}_{rk}$ respectively denote the number of baryons in the state defined by the spin r and momentum k.

If Λ is the charge of the baryon, then the antibaryon charge would be $-\Lambda$, and it is clear that the conserved quantity $\int j_t \, dV$ = constant is the total charge. That also explains why we call j_λ the charge current.

Consider now the case of interacting fields. The Lagrangian of the physical system becomes:

$$\mathcal{L} = \mathcal{L}_o + \mathcal{L}'_{interaction} .$$

If we assume that $\mathcal{L}'$ does not contain field derivatives and that the newly introduced fields are neutral as far as the original charged fields are concerned, then the expression of the charge current $j_\lambda = - i \, \Lambda \frac{\partial \mathcal{L}}{\partial \partial_\lambda \Psi_\alpha} \Psi_\alpha$ remains unchanged, although the field Ψ_α no longer satisfy the free field equation.

Up to here, we have always assumed Λ to be a constant. But in a localized field concept, as Yang and Mills remarked (1954), the relative phases at different space-time points should be completely unrelated. Therefore Λ has to be an arbitrary function of the coordinates x, or for convenience sake, we shall multiply the constant Λ by an arbitrary function $\chi(x)$. The generalized gauge transformation of the first kind is then defined by:

$$\Psi \longrightarrow e^{i\Lambda \chi(x)} \Psi \qquad (1')$$

The transformation (1') gives rise to:

$$\delta\mathcal{L} = \partial_\lambda \frac{\partial \mathcal{L}}{\partial\partial_\lambda \psi_\alpha} \delta\psi_\alpha$$

For an arbitrarily small Λ we get:

$$\delta\mathcal{L} = i\, \partial_\lambda \frac{\partial \mathcal{L}}{\partial\partial_\lambda \psi_\alpha} \Lambda\chi(x)\psi_\alpha$$

$$= i\, (\chi(x)\, \partial_\lambda j_\lambda + j_\lambda \partial_\lambda \chi(x))$$

Since the Lagrangian, being bilinear in ψ and ψ^+, should be invariant under (1), i.e. $\partial_\lambda j_\lambda = 0$, we get:

$$\delta\mathcal{L} = i\, j_\lambda \partial_\lambda \chi(x) \neq 0$$

The demand of gauge invariance of the first kind in the sense of (1') and the fact that the variation $\delta\mathcal{L}$ due to (1') is not zero has led some people* to raise the requirement of this gauge invariance to the necessary existence of a hermitian vector - say A_λ - which is coupled to the non-hermitian field ψ and which is subject to the "gauge transformation of the second kind"

$$A_\lambda \rightarrow A_\lambda + \partial_\lambda \chi(x) \tag{2}$$

We shall see how this comes about: Let us assume the interaction Lagrangian to be $\mathcal{L}' = -ij_\lambda A_\lambda$. The gauge transformation (2) will cause the following variation:

$$\delta\mathcal{L} = \delta\mathcal{L}' = -ij_\lambda \partial_\lambda \chi(x)$$

which compensates the variation $ij_\lambda\partial_\lambda\chi(x)$ due to (1'). The total Lagrangian $\mathcal{L}$ will then be invariant under the combined transformation

*Yang and Mill (1954); Sakurai (1960).

(1') and (2).

However this conjecture is not true in general. Indeed, that part of the variation of the Lagrangian specified to (1'), $ij_\lambda \partial_\lambda \chi(x)$ which equals $\partial_\lambda(ij_\lambda \chi(x))$ because $\partial_\lambda j_\lambda = 0$, has no physical consequence if $j_\lambda = \Lambda \frac{\partial \mathcal{L}}{\partial \partial_\lambda \Psi_a} \Psi_a$ does not depend on the field derivatives (Hill, 1951). For Dirac fields, the free Lagrangian is of first degree in $\partial_\lambda \Psi$, the charge current j_λ does not depend on $\partial_\lambda \Psi$, hence the gauge invariance of the first kind (1') does not necessitate the existence of some hermitian vector field.

For π-meson fields, which carry electric charge, the free Lagrangian is of second degree in $\partial_\lambda \phi$, the electric charge current j_λ contains $\partial_\lambda \phi$, therefore the existence of the electromagnetic field A_λ which carries no electric charge, is a consequence of the gauge invariance of the first kind (1').

We shall now turn our attention to the gauge transformation of the second kind which is assumed hereafter to apply to vector fields only.

As remarked by Ogievetski and Polubarinov (1962) the vector field B_λ of finite or zero mass quanta can be decomposed into spin 1 - part B_λ^1 and spin 0 - part B_λ^o as follows:

$$\begin{aligned} B_\lambda &= (B_\lambda - \Box^{-1}\partial_\lambda \partial_\nu B_\nu) + \Box^{-1}\partial_\lambda \partial_\nu B_\nu \\ &= \qquad B_\lambda^1 \qquad\qquad + \qquad B_\lambda^o \end{aligned}$$

where $\Box = \partial_\lambda \partial_\lambda$.

Under the gauge transformation of the second kind

$$B_\lambda \rightarrow B_\lambda + \partial_\lambda \chi$$

only the spin 0 part changes. The spin 1 part B_λ^1 is gauge independent.

$$B_\lambda^1 \rightarrow (B_\lambda - \Box^{-1}\partial_\lambda\partial_\nu B_\nu) + \underbrace{\partial_\lambda\chi - \Box^{-1}\partial_\lambda\partial_\nu\partial_\nu\chi}_{\shortparallel \atop 0}$$

Therefore, a theory which possesses gauge invariance of the second kind is a theory where spin zero quanta of the vector field do not have any physical significance. The Lorentz condition $\partial_\lambda B_\lambda = 0$ means the elimination of spin zero quanta.

In order to distinguish between the two kinds of gauge transformations, we shall consider an interaction where the vector field B_λ is coupled to a field f_α such that $\mathcal{L}'_{\text{interaction}} = i\, J_\lambda B_\lambda$; J_λ is a vector function of the fields f_α but not its derivatives.

Then gauge invariance of the second kind, $B_\lambda \rightarrow B_\lambda + \partial_\lambda\chi$ means $\delta S = \int \delta\mathcal{L}\, d^4x = 0$.

In this example:

$$\delta S = \int \delta\mathcal{L}\, d^4x = \int \delta\mathcal{L}'\, d^4x = i\int J_\lambda \partial_\lambda\chi\, d^4x$$

$$= -i\int (\partial_\lambda J_\lambda)\chi\, d^4x$$

The condition $\delta S = 0$ leads to $\partial_\lambda J_\lambda = 0$. We shall call the conserved vector J_λ the interacting current to distinguish it from the conserved charge current j_λ whose structure depends on the free Lagrangian.

Inversely, if $\partial_\lambda J_\lambda = 0$ then there exists gauge invariance of the second kind in this theory of interaction. In electromagnetic interactions, the interacting current J_λ is the electric charge current j_λ and the field B_λ is the photon field.

We note however that in a theory where the interacting current J_λ contains field derivatives, e.g. the π-meson current, the gauge invariance of the second kind does not lead to a continuity equation. The theory can be invariant under the combined transformation.

APPENDIX B

We want to calculate the following integral:

$$R = \int_o^{q-B} E^{1/2}\, I\, dE$$

where

$$I = \left(1 - \frac{q}{B - E_S}\right)^2 \frac{E_S}{E_S + E} + \frac{E_S}{B - E_S}\left(2q - \frac{q^2}{B - E_S}\right)\frac{1}{B+E} - \frac{q^2}{B - E_S}\,\frac{E_S}{(B + E)^2}$$

Put $E^{1/2} = x$, $E = x^2$, $dE = dxdx$, $E^{1/2}\,dE = dx^2dx$ and

$$\frac{1}{2}\,E^{1/2}\,IdE = \left[\left(1 - \frac{q}{B - E_S}\right)^2 \frac{E_S\,x^2}{E_S + x^2} + \frac{E_S}{B - E_S}\left(2q - \frac{q^2}{B - E_S}\right)\cdot \frac{x^2}{B + x^2} - \frac{q^2 E_S}{B - E_S}\,\frac{x^2}{(B + x^2)^2}\right]dx$$

Since

$$\frac{x^2}{E_S + X^2} = \frac{x^2 + E_S - E_S}{E_S + x^2} = 1 - \frac{E_S}{E_S + x^2}$$

$$\frac{x^2}{B + x^2} = 1 - \frac{B}{B + x^2}$$

$$\frac{x^2}{(B + x^2)^2} = \frac{1}{B + x^2} - \frac{B}{(B + x^2)^2}$$

we can rewrite:

$$\frac{1}{2}\,E^{1/2}\,IdE = \left(1 - \frac{q}{B - E_S}\right)^2 + \frac{E_S}{B - E_S}\left(2q - \frac{q^2}{B - E_S}\right) - \left(1 - \frac{q}{B - E_S}\right)^2 \frac{E_S{}^2}{E_S + x^2}$$

$$- \frac{E_S B}{B - E_S}\left(2q - \frac{q^2}{B - E_S}\right)\frac{1}{B + x^2} - \frac{q^2 E_S}{B - E_S}\,\frac{1}{B + x^2} +$$

$$+ \frac{q^2 B E_S}{B - E_S}\,\frac{1}{(B + x^2)^2}$$

$$= E_S - \left(1 - \frac{q}{B - E_S}\right)^2 E_S{}^2\,\frac{1}{E_S + x^2} + \left(\frac{q^2 E_S{}^2}{(B - E_S)^2} - \frac{2qBE_S}{B - E_S}\right)\cdot$$

$$\frac{1}{B + x^2} + \frac{q^2 B\, E_S}{B - E_S}\,\frac{1}{(B + x^2)^2}$$

We have:

$$\int \frac{dx}{B + x^2} = \frac{1}{\sqrt{B}}\,\mathrm{arctg}\,\frac{x}{\sqrt{B}}$$

$$\int \frac{dx}{E_S + x^2} = \frac{1}{\sqrt{E_S}}\,\mathrm{arctg}\,\frac{x}{\sqrt{E_S}}$$

$$\int \frac{dx}{(B + x^2)^2} = \frac{1}{2}\,\frac{1}{B^{3/2}}\quad \mathrm{arctg}\,\frac{x}{\sqrt{B}} + \frac{x/\sqrt{B}}{1 + \frac{x^2}{B}}$$

Thus:

$$R = \int_o^{q-B} E^{1/2}\, I\, dE$$

$$= 2E_S\sqrt{q - B} - 2E_S{}^2\left(1 - \frac{q}{B - E_S}\right)^2 \frac{1}{\sqrt{E_S}}\,\mathrm{arctg}\sqrt{\frac{q-B}{E_S}}$$

$$+ \frac{2q\,E_S}{B - E_S}\left(\frac{q\,E_S}{B - E_S} - 2B\right)\frac{1}{\sqrt{B}}\,\mathrm{arctg}\sqrt{\frac{q-B}{B}}$$

$$+ \frac{q^2\,E_S}{B - E_S}\,\frac{1}{\sqrt{B}}\,\mathrm{arctg}\sqrt{\frac{q-B}{B}} + \frac{q\,E_S}{B - E_S}\sqrt{q - B}$$

Finally:

$$R = \left(2E_S + \frac{q}{B/E_S - 1}\right)\sqrt{q-B} - 2\left(E_S - \frac{q}{B/E_S - 1}\right)^2 \frac{1}{\sqrt{E_S}} \operatorname{arctg}\sqrt{\frac{q-B}{E_S}}$$

$$+ \frac{q}{B/E_S - 1}\left(q - 4B + 2\frac{q}{B/E_S - 1}\right)\frac{1}{\sqrt{B}} \operatorname{arctg}\sqrt{\frac{q - B}{B}}$$

REFERENCES

R. K. Adair and E. C. Fowler, Strange Particles, Interscience Publishers (1963).

T. Ahrens and E. Feenberg, Physical Review 86, (1952) 64.

M. Anikina, et al., Journal of Experimental and Theoretical Physics (USSR) 15, (1962) 93.

G. Alexander et al., Physical Review Letters 9, (1962) 26.

M. Bardon et al., Phys. Rev. Lett. 7, (1961) 23.

W. Barkas et al., Phys. Rev. Lett. 9, (1962) 69.

E. Beall et al., Phys. Rev. Lett. 8, (1962) 75.

S. M. Berman, European Organization for Nuclear Research, Cern, 10-21 (1962).

J. Bernstein, The Cargese Lectures (1962).

H. A. Bethe and C. Longmire, Phys. Rev. 77 (1950) 647.

R. W. Birge et al., Phys. Rev. Lett. 11 (1963) 35.

J. L. Brown, Proceeding of the 1962 International Conference on High Energy Physics, p. 826.

N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531.

U. Camerini et al., Phys. Rev. Lett. 13 (1964) 318.

C. A. Coombes et al., Phys. Rev. Lett. 108 (1957) 1348.

B. Cork et al., Phys. Rev. 120 (1960) 1000.

C. L. Cowan and Reines, Phys. Rev., 113 (1959) 273.

F. S. Crawford, Proceedings of the 1962 International Conference on High Energy Physics, p. 827.

R. Davis, Phys. Rev. 97 (1955) 766.

G. Danby et al., Phys. Rev. Lett. 9, (1962) 36.

P. Depommier et al., Proceedings of the 1962 International Conference on High Energy Physics, p. 411; and Phys. Rev. Lett. 5, (1963) 61.

R. Ely et al., Phys. Rev. Lett. 8, (1962) 132.

W. F. Fry and Cline, Phys. Rev. Lett. 13, (1964) 101.

M. Gell-Mann, Phys. Rev. 125, (1962) 1067.

E. L. Hill, Reviews of Modern Physics 23, (1951) 253.

R. King, Phys. Rev. 121, (1961) 35.

L. Kirsch et al., Phys. Rev. Lett. 13, (1964) 35.

D. Luers et al., Phys. Rev. Lett. 7, (1961) 255.

T. D. Lee and C. N. Yang, Phys. Rev. 98, (1955) 1501; and Phys. Rev. Lett. 119, (1960) 1410.

T. D. Lee, Phys. Rev. Lett. 9, (1962) 319.

Y. K. Lee et al., Phys. Rev. Lett. 10, (1963) 253.

T. Mayer - KuKuck and F. Michel, Phys. Rev. Lett. 7, (1961) 167.

D. Neagu et al., JETP 40, (1961) 1618.

V. Ogievetski and Polubarinov, Nuovo Cimento 23 (1962) 173.

B. Pontecorvo, JETP 36, (1959) 1615.

J. J. Sakurai, Lectures of Theoretical Physics, Boulder, Interscience Publishers (1959).

D. Stern et al., Phys. Rev. Lett. 12, (1964) 459.

J. C. Taylor, Phys. Rev. 110 (1958) 1216.

V. Tegledi, Lectures Notes on Weak Interactions, Christian Fronsdal Editor (1962).

R. D. Trip et al., Phys. Rev. Lett. 9, (1962) 66.

W. Willis et al., Phys. Rev. Lett. 8, (1964) 291.

C. N. Yang and Mills, Phys. Rev. 96 (1954) 192.

C. N. Yang, Eastern Theoretical Physics Conference, Gordon and Breach, Science Publishers (1963).

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