Comparison of Thermoelectric and Stirling Type Cryocoolers Using Control Thermodynamic Model

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ABSTRACT

New efforts are underway to develop thermoelectric materials for cooling of infrared detectors at cryogenic temperatures. Stirling type cryocoolers routinely produce cooling at cryogenic temperatures with good efficiency, but challenges remain in their miniaturization, reliability and vibration reduction for space applications. In this study, a thermodynamic comparison of Stirling type and Thermoelectric (TE) cryocoolers is made for a typical second stage cryogenic refrigerator (30 K to 80 K). It is assumed that a reservoir at 80 K is available and a cooling load at 40 K is desired. It is shown that under the assumption of availability of TE materials with a reasonably high figure of merit, a multistage TE cryocooler is required. For comparison of the performance of the cryocoolers, thermodynamic models of the Stirling type and multistage TE cryocoolers are developed. The effect of important system parameters on the performance of the cryocoolers is presented. The thermal design challenges of miniaturization of Stirling Type cryocoolers and the development of multistage TE cryocoolers with high efficiency are discussed.

INTRODUCTION

Stirling Type Refrigerators (STRs) play an important role in satisfying the need for cryogenic cooling of space-based infrared detectors as well as electronics requiring coolers with high efficiency. Multistage STRs can be used for cooling at very low temperatures of about 3 K if required. Challenges exist in miniaturization of STRs and their vibration for space applications. STRs, referred to in this study, are a class of regenerative refrigerators that consists of Stirling Refrigerators (SRs) and Pulse Tube Refrigerators (PTRs).[1] Thermoelectric refrigerators possess great advantages for cooling of space-based infrared detectors because they are solid state devices having no moving parts and are miniature, highly reliable, and easy to integrate into the system. There have been many applications of the thermoelectric effect in both cooling and power generation.[2,3] The development of Thermoelectric Refrigerators (TERs) for application at cryogenic temperatures is ham-

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pered by the fact that thermoelectric materials for application at low temperatures are not available, and challenges exist in improving their cooling capacity and efficiency. New efforts are underway to develop thermoelectric materials for application at cryogenic temperatures. In this study we use control thermodynamic models of STRs and TERs to compare their performances with regard to the no-load temperature, cooling capacity, and efficiency. More specifically, we would like to estimate the figure of merit required for thermoelectric materials at the temperature range typical of the second stage cryocooler (30 K to 80 K) for a performance comparable to STRs. In addition, the effect of important global system parameters on the performance of STRs and TERs is evaluated.

Thermoelectric coolers are based on the Peltier phenomenon which exists when electric current is applied to the junction of two different conducting materials (typically semiconductors). One material, p-type, contains positive charge carriers (holes) and the other, n-type, contains negative charge carriers (electrons). In general, TE materials can be categorized as uniform bulk materials or thin films with periodic variation in structure and composition.[2,4] Bulk materials can pump heat fluxes on the order of 10 W/cm^2, while thin films can reach an order of magnitude higher heat fluxes with direct sensor cooling capabilities.[5] New efforts are underway to develop TE materials for cooling at cryogenic temperatures.[6] Assuming TE materials at cryogenic temperatures are available, control thermodynamic models of STRs and TERs for the temperature range of 30 K to 80 K are developed in this study. The performances of cryocoolers are compared using regenerator effectiveness for STRs and the figure of merit for TERs as the primary control parameters.

THERMODYNAMIC MODELS OF STRS

The control thermodynamic models of STRs using exergy analysis have been previously reported.[7,8] Figure 1 gives the schematic of STRs used in the model. The thermodynamic models of STRs include the most important control parameters for analysis of Stirling type cryocoolers. Assuming no external irreversibilities associated with heat transfer between the system and the reservoirs, no conduction heat transfer in the regenerator, and no heat leaks to the system, the cooling capacity of PTRs and SRs can be written respectively as

$$\dot{Q}_{c,PTR} = \frac{\dot{W}_{in}\eta_e\eta_{comp}\eta_r T_c}{T_o\eta_e + (1-\eta_e)T_c} - \frac{2\dot{W}_{in}P_a\eta_r\eta_{comp}\lambda\gamma(1/Mr - T_c/T_o)}{\pi p_1(\gamma-1)\Pr Cos(\phi_2-\theta_2)} \quad (1)$$

$$\dot{Q}_{c,SR} = \frac{\dot{W}_{in}\eta_e\eta_{comp}\eta_r}{1+\eta_e(T_o/T_c-\eta_r-1)} - \frac{2\dot{W}_{in}P_a\eta_r\eta_{comp}\lambda\gamma(\eta_e T_o/T_c-\eta_e+1)(1/Mr-T_c/T_o)}{\pi p_1(\gamma-1)\left[1+\eta_e(T_o/T_c-\eta_r-1)\right]\Pr Cos(\phi_2-\theta_2)} \quad (2)$$

where the parameters in the equations are defined as:

Mr = mass flow ratio across the regenerator
p_1 = pressure amplitude of the hot side of the regenerator
P_a = average pressure
Pr = pressure amplitude ratio across the regenerator
T_o = temperature of environment
$\dot{W}_{in}$ = compressor input power
η_e = exergetic efficiency of expansion process in pulse tube or the expander.
η_{comp} = exergetic efficiency of the compressor
η_r = exergetic efficiency of flow in the regenerator of PTRs or SRs
λ = ineffectiveness of the regenerator
γ = specific heat ratio of working fluid
$\phi_2-\theta_2$ = phase shift between the mass flow and pressure at the cold side of regenerator

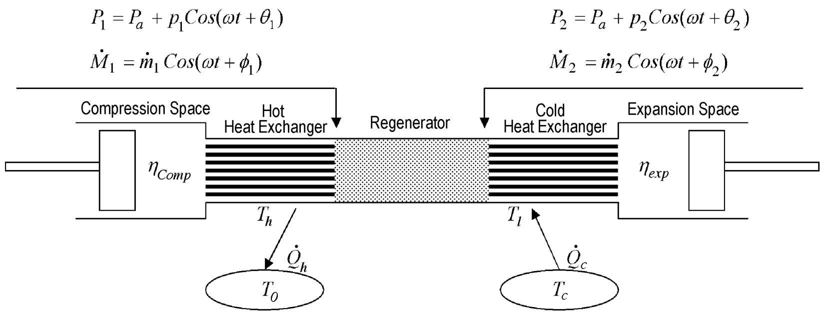

Figure 1. Schematic of Stirling type refrigerators and the parameters used in the model.

Defining the exergetic efficiency of flow in the regenerator makes the analytical expression for the cooling capacity of STRs possible.[8] For the ideal case of a thermally perfect regenerator (λ=0), using Eqs. (1) and (2), the ratio of the Coefficient of Performance (COP) of PTRs to the COP of SRs can be written as

$$\frac{(COP)_{PTR}}{(COP)_{SR}} = \frac{T_o\eta_e + (1-\eta_e(1+\eta_r))T_c}{T_o\eta_e + (1-\eta_e)T_c} = 1 - \frac{\eta_e\eta_r T_c}{T_o\eta_e + (1-\eta_e)T_c} \tag{3}$$

For the ideal case of $\eta_e = \eta_r = 1$, the ratio of COP given in Eq. (3) is $(1 - T_c / T_o)$. Thermal ineffectiveness of the regenerator plays an important role in the performance of STRs. For example, it can be shown that the thermodynamic bound for the no-load temperature of SRs can be obtained from the following expression:[8]

$$\frac{2P_a\lambda}{\pi\, p_1 \Pr}\frac{\gamma}{\gamma-1}(\frac{T_o}{T_{co}} - 1 + \frac{1}{\eta_e})(\frac{1}{Mr} - \frac{T_{co}}{T_o}) = Cos(\phi_2 - \theta_2) \tag{4}$$

where T_{co} is the no-load temperature. Eq. (4) shows the importance of the thermal ineffectiveness of the regenerator on the no-load temperature of SRs.

The thermodynamic model of thermoelectric refrigerators using average thermal and electrical properties of the TE couples in terms of electric current has been previously developed.[4,9] The cooling capacity and input power can be written respectively as

$$\dot{Q}_c = 2N[\alpha I T_c - 0.5 I^2 \rho / G - kG(T_o - T_c)] \tag{5}$$

$$\dot{W}_{in} = 2N[I^2 \rho / G - \alpha I (T_o - T_c)] \tag{6}$$

where the parameters in the equations are defined as:
G = geometric factor, the ratio of cross section to the length of TE element
I = electric current
k = thermal conductivity of TE couple
N = number of TE couples
$\dot{Q}_c$ = cooling capacity of TERs
$\dot{W}_{in}$ = input power to TERs
α = Seebeck coefficient of TE couple
ρ = electric resistivity of TE couple

Eqs. (5) and (6) can be combined to find the cooling capacity in terms of the input power for comparison to Eqs. (1) and (2) for STRs.

$$\dot{Q}_c = 0.5NZkG(T_o + T_c)(T_o - T_c)[(1 + \frac{2\dot{W}_{in}}{NZkG(T_o - T_c)^2})^{1/2} - 1] - 2NkG(T_o - T_c)] - 0.5\dot{W}_{in} \quad (7)$$

where Z is the figure of merit of the TE couple defined by $Z = \alpha^2 / k\rho$. Eq. (7) like Eqs. (1) and (2) assumes that there is no irreversibility due to the heat transfer between the refrigerator and the thermal reservoirs and the thermal and electrical properties of the TE material are independent of the temperature.

The maximum of cooling capacity of TERs can be obtained from Eq. (5) using $\partial \dot{Q}_c / \partial I = 0$ or from Eq. (7) using $\partial \dot{Q}_c / \partial \dot{W}_{in} = 0$. The maximum cooling capacity can be written as:

$$\dot{Q}_c = 2NGk[0.5ZT_c^2 - (T_o - T_c)] \quad (8)$$

The input work corresponding to the maximum cooling capacity of TERs is given by:

$$\dot{W}_{in} = 2NGkZT_cT_o \quad (9)$$

An important quantity to characterize the performance of TERs is its no-load temperature represented by:

$$T_{co,TER} = (\sqrt{1 + 2ZT_o} - 1) / Z \quad (10)$$

Eq. (10) for TERs corresponds to the result obtained for the no-load temperature of Stirling refrigerators given by Eq. (4). The figure of merit for TE materials has significant effect on the value of the no-load temperature of TERs. Another method for optimizing TERs is to evaluate the condition of the maximum COP. Using Eqs. (5) and (6), the optimum current can be written as:

$$I_{opt,COP} = \frac{\alpha G(T_o - T_c)}{\sqrt{1 + Z(T_o - T_c)/2} - 1} \quad (11)$$

Using the above equation the cooling capacity and the input power can be obtained from Eqs. (5) and (6), respectively.

THERMODYNAMIC MODELS OF MULTISTAGE TERS

One of the challenges of using TERs is the fact that the figure of merit of TE materials is not very high. Therefore, the no-load temperature of TERs given by Eq. (10) would not be low enough to reach cryogenic temperatures from the temperature of the environment in a single stage. One solution is to use multistage TERs between T_o and T_c. The thermodynamic analysis using an exo-reversible model of multistage TERs has been previously reported.[10] For the purpose of this study, two thermodynamic models for multistage TERs are developed. One model is based on the optimum cooling capacity at each stage using Eqs. (8) and (9). The other model is based on the optimum COP obtained from Eqs. (5), (6) and (11). To simplify the optimization process, the temperatures of the hot and cold sides for each stage are obtained assuming $T_{hi} / T_{ci} = (T_o / T_c)^{1/n}$ where n is the number of stages. In this manner the important quantities can be obtained for each stage successively. To find the thermodynamic bound for multistage TERs, it is assumed that the hot temperature of each stage is equal to the cold temperature of next stage. Therefore, it is assumed that the irreversibility due to heat transfer between the stages is zero. Finally the exergetic efficiency (fraction of Carnot efficiency) is calculated from the following equation.

$$\eta_{ex,TER} = \frac{\dot{Q}_c (T_o / T_c - 1)}{\sum_i \dot{W}_{in,i}} \quad (12)$$

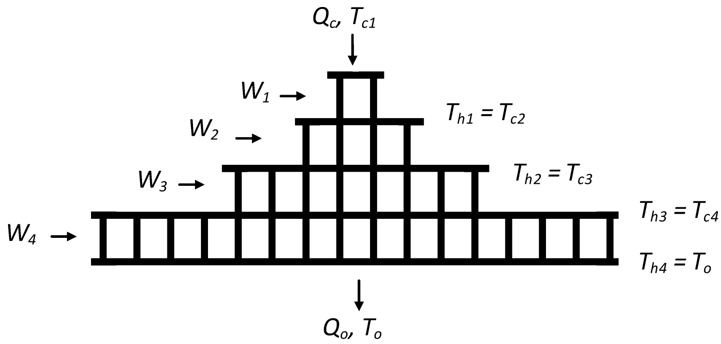

Figure 2. Schematic of a four-stage thermoelectric refrigerator and the parameters used in the model.

As an example, the schematic of a four-stage TER and parameters used in this study are shown in Fig. 2.

RESULTS AND DISCUSSION

Load curves are often used to characterize the performance of cryocoolers. Load curves ($\dot{Q}_c$ vs. T_c) are reported for different values of the environmental temperature and input power to the cryocoolers. Eqs. (1), (2) and (7) give the thermodynamic bound for cooling capacity of PTRs, SRs, and TERs, including important control parameters influencing the refrigerators. Figure 3 shows the load curves for the input power of 50 W and the environmental temperature of 300 K. The results of Eqs. (1) and (2) are given in the figure for selected parameters typical of STRs. The load curves for TERs can be obtained from Eq. (7) for a given number of TE couples. For comparison to STRs, the load curves are given for different figures of merit of TE couples calculated from Eqs. (8) and (9) with number of TE couples as a free parameter.

Therefore, the results for the cooling capacity are the optimum values and correspond to the thermodynamic bound of the load curves of TERs for the example under consideration. Since the number of TE couples is used as a control parameter, the smooth curves given in the figure should be interpreted as the interpolated values corresponding to an integer number of TE couples. The figure merit is the control parameter in this calculation, and it is assumed that the Seebeck coefficient can be changed while other parameters for the TE material are constant. From the figure it can be seen that the figure of merit of TERs must be high to produce load curves comparable to the high performance STRs at cryogenic temperatures. Such high

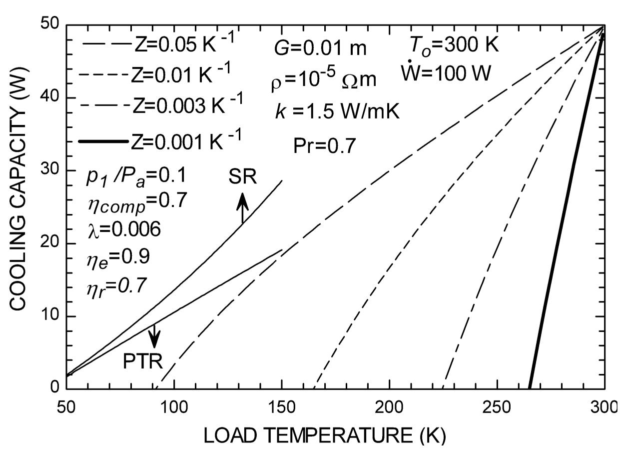

Figure 3. Load curves for TERs, SRs and PTRs for selected parameters.

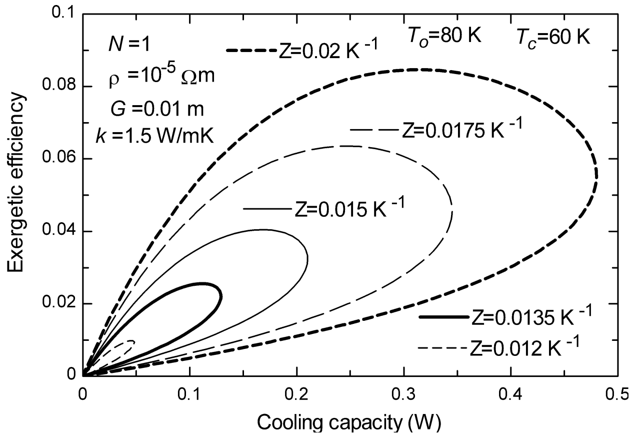

Figure 4. Cooling capacity and efficiency diagram of a thermoelectric refrigerator for different values of Z and T_c=60 K.

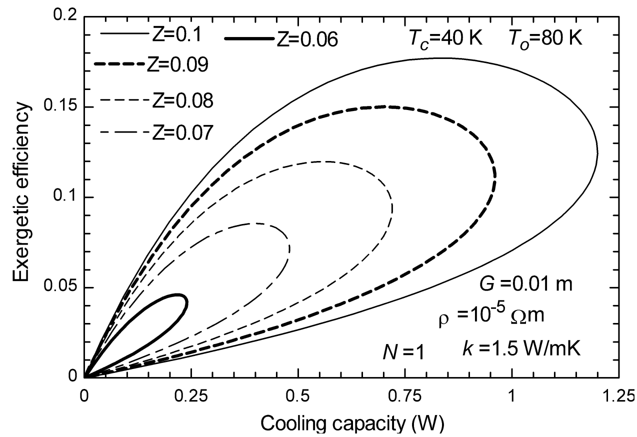

Figure 5. Cooling capacity and efficiency diagram of a thermoelectric refrigerator for different values of Z and Tc=40 K.

values for the figure of merit are an order of magnitude higher than what is achievable at the present time. The no-load temperature given in Fig. 3 corresponds to the values obtained from Eq. (10).

Figure 4 gives the power efficiency diagram ($\dot{Q}_c$ vs. η_{ex}) for the load temperature of 60 K and the environmental temperature of 80 K. The results are obtained using Eqs. (5) and (6) with the electric current as a free parameter. Other parameters are given in the figure. The figure shows that even for the small temperature difference across the TER, reasonably high values for the figure of merit are necessary to obtain acceptable values of efficiency. For example, the exergetic efficiency of typical SRs for the same temperature range obtained from Eq. (2) is about 0.25. It is interesting to note that the power efficiency diagram is a looped-shape curve indicating a compromise between cooling capacity and efficiency of TERs. Figure 5 shows the same result as in Figure 4 when the load temperature is reduced to 40 K. As expected, lowering the load temperature requires higher values for the figure of merit of the TE couple to produce reasonable cooling capacity and efficiency. For example, for values of Z lower than 0.012 K^{-1} the limit of cooling capacity of TERs is reached for the temperature range under consideration. To reach a load temperature of 40 K from the environmental temperature of 300 K, Eq. (10) shows that a very large figure of merit of the order of Z=0.3 is required.

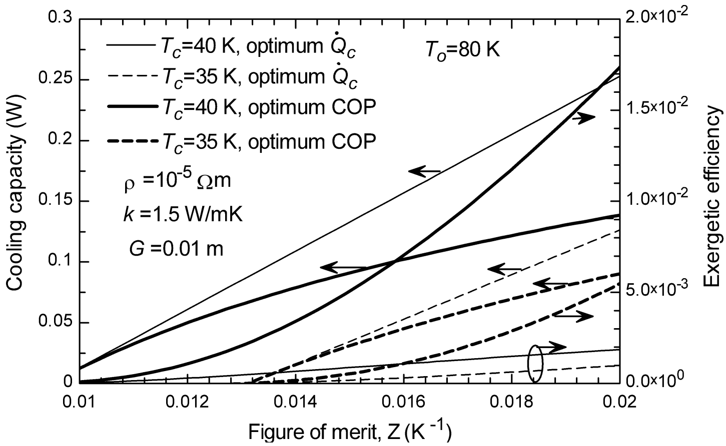

Figure 6. Comparison of cooling capacity and exergetic efficiency for a four-stage TER based on two different criteria of optimization.

A well-known solution to relax the requirement of a high figure of merit for TE couples in order to reach a reasonably low load temperature is to use multistage TERs. Each stage in the multistage TERs will require a lower value of figure of merit compared to the single stage for the same hot and cold temperature limits. Figure 6 shows the cooling capacity and exergetic efficiency for a four-stage TER shown schematically in Figure 2. All thermal and electrical properties are assumed to be the same in all stages. The results are given for two optimization criteria based on the maximum cooling capacity and maximum COP. The effect of the two optimization criteria on cooling capacity and efficiency of the multistage TER is clearly shown. In these calculations the number of TE couples in each stage is taken to be the control parameter. It should be pointed out that in the calculation of the thermodynamic bound of the multistage TER, it is assumed that there is no heat transfer irreversibility between the stages. In many applications this irreversibility is significant, which results in a substantial reduction in efficiency and cooling capacity of multistage TERs.

Even though the efficiency of STRs is much higher than the efficiency of TERs, especially at cryogenic temperatures, the latter has great advantage for spot cooling. Great challenges exit in the miniaturization of STRs, and thermal and physical limits exist in the design of such cryocoolers. Well designed STR microcoolers have a cooling capacity per unit volume of the order of 2 W/liter (2 mW/cm^3) at cryogenic temperatures of about 100 K.[11] In contrast, a recent three stage thin-film superlattice thermoelectric multistage TER can produce a cooling heat flux on the order of 100 W/cm^2 of cooling capacity.[12] This corresponds to a cooling capacity per unit volume of several orders of magnitude larger than STRs. It should be pointed out that the no-load temperature of such a cooler is much larger that STRs. High heat flux requirements of TERs for spot cooling creates thermal management issues for the design of such heat sink limited micro devices, especially in the multi-stage arrangement.

CONCLUSIONS

Using control thermodynamic models of Stirling type and thermoelectric refrigerators, their cooling performances and their efficiencies were compared with emphasis on their performance at cryogenic temperatures. It is shown that high values of figure of merits for thermoelectric materials are necessary even when they are used at cryogenic temperatures typical of second stage cryocoolers (80 K to 30 K). Thermodynamic bounds for a four-stage thermoelectric refrigerator typical of a second stage cryocooler under the optimum cooling capacity and optimum coefficient of performance are investigated. There are physical and thermal limits to miniaturization of Stirling type refrigerators, and thermoelectric refrigerators have clear advantages here due to their very small size. The resulting high cooling heat flux of thermoelectric refrigerators creates thermal management issues especially when applied in multistage configurations at cryogenic temperatures.

REFERENCES

1. Radebaugh, R., “Pulse tube cryocoolers for cooling infrared sensors,” *Proceedings of SPIE*, Vol. 4130 (2000), pp. 363-379.
2. Rowe, D.M., *CRC handbook of thermoelectric*, CRC, Boca Raton, FL, (1995).
3. DiSalvo, F.J., “Thermoelectric cooling and power generation,” Science, Vol. 285 (1999), pp. 703-706.
4. Rowe, D.M., *CRC handbook of thermoelectric: Macro to nano*, CRC, Boca Raton, FL, (2006)
5. Venkatasubramanian, R., Siivola, E., Colpitts, T., and O’Quinn, B., “Thin-film thermoelectric devices with high room-temperature figures of merit, *Nature*, Vol. 413 (2001), PP. 597-602.
6. Bentien, A., Johnsen, S., Madsen, G.K.H., Iversen, B.B., and Stegkich, F., “Colossal Seebeck coefficient in strongly correlated semiconductor $FeSb_2$,” *Europhysics letters*, Vol. 80, 17008 (2007), pp. 1-5.
7. Razani, A., T. Roberts and B. Flake. “A Thermodynamic Model Based on Exergy Flow for Analysis and Optimization of Pulse Tube Refrigerators,” *Cryogenics*, Vol. 47 (2007), pp. 166-173.
8. Razani, A., C. Dodson and T. Roberts. “A Model for Exergy analysis and Thermodynamic Bounds of Stirling Refrigerators,” *Cryogenics*, Vol. 50 (2010), pp. 231-238.
9. Phelan, P.E., Chiriac, V.A., Lee, T.T., “Current and future miniature refrigeration cooling technologies for high power electronics,” *IEEE Transactions on Components and Packaging Technologies*, Vol. 25 (2002), pp. 356-365.
10. Xuan, P.E. “Optimum staging of multistage exo-reversible refrigeration systems,” *Cryogenics*, Vol. 43 (2003), pp. 117-124.
11. Petach, M., Waterman, M., Pruitt, G. and Tward, E., “High frequency coaxial pulse tube microcoolers,” *Cryocoolers 15*, ICC Press, Boulder, CO (2009), pp. 97-102.
12. Bulman, G.E., Siivola, E.D., Wiitala, R., Venkatasubramanian, R. Acree, M., and Ritz, N. “Three-stage thin-film superlattice thermoelectric multistage microcoolers with a ΔTmax of 102 K,” *Journal of Electronic Materials*, Vol. 38 (2009), pp. 1510-1515.