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Contract value	$ 123000	123000
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PROJECT ADMINISTRATION DATA

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sor technical contact	Sponsor issuing office
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NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 02/21/91

ject No. G-37-624 Center No. R6379-0A0

ject Director STEPHAN E P School/Lab MATH

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tract/Grant No. DMS-8704463 Contract Entity GTRC

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Final Invoice or Copy of Final Invoice	N	
Final Report of Inventions and/or Subcontracts	Y	901115
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
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Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
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G-37-624

ANNUAL PROGRESS REPORT

Ernst P. Stephan November, 1988

Project Title: "Boundary Element Methods for Integral Equations of the First Kind Governing 3-D Problems"

Progress was made by Prof. Stephan "under NSF-Grant DMS-8704463" on a variety of topics in numerical analysis, especially concerning boundary element methods for the solution of elliptic problems. One post doc and 2 research assistants have been supported under this grant.

(a) p-version, h-p-version for Boundary Element Methods (BEM)

Over the last 10 years there has been a spectacular increase in research and applications of boundary element techniques. Almost all work on BEM has been performed with the h version, where the degree of the elements is fixed, usually at a low value, typically p = 0,1,2 and the accuracy is achieved by properly refining the mesh. Only recently has the p-version been introduced into the BEM. First theoretical results proving the convergence of the p-version for the Galerkin method for Fredholm integral equations of first kind and second kind are achieved by E. P. Stephan and M. Suri in the paper "On the convergence of the p-version of the boundary element Galerkin method" in Mathematics of Computation, January (1989). Meanwhile convergence results have also been derived for the h-p version of the boundary element method in Stephan and Suri [1], [2]. In these papers it has been proven that the rate of convergence for the p-version cannot be worse than that of the h-version with quasiuniform mesh. If the singularity of the solution is located at vertices, then the rate of convergence of the p-version is at least the double of the h-version with quasiuniform mesh. As shown in Babuška, Guo, Stephan [3] for a geometric mesh the h-p version has even exponential rate of convergence with respect to the number of degrees of freedom when solving problems with piecewise analytic data. These facts are meanwhile well-known for finite element approximations and even software packages for the p- and h-versions are commerically available (Probe). But for integral equations and boundary element methods there is extensive need of further numerical testing, especially for *adaptive* p- and h-p versions where very little is known on theoretical convergence results. We are currently working on those numerical implementations. Meanwhile the 2D results from [1], [2] are generalized in Stephan [6] to 3D problems.

(b) Mesh Refinement and BEM

Recently, improved convergence rates are obtained by Stephan and coauthors [5], [6] for BEM in connection with mesh refinement. These results are based on new sharp regularity results for the solutions of the underlying bvp's which are obtained in von Petersdorff, Stephan [4] and [5]. These regularity results enable us to introduce graded meshes which are refined towards the edges of the polyhedral domain such that the boundary element Galerkin solutions converge with the same high order as for smooth domains, i.e., the pollution effect (which yields a low rate of convergence) due to corner and edge singularities is completely erased. In [4], [5], [6], [9] we consider the Dirichlet and mixed boundary value problem for the Laplacian. Corresponding results for the elasticity equations and various 3D crack problems are currently under investigation. Furthermore detailed studies of the influence of other geometries like curved edges or exceptional angles on the explicit behavior of the exact solution of the given boundary value problem are done in [7], [8]. Numerical results with applications in electrostatics and elasticity are given in [10] and [11]. Furthermore, we consider collocation methods for hypersingular integral equations on polygons [12]. This is a continuation of recent work of the principal investigator. In [12] we use odd-degree splines as basis functions whereas in [13] we take Chebysheff polynomials.

(c) Multigrid Methods for Integral Equations of the First Kind

In [14] we prove the convergence of the multigrid method with damped Jacobi relaxation for a hypersingular integral equation of the first kind on polygons and pieces of curves (smooth arcs). Those equations arise in crack problems, for example. In a forthcoming paper [15] we prove the convergence of the multigrid method for a weakly singular integral equation of the first kind on polygons. Currently we are working (see [16]) on multigrid methods in higher dimensions also using as smoothers other relaxation methods (Gauss-Seidel, SOR). The numerical experiments in [16] were performed partly on the CRAY, in Pittsburgh, on the CYBER 205 at University of Georgia, Athens, GA, and on the CYBER 855 at Georgia Tech.

(d) Coupling of FEM and BEM

Following the theoretical papers [17], [18] on the convergence of various coupling methods for elliptic problems we have implemented those methods and derived experimental convergence rates in [19] and extended the method to fully discretized (time-dependent) elliptic-parabolic interface problems in [20]. Numerical experiments for higher dimensional problems or for the p-version of FE/BE coupling are to be done next.

The following is a list of publications by Prof. Stephan resulting from this research:

a)

[1] The h-p version of the boundary element method on polygonal domains with quasiuniform meshes, with M. Suri, submitted for publication.

In [1] we investigate the h-p version of the boundary element method for integral equation formulations for PDEs over polygonal domains where both the mesh size h and the polynomial degree p are changed to improve accuracy. Under the assumption of quasiuniform meshes, we obtain estimates for the rate of convergence which show the effects on the error of changing h and p either separately or together. Using precise results for the singular behavior of the solution near corners of the domain, it is shown that the rate of convergence for the p-version (h fixed) is twice that of the h version (p fixed) for most problems which agrees with computational results reported in the literature. Interior estimates are also derived.

[2] The h-p version of the Galerkin boundary element method for integral equations on polygons and open arcs, in Proc. Conference BEM-10, 1988 Southampton (ed. C. A. Brebbia)

In [2] we present the h-p version of the boundary element method when Dirichlet and Neumann problems are solved over polygonal domains using second kind integral equations. Also various crack problems which lead to integral equations of the first kind on open curves are considered. Asymptotic rates of convergence are given for both quasiuniform and geometric meshes.

[3] On exponential convergence of the h-p method for integral equations in polygons (with I. Babuška, B. Q. Guo), in preparation.

The paper [3] applies the technique of the h-p version to the boundary element method for boundary value problems on nonsmooth, plane domains with piecewise analytic boundary and data. The exponential rate of convergence of the boundary element Galerkin solution is proven when a geometric mesh refinement is used.

b)

[4] Decompositions in edge and corner singularities for the solution of the Dirichlet problem of the Laplacian in a polyhedron, with T. von Petersdorff, to appear in Mathematische Nachrichten.

The solution of the three-dimensional Dirichlet problem for the Laplacian in a polyhedral domain has special singular forms at corners and edges. The main result of paper [4] is a "tensor-product" decomposition of those singular forms along the edges. Such a decomposition with both edge singularities, additional corner singularities and a smoother remainder refines known regularity results for the solution where the edge singularities are of non-tensor product form or the remainder term belongs to an anisotropic Sobolev space for data given in an isotropic Sobolev space.

[5] Improved boundary element methods for mixed BVP's in $\mathbb{R}^3$, with T. von Petersdorff, submitted for publication.

The solution of the three-dimensional mixed boundary value problem for the Laplacian in a polyhedral domain has special singular forms at corners and edges. In [5] we derive a "tensor-product" decomposition of those singular forms along the edges. We present a strongly elliptic system of boundary integral equations which is equivalent ot the mixed boundary value problem. Sharp regularity results for the solution of this system of integral equations are given which allow us to analyze the influence of graded meshes on the rate of convergence of the corresponding boundary element Galerkin solutions. We show that it suffices to refine the mesh only towards the edges of the surfaces to regain the optimal rate of convergence.

[6] Improved Galerkin methods for integral equations on polygons and polyhedral surfaces, in Boundary Element Methods in Applied Mechanics (ed. M. Tanaka), Japan-U.S. Symposium on Boundary Element Methods, Tokyo (1988) 73-80.

In [6] we investigate the Galerkin boundary element method for the first kind integral equation with the single layer potential on polygons and on polyhedral surfaces. We derive quasi-optimal convergence rates for the h-p version when quasiuniform meshes are used. Furthermore, for the h-version we analyze the influence of graded meshes on the rate of convergence of the Galerkin solutions and show that it suffices to refine only towards the edges of the surfaces to regain the optimal rate of convergence.

[7] Singularities for domains with circular edges, with T. von Petersdorff, in preparation.

This paper generalizes the results from [4] to special domains with curved edges. We illustrate our results with several examples.

[8] Regularity near exceptional angles of the solution of boundary value problems in polygons, with T. von Petersdorff, in preparation.

It is well known that the solutions of elliptic boundary value problems exhibit singularities near corners. Since the solution of the Dirichlet problem of the Laplacian (for example) depends continuously on the geometry, it is interesting how the singularity decompositions change when the angle of a corner approaches an exceptional angle, say $\pi/2$. To investigate this behavior we employ in [8] the technique of Mellin transformation to an equivalent boundary integral equation as developed in earlier work by Stephan.

[9] Numerical solution of elliptic problems in three dimensions with unsmooth boundaries, with I. Babuška and T. von Petersdorff, in preparation.

In this paper we collect and review the regularity results for elliptic boundary value problems in polygonal and polyhedral domains. Especially crack problems in linear elasticity are considered. Also the corresponding integral equations are analyzed. Then the finite element and boundary element methods are presented together with sharp results on mesh refinements and improved convergence rates for the Galerkin solutions.

[10] A boundary element method with mesh refinement for a weakly singular integral equation, with V. J. Ervin, submitted for publication.

The paper [10] presents a boundary element Galerkin method using mesh refinement for a first kind integral equation (with the single layer potential) on an open surface piece Γ in $\mathbb{R}^3$. We present numerical results for the Galerkin method when Γ is a circular or an elliptic plate. The numerical experiments underline the given theoretical error estimates.

[11] A boundary element Galerkin method for a hypersingular integral equation on open surfaces, with V. J. Ervin, in preparation.

In [11] we present a boundary element Galerkin method using mesh refinement for a hypersingular integral equation on an open surface piece Γ in $\mathbb{R}^3$. For Γ being a circular or elliptic plate we give numerical experiments which underline our theoretical error estimates for the Galerkin solution.

[12] Collocation for a hypersingular integral equation on polygons, with M. Costabel, in preparation.

In [12] we prove convergence of the point collocation for a hypersingular integral equation when odd-degree splines are used. Such problems arise for example in crack problems. Mellin transformation and a localization principle are the bases of our analysis which considers the collocation scheme as a Galerkin-Petrov method with different test and trial spaces, where the first space consists of splines and the second of linear combinations of delta distributions. We derive asymptotic error estimates and analyze the effect of graded meshes.

[13] Collocation with Chebysheff polynomials for Symm's integral equation on open curves, with Ian Sloan, in preparation.

In [13] we prove convergence of the point collocation for a weakly singular integral equation when Chebysheff polynomials are used. The analysis is based on expanding the solution in a Fourier cosine-series and exploiting the orthogonality of the Chebysheff polynomials.

c)

[14] On the convergence of the multigrid method for a hypersingular integral equation of the first kind, with T. von Petersdorff, submitted for publication.

In [14] we prove the convergence of the multigrid method for a hypersingular integral equation of the first kind on polygons and open arcs. As pre- and post-smoothers we use damped Jacobi relaxation. We show convergence of the V-cycle and W-cycle when the discrete systems result from Galerkin equations for the integral equation on polygons. In case when the integral equations is given on an open arc, we show convergence of the W-cycle for the multigrid method. The proofs are based on positive definiteness of the hypersingular integral operator which is understood as a pseudodifferential equation.

[15] On the convergence of the multigrid method for a weakly singular integral equation of the first kind, with T. von Petersdorff and V. F. Postell, in preparation.

In [15] we analyze the convergence of the multigrid method for a weakly singular integral equation of the first kind on polygons and open arcs. Various relaxation methods are used as smoothers.

[16] Numerical experiments for multigrid methods for integral equations of the first kind on open surfaces in $\mathbf{R}^3$, with V. F. Postell, in preparation.

In [16] we solve the discrete systems by the multigrid method and compare the convergence factor ρ with those for various relaxation methods. The discrete systems originate from the boundary element Galerkin methods for either the hypersingular integral equation (with the normal derivative of the double layer potential) or for the weakly singular integral equation (with the single layer potential) on an open surface piece Γ in $\mathbf{R}^3$. Thus the problems have direct physical applications in electrostatics and elasticity. We have performed the V-cycle for the multigrid method with damped Jacobi and SOR as pre- and post-smoothers both on a uniform mesh and on graded meshes which are refined towards the edges of the surface piece Γ. Comparison of the multigrid method with pure relaxation (SOR, Jacobi) shows a tremendous saving of computing time in all cases. In case of the hypersingular integral equation our numerical experiments show that the convergence factor ρ of the 2 level multigrid method is strictly less than one ($\rho < 1$) for any relaxation parameters $\omega < 2$ if SOR is used and for $\omega \leq 1.5$ if Jacobi is used as smoothers. That means that ρ is independent of h, the mesh size parameter of the boundary element scheme. In case of the weakly singular integral equation our numerical experiments indicate the same dependence of ρ on ω. For the multilevel V-cycle a choice of smaller ω gives better results.

d)

[17] Integral equations for transmission problems in linear elasticity, with M. Costabel, to appear in Journal of Integral Equations and Applications.

In [17] the scattering of elastic waves by a penetrable homogeneous object is described by a system of integral equations for the field and its traction on the boundary of the scatterer. The system contains the operators of the single and double layer potentials, of the traction of the single layer potential, and of the traction of the double layer potential. It is a strongly elliptic system of pseudo-differential equations. Therefore every Galerkin scheme for its approximate solution is convergent. For Lipschitz boundaries we show strong ellipticity of the system of boundary integral operators and existence and uniqueness of the solution, and obtain quasioptimal error estimates for its Galerkin approximation.

[18] Coupling of finite element and boundary element methods for an elasto-plastic interface problem, with M. Costabel, submitted for publication.

In [18] we study a symmetric coupling method of finite elements and boundary elements for a class of nonlinear interface problems and we show convergence of Galerkin approximation schemes. The given coupling method is based on a reduction of the interface problem to the study of a functional involving domain integrals over a bounded domain Ω_1 where a nonlinear elliptic differential equation is supposed to hold, and boundary integral operators on the common boundary Γ_c with a second domain Ω_1 in which a linear elliptic differential equation is given. We apply our results to a quasilinear model problem from three dimensional elastoplasticity.

[19] Experimental convergence rates for various couplings of boundary and finite elements, with M. Costabel, V. J. Ervin, to appear in Computers and Mathematics with Applications.

In [19] we have developed an efficient computer code for the coupling of boundary elements and finite elements for 2D transmission problems in potential theory. The obtained experimental convergence rates are in agreement with the theoretical orders.

[20] Symmetric coupling of finite elements and boundary elements for a parabolic-elliptic interface problem, with M. Costabel and V. J. Ervin, submitted for publication.

In [20] we use our analysis from [17] and [18]. We show asymptotic convergence of a time-stepping method in connection with a coupling of finite elements and boundary elements where the trial functions are time-dependent and the test functions are time-independent. The problem under consideration here is the three-dimensional eddy current problem for non-ferromagnetic metals. Numerical experiments are also given.

List of Invited Lectures for Dr. E. P. Stephan in connection with the above publications:

1. NSF-CBMS Conference, University of Kentucky, Lexington, KY, on paper 4.

2. University of New South Wales, Sydney, Australia, Colloquium, on paper 1.

3. University of Queensland, Brisbane, Australia, Colloquium, on paper 4.

4. 10^{th} International Conference on Boundary Element Methods, Southampton, England, on paper 2.

5. Technische Hochschule Darmstadt, Workshop on Singularities, Germany, on paper 5.

6. 1^{st} Japan-U.S. Symposium on Boundary Element Methods, Tokyo, Japan, on paper 6.

7. Carnegie-Mellon University Pittsburgh, Colloquium, on paper 4.

8. Conference on "Numerical Solutions for Integral Equations," Oberwolfach, Germany, on paper 4.

The following researchers were brought to Georgia Tech for research consultations with E. P. Stephan.

1. Dr. M. Costabel, Fachbereich Mathematik, Technische Hochschule Darmstadt FRG (March-April 1988).

2. Prof. M. Suri, Department of Mathematics and Statistics, University of Maryland, Baltimore County, Baltimore, MD 21228.

3. Prof. J. C. Nedelec, Department of Mathematics, Ecole Polytechnique, Palaiseau, France

G-37-624

EXHIBIT VI-1

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
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PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Tech Research Corp Georgia Institute of Technology Atlanta, GA 30332	Applied Mathematics	DMS-8704463
	4. Award Period From 9/1/87 To 08/31/90	5. Cumulative Award Amount $ 123,000

6. Project Title: Mathematical Sciences (Numerical Analysis and Scientific Computing).: Boundary Element Methods for Integral Equations of the First Kind Governing 3D Problems

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

Research was carried out in numerical analysis, focusing mainly on boundary element methods (BEM).The primary objectives were to determine:(i) implementation of robust approximation schemes (development of BEM software) and study of fast solvers like Multigrid methods (MG),(ii) convergence of approximation schemes (Galerkin method and collocation for boundary integral equations and coupling of FEM and BEM, (iii) regularity of solutions of bvp's in irregular 3D domains.

The project involved one faculty researcher (PI) over a three-year period and one post doctoral associate for a one-year period and two graduate students with input from various colloboraters; the work was both theoretical and applied and did also involve computers(CRAY at Supercomputer Center, Pittsburgh, PA, CYBER 205 at Univ. of Georgia, Athens, GA; CYBER 855 at Georgia Tech).

The primary findings included (1) convergence of MG method for first kind integral equations on curves and surfaces (V and W-cycles on uniform and graded meshes) and development of MG codes, (2) exponential convergence of h-p version of BEM and development of robust computer codes, (3) h-p adaptive algorithms for BEM, (4) use of singular elements and mesh grading for BEM covering 3D problems, (5) corner- and edge singularities of solutions of 3D bvp's, (6) analysis of saddle-point structure of FEM/BEM coupling, (7) FEM/BEM coupling for time-dependent problems, (8) convergence of collocation with Chebyshev polynomials for first kind integral equations.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (√)	TO BE FURNISHED SEPARATELY TO PROGRAM: Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations		X			
c. Data on Scientific Collaborators	X				
d. Information on Inventions	X				
e. Technical Description of Project and Results					
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Ernst P. Stephan		11-6-90

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PART IV - SUMMARY DATA ON PROJECT PERSONNEL

NSF Division Mathematical Sciences

The data requested below will be used to develop a statistical profile on the personnel supported through NSF grants. The information on this part is solicited under the authority of the National Science Foundation Act of 1950, as amended. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. NSF requires that a single copy of this part be submitted with each Final Project Report (NSF Form 98A); however, submission of the requested information is not mandatory and is not a precondition of future awards. If you do not wish to submit this information, please check this box ☐

Please enter the numbers of individuals supported under this NSF grant.
Do not enter information for individuals working less than 40 hours in any calendar year.

*U.S. Citizens/ Permanent Visa	PI's/PD's		Post-doctorals		Graduate Students		Under-graduates		Precollege Teachers		Others	
	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.
American Indian or Alaskan Native												
Asian or Pacific Islander												
Black, Not of Hispanic Origin												
Hispanic												
White, Not of Hispanic Origin	1		1		2							
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umber of individuals who have a handicap that limits a major life activity.												

*Use the category that best describes person's ethnic/racial status. (If more than one category applies, use the one category that most closely reflects the person's recognition in the community.)

AMERICAN INDIAN OR ALASKAN NATIVE: A person having origins in any of the original peoples of North America, and who maintains cultural identification through tribal affiliation or community recognition.

ASIAN OR PACIFIC ISLANDER: A person having origins in any of the original peoples of the Far East, Southeast Asia, the Indian subcontinent, or the Pacific Islands. This area includes, for example, China, India, Japan, Korea, the Philippine Islands and Samoa.

BLACX, NOT OF HISPANIC ORIGIN: A person having origins in any of the black racial groups of Africa.

HISPANIC: A person of Mexican, Puerto Rican, Cuban, Central or South American or other Spanish culture or origin, regardless of race.

WHITE, NOT OF HISPANIC ORIGIN: A person having origins in any of the original peoples of Europe, North Africa or the Middle East.

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b. Publication Citations

1. "Decompositions in edge and corner singularities for the solution of the Dirichlet problem of the Laplacian", Math. Nachrichten 145 (1990)(with T.v. Petersdorff).

2. "Regularity of mixed boundary value problems in R^3 and boundary element methods on graded meshes", Math. Methods in Applied Sciences 12(1990) 229-249 (with T.v. Petersdorff).

3. "An h-p version of BEM for plane mixed boundary value problems". In: Advances in Boundary Elements Vol I (ed.C.A. Brebbia, J.J. Connor) (1989) 95-103 (with B. Q. Guo, T.v.Petersdorff).

4. "On the exponental convergence of the h-p version for boundary element Galerkin methods on polygons", Math. Methods in Applied Sciences 12 (1990) 413-427 (with I. Babuska, B. Q. Guo).

5. "On the h,p and h-p versions of the boundary element method - Numerical results", Computer Methods in Applied Mechanics and Engineering 83 (1990) 69-90 (with F. V. Postell).

6. "Coupling of finite and boundary element methods for an elastoplatic interface problem", SIAM J. Numer. Anal. (1990) (with M. Costabel).

7. "Integral equations for transmission problems in linear elasticity", J. Integral Equations and Applications 2 (1989) 1-13 (with M. Costabel).

8. "Experimental convergence rates for various couplings of boundary and finite elements", Computers and Mathematics with Applications, in press (with M. Costabel, V. J. Ervin).

9. "The h-p version of the boundary element method on polygonal domains with quasiuniform meshes", RAIRO Analys. Numer. (with M. Suri).

10. "Duality estimates for the numerical solution of integral equations", Numer. Math. 54 (1988) 339-353 (with M. Costabel.

11. "On the convergence of the multigrid method for a hypersingular integral equation of the first kind", Numer. Math. 57 (1990) 379-391 (with T. v. Petersdorff).

12. "A multigrid method on graded meshes for a hypersingular integral equation", submitted for publication (with T.v. Petersdorff).

13. "An augmented boundary element method for 3 D problems", submitted for publication

14. "Symmetric coupling of finite elements and boundary elements for a parabolic-elliptic interface problem",Quarterly of Applied Mathematics 48 (1990) 265-279 (with M. Costabel, V. J. Ervin).

15. "The h-p version of the boundary element method with geometric mesh on polygonal domains, Computer Methods in Applied Mechanics and Engineering", 80(1990) 319-325 (with I. Babuska, B.Q. Guo).

16. "Collocation with Chebyshev polynomials for a hypersingular integral equation on an interval, submitted for publication" (with V.J. Ervin).

17. "Collocation with Chebyshev polynomials for first kind integral equations on intervals", MAFELAP V, 1990, (ed. J.R. Whiteman) (with V.J. Ervin).

18. "Why multi-grid performs poorly for first kind integral equations with weakly singular kernels", in preparation (with T.v. Petersdorff).

19. "Numerical experiments for multigrid methods for first kind integral equations, submitted for publications" (with V.F. Postell).

20. "Regularity and boundary element solution of the h-p version for transmission problems with piecewise analytic data".
Part 1: Regularity of solution for transmission problems with piecewise analytic data, submitted for publication (with B. Guo)
Part 2: h-p version of boundary element method for transmission problems, in preparation (with B. Guo).

21. "On the h-p version of the boundary element method for Symm's integral equation on polygons, submitted for publication" (with V.J. Ervin, N. Heuer).

22. "Quadrature and collocation methods for the double layer potential on polygons, submitted for publication" (with M. Costabel, V.J. Ervin).

23. "The h-p version of the Galerkin boundary element method for integral equations on polygons and open arcs". In: Proc. Conference BEM-10, 1988, 205-216. Southampton (ed. C.A. Brebbia).

24. "Inproved Galerkin methods for integral equations on polygons and on polyhedral surfaces". Japan-US Symposium on Boundary Element Methods, Tokyo (1988) 73-80.

c. Scientific Collaborators

The principal investigator, E.P. Stephan, was collaborating with Prof. Dr. I. Babuska, IPST Univ. of Maryland, College Park; Prof. Dr. M. Suri, Univ. of Maryland, Baltimore County; Prof. Dr. B.Q. Guo, Univ. of Manitoba, Winnipeg (Canada); Dr. T. v. Petersdorff, Univ. of Maryland, College Park; Dr. M. Costabel, Technische Hochschule Darmstadt (FRG).

e. Technical Summary of Activities and Results

Results of research by E.P. Stephan (PI) which was supported in part by NSF Grant DMS-8704463 are summarized below. Abstracts of pertinent papers are given, followed by conferences, workshops, meetings and universities at which talks were presented and names of three researchers who were brought in for consultation.

E.P. Stephan (principal investigator) and V.J. Ervin (post doctoral associate) have conducted an active research program on a variety of theoretical and practical topics concerning Boundary Element Methods for 3-D problems. Areas under investigation have been:

(i) Galerkin and Collocation Methods for Boundary Integral Equations.

These methods were analyzed in [3,4,5,9,10,13,15,16,17,20,21,22,23,24] and a key role was played by the construction of robust approximation methods for boundary value problems using boundary elements. Efficient and reliable numerical solution procedures for boundary integral equations gain more and more importance due to their increased use in physics and engineering sciences. We have further developed the theoretical foundation of such approximation methods and have shown, e.g. how to speed up the asymptotic convergence of the Galerking error by use of mesh refinement, of special singular elements or via p version an h-p version. There are three verious of the BEM: the classical h-version which achieves accuracy by refining the mesh when using piecewise polynomials of low degree p. The p version uses a fixed mesh and increases p. The h-p version combines both methods. The p and h-p version are new and we have developed software packages for their implementation [5,21]. For the classical h version we have created robust codes for first kind integral equations (weakly singular and hypersingular) on surfaces which implement Galerkin and collocation schemes with piecewise constant or linear functions on triangles/rectangles for 3D problems. (See reference [48,50,51] of my CV. These three papers were supported by NSF Grants DMS-8501797 and DMS-8603954) Our boundary element codes for 2D and 3D problems include modifications which use mesh refinement and singular fructions to compensate the polluting effect of edge and corner singularities. We have also incorporated adaptive feedback algorithm for h and h-p version. Those algorithms are based on the fundamental principle to correct the computations only in such regions where the approximation is unsatisfactory.

(ii) Multigrid Methods (MG)

f) In the above mentioned approximation schemes often one has to solve linear systems of equations with several hundreds of unknowns. For elliptic differential equations and integral equations of the second kind multigrid methods have become very effective solvers. We have showm multigrid methods to be applicable also to Galerkin methods for first kind integral equations with hypersingular kernel [11], even when graded meshes are used [12]. In these papers we prove convergence of V and W cycles of the multigrid method applied to hypersingular integral equations. Numerical experiments with MG and various iteration methods (see [11,12,19]) were performed with the CRAY at Supercomputer Center Pittsburgh and the CYBER 855 at Georgia Tech. Currently we are working on multigrid methods for weakly singular integral equations and conjugate gradient methods. Also we are currently investigating domain substructuring for BEM and are working on vector processing for our boundary element codes. Care was taken in the development of our programs to make the codes highly vectorizable. The following things were implemented:

(i) accessing the matrices/vectors via columns;
(ii) minimal number of calls to subroutines and function evaluations;
(iii) choice of a linear solver (Gauss-Jordon Elimination) which can be vectorized.

Experiments were also performed with our codes for 3D boundary element methods on the Cyber 205 located at the University of Georgia in Athens. Items of computations were capacitance of electrified plates [24] and stresses and displacements of 3D elastic materials.

(iii) Regularity of Solutions of 3D Boundary Value Problems

The solutions of elliptic boundary value problems in nonsmooth domains have local losses of regularity, i.e. the solutions of the three-dimensional mixed value boundary problem, the Dirichlet or Neumann problem for the Laplacian in a polyhedral domain have special singular forms at corners and edges. In [1,2] we derive tensor-product decomposition of those singular forms along the edges and we present strongly elliptic systems of boundary integral equations which are equivalent to the bvp's. Then we give sharp regularity results for the solution of those systems of integral equations. Owing to the lack of regularity of the exact solution of the integral equations the convergence rate of the corresponding Galerkin solution is low if a uniform mesh is used. An improved numerical scheme to solve those bvp's approximately, eg. a Galerkin scheme with finite elements has obviously to make use of the special singular forms of the exact solution near corners and edges. For two-dimensional domains quasi-optimal error estimates for the Galerkin solution are well-known where high convergence rates are obtained either by augmenting the space of test and trial functions by special singular elements which imitate the corner behaviour of the exact solution or by using appropriate mesh refinement in the vicinity of the corners. For three-dimensional problems our sharp regularity results in [1,2] allow us to analyse the influence of graded meshes and of special singular elements on the rate of convergence of the corresponding boundary element Galerkin solutions. We show in [2,24] that it suffices to refine the mesh only towards the edges of the surfaces to regain the optimal rate of convergence. Based on our sharp regularity results we show in [13] how to reduce the pollution effect by use of special singular elements in the BEM.

(iv) Coupling of FEM and BEM

Our research on coupling of finite element and boundary element methods (see NSF Grant DMS-8603954) has been continued [8] and extended to time-dependent problems [14] and non-linear problems [6].

Abstracts of papers of E. P. Stephan which were partially supported by NSF Grant DMS-8704463.

1. "Decompositions in edge and corner singularities for the solution of the Dirichlet problem of the Laplacian", Math. Nachrichten 145 (1990) (with T.v. Petersdorff).

Abstract. The solution of the three-dimensional Dirichlet problem for the Laplacian in a polyhedral domain has special singular forms at corners and edges. The main result of this paper is a "tensor-product" decompositon of those singular forms along the edges. Such a decomposition with both edge and corner singularities and a smoother remainder refines known regularity results for the solution where either the edge singularities are of non-tensor product form or the remainder term belongs to an anisotropic Sobolev space for data given in an isotropic Sobolev space.

2. "Regularity of mixed boundary value problems in R^3 and boundary element methods on graded meshes", Math. Methods in Applied Sciences 12(1990) 229-249 (with T.v. Petersdorff).

Abstract. The solution of the three-dimensional mixed boundary value problem for the Laplacian in a polyhedral domain has special singular forms at corners and edges. A "tensor-product" decomposition of those singular forms along the edges is derived. We present a strongly elliptic system of boundary integral equations which is equivalent to the mixed boundary value problem. Regularity results for the solution of this system of integral equations are given which allow us to analyse the influence of graded meshes on the rate of convergence of the corresponding boundary element Galerkin solutions. We show that it suffices to refine the mesh only towards the edges of the surfaces to regain the optimal rate of convergence.

3. "An h-p version of BEM for plane mixed boundary value problems". In: Advances in Boundary Elements Vol I (ed.C.A. Brebbia, J.J. Connor) (1989) 95-103 (with B. Q. Guo, T.v.Petersdorff).

Abstract. We present the h-p version of the boundary element method for mixed boundary value problems on plane domains with piecewise analytic boundary and data. The exponential rate of convergence of the boundary element Galerkin solution is obtained when a geometric mesh refinement is used near the vertices,and near the collision points where the boundary conditions change.

4. "On the exponental convergence of the h-p version for boundary element Galerkin methods on polygons", Math. Methods in Applied Sciences 12 (1990) 413-427 (with I. Babuska, B. Q. Guo).

Abstract. This paper applies the technique of the h-p version to the boundary element method for boundary value problems on non-smooth, plane domains with piecewise analytic boundary and data. The exponential rate of convergence of the boundary element Galerkin solution is proved when a geometric mesh refinement towards the vertices is used.

5. "On the h,p and h-p versions of the boundary element method - Numerical results", Computer Methods in Applied Mechanics and Engineering 83 (1990) 69-90 (with F. V. Postell).

Abstract. In this paper we present numerical experiments with the h,p and h-p versions of BEM which are performed with our codes developed at Georgia Tech. We compute experimental convergence rates which underline the theoretical convergence results given in [4]. We consider two types of boundary integral equations of the first kind on the interval =(-1,1), namely (a) Symm's integral equation with logarithmic kernel and (b) a hypersingular integral equation having as kernel the normal derivative of the double layer potential. With a geometric mesh refinement towards the endpoints of we compute for the h-p version an exponential convergence rate. Further we present an adaptive feedback algorithm for the h-p version which converges also exponentally.

6. "Coupling of finite and boundary element methods for an elastoplatic interface problem", SIAM J. Numer. Anal. (1990) (with M. Costabel).

Abstract. A class of transmission problems is considered in which a nonlinear variational problem in one domain is coupled with a linear elliptic problem in a second domain. A typical example is a problem from three-dimensional elasticity theory where an elastoplastic material is embedded into a linear elastic material. The nonlinear problem is given in variational form with a strictly convex functional. The linear elliptic problem is described by boundary integral equations on the coupling boundary. The typical saddle point structure of such problems is analyzed. Galerkin approximations are studied which consist of a finite element approximation in the first domain coupled with a boundary element method on the coupling boundary. The convergence of the Galerkin approximation is based on the saddle-point structure which is shown to hold for the exact as well as the discretized problems.

7. "Integral equations for transmission problems in linear elasticity", J. Integral Equations and Applications 2 (1989) 1-13 (with M. Costabel).

Abstract. The scattering of elastic waves by a penetrable homogeneous object is described by a system of integral equations for the field and its traction on the boundary of the scatterer. The system contains the operators of the single and double layer potentials, of the traction of the single layer potential, and of the traction of the double layer potential. It is a strongly elliptic system of pseudodifferential equations. Therefore every Galerkin scheme for its approximate solution is convergent. For Lipschitz boundaries we show strong elipticity of the system of boundary integral operators. This implies existence and uniqueness of the solution and quasioptimal error estimates for its Galerkin solutions.

8. "Experimental convergence rates for various couplings of boundary and finite elements", Computers and Mathematics with Applications, in press (with M. Costabel, V. J. Ervin).

Abstract. Efficient computer codes are developed for the coupling of boundary elements and finite elements for 2 D transmission problems in potential theory. Piecewise linear finite elements and both piecewise constant/piecewise linear boundary elements are implemented. The obtained experimental convergence rates (h-version) are in agreement with the theoretical orders.

9. "The h-p version of the boundary element method on polygonal domains with quasiuniform meshes", RAIRO Analys. Numer. (with M. Suri).

Abstract. We investigate the h-p version of the boundary element method for integral equation formulations for PDEs over polygonal domains where both the mesh size h and the polynomial degree p are changed to improve accuracy. Under the assumption of quasiuniform meshes, we obtain estimates for the rate of convergence which show the effects on the error of changing h and p either separately or together. Using precise results for the singular behavior of the solution near corners of the domain, it is shown that the rate of convergence for the p-version (h fixed) is twice that of the h version (p fixed) for most problems, which agrees with computational results reported in the literature. Interior estimates are also derived.

10. "Duality estimates for the numerical solution of integral equations", Numer. Math. 54 (1988) 339-353 (with M. Costabel.

Abstract. We formulate and prove Aubin-Nitsche-type duality estimates for the error of general projection methods. Examples of applications include collocation methods and augmented Galerkin methods for boundary integral equations on plane domains with corners and three-dimensional screen and crack problems. For some of these methods, we obtain higher order error estimates in negative norms in cases where previous formulations of the duality arguments were not applicable.

11. "On the convergence of the multigrid method for a hypersingular integral equation of the first kind", Numer. Math. 57 (1990) 379-391 (with T. v. Petersdorff).

Abstract. We present a multigrid method to solve linear systems arising from Galerkin schemes for a hypersingular boundary integral equation governing three dimensional Neumann problems for the Laplacian. Our algorithm uses damped Jacobi iteration, Gauss-Seidel iteration or SOR as pre and post-smoothers. If the integral equation holds on a closed, Lipschitz surface we prove convergence of V- and W-cycles with any number of smoothing steps. If the integral equation holds on an open, Lipschitz surface (covering crack problems) we show convergence of the W-cycle. Numerical experiments are given which underline the theoretical results.

12. "A multigrid method on graded meshes for a hypersingular integral equation", submitted for publication (with T.v. Petersdorff).

Abstract. We consider an integral equation obtained by the "direct method" for the Neumann problem in a polygon which may contain "cracks", i.e., interior angles of 2 . By using a Galerkin method with piecewise linear functions on a suitable graded mesh one can optain the optimal convergence rate $h^{2/3}$. Using a Gauss solver for the arising linear system would require C h^{-6} operations. We show that a multigrid method can solve the linear system with an accuracy of the order of the Galerkin error with only C h^{-4} operations.

13. "An augmented boundary element method for 3 D problems", submitted for publication

Abstract. Here we introduce special singular elements which describe the behavior of the exact solution near corners and edges, as additional test and trial functions in the underlying Galerkin scheme for first kind integral equations on polyhedral surfaces. We analyze the influence of this augmentation on the accuracy of the boundary element method which yields improved error estimates.

14. "Symmetric coupling of finite elements and boundary elements for a parabolic-elliptic interface problem",Quarterly of Applied Mathematics 48 (1990) 265-279 (with M. Costabel, V. J. Ervin).

Abstract. We show asymptotic convergence of a time-stepping method (Crank-Nicolson) in connection with a coupling of finite elements and boundary elements where the trial functions are time-dependent and the test functions are time-independent. The problem under consideration here is the three-dimensional eddy current problem for non-ferromagnetic metals which is given by a transmission problem between the heat equation in a bounded domain and the Laplace equation in its unbounded exterior domain.

15. "The h-p version of the boundary element method with geometric mesh on polygonal domains, Computer Methods in Applied Mechanics and Engineering", 80(1990) 319-325 (with I. Babuska, B.Q. Guo).

Abstract. This paper introduces the techniques of the h-p version to the boundary element method for Dirichlet and Neumann boundary value problems on plane non-smooth domains with piecewise analytic boundary and data. The exponential rate of convergence of the boundary element Galerkin solution is given when a geometric mesh refinement is introduced near the vertices.

16. "Collocation with Chebyshev polynomials for a hypersingular integral equation on an interval, submitted for publication" (with V.J. Ervin).

Abstract. A hypersingular equation on the interval (-1,1) is solved via the collocation method using Chebyshev polynomials of the second kind. This procedure is one of the easiest methods to obtain numerical solutions. It correctly represents the endpoint behavior of the exact solution and yields exponentally fast convergence if the given data are smooth.

17. "Collocation with Chebyshev polynomials for first kind integral equations on intervals", MAFELAP V, 1990, (ed. J.R. Whiteman) (with V.J. Ervin).

Abstract. We present collocation methods with Chebyshev polynomials for Symm's integral equation and for a hypersingular equation on the interval (-1,1). The use of Chebyshev polynomials of first and second kind, respectively, yields exponentially fast convergence of the collocation solution for smooth given data. Numerical experiments are given which underline the theoretical results.

18. "Why multi-grid performs poorly for first kind integral equations with weakly singular kernels", in preparation (with T.v. Petersdorff).

Abstract. We give heuristic arguments for the performance of the multigrid method for differential operators, weakly and strongly singular integral operators. A key role is played by the distribution if eigenvalues of the corresponding Galerkin matrices. A strategy for construction of approximating subspaces is given to save the applicability of the multrigrid method for weakly singular integral equations for 3D problems.

19. "Numerical experiments for multigrid methods for first kind integral equations, submitted for publications" (with V.F. Postell).

Abstract. We present numerical results for the convergence factors of the V-cycle multigrid method for a hypersingular integral equation (normal derivative of double layer potential) and a weakly singular integral equation (single layer potential) on surfaces covering 3 D problems. The linear systems to be solved by the multigrd method arise from Galerkin schemes with piecewise linear or piecewise constant boundary elements, respectively. Our algorithm uses damped Jacobi or Gauss-Seidel interation or SOR as pre-and post smoothers. Numerical results are also given for graded meshes. The experiments were performed on the CRAY in Pittsburgh.

20. "Regularity and boundary element solution of the h-p version for transmission problems with piecewise analytic data".
Part 1: Regularity of solution for transmission problems with piecewise analytic data, submitted for publication (with B. Guo)
Part 2: h-p version of boundary element method for transmission problems, in preparation (with B. Guo).

Abstract. Part 1: This is the first of the series devoted to transmission problems with piecewise analytic data. The present paper analyzes regularity of solutions of partial differential equations and integral equations for transmission problems with piecewise analytic data in terms of countable weighted Sobolev spaces, which in Part 2 will be used for proper design of the boundary element solutions of the h-p version.

21. "On the h-p version of the boundary element method for Symm's integral equation on polygons, submitted for publication" (with V.J. Ervin, N. Heuer).

Abstract. We present numerical result for the h-p version of BEM on polygons. The problem under consideration is the Dirichlet problem of the Laplacian. Our numerical results, which are computed with our soft-ware packages, show exponential rate convergence for geometric mesh refinement. The same convergence behavior is obtained for adaptive versions based on a feedback algorithm which uses residuals as error indicators.

22. "Quadrature and collocation methods for the double layer potential on polygons, submitted for publication" (with M. Costabel, V.J. Ervin).

Abstract. The second kind integral equation with the double layer potential is considered on polygons. We prove convergence for the collocation method with piecewise constant trial functions on a quasi-uniform grid by first analyzing a quadrature method. We show stability of the approximating schemes by analyzing model problems on an infinite sector domain making use of a localization principle by Gohberg and Krupnick. We apply heavily Mellin transformation techniques together with properties of Toeplitz operators to handle the model problems.

23. "The h-p version of the Galerkin boundary element method for integral equations on polygons and open arcs". In: Proc. Conference BEM-10, 1988, 205-216. Southampton (ed. C.A. Brebbia).

Abstract. We present the h-p version of the boundary element method when Dirichlet and Neumann problems are solved over polygonal domains using integral equation techniques. Asymptotic rates of convergence are given for both quasiuniform and geometric meshes.

24. "Improved Galerkin methods for integral equations on polygons and polyhedral surfaces". Japan-US Symposium on Boundary Element Methods, Tokyo (1988) 73-80.

Abstract. We investigate the Galerkin boundary element method for the first kind equation with the single layer potential on polygons and on polyhedral surefaces. We derive quasi-optimal converence rates for the h-p version when quasiuniform meshes are used. Furthermore, for the h-version we analyze the influence of graded meshes on the rate of convergence of the Galerkin solutions and show that it suffices to refine only towards the edges of the surfaces to regain the optimal rate of convergence.

Talks were presented at the following meetings, conferences, and universities by E. P. Stephan

1. Workshop on PDEs, Univ. Chicago,	March	1990
2. Clemson University, SC,	Febr	1990
3. Univ.South Carolina, Columbia, SC.	March	1990
4. Conf. on Computational Mechanics, Austin,TX,	Oct	1989
5. Univ. New South Wales, Sydney (Australia)	July	1989
6. Finite Element Circus, Purdue Univ., West Lafayette, IN	April	1989
7. Conference on Numerical Methods for Integral Equation, Oberwohlfach (FRG),	Dec	1988
8. 10th Intern. Conf. on Boundary Element Methods, Southampton (UK),	Sept	1988
9. Workshop on Singularities, Technische Hochschule Darmstadt (FRG),	Sept	1988
10. Japan/US. Symposium on Boundary Element Methods, Tokyo (Japan),	Sept	1988
11. Finite Element Circus, Penn State Univ.,PA,	Sept	1988
12. Carnegie Mellon University, Pittsburgh, PA	Nov.	1988
13. NSF-CBMS Conference, Univ. of Kentucky, Lexington, KY	May	1988
14. Finite Element Circus, Univ. of Maryland, College Park, MD	April	1988

The following researchers were brought to Georgia Tech for research consultations with E. P. Stephan.

1. Prof. Dr. M. Suri, Univ. of Maryland, Baltimore County
2. Dr. M. Costabel, Technische Hochschule Darmstadt
3. Prof. Dr. B. Q. Guo, Univ. of Manitoba, Winnipeg

ERNST P. STEPHAN

CURRICULUM VITAE

OCTOBER 1990

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EDUCATIONAL BACKGROUND:

Staatsexamen	1970	Technical Univ., Darmstadt (FRG)	Mathematics & Physics
Ph.D.	1975	Technical Univ., Darmstadt (FRG)	Mathematics
Dr.habil.	1984	Technical Univ., Darmstadt (FRG)	Mathematics

Habilitationsschrift: Boundary integral equations for mixed boundary value problems, screen and transmission problems in $\mathbb{R}^3$, 136 pages.

EMPLOYMENT HISTORY:

Wissenschaftlicher Mitarbeiter, Technical University Darmstadt	1970-1983
Visiting Mathematician, Ecole Polytechnique, Paris-Palaiseau (France) (2 months)	1978
Visiting Mathematician, Carnegie-Mellon University, Pittsburgh (6 months)	1981
Visiting Mathematician, Israel Institute of Technology, Haifa (1 month)	1982
Visiting Mathematician, Institute for Applied Mathematics, Sonderforschungsbereich 72, University Bonn (2 months)	1984
Honorary Visiting Fellow, University of New South Wales, Sydney (Australia) (1 month)	1988
Associate Professor, School of Mathematics, Georgia Tech	1983-1988
Professor, School of Mathematics, Georgia Tech	1988-pres

CURRENT FIELDS OF INTEREST:

Numerical Analysis for problems arising in mathematical physics and in partial differential equations.

(i) Solution procedures for potential problems and time-harmonic scattering (Helmholtz, Maxwell) and problems in elasticity (crack problems) by means of pseudo-differential operators (boundary integral equations).

(ii) Error analysis for Galerkin, collocation and least squares methods; quadrature errors; finite elements, boundary elements. Numerical and theoretical aspects of boundary element methods: h,p and h-p versions, adaptive methods.

(iii) Regularity of elliptic boundary value problems in domains with corners and edges.

(iv) Coupling of finite elements and boundary elements.

(v) Multigrid methods, konjugate gradient methods.

(vi) Nonlinear elasticity, plasticity.

REFEREED PUBLICATIONS: (E.P. Stephan)

(a) Already published

1. On the h, p and h-p version of the boundary element method - Numerical results, Computer Methods in Applied Mechanics and Engineering (with F. V. Postell), 83 (1990) 69-90.

2. The h-p version of the boundary elemet method with geometric mesh on polygonal domains. Computer Methods in Applied Mechanics and Engineering 80 (1990) 319-325 (with I.Babuska,B.Q. Guo).

3. On the convergence of the multigrid method for a hyper-singular integral equation of the first kind, Numerische Mathematik (with T. von Petersdorff), 57, (1990) 379-391.

4. On the exponential convergence of the h-p version for boundary element Galerkin methods on polygons, Math. Methods in Applied Sciences (with I. Babuska, B. Guo) 12 (1990) 413-427.

5. Experimental asymptotic convergence of the collocation method for boundary integral equations on polygons, governing Laplace's equation, Journal of Computational Mechanics 6 (1990) 271-278 (with M. Costabel and V. J. Ervin).

6. A boundary integral equation procedure for the mixed boundary value problem of the vector Helmholtz equation, Applicable Analysis, 35 (1990) 59-74

7. A p-version boundary element method for screen and crack problems, Ramanujan Birth Centenary International Symposium on Analysis, Pune, India, 1987 (with M. Suri).

8. Decompositions in edge and corner singularities for the solution of the Dirichlet problem of the Laplacian in a polyhedron, Mathematische Nachrichten, 145 (1990) (with T. von Petersdorff).

9. Regularity of mixed boundary value problems in R^3 and boundary element methods on graded meshes, Mathematical Methods in Applied Sciences 12 (1990) (with T.Petersdorff), 229-249.

10. Symmetric coupling of finite elements and boundary elements for a parabolic-elliptic interface problem, Quarterly of Applied Mathematics (1990) (with M. Costabel, V.J. Ervin) 48, 265-279.

11. Integral equations for transmission problems in linear elasticity, Journal of Integral Equations and Applications (1990) (with M. Costabel) 2, 211-223.

12. Coupling of Finite Element and Boundary Element Methods for an Elasto-Plastic Interface Problem, SIAM Numerical Analysis 27 (1990) (with M. Costabel) (October).

13. On the convergence of the p-version for some boundary element Galerkin methods, Mathematics of Computation, 52 (1989) 31-48 (with M. Suri).

14. Duality estimates for the numerical approximation of boundary integral equations, Numerische Mathematik, 54 (1988) 339-353 (with M. Costabel).

15. Strongly elliptic boundary integral equations for electromagnetic transmission problems, Proc. Royal Society Edinburgh, 109A (1988) 271-296 (with M. Costabel).

16. On the convergence of collocation methods for Symm's integral equation on open curves, Mathematics of Computation, 51(1988) 167-179 (with M. Costabel and V. J. Ervin).

17. Singularities of the Laplacian at corners and edges of three dimensional domains and their treatment with finite element methods, Math. Meth. in Appl. Sciences, 10 (1988) 339-350 (with J. R. Whiteman).

18. On the convergence of collocation methods for boundary integral equations on polygons, Mathematics of Computation, 49 (1987) 461-478 (with M. Costabel).

19. An improved boundary element Galerkin method for three-dimensional crack problems, Journal of Integral Equations and Operator Theory, 10 (1987) 467-504 (with M. Costabel).

20. Boundary integral equations for screen problems in IR3, Journal of Integral Equations and Operator Theory, 10 (1987) 236-257.

21. Boundary integral equations for mixed boundary value problems in IR3, Math. Nachrichten, 134 (1987) 21-53.

22. A boundary integral equation method for three-dimensional crack problems in elasticity. Math. Meth. in Appl. Sciences 8 (1986) 609-623 (with M. Costabel).

23. Boundary integral equations for magnetic screens in IR3, Proc. Royal Soc.Edinburgh, 102 A, 189-210 (1986).

24. On the Finite Element-Least Squares Approximation to Higher Order Elliptic Systems, Archive Rat. Mech. Anal. 91 (1986) 137-151 (with G.J. Fix).

25. A skin effect approximation for eddy current problems, Archive Rat. Mech. Anal. 90, 87-98 (1985) (with R.C.MacCamy).

26. A direct boundary integral equation method for transmission problems. Journal of Mathematical Analysis and Applications, 106 (1985) 367-413 (with M. Costabel).

27. An augmented Galerkin procedure for the boundary integral method applied to mixed boundary value problems. Applied Numerical Mathematics 1 (1985) 121-143 (with W.L. Wendland).

28. An augmented Galerkin procedure for the boundary integral method applied to two-dimensional screen and crack problems. Appl.Analysis (1984), 18, 183-219 (with W.L. Wendland).

29. Galerkin collocation for an improved boundary element method for a plane mixed boundary value problem, Computing (1984), 33, 269-296 (with U. Lamp, T. Schleicher and W.L. Wendland).

30. Boundary integral equations for mixed boundary value problems in polygonal domains and Galerkin approximation, Mathematical Models and Methods in Mechanics. Banach Center Publications 15, 175-251, Warsaw, 1985 (with M. Costabel).

31. Solution procedures for three-dimensional eddy current problems. Journal Math. Anal.and Appl., 101, 348-379 (1984) (with R. C. MacCamy).

32. Boundary integral equations for Helmholtz transmission problems and Galerkin approximation, ZAMM 64, T356-T358 (1984) (with M. Costabel).

33. A boundary element method for a two-dimensional interface problem in electromagnetics. Numerische Mathematik, 42, 311-322 (1983) (with S. Hariharan).

34. On Boundary Integral Equations of the First Kind for the bi-Laplacian in a Polygonal Plane Domain, Annali Scuola Normale Superiore - Pisa, Ser. IV 10, 197-241 (1983) (with M. Costabel, W. L. Wendland).

35. The normal derivative of the double layer potential on polygons and Galerkin approximation. Applicable Analysis 16, 205-228 (1983) (with M. Costabel).

36. A boundary element method for an exterior problem for three-dimensional Maxwell's equations, Applicable Analysis, 16, 141-163 (1983) (with R.C. MacCamy).

37. Curvature terms in the asymptotic expansions for solutions of boundary integral equations on curved polygons, Journal Integral Equations, 5 (1983), 353-371 (with M. Costabel).

38. A modified Galerkin procedure for bending of beams on elastic foundations. SIAM Journal on Scientific and Statistical Computing, 4,(1983) 340-352 (with J. Bielak).

39. Boundary Element Method for Membrane and Torsion Crack Problems. Comput. Meth. Appl. Mech. Eng., 36, 331-358 (1983) (with W. L. Wendland).

40. Conform and Mixed Finite Element Schemes for the Dirichlet Problem for the Bilaplacian in Plane Domains with Corners, Math.Meth. in Appl. Sci. 1, 354-382 (1979).

41. On the Integral Equation Method for the Plane Mixed Boundary Value Problem of the Laplacian, Math. Meth. in the Appl. Sci. 1, 265-321, (1979) (with W.L. Wendland and G.C. Hsiao).

42. Zur Approximation von Schalen mit hybriden Elementen, Computing 20, 75-94 (1978) (with V. Weissgerber).

43. Remarks to Galerkin and Least Squares Methods with Finite Elements for General Elliptic Problems, manuscripta geodaetica 1, 93-123 (1976). Short version in: Proc. Conf. O. Partial Diff. Equa. Dundee 1976, Lecture Notes in Mathematics 564, 451-471, Springer 1976 (with W.L. Wendland).

44. Differenzenapproximation von Pseudo-Differentialgleichungen, ZAMM 56, T267-T268, (1976).

(b) Accepted for Publication

45. Experimental convergence rates for various couplings of boundary and finite elements, Computers and Mathematics with Applications (with M. Costabel and V.J. Ervin), in press.

46. A hyper-singular boundary integral method for two-dimensional screen and crack problems, Archive Rat. Mech. Anal. (with W.L. Wendland), in press.

47. The Dirichlet problem in elasticity for a domain exterior to an arc, J. Computational and Applied Mathematics (with G. C. Hsiao an W. L. Wendland).

48. An improved boundary element Galerkin method for the charge density of a thin, electrified, square plate in IR3, Math. Meth. in Appl.Sciences (with S. Abou El-Seoud and V. J. Ervin), in press.

49. The h-p version of the boundary element method on polygonal domains with quasiuniform meshes. RAIRO Analy. Numer. (with M. Suri).

50. A boundary element Galerkin method for a hypersingular integral equation on open surfaces, Math. Meth. in Appl. Sciences (with V. J. Ervin), in press.

51. A boundary element method with mesh refinement for a weakly singular integral equation, Communications in Applied Numerical Methods (with V.J. Ervin).

(c) Submitted for Publication

52. Regularity and boundary element solution of the h-p version for transmission problems with piecewise analytic data, Part 1: Regularity of solution. Part 2: h-p version of BEM, (with B.Q. Guo).

53. Am augmented boundary element method for 3D problems.

54. Collocation with Chebyshev polynomials for Symm's Integral Equation on an Interval, Australian Math. Society (with I. Sloan).

55. Quadrature and collocation methods for the double layer potential on polygons (with M. Costabel and V. J. Ervin) Zeitschrift für Analysis und ihre Anwendungen.

56. Singularities of the solution of the Laplacian in domains with circular edges, Complex Variables Theory and Applications (with T.von Petersdorff).

57. A multigrid method on graded meshes for a hypersingular integral equation (with T. von Petersdorff).

58. On the h-p Version of the Boundary Element Method for Symm's Integral Equation on Polygons (with V. J. Ervin, N. Heuer).

59. Collocation with Chebyshev Polynomials for a Hypersingular Integral Equation on an Interval (with V. J. Ervin).

60. Numerical experiments for multigrid methods for first kind integral equations, (with V. F. Postell).

(d) In Preparation

61. Numerical solution of elliptic problems in 3D with unsmooth boundaries (with I. Babiska, T. von Petersdorff).

62. A boundary element method for electromagnetic transmission problems.

63. Collocation for a hyper-singular integral equation on polygons (with M. Costabel and V. J. Ervin).

64. Regularity near exceptional angles of the solution of boundary value problems in polygons (with T.V.Petersdorff).

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66. An adaptive boundary element scheme for a weakly singular integral equation on surfaces (with V. J. Ervin).

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