A Closed Form Frequency Domain Model for Tangential Cutting Force in Peripheral Milling

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Abstract

An approach to develop a closed form frequency domain model for the tangential cutting force and torque is presented for peripheral milling processes. Based on a mechanistic local cutting force model, the total tangential cutting force is shown to be of a convolution integral form. The convolution integrands are defined in the context of local cutting force function and cutter chip width density function. The latter is related to cutter geometry and axial depth of cut, and the local cutting force function is determined by the radial cutting configuration. The convolution theorem of linear system theory is applied to obtain the Fourier transforms of total cutting force as the products of Fourier transforms of the elemental cutting and chip width density functions. Results are compared with other cutting force models reported.

Nomenclature:

t_x	feed per tooth
$t_c(\theta)$	uncut chip thickness
$\bar{t}_c$	average chip thickness
β, r, h	angular, radius and axial position variables for the cutting point in the cutter cylindrical coordinate system
xyz	the work coordinate system
ϕ	cutter angular displacement
$\theta(\phi,\beta)$	cutting point angular position in the work
d_a, d_r, c	axial and radial depth of cut, c: workpiece recess
N, R, α	number of flute, nominal radius and helical angle of the end mill
θ_{1r}, θ_{2r}, θ_{rr}	entry, exit angles and radial cutting range determined from d_r and c
$w(\theta)$	cutting window function for the radial cutting configuration
β_a	angular range of axial immersion of each flute within axial depth of cut
β_p	angular spacing between two adjacent flutes, $2\pi/N$
η_r, η_c	d_r/R and c/R

ω	normalized frequency with respect to spindle frequency
Ω	spindle frequency
C, p	cutting force coefficient and cutting force power law constant
K_t, K_r	tangential cutting pressure constant, K_r : ratio of radial to tangential cutting force
$cwd_k(\beta)$	chip width density of the kth flute
$cwd_c(\beta)$	chip width density of the cutter
f_t, $f_r(\theta)$	tangential and radial local cutting forces per unit chip width
f_x, $f_y(\theta)$	x and y component of the local cutting force per unit chip width
$\bar{f}_t(\phi)$	total tangential cutting force
$\bar{f}_x$, $\bar{f}_y(\phi)$	x and y component of the total cutting force
p_t	tangential elemental cutting function, the tangential cutting force f_t normalized with respect to $K_t t_x$,
p_1, p_2	x and y component of p_t
$A_t[k]$	Fourier coefficient at frequency k of the total tangential cutting force $\bar{f}_t(\phi)$
$A_x[k]$,$A_y[k]$	Fourier coefficient at frequency k of the total x and y components cutting force $\bar{f}_x$, $\bar{f}_y(\phi)$

Note: Function variables in upper case letter are the Fourier transforms of functions in lower case variables.

1. Introduction

A cutting force model is important for process planning, process control and machine design. The existing models are mostly in the time or the angular domain using either numerical or analytical approaches. Closed form analytical expressions for the total tangential cutting force on a single flute have been established as a function of the cutter angular position [Koenigsberger et al. 1961, Tlusty et al. 1975, Yucesan et al. 1990]. These cutting force models were based on the integration of local cutting forces with respect to the cutter angular position and different expressions were needed to evaluate forces at different angular conditions. Therefore the integration solutions were segmented into different formulations with different integration boundary points, which depend on the cutter/workpiece configuration as well as the cutter rotation. Most of the integration expressions of the cutting force system reported in the literature apply to one cutter flute only. For a multi-flute cutter, care has to be taken to determine which flutes are engaged in the cutting and which appropriate integration form is to be used. With the abundance of computing resources in recent years, numerical integration becomes a popular approach to modelling cutting forces. Kline et al. [1983] developed a mechanistic discrete models for end milling in which the cutter was treated as an aggregation of discretized thin disk cutters along the cutter axis, and summation of forces from all disks yielded the total cutting forces. This numerical scheme is representative of approaches used in other cutting force models by Zhou and Wang [1983], Ber et al. [1988] and Armarego and Deshpande [1989, 1991]. Numerical integrations were commonly used in these formulations to compute the total milling forces.

Closed form frequency domain models provide alternative approaches to the study of effects of various cutting parameters on the different components of the total cutting forces. A frequency domain model was first presented by Wang et al. [1991] through angular domain convolution analysis. The model starts by decomposing the milling process into three process component functions, namely the local cutting force function, the chip width density of each flute and the tooth sequence function. Both the elemental cutting process of a cutting point and the chip width density function of a cutter flute are recognized as the shift invariant process for each cutting point and each flute respectively, and the tooth sequence function is treated as the input to the milling process. Linear system theory is then invoked to model the total force generating process as the convolution of the three process functions. Convolution theorem is then applied to obtain the frequency domain representation of the total milling forces.

In this paper, milling forces model for an ideal rigid cutter is derived from a different perspective. This new approach represents the local cutting force as a function of both cutting point location on the cutter and the cutter angular displacement variables through the use of two coordinate systems, a cylindrical coordinate attached to the cutter and a rectangular coordinate for the work. Starting from the elemental force model for an infinitesimal chip width at a cutting point, the total tangential cutting force are formulated by integrating the local cutting force function along the axial depth of cut. The final total cutting force expressions are shown to be in a convolution integral form. This convolution integral quantitatively complies with the results obtained from numerical integration as in [Kline et al. 1982, Koenigsberger and Sabberwal 1961]. However, the convolution nature of the milling force generating process is unveiled in the total cutting force expression presented herein. In addition, the changes of integration limits as required in the other closed form analytical force model is also inherent in the convolution model and need not be updated as the cutter rotates. The main advantage of convolution model comes as the power of convolution theorem in linear system theory can be utilized to obtain the frequency domain force model. This allows different frequency components of cutting forces to be represented as functions of various cutting parameters, and the dynamics of cutting force pulsation analyzed.

The following section will set the stage for the model derivation by establishing two coordinate systems to represent the cutting point position. Section 3 presents the development of a convolution model for the tangential cutting force. In the discussion in Section 4, the tangential force model is compared with the force models for the cutting forces in the normal and feed directions. A summary in section 5 concludes the paper.

2. The position representations of a cutting point on the cutter and in the work

A cylindrical coordinate system, βrh, is attached to the cutter as shown in Fig. (1.a) to represent the geometry of the active cutting edges. Being active refers to that the cutting points are within the axial depth of cut and will be engaged in the cutting. The h axis of the coordinate coincides with the cutter rotation axis, and the origin is located at one end of the axial depth of cut with positive h pointing to the other end. The angular position of the cutting point at $h=0$ on an arbitrarily chosen first flute is defined to be $\beta=0$. The coordinate β increases in the positive h direction. Using this coordinate system, the active cutting edge of the kth flute on an N-flute end mill cutter can be described as

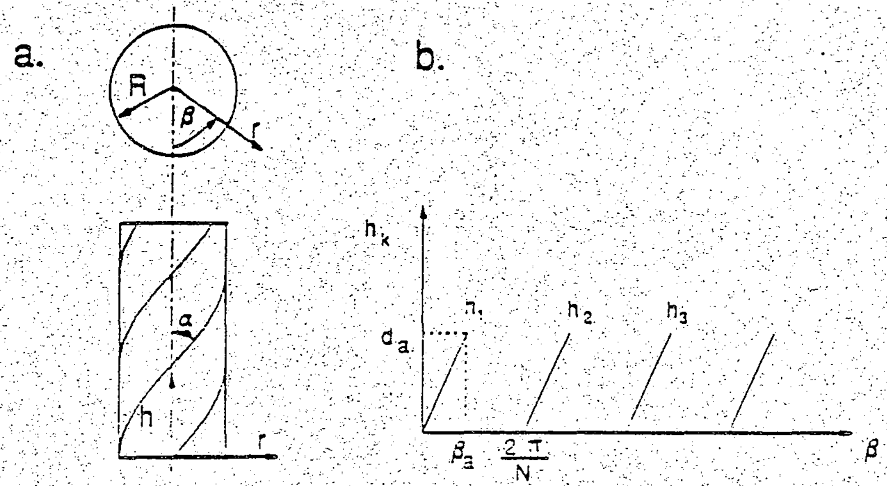

Figure 1 (a): The cylindrical cutter coordinate and (b): function h_k for cutter flute k.

$$h_k(\beta) = \frac{R}{\tan\alpha}(\beta - (k-1)\beta_p), \quad k = 1, 2...\infty, \quad (k-1)\beta_p \le \beta \le (k-1)\beta_p + \beta_a \tag{1}$$
$$r_k = R$$

where β, r and h represent the angular, radial and axial positions of each active cutting point. Fig. (1.b) illustrates their relationships. Note also $\beta_p=2\pi/N$ is the angular spacing for an N-flute cutter, and

$$\beta_a = \frac{d_a}{R}\tan\alpha$$

is the angular range of axial immersion of each cutter flute determined by d_a, R and α. In this expression, the circular N-flute cutter is treated mathematically as an unfolded cutter of rack type with infinite number of flutes. The flute number k ranges from 1 to ∞ with the understanding that flute number N+k is the same as flute number k. Also, $h_k(\beta)$ is defined only within a range of β determined by the flute number k and β_a such that h_k is constrained to lie between 0 and d_a. Only cutting points within that range are the active cutting points.

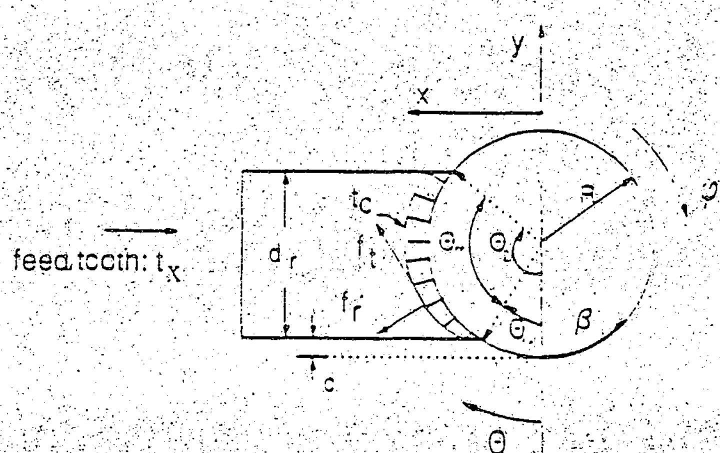

Figure 2 The radial cutting geometry in the work coordinate.

A nonrotating rectangular coordinate system xyz is also attached to the bottom center of the cutter with its origin coinciding with that of the βrh frame. The xyz coordinate moves in the x direction at the speed of t_x per tooth. In addition, a position variable θ in the xyz system is defined as the angular position of the cutting point in the workpiece as shown in Fig. (2). This xyz coordinate system is designated as the work coordinate. The cutter rotates about the z axis at angular velocity Ω, and the angular displacement of the cutter with respect to the xyz frame is represented by $\phi=\Omega t$. The angular position θ in the work of any cutting point at position β on the cutter is therefore

$$\theta = \phi - \beta \tag{2}$$

3. The Frequency Domain Model for Tangential Cutting force

With circular tooth path approximation [Matellotti 1941], the general chip thickness equation for an active cutting point on a cutter can be approximated as

$$t_c(\theta) = t_x \sin\theta \tag{3}$$

The cutter entry and exit angles, θ_{1r} and θ_{2r}, and radial range of cutting, θ_{rr} are dictated by the radial depth of cut, d, and cutter recess c as shown in Fig.(2) and are expressed by

$$\begin{aligned} \theta_{1r} &= \cos^{-1}(\eta_r + \eta_c - 1) \\ \theta_{2r} &= \cos^{-1}(\eta_c - 1) \\ \theta_{rr} &= \theta_{2r} - \theta_{1r} \end{aligned} \tag{4}$$

where

$$\eta_r = \frac{d_r}{D} \text{ and } \eta_c = \frac{c}{D}$$

By defining a rectangular window function,

$$w(\theta) = \begin{cases} 1, & \theta_{1r} \le \theta \le \theta_{2r} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

the chip thickness equation (3) can be rewritten to reflect the radial cutting configuration as follows:

$$t_c(\theta,\beta) = t_x \sin\theta\, w(\theta) \tag{6}$$

An empirical tangential cutting force equation for unit chip width by Sabberwal and Koenigsberger [1961] is written as

$$\begin{aligned} f_t(\theta) &= C\, t_c^{p}(\theta)\, t_c(\theta) \\ &= k_t(\theta)\, t_c(\theta) \quad , with\ k_t(\theta) = C\, t_c^{p}(\theta) \end{aligned} \tag{7}$$

The cutting pressure constant K_t corresponding to the average chip thickness t_c is often used in place of $k_t(\theta)$ to simplify the model and Eq. (7) is rewritten as

$$f_t = K_t t_x \sin\theta w(\theta), \text{ with } K_t = C\bar{t}_c^{\,p} \text{ and } \bar{t}_c = \frac{\int_{\theta_{1r}}^{\theta_{2r}} t_x \sin\theta \, d\theta}{\theta_{2r}-\theta_{1r}} = \frac{2t_x\eta_r}{\theta_{rr}} \tag{8}$$

A dimensionless function p_t is defined to be the normalized tangential cutting force per unit chip area per unit cutting pressure constant and is herein termed the elemental tangential cutting function,

$$p_t(\theta) = \frac{f_t}{K_t t_x} = \sin\theta w(\theta) \tag{9}$$

which is shown for a down milling case in Fig (3). Since $\theta=\phi-\beta$, $f_t(\theta)$ can also be written as $f_t(\phi-\beta)$, which expresses the tangential cutting force per unit chip width for a cutting point at position β as a function of angular displacement.

$$f_t(\phi-\beta) = K_t t_x p_t(\phi-\beta) \tag{10}$$

The total tangential cutting force associated with a cutter flute k can be obtained through an integration of Eq. (10) within the axial depth of cut in the axial direction

$$\bar{f}_{tk}(\phi) = \int_0^{d_a} f_t(\phi-\beta)dh_k \tag{11}$$

With change of variable from h_k to β using Eq. (1) and the change of integration limits from axial depth of cut to its corresponding angular range, Eq. (11) becomes

$$\bar{f}_{tk}(\phi) = \int_{(k-1)\beta_p}^{(k-1)\beta_p+\beta_a} f_t(\phi-\beta)\frac{R}{\tan\alpha}d\beta \tag{12}$$

Define the chip width density function of flute k, cwd_k, as

$$cwd_k(\beta) = \frac{dh_k}{d\beta} = \frac{R}{\tan\alpha}, \quad (k-1)\beta_p \le \beta \le (k-1)\beta_p + \beta_a \tag{13}$$
$$= 0, \text{ otherwise}$$

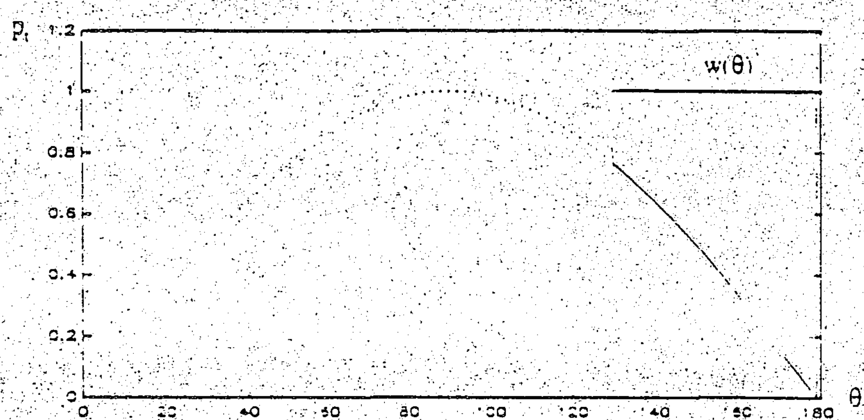

Figure 3. The elemental tangential cutting function $p_t(\theta)$ for a down cut configuration.

By substituting Eq. (13) into Eq. (12), Eq. (12) will take a convolution integral form as

$$\bar{f}_{tk}(\phi) = \int_{-\infty}^{\infty} f_t(\phi - \beta) cwd_k(\beta) d\beta \tag{14}$$

The total tangential cutting force of the cutter is the sum of forces of all flutes,

$$\begin{aligned} \bar{f}_t(\phi) &= \Sigma_{k=1}^{\infty} \bar{f}_{tk} = \int_{-\infty}^{\infty} f_t(\phi - \beta)\{\Sigma_{k=1}^{\infty} cwd_k(\beta)\} d\beta \\ &= \int_{-\infty}^{\infty} f_t(\phi - \beta) cwd_c(\beta) d\beta \\ &= K_t t_x \int_{-\infty}^{\infty} p_t(\phi - \beta) cwd_c(\beta) d\beta \end{aligned} \tag{15}$$

where the chip width density of the cutter cwd_c is defined to be

$$cwd_c(\beta) = \Sigma_{k=1}^{\infty} cwd_k(\beta) = \Sigma_{k=1}^{\infty} cwd_1(\beta - (k-1)\beta_p) \tag{16}$$

and is shown in Fig. (4).

The total cutting force is shown to be the convolution integral of local tangential cutting force and the cutter chip width density function. This convolution process is illustrated in Fig. (5) for a down-cut operation.

From the convolution theorem, the frequency domain representation of the tangential cutting force is the product of Fourier transforms of the local tangential cutting force and the cutter chip width density function. As the force expressions are in the angular domain, the frequency variable ω is referred to as the normalized frequency and its value of k is equivalent to k times the spindle frequency. The Fourier transform of the total tangential cutting force is expressed as,

$$\bar{F}_t(\omega) = F_t(\omega) CWD_c(\omega) = K_t t_x P_t(\omega) CWD_c(\omega) \tag{17}$$

The Fourier transform of p_t is

$$P_t(\omega) = \frac{1}{1-\omega^2}[(e^{-j\omega\theta_{1r}}(j\omega \sin\theta_{1r} + \cos\theta_{1r}) - e^{-j\omega\theta_{2r}}(j\omega \sin\theta_{2r} + \cos\theta_{2r})] \tag{18}$$

$P_t(\omega)$ can be reduced to simpler forms for up and down-milling processes and full-cut operation. For up-milling, $\theta_{1r}=0$,

$$P_t(\omega)\Big|_{\theta_{1r}=0} = \frac{1}{1-\omega^2}[1 - e^{-j\omega\theta_{2r}}(j\omega \sin\theta_{2r} + \cos\theta_{2r})] \tag{19}$$

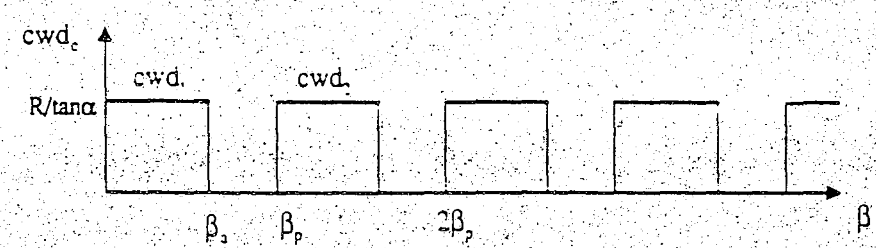

Figure 4. The chip width density function for a cutter flute, cwd_k, and the whole cutter, cwd_c

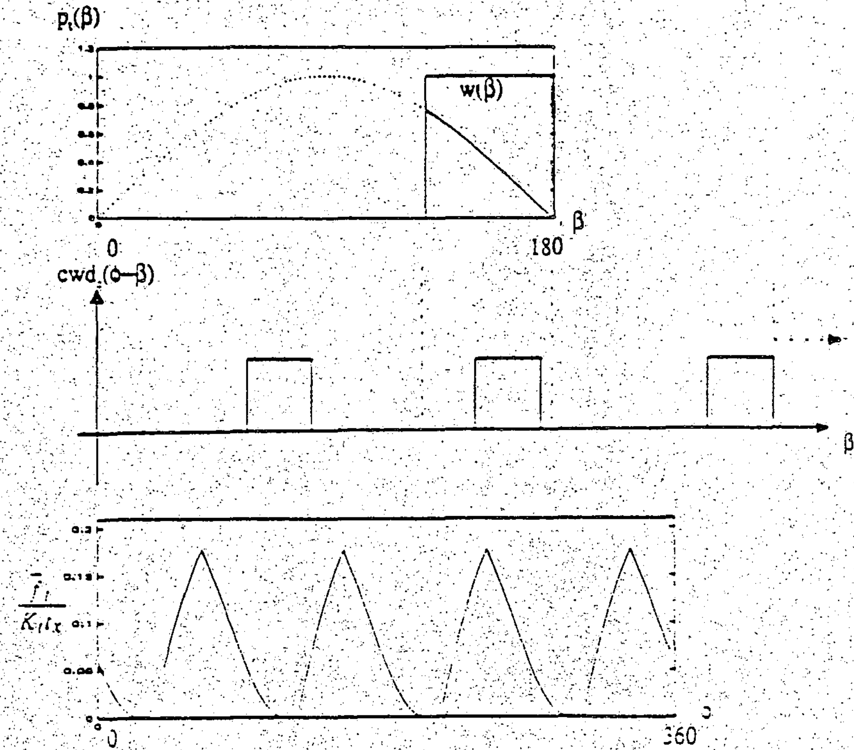

Figure 5 Angular convolution model of total tangential cutting force.

for down-milling, $\theta_{2r}=\pi$.

$$P_t(\omega)\Big|_{\theta_{2r}=\pi} = \frac{1}{1-\omega^2}[(e^{-j\omega\theta_{1r}}(j\omega\sin\theta_{1r}+\cos\theta_{1r})+e^{-j\omega\pi}] \tag{20}$$

and for full-cut operation,

$$P_t(\omega) = \frac{1}{1-\omega^2}(1+e^{-j\omega\pi}) \tag{21}$$

A three dimensional plot, Fig. (6), shows the frequency responses of Eq. (20) as functions of ω and η_r. The cross sections of Fig (6) for three values of η_r are presented in Fig (7), which indicates the DC value of P_t, $P_t(0)$ is equal to $2\eta_r$.

The Fourier transform of cwd_c can be shown to be

$$CWD_c(\omega) = N\sum_{k=-\infty}^{\infty}\delta(\omega-Nk)CWD_1(\omega) \tag{22}$$

$$\text{where } CWD_1(\omega) = \frac{2R}{\tan\alpha}\frac{\sin\frac{\omega\beta_a}{2}}{\omega}e^{-j\omega\frac{\beta_a}{2}}$$

$|CWD_1|$ is a sinc function with periodic zeros and a period of $2\pi/\beta_a$ as shown in Fig (8). Its magnitude is shown to have an envelope proportional to $1/\omega$, and its DC value is equal to the axial depth of cut. The frequency transform of cutter chip width density function is an impulse function with nonzero magnitude only at discrete frequencies Nk. Consequently, the total tangential cutting force will have frequency components only at normalized frequencies, Nk, and can be represented as a Fourier series function from Fourier integral formula.

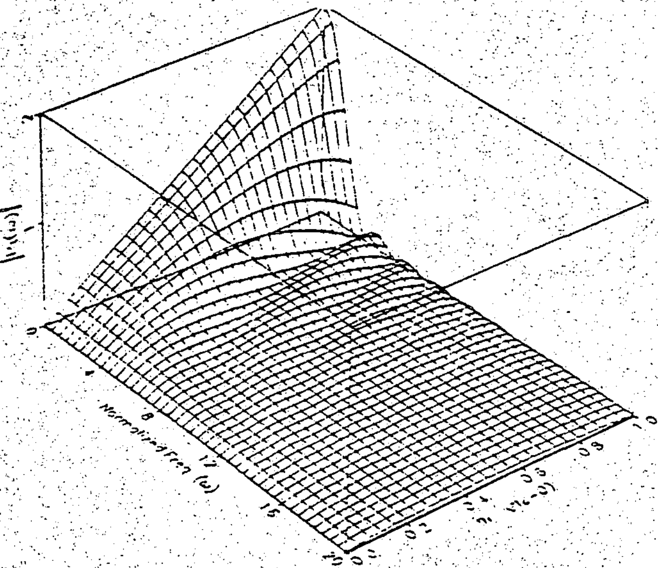

Figure 6. $|P_t(\omega)|$ as function of η_r and ω

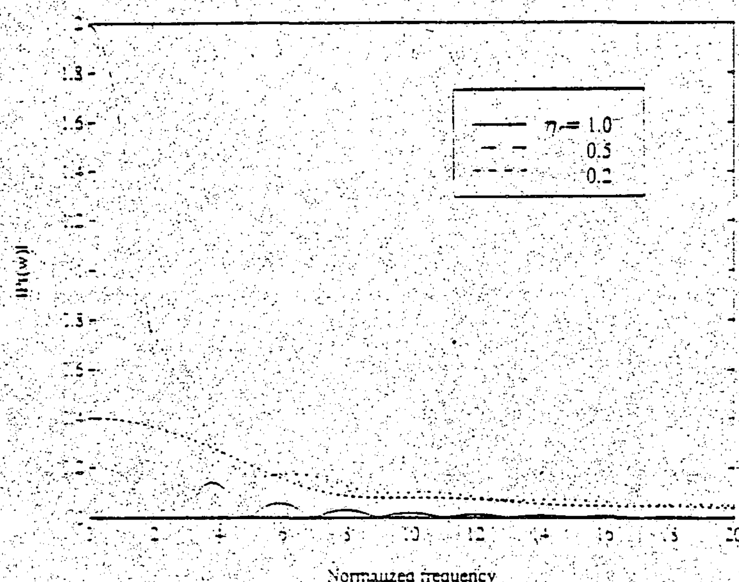

Figure 7. Cross sections of $P_t(\omega)$ in Figure 6 for three η_r's.

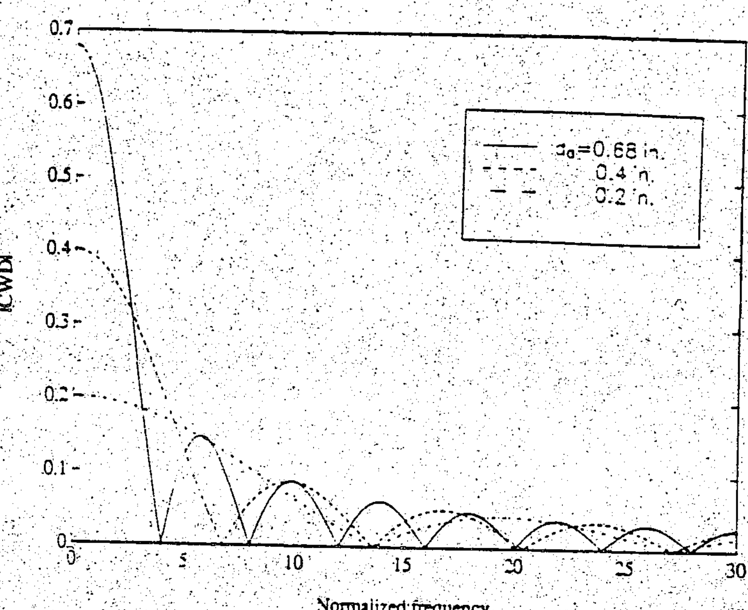

Figure 8. $|CWD_1(\omega)|$ for a cutter with R=0.25 in., α=30°

$$\bar{f}_t(\phi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{F}_t(\omega) e^{j\omega\phi} d\omega = \Sigma \sum_{k=-\infty}^{\infty} A_t[Nk] e^{jNk\phi} \quad (23)$$

where $A_t[Nk]$ is the Fourier series coefficients of the total tangential cutting force at normalized frequency Nk,

$$A_t[Nk] = \frac{NK_t t_x}{2\pi} CWD_1(Nk) P_t(Nk) \quad (24)$$

Also, the Fourier coefficients for the cutting torque follow from Eq. (24),

$$A_{tq}[Nk] = \frac{RNK_t t_x}{2\pi} CWD_1(Nk) P_t(Nk) \quad (25)$$

The different frequency components of total tangential cutting force in Eq. (17) and (24) is illustrated in Fig. (9) for a four flute cutter.

The average cutting force isimply the Fourier coefficient $A_t[0]$. With substitutions for $CWD_1(0)$ and $P_t(0)$, the average tangential cutting force is

$$A_t[0] = \frac{NK_t t_x}{2\pi} d_a \frac{d_r}{R}, \quad K_t = C(\frac{t_x \eta_r}{\theta_{rr}})^p \quad (26)$$

or the average cutting torque.

$$Tq_{avg} = \frac{NK_t t_x}{2\pi} d_a d_r \quad (27)$$

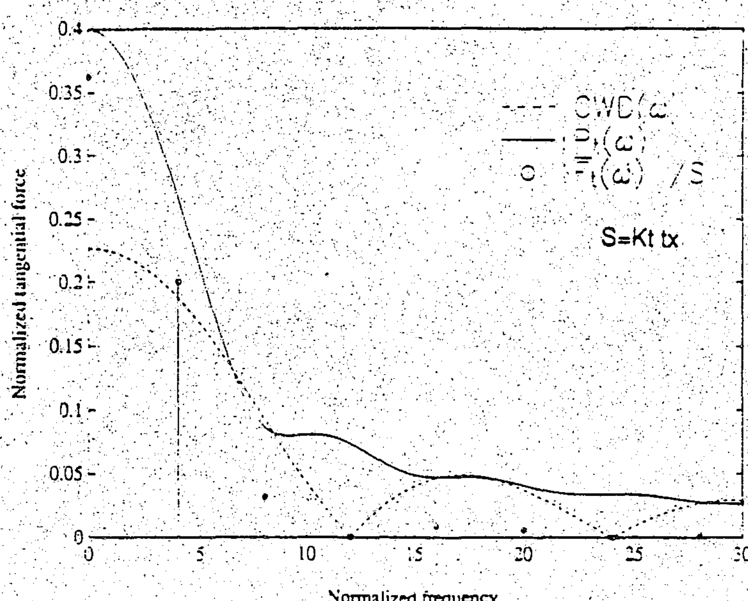

Figure 9. The frequency spectrum of total cutting force as the product of spectra of elemental tangential cutting function, $P_t(w)$, and chip width density function of the cutter, $CWD_1(\omega)$. The assumed cutting conditions are: cutter geometry with N=4, R=0.25 in., and $\alpha=30°$, da=0.226 in., dr=0.1 in and c=0 (down-cut).

The maximum cutting forces are more difficult to obtain. One approach is to compute the cutting forces for 1/Nth spindle revolution using the Fourier coefficients in Eq. (24) and identify the maximum value. Since the frequency transforms of cutting forces roll off inversely proportional to ω square, the higher harmonics will be relatively insignificant especially for cutter of large flute number. An approximate but more practical approach is to simply sum up the magnitude of Fourier coefficients of the first two harmonics components. That is

$$\left|\bar{f}_t\right|_{\max} = \sum_{k=-2}^{2} |A_t[Nk]| = |A_t[0]| + \sum_{k=1}^{2} 2|A_t[Nk]| \tag{28}$$

4. Model Verification and Discussions

To validate the closed form expression of Eq. (24), the values of At[Nk] as estimated from the expression and from stepwise integration are compared. For the instance of a cutting coefficient of

$$k_t = 569.14(t_c)^{-0.283} \quad \text{N/mm}^2 \tag{29}$$

which is obtained from actual cutting tests as reported in [Wang 1992] and within the range of values reported in [Kline et al. 1983, Altintas and Spence 1991] for 7075-T6 Aluminum. The true total tangential cutting forces were computed from numerical integration for four cutting configurations. The frequency spectra of these forces were then numerically computed using FFT software. These spectra, on the other hand, can be predicted from Eq. (24). This is accomplished by first finding the average cutting constant K_t and then computing the Fourier coefficients of the analytical model. The average cutting constant is obtained by rearranging Eq. (24) into

$$K_t = \frac{R2\pi}{Nt_x d_a d_r} A_t[0] \tag{30}$$

Table 1 lists the average cutting constant for each cutting configuration. True tangential force profiles and their frequency spectra are shown in Fig. (10) along with the predicted Fourier coefficients. Close agreement between the numerical integration results and the analytical model estimates supports the closed form expression for the Fourier coefficients of the tangential milling force.

The model validity were further examined in terms of the comparison between cutting forces in the normal and feed directions, which have been reported in previous work [Wang et al. 1991], and those obtained with the inclusion of a local radial cutting force model.

The radial local cutting force by Tlusty and MacNeil [1975] is expressed as

$$f_r(\theta) = K_r f_t(\theta) \tag{31}$$

Table 1. Assumed cutting conditions for comparisons of cutting force dynamic components from true cutting forces and analytical predictions in Figure 10. Cutter geometry: N=4, D=5/8 in.,(15.875 mm), Helical angle: 30 °. Material: 7075-T6 Aluminum. Down milling.

case	Cutting Conditions			Average Cutting
	d_r (mm)	d_a (mm)	t_x (10^{-3} mm)	K_t (N/mm sq.)
1	3.18	2.16	109.8	1,271.7
2	3.18	10.8	109.8	1,271.7
3	7.94	15.11	64.3	1350.0
4	15.88	6.48	35.7	1,917.9

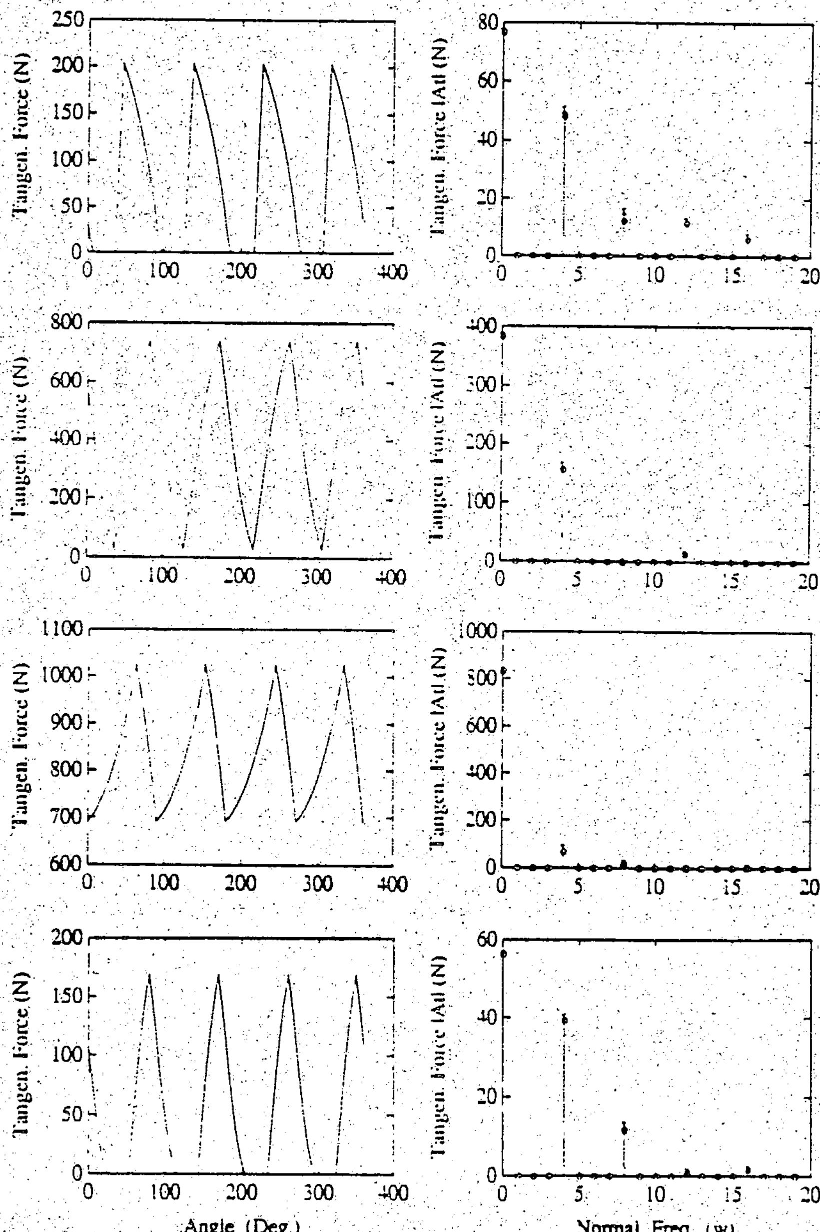

Figure 10. Cutting force profiles and spectra from numerical simulation for cases 1-4, Table 1 (from top). Circles are from simulation results. Analytical predictions of dynamic Fourier coefficients are given by 'x'.

where K_r, the ratio between local radial cutting force and tangential cutting force, is usually represented by an empirical expression similar to Eq. (7) for K_t, and is likewise assumed to be a constant corresponding to the average chip thickness. Expressing the local tangential and radial cutting forces in the xyz coordinate, the local cutting force f_x and f_y per unit chip width become

$$\begin{pmatrix} f_x(\theta) \\ f_y(\theta) \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{pmatrix} f_t(\theta) \\ f_r(\theta) \end{pmatrix} = K_t t_x \begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{pmatrix} p_1(\theta) \\ p_2(\theta) \end{pmatrix} \tag{32}$$

where

$$p_1 = \frac{\sin 2\theta}{2} w(\theta), \quad p_2 = \frac{1-\cos 2\theta}{2} w(\theta)$$

are the x and y components of the p_t function. Substituting these local cutting force functions into Eq. (15), the same convolution integral form as in [Wang et al. 1991] is obtained, and consequently, the same frequency domain model results. The frequency domain representations of the total x and y forces have been previously reported. Their Fourier series coefficients are shown here:

$$\begin{pmatrix} A_x[Nk] \\ A_y[Nk] \end{pmatrix} = \frac{NK_t t_x}{2\pi} CWD_1(Nk) \begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{pmatrix} P_1(Nk) \\ P_2(Nk) \end{pmatrix} \tag{33}$$

5. Summary

A frequency domain model for the tangential cutting force and cutting torque has been presented. Starting from the local mechanistic tangential cutting force model, the total cutting force in the angular domain is shown to be the convolution integral of the local cutting force function and the cutter chip width density function. The local cutting force function represents the chip thickness variation and the radial cutting configuration. The chip width density functions for a cutter flute and the whole cutter are related to the cutter geometry and axial depth of cut. The frequency domain model is obtained through convolution theorem as the product of Fourier transforms of local cutting force and chip width density functions. Fourier series coefficients of the total tangential cutting force are derived as algebraic function of cutting parameters. The effects of various cutting parameters on the total cutting forces thus can be explicitly characterized, and the total cutting forces can be computed efficiently using the Fourier coefficients. The frequency domain model is verified by numerical simulation and comparing the extended normal and feed force models with the ones reported in the previous work using the same local radial force model.

6. References

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