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PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. E-23-605 GTRI/GIT DATE 7 / 25 / 83

Project Director: Dr. John G. Papastavridis School/Lab ESM

Sponsor: National Science Foundation

Type Agreement: Grant No. MEA-8303663

Award Period: From 7/15/83 To 12/31/85* (Performance) 3/31/86 (Reports)

Sponsor Amount:

	This Change	Total to Date
Estimated:	$	$ 88,700
Funded:	$	$ 88,700

Cost Sharing Amount: $ 5,364 Cost Sharing No: E-23-312

Title: "The Dynamic Stability Analysis of Nonconservative Mechanical Systems Via Variational/Emergetic Methods: Theory and Application."

ADMINISTRATIVE DATA

OCA Contact Faith G. Costello Ext. 4820

1) Sponsor Technical Contact:

Elbert L. Marsh
Mechanical Systems Program
Div. of Mech. Eng. & App. Mech.
Directorate for Engineering
NSF
Washington, D. C. 20550

2) Sponsor Admin/Contractual Matters:

Lois A. Shapiro
Division of Grants & Contracts
Directorate for Administration
NSF
Washington, D. C. 20550

Defense Priority Rating: N/A Military Security Classification: N/A
(or) Company/Industrial Proprietary: N/A

RESTRICTIONS

See Attached NSF Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with Georgia Institute of Technology

COMMENTS:

*Includes a 6 month unfunded flexibility period.

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 5/12/86

Project No. E-23-605 | ~~School~~/Lab ESM

Includes Subproject No.(s) N/A

Project Director(s) J. G. Papastavridis | GTRC / ~~GIT~~

Sponsor National Science Foundation

Title The Dynamic Stability Analysis of Nonconservative Mechanical Systems Via Variational/Emergetic Methods: Theory and Application

Effective Completion Date: 12/31/85 (Performance) 3/31/86 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [X] Final Report of Inventions – Questionnaire sent to P.I.
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No. | Continued by Project No.

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NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

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PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology Atlanta Georgia, 30332	Mechanical Systems	MEA-8303663
	4. Award Period From 7/15/83 To 12/31/85	5. Cumulative Award Amount $88,700

6. Project Title The Dynamic Stability Analysis of Nonconservative Mechanical Systems Via Variational/Energetic Methods: Theory and Applications

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

A variational methodology for the approximate analysis of the periodic motions of nonlinear and/or nonconservative oscillators was developed. It is based on : i) the general variable time-endpoints "Hamilton's principle of varied action", ii) the general "Lagrangean 'least' action principle", and iii) the general Galerkin's (weighted residual) equation. In all three techniques one varies not only the unknown amplitude parameters, but also the oscillation frequency (-ies); i) and ii) can be viewed as the most general extensions of the Rayleigh-Ritz method to nonconservative systems. These analyses thus permit one to calculate not only amplitudes, but also unknown frequencies (e.g., as in the free or autonomous/self-excited oscillations of Van der Pol's oscillator).

Two additional problems were also completed: i) the investigation of orbital/trajectory stability of potential ("gradient") but nonconservative systems via the study of the extremum of an extension of the Lagrangean Action; that led to the "new" concept of nontautochronous and nonconservative kinetic foci, and ii) the detailed derivation of the general "Adiabatic" Theorem; the latter provides a conservation theorem, useful to the approximate calculations of amplitudes and frequencies of oscillating systems with slowly varying parameters. Finally, the important Virial equation and its frequency/amplitude derivative were shown to be useful alternatives for the approximate calculations of periodic solutions, and their stability, respectively.

All the above studies are reported through several (refereed) publications, and conference presentations.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (√)	TO BE FURNISHED SEPARATELY TO PROGRAM: Approx. Date
a. Abstracts of Theses					
b. Publication Citations					
c. Data on Scientific Collaborators					
d. Information on Inventions					
e. Technical Description of Project and Results		X			
f. Other *(specify)* Publication Reprints		X (11 Copies)			

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
John G. Papastavridis	[signature]	2/25/86

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FINAL TECHNICAL REPORT
Georgia Institute of Technology
Atlanta, Georgia 30332

by

DR. JOHN G. PAPASTAVRIDIS
Associate Professor
of
Engineering Science and Mechanics

on

NSF Grant MEA-8303663
(7/15/83 - 12/31/85)

entitled

THE DYNAMIC STABILITY ANALYSIS OF NONCONSERVATIVE MECHANICAL SYSTEMS VIA VARIATIONAL/ENERGETIC METHODS: THEORY AND APPLICATIONS

ABSTRACT

This final report covers the entire period from July 15, 1983 to December 15, 1985. During these two and a half years the following projects were completed:

i) A study of the extremum properties of the Lagrangean Action A_L and their connection with (orbital) stability of motion; see refs. [1], [2], and [7]. This was based on the study of A_L's second variable-endpoints variation; the latter led to the concept of (Lagrangean) Conservative and Non-Tautochronous Kinetic Foci: the presence of such foci on a trajectory signals its stability against isoenergetic disturbances, whereas their complete absence signifies its instability.

ii) The extension of this extremum method was next carried out, in ref [3], to potential but non-conservative systems: here the study of the second variation of the general Hamilton's characteristic Function/-al, includng time-dependent nonconservative forces, led to the concept of Nonconservative and Non-Tautochronous Kinetic Foci; their significance for the stability/instability of a trajectory, now against non-isoenergetic disturbances, is the same as in i).

With references [1], [2], [3] and [7], and the previously completed similar work of ref. [4] on the Hamiltonian Action (or Principal Function/-al) and its nonconservative but Tautochronous (i.e., fixed endpoints) Foci, the formulation of a hitherto missing comprehensive extremum theory of kinetic stability of discrete (holonomic), potential but nonconservative, mechanical systems has been completed. This constitutes the natural extension of the well-known method of the stability of equilibrium analysis via (minimum) potential energy, to kinetic phenomena;

here the Action's minimum, for any integration interval, denotes instability.

iii) The next task (ref. [5]) was, first, the general variational treatment of systems with time-dependent parameters (which were treated as additional generalized coordinates), and then its specialization to the case where the system undergoes periodic motion (libration or rotation) and its physical parameters change very slowly with respect to the period, i.e. adiabatically. A general variational proof of the Adiabatic theorem was supplied along with some applications to linear and nonlinear, one D.O.F., oscillators. The Virial theorem was also proved and recognized as a useful tool for approximate calculations; applications to several D.O.F. systems will be attempted in the future. Adiabatic theory has attracted a strong interest recently.

iv) Next, refs. [6], [8], and [12] extend the familiar method of Rayleigh-Ritz to the approximate calculation of unknown amplitude and frequencies of nonlinear and nonconservative oscillators (e.g., limit-cycles of van der Pol's oscillator). This is accomplished by combining the most general forms of "Hamilton's principle of varied action" (ref. [6]) and Lagrange's principle of "least action" with generalized virtual displacements that treat the unknown frequencies as additional generalized coordinates and subject them to similar variations. Reference [12] carries out the same task, not for variational, but for the weighted residual method of Galerkin; the latter's advantage is that it does not involve any boundary terms. Also, a stability/instability criterion based on the Virial theorem is developed in ref. [6]. Finally, the theory of ref. [8] is utilized to derive a more satisfactory variational proof of Rayleigh's theorem (see ref. [9]). In sum, the methods of refs. [6], [8], and [12] constitute the most general analytical methods available for the approximate calculations of the periodic motions of, one or several D.O.F., nonlinear and/or nonconservative, mechanical systems.

v) In ref. [10], some ad hoc derived variational statements by H. Lamb, for the small oscillations around relative equilibrium, were systematized and rederived in a clear and general fashion. The Virial theorem and the gyroscopic equivalent of the Lagrange-Rayleigh stationarity "principle" were shown to follow from general variational theorems; the proof of that "principle" also showed the way to the stability problem study.

vi) Currently, the principal investigator is involved with the formulation of differential and integral variational principles for the orbital (asymptotic) stability of periodic motions of autonomous (quasilinear) oscillators, i.e., of limit cycles. The just completed reference [11] is a first step in this direction; a very general time-integral second-order/variation criterion, and its connection with the very interesting method of LEE of ref. [13], is on the way. This, as well as some additional stability methods based on the extrumum of time-averages of energetic function, will be described in fuller details in a proposal soon to be submitted to NSF.

All of our papers that are mentioned in the "List of Publications" are attached to this report. Also attached is an excerpt from a review paper in the "Shock and Vibration Digest" by prof. K. Huseyin (U. of Waterloo, Canada) on the progress in vibrations since 1980: our most important contributions have been summarized there by this international known scholar. Judging from the enthusiastic responses of several colleagus, from the U. S. and from overseas, including requests for paper reprints and invitations for conferences and seminars, it can be concluded that this project has been an unqualified success.

SCIENTIFIC COLLABORATORS

1. Mr. Hwa-Soon Choi (Fall 1983 - Fall 1984): Doctoral Candidate School of Engineering Science and Mechanics, Georgia Institute of Technology.

2. Mr. Guang Chen (Fall 1984 - Present): Doctoral Candidate, School of Engineering Science and Mechanics, Georgia Institute of Technology.

LIST OF PUBLICATIONS

1. J. G. Papastavridis: "On a Lagrange Action-based Kinetic Instabilty Theorem of Kelvin and Tait," International Journal of Engineering Science, 24 (1), pp. 1-17, 1986.

2. J. G. Papastavridis: "On a Kelvin-Tait Instability of Motion Theorems," Proceedings of the SECTAM XII Conference, Vol. I, pp. 178-184, May 1984.

3. J. G. Papastavridis: "An Action-based Method for the Kinetic Stability of Potential but Nonconservative Systems," Journal of Applied Mechanics (ASME), 52, pp. 731-733, Sept. 1985.

4. J. G. Papastavridis: "Toward an Extremum Characterization of Kinetic Stability," J. of Sound and Vibration, 87(4), pp. 573-587, 1983.

5. J. G. Papastavridis: "The Variational Theory of Adiabatic Motions, and its Applications to Linear and Nonlinear Oscillators," J. of Sound and Vibration, 103(1), pp. 83-98, 1985.

6. J. G. Papastavridis: "The Variational and Virial-like Theory of Oscillations and Stability of Nonconservative and/or Nonlinear Mechanical Systems," J. of Sound and Vibration, 104(2), pp. 209-227, 1986.

7. J. G. Papastavridis: "The Principle of Least Action as a Lagrange Variational Problem: Stationarity and Extremality Conditions," to appear in Int'l J. of Engineering Science, Winter/Spring 1986.

8. J. G. Papastavridis and G. Chen: "The Principle of Least Action in Nonlinear and/or Nonconservative Oscillations," to appear in J. of Sound and Vibration, 109(1), September 1986.

9. J. G. Papastavridis: "Rayleigh's Principle via Least Action", submitted to J. of Sound and Vibration, Winter 1985/86.

10. J. G. Papastavridis: "On the Variational and Energetic Theory of Linear Gyroscopic Systems," submitted to J. of Sound and Vibration, January 1986.

11. J. G. Papastavridis: "On the Energetics of Limit-cycle Stability," to be submitted to J. of Sound and Vibration, Winter 1986.

12. G. Chen: "A Generalized Galerkin's Method for Nonlinear Oscillators," submitted to J. of Sound and Vibration, Winter 1985. Part of Mr. Chen's Ph.D. Thesis, under the supervision of Dr. J. G. Papastavridis.

13. L. H. N. Lee: "On Dynamic Stability and Quasi-Bifurcation," International Journal of Non-Linear Mechanics, 16(1), pp. 79-87, 1981.

PRESENTATIONS

1. J. G. Papastavridis: "The Variational Approach to Adiabatic Invariance: Theory and Some Applications to Linear and Nonlinear Oscillators," Proceedings of the 21st Annual Meeting of the Society of Engineering Science VPI & SU, Blacksburg, Va., October 15-17, 1984. [INVITED].

2. J. G. Papastavridis: "Time-Integral Methods for Nonlinear/ Nonconservative Oscillators," Proceedings of the 22nd Annual Meeting of the Society of Engineering Science, Pennsylvania State University, October 6-9, 1985. [INVITED].

3. J. G. Papastavridis: "On a Kelvin-Tait Instability of Motion Theorem," Proceedings of the 12th SECTAM Conference, Callaway Gardens, Pine Mountain, Georgia, May 10-11, 1984.

4. J. G. Papastavridis: "Kinetic Stability via Variational/Extremum Methods," College of Engineering/Georgia Institute of Technology, Structural Mechanics Seminar (Centennial Colloquium Series), January 31, 1985, [INVITED].

5. J. G. Papastavridis: "Adiabatic Invariance: The Variational Theory and its Applications to Linear and Nonlinear Oscillators, "School of Mechanical Engineering Research Seminar, Georgia Institute of Technology, May 23, 1985. [INVITED].

6. J. G. Papastavridis: "Least Action, Rayleigh's Principle and Nonlinear/Nonconservative Oscillations," Proceedings of the 13th SECTAM, University of South Carolina, Columbia, S.C., April 17-18, 1966. [INVITED].