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PROJECT NO. 1678
COOPERATOR Institute
REPORT NO. 1
DATE July 6, 1954
NOTE BOOK 379
PAGE 145 TO 157
SIGNED W. J. Whitsitt
W. J. Whitsitt

ABSTRACT

It is recognized that the Elmendorf and Torsion tear instruments propogate tears in different manners and so may bring different sheet properties into play. Little data have been gathered comparing the two instruments and no such information has been developed for boxboards. For this reason this study compares the evaluation of a number of grades of boxboard by these two instruments. Nine samples of boxboard were evaluated; four were white patent coated boards and five were solid kraft cylinder boards.

The two testers graded all samples in the same order. When the machine direction results were compared, it was noted that the two types of boxboard apparently defined separate regression lines. The cross-machine direction results for the two machines defined a linear relationship between the two tests that was not influenced by these two types of board. However, because the number of samples were limited, no statistical correlations were computed. The same fact must temper all conclusions.

INTRODUCTION

The Torsion tear instrument was devised in an effort to relate the tear strength of combined board to rough-handling box performance. However, the instrument was so designed that the tear strength of component materials could be measured by decreasing the mass of the pendulum. It is recognized that the Torsion and Elmendorf instruments propogate tears in different manners and so may bring different sheet properties into play.

To this date little data have been gathered comparing the two instruments and no such information was available for boxboards. For this reason this study was inaugurated to compare the evaluation of a number of greades of boxboard by these two instruments. The study is limited in scope because few samples were available and the interpretation of the data must be tempered by that fact.

MATERIALS

The materials for this study consisted of 4 samples of white patent coated boxboard and 5 samples of solid kraft cylinder board. The white patent coated samples ranged from 16 to 30 points in caliper and the solid kraft samples from 16 to 28 points in caliper.

CONDITIONING AND TESTING

All materials were preconditioned for at least 24 hours at 73° F. and less than 35% R.H. They were then conditioned for 48 hours at 50 ± 2% R.H. and 73 ± 3.5° F. before testing. All tests were performed in the conditioning atmosphere.

Twenty tests were performed on each sample and in each direction following current laboratory procedures. To enable the Torsion tear instrument to properly grip the specimens in the jaws, the old-style round cams were reinserted in the machine and the largest spacer blocks were used.

DISCUSSION OF RESULTS

The results for all 9 samples are summarized in Table I together with the identification of each sample. In the table it may be noted that the Elmendorf tear results show a marked tendency for the tear to progress beyond the 3/8-inch limits, particularly in the cross-machine direction. These deviations were so large that in some instances the specimen rubbed the sector, apparently resulting in greated readings than would be expected. (A few of the torn specimens were saved and placed in the research notebook for ready reference). This fact may well be of importance in evaluating the merits of the Elmendorf instrument for boxboard evaluation.

Examination of the table shows that the two testers graded all samples in the same manner. To illustrate this point, the relationship between the two testers has been graphed in Figures 1 and 2 for the machine and cross-machine direction results, respectively. Referring to the graph of the machine direction results, it may be noted that the two testers seem to be related in an approximately linear fashion but the two types of board represented in this study appear to define two separate lines. On the other hand in Figure 2 the cross-machine direction results appear to be linearly related but show no such differentiation between type of board.

Because the number of samples were so limited, no statistical correlations of the data were made. It is apparent, however, that such computations would show an exceptionally good relationship for the cross-machine direction results.

wjw/ls

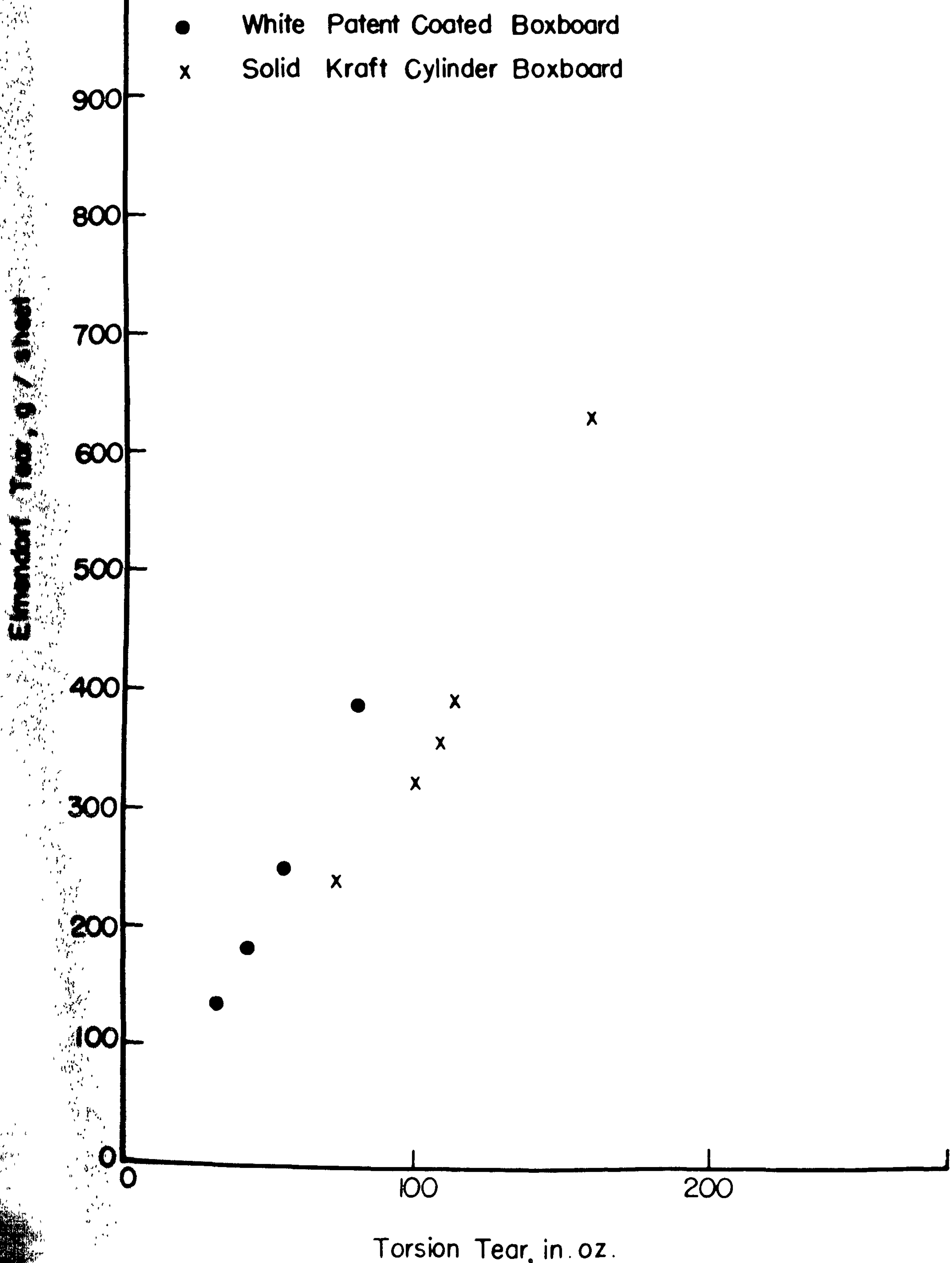
TABLE I

COMPARISON OF ELMENDORF AND TORSION TEAR RESULTS FOR VARIOUS GRADES OF BOXBOARD

Sample Number	Type of Boxboard	Elmendorf Tear,[a] g./sheet In	Across	Torsion Tear, in. oz. In	Across
521	0.016 White Patent Coated	135 (6/20)	218 (18/20)	33	45
522	0.020 White Patent Coated	184 (3/20)	293 (20/20)	43	62
523	0.026 White Patent Coated	249 (7/20)	369 (20/20)	56	81
524	0.030 White Patent Coated	383 (17/20)	527 (20/20)	78	118
520	0.028 Bleached Kraft Cylinder Board	626 (10/20)	885 (20/20)	159	199
525	0.016 Bleached Sulfate Manila Back	238 (0/20)	416 (15/20)	73	92
527	0.016 Unbleached Container Board	319 (0/20)	553 (20/20)	99	130
529	0.016 Bleached Sulfate Bleached Back	351 (8/20)	598 (20/20)	108	139
531	0.018 Bleached Sulfate Kraft Back	390 (2/20)	663 (20/20)	112	149

[a] The first figure in the parenthesis indicates the number of specimens which tore beyond the 3/8-inch limit. The second figure indicates the number of tests performed.

Machine Direction



Figure

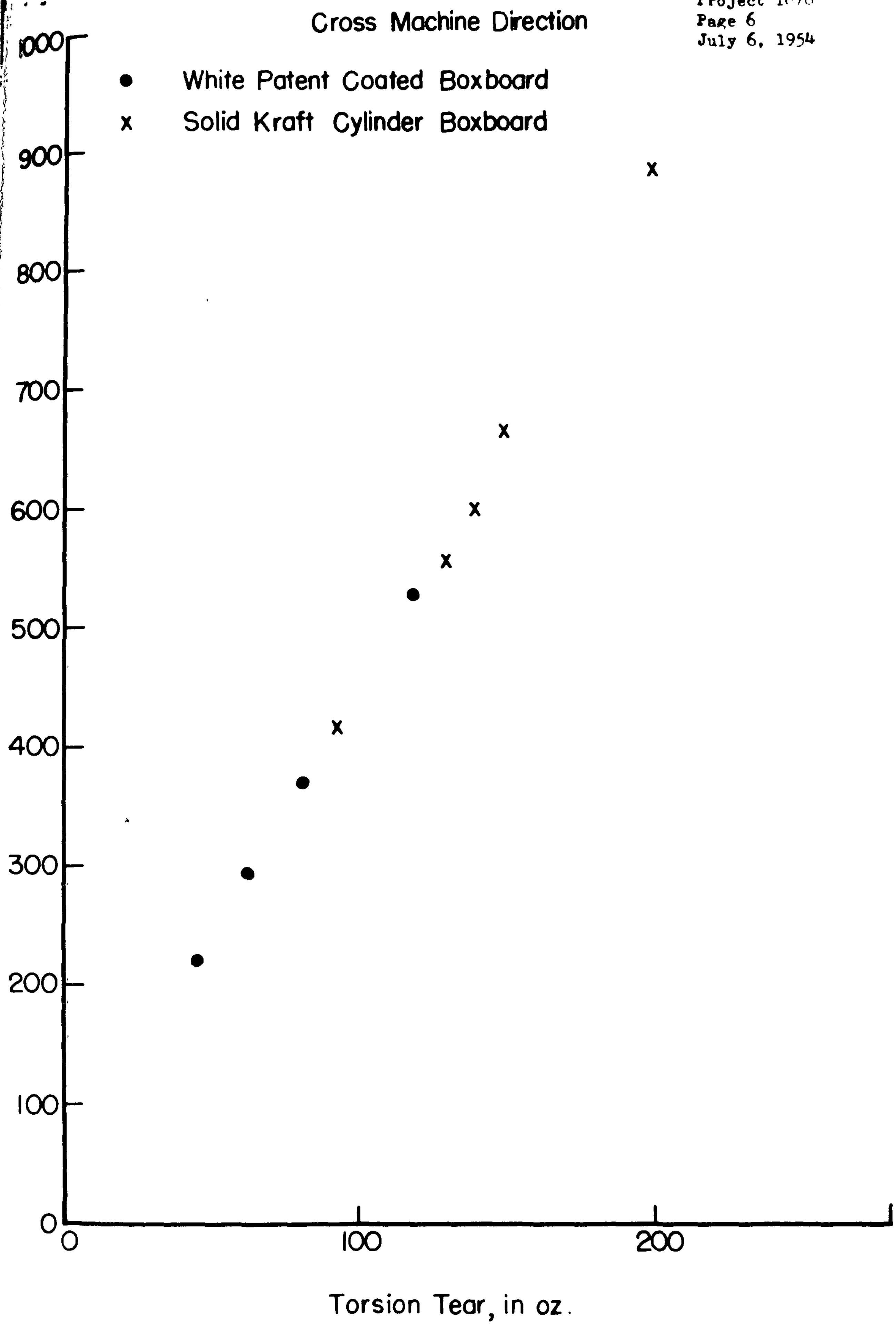

Cross Machine Direction
White Patent Coated Boxboard
x Solid Kraft Cylinder Boxboard
1000
900
800
700
600
500
400
300
200
100
0
0
100
200
Torsion Tear, in oz.

Figure 2

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COOPERATOR Institute
REPORT NO. Two
DATE November 29, 1956
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STATUS AS OF NOVEMBER 29

The writer requested and received a preliminary rough draft report on the results to date from Mr. Shoman on November 19, 1956. This work covers the testing and analysis of the data on 9 samples for a plate size of 6 by 9 inches.

The report referred to above has not been revised by the writer as yet. It is hoped this will be completed during the next month.

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PROJECT NO. 1678
COOPERATOR Institute of Paper Chemistry
REPORT NO. Three
DATE November 29, 1956
NOTE BOOK
PAGE TO
SIGNED John Shoman
John A. Shoman

A STUDY OF THE RELATIONSHIP BETWEEN MATERIAL CHARACTERISTICS OF COMPONENTS AND PERFORMANCE CHARACTERISTICS OF FOLDING CARTONS

A first draft of a progress report has been typed and revised. It is now in W. Whitsitt's possession for examination after which, with his help, it will be revised as needed.

In summary, the report brings out the following points:

1. Theory

As the deflection of a plate becomes large in comparison to its thickness, the major stress in the plate changes from that caused by bending conditions of the surfaces to the stress caused by the elongation of the middle plane, in tension.

2. Testing

Several tests were run on the boxboard for the purpose of allowing the reader to identify the material as he might choose. Determination of engineering properties were made for more direct use in the report for comparison with constants already derived, and for improvement of technique and apparatus for determining the constants.

3. Elastic properties

The classification of paper as an orthotropic material was more or less assumed.

4. Apparatus

The tank and frame used in the direct study of the plate was thought acceptable. Some modification to prevent slippage, and in the deflection measurement might be desirable.

5. Pressure-deflection relationship

The pressure and deflection quantities, in nondimensional form, were used in a correlation coefficient calculation and a method of least squares determination. The agreement between the calculated deflections and the observed deflections seemed to augment the correlation coefficient.

6. Future study

This study would suggest additional work might be done with different size plates, and with plates in different positions to eliminate any effect of the weight of the plate.

With the termination of the funds from Project 1675, the project was continued under Institute funds as 1678.

Additional study is being made to determine what, if any, effect the weight of the plate has on the deflection. This is done by placement of the plate in a vertical plane during pressure application.

The deflection of the center of the plate is being measured with the original method of differences from the first reading of the micrometer, and differences from a zero point as determined by contact of the micrometer on a plate placed on a plane surface.

A change has been made in the tank and monometer hookup to the vacuum source to see if the method of pressure control was having an effect on the plate.

An attempt is being made to note any rotation of the frame and tank. A slight rotation along the edge of the plate would cause a large increase in the central deflection.

At the present time no definite trends have been observed which can be related to the four preceding points of study. Additional data is to be taken before a more complete examination will be made.

An attempt was made to prevent slippage of the plate by fixing carborundum paper to the tank and frame with the use of double-faced, pressure-sensitive tape. This proved unsuccessful because the tape would creep. Rosin seems to prevent slippage.

Next month should bring a final revision of the progress report that is being examined. There may also be some additions made to it as results of the present data will be determined.

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PROJECT NO. 1678
COOPERATOR Institute of Paper Chem.
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SIGNED

W. J. Whitsitt

THE EFFECT OF LOCUST BEAN GUM ON THE TENSILE LOAD DEFORMATION CURVE OF PAPER

SUMMARY

This study was initiated to compare the influence of a chemical additive on the elastic and failure properties of paper or paperboard. For this purpose handsheets were formed with various concentrations of locust bean gum and evaluated for caliper, Taber stiffness, and tensile (load-deformation curves). The conclusions that might be drawn from this study are as follows:

1. The addition of locust bean gum significantly increased the maximum tensile load. Differences in stretch at failure were not significant.

2. Neither the apparent tensile modulus of elasticity nor the Taber bending modulus showed an appreciable increase with increasing amounts of additive.

INTRODUCTION

One common method of improving the bonding strength of paper and paperboard is through the use of various chemical additives. Among the classes of materials used as beater additives are gums and mucilages, synthetic latices, cellulose derivatives, starches, etc. Because these substances permit the preparation of paper of greater strength or paper of equivalent strength properties from lower grade pulps, they have been the subject of numerous experimental and commercial investigations. In general, however, investigators have focussed attention on the effect of such additives on ultimate strength properties such as tensile, bursting strength, stretch, etc. Little attention has been given to the properties associated with lower stresses such as the apparent elastic modulus, proportional limit, flexural stiffness, etc.

Previous work in this laboratory in connection with another study indicated that the apparent elastic properties of paper were not appreciably improved by the addition of chemical additives. In other words, it appeared that such additives increased the ultimate tensile strength without bringing about a corresponding increase in the apparent elastic modulus. With the above in mind, this study was initiated to compare the influence of an additive on both elastic and failure properties. The study was limited in scope and, consequently, the action of only one additive--locust bean gum--was studied. There is no intension to suggest that locust bean gum is the most effective agent for strength improvement as its selection was primarily based on the fact that its action on sheet performance is relatively well known.

MATERIALS AND SHEETMAKING PROCEDURE

For this study handsheets were prepared using a Weyerhaeuser bleached sulfite pulp at six levels of locust bean gum addition, namely, 0, 0.5, 1.0, 1.5, 3.0, and 5.0% locust bean gum. The handsheets were prepared by the Physical Chemistry group and their preparation was described in a memorandum from Swanson to McKee, dated Nov. 21, 1957. The following summary of the sheet-making procedures is taken from that memorandum.

The preparation of these sheets is set forth in Notebook 1339 on pages 115-117. Briefly, the sheets were prepared as follows: Weyerhaeuser bleached sulfite (390 g.) was soaked in six liters of tap water for one hour. The Valley beater was filled with tap water to the seventeen-liter mark. The soaked pulp was then added to the beater and the temperature was adjusted to 2[illegible]°C. and the pH to 7.0 with dilute sulphuric acid. The pulp was slurried for five minutes in the beater with the balance weight on the bedplate arm. The pulp was then beaten with 5500 g. weight on the bedplate to a freeness of 775 ml. at a beater consistency of 1.45%.

The locust bean gum was prepared by cooking a slurry of the gum in water at 0.5 solids. Two and one-half grams of locust bean gum were added to 497-1/2 ml. of deionized water at room temperature. This mixture was then heated on the steam bath for 15 min. at 95°C. Moderate agitation was provided by means of a Lightnin' stirrer. Aliquots (20 g. O.D.) of the beater pulp were measured into a bucket and 3% of rosin size stirred into the mixture for five minutes. The required quantity of gum dispersion was then added followed by stirring for 15 minutes. Finally 4% of alum based on the fiber was added to the mixture. After the alum had been stirred into the pulp for five minutes, the entire slurry was diluted to 0.5% solids and used in the preparation of handsheets in the conventional manner on a Valley sheet mold.

The handsheets should have a 45-pound basis weight (25 x 40/500). Because of the alkaline nature of our tap water, the pH of the water in the sheet mold was adjusted to 4.5 with 2% of sulphuric acid. The wet sheets were pressed for five minutes at 50 lb. pressure and dried for seven minutes sheet side up on the steam drum drier at 3-1/2 lb. steam pressure.

TEST PROCEDURE

The following test procedures were employed:

1. Caliper

Four caliper measurements were made on each of five sheets from each sample. The twenty readings were averaged to the nearest 0.1 point. The moment of inertia was computed from the equation $I = h^3/12$ where _h_ is the caliper.

2. Taber stiffness and Taber bending modulus

Ten determinations on each sample were made using a Taber V-5 instrument. The light weight attachment was used and the average of the ten readings was recorded to the nearest 0.01 unit. To convert Taber stiffness readings to a Taber bending modulus, the observed Taber reading was multiplied by the factor 0.001,465 and then divided by the moment of inertia.

3. Tensile load-deformation curves

One test was performed on each of ten specimens from each sample. Specimen width and test span were 1 inch and 3 inches, respectively. The Baldwin Southwark Universal tester was used at a test rate of 60 pounds per minute. A load-deformation curve was obtained and the maximum load indicated by the load dial was recorded with each curve. The average of the ten load readings were reported to the nearest 0.1 lb. per inch. The stretch at maximum load was computed from the load-deformation curves and the average was reported to the nearest 0.1% stretch.

To compute the modulus of elasticity in tension (E), tangent lines were fitted to the load-deformation curves in the initial linear portion and the deflection on the tangent at some arbitrary load was measured for each curve. After averaging the deflections, the load and average deflection at that load were substituted in the equation below, to compute the modulus.

$$E = PL/Ae$$

where E = modulus of elasticity, p.s.i.

P = load on tangent, lb.

A = cross-sectional area, sq. in.

L = test span, inches

e = deflection at load P, inches

The tangent modulus near failure (referred to as secondary modulus herein) was determined in a similar manner except that the tangent lines were fitted to the tensile curves in the region near failure. Calculations of the modulus so determined were made as described above.

The proportional limit was determined using an arbitrary "offset" of 0.005 inches (one smallchart division).

DISCUSSION OF RESULTS

The results obtained are summarized in Table I. In the table, it may be noted that increasing amounts of locust bean gum increased the maximum tensile strength in the expected manner. These increases in tensile strength were accompanied by much smaller increases in stretch which were not statistically significant. Thus, because the net effect of these two factors is a substantial increase in tensile load with little or no increase in stretch, it would be anticipated that the secondary modulus (slope near failure) would increase in much the same manner as the tensile load. Reference to the data indicates that such was the case.

While the above data indicate that a substantial change in the tensile load-deformation curves occurs at high load levels, neither the tensile modulus of elasticity nor the Taber bending modulus results show any appreciable increase with increasing amounts of additive. In other words, this data suggests that locust bean gum has little or no effect on the apparent elastic properties of the material.

Finally, an arbitrary measure of the proportional limit was calculated using an "offset" of 0.005 inches. It, therefore, corresponds to the load at which the curve has deviated 0.005 inches from the straight tangent line characteristic of the material's initial response to stress. In the table it may be observed that the proportional limit so measured increased as locust bean gum was added. On a percentage basis the increases are usually smaller than those recorded for the maximum load; however, it is felt they are significant.

TABLE I

PHYSICAL CHARACTERISTICS OF HANDSHEETS FORMED WITH VARIOUS AMOUNTS OF LOCUST BEAN GUM

Test	Locust Bean Gum, % 0	0.5	Diff.,%[c]	1.0	Diff.,%[c]	1.5	Diff.,%[c]	3.0	Diff.,%[c]	5.0	Diff.,%[c]
Caliper, points	4.3	4.2	-2.3	4.2	-2.3	4.2	-2.3	4.2	-2.3	4.1	-4.7
Moment of inertia, in.^{4} x 10^{-10}	66.3	61.7	-6.9	61.7	-6.9	61.7	-6.9	61.7	-6.9	57.4	-13.4
Taber stiffness, units	1.90	1.84	-3.2	1.80	-5.3	1.83	-3.7	1.78	-6.3	1.71	-10.0
Taber bending modulus, p.s.i. x 10^{+3}	420	437	+4.0	427	+1.7	435	+3.6	423	+0.7	436	+3.8
Tensile characteristics- Max. Load, lb.	25.5	27.9	+9.4	30.3	+18.8	30.7	+20.4	29.5	+15.7	31.1	+22.0
Stretch, %	3.7	3.8	+2.7	4.0	+8.1	4.0	+8.1	3.8	+2.7	3.9	+5.4
Proportional limit,[a] lb.	12.4	13.4	+8.1	14.2	+14.5	14.3	+15.3	14.2	+14.5	14.6	+17.7
Modulus of elasticity, p.s.i. x 10^{+3}	581	595	+2.4	560	-3.6	583	+0.3	583	+0.3	636	+9.5
Secondary modulus,[b] p.s.i. x 10^{+3}	81	90	+11.1	98	+21.0	101	+24.7	98	+21.0	106	+30.9

[a] Calculated at 0.005 inch offset.
[b] Slope of curve approaching failure.
[c] Based on 0% locust bean gum as reference.

To illustrate the above remarks in another way, the load-deformation curves for each run were re-analyzed in two ways. First, the loads corresponding to deflections of 0.010, 0.025, 0.050, and 0.075 inches were read from each curve. These results are summarized in Table II. Second, the deflections corresponding to load levels of 5, 10, 15 and 20 lb. were read from each curve. These results are summarized in Table III. In carrying out the above operations, it was apparent that there was a serious loss in curve reading accuracy in the initial portions of the curve. This is most noticeable in Table III at the 5 and 10 lb. load levels but may also be a disturbing factor in the results for 0.010 in. deflection in Table II. Thus, with either method of analysis, this difficulty prohibited measurements in the initial straight-line portions of the curve. To avoid this problem in any further investigations along these lines, the load and chart magnifications should be at least 2 to 5 times greater than were employed in this study. Such magnification ratios may be somewhat inconvenient for the entire curve. In this event, duplicate sets of specimens might be tested—one for modulus determinations and the other for maximum load and stretch determinations.

With the above in mind, the results in Table II appear to indicate that the effect of the additive is most noticeable at the higher load levels. An analysis of variance was performed to determine if the differences were significant with the results shown below in Table IV. Inspection of the results in Table IV indicates that significant increases in load were obtained at each deflection level as a result of the addition of locust bean gum.

In summary, therefore, the results of this study appear to indicate that an additive such as locust bean gum does not cause an appreciable increase in the apparent tensile elastic modulus of paper or paperboard. The additive does appear to substantially change the tensile load-deformation curve in the higher stress regions.

TABLE II

ANALYSIS OF THE TENSILE LOAD-DEFORMATION CURVE IN TERMS OF SELECTED DEFLECTIONS

Load at Indicated Deflection, lb.

Indicated Deflection, in.	Locust Bean Gum, %										
	0	0.5	Diff.*,%	1.0	Diff.*,%	1.5	Diff.*,%	3.0	Diff.*,%	5.0	Diff.*,%
0.010	8.2	8.5	+3.7	8.0	-2.4	8.5	+3.7	8.6	+4.9	9.2	+12.2
0.025	14.4	14.8	+2.8	14.7	+2.1	15.2	+5.6	15.2	+5.6	15.9	+10.4
0.050	18.8	19.8	+5.3	20.2	+7.4	20.5	+9.0	20.4	+8.5	21.4	+13.8
0.075	22.1	23.4	+5.9	24.2	+9.5	24.6	+11.3	24.4	+10.4	25.4	+14.9
Max. Load	25.5 at 0.110 in.	27.9 at 0.114 in.	+9.4	30.3 at 0.119 in.	+18.8	30.7 at 0.120 in.	+20.4	29.5 at 0.115 in.	+15.7	31.1 at 0.117 in.	+22.0

* Based on 0% locust bean gum as reference.

TABLE III

ANALYSIS OF THE TENSILE LOAD-DEFORMATION CURVE IN TERMS OF SELECTED LOADS

Deflection at Indicated Load, in.

	Locust bean gum, %										
Indicated Load, lb.	0	0.5	Diff.*, %	1.0	Diff.*, %	1.5	Diff.*, %	3.0	Diff.*, %	5.0	Diff.*, %
5	0.006	0.005	-16.7	0.006	0.0	0.005	-16.7	0.005	-16.7	0.005	-16.7
10	0.013	0.012	-7.7	0.013	0.0	0.012	-7.7	0.012	-7.7	0.012	-7.7
15	0.029	0.026	-10.3	0.025	-13.8	0.024	-17.2	0.024	-17.2	0.022	-24.1
20	0.060	0.052	-13.3	0.049	-18.3	0.047	-21.7	0.047	-21.7	0.043	-28.3
Max. defl.	0.110 at 25.5 lb.	0.114 at 27.9 lb.	+3.6	0.119 at 30.3 lb.	+8.2	0.120 at 30.7 lb.	+9.1	0.115 at 29.5 lb.	+4.5	0.117 at 31.1 lb.	+6.4

* Based on 0% locust bean gum as reference.

TABLE IV

STATISTICAL ANALYSIS OF TENSILE LOAD-DEFORMATION CURVE RESULTS

Source of Variance	Degrees of Freedom	Mean Square	F
	0.010 inch Deflection Level		
Between concentrations	5	1.8519	4.034**
Within concentrations	54	0.4591	
	0.025 inch Deflection Level		
Between concentrations	5	2.7836	6.020**
Within concentrations	54	0.4624	
	0.050 inch Deflection Level		
Between concentrations	5	7.1719	7.990**
Within concentrations	54	0.8976	
	0.075 inch Deflection Level		
Between concentrations	5	13.0986	9.163**
Within concentrations	54	1.4295	
	Maximum Load		
Between concentrations	5	44.8720	10.922**
Within concentrations	54	4.1083	

** Significant at the 1% level.

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James W. Gander

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COMPARISON OF ESTIMATED AND OBSERVED

FLEXURAL STIFFNESS OF FOUR-PLY BOXBOARD

SUMMARY

A comparison was made of observed and estimated flexural stiffness of seven arrangements of four-ply board comprised of kraft and news handsheets. The estimated stiffness was based on alternately the tensile moduli and the Taber bending moduli of the plies.

Inasmuch as a handsheet apparently suffered about 15% reduction in caliper when made into multi-ply board, the handsheet moduli were corrected for (a) change in caliper and (b) change in density. These two types of corrections yielded nearly identical results. Evidently, the change in caliper was the dominant effect, while the density correction was in the nature of a further refinement accounting for variations in basis weight between handsheet and board ply.

The handsheet tensile moduli led to estimates of flexural stiffness which were generally less than the observed board stiffness—about 9% less on the average. Corrected handsheet tensile moduli, on the other hand, gave estimates about 5% greater than the observed stiffness, on the average. The latter difference is in the expected sense since compression modulus was not employed in the estimate (provided tensile modulus is greater than compression modulus for these handsheets).

The Taber bending moduli gave estimates which were generally less than the observed board stiffness. When based on handsheet moduli, the estimates were about 19% low and when based on corrected moduli, about 7% low.

INTRODUCTION

Discussions with representatives of the Boxboard Research and Development Association have revealed that a fair degree of success was achieved in predicting the flexural stiffness of a multi-ply sheet from a knowledge of the elastic moduli and calipers of the individual handsheet plies. In two trials the differences between observed and calculated values were 2 and 9%.

The data of Tables XXIX, XXX and XXXIII of Reference (1) makes possible a similar comparison for seven four-ply arrangements of kraft and news handsheets. As discussed in greater detail in the remainder of this report, flexural stiffness was estimated by means of the equations of Reference (2), using alternately tensile modulus and Taber bending modulus. These moduli were adjusted for differences in (a) caliper and (b) density between handsheets and built-up board, as described under Methods of Calculation. A comparison of estimated flexural stiffness and Taber stiffness of multi-ply board is presented in Discussion of Results.

METHODS OF CALCULATION

Estimate of Ply Calipers

It may be noted in the data of Reference (1) that there is a decrease in average ply caliper of about 15% in going from handsheets to multi-ply board in the case of four-ply kraft (KKKK) and of four-ply news (NNNN). A first step of the calculation procedure, therefore, was to make an estimate of the individual ply calipers in each sample of multi-ply board.

This estimate was accomplished in the following way:

1. Calculate the average ply caliper of kraft from KKKK and of news from NNNN.

2. Calculate the expected total caliper of KNNK, for example, from the calipers of Step (1).

3. Determine the discrepancy between the expected and measured total caliper of KNNK.

4. Apportion the discrepancy among the K and N plies in the ratio of the average ply caliper of K and N, as determined in Step (1).

A numerical example of this procedure for KNNK follows:

1. From Table XXX of Reference (1), the average ply caliper of KKKK is 16.6/4 = 4.15 points. Similarly, the average news caliper is 18.3/4 = 4.58 points.

2. The expected total caliper of KNNK is 2 (4.15) + 2 (4.58) = 17.46 points.

3. The measured total caliper of KNNK is 17.8 points, leaving a discrepancy between expected and measured calipers of 0.34 point.

4. The two kraft plies are increased, therefore, by $\left(\frac{4.15}{4.15 + 4.58} \right)(0.34) =$ 0.16 point. That is, each kraft ply is increased by 0.08 point, so that its adjusted caliper is 4.15 + 0.08 = 4.23 points. Similarly, the two news plies are increased by $\left(\frac{4.58}{4.15 + 4.58} \right)$ (0.34) = 0.18 point. That is, a news ply is taken to be 4.58 + 0.09 = 4.67 points for this sample of board.

In the two cases where the number of kraft and news plies are unequal (KKKN and KNNN), the ratios used in the apportionment of Step (4) were weighted.

This method of estimating calipers makes use of the assumption that the reduction in caliper of a given type of handsheet is independent of its position in the multi-ply board.

Correction of Handsheet Modulus to Account for Caliper Reduction

Calculation of flexural stiffness of multi-ply board requires knowledge of the modulus of elasticity of each ply as well as caliper of each ply. Two types of moduli are available for these calculations: (a) tensile modulus for handsheets, and (b) Taber bending modulus, which is obtained from a Taber stiffness test on handsheets.

A correction of the tensile modulus was made to account for the reduction in caliper suffered by the handsheet when it was made into a multi-ply sheet. The correction equation employed was:

$$E' = \frac{t_o}{t'} E_o \qquad (1)$$

where E' = tensile modulus of ply of built-up board,

E_o = tensile modulus of handsheet

t' = caliper of ply of built-up board

t_o = caliper of handsheet

(A more elaborate correction--namely, a density correction--was made for both Taber bending modulus and tensile modulus and will be described in the next section of this report.)

Inasmuch as the ply caliper t' was less than the handsheet caliper t_o for the samples investigated, Equation (1) resulted in an increase in tensile modulus.

Equation (1) is tantamount to saying that at a given unit tensile strain, a tensile strip obtained from an individual ply will exhibit the same tensile force as a strip from the handsheet. That is, by Hooke's law, $P_o = \epsilon E_o t_o$ and $P' = \epsilon E' t'$ (P being tensile load per unit width and ϵ being unit strain). If it is assumed that the tensile forces are the same at a given strain; then $E_o t_o = E' t'$, which is Equation (1). This may be a reasonable assumption when the quantity of fiber (i.e., basis weight) remains constant. On the other hand, it may be argued that the pressing which leads to the caliper reduction might also alter the internal load-carrying characteristics of the sheet.

Correction of Handsheet Modulus to Account for Change in Density

The moduli correction involving caliper described above may have a rational basis provided the basis weight remains constant between handsheet and board ply. It may be reasoned that in the absence of a constant basis weight (B.W.), Equation (1) should be modified to:

$$E' = \frac{t_o}{t'} \frac{B.W.'}{B.W._o} E_o \qquad (2)$$

where the prime denotes board ply and subscript o denotes handsheet, as before. That is, if the ply has more fiber than the handsheet, one might expect an increase in modulus. Inasmuch as B.W./t is proportional to density, Equation (2) may be written as

$$E' = \frac{\rho'}{\rho_o} E_o \qquad (3)$$

where ρ denotes density, so that Equation (2) or (3) may be termed a density correction.

Comparison of Tables XXIX and XXX of Reference (1) reveals that there was an increase of about 19% in apparent density in going from handsheet to multi-ply board in the case of KKKK and NNNN. Therefore, a density correction was made. In the calculations of this report, Equation (2) rather than (3) was employed.

In the remaining five-ply combinations, an apportionment of basis weight between the two types of plies was required. This apportionment was accomplished by a method analogous to the apportionment of calipers described earlier. After the basis weight of each ply was estimated, a corrected modulus was determined by means of Equation (2).

A density correction was made for both tensile modulus and Taber bending modulus.

Calculation of Flexural Stiffness

Flexural stiffness, EI, was estimated by the equation of Reference (2), namely,

$$EI = \sum_{i=1}^{i=4} E_i I_i \qquad (4)$$

where E_i = modulus of the i^{th} ply

I_i = moment of inertia of the i^{th} ply about the neutral axis of the cross section,

Four of the ply arrangements were symmetrical constructions and the neutral axis, therefore, lies in the mid-plane. This assumes that the tension and compression moduli of each ply are equal. Compression moduli were not available for the calculations of this report. Thus, when only the tensile modulus is used in Equation (4), it is equivalent to making the above assumption. The Taber bending modulus, on the other hand, is of the nature of an average or effective modulus, involving both the tensile and compressive moduli. Since a given ply of built-up board is not acting in both tension and compression, in general, the Taber bending modulus is not rigorously applicable. By virtue of its "average" nature, however, it introduces compression behavior to the estimate. There is no other choice but to assume that the neutral axis is in the mid-plane of symmetrical constructions.

Three of the ply arrangements were unsymmetrical, requiring determination of the location of the neutral axis by means of the following equation from Reference (2):

$$d_{n.a.} = \frac{\sum_{i=1}^{i=4} A_i E_i d_i}{\sum_{i=1}^{i=4} A_i E_i} \qquad (5)$$

where $d_{n.a.}$ = distance from top of sheet to neutral axis

d_i = distance from top of sheet to mid-plane of i^{th} ply

A_i = cross-sectional area of i^{th} ply

E_i = modulus of i^{th} ply.

DISCUSSION OF RESULTS

Estimates were made of flexural stiffness of seven arrangements of kraft and news plies based on tensile modulus and Taber bending modulus of handsheets from Project 1986. Estimate of ply caliper and ply basis weight in the built-up board and correction of the handsheet moduli for changes in caliper and density were made as described in Methods of Calculation.

Table I lists the estimated caliper and basis weight of the kraft and news plies for each combination. The ply caliper is always less than the handsheet caliper--attributable to pressing. The ply basis weights are variously larger and smaller than the handsheet weights. These differences probably are associated with

... mean rather than process.

TABLE I

ESTIMATES OF CALIPER AND BASIS WEIGHT OF KRAFT AND NEWS PLIES IN MULTI-PLY SHEETS

Ply Arrangement	Caliper, 10^{-3} in. Kraft	News	Basis Weight, lb./25x40--500 Kraft	News
KKKK	4.15	—	46.90	—
NNNN	—	4.58	—	46.92
KNNK	4.23	4.67	46.95	46.97
NKKN	4.48	4.95	47.27	47.29
KKKN	4.17	4.61	46.75	46.77
KNKN	4.39	4.84	47.29	47.31
KNNN	4.15	4.58	45.48	45.50
Handsheet	4.9	5.4	46.7	47.2

Table II shows the several moduli (uncorrected and corrected) employed in the computation of flexural stiffness. With regard to tensile modulus, it may be noted that the caliper correction and density correction were nearly identical (5.48 vs. 5.49 for kraft and 3.20 vs. 3.17 for news, on the average. Reduction in caliper is the dominant effect in going from handsheet to build-up board, so that correction for caliper brings about the greatest change in modulus. The density correction is of the nature of a further refinement, accounting for variations in quantity of fiber.

The corrected tensile moduli are 15% greater than the handsheet moduli, corresponding to the 15% decrease in ply caliper relative to handsheet caliper. Similarly, the density correction increased the Taber bending modulus for each ply about 15%, relative to the handsheet modulus.

Table III presents a comparison of observed and estimated flexural stiffness for the seven ply arrangements. The observed stiffness was derived from Taber data in Table XXX and XXXIII of Reference (1) by converting to English units (or alternatively multiplying the Taber bending modulus of the built-up board by its moment of inertia).

It may be seen that, with one exception, the estimated flexural stiffness based on handsheet tensile modulus was less than the observed stiffness. The average percent difference was about -9%. The caliper- and density-corrected tensile moduli, on the other hand, gave estimates of flexural stiffness which exceeded the observed stiffness (with one exception); the difference was about +5%. Provided the compression modulus is less than the tension modulus, estimates

TABLE II

VALUES OF TENSILE MODULUS AND TABER BENDING MODULUS USED IN COMPUTATION OF FLEXURAL STIFFNESS OF 4-PLY BOARD

	Tensile Modulus of Elasticity, lb./in.^{2} (10^5)						Taber Bending Modulus, lb./in.^{2} (10^5)			
	Kraft			News			Kraft		News	
Ply Arrangement	Hand-sheet	Caliper Correction	Density Correction	Hand-sheet	Caliper Correction	Density Correction	Hand-sheet	Density Correction	Hand-sheet	Density Correction
KKKK	4.76	5.62	5.65	—	—	—	3.59	4.26	—	—
NNNN	—	—	—	2.78	3.28	3.26	—	—	2.91	3.41
KNNK	4.76	5.53	5.56	2.78	3.22	3.21	3.59	4.19	2.91	3.36
NKKN	4.76	5.20	5.27	2.78	3.03	3.04	3.59	3.97	2.91	3.18
KKKN	4.76	5.60	5.60	2.78	3.26	3.25	3.59	4.22	2.91	3.38
KNKN	4.76	5.31	5.38	2.78	3.10	3.10	3.59	4.06	2.91	3.25
KNNN	4.76	5.62	5.46	2.78	3.28	3.16	3.59	4.13	2.91	3.31
Av.	4.76	5.48	5.49	2.78	3.20	3.17	3.59	4.14	2.91	3.32

TABLE III

COMPARISON OF OBSERVED AND CALCULATED FLEXURAL STIFFNESS OF FOUR PLY BOXBOARD (KRAFT AND NEWS FURNISH)

Flexural Stiffness, lb. in.^{2}/in.

Ply Arrangement	Observed	Estimated									
		Based on Tensile Modulus						Based on Taber Bending Modulus			
	(Taber)	Handsheet	Diff., % [a]	Caliper Correction	Diff., %	Density Correction	Diff., %	Handsheet	Diff., %	Density Correction	Diff., %
KKKK	0.187	0.181	-3.2	0.214	+14.4	0.215	+15.0	0.137	-26.8	0.162	-13.4
NNNN	0.193	0.142	-26.4	0.168	-13.0	0.167	-13.5	0.149	-22.8	0.175	- 9.3
KNNK	0.240	0.209	-12.9	0.243	+ 1.2	0.244	+ 1.7	0.163	-32.1	0.190	-20.8
NKKN	0.160	0.167	+4.4	0.183	+14.4	0.183	+14.4	0.167	+ 4.4	0.183	+14.4
KKKN	0.173	0.157	-9.2	0.185	+ 6.9	0.185	+ 6.9	0.137	-20.8	0.160	- 7.5
KNKN	0.199	0.193	-3.0	0.215	+ 8.0	0.217	+ 9.1	0.169	-14.9	0.190	- 4.4
KNNN	0.190	0.167	-12.1	0.197	+ 3.7	0.191	+ 0.5	0.152	-20.2	0.174	- 8.7
Average Absolute			10.2		8.8		8.7		20.3		11.2
Algebraic			-8.9		+5.1		+4.9		-19.0		-7.1

[a] All percent differences are based on observed flexural stiffness.

based on tensile modulus only would be expected to be higher than the observed stiffness.

The estimates which employed the Taber modulus of the handsheets were markedly less than the observed stiffness (with one exception)—an average percent difference of -19%. The density-corrected Taber modulus, led to somewhat closer, though generally lower, estimates of flexural stiffness—an average difference of about -7%.

One may inquire why the Taber bending modulus which takes some account of compression behavior, generally led to underestimation of the flexural stiffness. An observation with possible bearing on this question is as follows: It may be expected that being a modulus, the Taber bending modulus (after density correction) would be the same as the Taber bending modulus of <u>built-up</u> board of one type of furnish. A comparison of Table II of this report and Table XXX of Reference (1) reveals this was not the case, as shown in Table IV below. In each instance the density-corrected handsheet modulus was lower than the built-up board modulus. Possible reasons for these differences are: (a) the density correction is not adequate to account for the change between handsheet and built-up board, or (b) the Taber test yields different test levels at these widely separated weights of board. Whatever may have been the reason for these apparent differences, they throw some light on why the estimates of flexural stiffness based on handsheet values generally did not attain the level of the observed stiffness.

It may be of interest to note that in the three unsymmetrical constructions, the location of the neutral axis was less than one point distant from the mid-plane of the board.

TABLE IV

COMPARISON OF TABER BENDING MODULUS OF HANDSHEETS (WITH DENSITY CORRECTION) AND OF ONE-FURNISH MULTI-PLY BOARD

	Taber Bending Modulus, lb./in.^{2}(10^5) Type of Furnish Kraft	News
Handsheet (with density correction)	4.26	3.41
4-Ply board (one furnish)	4.92	3.78
Difference, % (based on 4-ply value)	-13.4	-9.8

LITERATURE CITED

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2. Freas, A. D., Methods of computing strength and stiffness of plywood strips in bending, Forest Products Lab., Report No. 1304, USDA, 1946.