OPTIMIZATION METHODS FOR WILDFIRE-RESILIENT TRANSMISSION GRID OPERATIONS AND INFRASTRUCTURE PLANNING

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SUMMARY

Climate change-driven natural disasters pose an increasing threat to the power grid. Wildfires pose a unique challenge as power systems can ignite these destructive events, exposing utilities to liability. To mitigate the risk of ignitions, system operators proactively de-energize high-risk transmission lines in Public Safety Power Shutoff (PSPS) events. While effective for ignition risk mitigation, PSPS events can cause significant load shed. Utilities can also pursue infrastructure investments, such as line hardening or batteries, to mitigate the risk of ignition or maintain service but these decisions add complexity to already challenging optimization problems.

The contributions of this thesis include advances in modeling for optimally mitigating wildfire ignition risks, improved computational methods that leverage decomposition and machine-learning techniques for scalable algorithms on large and realistic test networks, and applications in infrastructure investments, climate resilience, and equity relevant to policymakers and utilities. This thesis provides detailed optimal power shutoff formulations in work evaluating sensitivity of decisions to ignition risk aggregation metrics and power flow formulations. Optimal power shutoff results achieve an order of magnitude reduction in load shed relative to methods comparable to industry standards. Extensions to infrastructure investment planning to support PSPS events are presented through tractable optimization algorithms, including flexibility to consider policy, equity, and alternative extreme weather events. Computational improvements are introduced through machine learning techniques and a novel temporal decomposition method that enables long time horizons to be modeled, resulting in fast, high-quality solutions that outperforms what was previously possible. These methods provide results on the order of an hour of computing time compared to days required under previous methods. In summary, this dissertation presents a detailed understanding of optimal transmission switching for PSPS events, offering further insights for engineers, utilities, and policymakers through flexible tools with realistic simulations.

CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

Climate change is increasing the prevalence of wildfire-prone conditions, leading to more severe and frequent wildfires [9, 10]. While most wildfires are not started by electric power infrastructure, wildfires ignited by power lines tend to be more destructive than those from other sources [11]. For instance, less than 10% of reported wildfire ignitions in California are due to power lines, but these account for nearly half of the most destructive fires [12], including the 2018 Camp Fire that is classified as the most destructive and deadliest fire in California history with over 150,000 burnt acres, 85 deaths, and over ten billion dollars in damage [13, 14]. Accordingly, engineers must operate and design power systems in a manner that mitigates the risk of wildfire ignitions [13, 15, 16].

Public Safety Power Shutoff (PSPS) events are one method for mitigating the chance that a power line fault will ignite a fire. During PSPS events, a utility preemptively de-energizes certain power lines to remove their ignition risk [17]. Utilities identify lines that are at high-risk of starting a fire based on line factors (condition, age, capacity, etc.), environmental conditions (temperature, wind velocity, humidity, etc.), wildfire spread models, and other considerations [18, 19]. Lines that exceed an acceptable risk level are de-energized to prevent a fault or sparks from this line igniting what could be a severe fire. PSPS events offer an effective and immediate way for utilities to temporarily remove excessively risky lines from operation. However, as lines in a network are de-energized, the utility's overall ability to transmit power is reduced, potentially leading to power outages for consumers. Power outages can have significant economic impacts and negatively affect communities and consumers relying on that power [20].

Since power utilities are held liable for damages caused by wildfires [21], they are investing in infrastructure for the transmission grid to mitigate the risk of wildfire ignitions or to aid in delivering power to consumers during PSPS events. This can involve measures like line maintenance (e.g., vegetation management) or line hardening (e.g., covered conductors or undergrounding) [22–25]. New methods can rapidly de-energize transmission lines when a break is detected along the conductor, preventing contact with the ground that can ignite fires [26]. While this method can prevent fires specifically from ground contact [27], vegetation contact or conductor clashing under high wind conditions can still lead to sparks that ignite fires [28]. Undergrounding lines is very effective as undergrounded lines can continue to operate with no risk of sparking a fire. Given the high budget and long timelines for these projects, it is important to consider the optimal way to harden a network [29].

Other infrastructure investments can reduce the extent of power outages from PSPS events. Options such as utility-scale solar can be installed to locally provide power during PSPS events. Outside of increasing resilience and reliability during grid outages, battery installations can also assist in increasing clean energy integration [30]. Similarly, grid-scale batteries can be used to locally maintain power during outages but also support renewables, frequency regulation, and price arbitrage during nominal operations [31]. Accordingly, utilities are also actively investing in grid-scale batteries [32]. Optimally sizing and siting these types of infrastructure will be helpful as investments continue to grow.

This research focuses on the development of optimization models to aid in decision making related to multiple multi-use investments (e.g., batteries supporting natural disaster driven outages and providing flexibility during nominal operations) under extreme weather conditions. To do this, models will need to consider long time horizons that capture impacts from varying natural disasters while also maintaining a level of temporal granularity that can capture the flexibility of the grid (e.g., hourly charging decisions for batteries and daily switching decisions in PSPS events). In addition, realistic data for wildfire ignition risk, load demand, renewable generation, and grid geography will need to be aggregated to inform decisions. Long time horizons, binary decision variables, temporal granularity, and large, realistic test cases all increase the computational complexity of an optimization problem, necessitating additional computational techniques.

1.2 Literature Review

This section reviews the literature related to work addressing the challenges introduced in Section 1.1. Section 1.2.1 reviews work related to wildfire resilience techniques. Section 1.2.1 discusses optimal transmission switching to reduce wildfire ignition risk. Section 1.2.1 discusses additional transmission planning to support PSPS events. Finally, Section 1.2.2 discusses techniques to aid in computational difficulty.

1.2.1 Wildfires and Power Infrastructure

The power grid faces an increased threat of power outages from weather-related events under worsening climate change [33]. A report from the National Oceanic and Atmospheric Administration found that extreme weather-related outages have had an average annual cost of $127.4 billion from 2019 to 2024 in the United States [9]. Sustained power outages, which can be largely attributable to extreme weather events [34], saw an increase in annual costs of 25% from 2002 to 2015, for a cost of over $40 billion in 2015 [35].

Wildfires pose a unique challenge to power infrastructure. Unlike other extreme weather events, power infrastructure can be damaged by extreme wildfire as well as be the source of the extreme wildfires. A recent climate model predicts that the land area burned due to wildfires to increase between 3% and 52% by 2050 based on a climate model ensemble in California [36]. Wildfires started by power system infrastructure tend to be severe and expansive compared to other ignition sources [11, 37]. In addition, power systems are a common ignition source [38]. This may be from high wind speeds and temperatures which are correlated with increased power line fault probability and fire spread.

Optimal Transmission Switching for Wildfire Resilience

To reduce the risk of power systems-ignited wildfires, utility operators will preemptively de-energize transmission lines, removing the risk of ignition but also eliminating the ability for the line to transmit power. Utilities have their own methods for making these de-energization decisions based on fire potential (including sources like the WFPI [39]), humidity, wind speeds, and fuel [18].

Rhodes et al. introduce the Optimal Power Shutoff (OPS) as an optimization problem to balance wildfire risk and load shed [40]. Optimizing the lines to be de-energized can reduce wildfire risk and load shed in a balanced manner. The key difference between the OTS formulation in [40] and previous OTS formulations (see, e.g., [41, 42]) is that the lines are switched off solely for the sake of de-energizing them to reduce wildfire ignition risk, not to achieve some other objective like reducing operating costs [41, 43, 44] or improving reliability [45, 46]. Prior OTS applications achieved these other objectives by focusing on the ability of transmission switching to ameliorate network congestion. In contrast, the sets of lines de-energized to mitigate wildfire ignition risks are not strongly related to network congestion effects. Moreover, in contrast to prior OTS applications, OTS problems in a wildfire context must consider the possibility of significant load shedding resulting from de-energization of non-negligible portions of the system. Extensions have been made to multi-period shutoff scheduling [1, 47, 48], security-constrained optimal power flow [49], stochastic unit commitment [50], alternative mitigation actions such as dynamic line ratings [51] and microgrid formation [52], power restoration [53], and equity considerations [54].

When formulated as mixed-integer linear programs (MILPs), OPS problems can require hours to days to solve for realistic large-scale networks, making them impractical for day-ahead operational planning. This computational challenge has motivated research into machine learning guided optimization techniques. Recent work has applied methods such as Predict-and-Search [55, 56] and Neural Diving [57] to accelerate MILP solution for OPS problems [7]. These approaches leverage graph neural networks to predict near-optimal line de-energization decisions, then use domain-informed refinement strategies to ensure feasibility while maintaining solution quality. By exploiting the fact that similar OPS problems are solved repeatedly with variations in wildfire risk, load, and renewable generation, ML-guided methods can achieve significant speedups compared to commercial solvers. This enables the use of more sophisticated optimization formulations in operational timescales, as discussed in Chapter 4.

Most OPS research relies on DC power flow approximations to ensure computational tractability. However, these approximations neglect reactive power and voltage magnitude considerations that can be critical for AC feasibility. Recent work has explored Linearized AC power flow formulations for OPS [8], which incorporate voltage magnitude variables and reactive power constraints while maintaining a linear or nearly linear problem structure. These formulations offer a middle ground between computationally efficient DC approximations and full AC power flow models, potentially yielding line de-energization decisions with better AC recoverability. Trade-offs between solution time, model fidelity, and operational feasibility across these different power flow representations are examined in Chapter 4.

Implementing OPS models requires geospatial wildfire risk data that can be mapped to transmission network components. The United States Geological Survey's (USGS) Wildland Fire Potential Index (WFPI) [39] provides a daily updating map of the United States with every 1 km^2 pixel associated with a wildfire potential in that area. This provides a proxy for wildfire ignition risk where the WFPI provides a value from 0 to 247 for each pixel. This gives more information for research as opposed to a binary threat indicator. Researchers can overlay risk maps with synthetic transmission networks to define risk values for individual system components. However, the method chosen to aggregate spatially granular risk data to line-level metrics can significantly impact OPS outcomes, as will be further discussed in Chapter 3.

Transmission Planning for PSPS Support

While PSPS events are an effective and immediate way to temporarily remove the risk of some transmission lines, they do reduce the ability to transmit power to consumers, causing economic impacts and negative effects on communities [20]. Utilities are in the process of investing in line hardening to minimize the extent of PSPS events via line undergrounding, covered conductors, and vegetation management [22–25]. Line undergrounding, in particular, is a compelling option as an undergrounded transmission line has effectively no risk of igniting a wildfire while still able to operate nominally, reducing load shed from PSPS events. Utilities in California, such as PG&E and SCE, are in the process of undergrounding sections of their transmission networks [24, 58, 59]. Undergrounding transmission lines is costly, on the order of $10 million USD per mile necessitating optimal planning to ensure the money is allocated effectively [29, 60, 61].

Another approach is to maintain power at isolated nodes during PSPS events. To do this, utilities may install grid-scale batteries [62, 63]. These allow consumers to continue to be served power during a PSPS event that severs their connections to the broader transmission network [31, 64]. Batteries have the additional benefits of being useful during nominal grid operations. For example, when there is low wildfire ignition risk, batteries can improve renewable integration or price arbitrage [65]. Some extensions have been made to model batteries in networks considering AC power flow formulations, however only on relatively small test networks [66]. When considering grid-scale battery investments, it is crucial to understand the hourly decisions made throughout the year under different conditions.

Researchers have investigated both of these wildfire mitigation options. Authors in [67] provide a survey of wildfire ignition risk mitigation techniques for transmission grids. In [68], Taylor et al. propose a method for undergrounding sections of transmission lines that may be most at risk for igniting wildfires. Similarly, Taylor et al. investigate a decomposition method to solve a multi-scenario undergrounding problem considering different grid and wildfire conditions in [69]. Authors in [70] evaluate various inputs to better estimate the potential damage associated with a wildfire ignited by each transmission line in the system to rank the priority of investment decisions.

Outside of undergrounding and batteries, other methods have been considered for wildfire ignition risk mitigation from power systems. Researchers in [71] considered distributed energy resources (DER) on a small test system. Authors in [52] evaluate microgrids in a distribution system to allow for load block switching to mitigate ignition risk. Tandon et al. consider dynamic line ratings as an operational way to minimize wildfire ignitions on a small test system [51]. Extensions have also been made to data-driven and machine learning techniques to better estimate lines to be de-energized [72, 73].

Similar work has been investigated outside of the wildfire context for other types of climate change driven natural disasters. Line hardening and undergrounding has been investigated as a resilience measure to outages from hurricanes [74]. Further work for hurricane resilience has considered stochastic distribution expansion planning [75] and capacity expansion planning [76]. Work has also been done considering winterization of generators [77]. Similarly, mobile battery systems have been considered for winterization [78]. Transmission expansion problems have been evaluated under extreme heat [79] and ongoing climate uncertainty [80].

1.2.2 Computational Techniques

Sizing and siting components on realistic test networks is a challenging problem. Transmission planning problems that take into account batteries quickly become computationally complex [65, 66]. Since it is necessary to incorporate operational decisions in detail across multiple scenarios to inform optimal placements, these problems scale poorly with both the number of scenarios and the size of the test network. While some research has been done to investigate stochastic techniques related to renewables [81] or line switching [82], neither consider both batteries and wildfires. Work has considered battery planning under wildfire risk at the distribution level, leveraging stochastic optimization techniques [83]. PSPS events will result in different topologies as different lines will be de-energized at different times of the year, depending on wildfire risk profiles. Since wildfire risk is correlated with factors like heat [18], one can also expect load demand to fluctuate with wildfire risk (e.g., more power used for air conditioning at times of high heat, aligning with ignition risk). As conditions vary over time and across the geographic region served by a network, the way the grid operates also changes greatly. This motivates a need for multi-scenario operation.

Progressive Hedging

Progressive Hedging is a scenario-based decomposition method developed for stochastic programming [84]. Through tools like Pyomo [85] and mpi-sppy [86], progressive hedging algorithms can be used to implement large optimization problems necessitating decomposition methods. Typically, progressive hedging algorithms allow for a set of common variables to be optimized across multiple scenarios, modeling uncertainty. In [87], Reiten and Mikkelsen develop a progressive hedging algorithm for long-term planning with consideration for operational dispatch in the context of lowering CO2 emissions. In [88], Kim and Dvorkin propose a progressive hedging algorithm for mobile battery deployment to aid in disaster resilience and recovery. The work discussed in Chapter 8 presents an approach to consider long-term and operational optimization for battery sizing and siting to support communities during PSPS events [3].

Benders Decomposition

Benders Decomposition is another decomposition technique that allows for optimization across different scenarios [89, 90]. This technique allows for investment decisions and operational constraints to be separated, aiding in computational efficiency. This method has been applied to battery planning problems, outside of the context of wildfires, for resilience and flexibility at the distribution [91] and transmission level [92]. Likewise, Yi et al. have considered battery planning in conjunction with line hardening for increased resilience under load and renewable uncertainties [93]. In the context of wildfires, this stochastic optimization approach allows for individual scenarios to model the wildfire and grid conditions seen at different times through the year.

1.3 Contributions

This dissertation advances the state-of-the-art in wildfire resilience planning for transmission systems through both methodological innovations and large-scale computational demonstrations. The key contributions are organized around three interconnected themes: operational decision-making under wildfire risk, infrastructure investment optimization, and scalable solution algorithms.

1.3.1 Wildfire Risk Quantification and Operational Decision-Making

- This work systematically compares six different methods for aggregating geospatial wildfire risk data into line-level risk metrics, demonstrating that the choice of aggregation method significantly impacts optimal line de-energization decisions and resulting load shed outcomes (Chapter 3). This analysis is the first to evaluate risk metric sensitivity on a large-scale, geographically realistic test system using real wildfire data. Optimal transmission switching provides order of magnitude improvements in load shed compared to thresholded method comparable to the industry standard. This work was originally presented at the Hawaii International Conference on System Sciences [4].

1.3.2 Infrastructure Investment Planning

- This work presents the first comprehensive infrastructure investment formulation for wildfire resilience that jointly considers battery storage, solar PV, and three line hardening options (undergrounding, covered conductors, vegetation management) within a multi-period optimal power shutoff framework (Chapter 5). Numerical results on test cases demonstrate that investment decisions are highly sensitive to both the available budget and the trade-off parameter between wildfire risk and load shed, with line hardening often dominating investment portfolios despite its high cost. This work was originally presented at the 11th Bulk Power System Dynamic and Control Symposium (IREP) [1].

- Work in Chapter 6 conducts the first equity-focused analysis of transmission line undergrounding investments for wildfire resilience, revealing critical limitations of vulnerability indices in protecting minority populations. This case study demonstrates that broadly defined climate-equity policy does not provide sufficient constraints to improve outcomes for some communities in regional areas affected by high wildfire ignition risk. This work was originally published in Applied Energy [2]
- Work in Chapter 6 extends the undergrounding investment analysis to evaluate resilience co-benefits across multiple hazard types (wildfires, hurricanes, wind events), providing the first empirical assessment of whether wildfire-focused investments provide protection against other extreme weather events. For this case study, results demonstrate that investments planned for a specific climate event (wildfire ignition risk mitigation) are not effective at mitigating load shed from other extreme weather events. This work was originally presented at the IEEE PowerTech 2025 Conference [5].

1.3.3 Scalable Computational Methods

- Work in Chapter 7 develops a Benders decomposition algorithm for the coupled battery sizing/siting and line undergrounding problem that enables scenario-based optimization across seasonal and yearly time horizons. This approach parallelizes the solution of operational subproblems and demonstrates that optimal investment decisions vary significantly across seasons, motivating year-round planning rather than worst-case scenario approaches. This work is published in the *Sustainable Energy, Grids, and Networks* journal and originally presented at the 12th IREP Symposium [6].
- Work in Chapter 8 presents a customized Progressive Hedging algorithm that enables

battery placement optimization over full-year horizons with hourly temporal granularity on large-scale transmission networks. Unlike standard Progressive Hedging implementations, this approach handles temporal coupling through battery state-of-charge variables to enable temporal decomposition. This represents the first solution of a year-long battery placement problem with hourly resolution. This framework provides order of magnitude speed ups, producing results in just over an hour that failed to build within 24 hours using traditional methods. This work is published in the Electric Power Systems Research journal and originally presented at the Power Systems Computation Conference [3].

1.3.4 Computational Demonstrations

Methodological contributions are demonstrated on large-scale, geographically realistic test systems including the California Test System (CATS, 8870 buses) and the Texas synthetic network (7000 buses), augmented with real-world data for daily wildfire risk, load and renewable generation profiles (CATS), demographic data (Texas), and extreme weather events (Texas). These demonstrations represent the largest-scale applications of wildfire-aware grid optimization in the academic literature and produce publicly available datasets to support future research.

This thesis furthers the field by providing an understanding of the impact of modeling decisions on network topology outcomes and impacts to communities. The flexible frameworks presented allow for considerations of policy, equity, and other extreme weather events in future infrastructure planning projects. The computational methods introduced here enable high quality solutions with low solution times. The completed work in this report is published or submitted to various conference proceedings [1, 4, 5, 7, 8] and journals [2, 3, 6].

1.4 Dissertation Organization

Chapter 2 introduces power flow and OPS modeling common throughout this work. Chapter 3 investigates how methods of aggregating real world data to synthetic networks impact outcomes of the OPS. Chapter 4 presents two extensions to the OPS framework: a machine learning guided optimization technique to achieve high quality results in shorter time frames, and an alternative power flow model that considers reactive power and voltage magnitude through Linearized AC formulations. Chapter 5 details models for planning infrastructure investments to support PSPS events, including line undergrounding, battery storage, and solar generation. Chapter 6 studies two real world applications: equitable outcomes of environmental justice policy applied to line undergrounding planning, and the efficacy of line undergrounding plans intended for wildfire risk mitigation when evaluated under other extreme weather event profiles. Chapter 7 compares investment decision outcomes when including different risk, load, and renewable profiles throughout the year in the planning horizon. Chapter 8 presents a temporal decomposition algorithm through Progressive Hedging to allow long time horizons to be modeled while maintaining consistency in temporally linked variables, such as battery state-of-charge. Chapter 9 concludes this dissertation.

CHAPTER 2

DC OPTIMAL POWER FLOW AND OPTIMAL POWER SHUTOFF MODEL

This chapter details the nomenclature and constrained optimization models used in projects throughout this dissertation. Specific chapters will have noted modifications to the models listed here.

2.1 Nomenclature

This section provides a unified nomenclature for the sets, parameters, and decision variables used throughout this dissertation. We first define the network component sets and temporal indices in Tables 2.1 and 2.2. Network parameters are provided in Table 2.3, with wildfire risk parameters detailed in Table 2.4. Finally, decision variables for operational models are defined in Table 2.5.

Table 2.1: Network Component Sets and Indices

Notation	Description
N	Set of buses in the network
L	Set of transmission lines
L_d^{switch}	Subset of lines that can be de-energized on day d, $L_d^{switch} \subseteq L$
$L^{n,fr}$	Subset of lines with bus n as the *from* bus
$L^{n,to}$	Subset of lines with bus n as the *to* bus
G	Set of generators
G^n	Set of generators located at bus n
n, m	Bus indices, $n, m \in N$
ℓ	Line index, $\ell \in L$
i	Generator index, $i \in G$

2.1.1 Conventions

The following conventions are used throughout this dissertation:

Table 2.2: Temporal Indices

Notation	Description
$\mathcal{T} = \{1, \ldots, T\}$	Set of time periods (typically hours) within a day
t	Time period index, $t \in \mathcal{T}$
T	Final time period in $\mathcal{T}$
$\mathcal{D}$	Set of days in the planning horizon
d	Day index, $d \in \mathcal{D}$

Table 2.3: Network Parameters

Notation	Description
Line parameters ($\ell \in \mathcal{L}$)	
b_ℓ	Susceptance in p.u.
$\overline{f}_\ell$	Power flow limit in p.u.
$\underline{\theta}_\ell, \overline{\theta}_\ell$	Lower and upper voltage angle difference limits in p.u.
$n^{\ell,\text{fr}}, n^{\ell,\text{to}}$	From and to buses
Generator parameters ($i \in \mathcal{G}$)	
$\underline{p}^i_g, \overline{p}^i_g$	Lower and upper power generation limits in p.u.
c^i_g	Generation cost coefficient in \$/MWh
n^i	Bus location of generator i
Bus parameters ($n \in \mathcal{N}$)	
$p^n_{l,t,d}$	Power demand at time t on day d in p.u.

- All power quantities use a 100 MVA base (1 p.u. = 100 MW or 100 MWh)
- All angles and angle differences are expressed in per unit
- Positive power flow on line ℓ flows from bus $n^{\ell,\text{fr}}$ to bus $n^{\ell,\text{to}}$
- Time periods $t \in \mathcal{T}$ represent one-hour intervals
- Days $d \in \mathcal{D}$ represent 24-hour periods

2.2 Optimal Power Shutoff

In this section, we introduce two optimization models: the DCOPF and an extension to the Optimal Power Shutoff (DCOPS) [40]. In this work, power flow is modeled using the DC approximation, which neglects reactive power, line losses, and voltage magnitudes [94]. Prior publications such as [95–98] have highlighted discrepancies between

Table 2.4: Wildfire Risk Parameters

Notation	Description
r^ℓ_d	Wildfire ignition risk for line ℓ on day d (unitless, non-negative)
R^{PSPS}	Network-wide risk threshold triggering PSPS (unitless)

Table 2.5: Operational Decision Variables

Notation	Description	Type
$p^i_{g,t,d}$	Power generation from generator i at time t on day d in p.u.	Continuous
$\theta^n_{t,d}$	Voltage angle at bus n at time t on day d in p.u.	Continuous
$f^\ell_{t,d}$	Active power flow on line ℓ at time t on day d in p.u.	Continuous
$p^n_{ls,t,d}$	Load shed at bus n at time t on day d in p.u.	Continuous
z^ℓ_d	Line energization status for $\ell \in \mathcal{L}^{switch}_d$ on day d $z^\ell_d = 1$ if line is energized $z^\ell_d = 0$ if line is de-energized	Binary

Optimal Transmission Switching (OTS) problems that use the DC approximation versus the AC power flow model. AC OTS problems are challenging mixed-integer nonlinear programming (MINLP) problems, and formulating scalable and computationally tractable solution methods is the focus of on-going research [95–97, 99, 100], with many approaches using power flow relaxations and approximations [101]. Therefore, we use the DC approximation for computational tractability in this work.

Equation (2.1b) enforces lower ($\underline{p}^i_g$) and upper ($\overline{p}^i_g$) generation limits for power generation ($p^i_{g,t,d}$) at all generators $i \in \mathcal{G}$ at all times $t \in \mathcal{T}$ and all days $d \in \mathcal{D}$. Equation (2.1c) constrains any load shedding ($p^n_{ls,t,d}$) to be nonnegative and less than the power demand ($p^n_{l,t,d}$) at that time at each bus $n \in \mathcal{N}$. Equation (2.1d) enforces line flow ($f^\ell_{t,d}$) to be within lower and upper limits ($\overline{f}^\ell$) for all lines $\ell \in \mathcal{L}$.

Equation (2.1e) constrains the difference in voltage angle across a line ℓ from the from-bus $n^{\ell,fr}$ to the to-bus $n^{\ell,to}$ at each time step to be within the lower ($\underline{\theta}^\ell$) and upper ($\overline{\theta}^\ell$) limits. Equation (2.1f) models the DC power flow approximation, enforcing the relationship between line flows and voltage angle differences via the line susceptance b^ℓ.

Equation (2.1g) ensures power balance at all buses in the network. The objective (2.1a)

Model 1 B DC Optimal Power Flow (DCOPF)

$$\min \sum_{d \in D} \sum_{t \in T} \sum_{n \in N} p^{n}_{ls;t;d} \tag{2.1a}$$

$$\text{s.t.} \quad \forall d \in D;\ \forall t \in T;$$

$$\underline{p}^{i}_{g} \leq p^{i}_{g;t;d} \leq \overline{p}^{i}_{g} \qquad \forall i \in G \tag{2.1b}$$

$$0 \leq p^{n}_{ls;t;d} \leq p^{n}_{l;t;d} \qquad \forall n \in N \tag{2.1c}$$

$$-\overline{f}^{\ell} \leq f^{\ell}_{t;d} \leq \overline{f}^{\ell} \qquad \forall \ell \in L \tag{2.1d}$$

$$\underline{\theta}^{\ell} \leq \theta^{n^{\ell;fr}}_{t;d} - \theta^{n^{\ell;to}}_{t;d} \leq \overline{\theta}^{\ell} \qquad \forall \ell \in L \tag{2.1e}$$

$$f^{\ell}_{t;d} = -b^{\ell}(\theta^{n^{\ell;fr}}_{t;d} - \theta^{n^{\ell;to}}_{t;d}) \qquad \forall \ell \in L \tag{2.1f}$$

$$\sum_{\ell \in L^{n;fr}} f^{\ell}_{t;d} - \sum_{\ell \in L^{n;to}} f^{\ell}_{t;d} = \sum_{i \in G^{n}} p^{i}_{g;t;d} - p^{n}_{l;t;d} + p^{n}_{ls;t;d} \qquad \forall n \in N \tag{2.1g}$$

minimizes the total load shed in the network. Alternative objectives may aim to minimize a cost function.

Similarly to Model 1, Equation (2.2b) enforces lower ($\underline{p}^{i}_{g}$) and upper ($\overline{p}^{i}_{g}$) generation limits for power generation ($p^{i}_{g;t;d}$) at all generators $i \in G$ at all times $t \in T$ and all days $d \in D$. Equation (2.2c) constrains any load shedding ($p^{n}_{ls;t;d}$) to be nonnegative and less than the power demand ($p^{n}_{l;t;d}$) at that time at each bus $n \in N$. Equations (2.2d) and (2.2e) enforce line flow ($f^{\ell}_{t;d}$) to be within lower and upper limits ($\overline{f}^{\ell}$) in accordance with the energization status z^{ℓ}_{d}, for lines $\ell \in L$. For lines not in the day-specific switchable set L^{switch}_{d}, note that z^{ℓ}_{d} is required to be set to one (indicating that the line is energized) instead of being a binary decision variable, thus reducing Equation (2.2d) to (2.2e). Lines with zero risk on day d are excluded from the switchable set L^{switch}_{d}.

Equations (2.2f), (2.2g), and (2.2h) constrain the difference in voltage angle across a line ℓ from the from-bus $n^{\ell;fr}$ to the to-bus $n^{\ell;to}$ at each time step to be within the lower ($\underline{\theta}^{\ell}$) and upper ($\overline{\theta}^{\ell}$) limits in accordance with the line energization status. Again, if a line is not in the switchable set for that day, the energization status z^{ℓ}_{d} is set to one, thus reducing Eqs. (2.2f) and (2.2g) to (2.2h).

Equations (2.2i), (2.2j), and (2.2k) model the DC power flow approximation with line

Model 2 B DC Optimal Power Shutoff (DCOPS)

$$
\begin{aligned}
&\min \sum_{d\in D}\sum_{t\in T}\sum_{n\in N} p^{n}_{ls;t;d} && && (2.2a)\\
&\text{s.t.}\quad \forall d\in D\,;\ \forall t\in T\,; && && \\
&\underline{p}^{i}_{g} \le p^{i}_{g;t;d} \le \overline{p}^{i}_{g} && \forall i\in G && (2.2b)\\
&0 \le p^{n}_{ls;t;d} \le p^{n}_{l;t;d} && \forall n\in N && (2.2c)\\
&-\overline{f}^{\ell} z^{\ell}_{d} \le f^{\ell}_{t;d} \le \overline{f}^{\ell} z^{\ell}_{d} && \forall \ell\in L^{switch}_{d} && (2.2d)\\
&-\overline{f}^{\ell} \le f^{\ell}_{t;d} \le \overline{f}^{\ell} && \forall \ell\in L \setminus L^{switch}_{d} && (2.2e)\\
&\theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d} \ge \underline{\theta}^{\ell} z^{\ell}_{d} + \underline{M}(1 - z^{\ell}_{d}) && \forall \ell\in L^{switch}_{d} && (2.2f)\\
&\theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d} \le \overline{\theta}^{\ell} z^{\ell}_{d} + \overline{M}(1 - z^{\ell}_{d}) && \forall \ell\in L^{switch}_{d} && (2.2g)\\
&\underline{\theta}^{\ell} \le \theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d} \le \overline{\theta}^{\ell} && \forall \ell\in L \setminus L^{switch}_{d} && (2.2h)\\
&f^{\ell}_{t;d} \ge -b^{\ell}(\theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d}) + |b^{\ell}|\underline{M}(1 - z^{\ell}_{d}) && \forall \ell\in L^{switch}_{d} && (2.2i)\\
&f^{\ell}_{t;d} \le -b^{\ell}(\theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d}) + |b^{\ell}|\overline{M}(1 - z^{\ell}_{d}) && \forall \ell\in L^{switch}_{d} && (2.2j)\\
&f^{\ell}_{t;d} = -b^{\ell}(\theta^{n_{\ell};fr}_{t;d} - \theta^{n_{\ell};to}_{t;d}) && \forall \ell\in L \setminus L^{switch}_{d} && (2.2k)\\
&\sum_{\ell\in L^{n;fr}} f^{\ell}_{t;d} - \sum_{\ell\in L^{n;to}} f^{\ell}_{t;d} = \sum_{i\in G^{n}} p^{i}_{g;t;d} - p^{n}_{l;t;d} + p^{n}_{ls;t;d} && \forall n\in N && (2.2l)\\
&\sum_{\ell\in L^{switch}_{d}} z^{\ell}_{d} r_{\ell;d} \le R^{PSPS}_{d} && \forall d\in D\,. && (2.2m)
\end{aligned}
$$

energization status. Eqs. (2.2f), (2.2g), (2.2i), and (2.2j) utilize big-M constants to allow for voltage angle differences to be unconstrained across de-energized lines, with $\overline{M}$ and $\underline{M}$ set to 2π and -2π respectively for results shown in this paper. Note that more sophisticated methods for selecting big-M values could be used, such as those in [102].

Equation (2.2l) ensures power balance at all buses in the network. Equation (2.2m) restricts the total risk on the network on day d to be below a value, R^{PSPS}_{d}, defined by the total remaining risk on the network resulting from the thresholded lines on the same day (see Section 3.3.1). Here $r_{\ell;d}$ represents the risk on line ℓ on day d based on the given metric. The objective shown in Equation (2.2a) minimizes total load shed in the network across all days and time periods. Other work may have different objectives as specified.

CHAPTER 3

QUANTIFYING METRICS FOR WILDFIRE IGNITION RISK FROM GEOGRAPHIC DATA IN POWER SHUTOFF DECISION-MAKING

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3.1 Introduction

To mitigate the risk of wildfire ignitions from power line faults, many utilities implement preemptive power shutoffs—called “Public Safety Power Shutoffs” (PSPS)—which involve selectively de-energizing certain power lines (via switching) to eliminate the possibility of these lines igniting fires. While this is an effective fire risk reduction strategy, it concurrently can lead to customer outages. Thus, operators face a complex environment in which they must avoid igniting wildfires by de-energizing lines while simultaneously ensuring reliable access to electricity. To better inform line de-energization planning, this paper characterizes how differences in wildfire ignition risk quantification can impact decisions in power grid operations.

To prioritize risk mitigation actions, many utilities use threshold-based methods that de-energize lines whose associated risk values exceed a pre-determined threshold [19]. Recently, researchers have proposed algorithms to better manage these trade-offs by considering both wildfire ignition risk and load shed due to power shutoffs when determining which lines to de-energize. Reference [40] proposes an optimization model, referred to as the "Optimal Power Shutoff" (OPS) problem, to balance wildfire risk and load shed. Similar works and extensions are proposed for multi-period shutoff scheduling [47, 48], security-constrained optimal power flow [49], stochastic unit commitment [50], alternative mitigation actions such as dynamic line ratings [51] and microgrid formation [52], power restoration [53], social equity considerations [54], and long-term investment planning [1, 3, 71]. Machine learning techniques have also been applied to predict ignitions from power lines [73] and to relate input wildfire scenarios and output mitigation strategies [72]. All of these models are sensitive to the specifics of wildfire risk parameters. Therefore, it is important to carefully consider how wildfire risk metrics for individual power lines are formulated.

As discussed in Section 3.2, this work defines wildfire risk based on wildfire potential, or the likelihood for an ignition to spread into a large and devastating wildfire. Numerous approaches are available to quantify wildfire potential, leveraging a combination of surface meteorological measurements and satellite data. This includes "Fire Weather Watch" and "Red Flag Warning" areas [103], Significant Fire Potential Outlooks [104], and the Wildland Fire Potential Index (WFPI) [39]. Other tools simulate fire spread, such as FSim [105] and FlamMap [106]. Another tool, Pyrecast, aggregates fire simulation results from millions of simulated ignition points to produce static maps of burned area risk [107].

This work leverages the availability of high-fidelity, real-world wildfire risk maps to characterize the risk of energized lines. WFPI and Pyrecast risk maps are good candidates for quantifying wildfire ignition risk from power equipment failure because of their temporal granularity (published daily and hourly, respectively), fine spatial granularity (1 square kilometer and 30 square meters, respectively), and the use of a range of potential levels (indices from 0 to 247 and the number of times each land area "pixel" is burned in the Monte Carlo sample set, respectively) rather than a binary "threat" categorization. Researchers

Figure 3.1: California's transmission line paths on a Wildland Fire Potential Index map for Oct. 26, 2020.

commonly overlay geospatial power grid data on top of such risk maps to obtain wildfire ignition risk values for individual power lines. Figure 3.1 shows California's transmission system superimposed on a WFPI map.

As seen in Figure 3.1, power lines generally intersect multiple wildfire potential pixels or values, thus raising the question of how to appropriately aggregate risk values along the length of a line to obtain a single risk value for the line. Past works have used the maximum intersecting risk value [50, 52, 53, 68] or a sum of intersecting risk values (i.e., a cumulative metric) [1, 2, 54]. However, it is not clear whether the maximum value accurately captures the risk of the entire line, as two lines with the same point-wise maximum value may have vastly different risk at other points; a decision-maker may be interested in prioritizing mitigation for the line with high risk along a significant fraction of the line length. Cumulative metrics, on the other hand, may not capture points of extreme risk, as the maximum value would. This motivates alternate strategies of aggregating risk, such as computing the mean of all intersecting values or thresholding out low intersecting risk values. A comparison of maximum and cumulative wildfire risk metrics in [68] demonstrates that different risk metrics produce significantly different optimal capital investments for long-term wildfire risk mitigation. Note that the analysis in [68] has some limitations: the optimal investment model does not include a power flow model and the test network (RTS-GMLC [108]) is relatively small and does not have realistic power line paths.

This paper addresses these limitations while also analyzing additional risk metrics. This work focuses on two challenges: (1) how to leverage wildfire risk maps to define the wildfire risk of power lines, and (2) how to leverage those metrics in power shutoff decision making for risk mitigation. This work compares the performance of six different risk metrics in two methods for determining line de-energizations to limit the wildfire risk in a power system: a threshold-based approach and an optimal power shutoff problem. Results are demonstrated on the California Test System (CATS) [109], a 9,000-bus synthetic test system with transmission line paths that represent the actual geographical locations of transmission lines in California (as shown in Fig. 3.1).

The primary contributions are as follows:

- The impacts of six different wildfire risk aggregation metrics are compared on the resulting power shutoff plans. The results find that these metrics result in significant differences among de-energization plans and load shed outcomes. To conduct this analysis, this work processed two years of real-world wildfire potential maps to compute risk parameters for the transmission lines in the California Test System. This dataset is publicly available upon request.
- Threshold- and optimization-based power shutoff planning are applied to the California Test System, a synthetic yet highly realistic grid model with power line paths accurately representing California infrastructure. Demand and renewable generation availability profiles based on real hourly time-series data from the California Independent System Operator (CAISO) are utilized to simulate varying operating conditions[109]. This is the first application of the optimal power shutoff problem to such a large and realistic test case in the academic literature.

- The results find that the optimization-based power shutoff method results in an approximately 80% reduction of the load shed compared to the threshold-based method while maintaining the same overall wildfire ignition risk.

The remainder of this chapter is organized as follows. Section 3.2 introduces the six risk metrics considered. Section 3.3 details the methods for planning power grid shutoffs. Section 3.4 describes the case study and presents computational results.

3.2 Wildfire Risk Metrics

This section presents six metrics that quantify the wildfire risk of individual power lines based on wildfire potential maps. To do this, the definition of wildfire risk is first discussed.

3.2.1 Wildfire Risk Definition

Consistent with the definitions in [68] and [110], this work considers wildfire risk from power lines in terms of two components: fault probability and wildfire potential. An electrical fault occurs when an abnormal event (e.g., contact with vegetation or animals, conductor clashing, or downed power lines) results in current flow outside of a power line conductor. These events might involve arcing, sparks, and burning equipment or vegetation. Fault probability is influenced by factors specific to power systems in addition to weather conditions (e.g., wind) and vegetation factors. The likelihood of a fault is a function of the age and condition of infrastructure, right-of-way, line loading and sagging, and voltage level. The energy release associated with a fault can ignite a wildfire. Wildfire potential, on the other hand, captures an ignition's impact, i.e., the subsequent potential for fire spread and intensity. This potential is dependent on factors that are not specific to power systems but rather is due to weather and vegetation conditions in the region surrounding an ignition.

While many faults occur, the probability of a fault occurring at any particular time and place is small and difficult to assess. Reference [111] proposes a risk metric that includes both fault probability and fire potential factors. Reference [112] analyzes fault probability due to conductor clashing, comparing the use of a nonlinear model of conductor vibrational physics under wind forces with machine learning methods for fault prediction. Among other data requirements, these studies require information about distances between conductors and vegetation, the ground, and other conductors, which is not commonly available in models of real or synthetic power systems. Even utilities may not have this data, as line inspections are costly and time-consuming. Thus, consistent with much of the literature on wildfire risk mitigation, this paper assumes that the probability of a fault occurring is constant throughout the power system and defines wildfire ignition risk based on wildfire potential only.

3.2.2 Aggregating Wildfire Potential Data

Risk metrics are derived from publicly available, real-world wildfire risk maps, specifically the WFPI maps from the U.S. Geological Survey [39]. The WFPI geographically represents the relative potential for large fires and fire spread, and is published daily. The index is calculated at a spatial granularity of 1 km^2 with a nominal range of 0-150. The index is enhanced by wind speed and can exceed 150 with very high wind speeds. Some land types such as desert and marshland do not have an associated value because they are considered "unburnable". Agricultural land also does not have a risk value because the vegetation type and moisture levels change often and are not readily available.

The wildfire risk metric for a power line is derived from the pixels of a wildfire risk map that a power line intersects. Lines typically cross many 1 km grid squares with potentially high variance in risk values, as shown in Fig. 3.1. Grid operators want to mitigate the risk of lines igniting fires, but are not able to de-energize just the high-risk segments of power lines. To assess the need for de-energization, the risk values that a power line passes through must be aggregated to obtain a single risk value for the entire line. The primary objective is to analyze how the choice of risk aggregation metric impacts line de-energization decision-making.

The following defines six power line wildfire risk metrics. The first three metrics are based on the mean, maximum, and cumulative values of the pixels the power lines intersect, respectively. The last three metrics use an additional pre-processing step to only consider the pixels whose risk values are above a particular threshold. Sections 3.2.3 and 3.2.4 mathematically define the six risk metrics, while Section 3.2.5 discusses the intuition and real-world significance of the metrics.

3.2.3 Baseline Wildfire Risk Metrics

For a power line ℓ, let P_ℓ denote the set of pixel indices p that the power line intersects. Let $R_{\ell;d;p}$ denote the set of pixel risk values on day d for line ℓ. For each line and on each day, this work aggregates these values as the maximum, mean, or cumulative value. The formal definitions for each metric are provided below.

Maximum (MA) Metric assigns a risk value for each line equivalent to the maximum risk of any pixel that the line intersects:

$$R^{MA}_{\ell;d} = \max_{p \in P_\ell} R_{\ell;d;p}. \tag{3.1}$$

Mean (ME) Metric assigns a risk value that is the average of the pixels that the line intersects:

$$R^{ME}_{\ell;d} = \frac{\sum_{p \in P_\ell} R_{\ell;d;p}}{|P_\ell|}. \tag{3.2}$$

Cumulative (CU) Metric assigns a risk that is the sum of the pixels that the line intersects:

$$R^{CU}_{\ell;d} = \sum_{p \in P_\ell} R_{\ell;d;p}. \tag{3.3}$$

3.2.4 Pre-Processing Wildfire Risk

The following describes a method of pre-processing wildfire risk data to better identify the highest risk pixels before aggregating the risk values into a metric. This method, adapted from [2], uses a risk threshold, where pixel p is removed from the set of pixels that a line traverses if its risk $R_{\ell dp}$ is below the threshold.[1] The purpose of this processing is to ignore sections of lines where the risk is low, and only keep pixel risk values that are above this threshold.[2]

Figure 3.2: A conceptual example of three transmission lines. Boxes represent pixels that lines intersect, with their risk values shown above. Red pixels represent high-risk pixels in the set $P^h_{\ell,d}$.

While there are many possible ways to compute the threshold value, this work determines a value based on statistics of the wildfire risk values of a historical year. The set of all risk values $R_{\ell,d,p}$ for all lines $\ell \in L$, for all days $d \in D$, and for all pixel indices $p \in P_\ell$ is taken. The average risk value $\bar{r}$ and the standard deviation of risk values σ of this set are computed. The threshold of interest is defined as any risk value greater than one standard deviation above the mean. The set of pixels above the threshold for each line on each day is defined as

$$P^h_{\ell,d} = \{p \in P_\ell \mid R_{\ell,d,p} \geq \bar{r} + \sigma\}, \quad \forall \ell \in L, \forall d \in D. \tag{3.4}$$

The aggregation metrics are recomputed with the high-risk thresholded risk values.

High-Risk Maximum (HRMA) Metric assigns a risk value equal to the maximum risk a line intersects in the high-risk pixel set:

---

[1] This new set of pixels is defined rather than set pixel values to zero for pixels below the threshold to better capture the intent of the high-risk mean metric. With this definition, only high-risk pixels are averaged rather than including zero values.

[2] Note, this pre-processing threshold is separate from the threshold-based de–energization method discussed in Section 3.3.1.

$$R^{HRMA}_{\ell,d} = \max_{p \in P^{h}_{\ell,d}} R_{\ell,d,p}. \tag{3.5}$$

High-Risk Mean (HRME) Metric assigns a risk value equal to the mean of the high-risk pixels that a line intersects:

$$R^{HRME}_{\ell,d} = \frac{\sum_{p \in P^{h}_{\ell,d}} R_{\ell,d,p}}{|P_{\ell}|}. \tag{3.6}$$

High-Risk Cumulative (HRCU) Metric assigns a risk value equal to the sum of the high risk pixels that a line intersects:

$$R^{HRCU}_{\ell,d} = \sum_{p \in P^{h}_{\ell,d}} R_{\ell,d,p}. \tag{3.7}$$

3.2.5 Risk Metric Discussion

The six risk metrics (MA, ME, CU, HRMA, HRME, and HRCU) each provide different incentives in a de-energization strategy, e.g. by focusing more or less on high versus average risk. The following provides a discussion and an illustrative example to demonstrate the similarities and differences between the metrics.

Figure 3.2 shows three example lines of different length, traversing areas with different wildfire risk. Table 1 summarizes the wildfire risk metrics for each line.

Table 3.1: The risk values for the three lines in Fig. 3.2 based on the six different risk metrics.

	MA	HRMA	ME	HRME	CU	HRCU
ℓ_1	100	100	50	33.3	150	100
ℓ_2	100	100	67	57	330	285
ℓ_3	100	100	60	28.6	420	200

Selecting the maximum (MA) metric directly emphasizes the worst-case risk of a wildfire ignition along a corridor. This metric ignores the length of a power line, and therefore how many pixels it traverses and the variance of those risk values. For instance, the shorter line ℓ_1 in Fig. 3.2 is treated the same as the longer line ℓ_3, as both have a maximum risk value of 100.

The mean (ME) metric averages risk values across the length of a line. While the MA metric is concerned with only the point of worst-case risk, the ME metric considers the likelihood of any potential points of failure along a power line. This helps to identify lines that have a greater proportion of their length in high-risk areas. For example, ℓ_2 line in Fig. 3.2 has a relatively high mean risk value, as a relatively high proportion of this line intersects high-risk pixels. Like MA, the ME metric does not capture the length factor of a power line's risk.

The cumulative (CU) metric sums all of the risk values along the length of the power line, thus accounting for more possible instances of failures occurring along longer lines. This captures the intuition that a long line is more likely to start a fire than a short line (e.g., line ℓ_3 is relatively long and has the highest CU risk of the lines in Fig. 3.2). One limitation of this metric is that there may be cases where long lines pass through predominantly low-risk areas but nevertheless have high CU-based risk due to their long length.

The high-risk metrics (HRMA, HRME, and HRCU) threshold out low-risk pixel values. These reflect the intuition that wildfire ignitions at relatively low-risk locations are not likely to spread into devastating fires. The high-risk maximum risk (HRMA) metric is the same as the baseline maximum for any line that has a risk value above the risk threshold (e.g., all three lines in Fig. 3.2 have equivalent MA and HRMA values). For any line whose maximum risk is below the risk threshold, the maximum value is zero rather than its baseline value. This removes any incentive to de-energize low- or moderate-risk power lines.

The high-risk mean (HRME) metric averages risk values across the length of a line after setting the low-risk pixels to a value of 0. Compared to the baseline mean metric, this metric decreases the risk value of long lines with large sections in low-risk regions (e.g., observe that line ℓ_3 in Fig. 3.2 has a significantly lower HRME than ME).

The high-risk cumulative (HRCU) metric sums all risk values above the threshold. Under this metric, a long line through a low-risk region will not be assigned a high risk value (note the lower HRCU than CU for line ℓ_3), but lines with long stretches through high-risk regions will still have higher risk values compared to short lines through these regions.

In general, for each metric, a higher metric value for a line indicates that a fault-induced ignition on that line has a higher potential to spread into a large fire. Importantly, the method of risk aggregation (i.e., the choice of risk metric) significantly impacts which line(s) are considered to have the greatest wildfire potential. For example, in Fig. 3.2, line ℓ_2 is riskiest based on the ME and HRCU metrics, but line ℓ_3 is riskiest based on the CU metric. For each metric, we expect that lines with high risk values would be better candidates for de-energization. In the remainder of this work, we show how selecting different risk metrics impacts power shutoff planning decisions.

3.3 Power Shutoff Decision-Making

Given risk values for all lines in a power system, an operator can make a decision concerning which lines should be de-energized in order to decrease the overall potential for power infrastructure to ignite wildfires. We now assess two common analytic methods for determining how to implement power shutoffs: thresholding and optimal power shutoffs.

3.3.1 Thresholding

Our first approach to planning PSPS events de-energizes any line that is considered "risky", i.e., any line that has a risk value that exceeds some predetermined level or threshold. This approach requires selection of a risk threshold that is low enough to turn off lines that are likely to ignite dangerous wildfires, yet high enough to avoid excessive de-energization and associated power outages.

A threshold for each metric is chosen using statistical measures, specifically the 95th percentile of risk values (across all lines and all scenarios). Lines are de-energized that have risk values above this threshold on each day. To determine how load is served under this shutoff plan, a modified version of the optimization problem in Model 2 is solved, which is described in Chapter 2. Specifically, we fix the line status binary decision variables, z_ℓ, to 0 for any de-energized line ℓ. The selected threshold results in lines being aggressively de-energized, one to two orders of magnitude more than seen historically. This leads to large quantities of load shed, allowing for clear comparison of our metrics and de-energization methods.

3.3.2 Optimal Power Shutoff

The optimal power shutoff (OPS) problem is an optimization problem that determines steady-state operations decisions (including generator outputs, line flows, loads served, and voltage angles) as well as binary line de-energization decisions in a way that balances wildfire risk mitigation with load that is shed due to those de-energizations. The OPS was first proposed in [40] and is studied and extended in [1–3, 47–54, 71].

There are several ways to formulate the risk and load shed mitigation strategies in the OPS problem. For example, we can formulate a multi-objective problem that minimizes wildfire risk and load shed, as in [1, 40, 54], or we can minimize wildfire risk while constraining load shed to some acceptable level as in [49] (or vice versa as in [2]). In this work, given an interest in comparing risk metrics, the objective minimizes load shed while wildfire risk is constrained in the model. This allows us to maintain a safe level of wildfire risk in the network while optimizing to reduce negative impact on loads. The OPS formulation is outlined in Model 2. Here, Model 2 is modified to include the objective detailed in Eq. (3.8)

$$\min \sum_{t \in T} \sum_{n \in N} p^n_{ls;t} + {}_{switch} \sum_{\ell \in L^{switch}} (1 \quad z_\ell) \tag{3.8}$$

The objective (3.8) minimizes total load shed in the network with an associated penalty term on the number of de-energized lines. For the numerical results in Section 3.4, ${}_{switch}$ is set to 0.01 to avoid the line switching decisions dominating the objective.

The parameter R^{PSPS} in constraint Eq. (2.2m) is defined as the total remaining risk on the network resulting from the thresholded lines on the same day (see Section 3.3.1). Here, $R_{\ell,d}$ represents the risk on line ℓ based on the given metric.

3.4 Test Case Results

We compare the performance of the six wildfire risk metrics and two shutoff methods utilizing CATS [109], a 9000-bus, 11000-line test system with transmission line corridors that reflect the actual grid in California, but with synthetic parameters so as not to reveal any critical information about the real grid. The geographic realism and large scale of CATS make it a compelling test case. However, the results derived here are not necessarily an indication of how California's actual power grid operates. We use 2019 WFPI daily risk data to determine high-risk pixel thresholds, and study de-energization plans using 2020 WFPI data. All models were implemented using Julia 1.9.2 [113] and solved using Gurobi 10.0.1 [114].

To reduce solve times (due to the scale of CATS), we take a number of approaches. First, when finding optimal switching decisions, we warm-start the binary variables in the problem with the status of lines de-energized under the corresponding thresholded case. The number of binary variables are reduced by only allowing lines with non-zero wildfire risk to be included in the switchable line set, $\mathcal{L}^{switch}$. Additionally, relaxing the lower bound of generators to be 0 p.u. avoids binary decision variables associated with generator on/off statuses. Finally, to reduce the problem size, optimal switching decisions are made for the entire day based on the worst-case hour of the day. We define the worst-case hour as the hour with the most load shed from thresholded de-energization decisions from the same day with the same metric. After optimal de-energization decisions are made on the given hour, these decisions are fixed for the full 24-hour period, with hourly load shed and operational decisions found for the full day.

3.4.1 De-energization Decisions based on Thresholding

Table 3.2: The number of unique lines de-energized in 2020 for each metric in the threshold- and optimization-based methods.

	MA	HRMA	ME	HRME	CU	HRCU
Threshold	6376	6376	5204	5005	1206	2505
OPS	6276	6398	5564	5859	3568	5649

Thresholding decisions differ between the normal and high-risk methodologies. As shown in Fig. 3.3a, each method provides different line de-energization outcomes throughout the year. Both the CU and HRCU see a more uniform number of de-energized lines throughout the year while MA, ME, HRMA, and HRME all see more variation in numbers of de-energized lines. Figure 3.3a also shows that the the number of de-energized lines resulting from metrics with the high-risk pixel threshold (HRMA, HRME, HRCU) are more closely aligned throughout the year than those from the metrics which use the raw wildfire data (MA, ME, CU).

The first row of Table 3.2 shows the number of unique lines de-energized by the thresholded method in 2020. Note that while the MA and ME metrics de-energize nearly the same number of unique lines as the HRMA and HRME metrics, the HRCU metric de-energizes nearly twice as many unique lines as the CU metric over the span of the year. The resulting daily load shed for each metric is shown in Fig. 3.4a. While the HRCU metric de-energizes fewer lines than the max- or mean- derived metrics, it results in the most load shed throughout the year. Also note that the MA and HRMA metrics de-energize the same number of unique lines (and the most by the thresholding method) but result in less load shed compared to other metrics for most of the year.

3.4.2 De-energization Decisions based on OPS

When looking at the number of optimally de-energized lines in Fig. 3.3b, the overall quantity of lines being de-energized is similar to the thresholded method. However, the optimal

(a) Number of lines de-energized daily by metric under the 95th-percentile thresholding method.

(b) Number of lines de-energized daily, by metric, under the optimal power shutoff method.

Figure 3.3: Comparison of daily line de-energization counts under (a) threshold-based and (b) optimization-based methods.

case de-energizes slightly more lines across all metrics. This is likely due to more lower-risk lines being de-energized to allow energization of some high-risk lines that are crucial for power delivery. By strategically de-energizing more low-risk lines, the overall risk in the network can be maintained while reducing the amount of load shed. Figure 3.4b shows that the optimal line switching decisions achieve approximately 20% of the load shed resulting from the thresholded method. Note that the scale of the y-axis differs from that in Fig. 3.4a to allow for variations in the load shed to be visible. Similar to the thresholded method, we again see that the CU and HRCU methods, while de-energizing fewer lines, result in larger amounts of load shed.

3.4.3 Comparing Thresholding and Optimal Decisions

While both thresholded and optimal de-energization decisions result in a similar number of lines being switched off, they do not select the same lines to be turned off throughout

(a) Seven-day rolling average load shed from thresholded line de-energizations.

(b) Seven-day rolling average load shed from optimal power shutoff line de-energization decisions.

Figure 3.4: Comparison of seven-day rolling average load shed under (a) threshold-based and (b) optimization-based line de-energization methods.

the year. Figure 3.5 shows the similarity between decisions made by each method/metric pairing. For each method/metric pair, we sum the number of times each individual line is de-energized and then normalize each of these vectors and take the dot product. Thus, a value of 1.0 in the heat map indicates that each line was de-energized the same number of times throughout the year.

Figure 3.5 shows that thresholded MA and HRMA metrics produce the same results, and that the optimal MA and HRMA metrics produce nearly identical results. The decisions made by the ME or HRME metrics have very weak correlations with those made by the CU or HRCU metrics. Also note that the CU and HRCU metrics result in dissimilar decisions and optimal load shed. This might be caused by the HRCU method de-energizing nearly 60% more lines compared to the CU method in the optimal case (as shown in Table 3.2).

3.5 Conclusions

This chapter characterizes the impacts of different approaches to aggregating wildfire risk data for power lines using a realistic large-scale synthetic power system. This work defines and compares six wildfire risk aggregation metrics using two line de-energization decision methods: thresholding and the optimal power shutoff. Compared to thresholding, the op-

Figure 3.5: This heatmap shows how similar the line de-energization decisions are between the thresholded method and optimal methods with each metric over the full year. A value of 1.0 (yellow) indicates each line was de-energized the same number of times over the year while a value of 0.0 (black) indicates none of the same lines were de-energized over the year.

timal power shutoff formulation drastically reduces load shed, despite a similar extent of line de-energizations and similar overall network risk. The numerical results also clearly show that the choice of metric significantly alters the de-energization decisions and associated load shed. If it was found that all six metrics produced similar de-energization results, then the choice of risk quantification metric in future analyses would not matter. However, since the results significantly differ, modelers and decision-makers should be careful when choosing how to aggregate risk.

Based on our results alone, it is not clear which risk metric or method is the most effective or fair to use in planning pre-emptive de-energizations. To determine this, more research is needed. For example, high-fidelity fire simulations could achieve a thorough analysis of the impact of ignitions that would occur (or be avoided), which could help determine if focusing on high risk locations or average risk across the line span is more important. Further, while optimization-based methods achieve lower load shed at comparable wildfire risk compared with the threshold-based method, they may leave certain high risk areas unaddressed to avoid load shedding in further away locations. Understanding the effects of trade-offs and metrics on both local communities and the overall grid is a topic that deserves more attention in future work.

CHAPTER 4

EXTENSIONS OF THE OPTIMAL POWER SHUTOFF PROBLEM

4.1 ML Enhanced Optimization for High Quality Solutions in Less Time

Typically formulated as Mixed-Integer Linear Programs (MILP), OPS problems pose significant computational challenges that we address in this paper via machine learning (ML) guided MILP solving techniques. FPIs and other conditions change daily, necessitating quick responses from utilities. However, solving OPS problems with realistic large-scale power system models can require hours to days of computation time, as discussed in Section 3.4, rendering the resulting solutions operationally irrelevant. Thus, new computational methods are needed to operationalize OPS formulations for practically relevant system models.

Recent advances in ML-guided MILP solving [115, 116] hold significant promise for addressing the computational challenges of OPS problems. Recognizing that MILPs with similar structures (i.e., MILPs of the same problem formulation constructed from data parameters sampled from a given distribution) are solved repeatedly in many applications, this body of research focuses on using ML to guide existing algorithmic policies or build new ones customized for specific instance distributions.

Two key features of OPS problems make the application of these recent ML advances particularly attractive. First, practical implementations of OPS problems would require repeated solution of very similar problems to determine daily line de-energization decisions. With limited changes to the network and conventional generators' characteristics, differences between OPS problems on a daily basis are primarily driven by varying wildfire ignition risks, load demands, and renewable generation, all of which have temporal and spatial patterns. Second, OPS formulations inherently involve significant modeling assumptions (e.g., the use of power flow approximations [98]) and parameter uncertainties associated with forecasts for wildfire ignition risk, load demands, and renewable generator outputs [117–119]. By permitting the use of shorter forecast horizons, nearly optimal decisions obtained more quickly can potentially achieve better outcomes in practice than high-precision optima that require long computing times. The limited variation across OPS problems and modest value of high-precision global optima make it worthwhile to spend computational time in offline training of ML-guided MILP solvers to speed up day-of solution times when uncertainties are realized close to real-time operations. Thus, based on these two features, recent advances in ML-guided MILP solving such as those in [55–57, 120] have the potential to determine OPS de-energization decisions much more quickly than traditional MILP solvers while significantly outperforming traditional threshold-based de-energization methods.

For large-scale MILPs, a large body of ML-guided MILP solving research focuses on ML-guided primal heuristics, where the goal is to obtain high-quality solutions within a short amount of time, instead of proving optimality. These approaches typically learn to predict the near-optimal assignment for variables in the problem [55–57]. Since an ML model's direct prediction may be infeasible, these methods refine the prediction by solving another MILP at inference time. Two major frameworks for this refinement are Neural Diving (ND) [57] and Predict-and-Search (PS) [55, 56]. ND fixes a subset of variables to their predicted values, creating a smaller, easier-to-solve sub-MILP. While computationally efficient, this aggressive reduction of the search space may yield highly suboptimal solutions. In contrast, PS searches for near-optimal solutions within a neighborhood based on the prediction. PS operates on the original MILP and has a neighborhood parameter that specifies the number of variables that are allowed to change from their prescribed assignment suggested by the ML model [55, 56]. This approach offers more flexibility to correct prediction errors but is more computationally demanding, as the problem size remains unchanged.

While ML has previously been used for other wildfire applications (see, e.g., [73] for ML predicted ignitions and [72] for data-driven operation predictions under PSPS events), this work is, to the best of our knowledge, the first to apply ML-guided MILP methods to quickly identify high-quality line de-energization decisions. As most lines are energized in the context of OPS, many binary variables take 1 as the solution value (as opposed to 0). This creates a class imbalance in the ML-guided framework, which is uncommon in other domains where ML-guided primal heuristics such as PaS and ND have been applied. Thus, this chapter extends the PaS framework used in [55, 56] by introducing two separate neighborhood parameters for the number of variables that are allowed to change from their ML-prescribed assignment, one for the variables predicted to be 0 and another one for the variables predicted to be 1. This allows the integration of domain knowledge on the number of binary variables that take 0 and 1, respectively, as their solution value in the OPS problem. Finally, this work presents a variant that combines PaS and ND, where the domain-informed PaS constraint acts as a safety net to restore feasibility. Results on the CATS network [109] show that the ML-guided method significantly outperforms Gurobi.

4.1.1 Overview of Domain Informed ML-guided Methods

Each instance of the OPS problem is parameterized by the wildfire risk per line r_ℓ, renewable generation availability impacting the upper limit of some generation $\overline{p}_g^{l}$, and load demand at each bus $p^{n}_{l,t,d}$. Parameters of Model 2 drawn from the same distribution result in a set of MILP instances with similar structures, which motivates the use of ML to accelerate MILP solvers. Building on existing techniques [55–57], this section introduces an ML-guided framework for faster solution of OPS problems.

ML is used to predict the solution for the set of de-energization variables $z = [z_d^{\ell}]_{\forall \ell \in L_d^{\text{switch}}}$ in (2.2d), as MILP complexity significantly depends on these binary decisions. Following established approaches [55, 57], we learn a policy $(z \mid \omega; M)$ using a Graph Attention Network (GAT) that outputs prediction probabilities for each binary variable. The policy is trained to approximate the conditional distribution of solutions given a problem instance, using a weighted cross-entropy loss function that assigns higher probabilities to solutions with better objective values. Since ML predictions can be infeasible after rounding, we employ inference-time refinement using domain-informed variants of Predict-and-Search (PaS) and Neural Diving (ND) methods. These approaches constrain the MILP solution to remain within a neighborhood of the ML prediction, with carefully chosen parameters that leverage domain knowledge about the OPS problem structure.

Our domain-informed approach exploits two key characteristics of OPS problems: (1) the number of de-energized lines (zero entries in z) must fall within a known feasible range $[N_0^{min}; N_0^{max}]$, and (2) most lines remain energized during typical PSPS events. We modify the standard PaS framework by splitting the neighborhood parameter into separate parameters $_0$ and $_1$ for variables predicted to be zero and one, respectively, with $_0 = N_0^{max} \quad N_0^{min}$ to prevent infeasibility. Additionally, we introduce a PaS+ND hybrid that fixes high-confidence predictions while allowing flexibility for uncertain variables, using the PaS constraint as a safety net to maintain feasibility. These modifications, detailed in [56], enable the ML-guided solver to find high-quality solutions significantly faster than commercial solvers while respecting the physical constraints of the power system. For a full discussion of the ML methods, see [7]

4.1.2 ML Dataset

Similar to Section 3.3, this ML method is trained on single-hour OPS events with a risk constraint determined by a naive thresholded de-energization method. The single-hour OPS problems are solved for 2020 and 2021, resulting in 651 optimization for problems in the use of the ML training. Some days have no wildfire risk or resulting load shed so not every day warrants an OPS problem.

For the ML method described in Section 4.1.1, each daily problem instance is classified as either “easy” or “hard”. Easy problems can be solved using conventional MILP solvers such as Gurobi in a reasonable amount of time (on the order of an hour). Hard problems under this classification tend to take hours to days of computing time to reach an optimal solution, making them a good target for ML-guided optimization techniques to provide high-quality switching decisions on a day-ahead operational time scale.

The determination of an easy/hard classification is based on the amount of load shed seen in the thresholded problem since, as an LP, this problem can be solved within minutes. If the thresholded de-energization problem yields relatively low load shed (less than 100 MWh based on prior experience in [4]), the OPS problem is typically able to find an optimal solution quickly. Classifying days based on the initial load shed leads to 111 easy problems and 540 hard problems.

The ML methods are applied to problems classified as hard, as the easy problems can be solved to optimality with Gurobi within a short amount of time. The hard instances are split 80%, 10%, and 10% for training, validation, and test. This results in 432, 54, and 54 instances for train, validation, and test, respectively.

4.1.3 Results

This section presents our evaluation metrics, implementation setup, the results for the ML-guided optimization method alongside several benchmark methods.

Evaluation Metrics

The following metrics are used to evaluate the effectiveness of different methods: (1) The *Objective Value* (OV) is the best known objective value of the MILP found by the time cutoff. (2) The *Primal Gap* (PG) [121] is the normalized difference between the objective value v found by a method and a best known objective value v^*, defined as PG $= \frac{|v - v^*|}{\max(|v|;|v^*|)}$, when $v v^* > 0$. When no feasible solution is found or when $v v^* < 0$, PG is defined to be 1. PG is 0 when $|v| = |v^*| = 0$. (3) The *Primal Integral* (PI) [121] is the integral of the primal gap over time, which captures the speed at which better solutions are found.

(4) The number of wins in terms of PI (# wins) is the number of MILP instances in the test set for which the method results in the lowest PI across all other methods. Note that lower values for the objective, primal gap, and primal integral indicate superior performance. For the the domain-informed PaS method (PaS), the final hyperparameters used are $= 0{:}9$ and $^{0} = 0{:}05$. For the domain-informed PaS+ND with safety net (PaS+ND), $= 0{:}7$ is used.

Results and Discussions

Table 4.1: Results of ML-guided methods vs Gurobi. Average Objective Value (OV), Primal Gap (PG), Primal Integral (PI), and number of wins in terms of PI (# wins) across 54 hard test instances.

Method	OV	PG	PI	# wins
Gurobi (30-min)	29.60	0.29	627.63	11
PaS (30-min)	29.66	0.27	686.91	1
PaS+ND (30-min)	15.14	0.02	433.46	42

This work compares the ML guided approaches (PaS and PaS+ND) against Gurobi. As shown in Table 4.1, PaS+ND performs best among the three methods in all evaluation metrics, achieving the lowest average objective value, primal gap, and primal integral and the highest number of wins in terms of primal integral on the test instances. PaS+ND significantly outperforms Gurobi, reducing the primal integral by 30.94% relative to Gurobi. 4.1a shows a scatter plot of the primal integral with PaS+ND and with Gurobi, where each point represents an MILP instance in the test set. Many points in the scatter plot lie below the 45-degree line, indicating that PaS+ND often outperforms Gurobi. Notably, the biggest gains of PaS+ND are on the hardest instances. Additionally, 4.1b shows the primal gap, averaged over the test set, as a function of time. PaS+ND produces high-quality solutions at a faster speed across most time steps compared to Gurobi.

PaS performs worse than PaS+ND and Gurobi in most of the metrics considered. A possible explanation is that PaS solves an MILP that contains the same number of binary variables as the original MILP at inference time, discussed more in [7]. Therefore, it takes

(a) Scatter plot of primal integral with Gurobi vs. primal integral with PAS+ND.

(b) Average primal gap as a function of time (seconds).

Figure 4.1: Comparison of solver performance metrics. (a) Scatter plot comparing primal integrals between Gurobi and PAS+ND. (b) Evolution of average primal gap over time.

a longer time for PAS to produce solutions with lower objective values, although it provides more flexibility to correct errors from the ML predictions by solving a larger MILP, compared to PAS+ND. The success of PAS+ND on this problem can likely be attributed to the high *recall* value of the ML prediction, which is the ratio of true positives to all actual positives. The *recall* of the ML model with respect to the best ground truth solutions is 98.99%, meaning that a high number of variables that take 1 as the solution value in the ground truth data are correctly predicted to 1 by the ML model. Therefore, aggressively fixing a large percentage of variables that has high prediction scores leads to good performance.

PSPS Outcomes

To better characterize the physical impacts of the solutions from each solution approach, we detail results from the thresholded version of this problem (denoted as *threshold*, see Section 3.3.1), the ML guided outcomes at a 30-minute time limit (denoted as *ML guided 30-min*, see Section 4.1.1), the Gurobi output at a 30-minute time limit (denoted as *Gurobi 30-min*), and extended results at a 24-hour time limit or optimality at a 1% MIP gap (denoted as *Gurobi extended*, see 3.3). For the ML guided results in this section, we consider

(a) Thresholded line statuses.

(b) Line statuses from the Gurobi solution at 30 minutes.

(c) Line statuses from the ML-guided solution at 30 minutes.

(d) Line statuses from Gurobi solution at 24 hours

Figure 4.2: Geographic plots showing the line statuses and load shed from different solution methodologies for March 2nd, 2021, one of the hard instances. Red dashed lines show lines de-energized under that solution methodology. Black solid lines show lines that are energized across all solution methodologies for this day. Grey lines show lines that are energized in that specific methodology. Red circles indicate load shed at a bus with the size of the circle corresponding to the amount of load shed.

the PAS+ND outcomes as this ML methodology achieves better performance, as shown in Section 4.1.3.

In Fig. 4.2, we show the switching outcomes from these four different solution methodologies on a representative day (March 2, 2021) with high wildfire ignition risk in southern California. We can see that the switching decisions from the Gurobi solution at a 30-minute time limit (Fig. 4.2b) are similar to those from the thresholded version (Fig. 4.2a) with large amounts of load shed. The ML-guided outcomes (Fig. 4.2c) have different switching decisions with much lower load shed when compared to the thresholded and 30-minute Gurobi solutions. The extended Gurobi solution for March 2nd, 2021, run to a 24-hour time limit (3.23% MIP gap), achieves much lower load shed with a different network topology. Across the network, there are a large number of lines that remain energized under all solution methodologies, highlighting that the choice of line energization status can greatly impact the amount of load shed.

The box plots in Fig. 4.3 show the distribution of objective values across the four different methodologies. The thresholded and 30-minute Gurobi results have similar values with the 30-minute Gurobi solutions having a slightly lower objective on average. The ML-guided and extended Gurobi solutions have similar distributions of objective values. While the extended Gurobi does reach more optimal solutions on average, this requires a 24-hour time limit as opposed to the ML guided solutions which are only run for 30 minutes.

4.1.4 Conclusions on ML-OPS

This work presents a domain-informed ML-guided solution method for OPS problems. Compared to Gurobi, the domain-informed PAS+ND method finds high-quality de-energization decisions at a faster speed and reduces the primal integral by 30.94% and primal gap by 93.10% at the 30-minute computational time cutoff. PAS+ND also achieves large reductions in load shedding compared to traditional optimization methods with the same amount of time.

Figure 4.3: Box plots showing the distribution of objective outcomes across the four considered solution methodologies. Only objective values from dates in the hard test data set (see Section 4.1.2) are included in the distribution.

4.2 Linearized AC Optimal Power Shutoff

In this section, an extension of the OPS is introduced that considers reactive power in the network. The Linearized AC Power Flow (LACPF), detailed further in [101], linearizes the non-convex AC power flow equations to allow the introduction of voltage magnitude decision variables that better approximate the AC power flow model. This can be extended to include switching decisions for wildfire risk as shown in Model 3. Work in this section is exploratory and first presented in [8]. Future directions of this research are further discussed in 9.3

Many constraints in the LACOPS model are similar to those in the DCOPS. Active generation, load shedding, power flow (relabeled here as $p_{\ell,t,d}$), and power balance are handled similarly to the DCOPS model in Eqs. (4.1b), (4.1d), (4.1i), and (4.1p). Additionally, reactive generation ($q^{i}_{g,t,d}$), load shedding ($q^{n}_{s,t,d}$), power flow ($q_{\ell,t,d}$), and power balance are constrainted similarly in Eqs. (4.1c), (4.1e), (4.1j), and (4.1q). The reactive load shed is also bounded to be proportional to the active load shed in Eq. (4.1f). This is to prevent over excessive reactive load shed since this term does not appear in the objective. The voltage magnitude is bounded by network limits in Eq. (4.1g), similar to the voltage angle difference in Eq. (4.1h). An additional non-linear term is introduced in Eq. (4.1k). This can be relaxed for a fully linear model as will be discussed in Section 4.2.1. Linearized AC power flow constraints are shown in Eqs. (4.1l), (4.1m), (4.1n), (4.1o). These include big-M terms to ensure the voltage angles and voltage magnitude are unbounded when lines are de-energized ($z_{\ell d} = 0$) through big-M values $\overline{\ }$ and $\overline{V}$, respectively. Finally, similar to Model 2, we include a risk constraint in Eq. (4.1r). The objective (4.1a) is the same objective in Eq. (3.8), aiming to minimize active load shed with a penalty term to minimize the total number of de-energized lines.

Model 3 Linearized AC Optimal Power Shutoff (LACOPS)

$$\min \sum_{d\in D}\sum_{t\in T}\sum_{n\in N} p^n_{ls,t,d} + \lambda_{switch}\sum_{\ell\in L_{switch}}(1 - z_\ell) \tag{4.1a}$$

s.t. $\forall d \in D; \forall t \in T;$

$$\underline{p}^i_g \leq p^i_{g,t,d} \leq \overline{p}^i_g \qquad \forall i \in G \tag{4.1b}$$

$$\underline{q}^i_g \leq q^i_{g,t,d} \leq \overline{q}^i_g \qquad \forall i \in G \tag{4.1c}$$

$$0 \leq p^n_{ls,t,d} \leq p^n_{l,t,d} \qquad \forall n \in N \tag{4.1d}$$

$$0 \leq q^n_{ls,t,d} \leq q^n_{l,t,d} \qquad \forall n \in N \tag{4.1e}$$

$$q^n_{ls,t,d} \leq 0.1\, p^n_{ls,t,d} \qquad \forall n \in N \tag{4.1f}$$

$$\underline{v}^n \leq v^n_{t,d} \leq \overline{v}^n \qquad \forall n \in N \tag{4.1g}$$

$$\underline{\theta}^\ell \leq \theta^{n_{\ell,fr}}_{t,d} - \theta^{n_{\ell,to}}_{t,d} \leq \overline{\theta}^\ell \qquad \forall \ell \in L \tag{4.1h}$$

$$-\overline{f}^\ell z^\ell_d \leq p^\ell_{t,d} \leq \overline{f}^\ell z^\ell_d \qquad \forall \ell \in L \tag{4.1i}$$

$$-\overline{f}^\ell z^\ell_d \leq q^\ell_{t,d} \leq \overline{f}^\ell z^\ell_d \qquad \forall \ell \in L \tag{4.1j}$$

$$(p^\ell_{t,d})^2 + (q^\ell_{t,d})^2 \leq (\overline{f}^\ell)^2 \qquad \forall \ell \in L \tag{4.1k}$$

$$p^\ell_{t,d} \geq -b^\ell(\theta^{n_{\ell,fr}}_{t,d} - \theta^{n_{\ell,to}}_{t,d}) + g^\ell(v^{n_{\ell,fr}}_{t,d} - v^{n_{\ell,to}}_{t,d}) + b^\ell\overline{\theta}(1 - z^\ell_d) - g^\ell\overline{V}(1 - z^\ell_d) \qquad \forall \ell \in L \tag{4.1l}$$

$$p^\ell_{t,d} \leq -b^\ell(\theta^{n_{\ell,fr}}_{t,d} - \theta^{n_{\ell,to}}_{t,d}) + g^\ell(v^{n_{\ell,fr}}_{t,d} - v^{n_{\ell,to}}_{t,d}) + b^\ell\overline{\theta}(1 - z^\ell_d) + g^\ell\overline{V}(1 - z^\ell_d) \qquad \forall \ell \in L \tag{4.1m}$$

$$q^\ell_{t,d} \geq -g^\ell(\theta^{n_{\ell,fr}}_{t,d} - \theta^{n_{\ell,to}}_{t,d}) - b^\ell(v^{n_{\ell,fr}}_{t,d} - v^{n_{\ell,to}}_{t,d}) + b^\ell\overline{\theta}(1 - z^\ell_d) - g^\ell\overline{V}(1 - z^\ell_d) \qquad \forall \ell \in L \tag{4.1n}$$

$$q^\ell_{t,d} \leq -g^\ell(\theta^{n_{\ell,fr}}_{t,d} - \theta^{n_{\ell,to}}_{t,d}) - b^\ell(v^{n_{\ell,fr}}_{t,d} - v^{n_{\ell,to}}_{t,d}) + b^\ell\overline{\theta}(1 - z^\ell_d) + g^\ell\overline{V}(1 - z^\ell_d) \qquad \forall \ell \in L \tag{4.1o}$$

$$\sum_{\ell\in L^{n,fr}} p^\ell_{t,d} - \sum_{\ell\in L^{n,to}} p^\ell_{t,d} = \sum_{i\in G^n} p^i_{g,t,d} - p^n_{l,t,d} + p^n_{ls,t,d} \qquad \forall n \in N \tag{4.1p}$$

$$\sum_{\ell\in L^{n,fr}} q^\ell_{t,d} - \sum_{\ell\in L^{n,to}} q^\ell_{t,d} = \sum_{i\in G^n} q^i_{g,t,d} - q^n_{l,t,d} + q^n_{ls,t,d} \qquad \forall n \in N \tag{4.1q}$$

$$\sum_{\ell\in L} z^\ell_d r_{\ell,d} \leq R^{PSPS}_d \qquad \forall d \in D. \tag{4.1r}$$

4.2.1 LACOPS Case Study and Comparison

This work evaluates the RTS network, a 73-bus network with 120 transmission lines geolocated in southern California. This network has hourly updating load information. A period from June to October of 2021 is assessed with real world USGS WFPI associated risk. Specifically a cumulative metric is used for wildfire risk aggregation. Three de-energization methods are evaluates (1) the DCOPS with objective (3.8), (2) the linearized LACOPS formulation which ignores constraint (4.1k), and (3) the LACOPS formulation described in Model 3.

Figure 4.4: Comparison of load shed across different OPS formulations under a period of high wildfire risk.

Figure 4.4 shows the difference in load shed from a high risk period in June. As can be seen, the DCOPS formulation and the linearized LACOPS formulations typically find similar quantities of load shed while the non-linear LACOPS formulation sees more load due to the presence of more constrainted power flows. The increased load shed under the non-linear LACOPS may indicate this result is closer to the full AC solution which typically sees higher levels of load shed during PSPS events as discussed in [98]. In Fig. 4.5, the

Figure 4.5: Comparison of line switching decisions on June 11, 2021 from different OPS formulations.

switching decisions are shown with dark blue marks indicating a given line is energized and a paler mark indicates a line is de-energized. Here, similarity between line switching decisions can be seen between the linearized and non-linear LACOPS formulations which are distinct from the decisions made under the DCOPS. This indicates that the linearized LACOPS may be a good approximation to achieve a network topology with better AC recoverability compared to the DCOPS, which gives fast but approximated solutions, or the non-linear LACOPS, which gives much slower solutions that may be closer to AC feasibility. Future work can investigate the efficacy of these alternative power flow models int terms of AC feasibility and compare impacts to planning decisions.

CHAPTER 5

TRANSMISSION INFRASTRUCTURE INVESTMENT MODELING FOR WILDFIRE IGNITION RISK MITIGATION

5.1 Introduction

To address imminent wildfire conditions, system operators use so-called “Public Safety Power Shutoffs” (PSPS) that temporarily de-energize power lines located in wildfire-prone regions during severe wildfire conditions [12]. De-energized lines cannot ignite wildfires, so PSPS strategies are effective at quickly reducing acute wildfire ignition risks. However, risk reduction from PSPS events comes at the cost of load shedding if the partially de-energized system cannot supply all load demands. A cost/benefit analysis in [122] regarding various strategies for mitigating wildfire ignition risks indicates that PSPS events are a cost-effective mechanism for reducing acute wildfire risks. Since they are likely to continue being employed during severe wildfire-prone conditions, PSPS events deserve further research to achieve system operators' goals of making PSPS events smaller in scope, shorter in duration, and smarter in performance [123].

Over longer time scales, infrastructure investments can reduce the amount of load shedding needed to achieve desired risk reductions during PSPS events. For instance, utilities can:

- Add batteries that supply loads during PSPS events to reduce the amount of load shedding,
- Install distributed energy resources such as solar photovoltaic generators to provide local supplies of power,
- Harden power lines via undergrounding, installing covered conductors, and performing intensive vegetation management in order to reduce wildfire ignition risk without

the need for de-energization.

The severe impacts of wildfires justify substantial infrastructure investments. For instance, the Infrastructure Investment and Jobs Act recently passed in the United States allocates $5 billion towards resiliency measures like undergrounding power lines to prevent wildfire ignitions and installing microgrids to reduce the impacts of PSPS events [124]. Likewise, in April 2021, the state of California funded $536 million for wildfire resilience projects [125]. Moreover, the California utility PG&E plans to underground 10,000 miles of power lines to prevent wildfire ignitions, which will require a substantial increase in the utility's current rate of 70 miles of newly undergrounded lines per year [22, 126].

The scale of these investments motivates the development of algorithms for optimally siting and sizing new power system infrastructure in a wildfire context. There is an extensive literature on siting and sizing various power system components with the aim of reducing generation costs and providing ancillary services [66, 127–132], improving reliability [128], deferring capital investments [128, 132], etc. However, the wildfire setting presents a key challenge that differs from this prior literature, namely, that the infrastructure investments will be sited to support future PSPS events. Thus, choices for the locations and sizes of the infrastructure investments should be cognizant of the underlying line de-energizations associated with PSPS events.

To the best of our knowledge, none of the existing wildfire risk mitigation literature (see [16] for a recent survey) proposes infrastructure investment algorithms that consider OTS-based line de-energization using a power flow model as in [40]. In other words, none of the existing literature discusses how to optimally plan infrastructure investments in order to support system operations during PSPS events. A cost/benefit analysis by Williamson indicates that solar PV generation could be effective at reducing load shedding during PSPS events in Australia [133], Haces-Fernandez studies the suitability of wind generators to locally supply power during wildfire-prone conditions [134], and Taylor and Roald consider line undergrounding investments in the context of various wildfire risk metrics [68]. While providing many valuable insights, the formulations in these papers do not incorporate a power flow model and may therefore miss important spatial interactions and network constraints inherent to power systems. Other papers focus on methods for operating power systems during wildfire-prone conditions. For instance, both Nazemi et al. and Tandon, Grijalva, and Molzahn study the impact of dynamic line ratings to increase operational flexibility [51, 135], Hong et al. propose data-driven techniques for minimizing load shedding after switching off high-risk lines while considering the possibility of cascading failures [72], Zhou et al. use data-mining techniques to assess and mitigate wildfire ignition risks [136], Haseltine and Roald analyze how recloser operation affects both wildfire risks and system reliability [137], and Kadir et al. describe a reinforcement learning approach to line de-energization and other operational decisions [138]. However, none of these papers incorporate an infrastructure investment model. Many other papers propose methods for enhancing power system resilience to extreme events such as wildfires, but do not explicitly consider a line de-energization model based on wildfire ignition risks; see [139–142] for recent surveys of the power system resilience literature.

Accordingly, this chapter presents optimization formulations that augment OTS problems which minimize wildfire ignition risks with models for optimally siting and sizing various infrastructure investments. This work specifically considers investments in batteries, solar photovoltaic (PV) generators, and infrastructure hardening via undergrounded lines, covered conductors, and intensive vegetation management. This chapter first presents a multi-period extension of the OTS problem presented in [40] for mitigating the risk of wildfire ignitions. The time periods in this problem are coupled by both the selection of a fixed network topology across all periods and the batteries' state-of-charge dynamics. This work then extends this multi-period OTS problem to an infrastructure investment formulation that incorporates models of batteries, solar PV generators, and infrastructure hardening, using discrete variables to represent the presence of these investments at each location. Finally, the infrastructure investment formulation is numerically demonstrated using realistic test cases with actual wildfire risk and infrastructure investment data.

As in much of the other existing literature on mitigating wildfire ignition risks (e.g., [40, 51, 68, 72, 136, 138]), we focus on transmission systems. Transmission systems have ignited major fires, such as the 2018 Camp Fire in California [13, 14], and are thus deserving of significant research attention. Additionally, we note that distribution systems can also ignite wildfires [143]. Studying analogous formulations that aim to further mitigate wildfire risks and load shedding during PSPS events via infrastructure investments in distribution systems is an important direction for future research.

This chapter is organized as follows. Section 5.2 discusses modifications to Model 2 to a wildfire-risk-minimizing OPS problem. Section 5.3 then presents models for infrastructure investments in batteries, solar PV generators, and infrastructure hardening. Sections 5.4 and 5.5 next describe the test cases and procedures, respectively, that are used to evaluate these models, followed by the numerical results in themselves in Section 5.6.

5.2 Multi-Time-Period Public Safety Power Shutoffs

In this section, a framework for optimized multi-time-period PSPS events is presented. Past work [40] formulates optimized shutoffs for a single time period. Here, we extend this existing work, and consider how varying demand throughout a day may affect the set of lines selected for de-energization. In Section 5.3, we investigate the placement of batteries to support line de-energization, which couples the time periods together.

Our framework models a single day of operation when the risk of wildfire ignition via power systems infrastructure is high. Hence, system operators no longer operate the network to minimize generation cost, but rather to reduce the risk of wildfire ignition while minimizing load shedding. As is consistent with United States Geological Survey (USGS) wildfire forecasts [117], we assume that the risk associated with a single energized line is static over a day. Therefore, given a forecast for wildfire risk and multi-time-period demands, our framework selects lines to de-energize for an entire 24-hour period. Although switching lines multiple times per day would provide more flexibility for reducing load shedding, utilities must perform inspections of de-energized lines to ensure safe re-energizations [144]. The time required for these inspections precludes the use of intraday switching in our formulation.

5.2.1 Objective Function

Our goal is to simultaneously minimize wildfire risk and load shedding, which are often competing objectives. Let D be the total demand in the network over all time periods, i.e.:

$$D = \sum_{t \in T} \sum_{n \in N} p^n_{d;t};$$

and let R be the total wildfire risk the network poses if all lines $\ell \in L$ are energized, i.e.:

$$R = \sum_{\ell \in L} r^{\ell}:$$

Now, let $\lambda \in [0;1]$ be a parameter that quantifies the priority of the user between these two competing objectives. If $\lambda = 1$, then the user is solely interested in reducing load shedding. If $\lambda = 0$, then the user is solely interested in reducing wildfire risk. For $\lambda \in (0;1)$, the user seeks a weighted balance of the two objectives. For a given value of λ, let $C_\lambda(\cdot)$ be the objective that the user wishes to minimize, which is a function of the load shedding and line de-energizing variables:

$$C_\lambda(z; p_{ls}) = \frac{\lambda}{D} \left(\sum_{t \in T} \sum_{n \in N} p^n_{ls;t;d} \right) + \frac{(1-\lambda)}{R} \left(\sum_{\ell \in L} r^{\ell}_d z^{\ell}_d \right): \tag{5.1}$$

Observe that dividing the first and second terms in (5.1) by the total demand D and the total risk R enables the interpretation of these terms as the fractions of load shed and wildfire risk remaining after the line switching operations. Note, the models described in Chapter 2 details a multi-day multi-time period model. In this chapter, we only consider a single day with 24 hour-long time periods.

5.2.2 Multi-Time-Period PSPS Formulation

Now, we can formulate the multi-time-period PSPS optimization problem as:

$$\min_{p_g;\,;f;p_{ls};z} \quad (5.1) \quad \text{s.t. } (2.2b) \quad (2.2l); \qquad \text{(MTP-PSPS)}$$

where our goal is to minimize a weighted combination of the total load shedding and wildfire risk over the traditional operational variables (p_g, , and f), load shedding (p_{ls}), and the line switching decisions (z). Differing from Model 2, here risk is minimized in the objective instead of being handled through the constraint (2.2m)

5.3 Infrastructure Investments

The (MTP-PSPS) problem presented in the preceding section can help operators manage the trade-off between wildfire risk reduction and load shedding. However, as the threat of wildfire ignition becomes more severe and the length of the wildfire season extends, it may be the case that no trade-off (i.e., no value of) provides acceptable system performance. In this situation, communities may need to invest in new infrastructure that can either reduce wildfire risk directly (e.g., undergrounding lines) or support de-energizing additional lines via load shed reduction (e.g., installing grid-scale batteries). California, for instance, is currently investing billions of dollars in such wildfire resilience infrastructure through state and federal funding [22, 124–126].

In this section, we extend the (MTP-PSPS) problem formulation to consider the placement and operation of new infrastructure. Although infrastructure placement decisions would ideally be made in a manner that accounts for many possible realizations of wildfire risk, in this investigative work, we make infrastructure placement decisions based on a representative wildfire risk realization due to the modeling and computational challenges that exist when jointly considering optimal switching and an infrastructure investment model. We present our development of a representative realization in Section 5.5.1.

We explore three types of investments: (1) grid-scale batteries, (2) solar PV, and (3) line hardening or line maintenance measures. We assume that a user of this investment framework is provided a budget for infrastructure improvements, and each investment has an associated cost. Therefore, investment decisions are based on the load shedding versus wildfire risk trade-off parameter , the total budget, the cost of the individual investments, and the representative wildfire risk realization. We first discuss the three considered types of investments and then formulate the infrastructure investment problem.

5.3.1 Grid-Scale Batteries

First, we consider the placement of grid-scale batteries. We allow any number of batteries to be placed at a subset of the buses $N^{batt} \subseteq N$. We assume all batteries in the network have the following characteristics:

- $\overline{E}$ and $\underline{E}$, the maximum and minimum energy storage limits of the battery, respectively, in p.u.,
- E_0^n, the sum of the initial charges of the batteries at bus $n \in N^{batt}$ in p.u.,
- $e \in (0,1]$ charge efficiency, and $\frac{1}{e}$ is the discharge efficiency,
- $\overline{p}_+$ and $\underline{p}_+$, the maximum and minimum charge rate limits for a single battery, respectively, in p.u. per considered time interval,
- $\overline{p}$ and $\underline{p}$, are the maximum and minimum discharge rate limits for a single battery, respectively, in p.u. per considered time interval,
- ^{batt}, price of a single battery in millions of dollars.

For each bus $n \in N^{batt}$, we introduce the following variables:

- $x^n \in Z$, number of batteries placed at bus n,
- $u_{t,d}^n \in \{0,1\}$, state of batteries located at bus n at time $t \in T$, where $u_t^n = 1$ indicates that the batteries at bus n are charging, and $u_t^n = 0$ indicates discharging,

Model 4 Battery Storage Constraints with Binary Exclusion Variable

$$\text{s.t.}\quad \forall d \in D;\ \forall t \in T;\ \forall n \in N^{batt},$$

$$E^n_{t+1}(x; p_+, p_-) = \tilde{E}^n_{t,d}(x; p_+, p_-) + e p^n_{+,t,d} - \frac{1}{e} p^n_{-,t,d} \qquad (5.2a)$$

$$x^n \underline{E} \le E^n_{t+1,d} \le x^n \overline{E} \qquad (5.2b)$$

$$p^n_{+,t,d} \le \overline{p}_+ M^{batt} u^n_{t,d} \qquad (5.2c)$$

$$p^n_{-,t,d} \le \overline{p}_- M^{batt}(1 - u^n_{t,d}) \qquad (5.2d)$$

$$0 \le p^n_{+,t,d} \le \overline{p}_+ x^n \qquad (5.2e)$$

$$0 \le p^n_{-,t,d} \le \overline{p}_- x^n \qquad (5.2f)$$

- $p^n_{+,t,d}$, charging rate at bus n and at time $t \in T$ in p.u.,
- $p^n_{-,t,d}$, discharging rate at bus n and at time $t \in T$ in p.u.,
- $E^n_{t,d}$, the battery state-of-charge at bus n and at time $t \in T$ in p.u.

Note, for consistency with other notation in this thesis, we index variables over days as well. Extensions can be made to a multi-day framework but this work considers $d \in D = [1]$.

Equation (5.2a) defines the state-of-charge update for batteries at bus $n \in N^{batt}$, where the stored energy at time $t+1$ on day d equals the initial charge $x^n E^n_0$ plus the cumulative net energy from charging (with efficiency e) and discharging (with efficiency $1/e$) over all previous time periods. Equation (5.2b) enforces lower ($x^n \underline{E}$) and upper ($x^n \overline{E}$) storage limits for the total energy stored across all batteries at bus n. Equations (5.2c) and (5.2d) use big-M constraints (with M^{batt} equal to the maximum number of batteries that can be placed) to enforce that batteries can only charge when $u^n_{t,d} = 1$ and only discharge when $u^n_{t,d} = 0$, preventing simultaneous charging and discharging. Equations (5.2e) and (5.2f) enforce that the charging and discharging rates are proportional to the number of batteries installed at bus n, ensuring that the rates do not exceed the capacity of the installed batteries.

5.3.2 Solar PV

Second, we consider the placement of solar PV to support line de-energization. We assume solar PV can be placed at a subset of the buses $N^{solar} \subseteq N$, and the user has access to the following parameters:

- S^n_t, the maximum possible output of a unit of solar PV at bus $n \in N^{solar}$ and time interval $t \in T$ in p.u.,
- ^{solar}, price of 1 unit of solar PV in millions of dollars.

For all $n \in N^{solar}$, we introduce the following variables:

- $a^n \geq 0$, the number of 1-unit installations of solar PV at bus n,
- $p^n_{s,t} \geq 0$, the solar PV output at bus n and time interval $t \in T$.

We require that the solar PV output does not exceed its upper bound on the possible power production, which is a function of the amount of solar PV installed at that bus as well as the location and time of day:

$$p^n_{s,t} \leq S^n_t a^n, \qquad \forall n \in N^{solar}, \ \forall t \in T. \tag{5.3}$$

Observe that Eq. (5.3) permits solar production below the maximum possible power production, i.e., “spilling” solar.

5.3.3 Line Hardening and Maintenance

The final type of investment we consider is line hardening or maintenance for wildfire risk reduction. Let the subset $L^{harden} \subseteq L$ be the set of lines that are candidates for hardening/maintenance. We assume users have access to the following parameters:

- $\in [0, 1]$, the reduction in wildfire risk due to line hardening or maintenance measures,
- ^{harden}, price of line hardening or maintenance in millions of dollars per mile of line length.

For all $\ell \in L^{harden}$, we introduce the following variable:

- $y_\ell \in \{0, 1\}$, a state of the line indicating whether the line has been hardened or maintained ($y_\ell = 1$) or no measures are enacted on the line ($y_\ell = 0$).

We assume that the entire length of line ℓ is hardened or has increased maintenance. Hardening or performing increased maintenance on partial segments of lines may provide better outcomes by targeting improvements in specific areas; however, this does not change the fundamental characteristics of the problem and our formulation could be extended accordingly. We note that reference [68] studies line upgrades on partial segments for wildfire risk mitigation, but does not include a power flow model.

Note that there is no benefit to simultaneously hardening and de-energizing a line since de-energizing a line reduces its risk to zero while hardening a line reduces the risk at by a cost of c^{harden} per mile. We impose the following constraint so that the solver does not consider simultaneously de-energizing and hardening a line:

$$(1 - z_\ell) + y_\ell \leq 1, \qquad \forall \ell \in \mathcal{L}^{\text{harden}} \cap \mathcal{L}^{\text{switch}}. \tag{5.4}$$

5.3.4 Infrastructure Investment Formulation

Now we can formulate the infrastructure investment problem. We require power balance at all buses:

$$\begin{aligned}\sum_{\ell \in \mathcal{L}^{n,\text{fr}}} f_{\ell,t,d} - \sum_{\ell \in \mathcal{L}^{n,\text{to}}} f_{\ell,t,d} = {} & -p^n_{l,t,d} + p^n_{ls,t,d} - p^n_{+,t,d} + p^n_{-,t,d} \\ & + p^n_{s,t,d} + \sum_{i \in \mathcal{G}^n} p^i_{g,t,d}, \quad \forall n \in \mathcal{N}, \forall t \in \mathcal{T}, \forall d \in \mathcal{D}, \end{aligned} \tag{5.5}$$

where, for notational simplicity, we assume that $p^n_{+,t,d} = 0$ and $p^n_{-,t,d} = 0$, $\forall t \in \mathcal{T}$ if $n \notin \mathcal{N}^{\text{batt}}$. Similarly, we assume $p^n_{s,t,d} = 0$, $\forall t \in \mathcal{T}$ if $n \notin \mathcal{N}^{\text{solar}}$.

Let B be the infrastructure investment budget in millions of dollars. We require the total investment to be within this budget, i.e.:

$$\sum_{\ell \in \mathcal{L}^{\text{harden}}} c^{\text{harden}} l_\ell y_\ell + \sum_{n \in \mathcal{N}^{\text{batt}}} c^{\text{batt}} x^n + \sum_{n \in \mathcal{N}^{\text{solar}}} c^{\text{solar}} a^n \leq B. \tag{5.6}$$

We modify the objective function in (5.1) to include the reduction in wildfire risk via line hardening or maintenance. Let $C_{invest}(\ )$ be the modified objective function, which we define as:

$$C_{invest}(p_{ls}; z; y) = C\ (p_{ls}; z)\quad \frac{(1\quad)}{R} \sum_{j \in L_{harden}} y^{j}\ : \tag{5.7}$$

We can now formulate the multi-time-period PSPS infrastructure investment problem as:

$$\min_{p_g;\ ;f;p_{ls};z;x;u;p_+;p\ ;a;p_S;y} \quad (5.7) \qquad \text{(Invest-Opt)}$$
$$\text{s.t. } (2.2b)\quad (2.2k);\ (5.2)\quad (5.6);$$

where our goal is now to minimize a weighted combination of the total load shedding and wildfire risk taking into account the reduction of risk via line hardening or maintenance. We optimize over the traditional operational variables (p_g, , and f), load shedding (p_{ls}), line switching decisions (z), battery variables (x, u, p_+, and p), solar PV variables (a and p_S), and line hardening/maintenance variables (y).

5.4 Test Networks and Parameter Values

In this section, two networks are presented for use in numerical tests. Values are also assigned to the parameters described in Sections 5.2 and 5.3 for the (Invest-Opt) problem.

5.4.1 Networks

We demonstrate our algorithm using two synthetic transmission networks geolocated in parts of the western United States that intersect with historically high wildfire risk areas:

1. RTS: 73-bus RTS-GMLC test case, Active Power Increase (API) version,
2. WECC: 240-bus test case representing the Western Interconnect.

The network topologies and electrical information associated with these test cases are