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Results since the first annual report are summarized in the attached report "Study of a Numerical Simulation of Atmospheric Turbulent Diffusion", which was prepared by Mr. H. B. Mebane as a special problem report as part of his M.S. degree requirements in the School of Aerospace Engineering at Georgia Tech.

Publications and Reports:

"Modeling the Anisotropy in Stratified Shear Flow", Symposium on Air Pollution, Turbulence, and Diffusion, Las Cruces, N. M., December, 1971, C. G. Justus and J. E. Hicks.

"Diffusion Simulation Results in Stratified Shear Flow", AMS Annual Meeting, New Orleans, La., January, 1972, C. G. Justus and J. E. Hicks.

Theses:

"A Numerical Simulation of Nearly Incompressible Stably Stratified Atmospheric Turbulent Diffusion", Ph.D. Thesis School of Aerospace Engineering, Georgia Tech, September, 1971, J. E. Hicks.

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STUDY OF A NUMERICAL SIMULATION MODEL OF ATMOSPHERIC TURBULENT DIFFUSION

by

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I. INTRODUCTION

Hicks and Justus (1971) developed a numerical simulation model for atmospheric turbulent diffusion in which fluid point pairs are traced during random walks in a background correlation field. Specifically, the effect of turbulent pressure forces is modeled by forcing space-time correlations between random turbulent acceleration components a_i. The longitudinal correlation for acceleration component parallel to the vector joining the two points is assumed to be of the form

$$f_1(r) = \exp - (\pi r^2/L^2),$$

where r is the magnitude of the separation vector between the fluid points and L is the characteristic length scale of the flow. For acceleration components normal to the separation vector, the transverse correlation is given in terms of the longitudinal correlation as

$$f_2(r) = f_1(r) + \frac{r}{2}\frac{df_1(r)}{dr}$$

and the time correlation is assumed to be

$$g(t) = \exp (- \pi t^2/T^2),$$

where T is the characteristic time scale of the flow.

In the present analysis, the assumed Gaussian form of the correlation functions was replaced by a linear exponential form so that

$$f_1(r) = \exp (- r/L),$$

and

$$g(t) = \exp(-t/T),$$

with the transverse correlation $f_2(r)$ given in terms of $f_1(r)$ as before.

The method used to force correlations between the turbulent acceleration components a_i is explained in detail by Hicks and Justus, and the entire explanation need not be repeated here.

Very briefly, it is assumed that the turbulent acceleration components due to pressure changes may be written in terms of components at the previous time step as

$$a_{i1}(t_{k+1}) = \alpha_i a_{i1}(t_k) + \beta_i a_{i2}(t_k) + \gamma_i z_i.$$

The z_i's are random numbers chosen from a standard normal distribution; the coefficients α_i, β_i, γ_i depend on the values of the correlation functions at time step t_k, and are chosen so as to insure the proper value of the space-time correlation coefficients for the acceleration components. Here the a_{i1}'s and the a_{i2}'s are the turbulent acceleration components of the two fluid points considered in the i direction.

The relative diffusion of the two fluid points is computed at each time step from finite difference approximations to the equations of motion, with the pressure force terms modeled by the a_i's.

Qualitatively, the effect of changing the form of the correlation functions described above would seem to be a decrease in the correlation of longitudinal components at small separations; the longitudinal correlations should tend to increase at larger relative displacements. However, the effect on the transverse correlation function and on the actual space-time correlation coefficients are not obvious, so that the precise effect on the relative diffusion of the fluid points is difficult, if not impossible, to predict before running the model.

In addition to changing the form of the correlation functions, a more basic change was also considered whereby a desired correlation is forced between components of turbulent velocity rather than turbulent acceleration.

The original program developed by Hicks and Justus contained an option to allow for such an approach; this portion of the program was rewritten in order to make the approach more physically consistent and more analogous to the correlated acceleration case.

In the present approach, we assume that the turbulent velocity components u_i may be decomposed as

$$u_i = u_{pi} + u_{si},$$

where u_{pi} is the contribution made to the turbulent velocity by pressure forces, and u_{si} is the contribution of shear and buoyancy effects.

The assumed form for the Eulerian space-time correlations is forced between the u_{pi}'s, which are calculated from

$$u_{p1i}(t_{k+1}) = \alpha_i u_{p1i}(t_k) + \beta_i u_{p2i}(t_k) + \gamma_i z_i,$$

where the notation is completely analogous to the correlated acceleration field analysis described above. If the same linear exponential forms are assumed for the correlation functions, the α_i's, β_i's, and γ_i's are equivalent to those computed for the correlated acceleration field case.

The u_{si}'s may be computed at each time step from the equations of motion and are found to be

$$u_{si}(t_{k+1}) = u_{si}(t_k) - [K_i u_3(t_k) + \omega u_i(t_k)](t_{k+1} - t_k) + \frac{1}{2}K_i \{\Gamma[x_3(t_k) - x_3(t_o)] + \omega u_3(t_k)\} (t_{k+1} - t_k)^2.$$

$$(i = 1, 2)$$

$$u_{s3}(t_{k+1}) = u_{s3}(t_k) - \{\Gamma[x_3(t_k) - x_3(t_o)] - \omega u_3(t_k)\} (t_{k+1} - t_k).$$

Here, K_i is the mean wind shear $\partial\bar{U}_i/\partial x_3$, ω is the viscous damping coefficient, $\omega = \epsilon/\langle u^2\rangle$, and Γ is the logarithmic vertical potential temperature gradient, $\Gamma = (g/\theta)\, d\theta/dx_3$.

The above changes have been incorporated into the diffusion model program so that the user has the option of specifying either a background correlated turbulent acceleration field or velocity field.

Using the modified program, an attempt was made to investigate the effect on the relative diffusion of the two fluid points resulting from varying the magnitude and direction of the initial separation vector. This analysis was first carried out using the background correlated turbulent acceleration field, and then using the correlated velocity field approach; the results are examined in some detail in Section II of this report.

Finally, the power spectrum of the turbulent velocity components generated by the model is calculated using a computer program developed by NOAA; the power spectra for the correlated acceleration field case and for the correlated velocity field case are compared in Section III.

II. INITIAL SEPARATION EFFECTS

The effect of altering the initial separation vector between the two fluid points on their relative diffusion was studied by systematically varying the magnitude and direction of the vector while keeping all other input flow parameters constant. Specifically, relative displacement vs. time plots were obtained for initial separations of 1.0, 0.1, and 0.01 integral length scales in each of the three coordinate directions. The logarithmic potential temperature gradient Γ was 0.01 (time scales)$^{-2}$, and the mean wind shear $\partial U/\partial z$ was 1.0 (time scales)$^{-1}$ in all cases. The analysis was carried out for both correlated turbulent acceleration and velocity fields.

It should be carefully noted that the relative diffusion of the two fluid points is defined as the separation of the points relative to their initial separation at $t = 0$, so that all relative diffusion components are initially zero, regardless of the magnitude or direction of the initial separation vector.

As expected, the orientation of the x - axis in the direction of the mean wind led to the emergence of the x - component of the relative displacement as the dominant component for large time scales regardless of initial displacement.

Generally, there was a marked difference between the relative displacements for correlated velocity and correlated acceleration fields in that the y and z - components tended to an eddy diffusion-type variation at large times for the acceleration field case, but continued to increase sharply, although less rapidly than the x - component, in the correlated velocity case. The initial displacement effects are now examined separately for the correlated velocity field and correlated acceleration field cases.

Correlated Turbulent Acceleration Field

The magnitude of the initial separation vector significantly affects the relative diffusion of the two fluid points for small times after their release; the effect of varying the direction of the initial displacement, however, is evident only for an initial separation of one integral length scale (or greater, presumably).

The relative separation at very small times (t = 0.1 time scale) decreases sharply with a decrease in initial displacement magnitude as shown in Table 1. At large times (roughly 5 to 10 time scales), however, the relative diffusion in a given direction is of the same magnitude regardless of initial separation.

The relative diffusion of the fluid points tends to follow a t^2 power law for a short period after release; the exact length of this period appears to increase with an increase in initial separation magnitude in a roughly linear fashion (Table 2).

Following the initial t^2 period, the x - component of diffusion gradually undergoes a transition to a power law of t^3 for the case of a one length scale initial separation, and t^4 for the 0.1 and 0.01 length scale cases. These variations appear to persist for times at least as great as those considered here, and may be attributed to the mean shear as discussed by Hicks and Justus (1971).

The y and z components tend to fluctuate, and hence follow no discernible power variation. At times near 10 time scales, the largest time con-

Table 1. Effect of Initial Separation Magnitude on Diffusion in a Correlated Turbulent Acceleration Field

initial separation magnitude (length scales)	x - component of relative separation at t = 1 time scale (length scales)	x - component of relative separation at t = 10 time scales (length scales)
0.01	0.0001	100
0.10	0.005	100
1.00	0.01	100

Table 2. Effect of Initial Separation Magnitude on t^2 Variation of Relative Diffusion

initial separation magnitude (length scales)	duration of t^2 power law for relative diffusion (time scales)
0.01	0.5
0.10	0.7
1.00	1.0

sidered in the present analysis, the y - component tends toward a t^1 (eddy diffusion) variation. The vertical component begins to level off at large times as a result of the stable density stratification; a similar effect was noted by Hicks and Justus (1971).

As mentioned earlier, a change in the direction of the initial separation vector has no measurable effect on the relative diffusion for small initial displacement magnitudes (0.1 or 0.01 integral length scales).

Although small differences are apparent for a given magnitude and varying direction, especially at very small times, they appear to follow no consistent pattern and are indistinguishable from the random fluctuations characteristic of the model.

The effect of varying the direction of initial displacement for the large separation magnitude (one length scale) is more significant. For the case of an initial separation of one length scale in the x - direction, shown in Figure 1, the relative diffusion components are very nearly equal for times out to one integral scale, where the y and z components begin to fluctuate as described earlier. With the initial separation in the y - direction, the x - component of diffusion is roughly twice the value of the y and z components for times less than one time scale. This behavior is unexpected and the explanation is not obvious. An initial displacement of one length scale in the vertical results in a diffusion pattern clearly dominated by the effects of mean wind shear, as shown in Figure 2. The x - component of relative diffusion is greater than the y and z - components by a factor of at least 2 for very small times (0.1 time scales), and the gap

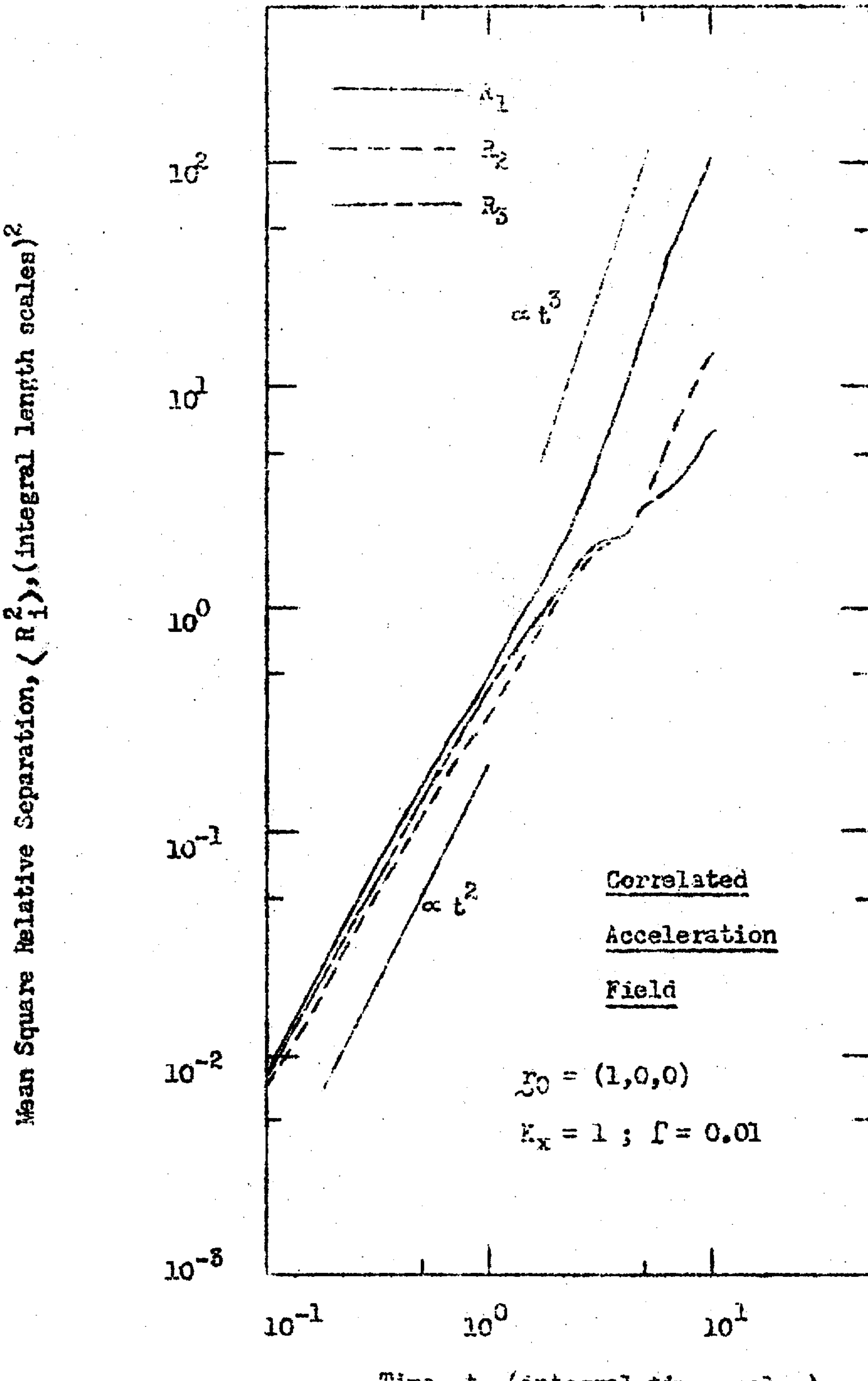

Figure 1. Fluid Point Pair Dispersion in a Correlated Turbulent Acceleration Field.

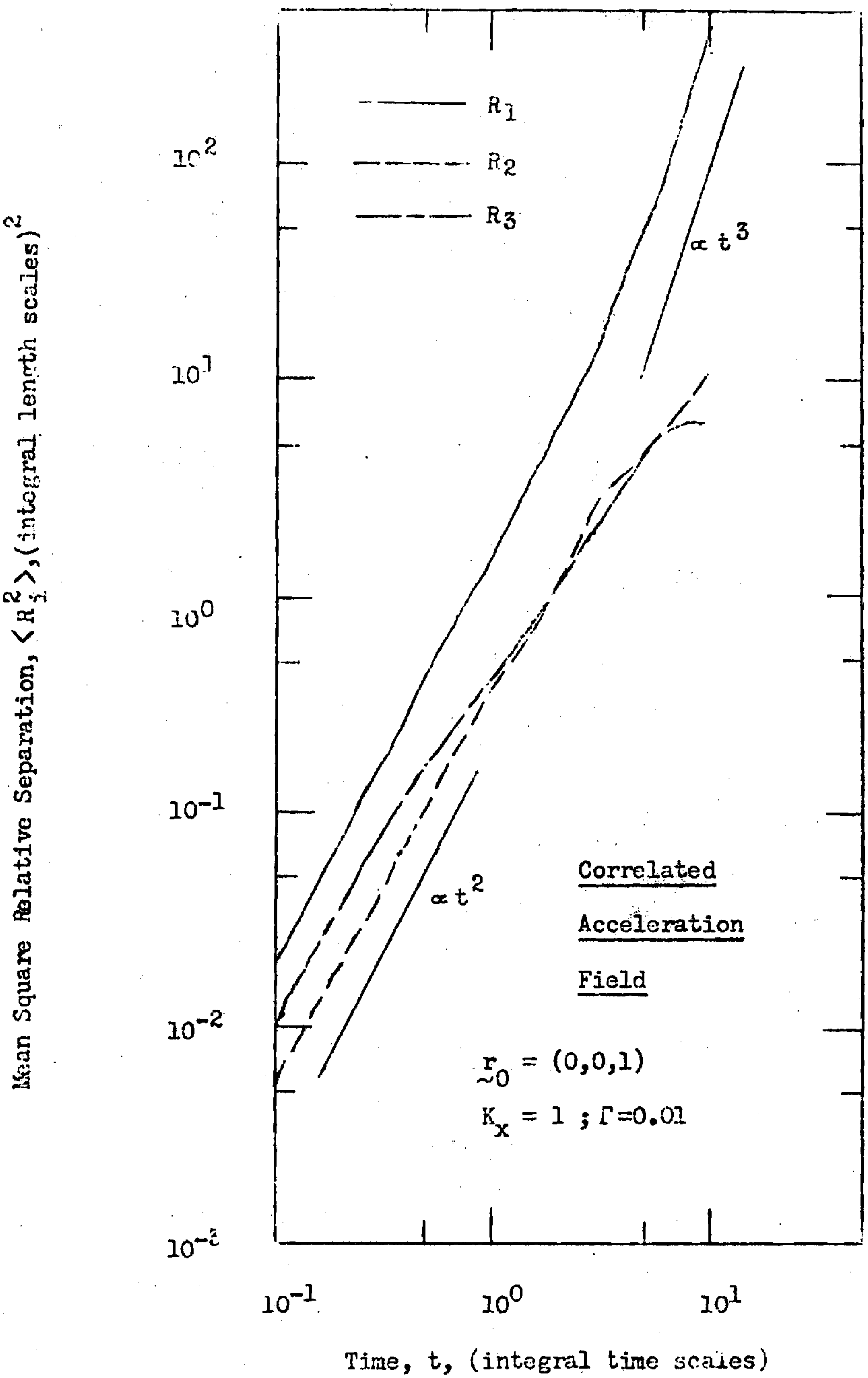

Figure 2. Fluid Point Pair Dispersion in a Correlated Turbulent Acceleration Field.

widens steadily with increasing time. Due to the effect of mean shear, the diffusion in the x - direction is larger than the y and z components for sufficiently large times regardless of initial separation; the distinction of the above case lies in the domination of the x - component even for small times. Also in the above case, the reason for the relatively larger difference in the y and z components at small times is not readily explained.

Correlated Turbulent Velocity Field

Generally, the effect of initial separation for the correlated velocity field case is less clearly defined and does not lend itself as readily to physical or intuitive explanations. The diffusion appears to be dominated by the random pressure contributions to the turbulent velocity, and hence shows less effect of flow characteristics such as mean shear and density stratification.

The magnitude of the initial separation has a strong effect on all diffusion components for very small times after release, as was the case for the correlated acceleration field, and in addition affects the magnitude of diffusion for even the largest times considered (10 integral time scales), as shown in Table 3.

The relative diffusion components vary according to a t^2 power law for times up to approximately one integral time scale. The x - component subsequently undergoes a very gradual transition to a t^3 power law. The time at which this transition occurs, unlike the correlated acceleration field case, does not depend on initial separation. Flunctuations in the y and z components are much less pronounced than those in the acceleration field

Table 3. Effect of Initial Separation Magnitude on Diffusion in a Correlated Velocity Field

initial separation magnitude (length scales)	x - component of relative separation at t = 1 time scale (length scales)	x - component of relative separation at t = 10 time scales (length scales)
0.01	0.0005	10
0.10	0.005	50 to 100
1.00	0.01	500

case, and they obey essentially the same power laws described above for the x - component, although the transition to the t^3 variation is somewhat more rapid. Unlike the acceleration field case, the y - component does not tend toward an eddy diffusion (t^1) variation. Also, the vertical diffusion component shows no marked tendency to level off, so that there is no visible evidence of the effect of stable density stratification.

Just as for the acceleration field case, there appears to be no visible effect of varying initial separation direction for initial displacements of 0.1 or 0.01 length scales. While the relative diffusion vs. time plots are not identical for different initial separation directions, the differences noted are generally small, and follow no logical pattern. For the case of an initial displacement of 1.0 integral length scales in the vertical, the effects of mean wind shear are again evident, as the x - component of relative diffusion is dominant for the entire time span considered, as shown in Figure 3.

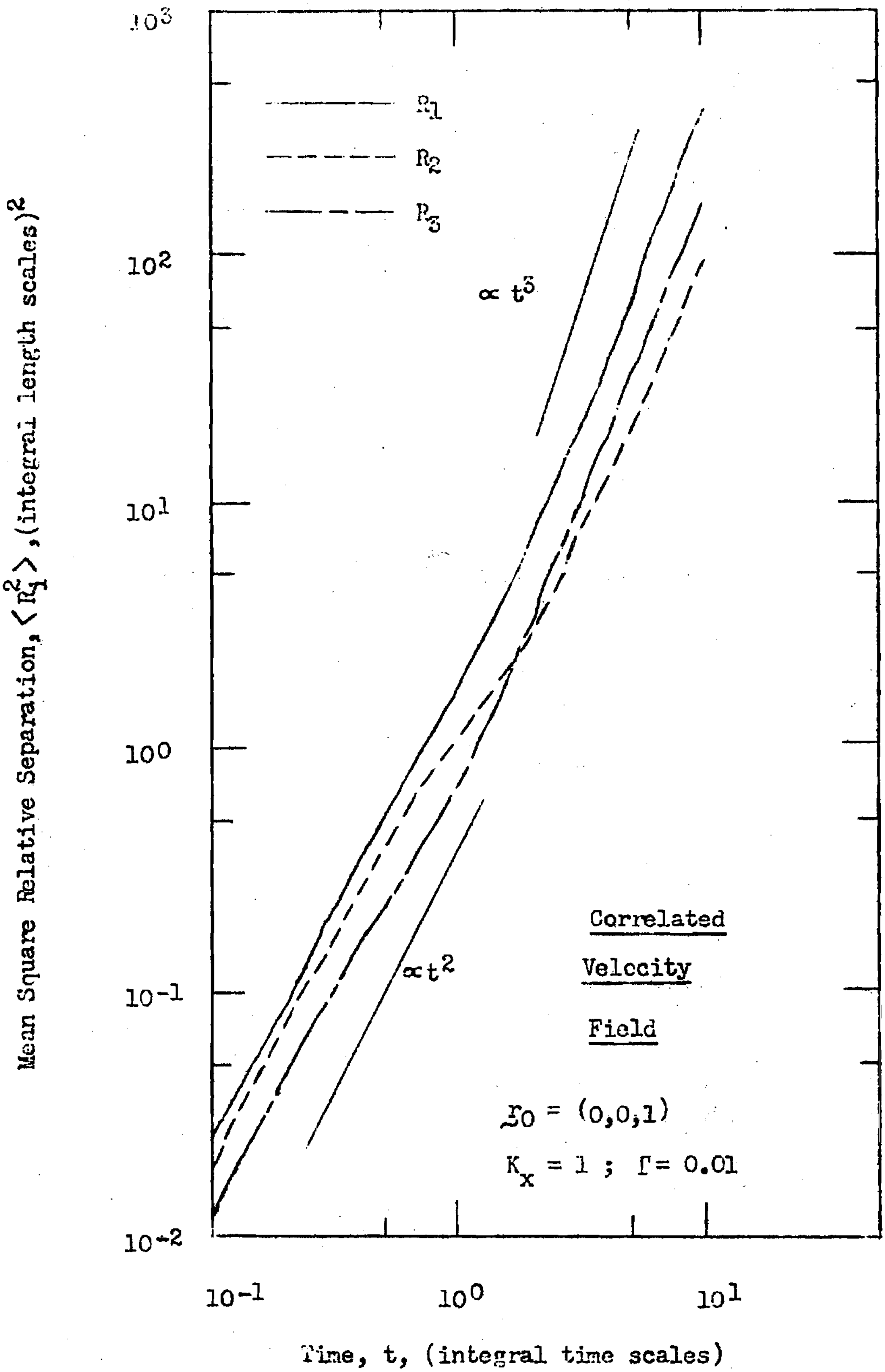

Figure 3. Fluid Point Pair Dispersion in a Correlated Turbulent Velocity Field.

III. POWER SPECTRUM ANALYSIS

A power spectrum for the turbulent velocity components was generated using a computer program developed by Mr. Larry Wendell of NOAA. The program was modified slightly to accept strings of turbulent velocity values generated by the diffusion model program as input data.

The program accepts strings of complex data values and computes complex Fourier coefficients using a Fast Fourier Transform technique; the coefficients are in turn used to compute the power spectrum, and, if desired, the cospectrum and quadrature spectrum for the data string. The complex data values are input in the form of two data strings which are interpreted as the real and imaginary parts of the complex quantities. In order to obtain a power spectrum of a set of real-valued data, say $\{u_i\}$, it is necessary to represent each data value as a complex quantity, so that the u_i's are input as data pairs of the form $(u_i, 0)$. The program includes options to remove the mean and a linear trend from the original data, to taper the ends of the data strings, to smooth the spectra, and to plot the results.

For the present analysis, an option was added to calculate the average spectra and standard deviations for a group of data sets.

Turbulent velocity components were generated for 10 realizations (loops) of the diffusion program, comprising 600 time steps each. The x velocity components were stored at each time step and used as input data for the power spectrum program as described earlier; 512 velocity values were stored for each realization, beginning with the fifth time step.

The process was repeated for various values of mean wind shear and buoyancy factor Γ for both the correlated acceleration field and correlated velocity field cases in order to determine the effect on the power spectrum curves.

Correlated Turbulent Acceleration Field

For higher frequency values, the energy spectrum varied as

$$\phi\ (\omega) \propto \omega^{-a},$$

where a is slightly greater than 10/3, as shown in Figure 4.

The inclusion of a moderate wind shear (1.0 time scales^{-1}) had the effect only of increasing the coefficient of the power law very slightly.

A stable density stratification (Γ = 0.1) had no measurable effect whatever on the high frequency portion of the curve.

Correlated Turbulent Velocity Field

Power spectral analysis of the x - component of turbulent velocity for the case of a background correlated velocity field yields a spectrum which obeys a power law with an exponent of roughly -7/4 except at the lower end of the frequency spectrum. This variation is remarkably similar to the well-known isotropic power law (cf. Tennekes and Lumley, 1972)

$$\phi\ (\omega) \propto \omega^{-5/3}.$$

The -7/4 exponent was obtained from a diffusion analysis with zero mean shear and Γ = 0. The inclusion of a strong stable density stratification, Γ = 0.1 (time scales)$^{-2}$, produced no visible changes in the power spectrum

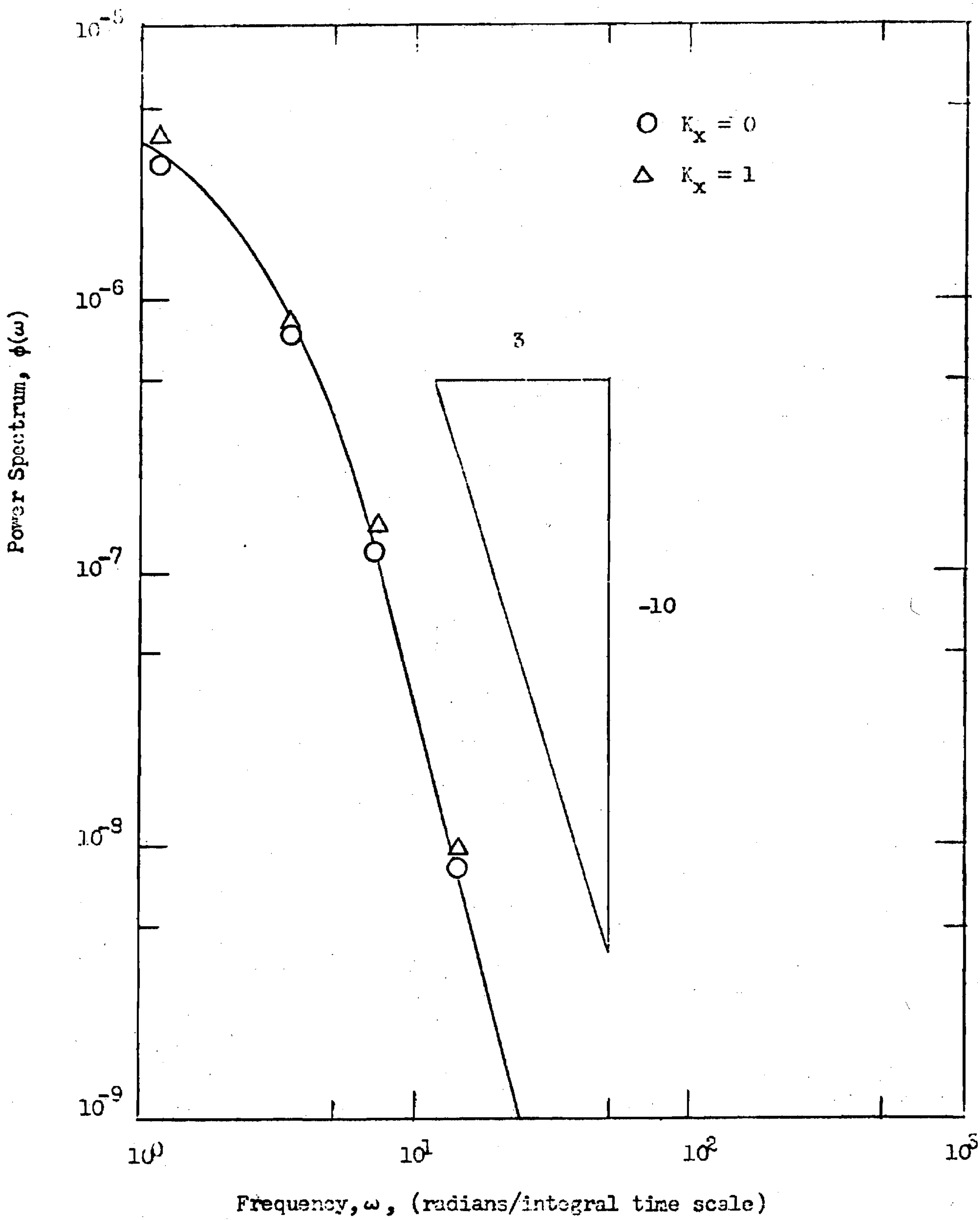

Figure 4. Power Spectrum of x-component of Turbulent Velocity for Fluid Point Pair Dispersion in a Correlated Turbulent Acceleration Field.

(as was the case for the correlated acceleration field). The inclusion of mean wind shear results in an increase in the coefficient of the power law, and also slightly decreased the exponent to -8/5, as shown in Figure 5.

Lumley and Panofsky (1964) have noted that, although the exact effects of stability on the energy spectrum of longitudinal velocity components are not totally understood, a variation in stability does not significantly alter the shape of the spectral curve; as noted above, this seems to be the case in the present analysis. Also, since mean wind shear can serve to transfer energy from the mean wind into the turbulence, it appears that the larger spectral values for $K_x > 0$ are also reasonable.

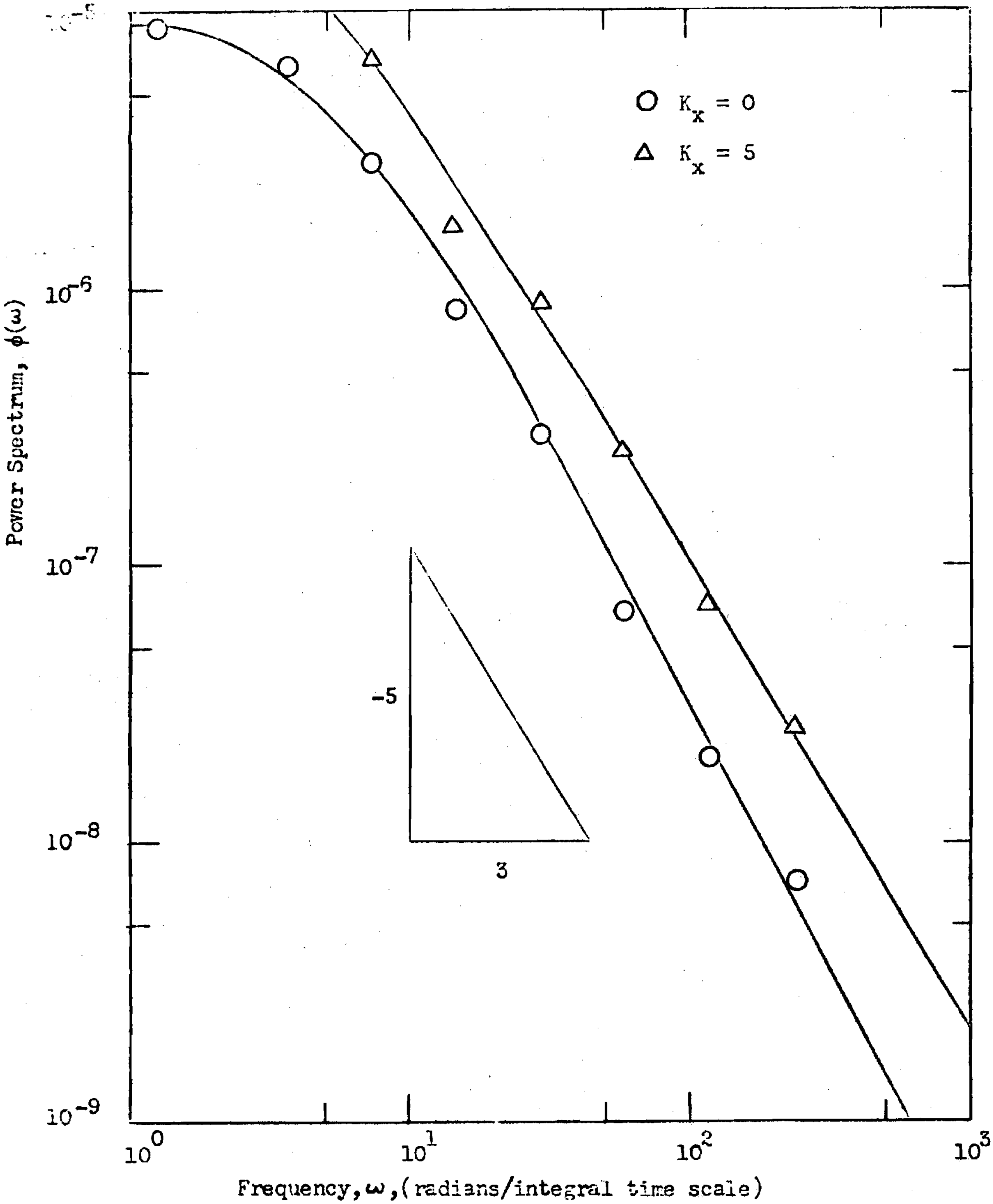

Figure 5. Power Spectrum of x-component of Turbulent Velocity for Fluid Point Pair Dispersion in a Correlated Turbulent Velocity Field.

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