MATHEMATICAL MAN-MACHINE ANALYSIS FOR A WEAVE ROOM WITH COLLABORATING WEAVERS

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MATHEMATICAL MAN-MACHINE ANALYSIS FOR A WEAVE ROOM

WITH COLLABORATING WEAVERS

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SUMMARY

This research presents an approximate analytical method for finding the mean idle time in a weave room of certain size when a number of s servers are employed to tend these looms and the servers are collaborating.

The mathematical model presented is based on the birth-and-death process and is a series of "birth-and-death" equations at "steady state", which, when solved, yield the probability of being in any of the feasible states. It allows the user to specify the arrival rate, the three service rates, the mean walk time, the occurrence rate of each type of break, the number of machines and the number of servers, gives the mean waiting time, and the probabilities of having n breaks unattended at a particular time.

The methodology and the computer results presented were developed for use by the weave room manager in determining the optimum assignment of weavers needed for a certain number of looms. The research concludes that the program produces accurate and useful estimates when the rate of arrivals does not exceed the sum of the rate of service and walk rate, that is, when steady state conditions exist. It also provides evidence, judging from the execution time for each run, that an analytical method may be less costly than simulation in further studies of the same type.

CHAPTER I

INTRODUCTION

Weaving started over six thousand years ago in Europe and Egypt, but until recent times, weaving was a skill associated with domestic life. The Industrial Revolution brought weaving from home to the factories and the first weaving mills appeared around 1790. Much progress has been made since then in the technology of weaving, not only in the area of new machine development, but in the economic and human aspects of it as well.

Knitted and "non-woven" materials are challenging the weaving in recent years. This creates the need for rapid development of new technology and improvement of weaving efficiencies in order to reduce weaving costs, which in turn will reduce the cost of woven products.

In deciding on the type of work the weaver has to perform, two systems are commonly used. In the first, the "one man" system, a relatively small number of looms is assigned to the weaver and he is expected to perform any type of work related to weaving. In the second system, the "helper-weaver" system, the number of different things the weaver has to do is very limited and the amount of time taken on each thing is very small (see table on page 2). The weaver has a "helper-weaver" for those jobs which take more than one to two minutes in repair time (for example, changing a warp beam, getting a new warp going, fixing warp breaks when a number of warp ends break at the same time).

Table 1. Typical Weaver's Tasks and Repair Times Required for a Shuttle Loom

Type of Stoppage	Occurrence Rate per Loom Hour	Repair Time (Min.)
Warp Break*	1.00	0.85
Weft Break	0.30	0.33
Slack End and Others	0.15	0.58

* See Appendix 1 for definitions.

"Interference" is the term used for the time that is lost by a loom which is stopped while the weaver is working on other looms, and it is this interference that limits the number of looms that can be run efficiently. Efficiency also depends on the reliability of the looms, however, interference is what changes with manpower. Events occurring at random, such as breaks, cannot be calculated from engineering data. They must be observed and counted over a period of time to obtain the average frequency of occurrence. Then it may be assumed that the system of chance causes will continue to operate in the same manner. Since the stops occur at random, prediction of the weaving efficiency is complicated. The weave room manager needs to be able to estimate the weaving efficiency before assigning looms to a weaver, and he should be in a position to calculate the weaving efficiency for any new assignment.

1.1 Objective and Procedure

The primary objective of this research is to develop a usable analytical model for the loom-assignment problem that will determine the average number of idle looms, given the size of the room and the number of collaborating weavers tending these looms. Representative data for arrival and service rates are used. One of the features of the model is that it takes into account the walking time (transit time) from one break to the next. The mean of this walking time distribution is a function of the dimensions of the particular room under consideration. A secondary goal is to estimate the probability of being in a particular state for all the possible states, where a state is defined as the number of idle and unattended machines and the number of idle machines being serviced.

If there are s servers and s + 1 or more machines are stopped at the same time, only s machines can be attended at once, and thus, production is lost while machines are awaiting attention. This phenomenon is called interference. Production is lost both while machines are being repaired and while the machines are awaiting attention. The rate of production depends on the frequency of repair, repair times, and total number of machines. The machines break basically for three reasons and there are three types of service rates, all of which are less than one minute.

The procedure of this investigation is as follows:

(1) Write the general form of the birth-and-death equations for this particular problem with all of the assumptions and constraints.

(2) Write a computer program that will generate all of the possible states and all of the birth-and-death equations explicitly and store all of the nonzero coefficients of these equations.

(3) Solve these equations simultaneously to come up with the

probabilities of being in the different states and also estimate the average number of idle machines for a given number of operators.

1.2 Review of Literature

Numerous studies have attempted to show how to determine what number of machines to assign to one or more operators for best results. In 1928, Tronton C. Fry of the Bell Telephone Laboratories solved a problem of congestion of telephone lines. He was interested in calculating the expected delay in service if telephone calls all of the same length are accommodated through a single channel (23). In 1932, Pinkerton (55) developed a solution to the general problem of machine interference, based on the laws of probability. Wright (60) and Duvall (19) converted Dr. Fry's solution of distribution of "calls" into terms of interference and in the same year we have the first "work assignment" study by Alford (1). One year later, in 1937, Deuel (18) stressed the analysis of job and time studies as a means to increase both profits and wages.

For approximately the next ten years, nothing major was done in this area, until 1949, when W. Dale Jones (28) at Georgia Institute of Technology developed a wage incentive plan for multiple machine assignments tended by one operator. Jones (29), (30), (31) also presented a mathematical method for determining machine interference. In addition, he provided the means of preparing wage incentive curves for interference conditions. The years 1950 to 1970 have seen the publication of several papers dealing with the effective rate of production of a group of machines, each liable to break down and need repair, which are attended by one or more operators. Henry Ashcroft (3), in England, developed tables for the average number of machines running, and in the case of breakdowns requiring constant repair time, the machines being attended by one operator. He also showed how to solve this problem for a general distribution repair time. O'Connor (43 through 52) popularized the use of the Ashcroft tables and applied them in many phases of the Textile Industry, particularly in weaving. He tried to determine what number of machines to assign to one operator for best results.

The next contributions of note in this area of model development were made by Denholm, Benson, Cox, Stout, Brunnschweiler, Fetter, Bowman, Lomicka and Allderige. Stout (59) directed attention only toward illustrating in a general way the order of time lost through synchronizations, and how this may be expected to vary with the operating conditions. Using a rather different method, Benson and Cox (6) gave tables of efficiency and operator utilization in the case of exponentially distributed repair times. They also considered the situation where a team of operators attend the machines and the case of a team of m operators specializing in one of m types of stoppage, but there was no account for patrolling time. Brunnschweiler (10) first discussed patrolling problems in 1954, and Mack gave tables of efficiency for constant walking time, constant repair time and unidirectional patrolling. Fetter (20) was the first to treat the machine assignment problem for the case of collaborating servicemen, but he ignored the walking times involved.

In the area of machine delay time, considered both a random and systematic occurrence, models were first developed by Lomicka and Allderige (37). Palm (53), a Swedish writer, demonstrated that one can always obtain better usage of workers (at least, theoretically) if several workers cooperate in servicing a larger group of machines. Kemp and Mack (34) in 1961, gave tables for calculating machine interference in automatic weaving. King (36) looked at the problem of optimum assignment of machines in 1966 and provided a quick and ready means of solution to such problems in practice. A very interesting article by Bawa and Nair (4) followed in 1966. The article considered the assignment of cooperating workers to a group of machines when the service time follows exponential distribution. The computation of the economic advantage of this assignment over that where there is no cooperation is illustrated with an example, but the effect of walk times is not included in the analysis. In 1970, King (35) examined the problem of determining the optimum size of a repair servicing crew for a given number of automatic machines and presented charts applicable to the Poisson arrival-exponential service time model.

From 1970 to the present time, contributions of note in this area were made by Bunday and Jackson, Mack, Crabill, Horn, Hoover and Freeman, and Maritas. Bunday and Jackson (11) looked at the efficiency of a group of machines bi-directionally traversed by one operator when walking time is constant. Crabill (16) developed a continuous-time Markovian model to consider the costs dependent on the service rates and costs due to lost production. The model seeks to minimize the long-run expected average cost of the system. Bunday and Mack (12) solved the problem of finding the effective rate of production of a group of n machines bi-directionally traversed by one operator. In 1973, Freeman (21) presented a general machine interference problem and described a general purpose computer simulation which estimates interference times and work load.

In 1977, Maritas (39) studied the machine interference problem with more than one operator, when the machine running times follow the negative exponential distribution and the repair times follow the Erlang distribution.

While all the above papers have dealt with some aspect of the machine-assignment problem, they provide no comprehensive model for the machine assignment problem that involves team-work and takes into account walking times involved. A number of papers (20), (4), (34), (53) mentioned that walking times can become important if the number of machines in the assignment is large, but no one discussed the effect of walking times on the optimum assignment.

Considering that one can walk approximately 240 feet per minute, one can see how walking can become important in a weave room of 20,000 square feet area (200 feet by 100 feet), for example. It will be shown in Appendix B that the mean walk rectilinear distance for a room of this size is approximately 100 feet. This means that one can complete 150 walks per hour, which is comparable to the number of service completions per hour. This fact says that one cannot neglect the walk times, because they are of the same magnitude as service times.

Prior to 1950, one could neglect the walk times because the assignments were small (less than 15 looms per weaver). Today the vast majority of looms have been automated, and a weaver can handle anywhere from 25 to 50 or more looms. In this case, one can see how walk times can become important in determining the optimum number of operators for a given number of machines.

Throughout this study, the terms operator, server, weaver, and worker will be used indiscriminately.

CHAPTER II

DEVELOPMENT OF THE METHODOLOGY

2.1 Problem Statement and Background

The theory of waiting lines or queues concerns situations in which units require a particular service on a "random" basis, i.e., at intervals that can be described only through a probability distribution. Usually the service facilities are limited, such that at times, a waiting line of units builds up. Looms stopping at random, waiting for weaver attention, is one example of industrial queuing situation.

The term "at random" is a key feature of queuing problems. If units need service on a regular, production basis, service facilities can be planned to be available on a production basis with little waiting involved. However, in the typical queuing problem, one does not know how many units, if any, will need service in the next hour; the average number that will need service in a shift may be known. Likewise, the service time may vary widely and may be described as more or less "random" with some approximately known average.

In solving a queuing problem, one seeks at least the average number of idle units and the average percentage of idle time of the service channels. As an alternative to analytical solutions, one can always simulate the queuing process, using computer methods, like the General Purpose System Simulator (GPSS). Unfortunately, simulation has been time-consuming and costly even when a computer was used, due to the initial programming effort. As a result, queuing problems in Textiles are normally approached by guesswork or by trial and error methods.

The main advantages of the queuing theory approach are its simplicity and the availability of computational procedures to determine solutions. The main disadvantage is the assumptions that are forced upon the problem in order to facilitate the mathematical solution. The assumptions for this particular model will be stated in section 2.3.

This study considers a production system consisting of a finite number of productive units —looms— that are subject to failure during use. We assume that the machines fail, or break down, with an exponential time-to-failure distribution, i.e., according to a Poisson process.

We have three types of stoppage and to each type of stoppage corresponds a particular frequency and duration. Both frequency and duration of stoppages vary with different conditons of working, being determined partly by the skill of the operators and partly by the intrinsic properties of the yarn. Typical values were given in Table 1 of Chapter 1.

Walking time is important in this model. This is the time it takes for a weaver to move (with his tools) from one machine to another which requires servicing. It also includes the time needed to recognize a call for service and the time for taking the correct position at the machine.

2.2 Walk Time Distribution

A typical weave room will look something like Figure 1. The looms stop for one of three reasons given in Table 1 (warp breaks, weft break, slack end). When the operator sees a break, he walks to that particular loom, positions himself along the front or the back of the loom (depending on the type of break), and starts working on it. Front and back of a loom in this picture will be the two sides parallel to the x-axis. The weaver restarts the loom and stays there looking around watching for the next break.

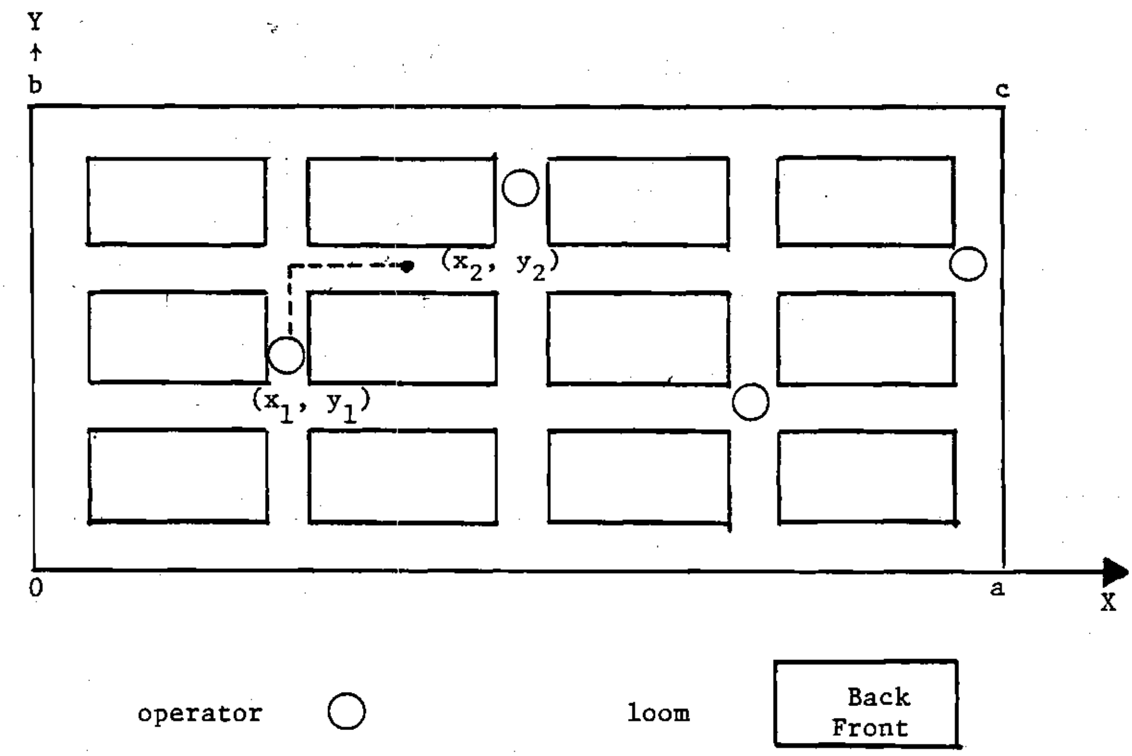


Figure 1. Typical Weave Room

It was explained in Chapter I why the walk time from one break to the next one is important. The question of how one should go about estimating the average walk time now arises. The problem can be stated as follows: Given a weave-room A, with dimensions a by b ($a \geq b$) as indicated in Figure 1; if (x_1, y_1) is a random location of an operator and (x_2, y_2) is a random location of a break and both points are distributed uniformly in the rectangle Oacb, what is the distribution of

$z=|x_1-x_2|+|y_1-y_2|$ where z is the rectilinear distance between the two points.

It is assumed that the operators that are not busy are randomly located in space because an operator remains at his job site when he finishes the job.

It turns out (Appendix B) that z has the following probability density function:

$$f_z(z)=\begin{cases} \dfrac{4}{a^2b^2}\left[abz - \dfrac{az^2}{2} - \dfrac{bz^2}{2} + \dfrac{z^3}{6}\right] & a\le z\le b \\[2ex] \dfrac{4}{a^2b^2}\left[-\dfrac{b^2z}{2} + \dfrac{ab^2}{2} + \dfrac{b^3}{6}\right] & b\le z\le a \\[2ex] \dfrac{-4}{a^2b^2}\left[-\dfrac{a^2b}{2} - \dfrac{ab^2}{2} + abz + \dfrac{a^2z}{2} - \dfrac{a^3}{6} + \dfrac{zb^2}{2} - \dfrac{bz^2}{2} - \dfrac{az^2}{2} + \dfrac{z^3}{6} - \dfrac{b^3}{6}\right] & a\le z\le a+b \end{cases}$$

Figures 2, 3, and 4 show plots of $f_z(z)$ for a=80, b=20; a=50, b=50 and a=99, b=1 respectively.

In the model below it will be assumed that the walk-time distribution is exponential. This assumption becomes more and more realistic as a>>b. However, it clearly leads to some inaccuracy when a≅b. In the limit this distribution is exponential in the one dimension case. A two dimensional walk model is a better suggestion. In this case the walk-

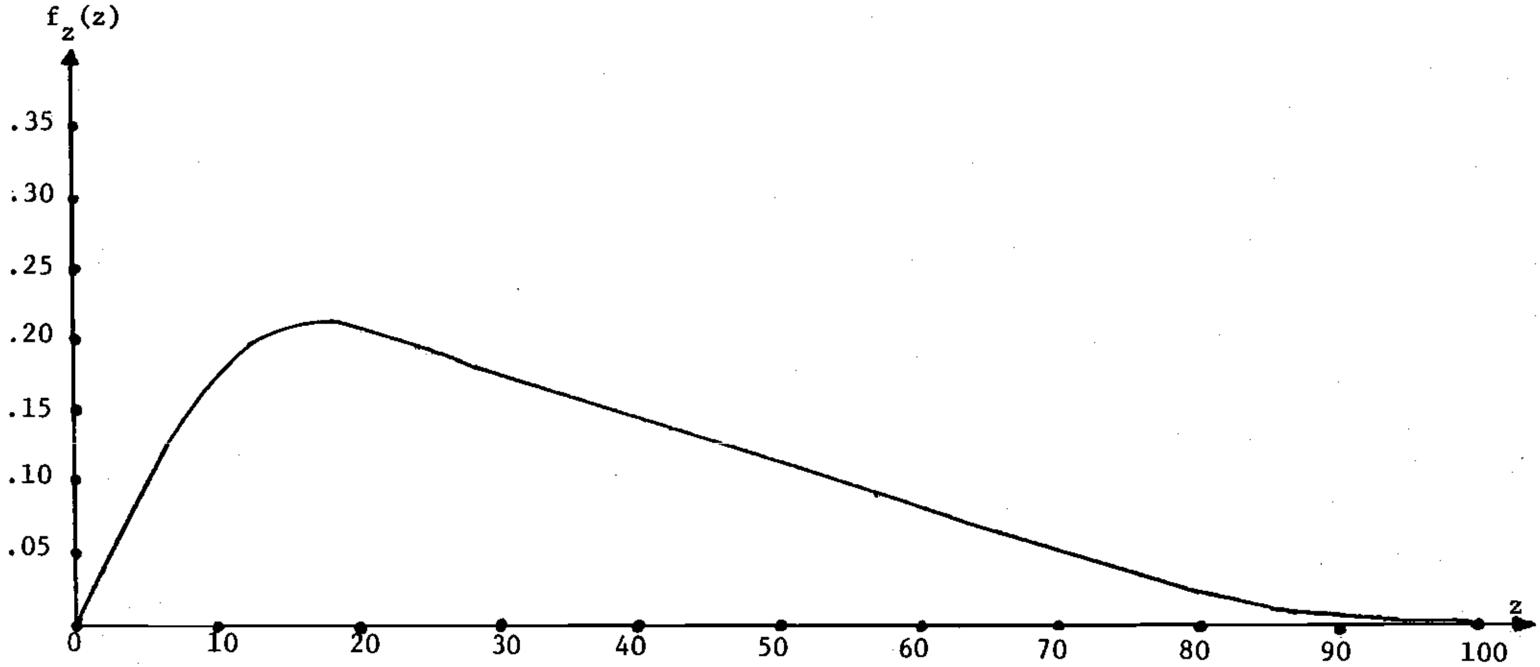


Figure 2. $f_z(z)$ for $a = 80$ $b = 20$

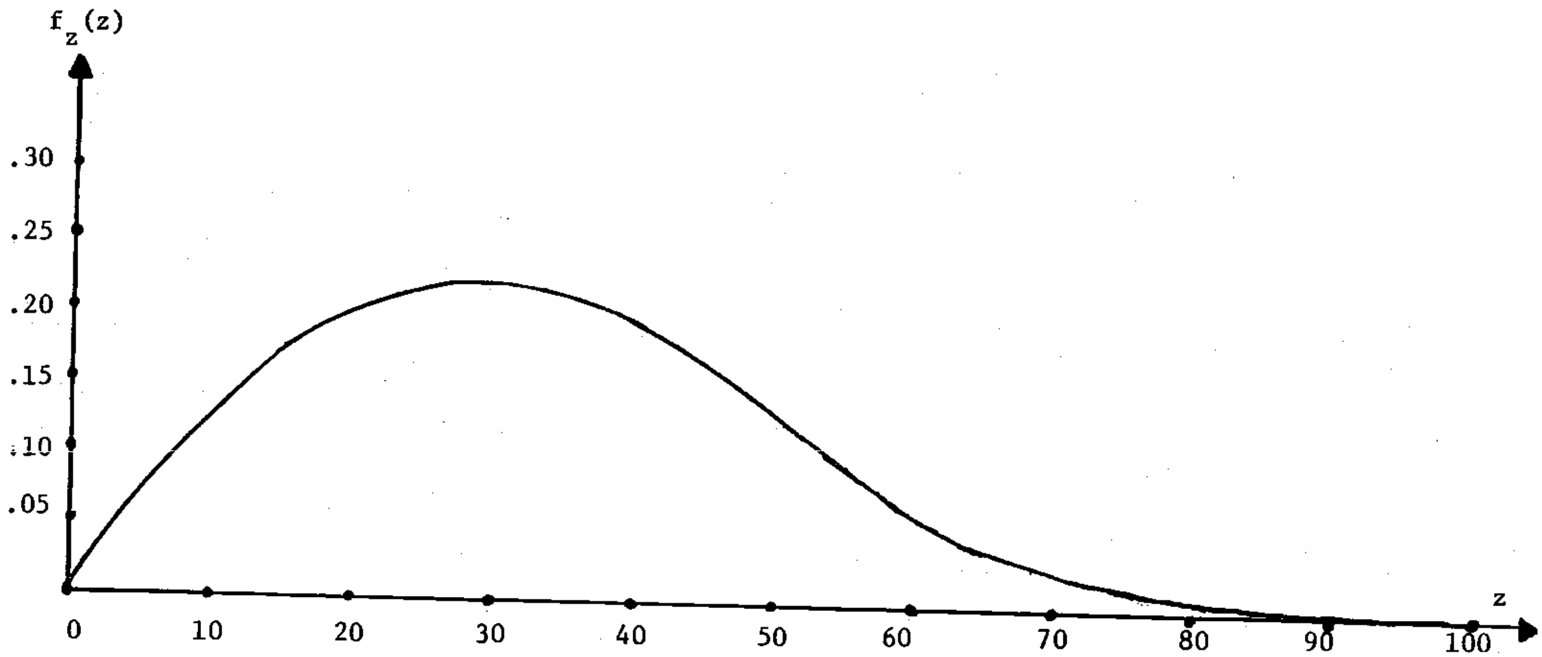


Figure 3. $f_z(z)$ for a = 50 b = 50

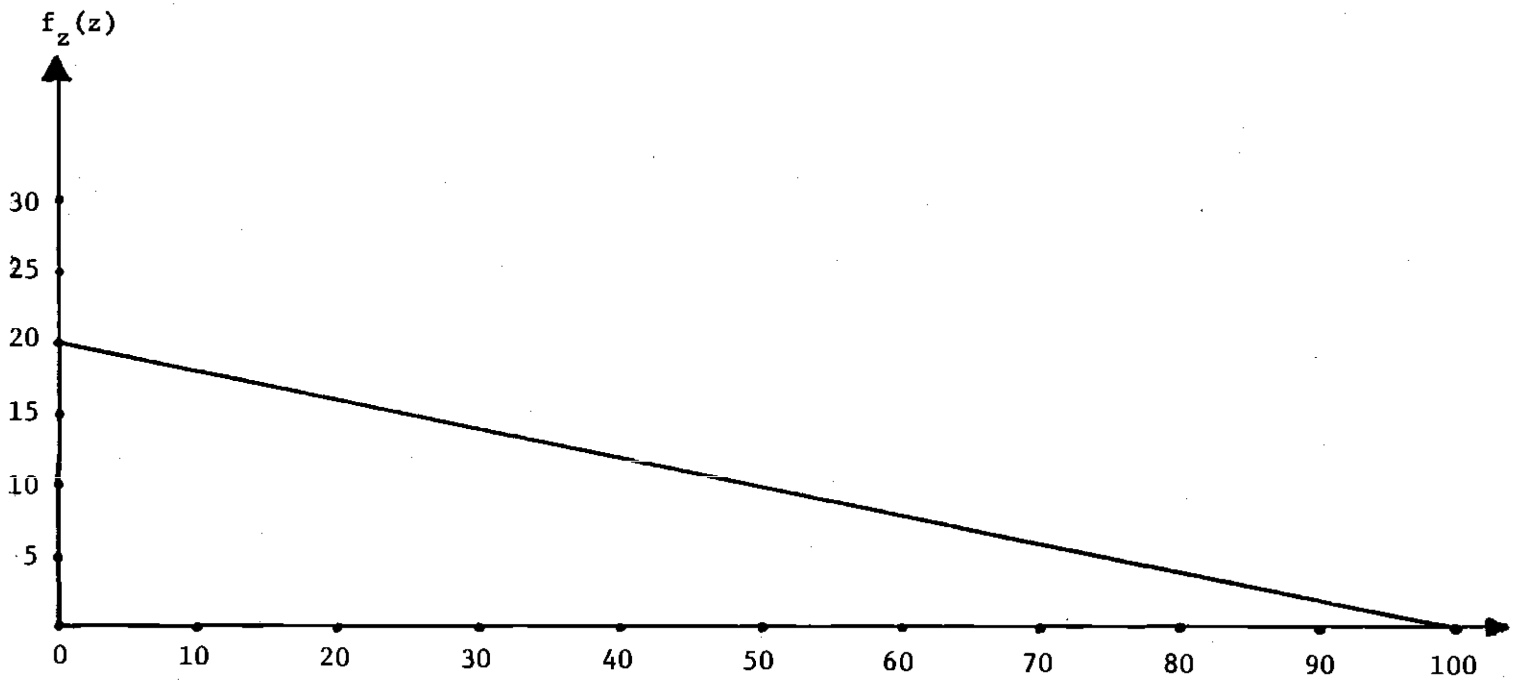


Figure 4. $f_z(z)$ for $a = 99$ $b = 1$

time distribution is the sum of two exponential distributions (i.e. a two phase exponential) which obviously is a better walk model but this is at the expense of an increased number of states. In deriving the walk-time distribution, the implicit assumption is that the servers are uniformly distributed in the room at all times. Another factor that enters the walk-time distribution calculations is the shape of the particular room under consideration.

2.3 Assumptions

When applying mathematics to the solution of a waiting line problem, the system to be studied must be described mathematically, and the result is often referred to as a mathematical model. The usual procedure is to construct a mathematical model of the system and then to study its properties. Certain approximations and simplifications must be made when constructing a mathematical model since it is not possible to accurately represent the real world. There are many reasons for this. One is that it is essentially impossible to find out what the "real" world is actually like. Another is that a very accurate model of the "real" world can become extremely difficult to work with mathematically. A final reason is that accurate models often cannot be justified on economic grounds. In order to develop an approximate analytical model for this system, with sufficient detail and relatively short set up and execution times, the following assumptions were imposed on it.

It was assumed that:

1. Variations in operator effort are neglected.

2. The service times and the walk times follow exponential distributions.

3. Time between breakdowns of the machines, all types together, are assumed to occur exponentially during the running time at an average rate of $\frac{1}{\lambda}$ per machine per hour.

4. There is no argument as to who should attend to a call when it arises. It is assumed that the closest free server will take it, and if there is no free server the call will have to wait. Since calls occur at random, there will be an equitable distribution of the work load among the weavers in the long run, and, hence, this assumption is justified.

5. In the states where the number of machines requiring service exceeds the number of weavers, a queue will exist.

6. No machine is re-started twice for different stops without the restarting of another machine in between.

7. The probability of a particular machine failing does not depend on the state of other machines nor on the time which has elapsed since it was last repaired.

8. All of the breakdowns require the service of only one weaver.

2.4 Birth-and-Death Equations

A careful analysis showed that the problem here could best be described by a system of birth-and-death equations. The theory of birth-and-death processes, developed mostly by Feller, is part of the subject matter commonly called stochastic processes. Typical birth-and-death process examples would be (a) a city whose population is $N(t)$ at time t, (b) a telephone switchboard where $N(t)$ is the number of calls occuring in an interval of length t, (c) a queue, where $N(t)$ is the number of customers waiting or in service at time t. A variation of this last example is the problem that this thesis is attempting to solve. An important part of the birth-and-death processes, and the queuing problems in general, is being able to describe the state of the system at each time interval of length t. The state of the system is the number of customers in the queuing system.

The state of the system at time t ($t \geq 0$) is given by (i,j1,j2,j3)= N(t) where i = number of idle machines. The machines are idle for one of three reasons (warp break, weft break, other) and are waiting for the operator(s) to work on them. j1 = the number of machines that are down because of a warp break, and are now being worked on. j1 is also the number of operators working on j1 machines that are down because of a warp break. One operator is required per machine per break. Likewise, j2 is the number of operators working on j2 machines that broke down for reason number two, a weft break, and j3 is the number of operators working on j3 machines that are idle because of a slack end or other reason. Walk and repairs are separated because walk is differentiated by the number of available servers (closest server notion) and repairs are differentiated by the job type (three types of jobs).

P(i,j1,j2,j3) is the probability of being in state (i,j1,j2,j3). For example, P(3,1,2,0) will mean that there are three idle machines that nobody is working on (simply because there is no server available); one server is working on a machine that has stopped because of a warp break; two servers are working on two machines that are down because of weft breaks, and there is no machine idle because of a slack end or any other reason.

The birth-and-death process describes probabilistically how (i,j1,j2,j3) change as time increases. Birth refers to the arrival of a new break into the queuing system, and death refers to a service completion of a break and the restarting of a machine. Generally speaking, the birth-and-death process says that individual births and deaths occur randomly and their mean occurrence rates depend only upon the current state of the system.

The general assumptions imposed on all birth-and-death processes will apply here, too.

2.5 Derivation of the Steady State Equations

The key principle of the birth-and-death process is the RATE OUT = RATE IN principle, which says that for any given state of the system, the mean rate (expected number of occurrences per unit time) at which the system enters that state must equal the mean rate at which it leaves. The equation expressing this principle of "conservation of flow" is called the balance equation for the state.

Before any attempt is made to write the "steady state" birth-and-death equations, one must have a clear understanding of what a state is and what the possible states are. We defined the state of the system as N(t)=(i,j1,j2,j3).

In order to come up with all of the possible states, one must keep in mind the following simple rule: $j1 + j2 + j3 \leq s$ (See Appendix C for definitions). This rule states that if the number of weavers for a particular weave room is s, then at any time the number of busy weavers is less than or equal to s. By knowing the values of s and i, one can come up with all of the possible states, (i, j1, j2, j3). For example, if i=0 and s=2, there are a total of 27 states, 10 of which are possible, namely the states underlined in Table 2. State nine, for example, is not a valid state because if there are only two servers, four of them can not be working at one particular time.

As i increases, the number of states also increases linearly, so for the same value of s and i=0, 1, 2, 3, the number of possible states is 10, 20, 30, 40 respectively. When s increases and i remains constant, the number of states increases in combinatorial fashion.

Table 2. Possible States for the Case of Two Servers and No Idle Machines

1.	0000	10.	0100	19.	0200
2.	0001	11.	0101	20.	0201
3.	0002	12.	0102	21.	0202
4.	0010	13.	0110	22.	0210
5.	0011	14.	0111	23.	0211
6.	0012	15.	0112	24.	0212
7.	0020	16.	0120	25.	0220
8.	0021	17.	0121	26.	0221
9.	0022	18.	0122	27.	0222

Once the possible number of states is known, the next logical step would be to find how one can go from one state to the other. Figure 5 gives a schematic of the sequence of the elements involved in the process of fixing a break.

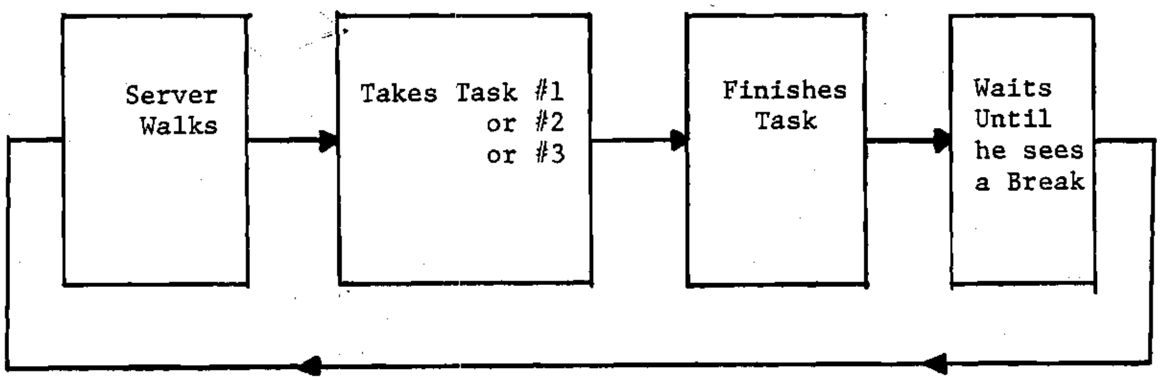


Figure 5. Process of Fixing a Break

One can see that a server can not go from one task to another directly, because there is a walk involved. The ways to enter a state are:

1. Via an arrival.
2. Via a service completion.
3. Via a walk completion.

The ways to exit a state are:

1. Via an arrival.
2. Via a service completion.
3. Via a walk completion.

Consider, for example, state (0, 0, 0, 0) for the case of s=2. The only way to get out of this state is by an arrival. An arrival would mean transition from state (0, 0, 0, 0) to (1, 0, 0, 0). There can not be any service completion or walk completion since no one is working or walking. The ways to get into this state are: from state (0, 1, 0, 0) via a service completion of a warp break, from state (0, 0, 1, 0) via a service completion of a weft break, and from state (0, 0, 0, 1) via a service completion of a slack end. These are the only ways.

Utilizing the RATE OUT = RATE IN principle, one can write the "steady state" birth-and-death equation for this state as follows:

$$\lambda P(0, 0, 0, 0) = \mu_1 P(0, 1, 0, 0) + \mu_2 P(0, 0, 1, 0) + \mu_3 P(0, 0, 0, 1)$$

As a second example, let us consider state (0, 1, 0, 0). The ways to exit this state are by an arrival, which would mean transition from state (0, 1, 0, 0) to state (1, 1, 0, 0), or by service completion of a warp break, which means going from state (0, 1, 0, 0) to state (0, 0, 0, 0). There are four ways to enter state (0, 1, 0, 0). One way is by having both weavers working on two warp breaks and one has just finished. This can be expressed as $2\mu_1 P(0, 2, 0, 0)$ and is an expression of the rate of transition from state (0, 2, 0, 0) to state (0, 1, 0, 0). The second way is from state (0, 1, 1, 0) by completing a weft break service. This is expressed as $\mu_2 P(0, 1, 1, 0)$. The third way is from state (0, 1, 0, 1) by completing a slack end service and the rate of transition is $\mu_3 P(0, 1, 0, 1)$. The final way is by completing a walk from state (1, 0, 0, 0). The rate of this transition from state (1, 0, 0, 0) to state (0, 1, 0, 0) is $2\alpha_1 P(1, 0, 0, 0)$ where 2 is the number of available servers plus one and $\alpha_1 = P_1 \alpha$ as defined in section 3.2. Therefore, the balance equation for state (0, 1, 0, 0) would be

$$P(0, 1, 0, 0)(\lambda+\mu_1) = 2\mu_1 P(0, 2, 0, 0) + \mu_2 P(0, 1, 1, 0) + \mu_3 P(0, 1, 0, 1) +$$

$$2\alpha_1 P(1, 0, 0, 0)$$

Similarly, one can write the balance equation for state (1, 0, 0, 0) as

$$P(1, 0, 0, 0)\ (\lambda+2\alpha) = \lambda P(0, 0, 0, 0) + \mu_1 P(1, 1, 0, 0) + \mu_2 P(1, 0, 1, 0) + \mu_3 P(1, 0, 0, 1)$$

The 2α on the left side of this equation represents the number of available servers times the rate of walk.

With the conservation of flow principle in mind and a few other simple rules that appear in the next section, one can proceed to write the general form of the birth-and-death equations.

2.6 Model Formulation

For a given state (i, j1, j2, j3) the service rates for the three different breaks (warp, weft, slack end) are $\mu_1 j1$, $\mu_2 j2$ and $\mu_3 j3$, respectively. The walk completion rate (WCR) is

$$\text{WCR} = (\text{number of servers now walking})\ \alpha_k\ (\text{number of available servers} + 1)$$

where $\alpha_k = P_k\alpha$ for k=1, 2, 3. (2.1a)

The number of servers now walking is given by min (i, s-j1-j2-j3) and the number of available servers plus one is equal to

(1 + max (s-j1-j2-j3-i,0)). Equation (2.1a) now becomes

$$\text{WRC} = \min(i, s-j1-j2-j3)\ \alpha_k\ (1 + \max(s-j1-j2-j3-i, 0)) \qquad (2.1b)$$

What is sought here is the number of walk completions. The servers are counted at the beginning (i.e. before the walk). Therefore, assuming that there can only be one walk completion at a time, there are 1+ the number of free servers available before the walk starts. This explains why the number of available servers plus one is used in computing the walk completion rate.

It was assumed in section 2.3 that the closest server will take the call. This does not violate the assumption that the walk time is exponential. Fisher and Tippett (22) showed that the limit of the minimum value of a series of exponential distributions is also exponential. Therefore, the distribution of the walk time of the closest server will still be exponential. No attempt is made here to figure out who the closest server is each time. It is assumed that it is obvious to all servers who should take a particular call when it arises. In other words, they know who is closest to the call. Again, this is one of the points where further research can be done.

Knowing the service rates and the walk completion rates, and keeping in mind that there can only be one arrival or service completion or walk completion at a time, one can write the general form of the birth-and-death equations at "steady" state for this particular model, following the RATE OUT = RATE IN principle discussed earlier.

For the state (i, j1, j2, j3) the rate out would be:

$P(i,j1,j2,j3)(\lambda+j1\mu_1 + j2\mu_2 + j3\mu_3) +$

arrival rate + rate of completion for the three types of tasks

$P(i,j1,j2,j3)(\alpha_1(\min(i,s-j1-j2-j3))(1 + \max(s-j1-j2-j3-i,0))) +$

walk completion for task 1

$P(i,j1,j2,j3)(\alpha_2(\min(i,s-j1-j2-j3))(1 + \max(s-j1-j2-j3-i,0))) +$

walk completion for task 2

$P(i,j1,j2,j3)(\alpha_3(\min(i,s-j1-j2-j3))(1 + \max(s-j1-j2-j3-i,0)))$

walk completion for task 3 (2.2)

but $\alpha_1=P_1\alpha$, $\alpha_2=P_2\alpha$, $\alpha_3=P_3\alpha$ and $P_1 + P_2 + P_3 = 1$, so

$\alpha_1 + \alpha_2 + \alpha_3=\alpha$. Therefore, equation (2.2) simplifies to

$P(i,j1,j2,j3)(\lambda+j1\mu_1+j2\mu_2+j3\mu_3+\alpha(\min(i,s-j1-j2-j3))(1+\max(s-j1-j2-j3-i,0)))$.

The rate into state (i,j1,j2,j3) is equal to

$\lambda P(i-1,j1,j2,j3) + (j1+1)\ \mu_1 P(i,j1+1,j2,j3) +$

arrival + service completion of task 1

$(j2+1)\ \mu_2 P(i,j1,j2+1,j3) + (j3+1)\mu_3 P(i,j1,j2,j3+1) +$

service completion of task 2 + service completion of task 3

$\alpha_1(\min(i+1,s-j1-j2-j3+1))(1+\max(s-j1-j2-j3-i,0))P(i+1,j1-1,j2,j3) +$

walk completion for task 1

$\alpha_2(\min(i+1,s-j1-j2-j3+1))(1+\max(s-j1-j2-j3-i,0))P(i+1,j1,j2-1,j3) +$

walk completion for task 2

α_3(min(i+1,s-j1-j2-j3+1))(1+max(s-j1-j2-j3-i,0))P(i+1,j1,j2,j3-1)

walk completion for task 3 (2.3)

Table 3 shows the balance equation (RATE OUT = RATE IN) for the state (i,j1,j2,j3).

One can see that there are eight probability terms involved in this model. They have been numbered as follows:

1. P(i-1,j1,j2,j3)
2. P(i,j1,j2,j3)
3. P(i,j1,j2,j3+1)
4. P(i,j1,j2+1,j3)
5. P(i,j1+1,j2,j3)
6. P(i+1,j1-1,j2,j3)
7. P(i+1,j1,j2-1,j3)
8. P(i+1,j1,j2,j3-1)

The reason for numbering these probabilities this way is for easier reference in the rest of this section as well as for programming reasons (Appendix D).

The conditions imposed on this model are:

1. In all P(i,j1,j2,j3) combinations, the sum of all j's can not exceed s, the number of servers. For example, for P(i,j1,j2,j3), j1+j2+j3$\leq$s; for P(i,j1+1,j2,j3), j1+1+j2+j3$\leq$s; for P(i,j1,j2+1,j3), j1+j2+1+j3), j1+j2+1+j3$\leq$s, etc.

2. Probability terms 3, 4 and 5 are zero is j1+j2+j3$\geq$s.

3. Probability term 2 exists if i$\geq$1.

4. If any of the j's or i's are negative, in any of the eight probability terms, then that probability term is equal to zero. For example, P(i+1,j1-1,j2,j3) = 0 if j1 = 0.

5. $$\sum_{i=0}^{i} \sum_{j1=0}^{j1} \sum_{j2=0}^{j2} \sum_{j3=0}^{j3} P(i,j1,j2,j3) = 1,$$ which is the normalization equation, states that the sum of all the probabilities must be equal to one.

Table 3. RATE OUT = RATE IN Equations

$P(i,j1,j2j3)(\lambda+j1\mu_1+j2\mu_2+j3\mu_3+\alpha(\min(i,s-j1-j2-j3))(1+\max(s-j1-j2-j3-i,0)))$

arrival rate + rate of service completion + rate of walk completion

$= \lambda P(i-1,j1,j2,j3) + (j1+1)\mu_1 P(i,j1+1,j2,j3)$

arrival + service completion of task 1

$+ (j2+1)\mu_2 P(i,j1,j2+1,j3) + (j3+1)\mu_3 P(i,j1,j2,j3+1)$

service completion of task 2 + service completion of task 3

$+ \alpha_1(\min(i+1,s-j1-j2-j3+1))(1+\max(s-j1-j2-j3-i,0))P(i+1,j1-1,j2,j3)$

walk completion for task 1

$+ \alpha_2(\min(i+1,s-j1-j2-j3+1))(1+\max(s-j1-j2-j3-i,0))P(i+1,j1,j2-1,j3)$

walk completion for task 2

$+ \alpha_3(\min(i+1,s-j1-j2-j3+1))(1+\max(s-j1-j2-j3-i,0))P(i+1,j1,j2,j3-1)$

walk completion for task 3

CHAPTER III

DEMONSTRATION AND APPLICATION OF THE METHODOLOGY

3.1 Explanation of Programming

Chapter II was concluded with the development of the general model for the "steady state" equations for this system. This model appears in Table 3.

The next logical step would be to write explicitly all of the equations for a particular NM and NS combination and to come up with some actual values for the probability of being in a particular state for all of the possible states. It should be remembered that NM is the maximum number of idle machines that one can have at any particular time, and NS is the number of servers. It turns out, for example, that for NM = 10 and NS = 5, there are 616 possible states, which means 616 equations with 616 unknowns. This creates the need for a computer program that will write all of the feasible states according to the conditions of section 2.6, write the equation for each state, and solve these equations to obtain numerical solutions.

Program "QUE" which appears in Appendix D does this. The program was written in FORTRAN language and was tested on a Cyber 74 computer. The program is divided into eight parts.

PART 1. The user assigns values to the service rates μ_1, μ_2, μ_3, to the arrival rate λ, the walk rate α, and the α_1, α_2, α_3 terms. He also reads in the maximum number of idle machines that can be expected at one time, as well as the number of servers that will be employed to attend the machines.

PART 2. This part of the program creates all of the possible states, prints them in ascending order and assigns them an identifying number.

PART 3. This part creates the coefficients of the first row, which is the normalization equations (that is, the sum of all probabilities must be equal to one), and it also creates the P(i,j1,j2,j3) combinations for all of the other rows.

PART 4. This part generates all of the constant terms for all P(i,j1,j2,j3) combinations of Part 3.

PART 5. This part matches each variable (i.e. each P(i,j1,j2,j3) combination) with its nonzero coefficient (if there is one).

PART 6. This part consists of a subroutine called Linel 3(N). This subroutine solves a large system of linear equations by Gaussian elimination with partial pivoting.

PART 7. This part of the program writes out the solution, which Linel 3(N) produces, and sums up all of the probabilities with the same NM value.

PART 8. This is the final part of the program. It calculates and prints the mean idle time for a given NM and NS and also prints the execution time for the particular run. The complete program, with documentation and further explanations of the programming techniques used, appears in Appendix D.

Since there is one equation for each state and the normalization equation must be included in the system, one can observe that for a given NM and NS there are "n+1" equations with "n" unknowns. In other words, one of the equations is redundant. This means that one can leave the equation of one (anyone) state out of the system and solve the system with all of the other equations and the normalization equation.

3.2 Model Parameters

It was mentioned in section 3.1 that in the first part of the computer program the user has to specify the values for λ, μ_1, μ_2, μ_3, α, α_1, α_2, α_3 from which LM, M1, M2, M3, AA, A1, A2 and A3 can be computed as follows:

LM = λ (Total number of looms)

M1 = μ_1

M2 = μ_2

M3 = μ_3

AA = Rate of walk (ft. per hour) $\frac{1}{\alpha}$

A1 = P1 AA

A2 = P2 AA

A3 = P3 AA

Table 1 in Chapter I gives typical values for the occurrence rates for the various breakdowns as well as their typical repair times. This table is repeated below for convenience.

Table 1. Typical Weaver's Tasks and Repair Times Required for a Shuttle Loom

Type of Stoppage	Occurrence Rate per Loom Hour	Repair Time (Min.)
Warp Break	1.00	0.85
Weft Break	0.30	0.33
Slack End and Others	0.15	0.58

From Table 1, one can see that λ=1.45 breaks per loom per hour.

$$\mu_1 = \frac{1}{\text{repair time (min)}} \quad \frac{60 \text{ min}}{\text{hr}} = \frac{60}{.85} = 70.588$$

warp break repairs per hour. Similarly μ_2 = 181.818 weft break repairs per hour and μ_3 = 103.448 slack end break repairs per hour.

The mean walk time is α and it is a function of the size of the particular weave room under consideration. It turns out, for example, that for a weave room of 200 ft. by 100 ft., the mean walk distance is approximately 101 feet.

Appendices E and F show how this value was derived. Appendix E shows a computer program for the computation of the distribution of the walk distance, which is a function of the dimensions of the room and has probability density function $f_z(z)$ (See section 2.2 and Appendix B for the derivation of $f_z(z)$). The program in Appendix F finds the mean walk distance for a particular room based on the results produced by the program in Appendix E.

Given that one can walk about 240 feet per minute, one can calculate the number of walk completions per hour as follows:

$$AA = \frac{240 \text{ feet}}{\text{min}} \quad \frac{1}{101 \text{ feet}} \quad \frac{60 \text{ min}}{\text{hr}} \cong 140 \text{ walks per hour}$$

In section 2.5 α_1, α_2 and α_3 were defined as follows: $\alpha_1=P_1\alpha$, $\alpha_2=P_2\alpha$ and $\alpha_3=P_3\alpha$.

From the values of Table 1 and the definitions of P1, P2 and P3, one can calculate the conditional probabilities P1, P2 and P3 as P1≅.689, P2≅.207 and P3≅.103.

NS in this model is the number of collaborating servers assigned to tend the machines. This number has to be specified for each particular trial. The user needs to read in NS and NM each time. NM = max i is the maximum number of machines expected to be idle at one time. Theoretically, NM can be equal to the total number of machines, but the probability of this happening is so small that one can safely say that it will never happen in a real life situation. The choice of NM will depend on the total number of machines, the service rates, the arrival rate and the number of operators available. In choosing NM, one can start with small values and gradually increase them until the probability of a particular NM happening is close to zero. We can then say that for this particular system NM can not practically exceed this value. This will become clear in the examples of the next section.

3.3 Examples

It was mentioned at the beginning of this chapter that in order to run this computer program, one must specify the maximum expected number of idle machines, NM. Based on this and the total number of servers, NS, we come up with the mean number of idle machines. Specifications of NM makes the execution time smaller, but it limits the problem somewhat, because it provides no information on the probability of having more idle machines than were expected. It is obvious that if one wants to look at all possible combinations for a range of NS's (i.e. NS = 1, 2, 3. . .6) and a range of NM's (i.e. NM = 1, 2, 3. . .20), it will take a large number of runs, making the analysis and the interpretation of the results more complicated. To avoid such complications and still accomplish the objectives of this thesis, it was decided that only two different sizes of weave rooms be considered and that five runs be made on each one of them.

Weave room I has dimensions of 200 feet by 100 feet and a capacity of 200 looms. Weave room II is 125 feet by 80 feet and contains 100 looms. Table 4 summarizes the values of the various parameters for the two rooms.

A run consists of one NS value and one NM value. For example, NS = 2 and NM = 20 is one run. The five runs considered here are the following:

	NM	NS
Run 1	20	2
Run 2	16	3
Run 3	12	4
Run 4	8	5
Run 5	4	6

The choice of these combinations makes practical sense for these particular weave rooms due to their dimensions. Two more runs were made for the smaller room (room II) in order to show that a small increase in NM has almost no effect in the mean idle time. The relative error though for these two runs can be quite large. Table 5 and Figures 6 and 7 show the results of these runs for the two rooms.

Table 4. Characteristics and Parameter Values
For the Two Rooms

Characteristics and Parameter Values	Room I	Room II
Area (feet squared)	20,000	10,000
Number of looms	200	100
Average Number of Breaks per Hour (LM)	308	154
Average Number of Walk Completions Per Hour (AA)	100	125
Average Walking Rate for Break Type 1 (A1)	68.96	86.20
Average Walking Rate for Break Type 2 (A2)	20.70	25.86
Average Walking Rate for Break Type 3 (A3)	10.34	12.92
Average Service Rate for Break Type 1 (M1)	70.59	70.59
Average Service Rate for Break Type 2 (M2)	181.82	181.82
Average Service Rate for Break Type 3 (M3)	103.44	103.44

3.4 Results

Figures 6 and 7 illustrated the probabilities for the number of idle machines for the two weave rooms considered here. Each figure is a plot of the number of idle machines versus their probabilities of being idle (for a specific number of servers). Figure 7, Run 1 (NM = 20, NS = 2), for example, can be interpreted as follows: Given a room 125 feet by 80 feet with 100 looms, and given the occurrence rates for the various breaks and their repair times as in table 1, with only two weavers assigned to take care of these machine, then: 25% of the time there will be no idle machine, 16.5% of the time there will only be one idle machine, 11.8% of the time there will only be 2 idle machines, and so on. It is seen that only 0.1% of the time one can expect 20 idle machines at the same time. The mean number of idle machines for this set up is approximately 3.6. One can now interpret the other figures in a similar fashion.

It was found, empirically, that for Room I the probabilities of

Table 5. Probabilities of Idle Machines

	ROOM I					ROOM II						
	Probabilities					Probabilities						
NM	NS=2	NS=3	NS=4	NS=5	NS=6	NS=2	NS=3	NS=4	NS=5	NS=5	NS=5	NS=6
0	.1178	.1515	.1774	.1936	.3386	.2506	.3535	.4625	.5869	.5858	.5853	.7110
1	.0978	.1252	.1534	.1915	.2973	.1650	.2151	.2496	.2484	.2481	.2480	.2179
2	.0838	.0974	.1178	.1537	.1926	.1188	.1303	.1239	.0917	.0917	.0917	.0527
3	.0736	.0836	.0973	.1263	.1159	.0907	.0882	.0687	.0396	.0396	.0397	.0144
4	.0658	.0732	.0842	.1053	.0554	.0716	.0614	.0397	.0184	.0185	.0186	.0037
5	.0595	.0650	.0734	.0865	NA	.0576	.0435	.0232	.0086	.0087	.0088	NA
6	.0544	.0583	.0643	.0681	NA	.0468	.0311	.0137	.0039	.0041	.0042	NA
7	.0500	.0526	.0565	.0485	NA	.0382	.0224	.0081	.0016	.0018	.0019	NA
8	.0462	.0476	.0496	.0261	NA	.0313	.0162	.0047	.0005	.0008	.0009	NA
9	.0428	.0432	.0432	NA	NA	.0257	.0117	.0027	NA	.0002	.0003	NA
10	.0397	.0393	.0366	NA	NA	.0211	.0084	.0015	NA	NA	.0001	NA
11	.0369	.0358	.0286	NA	NA	.0173	.0061	.0008	NA	NA	NA	NA
12	.0343	.0326	.0171	NA	NA	.0142	.0043	.0003	NA	NA	NA	NA
13	.0319	.0296	NA	NA	NA	.0117	.0031	NA	NA	NA	NA	NA
14	.0297	.0266	NA	NA	NA	.0096	.0021	NA	NA	NA	NA	NA
15	.0276	.0225	NA	NA	NA	.0079	.0013	NA	NA	NA	NA	NA
16	.0257	.0151	NA	NA	NA	.0064	.0006	NA	NA	NA	NA	NA
17	.0240	NA	NA	NA	NA	.0053	NA	NA	NA	NA	NA	NA
18	.0222	NA	NA	NA	NA	.0042	NA	NA	NA	NA	NA	NA
19	.0201	NA	NA	NA	NA	.0031	NA	NA	NA	NA	NA	NA
20	.0153	NA	NA	NA	NA	.0018	NA	NA	NA	NA	NA	NA

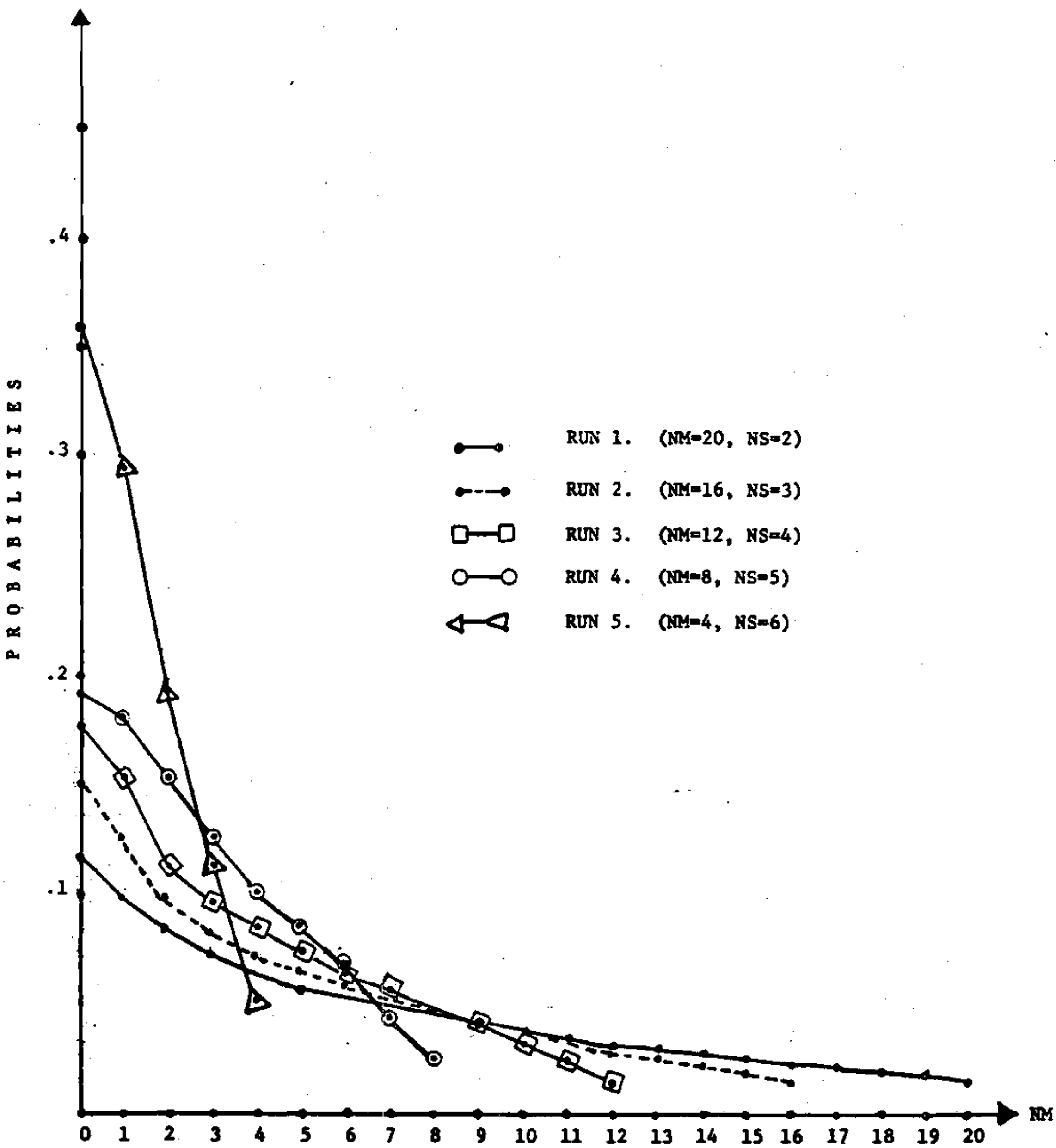


Figure 6. Results for Room I

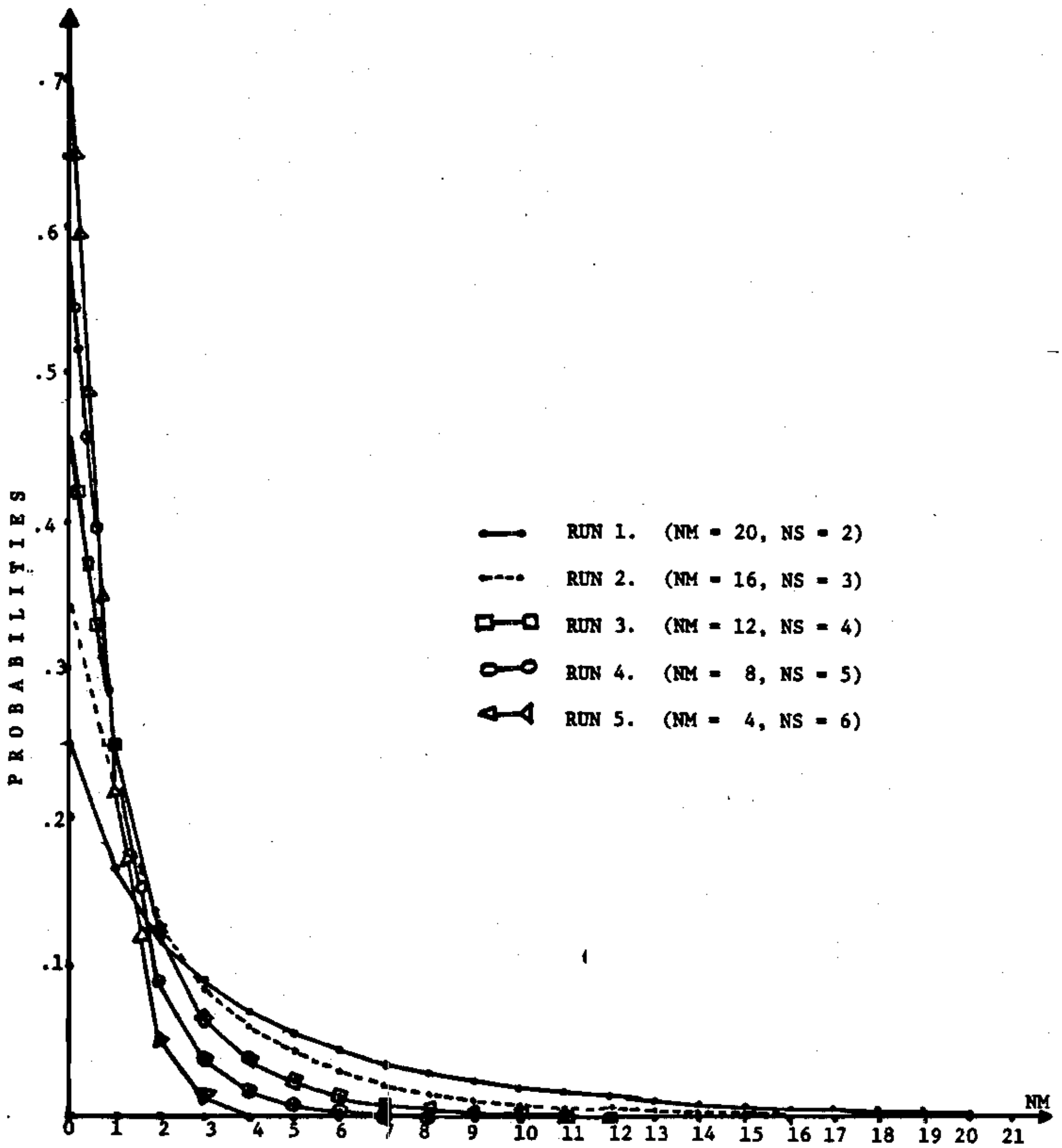


Figure 7. Results for Room II

Table 6. Mean Number of Idle Machines

Run NM	NS	Room I	Room II
20	2	6.85	3.63
16	3	5.19	2.08
12	4	3.84	1.21
8	5	2.69	.70
9	5	NA	.71
10	5	NA	.38
4	6	1.25	

Table 7. Execution Time for Cyber 76 (Seconds)

Run NM	NS	Room I	Room II
20	2	6.153	5.775
16	3	14.344	13.719
12	4	26.383	27.176
8	5	42.851	46.057
9	5	NA	49.973
10	5	NA	52.225
4	6	51.415	51.337

having a high number of idle machines are relatively large. For example, Figure 6, Run 1 (NM = 20, NS = 2) says that 4% of the time there will be 10 idle machines at the same time, 3% 14 machines, 21% 18 and 1.7% there will be 20 idle machines at the same time. This can be interpreted as lack of "steady state" in the infinite case model (i.e. in the case where there is an infinite number of states because NM is not set to a particular number). In other words, the rate of break occurrence is higher than the service rate and for this kind of set up there is always going to be a number of idle machines. The addition of a few more servers will take care of this problem. However, there is a trade off here between the cost of having some machines idle and the cost of hiring additional people, and it might make economical sense to have a number of machines idle a certain percentage of the time, as opposed to paying additional weavers. This will be discussed further in the next section. Figure 6, Runs 2, 3, 4, and 5 also refer to Room I and they indicate the lack of "steady state", too.

Table 6 shows a reduction in the mean idle time as the number of servers increases. This is true for both rooms and is something that should be expected. Also, the mean idle time for each case for Room I was twice or more than the mean idle time for the same case for Room II.

As the number of servers increases, the execution times increase in a combinatorial fashion. Comparison of the execution times for the same case for the two different weave rooms reveals that the execution times are insensitive to changes in the size of the room, and therefore, in the mean walk time.

3.5 Discussion of the Results

This thesis examined one of the two aspects of the problem; that of finding the average number of idle machines according to the various numbers of servers attending these machines. The other aspect of the problem is the estimation of the service costs. One needs to examine both aspects of the problem in order to decide on the optimum assignment, because decisions regarding the amount of service capacity to provide are based primarily on two considerations:

1. The cost incurred by providing the service.
2. The amount of waiting for that service.

These two considerations create conflicting pressure on the decision maker. The objective of reducing service costs recommends a minimal level of service. On the other hand, long waiting times are undesirable, which recommends a high level of service. Therefore, it is necessary to strive for some type of compromise.

In order to compare service costs and waiting costs, one has to adopt (explicitly or implicitly) a common measure of their impact. It is possible to directly identify some or all of the costs associated with the idleness of the looms. The estimating process would be a good topic of further research on this study. When the cost of waiting has been evaluated explicitly, one can calculate the level of service which minimizes the total of the expected cost of service and the expected cost of waiting for service, as suggested in Figure 8. As a final conclusion, one might state that the method outlined in this thesis, along with the knowledge of the service costs and waiting costs, can help a decision maker make an intelligent decision as to the proper balance between delays and service costs and come up with the economically optimal number of servers for a given number of machines. In practice, such items as imperfect communications and sensitivity of workers to the queue length would tend to introduce differences between predicted and actual values.

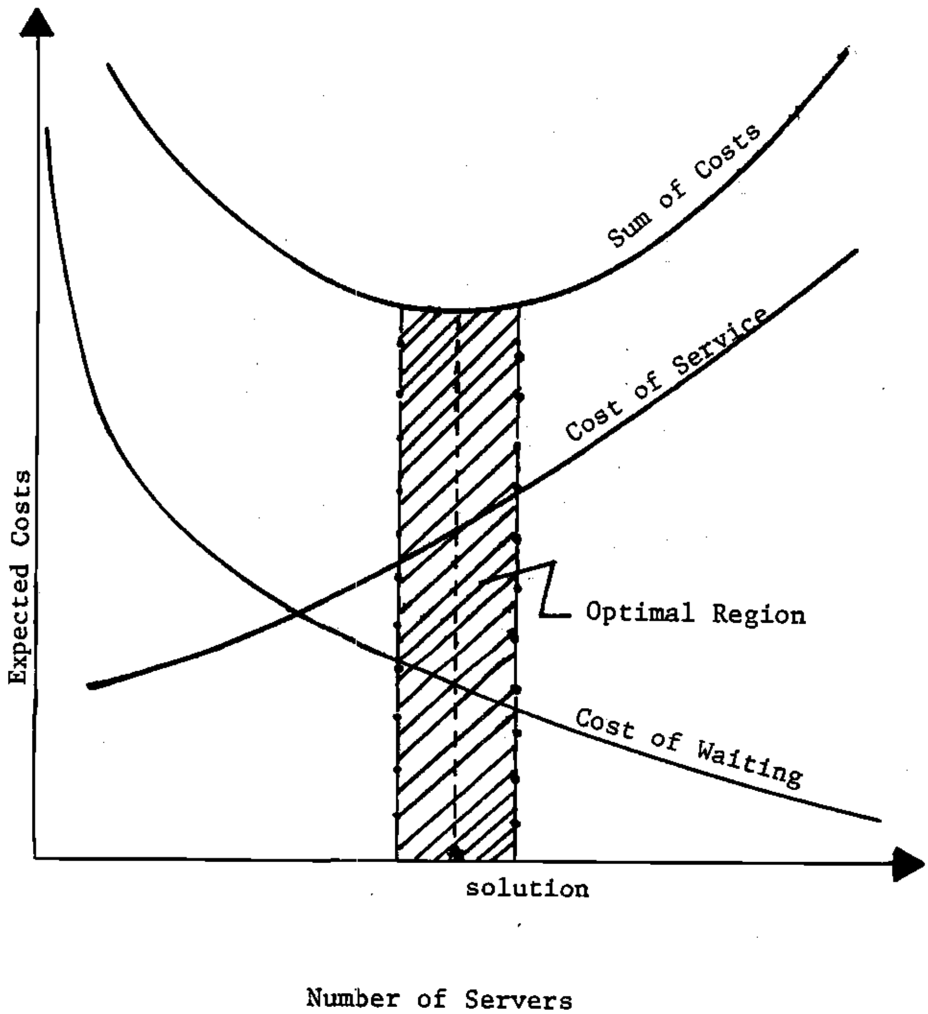


Figure 8. Solution Procedure for Optimum Assignment

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

The model formulated in Chapter III, while not being the only approach, does present a logical approach to solving loom assignment problems. The application of the birth-and-death principles for the solution of this problem is a viable methodology. The approach presented incorporates both sufficient detail and low set up and run times. A single family of curves covering a majority of situations would constitute an ideal solution for finding the average number of idle machines. But as was mentioned in Chapter III, this is not possible because the various parameters change for different kinds of looms, weave room sizes, and different operators.

In practice, of course, before using the curves presented here, it is necessary to ensure that the assumptions and characteristics of the model on which they are based are applicable (at least approximately) to the actual production system under study. However, the problem in all real life situations is not simply one of trying to maximize machine utilization, and hence, output (obviously this can be achieved with one serviceman assigned to each machine), but rather one of determining an economic balance between the losses due to machine downtime on the one hand, and the costs of providing a higher level of service on the other. It is not proposed to discuss here the detailed calculations necessary to determine this point of optimum balance, although a general outline of how one can decide on the optimum assignment was given in the last section of Chapter III.

4.2 Recommendations and Possible Extensions

Recommendations for further work have generally been mentioned as they occurred in the text. They are repeated here with some amplification.

1. It was assumed that the operators have no duties other than attending to stopped machines. In fact, in machine tending problems, some allowance must always be made for relaxation and personal needs. Demands on the operator's time, other than the restarting of stopped machines and walking, can be called "ancillary work". One could study the effects of "ancillary work" on idle time. This, of course, requires knowledge of the incidence and duration of "ancillary work".

2. Comparisons of this arrangement should be made with the set up where an operator takes care of a certain number of machines and there is no collaboration. In this case, the walking times involved should be smaller, but one can expect higher idle times and less efficient use of labor.

3. The conditions for the existence of "steady state" should also be investigated. The utilization factor for the service facility can be expressed as $\rho = \frac{L}{SM}$. This is the expected fraction of time the servers are busy, because L/SM represents the fraction of the system's capacity (SM) that is being utilized on the average by arriving customers (L). The requirement is that $\rho<1$. Otherwise, the state of the system tends to grow continually larger as time goes on. M is a function of the three service rates and the mean walk rate. One can investigate

this relationship and derive an expression of M in terms of the service rates and the mean walk rate.

4. Another modification would be to consider a model with both random and deterministic stops. This would come closer to describing the "real world" because one has to plan certain stops for maintenance, or in order to cut off the woven fabric, or to supply the machines with raw material.

5. One could compare the results of the procedure with the results of one of the other two well-known methods for numerical solutions of large linear systems, namely the Gauss-Seidel method and the over-relaxation method.

6. The fact that this birth-and-death model has not been validated and because room dimensions affect the walk-time a study is recommended to compare model results with either simulation or actual data.

7. Another modification would be to generalize the service-time, failure time, and walk time distributions to a larger class than exponential (i.e. Erlang model with birth-and-death characteristics).

8. The strongest recommendation to be made is that this model be utilized and studied with actual "real world" examples, accounting for variations in operator effort level and the necessary allowances for the operator's relaxation times. Testing the model with real world examples will also point out the difficulties in organization, if any, that cooperation is going to cause.

A P P E N D I C E S

Appendix A. Definitions of Terms Applicable to Weaving

Warp: * The longitudinal yarns in a woven fabric.

Warp Breaks: Breaks in the warp which cause the loom to be stopped.

Weft or Filling: A yarn which is interlaced with warp ends to make a fabric.

Weft Breaks: Breaks in the weft which cause the loom to be stopped.

Slack End: When a warp yarn becomes slack a device will stop the loom.

Reed: A comb-like device used to separate yarns on a loop and to beat-up filling during weaving.

Dent: A term to describe the space between adjacent reed wires.

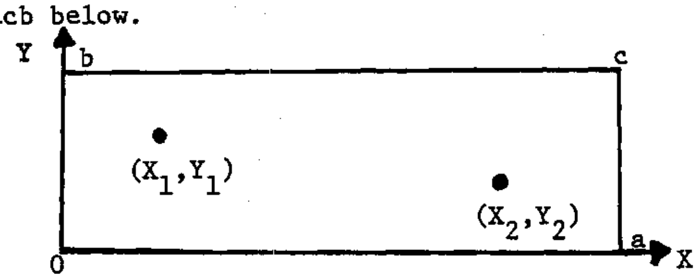
Heddle or Heald: A wire or thin perforated plate through which a warp end is threaded.

Drawing-In: The operation of threading warp yarns through the correct heddle and dents.

* These definitions were taken from P. R. Lord and M. H. Mohamed, "Weaving: Conversion of Yarn to Fabric", Merrow Publishing Co. Ltd., Watford-Herts, England, 1973.

Appendix B. Derivation of the Probability Density Function of $Z = |X_1-X_2| + |Y_1-Y_2|$

Points (X_1,Y_1) and (X_2,Y_2) are uniformly distributed in the rectangle 0acb below.

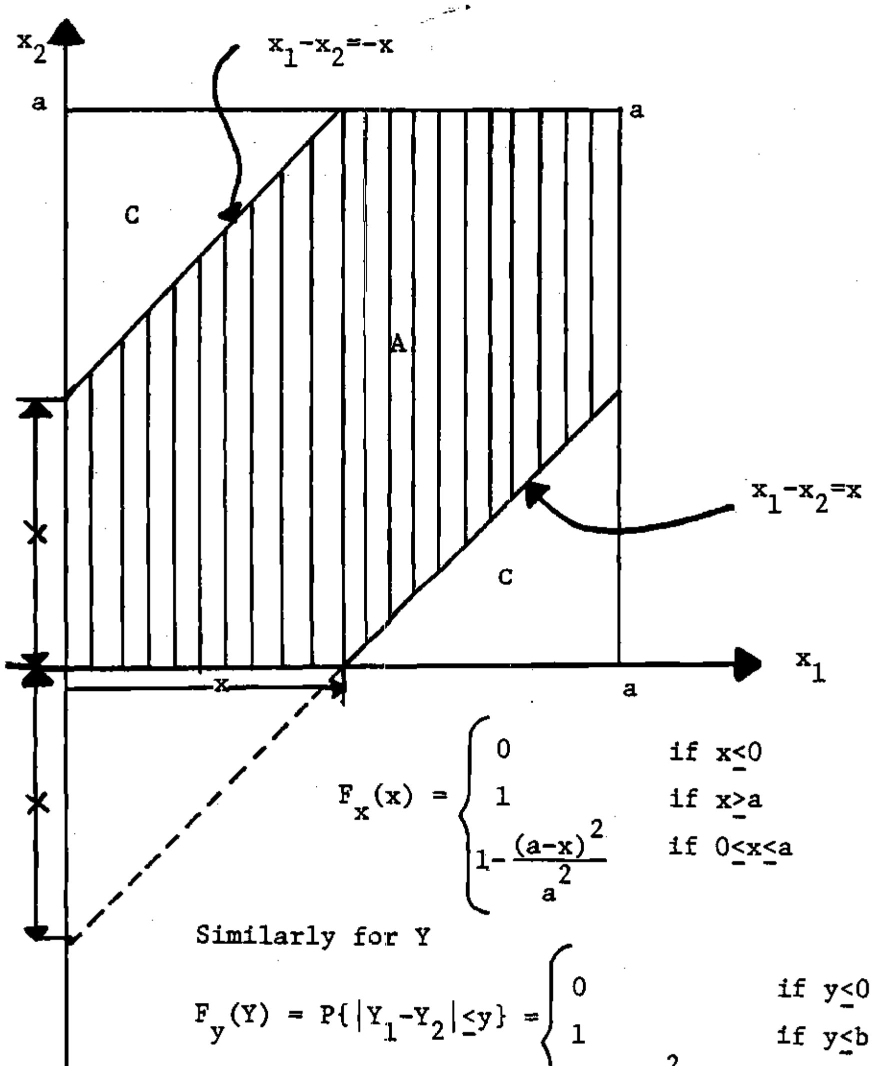


$Z = |X_1-X_2| + |Y_1-Y_2|$ is the rectilinear distance between the two points. We want the probability density function of Z. X and Y are independent and (X_1, Y_1) is independent of (X_2, Y_2) therefore the distribution of (X_1-X_2) will be independent of the distribution of (Y_1-Y_2) and Z will be the sum of two independent random variables. The cumulative distribution function for X will be

$$F_x(x) = P\{|X_1-X_2| \leq x\} = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x \geq a \\ \frac{1}{a^2} \text{ (area A)} & \text{if } 0 \leq x \leq a \end{cases}$$

By direct geometry [see top of next page]

$$F_x(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x \geq a \\ \frac{1}{a^2} (a^2 - 2\text{(area C)}) & \text{if } 0 \leq x \leq a \end{cases}$$



$$F_x(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 & \text{if } x \ge a \\ 1-\dfrac{(a-x)^2}{a^2} & \text{if } 0 \le x \le a \end{cases}$$

Similarly for Y

$$F_y(Y) = P\{|Y_1-Y_2| \le y\} = \begin{cases} 0 & \text{if } y \le 0 \\ 1 & \text{if } y \le b \\ 1-\dfrac{(b-y)^2}{b^2} & \text{if } 0 \le y \le b \end{cases}$$

Differentiating:

$$f_x(x) = \begin{cases} \dfrac{2(a-x)}{a^2} & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_y(y) \begin{cases} = \frac{2(b-y)}{b^2} & 0 \leq y \leq b \\ 0 & \text{otherwise} \end{cases}$$

By independence the joint probability density function is

$$f(x,y) = \begin{cases} \frac{4}{a^2 b^2} \quad (a-x)(b-y) & 0 \leq x \leq a \\ & 0 \leq y \leq b \\ 0 & \text{otherwise} \end{cases}$$

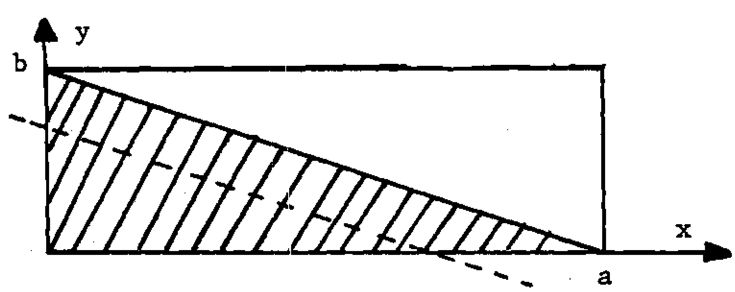
We want to integrate this over this region where $x+y \leq z$ to obtain $F_z(z)$

$$F_z(z) = P\{|X_1-X_2|+|Y_1-Y_2| \leq z\}$$

Clearly

$$F_z(z) = \begin{cases} 0 & \text{if } z \leq 0 \\ 1 & \text{if } z \geq a+b \end{cases}$$

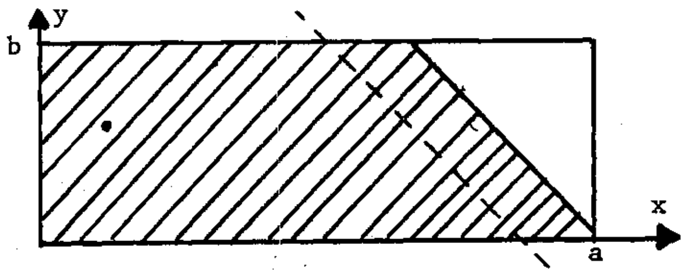
For other cases we want to integrate f(x,y) over the region where $x+y \_ z$. We will assume that $b \leq a$. Then the 1st case arises where $0 \leq z \leq b$ (shaded region)



There we integrate over the triangle below $z=x+y$

For this region $$F_z(z) = \int_{x=0}^{z} \int_{y=0}^{z-x} f(x,y)\, dydx$$

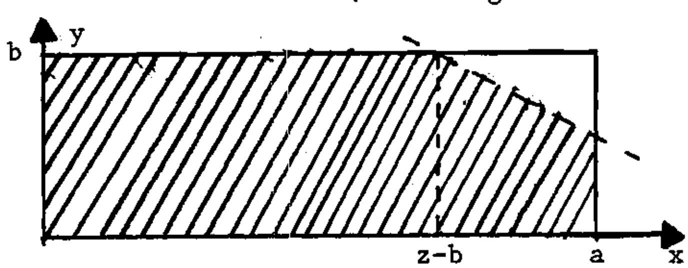
The 2nd case occurs when $b \le z \le a$. There (the shaded region below) we add the rectangle on the left and the lower triangle below $z=x+y$.



Thus in this region we have

$$F_z(z) = \int_{x=0}^{z-b} \int_{y=0}^{b} f(x,y)dydx + \int_{x=z-b}^{z} \int_{y=0}^{z-x} f(x,y)dydx$$

The 3rd case occurs when $a \le z \le a+b$. There (the shaded region below) we can do the calculation as 1-(the integral over the upper triangle).



For this region $$F_z(z) = 1 - \int_{x=z-b}^{a} \int_{y=z-x}^{b} f(x,y)dydx$$

For the 1st case

$$F_z(z) = \frac{4}{a^2b^2} \int_0^z \int_0^{z-x} (ab-ay-bx+xy)dydx=$$

$$= \frac{4}{a^2b^2}\left[\frac{abz^2}{2} - \frac{az^3}{6} - \frac{bz^3}{6} + \frac{z^4}{24}\right] \quad \text{and}$$

$$f_z(z) = \frac{4}{a^2b^2}\left[abz - \frac{az^2}{2} - \frac{bz^2}{2} + \frac{z^3}{6}\right]$$

For the 2nd case

$$F_z(z) = \int_{x=0}^{z-b}\int_{y=0}^{b} f(x,y)dydx + \int_{x=z-b}^{z}\int_{y=0}^{z-x} f(x,y)dydx =$$

$$= \frac{4}{a^2b^2}\left[\frac{ab^2z}{2} - \frac{ab^3}{6} - \frac{z^2b^2}{4} - \frac{b^4}{24} + \frac{b^3z}{6}\right]$$

$$\text{and } f_z(z) = \frac{4}{a^2b^2}\left[\frac{ab^2}{2} - \frac{b^2z}{2} + \frac{b^3}{6}\right]$$

For the 3rd case

$$F_z(z) = \int_{x=z-b}^{a}\int_{y=z-x}^{b} f(x,y)dydx =$$

$$=1 - \frac{4}{a^2b^2}\left[\frac{a^2b^2}{4} - \frac{a^2bz}{2} - \frac{ab^2z}{2} + \frac{abz^2}{2} + \frac{a^2z^2}{4} - \frac{a^3z}{6} + \frac{a^3b}{6}\right.$$

$$\left. + \frac{ab^3}{6} + \frac{b^2z^2}{4} - \frac{zb^3}{6} - \frac{bz^3}{6} - \frac{az^3}{6} + \frac{a^4}{24} + \frac{z^4}{24} + \frac{b^4}{24}\right] \quad \text{and}$$

$$f_z(z) = \frac{-4}{a^2b^2}\left[-\frac{a^2b}{2} - \frac{ab^2}{2} + abz + \frac{a^2z}{2} - \frac{a^3}{6} + \frac{zb^2}{2} - \frac{b^3}{6} - \frac{bz^2}{2} - \frac{az^2}{2} + \frac{z^3}{6}\right]$$

Therefore:

$$f_z(z)=\begin{cases} \dfrac{4}{a^2b^2}\left[abz - \dfrac{az^2}{2} - \dfrac{bz^2}{2} + \dfrac{z^3}{6}\right] & (0\leq z\leq b) \\ \dfrac{4}{a^2b^2}\left[\dfrac{-b^2z}{2} + \dfrac{ab^2}{2} + \dfrac{b^3}{6}\right] & (b\leq z\leq a) \\ \dfrac{-4}{a^2b^2}\left[\dfrac{-a^2b}{2} - \dfrac{ab^2}{2} + abz + \dfrac{a^2z}{2} - \dfrac{a^3}{6} + \dfrac{zb^2}{2} - \dfrac{bz^2}{2} + \right. & \\ \left. \dfrac{-az^2}{2} + \dfrac{z^3}{6} - \dfrac{b^3}{6}\right] & (a\leq z\leq a+b) \end{cases}$$

Appendix C. Symbols and Definitions

λ = arrival rate. The arrival rate is the same for all three types of breaks.

μ_1 = service rate per server for fixing a warp break.

μ_2 = service rate per server for a weft break.

μ_3 = service rate per server for slack ends or other types of stops.

s = the number of weavers (servers).

α = mean walk rate. This is assumed to be the same for each server.

P_1 = probability that there is a warp break given that there is a break.

P_2 = probability that there is a weft break, given that there is a break.

P_3 = probablity that there is a slack end, given that there is a break.

$\alpha_1 = P_1\alpha$

$\alpha_2 = P_2\alpha$

$\alpha_3 = P_3\alpha$

Appendix D

This appendix contains the complete FORTRAN program "QUE", which generates the equation for all the possible states and solves them to come up with the probablity of being in a particular state for all possible states. It also gives the probablities of having I idle machines (where I=0,1,2 . . . NM) and the mean number of idle machines.

```
      PROGRAM QUE  (INPUT,OUTPUT,TAPE6=OUTPUT )
      COMMON /LS/ A(25000),IC(25000),B(1000),X(1000),
     *  LR(1000)
      DIMENSION ID(1000),JD(8),C(1000)
      REAL ISUM(1000)
      REAL LM,M1,M2,M3
      M1=70.59
      M2=181.82
      M3=103.44
      LM=154.
      AA=125.
      A1=86.20
      A2=25.86
      A3=12.92
      CALL SECOND(Q)
C     READ IN THE MAXIMUM NUMBER OF IDLE MACHINES  AND THE
C     NUMBER OF SERVERS
      READ*,NM,NS
C     CREATES ALL THE POSSIBLE STATES AND PUTS THEM IN
C     ASCENDING ORDER,AND ASSIGNS THEM AN ID NUMBER
      NI=NM+1
      NJ=NS+1
      WRITE(6,117) NM
  117 FORMAT(10X,"MAX I=",I3)
      WRITE(6,118) NS
  118 FORMAT(10X,"THE NUMBER OF SERVERS=",I3//)
      WRITE(6,119)
  119 FORMAT(//,7X,"STATE NUMBER",9X,"STATE ID",/)
      N=0
      DO 10 I=1,NI
      ION=1000*(I-1)
      DO 5 J1=1,NJ
      DO5 J2=1,NJ
      DO5 J3=1,NJ
      IF(J1+J2+J3.GT.NS+3) GO TO 5
      N=N+1
      ID(N)=ION+J3-1+10*(J2-1+10*(J1-1))
      WRITE(6,101) N,ID(N)
  101 FORMAT(10X,I5,10X,I8)
    5 CONTINUE
   10 CONTINUE
C     CREATES THE COEFFICIENTS OF THE FIRST ROW,WHICH IS
C     THE NORMALISATION EQUATION: THAT IS THE SUM OF ALL
C     PROBABILITIES MUST BE EQUAL TO ONE.
      DO 12 I=1,N
      A(I)=1.
   12 IC(I)=I
      LR(1)=N
      B(1)=1.
      K=N
```

```
C       CREATES ALL THE OTHER ROWS
        DO 20 L=2,N
        I=ID(L)/1000
        J1=MOD(ID(L),1000)/100
        J2=MOD(ID(L),100)/10
        J3=MOD(ID(L),10)
C       CONSTANT TERMS FOR EACH OF THE EIGHT P(I,J1,J2,J3)
C         COMBINATIONS
        C(1)=LM
        IF(I-1.LT.0) C(1)=0.
        IF(J1+J2+J3.GT.NS) C(1)=0.
        IF(J1.GT.NS.OR.J2.GT.NS.OR.J3.GT.NS) C(1)=0.
        C(2)=-(LM+J1*M1+J2*M2+J3*M3)-AA*(AMIN0(I,NS-J1-J2-J3))
     *  *
     *(1+AMAX0(NS-J1-J2-J3-I,0))
        IF(J1+J2+J3.GT.NS)C(2)=0.
        IF(J1.GT.NS.OR.J2.GT.NS.OR.J3.GT.NS) C(2)=0.
        C(3)=(J3+1.)*M3
        IF(J1+J2+J3.LT.NS) C(3)=FLOAT(J3+1)*M3
        C(4)=(J2+1.)*M2
        IF(J1+J2+J3.LT.NS) C(4)=FLOAT(J2+1)*M2
        C(5)=(J1+1.)*M1
        IF(J1+J2+J3.LT.NS) C(5)=FLOAT(J1+1)*M1
        C(6)=A1*(AMIN0(I+1,NS-J1-J2-J3+1))*(1.+AMAX0(NS-J1-J2-
     *   J3-I,0))
        IF(J1-1.LT.0) C(6)=0.
        IF(J1-1+J2+J3.GT.NS) C(6)=0.
        IF(J1-1.GT.NS.OR.J2.GT.NS.OR.J3.GT.NS) C(6)=0.
        IF(I.GT.NI)C(6)=0.
        C(7)=A2*(AMIN0(I+1,NS-J1-J2-J3+1))*(1.+AMAX0(NS-J1-J3-
     *   J3-I,0))
        IF(J2-1.LT.0) C(7)=0.
        IF(J1+J2-1+J3.GT.NS) C(7)=0.
        IF(J1.GT.NS.OR.J2-1.GT.NS.OR.J3.GT.NS) C(7)=0.
        IF(I.GT.NI) C(7)=0.
        C(8)=A3*(AMIN0(I+1,NS-J1-J2-J3+1))*(1.+AMAX0(NS-J1-J2-
     *   J3-I,0))
        IF(J3-1.LT.0) C(8)=0.
        IF(J1+J2+J3-1.GT.NS) C(8)=0.
        IF(J1.GT.NS.OR.J2.GT.NS.OR.J3-1.GT.NS) C(8)=0
        IF(I.GT.NI) C(8)=0.
C       TO GET THE EIGHT P(I,J1,J2,J3)COMBINATIONS THAT MAY
C         APPEAR IN A RO
        JD(1)=ID(L)-1000
        JD(2)=ID(L)
        JD(3)=ID(L)+1
        JD(4)=ID(L)+10
        JD(5)=ID(L)+100
        JD(6)=ID(L)+900
        JD(7)=ID(L)+990
```

```
      JD(8)=ID(L)+999
C     AT THIS POINT WE TAKE THE N VARIABLES ,THAT IS THE N
C       P(I,J1,J2,J3)
C     COMBINATIONS AND TRY TO MATCH THEM AGAINST THE EIGHT
C       P(I,J1,J2,J3)
C     AT THIS POINT WE HAVE BOTH OF THEM IN ASCENDING
C       ORDER. NOW IF
C     THERE IS A MATCH WE ASSIGN THAT VARIABLE ITS NONZERO
C       COEFFICIENT.
C     IF ID(MI)-ID(MJ) IS NEGATIVE CONTINUE IF MORE
C       VARIABLES ARE LEFT,
C     IF POSITIVE GO TO THE NEXT P(I,J1,J2,J3) COMBINATION
      MI=0
      MJ=0
   14 MJ=MJ+1
   15 MI=MI+1
      IF(ID(MI)-JD(MJ)) 17,16,14
   16 IF (C(MJ).EQ.0.) GO TO 14
      K=K+1
      A(K)=C(MJ)
      IC(K)=MI
      IF(MJ.LT.8) GO TO 14
      GO TO 18
   17 IF(MI.LT.N) GO TO 15
   18 LR(L)=K
   20 B(L)=0.
      NN=LR(N)
      WRITE(6,109)
  109 FORMAT(//,T14,"I",T30,"IC(I)",T48,"A(I)",/)
      DO 30 I=1,NN
C     WRITE(6,102) I,IC(I),A(I)
C 102 FORMAT(10X,I5,10X,I8,10X,F12.5)
   30 CONTINUE
      WRITE(6,107)
107   FORMAT("1"///)
      WRITE(6,108)
  108 FORMAT(T15,"I",T30,"B(I)",T49,"LR(I)",/)
      DO 31 I=1,N
      WRITE(6,103)I,B(I),LR(I)
  103 FORMAT(10X,I5,10X,F10.5,10X,I8)
   31 CONTINUE
      CALL LINEL3(N)
      WRITE(6,104)
  104 FORMAT("1"///)
C     WRITES THE SOLUTIONS WHICH SUBROUTINE LINEL GIVES
      WRITE(6,106)
  106 FORMAT(10X,"STATE(I)",10X,"X(I)=PROBABILITIES",/)
      DO 32 I=1,N
      WRITE(6,105) N,X(I)
  105 FORMAT(10X,I5,10X,F15.10)
```

```
   32   CONTINUE
        WRITE(6,112)
  112   FORMAT(///,T14,"I",T22,"DENSITIES")
C       SUMS UP ALL THE PROBABILITIES WITH THE SAME I
        DO 50 I=1,NI
   50   ISUM(I)=0
        DO 60 J=1,N
        I=ID(J)/1000+1
   60   ISUM(I)=ISUM(I)+X(J)
        WRITE(6,111)(I-1,ISUM(I),I=1,NI)
  111   FORMAT(/,10X,I4,5X,F10.6)
C       FINDS THE MEAN IDLE TIME FOR A GIVEN NUMBER OF
        SUM=0.
        XMEAN=0.
        DO 7 I1=1,NI
        I=I1-1
        SUM=SUM+I*ISUM(I+1)
    7   CONTINUE
        XMEAN=SUM
        WRITE(6,114)XMEAN
  114   FORMAT(///,10X,"THE MEAN IS  ",F8.6,///)
        CALL SECOND(QQ)
        Z=QQ-Q
        WRITE(6,113)Z
  113   FORMAT(10X,"EXECUTION TIME=",F7.4,"    SECONDS")
        STOP
        END
```

```
      SUBROUTINE LINEL3(N)
C     THIS SUBROUTINE SOLVES A SYSTEM OF N LINEAR EQUATIONS
C       WITH N UNKNOWNS
      COMMON /LS/A(25000),IC(25000),B(1000),X(1000),LR(1000)
      DIMENSION LA(1000),W(1000),ICB(1000),IBC(1000)
      DATA LA(1)/1/
      NC=LR(N)
      DO 1 I=1,NC
      J=25000-NC+I
      A(J)=A(I)
    1 IC(J)=IC(I)
      DO 2 I=1,N
      ICB(I)=I
      IBC(I)=I
    2 LR(I)=LR(I)+25000-NC
      L=0
      LL=1+25000-NC
      DO 14 K=1,N
      MM=LR(K)
      DO 3 I=1,N
    3 W(I)=0.
      DO 4 J=LL,MM
    4 W(ICB(IC(J)))=A(J)
      DO 13 I=1,N
      IF(W(I).EQ.0..AND.K.NE.I) GO TO 13
      IF(I-K) 10,5,12
    5 BIG=0.
      DO 6 J=K,N
      ABSWJ= ABS(W(J))
      IF(BIG.GT.ABSWJ) GO TO 6
      BIG=ABSWJ
      JP=J
    6 CONTINUE
      IF(BIG.EQ.0.) GO TO 8
      BB=1./W(JP)
      B(K)=B(K)*BB
      W(JP)=W(K)
      ICB(IBC(K))=JP
      ICB(IBC(JP))=K
      IBC(JP)=IBC(K)
      GO TO 13
    8 WRITE (6,21)
      RETURN
   10 B(K)=B(K)-W(I)*B(I)
      J=LA(I)
   11 IF (J.GE.LA(I+1)) GO TO 13
      W(ICB(IC(J)))=W(ICB(IC(J)))-W(I)*A(J)
      J=J+1
      GO TO 11
```

```
12 L=L+1
   IC(L)=IBC(I)
   A(L)=W(I)*BB
13 CONTINUE
   LL=LR(K)+1
   LA(K+1)=L+1
   IF(LR(K).GE.L) GO TO 14
   WRITE (6,22)
   RETURN
14 CONTINUE
   DO 16 I=2,N
   K=N-I+1
   J=LA(K)-1
   MM=LA(K+1)-1
15 J=J+1
   IF(J.GT.MM) GO TO 16
   B(K)=B(K)-A(J)*B(ICB(IC(J)))
   GO TO 15
16 CONTINUE
   DO 17 I=1,N
17 X(I)=B(ICB(I))
   RETURN
21 FORMAT (4X,15HSINGULAR MATRIX)
22 FORMAT (4X,23HDYNAMIC STORAGE OVERLAP)
   END
```

Appendix E. Program for $f_z(z)$ Calculation

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL I
      DO 50 J=1,301
      I=J-1
      A=200.
      B=100.
      IF(I.LE.100) GO TO 10
      IF(I.GT.100.AND.I.LE.200) GO TO 20
      IF(I.GT.200.AND.I.LE.300) GO TO 30
   10 W=4./(A*A*B*B)*((A*B*I)-(A*I*I)/2.-(B*I*I)/2.+(I*I*I)/
     *  6.)
      GO TO 40
   20 W=4./(A*A*B*B)*((-B*B*I)/2.+(A*B*B)/2.+(B*B*B)/6.)
      GO TO 40
   30 W=-4./(A*A*B*B)*((-A*A*B)/2.-(A*B*B)/2.+(A*B*I)+
     *  (A*A*I)/2.-
     *(A*A*A)/6.+(B*B*I)/2.-(B*I*I)/2.-(A*I*I)/2.+(I*I*I)/
     *  6.-(B*B*B)/6.)
 40   WRITE(6,100) I,W
  100 FORMAT(10X,F8.4,10X,F10.5)
   50 CONTINUE
      STOP
      END
```

Appendix F. Program for Mean Walk Distance Calculation

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL I
      XNUMB=0.
      SUMX=0.
      XPROD=0.
 1    READ(5,101) I,W
      XNUMB=XNUMB+1.
      XPROD=I*W
      SUMX=SUMX+XPROD
      IF(XNUMB-300.)1,2,2
2     XMEAN=SUMX
      WRITE(6,100)XMEAN
100   FORMAT(F10.5)
101   FORMAT(10X,F8.4,10X,F10.5)
      STOP
      END
```

Appendix G. An Example Run (Run 1 NM=16, NS=3)

```
MAX I= 16
THE NUMBER OF SERVERS=  3
```

STATE NUMBER	STATE ID
1	0
2	1
3	2
4	3
5	10
6	11
7	12
8	20
9	21
10	30
11	100
12	101
13	102
14	110
15	111
16	120
17	200
18	201
19	210
20	300
21	1000
22	1001
23	1002
24	1003
25	1010
26	1011
27	1012
28	1020
29	1021
30	1030
31	1100
32	1101
33	1102
34	1110
35	1111
36	1120
37	1200
38	1201
39	1210
40	1300
41	2000
42	2001

43	2002
44	2003
45	2010
46	2011
47	2012
48	2020
49	2021
50	2030
51	2100
52	2101
53	2102
54	2110
55	2111
56	2120
57	2200
58	2201
59	2210
60	2300
61	3000
62	3001
63	3002
64	3003
65	3010
66	3011
67	3012
68	3020
69	3021
70	3030
71	3100
72	3101
73	3102
74	3110
75	3111
76	3120
77	3200
78	3201
79	3210
80	3300
81	4000
82	4001
83	4002
84	4003
85	4010
86	4011
87	4012
88	4020
89	4021
90	4030
91	4100
92	4101

93	4102
94	4110
95	4111
96	4120
97	4200
98	4201
99	4210
100	4300
101	5000
102	5001
103	5002
104	5003
105	5010
106	5011
107	5012
108	5020
109	5021
110	5030
111	5100
112	5101
113	5102
114	5110
115	5111
116	5120
117	5200
118	5201
119	5210
120	5300
121	6000
122	6001
123	6002
124	6003
125	6010
126	6011
127	6012
128	6020
129	6021
130	6030
131	6100
132	6101
133	6102
134	6110
135	6111
136	6120
137	6200
138	6201
139	6210
140	6300
141	7000
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143	7002
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146	7011
147	7012
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158	7201
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161	8000
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163	8002
164	8003
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166	8011
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6	.031171
7	.022442
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11	.006121
12	.004391
13	.003104
14	.002115
15	.001311
16	.000511

B I B L I O G R A P H Y

BIBLIOGRAPHY

1. Alford, L. P., "Looms Per Weaver," Textile World, v. 86, January 1936, 72-73.

2. Anson, C. J., "Determining the Number of Machines to be Attended by an Operator in Order to Minimize the Total Cost Per Unit of Production," Time and Motion Study, April 1957, 12-27.

3. Ashcroft, H., "The Productivity of Several Machines Under the Care of One Operator," Royal Statistical Society, v. 12, No. 1, 145-151.

4. Bawa, Vijay S., and Nair, K.P.K., "The Assignment of Cooperating Workers to Service Automatic Machines," The Journal of Industrial Engineering, v. 17, No. 9, September 1966.

5. Benson, F., Miller, J. G., and Townsend, M.W.H., "Machine Interference," The Journal of the Textile Institute, v. 44, December 1953, T619-T644.

6. Benson, F., and Cox, D. R., "The Productivity of Machines Requiring Attention at Random Intervals," J. R. Statistical Society, B, 14, 200-210, 1951.

7. Black, D. N., Pilot Models for Man Multi-Machine Jobs Involving Systematic Delays, M.S. Thesis, Georgia Institute of Technology, 1964.

8. Bowman, E. H., and Fetter, R. B., Analysis of Industrial Operations, Irwin, Illinois, 257-294.

9. Brand, E. B., and Limay, D. R., "MAD: Mathematical Analysis of Downtime," Naval Research Logistics Quarterly, 17, 525-534 (1970).

10. Brunnschweiler, D., "Machine Interference," The Journal of the Textile Institute, v. 45, 1954, T886-895.

11. Bunday, B. D., and Jackson, A. H., "The Efficiency of a Set of Machines Bidirectionally Traversed by One Operative When Walking Time is Constant," The Journal of the Textile Institute, v. 63, 1972, T281-287.

12. Bunday, B. D., and Mack, C., "Efficiency of Bi-Directionally Traversed Machines," Royal Statistical Society Journal C, 22(1), 74-81, (1973).

13. Cooper, Robert B., Introduction to Queueing Theory, MacMillan, New York, 1972.

14. Cotterill, P. J., "It's Not Our Fault," Textile Institute and Industry, v. 8, January 1970, 16-18.

15. Cox, D. R., and Smith, W. L., Queues, John Wiley and Sons, Inc., New York, 1961.

16. Crabill, T. B., "Optimal Control of a Maintenance System with Variable Service Rates," Operations Research, v. 22, 1974, 736-745.

17. Denholm, D. H., "How Many Machines Per Operator," Factory Management and Maintenance, v. 112, No. 7, July 1954.

18. Deuel, H. O., "Spindles per Spinner," Textile World, v. 87, October 1937.

19. Duvall, William C., "Machine Interference Part II," Mechanical Engineering, v. 58, No. 8, August 1936, 510ff.

20. Fetter, R. N., "The Assignment of Operators to Service Automatic Machines," The Journal of Industrial Engineering, September-October 1955, 22-30.

21. Freeman, D. R., Hoover, S. V., and Satia, J., "Solving Machine Interference by Simulation," The Journal of Industrial Engineering, July 1973, 32-38.

22. Fisher, R. A., and Tippett, L.H.C., "Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample," Cambridge Philosophical Society (Proceedings), v. 24, 1928, 180-190.

23. Fry, T. C., Probability and its Engineering Uses, Van Nostrand, New York, 1928.

24. Giffin, Walter C., Transform Techniques for Probability Modeling, Academic Press, New York, 1975.

25. Goddard, L. S., Mathematical Techniques of Operational Research, Pergamon Press, Oxford, 1963.

26. Gross, Donald, Fundamentals of Queueing Theory, John Wiley & Sons, Inc., New York, 1974.

27. Horn, W. A., "Some Simple Scheduling Algorithms," Naval Research Logistics Quarterly, v. 22, 1974, 177-185.

28. Jones, W. Dale, Wage Incentive Payment for Multiple Machine Assignments, M.S. Thesis, Georgia Institute of Technology, 1949.

29. Jones, W. Dale, "Machine Interference," Journal of Industrial Engineering, v. 19, September 1949, 10-11.

30. Jones, W. Dale, "Mathematical and Experimental Calculations of Machine Interference Time," Research Engineer, Georgia Institute of Technology, January 1, 1949, 9, 10, 20-23.

31. Jones, W. Dale, "Determining Interference in Multiple Machine Assignments," Research Engineer, Georgia Institute of Technology, April 1953, 7-14.

32. Karlin, S., and McGregor, J. L., The Classification of Birth and Death Processes, Transactions of the American Mathematical Society, v. 86, 1957, 366-400.

33. Keller, Joseph B., "Optimum Checking Schedules for Systems Subject to Random Failure," Management Science, v. 21, No. 3, November 1974.

34. Kemp, A., and Mack, C., "Tables for Calculating Machine Interference in Automatic Weaving," Textile Institute Journal, v. 52, 1961, T471.

35. King, J. R., "On the Optimum Size of Workforce Engaged in the Servicing of Automatic Machines," The International Journal of Production Research, v. 8, No. 3, 1970, 207-220.

36. King, J. R., "Machine Supervision," The Production Engineer (Special issue on Operational Research), v. 46, No. 2, 105-111.

37. Lomicka, R. F., and Allderige, J. M., "Mathematical Man-Machine Analysis," The Journal of Industrial Engineering, v. 8, No. 3, May-June 1957, 157-164.

38. Lucey, E. A., "How Many Looms Should a Weaver Run," Textile World, v. 91, 1941, 62-63.

39. Maritas, D. G., and Xirokostas, D. A., "The M/Ek/R Machine Interference Model," European Journal of Operational Research, v. 1, 1977, 112-123.

40. McTeer, H. C., "Controlled Maintenance," Textile Industries, September 1974, 70-71.

41. Nair, K.P.K., and Chandrasekaran, R., "Optimal Location of a Single Service Center of Certain Types," Naval Research Logistics Quarterly, v. 21, 1974, 503-510.

42. Nass, A. W., An Investigation of the Effects of Non-Preemptive Priority and Operator Interference in a Textile Weaving Process, M.S. Thesis, Georgia Institute of Technology, 1965.

43. O'Connor, T. F., "Multi-Machine Assignments No. 1., How to Cut Machine Interference," Modern Textile Magazine, v. 40, May 1959, 34-37.

44. O'Connor, T. F., "Multi-Machine Assignments No. 2., Determining Cycle Times and Ratios," Modern Textile Magazine, v. 40, August 1959, 65-66.

45. O'Connor, T. F., "Multi-Machine Assignments No. 3., Time Study," Modern Textile Magazine, v. 40, December 1959, 54, 60, 83.

46. O'Connor, T. F., "Multi-Machine Assignments No. 4., Determining Task Frequencies," Modern Textile Magazine, v. 41, February 1960, 27-30.

47. O'Connor, T. F., "Multi-Machine Assignments No. 5., How to Use the Tables," Modern Textile Magazine, v. 41, March 1960, 27-30.

48. O'Connor, T. F., "Multi-Machine Assignment," Modern Textile Magazine, v. 41, April 1960, 34, 48.

49. O'Connor, T. F., "An Easy Way to Allow for Machine Interference," Textile World, v. 102, December 1952, 100-101, 246-248.

50. O'Connor, T. F., "How Four Factors Affected Ashcroft Interference Tables," Textile World, v. 105, December 1955, 134-135, 198-200.

51. O'Connor, T. F., "Compare Cost of Cylce Vs. Random Service," Textile Industries, v. 123, March 1959, 81-85.

52. O'Connor, T. F., "Set Up Loom Assignments for Maximum Profit," Textile World, v. 103, No. 2, December 1953, 135, 248-249.

53. Palm, D. C., "The Assignment of Workers in Servicing Automatic Machines," Journal of Industrial Engineering, v. 9, No. 1, January-February 1958, 28-42.

54. Phillips, C. R., "A New Approach to Waiting-Line Problems (Queueing)," Georgia Tech Memorandum, TM 98160, September 1967.

55. Pinkerton, D. W., "When Pieceworker Runs Several Machines," American Machinist, v. 76, July 1932, 816-818.

56. Riordan, J., Delay Curves for Calls Served at Random, Bell System Technical Journal, v. 32, 1953, 100-119.

57. Saaty, T., Elements of Queueing Theory with Applications, McGraw-Hill Company, Inc., 1961.

58. Shah, S. J., "Productivity in Mechanical Processes in Textile Mills," Manmade Textiles, India, v. 17, October 1974, 306-307.

59. Stout, H. P., "Synchronization Time and Its Effect on Machine Efficiency," Textile Institute Journal, 1950, T225-T235.

60. Wright, W. R., "Machine Interference Part I," Mechanical Engineering, v. 58, No. 8, August 1936, 510ff.