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PILOT MODELS FOR MAN MULTI-MACHINE JOBS INVOLVING SYSTEMATIC DELAYS

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PILOT MODELS FOR MAN MULTI-MACHINE JOBS INVOLVING SYSTEMATIC DELAYS

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CHAPTER I

INTRODUCTION

To be of benefit to mankind, the profession of industrial engineering must advance. To advance, new methods and techniques must be developed which can be applied to the problems presented by advances in technology. At the same time, better methods and techniques must be developed which can be applied to problems that exist today.

This study will be devoted to developing a better method for solving an existing problem, that of the development of a better method for analyzing the relationships between an operator and two machines in the performance of a man-machine sub-system. At present, this problem is solved by using the man-machine chart.

The man-machine chart is a graphic representation on a time scale of productive work and idle time which are encountered in the performance of a man-machine sub-system. The productive operations portrayed are related in that one or more of them usually have control over the others. This relationship creates idle time by preventing operations being performed at will. The types of operations usually portrayed by the man-machine chart are those which are performed in a sequence which is repeated time after time. The man-machine chart is used to portray the operations of one worker or a group of workers servicing one or more machines.

The man-machine chart is used to analyze the relationships among the operations. The primary objectives of such an analysis are to determine the optimum utilization of equipment and personnel and the standard time for the operation. Man-machine charts can be classified as follows:

1. Man single machine charts--charts for one operator servicing one machine.

2. Man multi-machine charts--charts for one operator servicing two or more machines.

3. Man multi-machine charts--charts for one operator servicing two or more machines.

4. Multi-man single machine charts--charts for two or more operators servicing two or more machines.

The development of mathematical models which can be used in place of the man-machine chart can be divided into three areas, according to the way in which machine delay time is considered, as follows:

1. Machine delay time considered to be a systematic occurrence. Algebra is the primary method used in developing models for this type of machine delay time.

2. Machine delay time considered to be a random occurrence. Statistical methods are used in developing models based on this type of machine delay time.

3. Machine delay time considered to be both a systematic and random occurrence. Both algebraic and statistical methods are used in developing models based on this type of machine delay time.

In the present study, models will be developed which can be used in place of the man-machine chart. In developing these models, machine delay time will be considered to be a systematic occurrence, and algebraic methods will be used. The models will be restricted to the class of man multi-machine jobs in which one operator services two machines. The machines considered herein have unlike machining and servicing times. The models will be based on the sequence in which the machines are initially placed in operation. A decision model for determining which machine to place into operation first will be presented.

The development of these models for this class of man-machine charts was selected because it presented the most challenging task that could be accomplished within the limits of a study of this type. The author was also motivated in selecting the development of these models because of the practical application that can be made of the results.

CHAPTER II

DEFINITION OF TERMS

Cycle time is the interval of time between the points in a repetitive cycle when the machines and the operator are at a given phase of relationship with each other, and when that exact phase of relationship occurs again as time progresses. Cycle time includes operator and machine delay times which are part of the standard method.

External work refers to the work performed by the operator in servicing a machine which is in a nonproductive state, such as loading or unloading a machine. This term relates only to the machine being serviced. Included in external work is stopping the machine, if necessary, and the tasks which are required to place the machine back into operation.

Operator delay time refers to those intervals of time in which the operator is idle, because all the machines are operating properly and there is no work for the operator to perform. The operator, however, cannot leave the work station because his services may be required at any moment.

Internal work refers to the work performed by the operator on a machine or the products from a machine while the machine is in a producing state. The work is internal to but one machine at a time. Internal work does not interfere with the producing state of the machine to which it is internal.

Machine time refers to the time required by a machine to perform the required operations on one unit of production at one stage of production.

Machine time is restricted to those machining operations which are in no way operator controlled. Operations which are part machining time and part external work but are under the control of the operator are considered to be external work.

Machine delay time is the interval of time during which a machine is idle awaiting the services of the operator. During this interval of time, the operator is performing other required work.

Effort level is a measure of the time rate at which productive work is done by the operator. This measure is stated in relation to "normal".

Sub-cycle time is the total time required by the operator and a machine to perform the required operations on one unit of production at one work station.

Transient time is the time interval between the initial start of operations of the machines and the instant at which the operation enters its repetitive cycling. Transient time occurs when all the machines are stopped, cleared of work, and started in the prescribed order.

The period is the elapsed time between the initial start of the first machine and the time operations are completed by stopping each machine independently as each work piece is finished.

CHAPTER III

LITERATURE REVIEW

As previously stated, the development of models for the man-machine chart can be divided into three areas. These areas are machine delay time considered a systematic occurrence, machine delay time considered a random occurrence, and machine delay time considered both a random and systematic occurrence. The majority of the work in model development has been devoted to the area of machine delay time considered a random occurrence.

In the area of machine delay time considered a systematic occurrence, algebraic methods are primarily used in developing the models. In this area, the most popular techniques are to develop equations, graphs, or ratios for determining the number of machines per operator and the amount of machine delay and operator delay times. The models developed by Benson, Miller, Townsend, and Cook best illustrate this type of model development (1). Benson, Miller, and Townsend used a combination of graphs, ratios and equations to determine the number of machines per operator and the amount of machine and operator delay times (2).

In the area of machine delay time considered a random occurrence, statistical methods were used to develop the models. This area of model development was pioneered by Bernstein and Wright in the 1930's. Bernstein employed the binomial expansion and Wright the telephone line delay

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* Numbers in parentheses refer to items in the Bibliography.

solution developed by T. C. Fry. The next contributions of note in this area of model development were made by Jones, O'Conner, Palm, and Fitters. Jones introduced models based on the binomial theorem in 1949 (3). In the 1950's, O'Conner popularized the use of the Ashcroft Tables for determining the number of machines per operator and the amount of machine delay time. The Ashcroft Tables were developed in England by Henry Ashcroft (4). Palm, a Swedish writer, developed models based on the procedure used for waiting line problems in telephony. These models were introduced in this country by the Journal of Industrial Engineering in 1958 (5). Fitters developed models based on the waiting line theory. Fitters has also contributed to the area of model development by analyzing the differences between the models based on machine delay time considered a random occurrence (6).

In the area of machine delay time considered both a random and systematic occurrence, algebraic and statistical methods were used to develop the models. In this area, Lomicka and Allderige have developed models (7).

Denholm and Shaughnessy have contributed to the development of models by applying different techniques to the basic models developed by others. Denholm used Jones's models in developing models based on a minimum operating cost technique (8). Shaughnessy applied the technique of marginal cost analysis to the Wright-Fry Formula for determining machine delay time (9).

In this review of the literature on models for the man-machine chart, a variety of models were found. However, no models were found for machines with unlike machining and servicing times which were based on the sequence in which the machines are initially placed into operation. Nor were models found which considered the effects that the sequence of starting the machines has on the machine delay time.

CHAPTER IV

OBJECTIVE AND METHOD

The objective of this study is to develop algebraic models that consider two machines with different machining and servicing times. The models will be based on the sequence in which the machines are initially started. These models will be used to determine operator delay time, machine delay time, cycle time, transient period, utilization of operator and machines during the period, and the number of parts produced during the period. A decision model will be developed for determining the sequence in which the machines are initially started.

The models will be restricted to the consideration of systematic machine delay time. Systematic machine delay time is the normal delay time which occurs when the sub-system of men and machines is operating according to the standard method. This means that foreign elements and avoidable delays will be excluded. Variations in operator effort level are neglected. Also, the models are to be restricted to machines that require no servicing during the machining time.

The method used in developing the models presented in this study was to identify all the possible man-machine charts for two machines with unlike machining and servicing times serviced by one operator. This included the identification of the charts for initially starting each machine first. Algebraic equations were developed for each of the man-machine charts. A rule was devised for selecting the proper models for the man multi-machine job to be analyzed. These models are called models for man multi-machine jobs.

Based on the models for man multi-machine jobs, algebraic models were developed for man multi-machine jobs for a period. Based upon the models for man multi-machine jobs for a period, a decision model was developed for determining which machine to initially place into operation first.

The models for man multi-machine jobs are based on the models for completed transient periods and cycle times. In developing the models for man multi-machine jobs for a period, it was necessary to assume that if the transient period or the cycle time cannot be completed, the delay time that would be experienced if the transient period or the cycle time were completed can be applied as a percentage. The percentage applied would be based on the percentage of the transient period or the cycle time completed. This assumption is required because to determine the exact ending point of the man-machine sub-system at the end of a period proved to be a task more complicated than the identification of all the possible man-machine charts.

Algebra was selected as the mathematical method to be used in developing the models presented herein because the author is most familiar with the use of this method and algebra is a very versatile mathematical tool.

CHAPTER V

MODELS FOR MAN MULTI-MACHINE JOBS

Symbols and Definitions

M_A. If a machine time is the largest element, it is defined as M_A. If an external work time is the largest element, the corresponding machine time is defined as M_A.

E_A. If an external work time is the largest element, it is defined as E_A. If a machine time is the largest element, the corresponding external work time is defined as E_A.

M_B. The machine time for the machine for which neither the machine time nor the external work is the largest element.

E_B. The external work time of the machine for which neither the machine time nor the external work time is the largest element.

C_A. Sub-cycle time for the machine for which either the external work time or machine time is the largest element. $C_A = M_A + E_A$.

C_B. Sub-cycle time for the machine for which neither the external work time nor the machine time is the largest element. $C_B = M_B + E_B$.

MDT. Machine delay time.

C_T. Cycle time.

T_P. Transient period.

A. Machine A.

B. Machine B.

ODT. Operator delay time.

Models

The development of models is presented in the Appendix. The method used in developing these models has necessitated their division into four categories. These categories are based on the conditions governing the use of the models. The conditions are $C_A > C_B$, $C_A = C_B$, $C_A < C_B$ and M_A the largest element and $C_A < C_B$ and E_A the largest element.

CONDITION: $C_A > C_B$

Starting Mach A First

Find the smallest N of the following:*

a. $N(C_A - C_B) - XC_B + M_A \leq E_B$
where $N = 0, 1, 2 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B + M_A > 0$

b. $N(C_A - C_B) = XC_B - M_A$
where $N = 0, 1, 2 \ldots$
$X = 1, 2, 3 \ldots$

c. $N(C_A - C_B) - XC_B < E_A$
where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B \geq 0$

(1) If $N(C_A - C_B) - XC_B + M_A \leq E_B$ is smallest, find the smallest K of the following:*

Starting Mach B First

Find the smallest N of the following:*

a. $N(C_A - C_B) - XC_B - M_B \leq E_B$
where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B - M_B > 0$

b. $N(C_A - C_B) = XC_B + M_B$
where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$

c. $N(C_A - C_B) - XC_B - M_B + E_A < E_A$
where $N = 0, 1, 2 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B - M_B + E_A \geq 0$

(1) If $N(C_A - C_B) - XC_B - M_B \leq E_B$ is smallest,

(a) $K(C_A-C_B) - VC_B - M_B \le E_B$
where $K = 1, 2, 3 \ldots$
and $V = 0, 1, 2 \ldots$
$K(C_A-C_B) - VC_B - M_B > 0$

(b) $K(C_A-C_B) = VC_B + M_B$
where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$

(c) $K(C_A-C_B) - VC_B - M_B + E_A < E_A$
where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A-C_B) - VC_B - M_B + E_A \ge 0$

1. If $K(C_A-C_B) - VC_B - M_B \le E_B$ is smallest,
$T_P = E_A + (N+X)C_B + E_B$
$C_T = (K+V+1)C_B$
MDT A for $T_P = (N+X)C_B + E_B - NC_A - M_A$
MDT B for $T_P = E_A$
ODT for $T_P = (N+X)M_B - NE_A$
ODT/Cycle = $(K+V+1)M_B - KE_A$

2. If $K(C_A-C_B) = VC_B + M_B$ is smallest,
$C_T = (N+X+K+V+1)C_B + E_A$
MDT/Cycle A = $(N+X)C_B + E_B - NC_A - M_A$
ODT/Cycle = $(N+X+K+V+1)M_B - (N+K)E_A$

$T_P = E_B$
$C_T = (N+X+1)C_B$
MDT A for $T_P = E_B$
MDT B for $T_P = 0$
MDT/Cycle A = $(N+X+1)C_B - NC_A$
MDT/Cycle B = 0
ODT for $T_P = 0$
ODT/Cycle = $(N+X+1)M_B - NE_A$

(2) If $N(C_A-C_B) = XC_B + M_B$ is smallest,

$C_T = NC_A + E_B$
MDT/Cycle A = E_B
MDT/Cycle B = 0
ODT/Cycle = $NM_A - (N+X)E_B$

(3) If $N(C_A-C_B) - XC_B - M_B + E_A < E_A$ is smallest, find the smallest K of the following:

(a) $K(C_A-C_B) - VC_B + M_A \le E_B$
where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A-C_B) - VC_B + M_A > 0$

(b) $K(C_A-C_B) = VC_B - M_A$
where $K = 0, 1, 2 \ldots$
$V = 1, 2, 3 \ldots$

3. If $K(C_A - C_B) - VC_B - M_B + E_A \leq E_A$ is smallest,

$T_P = E_A$

$C_T = (N+X)C_B + E_B + KC_A + E_A$

MDT A for $T_P = 0$

MDT B for $T_P = E_A$

MDT/Cycle A $= (N+X)C_B + E_B - NC_A - M_A$

MDT/Cycle B $= KC_A + E_A - (K+V)C_B - M_B$

ODT for $T_P = 0$

ODT/Cycle $= (N+X)M_B - NE_A + KM_A - (K+V)E_B$

(2) If $N(C_A - C_B) = XC_B - M_A$ is smallest,

$C_T = (N+1)C_A$

MDT/Cycle A $= 0$

MDT/Cycle B $= E_A$

ODT/Cycle $= (N+1)M_A - (N+X)E_B$

(3) If $N(C_A - C_B) - XC_B < E_A$ is smallest,

$T_P = E_A$

$C_T = NC_A$

MDT A for $T_P = 0$

MDT B for $T_P = E_A$

MDT/Cycle A $= 0$

MDT/Cycle B $= NC_A - (N+X)C_B$

ODT for $T_P = 0$

ODT/Cycle $= NM_A - (N+X)E_B$

(c) $K(C_A - C_B) - VC_B < E_A$

where $K = 1, 2, 3 \ldots$

$V = 0, 1, 2 \ldots$

and $K(C_A - C_B) - VC_B \geq 0$

1. If $K(C_A - C_B) - VC_B + M_A \leq E_B$ is smallest,

$T_P = E_B$

$C_T = NC_A + E_A + (K+V)C_B + E_B$

MDT A for $T_P = E_B$

MDT B for $T_P = 0$

MDT A/Cycle $= (K+V)C_B + E_B - KC_A - M_A$

MDT B/Cycle $= NC_A + E_A - (N+X)C_B - M_B$

ODT for $T_P = 0$

ODT/Cycle $= NM_A - (N+X)E_B + (K+V)M_B - KE_A$

2. If $K(C_A - C_B) = VC_B - M_A$ is smallest,

$C_T = (N+K+1)C_A + E_B$

MDT/Cycle A $= E_B$

MDT/Cycle B $= NC_A + E_A - (N+X)C_B - M_B$

ODT/Cycle $= (N+K+1)M_A - (N+X+K+V)E_B$

3. If $K(C_A - C_B) - VC_B < E_A$ is smallest,

$T_P = E_B + NC_A + E_A$

$C_T = KC_A$

MDT A for $T_P = E_B$

MDT B for $T_P = NC_A + E_A - (N+X)C_B - M_B$

MDT/Cycle A $= 0$

MDT/Cycle B $= KC_A - (K+V)C_B$

ODT for $T_P = NM_A - (N+X)E_B$

ODT/Cycle $= KM_A - (K+V)E_B$

CONDITION: $C_A = C_B$

Starting Mach A First

$T_P = E_A$
$C_T = C_A$
MDT A for $T_P = 0$
MDT B for $T_P = E_A$
MDT/Cycle A = 0
MDT/Cycle B = 0
ODT for $T_P = 0$
ODT/Cycle = $M_A - E_B$

Starting Mach B First

$T_P = E_B$
$C_T = C_B$
MDT A for $T_P = E_B$
MDT B for $T_P = 0$
MDT/Cycle A = $C_A - C_B$
MDT/Cycle B = 0
ODT for $T_P = 0$
ODT/Cycle = $M_B - E_A$

CONDITION: $C_A < C_B$ and M_A the largest Element

Find the smallest N
for $N(C_B - C_A) + E_B - M_A < E_A$
where N = 1, 2, 3 ...
and $N(C_B - C_A) + E_B - M_A > 0$

$T_P = E_A + NC_A + E_B$
$C_T = C_B$
MDT A for $T_P = NC_B + E_B - NC_A - M_A$
MDT B for $T_P = E_A$
MDT/Cycle A = $C_B - C_A$
MDT/Cycle B = 0
ODT for $T_P = N(M_B - E_A)$
ODT/Cycle = $M_B - E_A$

$T_P = E_B$
$C_T = C_A$
MDT A for $T_P = E_B$
MDT B for $T_P = 0$
MDT/Cycle A = $C_A - C_B$
MDT/Cycle B = 0
ODT for $T_P = 0$
ODT/Cycle = $M_B - E_A$

CONDITION: $C_A \leq C_B$ and E_A the largest Element

$T_P = E_A$
$C_T = E_A + E_B$
MDT A for $T_P = 0$
MDT B for $T_P = E_A$
MDT/Cycle A = $E_B - M_A$
MDT/Cycle B = $E_A - M_B$
ODT for $T_P = 0$
ODT/Cycle = 0

$T_P = E_B$
$C_T = E_B + E_A$
MDT A for $T_P = E_B$
MDT B for $T_P = 0$
MDT/Cycle A = $E_B - M_A$
MDT/Cycle B = $E_A - M_B$
ODT for $T_P = 0$
ODT/Cycle = 0

* NOTE: If two values for N or K are equal and the smallest, the smallest X or V will determine the N or K to be used.

CHAPTER VI

MODELS FOR MAN MULTI-MACHINE JOBS FOR A PERIOD

A period has been defined as the elapsed time between the initial start of the first machine and the time operations are completed by stopping each machine independently as each work piece is finished.

$$\text{Period } (P) = T_P + QC_T, \text{ where } Q = 0 \text{ if } P \leq T_P$$

It has been assumed that when the transient period or the cycle time cannot be completed during the period, the machine and operator delay times that would be experienced if the period were completed can be applied as a percentage.

Models

If $P > T_P$

1. MDT A for P = MDT A for T_P + Q(MDT A/Cycle)
2. MDT B for P = MDT B for T_P + Q(MDT B/Cycle)
3. ODT for P = ODT for T_P + Q(ODT/Cycle)
4. Parts A for P = $\dfrac{P - \text{MDT A for P}}{C_A}$
5. Parts B for P = $\dfrac{P - \text{MDT B for P}}{C_B}$
6. Percent utilization of A = $\dfrac{P - \text{MDT A for P}}{P} \times 100$
7. Percent utilization of B = $\dfrac{P - \text{MDT B for P}}{P} \times 100$
8. Percent utilization of Operator $\dfrac{P - \text{ODT for P}}{P} \times 100$

If $P \leq T_p$

1. $\text{MDT A for P} = \frac{P}{T_p} \text{ MDT A for } T_p$

2. $\text{MDT B for P} = \frac{P}{T_p} \text{ MDT B for } T_p$

3. $\text{ODT for P} = \frac{P}{T_p} \text{ ODT for } T_p$

4. $\text{Parts A for P} = \frac{\text{P - MDT A for P}}{C_A}$

5. $\text{Parts B for P} = \frac{\text{P - MDT B for P}}{C_B}$

6. $\text{Percent utilization of A} = \frac{\text{P - MDT A for P}}{P} \times 100$

7. $\text{Percent utilization of B} = \frac{\text{P - MDT B for P}}{P} \times 100$

8. $\text{Percent utilization of Operator} = \frac{\text{P - ODT for P}}{P} \times 100$

CHAPTER VII

DECISION MODEL

Many different decision models for determining which machine to start first can be developed from the models for man multi-machine jobs for a period. These decision models would depend on the assumption under which they are developed and the objectives of the firm or organization involved.

The decision model developed in this study is based on the following assumptions:

1. The objective of the firm is to obtain the required production at a minimum cost.

2. Machine and operator delay times can occur at any time during the transient period or the cycle time. Therefore, if the transient period or the cycle time cannot be completed during the period of operation, the delay time that would be experienced if the transient period or cycle time were completed can be applied as a percentage. The percentage applied would be based on the percent of the transient period or the cycle time completed.

Decision Model

The smallest delay costs for the period will determine the machine which should be placed into operation first. Compute the delay costs for the period for starting both Machine A and Machine B first, as follows:

Delay costs for P = (MDT A for P) Cost A/Hour + (MDT B for P)Cost B/Hour
+ (ODT for P) Cost of Operator/Hour

CHAPTER VIII

MODEL USAGE

The models developed in Chapters V, VI, and VII can be used to analyze the relationships between two machines with unlike machining and servicing times serviced by one operator. The procedure to be followed is presented below and an example is presented in the Appendix.

Step One - Determine the applicable model for man multi-machine jobs for starting Machine A first as outlined in Chapter V.

Step Two - From the model for man multi-machine jobs determined in Step One, determine the transient period, cycle time, machine delay time for Machines A and B for the transient period and for each cycle, and operator delay time for the transient period and for each cycle for starting Machine A first.

Step Three - Determine the machine delay time for a period for Machines A and B and the operator delay time for a period as outlined in Chapter VI for starting Machine A first.

Step Four - Determine the delay cost for a period for starting Machine A first as outlined in Chapter VII.

Step Five - Repeat Step One for starting Machine B first.

Step Six - Repeat Step Two for starting Machine B first.

Step Seven - Repeat Step Three for starting Machine B first.

Step Eight - Repeat Step Four for starting Machine B first.

Step Nine - Compare the delay costs for a period computed in Steps Four and Eight and apply the decision model outlined in Chapter VII.

Step Ten - For the machine to be started first, determine the parts for the period for Machines A and B, percent utilization of Machines A and B and the operator as outlined in Chapter VI.

CHAPTER IX

CONCLUSIONS

In this study, algebraic models were developed for man multi-machine jobs involving two machines with unlike servicing and machining times serviced by one operator. The models can be used to determine the amount of machine and operator delay times, cycle time, transient period, utilization of operator and machines during the period, and the number of parts produced during the period. The models point up the need for an awareness of the effects of the sequence in which machines with unlike machining and servicing times are initially placed into operation. A decision model was developed for determining which machine should be placed into operation first. These models were restricted to the consideration of systematic delay time and machines which require no servicing during the machining time.

The use of the models developed in this study provide a method for analyzing the relationships between the operator and the machines, without the need for constructing man-machine charts. The use of these models should reduce operating costs because they provide a method for determining the initial starting sequence which results in the lowest delay costs. The example in the Appendix demonstrates these conclusions.

CHAPTER X

DISCUSSION OF METHOD

The application of the method presented herein to develop all possible man-machine charts for starting each machine first proved to be an almost impossible task. The objective of the study was accomplished by developing the definitions for machine time and external work and by finding all the possible ending points for the charts. To apply this method to the development of models for three or more machines with unlike servicing and machining times serviced by one operator would be a difficult task. The difficulty arises in determining all the possible ending points for the charts and the number of definitions of machining and servicing times that would be required.

Developing algebraic models for each man-machine chart for two machines and a rule for selecting the proper model for a particular man-machine job presented no difficulties. It is believed that the development of models for the man-machine charts and a rule for selecting the proper model for particular man-machine jobs for a system involving three or more machines serviced by one operator would present some problems.

CHAPTER XI

RECOMMENDATIONS

This study has demonstrated the need for models for man multi-machine jobs which involve machines with unlike machining and servicing times. The industrial engineer should be aware of the effects that the sequence in which such machines are initially started has on the machine delay time. Models should be developed to determine these effects for more than two machines serviced by one operator. For the reasons previously stated, the methods used in this study are not recommended for developing such models.

The effects that random delays have on the models developed in this study should be investigated. Such an investigation may provide the key for developing models for determining the effects that the sequence of initially starting the machine has on machine delay time for more than two machines serviced by one operator.

APPENDIX

Model Development

CONDITION I: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$, and $C_A > C_B$

Starting Mach A First

```
                                                             4 3 2 1  B A
Mach A : E :      M          :E: . . . :E: M  :E:. . . :E:  M  : : : :  :E:

       E_A
Mach B : x : E : M  : E : M  :...: M  :...: M  : E :  M :...: M  :E  : M  : E : M  :
```

If M_A is the largest element and $E_A < M_B$, the system will continue to cycle under the following condition:

A. E_A ends before Point "A" and M_A ends between Point "1" and Point "B".

The system will cycle until one of the following conditions occur:

1. M_A and E_B end at the same time.
2. M_A ends before E_B.
3. M_A and M_B end at the same time.
4. M_A ends before M_B, and (A) E_A and M_B end at the same time, or (B) M_B ends before E_A.

If M_A is the largest element and $E_A > M_B$, Condition 4B will apply and the cycle will terminate at the end of the first E_A. If E_A is the largest element and $M_A > E_B$, Condition 4B will apply and the cycle will terminate at the end of the first E_A. If E_A is the largest element and $M_A < E_B$, Condition 2 will apply and the cycle will terminate at the end of the first E_B. (Reference Note IIA.)

Condition 1: M_A and E_B end at the same time.

P_1 P_2

Mach A :E: M :E: :E: M :E:... : M :

E_A

Mach B :x: E : M : E : M :...: M :...: M : E : M :...: M : E :

:——————————P_1P_2——————————:

Find $N(C_A - C_B) - XC_B + M_A = E_B$ where N = 0, 1, 2 ...
x = 1, 2, 3 ...

$P_1P_2 = (N + X)C_B + E_B$ MDT A for $P_1P_2 = 0$

ODT for $P_1P_2 = (N+X)M_B - NE_A$ MDT B for $P_1P_2 = 0$

The first point at which M_A can end, therefore, $N \geq 0$ and $X \geq 1$.

Condition 2: M_A ends before E_B.

P_1 P_2

Mach A :E: M :E: . . . :E: M :E:... : M :x:
MDT

E_A

Mach B :x: E : M : E : M :...: M :...: M : E : M :...: M : E :

:——————————P_1P_2——————————:

Find $N(C_A - C_B) - XC_B + M_A < E_B$ where N = 0, 1, 2 ...
X = 0, 1, 2 ...
and $N(C_A - C_B) - XC_B + M_A > 0$

$P_1P_2 = (N + X)C_B + E_B$ MDT A for $P_1P_2 = (N + X)C_B + E_B - NC_A - M_A$

ODT for $P_1P_2 = (N+X)M_B - NE_A$ MDT B for $P_1P_2 = 0$

NOTE: If E_A is the largest element and $E_B > M_A$, the cycle would terminate at the end of the first E_B, therefore, $N \geq 0$ and $X \geq 0$.

Summary of Conditions 1 and 2 for P_1P_2.

Find $N(C_A - C_B) - XC_B + M_A \leq E_B$ where $N = 0, 1, 2 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B + M_A > 0$

$P_1P_2 = (N + X)C_B + E_B$ MDT A for $P_1P_2 = (N + X)C_B + E_B - NC_A - M_A$

ODT for $P_1P_2 = (N+X)M_B - NE_A$ MDT B for $P_1P_2 = 0$

Conditions 1 and 2 after P_2.

```
           P2                                                            Z Y X W  2 1
Mach A :E:        M           :E: . . . :E:  M  :E:. . . :E:   M     : : : :  :E:

Mach B : M  : E : M  : E : M  :...:  M :...: M  : E : M  :...: M  : E : M  : E : M  :
```

If M_A is the largest element and $E_A < M_B$, the system will continue to cycle under the following condition:

A. E_A ends before Point "1" and M_A ends between Point "W" and Point "2".

The system will cycle until one of the following conditions occur:

W. M_A and E_B end at the same time.

X. M_A ends before E_B.

Y. M_A and M_B end at the same time.

Z. M_A ends before M_B, and (1) E_A and M_B end at the same time, or (2) M_B ends before E_A.

If E_A is the largest element or if M_A is the largest element and $E_A > M_B$, Condition Z2 will apply at the end of the first E_A and the cycle will terminate. (Reference Note IIB).

Condition W for Conditions 1 and 2 after P_2, M_A and E_B end at the same time.

```
       P2                                                    P3
Mach A :E:     M            :E: . . . :E:  M  :E:. . .:    M    :
                                *
Mach B : M  : E : M  : E :...: M  :...: M  : E : M  :...: M  : E :

       : ——————————————————— P2P3 ——————————————————————————————:
```

Find $K(C_A - C_B) - VC_B - M_B = E_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$

$P_2P_3 = (K + V + 1)C_B$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = (K+V+1)M_B - KE_A$ MDT B for $P_2P_3 = 0$

* The first point at which M_A can end, therefore, $K \geq 1$ and $V \geq 0$.

Condition X for Conditions 1 and 2 after P_2, M_A ends before E_B.

```
       P2                                                    P3
Mach A :E:       M          :E: . . . :E:  M  :E:...:    M     :x:
                             *                                 MDT
Mach B : M  : E :  M : E :...:  M :...: M  :E  : M  :...: M  : E :

       : ———————————————————P2P3 ———————————————————————————————:
```

Fine $K(C_A - C_B) - VC_B - M_B < E_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B > 0$

$P_1P_2 = (K + V + 1)C_B$ MDT A for $P_2P_3 = (K + V + 1)C_B - KC_A$

ODT for $P_2P_3 = (K+V+1)M_B - KE_A$ MDT B for $P_2P_3 = 0$

* The first point at which M_A can end, therefore, $K \geq 1$ and $V \geq 0$.

Summary of Conditions W and X for Conditions 1 and 2 after P_2.

Find $K(C_A - C_B) - VC_B - M_B \leq E_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B > 0$

$P_2P_3 = (K + V + 1)C_B$ MDT A for $P_2P_3 = (K + V + 1)C_B - KC_A$

ODT for $P_2P_3 = (K+V+1)M_B - KE_A$ MDT B for $P_2P_3 = 0$

Condition Y for Conditions 1 and 2 after P_2, M_A and M_B end at the same time.

```
        P2                                                              P3
Mach A :E:          M           :E: . . . :E:  M  :E:. . .:  M  :

Mach B : M  : E :  M : E : M  :...:  M :...:  M : E :  M :...: M  :

        :------------------P2P3 --------------------------:
```

Find $K(C_A - C_B) = VC_B + M_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$

$P_2P_3 = (K + V)C_B + M_B$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = (K+V+1)M_B - KE_A$ MDT B for $P_2P_3 = 0$

The first point at which M_A can end, therefore, $K \geq 1$ and $V \geq 0$.

Condition Z1 for Conditions 1 and 2 after P_2, M_A ends before M_B and E_A and M_B end at the same time.

```
      P2                                                      P3
Mach A:E:       M              :E: . . . :E:  M  :E:...:   M  :E:

Mach B: M  : E : M  : E :....: M  :....: M  : E : M   :....: M  :

      :________________P2P3________________________________:
```

Find $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ where $K = 1, 2, 3 \ldots$

$V = 0, 1, 2 \ldots$

and $K(C_A - C_B) - VC_B - M_B + E_A = 0$

$P_2P_3 = KC_A + E_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = KM_A - (K+V)E_B$ MDT B for $P_2P_3 = 0$

The first point at which E_A can end, therefore, $K \geq 1$ and $V \geq 0$.

Condition Z2 for Conditions 1 and 2 after P_2, M_A ends before M_B and M_B ends before E_A.

```
      P2                                                       P3
Mach A :E:       M            :E:       :E: M  :E: . . .:  M :  E :
                                                               MDT
Mach B : M  : E : M  : E :....:  M :....: M  : E : M  : . . .: M  :x:

      :______________________P2P3__________________________:
```

Find $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ where $K = 0, 1, 2 \ldots$

$V = 0, 1, 2 \ldots$

and $K(C_A - C_B) - VC_B - M_B + E_A > 0$

$P_2P_3 = KC_A + E_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = KM_A - (K+V)E_B$ MDT B for $P_2P_3 = KC_A + E_A - (K + V)C_B - M_B$

NOTE: If E_A is the largest element or $E_A > M_B$, the cycle would terminate at the end of the first E_A, therefore, $K \geq 0$ and $V \geq 0$.

Summary of Conditions Z1 and Z2 for Conditions 1 and 2 after P_2.

Find $K(C_A - C_B) - VC_B - M_B + E_A > E_A$ where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B + E_A \geq 0$

$P_2P_3 = KC_A + E_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = KM_A - (K+V)E_B$ MDT B for $P_2P_3 = KC_A + E_A - (K + V)C_B - M_B$

Summary of Conditions 1 and 2.

Find the smallest K of the following:

1. $K(C_A - C_B) - VC_B - M_B \leq E_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B > 0$

2. $K(C_A - C_B) = VC_B + M_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$

3. $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B + E_A \geq 0$

If $K(C_A - C_B) - VC_B - M_B \leq E_B$ is smallest

```
           P1                                   P2                                  P3
Mach A :E:          M          :E:....:   M  :x:E:         M         :E: . . .:  M    :x:
       EA                                   MDT
Mach B :x: E : M  : E :  M :...: M  :...: M  : E : M  : E :  M :...:  M : . . .: M  : E :
       :______________Tp________________________:__________________________________:
```

$T_P = E_A + P_1P_2 = E_A + (N + X)C_B + E_B$ ODT for T_P = ODT for $P_1P_2 = (N + X)M_B - NE_A$

$C_T = P_2P_3 = (K + V + 1)C_B$ ODT/Cycle = ODT for $P_2P_3 = (K + V + 1)M_B - KE_A$

MDT A for T_P = MDT A for $P_1P_2 = (N + X)C_B + E_B - NC_A - M_A$

MDT B for $T_P = E_A$ + MDT B for $P_1P_2 = E_A$

MDT/Cycle A = MDT A for $P_2P_3 = (K + V + 1)C_B - KC_A$

MDT/Cycle B = MDT B for $P_2P_3 = 0$

If $K(C_A - C_B) = VC_B + M_B$ is smallest

```
           P1                                    P2                                 P3
Mach A :E:           M           :E:...:   M   :x:E:        M          :E:. . .:   M    :
       EA                                    MDT
Mach B :x:  E :  M : E : M   :...: M   :...:  M : E : M   : E :  M :...:  M : . . .:  M :
       :_________________________CT______________________________________________:
```

$C_T = E_A + P_1P_2 + P_2P_3 = E_A + (N + X)C_B + E_B + (K + V)C_B + M_B = (N + X + K + V + L)C_B + E_A$

MDT/Cycle A = MDT A for P_1P_2 + MDT A for $P_2P_3 = (N + X)C_B + E_B - NC_A - M_A$

MDT/Cycle B = E_A + MDT B for P_1P_2 + MDT B for $P_2P_3 = E_A$

ODT/Cycle = ODT for P_1P_2 + ODT for $P_2P_3 = (N + X)M_B - NE_A + (K + V + 1)M_B - KE_A = (N + X + V + 1)M_B - (N + K)E_A$

If $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ is smallest

```
          P1                                          P2                                    P3
Mach A :E :        M                :E:        M    :x:  E:      M           :E: . . .:  M  : E:
                                                    MDT
       EA                                                                                   MDT
Mach B :xx: E : M  : E :  M :...: M  :...: M  : E : M  :E  :  M :...: M  :. . . : M  :x:

       :Tp:---------------------------------  CT -----------------------------------------:
```

$T_P = E_A$ MDT A for $T_P = 0$ ODT for $T_P = 0$

MDT B for $T_P = E_A$ ODT/Cycle = ODT for P_1P_2 + ODT for $P_2P_3 = (N+X)M_B - NE_A + KM_A - (K+V)E_B$

$C_T = P_1P_2 + P_2P_3 = (N + X)C_B + E_B + KC_A + E_A$

MDT/Cycle A = MDT A for P_1P_2 + MDT A for $P_2P_3 = (N + X)C_B + E_B - NC_A - M_A$

MDT/Cycle B = MDT B for P_1P_2 + MDT B for $P_2P_3 = KC_A + E_A - (K + V)C_B - M_B$

Condition 3: M_A and M_B end at the same time.

```
Mach A :E:             M             :E: . . .:      :

       EA
Mach B :x: E : M  *: E : M  : E :...: M  : . . .: M  :

       :---------------CT-----------------------------:
```

Find $N(C_A - C_B) = XC_B - M_A$ where $N = 0, 1, 2 \ldots$
$X = 1, 2, 3 \ldots$

$C_T = (N + 1)C_A$ MDT/Cycle A = 0 ODT/Cycle $= (N + 1)M_A - (N + X)E_B$

MDT/Cycle B = E_A

*The first point at which M_A can end, therefore, $N \geq 0$ and $X \geq 1$.

Condition 4A: M_A ends before M_B and E_A and M_B end at the same time.

```
Mach A :E:        M                :E:. . .:  M  :E:

       EA
Mach B :x: E : M  : E : M  *:...: M  : . . .: M  :

       :TP:___________CT______________________:
```

Find $N(C_A - C_B) - XC_B < E_A$ where $N = 1, 2, 3 \ldots$
$X = 1, 2, 3 \ldots$
and $N(C_A - C_B) - XC_B = 0$

$T_P = E_A$ MDT A for $T_P = 0$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = NC_A$ MDT B for $T_P = E_A$ MDT/Cycle B = 0 ODT/Cycle = $NM_A - (N + X)E_B$

*The first point at which E_A can end, therefore $N \geq 1$ and $X \geq 1$.

Condition 4B: M_A ends before M_B and M_B ends before E_A.

```
Mach A :E :        M                :E :. . .:  M  :E :

       EA                                      MDT
Mach B :xx: E : M  : *E : M  :...: M  : . . .: M  :x:

       :TP_____________CT_______________________:
```

Find $N(C_A - C_B) - XC_B < E_A$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B > 0$

$T_P = E_A$ MDT A for $T_P = 0$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = NC_A$ MDT B for $T_P = E_A$ MDT/Cycle B = $NC_A - (N + X)C_B$ ODT/Cycle = $NM_A - (N + X)E_B$

*The first point at which E_A can end, therefore, $N \geq 1$ and $X \geq 0$.

Summary of Conditions 4A and 4B

Find $N(C_A - C_B) - XC_B < E_A$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B \geq 0$

$T_P = E_A$ MDT A for $T_P = 0$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = NC_A$ MDT B for $T_P = E_A$ MDT/Cycle B = $NC_A - (N+X)C_B$ ODT/Cycle = $NM_A - (N+X)E_B$

Summary of Condition I: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$, and $C_A > C_B$.

Starting Mach A First

Find the smallest N of the following:

1. $N(C_A - C_B) - XC_B + M_A \leq E_B$ where $N = 0, 1, 2 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B + M_A > 0$

2. $N(C_A - C_B) = XC_B - M_A$ where $N = 0, 1, 2 \ldots$
$X = 1, 2, 3 \ldots$

3. $N(C_A - C_B) - XC_B < E_A$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B \geq 0$

1. If $N(C_A - C_B) - XC_B + M_A \leq E_B$ is smallest

Find the smallest K of the following: *

A. $K(C_A - C_B) - VC_B - M_B \leq E_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B > 0$

B. $K(C_A - C_B) = VC_B + M_B$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$

C. $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B - M_B + E_A \geq 0$

A. If $K(C_A - C_B) - VC_B - M_B \leq E_B$ is smallest

$T_P = E_A + (N + X)C_B + E_B$ ODT for $T_P = (N + X)M_B - NE_A$

$C_T = (K + V + 1)C_B$ ODT/Cycle $= (K + V + 1)M_B - KE_A$

MDT A for $T_P = (N + X)C_B + E_B - NC_A - M_A$ MDT/Cycle A $= (K + V + 1)C_B - KC_A$

MDT B for $T_P = E_A$ MDT/Cycle B $= 0$

B. If $K(C_A - C_B) = VC_B + M_B$ is smallest

$C_T = (N + X + K + V + 1)C_B + E_A$ ODT/Cycle $= (N + X + K + V + 1)M_B - (N + K)E_A$

MDT/Cycle A $= (N + X)C_B + E_B - NC_A - M_A$

MDT/Cycle B $= E_A$

C. If $K(C_A - C_B) - VC_B - M_B + E_A < E_A$ is smallest

$T_P = E_A$ ODT for $T_P = 0$

$C_T = (N + X)C_B + E_B + KC_A + E_A$ ODT/Cycle $= (N + X)M_B - NE_A + KM_A - (K + V)E_B$

MDT A for $T_P = 0$ MDT/Cycle A $= (N + X)C_B + E_B - NC_A - M_A$

MDT B for $T_P = E_A$ MDT/Cycle B $= KC_A + E_A - (K + V)C_B - M_B$

2. If $N(C_A - C_B) = XC_B - M_A$ is smallest

$C_T = (N + 1)C_A$ ODT/Cycle $= (N + 1)M_A - (N + X)E_B$

MDT/Cycle A = 0

MDT/Cycle B $= E_A$

3. If $N(C_A - C_B) - XC_B < E_A$ is smallest

$T_P = E_A$ ODT for $T_P = 0$

$C_T = NC_A$ ODT/Cycle $= NM_A - (N + X)E_B$

MDT A for $T_P = 0$ MDT/Cycle A $= 0$

MDT B for $T_P = E_A$ MDT/Cycle B $= NC_A - (N + X)C_B$

* NOTE: If two values for N or K are equal and the smallest, the smallest X or V will determine the N or K to be used.

CONDITION I: M_A or E_A is the largest element,
$M_A \neq E_A \neq M_B \neq E_B$, and $C_A > C_B$

Starting Mach B First

```
Mach B : E : M  : E : M  :...: M  :...: M  : E : M  :...: M  : E : M  : E :  M :

        E_B                                                     4 3 2 1  B A
Mach A :xxx:        M        :E: . . . :E:  M  :E: . . .:E:   M    : : : :  :E:
```

If M_A is the largest element and $E_A < M_B$, the system will continue to cycle under the following condition:

A. E_A ends before Point "A" and M_A ends between Point "1" and Point "B".

The system will cycle until one of the following conditions occur:

1. M_A and E_B end at the same time.
2. M_A ends before E_B.
3. M_A and M_B end at the same time.
4. M_A ends before M_B, and (A) E_A and M_B end at the same time, or (B) M_B ends before E_A.

If E_A is the largest element or if M_A is the largest element and $E_A > M_B$, Condition 4B will apply and the cycle will terminate at the end of the first E_A. (Reference Note IA.).

Condition 1: M_A and E_B end at the same time.

```
Mach B : E : M  : E : M  : E *:...: E :  M :...: M  : E : M  :...: M  : E :

          E_B
Mach A :xxx:E:           M           :E: . . . :E:  M  :E:. . .:    M   :

       :T_P-:-------------------------- C_T ----------------------------:
```

Find $N(C_A - C_B) - XC_B - M_B = E_B$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = (N + X + 1)C_B$ MDT B for $T_P = 0$ MDT/Cycle B = 0 ODT/Cycle = $(N+X+1)M_B - NE_A$

*The first point at which M_A can end, therefore $N \geq 1$ and $X \geq 0$.

Condition 2: M_A ends before E_B.

```
Mach B : E :  M : E : M  : E* :...: E : M  :...: M  : E : M  :...: M  : E :

          E_B                                                       MDT
Mach A :xxx:E:           M           :E: . . . :E:  M  :E:...:  M   :x:

       :T_P-:----------------------- C_T -------------------------------:
```

Find $N(C_A - C_B) - XC_B - M_B < E_B$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B - M_B > 0$

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A = $(N+X+1)C_B - NC_A$ ODT for $T_P = 0$

$C_T = (N + X + 1)C_B$ MDT B for $T_P = 0$ MDT/Cycle B = 0 ODT/Cycle = $(N+X+1)M_B - NE_A$

*The first point at which M_A can end, therefore $N \geq 1$ and $X \geq 0$.

Summary of Conditions 1 and 2.

Find $N(C_A - C_B) - XC_B - M_B \leq E_B$ where $N = 1, 2, 3, 4 \ldots$
$X = 0, 1, 2, 3 \ldots$
and $N(C_A - C_B) - XC_B - M_B > 0$

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A = $(N+X+1)C_B - NC_A$ ODT for $T_P = 0$

$C_T = (N + X + 1)C_B$ MDT B for $T_P = 0$ MDT/Cycle B = 0 ODT/Cycle = $(N+X+1)M_B - NE_A$

Condition 3: M_A and M_B end at the same time.

```
                                 *
Mach B : E :  M : E :  M :...: M  :...: M  : E :  M :...:  M :


Mach A :xxx:E:       M       :E: . . . :E:  M  :E:...:  M   :

       :------------------------ C_T ---------------------------:
```

Find $N(C_A - C_B) = XC_B + M_B$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$

$C_T = NC_A + E_B$ MDT/Cycle A = E_B ODT/Cycle = $NM_A - (N + X)E_B$

MDT/Cycle B = 0

*The first point at which M_A can end, therefore $N \geq 1$ and $X \geq 0$.

Condition 4A: M_A ends before M_B and E_A and M_B end at the same time.

```
             P1                                                   P2
Mach B : E :  M : E : M  *:...:  M :...: M  : E :  M : . . .:  M :

        EB
Mach A :xxx:E:     M        :E: . . . :E:  M  :E:. . .:  M  :E:

          :--------------P1P2---------------------------:
```

Find $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where $N = 1, 2, 3 \ldots$
$X = 0, 1, 2 \ldots$
and $N(C_A - C_B) - XC_B - M_B + E_A = 0$

$P_1P_2 = NC_A + E_A$ MDT A for $P_1P_2 = 0$

ODT for $P_1P_2 = NM_A - (N+X)E_B$ MDT B for $P_1P_2 = 0$

*The first point at which E_A can end, therefore $N \geq 1$ and $X \geq 0$.

Condition 4B: M_A ends before M_B and M_B ends before E_A.

```
             P1                                                 P2
Mach B : E :  M : E : M  :...:  M :...:  M : E : M  :...: M  :x:
        EB                                                 MDT
Mach A :xxx:E:     M        :E: . . . :E:  M  :E:...:  M  :E :

          :-----------------P1P2------------------------:
```

Find $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where $N = 0, 1, 2 \ldots$
$X = 0, 1, 2 \ldots$
and $N(CA - C_B) - XC_B - M_B + E_A > 0$

$P_1P_2 = NC_A + E_A$ MDT A for $P_1P_2 = 0$

ODT for $P_1P_2 = NM_A - (N+X)E_B$ MDT B for $P_1P_2 = NC_A + E_A - (N + X)C_B - M_B$

NOTE: If E_A is the largest element or $E_A > M_B$, the cycle would terminate at the end of the first E_A, therefore $N \geq 0$ and $X \geq 0$.

Summary of Conditions 4A and 4B for P_1P_2.

Find $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where $N = 0, 1, 2 \ldots$

$X = 0, 1, 2 \ldots$

and $N(C_A - C_B) - XC_B - M_B + E_A \geq 0$

$P_1P_2 = NC_A + E_A$ MDT A for $P_1P_2 = 0$

ODT for $P_1P_2 = NMA - (N+X)E_B$ MDT B for $P_1P_2 = NC_A + E_A - (N + X)C_B - M_B$

Conditions 4A and 4B after P_2.

```
Mach B : E :  M :...: M :...: M  : E :  M :...:  M : E : M  : E : M  :

                                                    Z Y X W  R S
Mach A :    M      :E: . . . :E:  M  :E: . . .:E:  M    : : : :  :E:
```

If M_A is the largest element and $E_A < M_B$, the system will continue to cycle under the following condition:

A. E_A ends before Point "S", and M_A ends between Point "W" and Point "R".

The system will cycle until one of the following conditions occur:

W. M_A and E_B end at the same time.

X. M_A ends before E_B.

Y. M_A and M_B end at the same time.

Z. M_A ends before M_B, and (A) E_A and M_B and at the same time, or (B) M_B ends before E_A.

If E_A is the largest element and $M_A > E_B$ or if M_A is the largest element and $E_A > M_B$, Condition ZB would apply and the cycle will terminate at the end of the first E_A. If E_A is the largest element and $M_A < E_B$, Condition X would apply and the cycle would terminate at the end of the first E_B. (Reference Note IB.)

Condition W for Conditions 4A and 4B after P_2, M_A and E_B end at the same time.

```
       P2                                                  P3
Mach B : E : M  : E *: M  :...:  M :...:  M : E : M  : . . .: M  : E :

Mach A :         M              :E: . . . :E:  M  :E:. . .:    M     :

       :------------------------ P2P3 -----------------------------:
```

Find $K(C_A - C_B) - VC_B + M_A = E_B$ where $K = 0, 1, 2 \ldots$
$V = 1, 2, 3 \ldots$

$P_2P_3 = (K + V)C_B + E_B$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = (K+V)M_B - KE_A$ MDT B for $P_2P_3 = 0$

*The first point at which M_A can end, therefore $K \geq 0$ and $V \geq 1$.

Condition X for Conditions 4A and 4B after P_2, M_A ends before E_B.

```
         P2                                                       P3
Mach B : E :  M : E : M  :...: M  :...: M  : E : M : . . .:  M : E :

                                                                 MDT
Mach A :        M          :E: . . . :E:  M  :E:. . .:     M   :x:

         :-------------------- P2P3 -----------------------------:
```

Find $K(C_A - C_B) - VC_B + M_A < E_B$ where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B + M_A > 0$

$P_2P_3 = (K + V)C_B + E_B$ MDT A for $P_2P_3 = (K + V)C_B + E_B - KC_A - M_A$

ODT for $P_2P_3 = (K+V)M_B - KE_A$ MDT B for $P_2P_3 = 0$

NOTE: If E_A is the largest element and $M_A < E_B$, the cycle will terminate at the end of the first E_B, therefore $K \geq C$

Summary of Conditions W and X for Conditions 4A and 4B after P_2.

Find $K(C_A - C_B) - VC_B + M_A \leq E_B$ where $K = 0, 1, 2, 3 \ldots$
$V = 0, 1, 2, 3 \ldots$
and $K(C_A - C_B) - VC_B + M_A > 0$

$P_2P_3 = (K + V)C_B + E_B$ MDT A for $P_2P_3 = (K + V)C_B + E_B - KC_A - M_A$

ODT for $P_2P_3 = (K+V)M_B - KE_A$ MDT B for $P_2P_3 = 0$

Condition Y for Conditions 4A and 4B after P_2, M_A and M_B end at the same time.

```
       P2                                                      P3
Mach B : E :  M *: E :  M :...:  M :...: M  : E : M  :. . .: M  :

Mach A :       M             :E: . . . :E:  M  :E:. . .:   M  :

       :------------------- P2P3 --------------------------:
```

Find $K(C_A - C_B) = VC_B - M_A$ where $K = 0, 1, 2 \ldots$
$V = 1, 2, 3 \ldots$

$P_2P_3 = KC_A + M_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = (K+1)M_A - (K+V)E_B$ MDT B for $P_2P_3 = 0$

*The first point at which M_A can end, therefore $K \geq 0$ and $V \geq 1$.

Condition ZA for Conditions 4A and 4B after P_2, M_A ends before M_B and E_A and M_B end at the same time.

```
       P2                                                       P3
Mach B : E :  M : E :  M *:...: M  :...: M  : E : M  : . . .: M  :

Mach A :         M           :E: . . . :E:  M  :E:. . .:  M  :E:

       :------------------ P2P3 ----------------------------:
```

Find $K(C_A - C_B) - VC_B < E_A$ where $K = 1, 2, 3 \ldots$
$V = 1, 2, 3 \ldots$
and $K(C_A - C_B) - VC_B \geq 0$

$P_2P_3 = KC_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = KM_A - (K+V)E_B$ MDT B for $P_2P_3 = 0$

*The first point at which E_A can end, therefore $K \geq 1$ and $V \geq 1$.

Condition ZB for Conditions 4A and 4B after P_2, M_A ends before M_B and M_B ends before E_A.

```
        P2                                                           P3
Mach B : E : M  : E* :  M :...: M  :...: M  : E : M  :. . .: M  :x:
                                                                MDT

Mach A :            M           :E: . . . :E:  M  :E:. . .:  M   :E :

       :-------------------------P2P3-----------------------------:
```

Find $K(C_A - C_B) - VC_B < E_A$ where $K = 1, 2, 3 \ldots$
$V = 0, ., 2 \ldots$
and $K(C_A - C_B) - VC_B > 0$

$P_2P_3 = KC_A$ MDT A for $P_2P_3 = 0$

ODT for $P_2P_3 = KM_A - (K+V)E_B$ MDT B for $P_2P_3 = KC_A - (K + V)C_B$

*The first point at which E_A can end, therefore, $K \geq 1$ and $V \geq 0$.

Summary of Conditions ZA and ZB for Conditions 4A and 4B after P_2.

Find $K(C_A - C_B) - VC_B < E_A$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B \geq 0$

$P_2P_3 = KC_A$ MDT A for $P_2P_3 = 0$

ODT $= KM_A - (K+V)E_B$ MDT B for $P_2P_3 = KC_A - (K + V)C_B$

Summary of Conditions 4A and 4B.

Find the smallest K of the following:

1. $K(C_A - C_B) - VC_B + M_A \leq E_B$ where $K = 0, 1, 2 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B + M_A > 0$

2. $K(C_A - C_B) = VC_B - M_A$ where $K = 0, 1, 2 \ldots$
$V = 1, 2, 3 \ldots$

3. $K(C_A - C_B) - VC_B < E_A$ where $K = 1, 2, 3 \ldots$
$V = 0, 1, 2 \ldots$
and $K(C_A - C_B) - VC_B \geq 0$

If $K(C_A - C_B) - VC_B + M_A \leq E_B$ is smallest

```
            P1                                  P2                                       P3
Mach B : E : M  : E : M  :...: M  :...: M  :x: E : M  : E : M  :...: M  : . . .: M  : E :
                                           MDT
       EB                                                                            MDT
Mach A :xxx: E:      M       :E:...:  M  : E:       M            :E :. . .:  M    :x:

       :Tp-:---------------------------- CT -------------------------------------------:
```

$T_P = E_B$ MDT A for $T_P = E_B$

$C_T = P_1P_2 + P_2P_3 = NC_A + E_A + (K + V)C_B + E_B$ MDT B for $T_P = 0$

MDT A/Cycle = MDT A for P_1P_2 + MDT A for P_2P_3 = $(K + V)C_B + E_B - KC_A - M_A$

MDT B/Cycle = MDT B for P_1P_2 + MDT B for P_2P_3 = $NC_A + E_A - (N + X)C_B - M_B$

ODT for $T_P = 0$

ODT/Cycle = ODT for P_1P_2 + ODT for P_2P_3 = $NM_A - (N + X)E_B + (K + V)M_B - KE_A$

If $K(C_A - C_B) = VC_B - M_A$ is smallest

```
            P1                                     P2                                P3
Mach B : E : M   : E : M   :...:  M : . . .: M :x: E : M   : E : M   :...: M   :. . .:  M :
                                                MDT

Mach A :xxx:E :      M          :E :. . .:  M   :E :         M              :E :. . .:   M  :

       :---------------------------- CT ------------------------------------------:
```

$C_T = E_B + P_1P_2 + P_2P_3 = E_B + NC_A + E_A + KC_A + M_A = (N + K + 1)C_A + E_B$

MDT/Cycle A $= E_B$ + MDT A for P_1P_2 + MDT A for $P_2P_3 = E_B$

MDT/Cycle B = MDT B for P_1P_2 + MDT B for $P_2P_3 = NC_A + E_A - (N + X)C_B - M_B$

ODT/Cycle = ODT for P_1P_2 + ODT for $P_2P_3 = NM_A-(N+X)E_B+(K+L)M_A-(K+V)E_B=(N+K+1)M_A-(N+X+K+V)E_B$

If $K(C_A - C_B) - VC_B < E_A$ is smallest

```
            P1                                        P2                                   P3
Mach B : E : M   : E : M   :...: M   : . . .: M   :x: E : M   : E : M   :...: M   :. . .: M   :x:
       EB                                          MDT                                     MDT
Mach A :xxx:E :      M          :E :. . .:  M  :E :          M              :E :. . .:  M  :E :

       :--------------- TP ---------------------:------------------ CT -------------------:
```

$T_P = E_B + P_1P_2 = E_B + NC_A + E_A$ ODT for T_P = ODT for $P_1P_2 = NM_A - (N + X)E_B$

$C_T = P_2P_3 = KC_A$ ODT/Cycle - ODT for $P_2P_3 = KM_A - (K + V)E_B$

MDT A for $T_P = E_B$ + MDT A for $P_1P_2 = E_B$

MDT B for T_P = MDT B for $P_1P_2 = NC_A + E_A - (N + X)C_B - M_B$

MDT/Cycle A = MDT A for $P_2P_3 = 0$

MDT/Cycle B = MDT B for $P_2P_3 = KC_A - (K + V)C_B$

Summary of Condition I: M_A or E_A is largest element, $M_A \neq M_B \neq E_A \neq E_B$ and $C_A > C_B$

Starting Mach B First

Find the smallest N of the following:*

1. $N(C_A - C_B) - XC_B - M_B \leq E_B$ where $N = 1, 2, 3 \ldots$
   $X = 0, 1, 2 \ldots$
   and $N(C_A - C_B) - XC_B - M_B > 0$

2. $N(C_A - C_B) = XC_B + M_B$ where $N = 1, 2, 3 \ldots$
   $X = 0, 1, 2 \ldots$

3. $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where $N = 0, 1, 2 \ldots$
   $X = 0, 1, 2 \ldots$
   and $N(C_A - C_B) - XC_B - M_B + E_A \geq 0$

If $N(C_A - C_B) - XC_B - M_B \leq E_B$ is smallest

$T_p = E_B$ MDT A for $T_p = E_B$ MDT/Cycle A $= (N + X + 1)C_B - NC_A$

$C_T = (N + X + 1)C_B$ MDT B for $T_p = 0$ MDT/Cycle B $= 0$

ODT for $T_p = 0$

ODT/Cycle $= (N + X + 1)M_B - NE_A$

If $N(C_A - C_B) = XC_B + M_B$ is smallest

$C_T = NC_A + E_B$ $\quad$ MDT/Cycle A $= E_B$

ODT/Cycle $= NM_A - (N+X)E_B$ $\quad$ MDT/Cycle B $= 0$

If $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ is smallest.

Find the smallest K of the following:

1. $K(C_A - C_B) - VC_B + M_A \leq E_B$ $\quad$ where $K = 0, 1, 2 \ldots$
   $V = 0, 1, 2 \ldots$
   and $K(C_A - C_B) - VC_B + M_A > 0$

2. $K(C_A - C_B) = VC_B - M_A$ $\quad$ where $K = 0, 1, 2 \ldots$
   $V = 1, 2, 3 \ldots$

3. $K(C_A - C_B) - VC_B < E_A$ $\quad$ where $K = 1, 2, 3 \ldots$
   $V = 0, 1, 2 \ldots$
   and $K(C_A - C_B) - VC_B \geq 0$

If $K(C_A - C_B) - VC_B + M_A \leq E_B$ is smallest

$T_P = E_B$ $\quad$ MDT A for $T_P = E_B$ $\quad$ ODT for $T_P = 0$

$C_T = NC_A + E_A + (K + V)C_B + E_B$ $\quad$ MDT B for $T_P = 0$ $\quad$ ODT/Cycle-$NM_A-(N+X)E_B+(K+V)M_B-KE_A$

MDT A/Cycle $= (K + V)C_B + E_B - KC_A - M_A$

MDT B/Cycle $= NC_A + E_A - (N + X)C_B - M_B$

If $K(C_A - C_B) = VC_B - M_A$ is smallest

$C_T = (N + K + 1)C_A + E_B$ MDT/Cycle $A = E_B$ ODT/Cycle $= (N+K+1)M_A - (N+X+K+V)E_B$

MDT/Cycle $B = NC_A + E_A - (N + X)C_B - M_B$

If $K(C_A - C_B) - VC_B < E_A$ is smallest

$T_P = E_B + NC_A + E_A$ MDT A for $T_P = E_B$ ODT for $T_P = NM_A - (N + X)E_B$

$C_T = KC_A$ MDT B for $T_P = NC_A + E_A - (N + X)C_B - M_B$ ODT/Cycle $= KM_A - (K + V)E_B$

MDT/Cycle $A = 0$

MDT/Cycle $B = KC_A - (K + V)C_B$

NOTE: If two values for N or K are equal and the smallest, the smallest X or V will determine the N or K to be used.

CONDITION II: M_A or E_A is the largest element,
$M_A \neq M_B \neq E_A \neq E_B$, and $C_A \leq C_B$

Starting Mach A First

Condition 1: M_A is the largest element, $E_A < M_B$ and $C_A \leq C_B$

```
                    B A
Mach A : E :  M   :E: :

        E_A
Mach B :xxx:  E  : M  :
```

NOTE: E_A cannot be greater than Point "A" because $C_A \leq C_B$.

Conditions A. $C_A = C_B$

B. $C_A < C_B$

Condition 1A: M_A is the largest element, $E_A < M_B$ and $C_A = C_B$.

```
Mach A : E :   M  : E :

        E_A
Mach B :xxx:  E  : M  :

       :T_P-:--- C_T-----:
```

$T_P = E_A$ MDT A for $T_P = 0$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = C_A$ MDT B for $T_P = E_A$ MDT/Cycle B = 0 ODT/Cycle = $M_A - E_B$

Condition 1B: M_A is the largest element, $E_A < M_B$, and $C_A < C_B$

```
Mach A :E:    M     :E:   M      :E:   M      :E:     M    :

                                              D     C     B A
Mach B :x: E  :    M    : E  :    M    : E  : M  :     :     : :
```

NOTE: M_B and E_B move to the right with respect to E_A and M_A because $C_A < C_B$. If M_B ends before Point "C" and E_B ends before Point "B", another cycle will occur. M_B must be greater than Point "D" because $C_B > C_A$. M_B must be less than Point "B" because M_A is the largest element. The system will cycle until one of the following conditions occur:

Conditions: A. M_A ends before E_B.

B. M_A and E_B end at the same time.

A. M_A ends before E_B

```
                                                MDT             MDT
Mach A :E:   M      :E:    M     :E:...:     M     :xx:E:    M      :x:

Mach B :x:  E :    M    : E  :    M     : . . . : E  :    M    : E  :

       :---------- Tp ------------------------------- CT -------:
```

Find $N(C_B - C_A) + E_B - M_A < E_B$ where $N = 1, 2, 3 \ldots$ and $N(C_B - C_A) + E_B - M_A > 0$

$T_P = E_A + NC_B + E_B$ MDT A for $T_P = NC_B + E_B - NC_A - M_A$ ODT for $T_P = N(M_B - E_A)$

$C_T = C_B$ MDT B for $T_P = E_A$ ODT/Cycle $= M_B - E_A$

MDT/Cycle A $= C_B - C_A$

MDT/Cycle B $= 0$

B. M_A ends before E_B.

```
                                                                    MDT
Mach A :E:   M     :E:   M     :E: . . . :    M     :E:   M         :x:

Mach B :x: E  :   M    :  E :    M    : . . .  : E  :  M :    E     :

       :_______________T_P_________________________:_____C_T______:
```

Find $N(C_B - C_A) + E_B - M_A < E_B$ where $N = 1, 2, 3 \ldots$ and $N(C_B - C_A) + E_B - M_A = 0$ ODT for $T_P = N(M_B - E_A)$

$T_P = E_A + NC_B + E_B$ MDT A for $T_P = 0$ MDT/Cycle A $= C_B - C_A$ ODT/Cycle - $M_B - E_A$

$C_T = C_B$ MDT B for $T_P = E_A$ MDT/Cycle B = 0

Summary of Condition 1B

Find $N(C_B - C_A) + E_B - M_A < E_B$ where $N = 1, 2, 3 \ldots$ and $N(C_B - C_A) + E_B - M_A \geq 0$

$T_P = E_A + NC_B + E_B$ MDT A for $T_P = NC_B + E_B - NC_A - M_A$ ODT for $T_P = N(M_B - E_A)$

$C_T = C_B$ MDT B for $T_P = E_A$ ODT/Cycle $= M_B - E_A$

MDT/Cycle A $= C_B - C_A$

MDT/Cycle B = 0

Condition 2: M_A is the largest element, $E_A > M_B$ and $C_A \leq C_B$.

```
Mach A :  E  :   M  :  E  :

        E_A           MDT
Mach B :xxxxx: E : M  :xxx:
```

If M_A is the largest element and $E_A > M_B$, $C_A > C_B$.

Condition 3: E_A is the largest element $M_A > E_B$, and $C_A \leq C_B$.

```
Mach A : E   : M  :  E  :

         E_A        MDT
Mach B :xxxxx:E : M :xxx:
```

If E_A is the largest element and $M_A > E_B$, $C_A > C_B$.

Condition 4: E_A is the largest element $M_A < E_B$, and $C_A \leq C_B$.

```
                MDT
Mach A :  E  :M :x:  E  :

         E_A          MDT
Mach B :xxxxx:  E : M :x:

       :-T_P-:----- C_T -----:
```

$T_P = E_A$	MDT A for $T_P = 0$	MDT/Cycle A = $E_B - M_A$	ODT for $T_P = 0$
$C_T = E_A + E_B$	MDT B for $T_P = E_A$	MDT/Cycle B = $E_A - M_B$	ODT/Cycle = 0

Summary of Condition II: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$, and $C_A \leq C_B$.

Starting Mach A First

If M_A is the largest element

1. If $C_A = C_B$

$T_P = E_A$ MDT A for $T_P = 0$ MDT/Cycle A = 0 ODT for $T_P = 0$

$C_T = C_A$ MDT B for $T_P = E_A$ MDT/Cycle B = 0 ODT/Cycle = $M_A - E_B$

2. If $C_A < C_B$

Find $N(C_B - C_A) + E_B - M_A < E_A$ where N = 1, 2, 3 ... and $N(C_B - C_A) + E_B - M_A \geq 0$

$T_P = E_A + NC_B + E_B$ MDT A for $T_P = NC_B + E_B - NC_A - M_A$

$C_T = C_B$ MDT B for $T_P = E_A$

MDT/Cycle A = $C_B - C_A$ ODT for $T_P = N(M_B - E_A)$

MDT/Cycle B = 0 ODT/Cycle = $M_B - E_A$

If E_A is the largest element

$T_P = E_A$ MDT A for $T_P = 0$ ODT for $T_P = 0$

$C_T = E_A + E_B$ MDT B for $T_P = E_A$ ODT/Cycle = 0

MDT/Cycle A = $E_B - M_A$

MDT/Cycle B = $E_A - M_B$

CONDITION II: M_A or E_A is the largest element,
$M_A \neq M_B \neq E_A \neq E_B$, and $C_A \leq C_B$

Starting Mach B First

Condition 1: M_A is the largest element and $E_A < M_B$

```
Mach B : E : M  : E :

        EB       B A
Mach A :xxx:E: M  : :
```

NOTE: M_A cannot be greater than Point "A" because $C_A \leq C_B$.

Conditions: A. $C_A = C_B$

B. $C_A < C_B$

Condition 1A: M_A is the largest element, $E_A < M_B$ and $C_A = C_B$.

```
Mach B : E : M  : E :

        EB
Mach A :xxx:E:  M   :

       :TP-:___CT___:
```

$T_P = E_B$	MDT A for $T_P = E_B$	MDT A/Cycle = 0	ODT for T_P = 0
$C_T = C_B$	MDT B for T_P = 0	MDT B/Cycle = 0	ODT/Cycle = $M_B - E_A$

Condition 1B: M_A is the largest element, $E_A < M_B$ and $C_A < C_B$

```
Mach B :  E  : M  :  E  :

        E_B          MDT
Mach A :xxxxx:E:  M   :x:

       :_T_P_:_____C_T_____:
```

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A = $C_B - C_A$ ODT for $T_P = 0$

$C_T = C_B$ MDT B for $T_P = 0$ MDT/Cycle B = 0 ODT/Cycle = $M_B - E_A$

Summary of Conditions 1A and 1B.

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A = $C_B - C_A$ ODT for $T_P = 0$

$C_T = C_B$ MDT B for $T_P = 0$ MDT/Cycle B = 0 ODT/Cycle = $M_B - E_A$

Condition 2: M_A is the largest element, $E_A > M_B$ and $C_A \leq C_B$

```
                  MDT          MDT
Mach B : E : M  :x: E : M  :xxxxx:

       E_B
Mach A :xxx:  E   :   M   :  E   :
```

NOTE: If M_A is the largest element and $E_A > M_B$, then $C_A > C_B$.

Condition 3: E_A is the largest element, $M_A > E_B$ and $C_A \leq C_B$

```
              MDT        MDT
Mach B :E : M :xx:E : M :xxxx:

        E_B
Mach A :xx:  E   : M  :  E   :
```

NOTE: If E_A is the largest element and $M_A > E_B$, then $C_A > C_B$.

Condition 4: E_A is the largest element, $M_A < E_B$ and $C_A \leq C_B$

```
                   MDT
Mach B :   E  :  M :xx:  E   :

          E_B           MDT
Mach A :xxxxxx:   E   : M :xx:

       :__T_P__:_____C_T_______:
```

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A $= E_B - M_A$ ODT for $T_P = 0$

$C_T = E_A + E_B$ MDT B for $T_P = 0$ MDT/Cycle B $= E_A - M_B$ ODT/Cycle $= 0$

Summary of Condition II: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$ and $C_A \leq C_B$.

If M_A is the largest element

$T_P = E_B$ MDT for $T_P A = E_B$ MDT/Cycle A $= C_A - C_B$ ODT for $T_P = 0$

$C_T = C_B$ MDT for $T_P B = 0$ MDT/Cycle B $= 0$ ODT/Cycle $= M_B - E_A$

If E_A is the largest element

$T_P = E_B$ MDT A for $T_P = E_B$ MDT/Cycle A $= E_B - M_A$ ODT for $T_P = 0$

$C_T = E_B + E_A$ MDT B for $T_P = 0$ MDT/Cycle B $= E_A - M_B$ ODT/Cycle $= 0$

Notes

I. CONDITION I: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$, and $C_A > C_B$.

Starting Mach B First

A. General Condition

(1) E_A the largest element

(a) $M_A < E_B$

```
                MDT
Mach B :E : M  :xxx:

       E_B                      Condition 4B:  M_B ends before E_A.
Mach A :xx:    E    :
```

(b) $M_A > E_B$

```
                MDT
Mach B :E : M  :xxx:

       E_B                      Condition 4B:  M_B ends before E_A.
Mach A :xx:  E      :
```

(2) M_A the largest element

(a) $E_A > M_B$

```
                MDT
Mach B :E : M  :xxx:

       E_B                      Condition 4B:  M_B ends before E_A.
Mach A :xx:   E     :
```

Conclusion: If E_A is the largest element or if M_A is the largest element and $E_A > M_B$, Condition 4B will apply and the cycle will terminate at the end of the first E_A.

B. Conditions 1 and 2 after P_2.

(1) E_A is the largest element

(a) $M_A > E_B$

```
          P2    P3
Mach A :  E   :
                                    Condition Z2:  MB ends before EA.
             MDT
Mach B : M:xx:
```

(b) $M_A > E_B$

```
          P2    P3
Mach A :  E   :
                                    Condition Z2:  MB ends before EA.
             MDT
Mach B : M:xx:
```

(2) M_A is the largest element

(a) $E_A > M_B$

```
          P2    P3
Mach A :  E   :
                                    Condition Z2:  MB ends before EA.
             MDT
Mach B :M :xx:
```

Conclusion: If E_A is the largest element or if M_A is the largest element and $E_A > M_B$, Condition Z2 will apply and the cycle will terminate at the end of the first E_A.

II. CONDITION I: M_A or E_A is the largest element, $M_A \neq M_B \neq E_A \neq E_B$, and $C_A > C_B$.

Starting Mach A First

A. General Condition

(1) E_A is largest element

(a) $M_A < E_B$

```
                    MDT
Mach A :   E    : M :xx:
             E_A
Mach B :xxxxxxx:  E    :
```

Condition 2: M_A ends before E_B.

(b) $M_A > E_B$

```
Mach A : E  : M : E   :
           E_A        MDT
Mach B :xxxx:E:M :xxx:
```

Condition 4B: M_A ends before M_B and M_B ends before E_A.

(2) M_A is largest element

(a) $E_A > M_B$

```
Mach A :E : M  : E:
          E_A      MDT
Mach B :xx:E: M :x:
```

Condition 4B: M_A ends before M_B and M_B ends before E_A.

Conclusion: If M_A is the largest element and $E_A > M_B$, Condition 4B will apply and the cycle will terminate at the end of the E_A. If E_A is the largest element and $M_A > E_B$, Condition 4B will apply and the cycle will terminate at the end of the first E_A. If E_A is the largest element and $M_A < E_B$, Condition 2 will apply and the cycle will terminate at the end of the first E_B.

B. Conditions 4A and 4B after P_2

(1) E_A is the largest element

(a) $M_A < E_B$

```
        P2      P3
Mach B :   E    :
                                        Condition X:  MA ends before EB.
           MDT
Mach A : M :xxx:
```

(b) $M_A > E_B$

```
        P2          P3
Mach B :E : M   :xxx:
                MDT                     Condition Z2:  MA ends before MB and MB ends before EA.

Mach A : M :    E    :
```

(2) M_A is the largest element

(a) $E_A > M_B$

```
        P2          P3
Mach B : E: M   :xxx:
                MDT                     Condition Z2:  MA ends before MB and MB ends before EA.

Mach A : M :    E    :
```

Conclusion: If E_A is the largest element and $M_A > E_B$ or if M_A is the largest element and $E_A > M_B$, Condition Z2 would apply and the circle will terminate at the end of the first E_A. If E_A is the largest element and $M_A < E_B$, Condition X would apply and the cycle would terminate at the end of the first E_B.

Example

Conditions: Both machines are started independently by the operator at 7:30 A.M. The operations are such that the operator can stop the machines at anytime and not change the normal sequence. The operator has scheduled breaks at 9:45 to 9:55 A.M. and 2:45 to 2:55 P.M., and lunch period from 12:00 to 1:00 P.M. Just prior to or at 4:30, the operator stops each machine independently as the work pieces are completed and clears the machines. The standard time for the operations involved are as follows:

1. Machine time for Machine A is 20 minutes ($M_A = 20$).
2. Machine time for Machine B is 18 minutes ($M_B = 18$).
3. External work for Machine A is 6 minutes ($E_A = 6$).
4. External work for Machine B is 7 minutes ($M_B = 7$).

The operating costs are as follows:

1. Machine A - $5.00/hour.
2. Machine B - $10.00/hour.
3. Operator - $3.00/hour.

The period (P) is 460 minutes.

Calculations:

Step One.--Determine the applicable model for man multi-machine jobs for starting Machine A first as outlined in Chapter V.

1. $N(C_A - C_B) - XC_B + M_A \leq E_B$

$$N(26 - 25) - 25X + 20 \leq 7$$

$$N - 25X + 20 \leq 7$$

$$X = 1$$

$$N = 6$$

2. $N(C_A - C_B) = XC_B - M_A$

$N(26 - 25) = 25X - 20$

$N = 25X - 20$

$X = 1$

$N = 5$

3. $N(C_A - C_B) - XC_B < E_A$

$N(26 - 25) - 25X < 6$

$N - 25X < 6$

$X = 0$

$N = 1$

The model for man multi-machine jobs to be used is $N(C_A - C_B) - XC_B < E_A$ where $N = 1$ and $X = 0$.

Step Two.--From the model for man multi-machine jobs determined in Step One, determine the following for starting Machine A first.

$T_P = E_A = 6$ minutes

$C_T = NC_A = (1)(26) = 26$ minutes

MDT A for $T_P = 0$

MDT B for $T_P = E_A = 6$ minutes

MDT/Cycle A = 0

MDT/Cycle B = $NC_A - (N + X)\ C_B = 26 - 25 = 1$ minute

ODT for $T_P = 0$

ODT/Cycle = $NM_A - (N + X)\ E_B = 20 - 7 = 13$ minutes

Step Three.--Determine the following as outlined in Chapter VI for starting Machine A first.

$P = T_P + QC_T$

$460 = 6 + 26Q$

$Q = \frac{460 - 6}{26} = 17.5$

MDT A for P = MDT A for T_p + Q(MDT/Cycle A)

= 0 + (17.5)(0)

= 0

MDT B for P = MDT B for T_p + Q(MDT/Cycle B)

= 6 + (17.5)(1)

= 23.5 minutes

ODT for P = ODT for T_p + Q(ODT/Cycle)

= 0 + (17.5)(13)

= 227.5 minutes

Step Four.--Determine the delay costs for a period for starting Machine A first as outlined in Chapter VII.

Delay costs for P = (MDT A for P) Cost A/hour + (MDT B for P) Cost B/hour + (ODT for P) Cost of operator/hour = $(0)(\frac{\$5.00}{60}) + (23.5)(\frac{\$10.00}{60})$ + $(227.5)(\frac{\$3.00}{60})$ = \$3.92 + \$11.38 = \$15.30

Step Five.--Determine the applicable model for man multi-machine jobs for starting Machine B first as outlined in Chapter V.

1. $N(C_A - C_B) - XC_B - M_B \leq E_B$

$N(26 - 25) - 25X - 18 \leq 7$

$N - 25X - 18 \leq 7$

$X = 0$

$N = 19$

2. $N(C_A - C_B) = XC_B + M_B$

$N(26 - 25) = 25X + 18$

$N = 25X + 18$

$X = 0$

$N = 18$

3. $N(C_A - C_B) - XC_B - M_B + E_A < E_A$

$$N(26 - 25) - 25X - 18 + 6 < 6$$

$$N - 25X - 12 < 6$$

$$X = 0$$

$$N = 12$$

The model for man multi-machine jobs to be used is $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where N = 12 and X = 0.

4. $K(C_A - C_B) - VC_B + M_A \leq E_B$

$$K(26 - 25) - 25V + 20 \leq 7$$

$$K - 25V + 20 \leq 7$$

$$V = 1$$

$$K = 6$$

5. $K(C_A - C_B) = VC_B - M_A$

$$K(26 - 25) = 25V - 20$$

$$K = 25V - 20$$

$$V = 1$$

$$K = 5$$

6. $K(C_A - C_B) - VC_B < E_A$

$$K(26 - 25) - 25V < 6$$

$$K - 25V < 6$$

$$V = 0$$

$$K = 1$$

The model for man multi-machine jobs to be used is $N(C_A - C_B) - XC_B - M_B + E_A < E_A$ where N = 12 and X = 0 and $K(C_A - C_B) - VC_B < E_A$ where K = 1 and V = 0.

Step Six.--From the model for man multi-machine job determined in Step Five, determine the following for starting Machine B first.

$T_P = E_B + NC_A + E_A = 7 + (12)(26) + 6 = 325$ minutes

$C_T = KC_A = (1)(26) = 26$ minutes

MDT A for $T_P = E_B = 7$ minutes

MDT B for $T_P = NC_A + E_A - (N+X)C_B - M_B = (12)(26) + 6 - (12)(26) - 18 = 0$

MDT/Cycle A = 0

MDT/Cycle B = $KC_A - (K + V)C_B = (1)(26) - (1)(25) = 1$ minute

ODT for $T_P = NM_A - (N + X)E_B = (12)(20) - (12)(7) = 156$ minutes

ODT/Cycle = $KM_A - (K + V)E_B = (1)(20) - (1)(7) = 13$ minutes

Step Seven.--Determine the following as outlined in Chapter VI for starting Machine B first.

$$P = T_P + QC_T$$

$$460 = 326 + 26Q$$

$$Q = \frac{460 - 326}{26}$$

$$= 5.2$$

MDT A for P = MDT A for T_P + Q(MDT/Cycle A) = 7 + (5.2)(0) = 7 minutes

MDT B for P = MDT B for T_P + Q(MDT/Cycle B) = 0 + (5.2)(1) = 5.2 minutes

ODT for P = ODT for T_P + Q(ODT/Cycle) = 156 + (5.2)(13) = 223.6 minutes

Step Eight.--Determine the delay costs for a period for starting Machine B first as outlined in Chapter VII

Delay costs for P = (MDT A for P) Cost A/hour + (MDT B for P) Cost B/hour + (ODT for P) Cost of operator/hour = $(7)(\frac{\$5.00}{60}) + (5.2)(\frac{\$10.00}{60}) + (223.6)(\frac{\$3.00}{60})$ = \$.58 + \$.87 + \$11.18 = \$12.63

Step Nine.--Compare the delay costs for the period computed in Steps Four and Eight and apply the decision model outlined in Chapter VII.

$\text{Delay costs for A for P} = \$15.30$

$\text{Delay costs for B for P} = \$12.63$

$\text{Delay costs for B for P} < \text{Delay costs for A for P}$

The delay costs for B for P are the smallest; therefore, Machine B should be started first.

Step Ten.--Determine following as outlined in Chapter VI for starting Machine B first.

$$\text{Parts A for P} = \frac{\text{P - MDT A for P}}{C_A} = \frac{460 - 7}{26} = 17$$

$$\text{Parts B for P} = \frac{\text{P - MDT B for P}}{C_B} = \frac{460 - 5.2}{25} = 18$$

$$\text{Percent utilization of A} = \frac{\text{P-MDT A for P}}{\text{P}} \text{ X } 100 = \frac{460 - 7}{460} \text{ X } 100 = 98.5\%$$

$$\text{Percent utilization of B} = \frac{\text{P - MDT B for P}}{\text{P}} \text{ X } 100 = \frac{460 - 5.2}{460} \text{ X } 100$$

$$= 98.5$$

$$\text{Percent utilization of operator} = \frac{\text{P - ODT for P}}{\text{P}} \text{ X } 100 = \frac{460 - 223.6}{460} \text{ X } 100$$

$$= 51.4\%$$

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