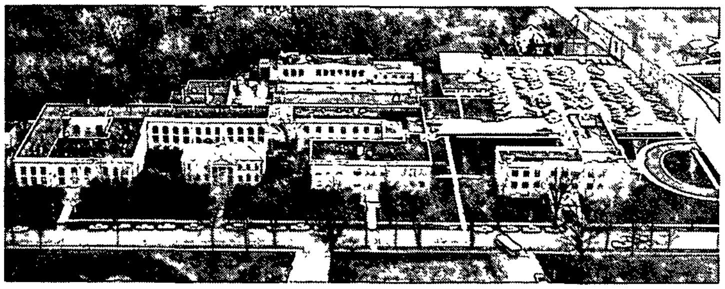

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NUMERICAL ANALYSIS OF THE INFLUENCE OF FORMATION ON THE OPTICAL PROPERTIES OF PAPER

DOUGLAS WAHREN

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Douglas Wahren

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NUMERICAL ANALYSIS OF THE INFLUENCE OF FORMATION ON THE OPTICAL PROPERTIES OF PAPER

Douglas Wahren
The Institute of Paper Chemistry*
Appleton, WI 54912 U.S.A.

ABSTRACT

Reported here are results of numerical studies of the detrimental effects of uneven mass distribution on the optical properties of paper, based on the assumption that the Kubelka-Munk equations are valid. The effects can be quite large. It is shown that if the Kubelka-Munk equations are valid at a point in a sheet, then even very intense unevenness will not cause significant deviations from the results predicted by these equations.

The data also support the hypothesis that, on a microscopic scale, the nonuniformity of mass distribution in paper is so intense that nonuniformities on a larger, easily visible scale are of limited importance to macroscopic optical properties such as opacity and transmittance.

If the hypothesis is true, then there might exist real opportunities to engineer improved overall optical properties at the microscopic level, while the macroscopic structure can be engineered with other objectives in mind.

INTRODUCTION AND HYPOTHESIS

The main components of paper furnishes are discrete particles, mainly fibers. Particles in suspension tend to flocculate, and such effects cause nonuniform distribution of basis weight (mass per unit area) in paper sheets. This causes deterioration of most properties, notably strength, printing and optical properties, and general appearance.

Experimental quantification of these effects is scarce. This is due, in part, to the difficulties involved in producing sheets differing only in uniformity of mass distribution. In part it is also due to the difficulties involved in quantitative measurements of uniformity of mass distribution with high resolution.

> **The HYPOTHESIS set forth here is that, on a microscopic scale, the intensity of nonuniformity of mass distribution, or the intensity of optical inhomogeneities, in paper may be so high that nonuniformities on a larger, easily visible scale are of limited importance to macroscopic optical properties such as opacity, transmittance, and light scattering and absorption coefficients.**

The optical properties of paper, naturally, are determined by the properties and nonuniformities of the paper in a size range extending from the size of the sample down to dimensions comparable to the wavelength of light. The essence of the hypothesis is that most of the nonuniformities which determine the light scattering properties of the paper are so small that additional inhomogeneities, typically the size of visible "flocs", exert little additional influence on properties such as reflectance and opacity.

If the hypothesis is true, then there might exist real opportunities to engineer improved overall optical properties at the microscopic level, while the macroscopic structure can be engineered with other objectives in mind.

It has not been possible, yet, to prove this hypothesis by direct measurement because it requires determination of nonuniformity of mass distribution on a much smaller scale than is possible with presently available methods.

SOME PREVIOUS WORK

Transmission beta radiography has been used by a number of investigators. High resolution scanning with a very fine, reasonably collimated beam of beta rays is very slow, and investigators therefore have limited the resolution and/or the degree of collimation in order to obtain enough coverage and enough data to achieve statistically significant results. Using a beta-emitting film pressed in contact with the sheet and scanning the resulting radiogram permits statistical significance to be obtained in a reasonable time-frame, but the resolution is limited, approximately, to the thickness of the sheet, i.e., far from the wavelength of light.

The extremely strong influence of resolution on the measured value of formation was pointed out and modeled by Norman and Wahren (1). Even in the simplified case of uniform fibers of dimensions typical of Nordic softwoods, the measured values of formation using an aperture of 1/10th of a millimeter might well differ by a factor of 2 to 5 from the ones that would be obtained at infinite resolution. A mitigating factor seemed to be that the forming process, for handsheets at least, appears to reduce the unevenness. On the other hand, real wood fibers are certainly not uniform, and fillers and fines have very small dimensions.

*Present address: STORA Technology,
S-791 80 Falun, Sweden.

The highest measured values of formation reported in the literature have increased somewhat as techniques have been refined. Norman and Wahren (1) reported values up to about 15 percent for newsprint while Sara (2) reported values up to 20 percent for the same grade. Both used a circular measuring area with a nominal diameter of 1/10th of a millimeter.

Methods based on the use of soft X-rays show promise for determination of nonuniformity of basis weight with much improved resolution and may eventually be used to prove the hypothesis by direct measurement. For the time being, however, the correctness of the hypothesis can only be inferred from calculations. Results of such calculations constitute the main body of this article.

W. J. Foote, working under the direction of J. A. Van den Akker, performed experiments (3,4) aimed at studying the validity of the Kubelka-Munk equations (5) over a wide range of absorption and scattering coefficients. Some of the results were later used by Van den Akker (6) as a basis of comparison with theoretical models of nonuniformly distributed light scattering and light absorbing centers. He studied various modes of nonuniform distributions, namely layering of different kinds and in-plane nonuniformities. The latter were modeled both as unrelated and related distributions of light scattering and absorption centers. Van den Akker concluded that the system "horizontally concentrating the light-scattering centers and absorbing matter into the same randomly distributed patches.....yields changes in the apparent Kubelka-Munk coefficients that are most nearly in accord with laboratory data".

B. Norman and D. Wahren (1) used a similar approach but, by treating only a special class of cases, managed to arrive at an explicit expression for the influence of formation, F, on the apparent (measured) light scattering coefficient:

$$\Delta s/s = - F^2 bsW/2 \qquad (1)$$

Here $\Delta s/s$ is the relative change of the light scattering coefficient. The formation, F, is defined according to Wahren (7) as the coefficient of variation of local basis weight, W, and b has its usual meaning in the context of the Kubelka-Munk equations. The term bsW on the right hand side denotes the average value of absorbance for a uniform sheet being part of a stack of identical sheets. Due to the assumptions and simplifications made this formula was expected to underestimate the influence of formation to some degree. It was found during this investigation that, when compared to data generated by using a Poisson distribution of basis weight, the formula underestimates the influence of formation significantly, particularly for transparent grades. It happens to give a closer estimate of the effects produced by a log-normal distribution of basis weight.

Wahren noted (1) that "the term F^2 is quite small for well-formed papers. For example, newsprint has an absorbance value slightly greater than unity and an estimated total intensity of mass distribution of some 35 percent (including all wavelengths down to the wavelength of light, but only as extrapolated from measured wavelength spectra). This leads to a decrease of only 6 percent of the effective scattering coefficient. It may be concluded that the influence of mass distribution on the average optical properties of paper is quite small, except in extreme cases. At very low basis weights the influence may be considerable."

Hence, it was surprising to read, in an article by B. D. Jordan (8), that, for newsprint, the effects of formation on opacity and light scattering coefficient are quite significant. His formulas 1 and 3 are correct but the method actually employed in the calculations is based on a procedure outlined in his Figure 2. Additional normalization would be required in order to obtain correct results. The results reported below for Gaussian distribution of basis weight at modest levels of formation are identical to those which would be obtained by using the above mentioned formulas.

ASSUMPTIONS AND SELECTIONS

The following treatment is based on two basic assumptions:

1. The Kubelka-Munk formulas are valid
2. The nonuniformities of distribution of light scattering and light absorption centers are correlated.

One of the many implications of the first assumption is that it is sufficient to calculate the influence of formation on reflectance, R_o, and transmittance, T, to obtain complete information. This follows from the facts that, by definition, the reflectivity, R_∞, is not influenced by formation and that any three optical parameters suffice for complete characterization.

Designating the light absorption and scattering coefficients as k and s respectively, the second assumption can be restated as follows.

Define:

$$q = 2bs = 2*(k^2 + 2ks)^{1/2} \qquad (2)$$

and then assume that, for each particular sheet, q is a constant. Let W be the local basis weight (mass per unit area) and f(W) the continuous amplitude (relative frequency) distribution of basis weight. Referring to the basic Kubelka-Munk formulas (M), one can write:

Average Reflectance =

$$R_o = R_\infty \cdot \int_o^\infty \frac{e^{qw} - 1}{e^{qw} - R_\infty} \cdot f(W) \cdot dW \qquad (3)$$

Average Transmittance =

$$T_o = \int_o^\infty \frac{1-R_\infty^2}{e^{qw/2} - R_\infty^2 / e^{qw/2}} \cdot f(w) \cdot dW \qquad (4)$$

If the distribution of basis weight is discontinuous:

$$W = n*W_1, \text{ where } n = 0, 1, 2, 3, \ldots.$$

and f(n) the amplitude (relative frequency) distribution of n ("the number of layers"), the corresponding expressions are:

$$R_o = R_\infty \cdot \sum_{n=1}^{\infty} \frac{e^{qw_1 \cdot n} - 1}{e^{qw_1 n} - R_\infty} \cdot f(n) \qquad (5)$$

$$T = R_\infty \cdot \sum_{n=0}^{\infty} \frac{1 - R_\infty^2}{e^{qw_1 n/2} - R_\infty^2 \cdot e^{-qw_1 n/2}} \cdot f(n) \qquad (6)$$

Various forms of the amplitude distribution of basis weight can be assumed. The results presented below show that the shape of the amplitude distribution is important. From experiments reported in the literature (1,2) the distributions appear to be fairly close to the Gaussian or normal distribution, but Sara (2) reported consistently skewed distributions. His results may be taken as a suggestion that a Poisson or log-normal type of distribution might be involved at a microscopic level of resolution. Note, however, that the reported formation values do not exceed 20 percent and at such low levels there is not much difference between a Gaussian and a Poisson distribution. Experimentally, it might be difficult at these levels to distinguish with certainty even the difference between any of these distributions and a log-normal distribution.

Norman and Wahren (1) assumed a Poisson distribution in random sheets made from uniform fibers. Real sheets, however, are made from particles having a wide distribution of sizes (widths). Hence, a "continuous Poisson" distribution (9,10) might be expected in real sheets. The analytical difficulties involved in such an approach are rather formidable, but numerical spot checks made at IPC by R. Halcomb (11) show that using such a distribution generates results identical to those obtained using a "regular", i.e., discrete Poisson distribution.

I concluded at this point that not enough was known about the shape of the amplitude distributions of basis weight in real sheets and that, therefore, it would be necessary to review the potential importance of the shape.

Several different shapes were used and their influence on the optical properties studied at various levels of formation. It was found that most "strange" shapes, such as those having dual peaks or an asymptote at the origin, and all extending to negative basis weights, give unreasonable results over some range, usually at high formation values. Selected for inclusion here were three different amplitude distribution functions, namely, the Poisson distribution, the log-normal distribution, and the Gaussian distribution truncated at the origin (i.e., excluding negative basis weights) and properly normalized. At high intensities of formation, the truncation leads to a shift of the mean basis weight away from the type value, i.e., the mean basis weight is not the most commonly occurring basis weight. This effect limits the formation intensity which can be described by a truncated Gaussian distribution to values between zero and just over 75 percent. There are no such limitations with the log-normal, the Poisson, or the "continuous Poisson" distributions.

IMPLEMENTATION

The Poisson distribution can be implemented in the form:

$$f(W) = f(n) = (1/F^2)^n / [n! * \exp(1/F^2)] \qquad (7)$$

$$W_1 = \overline{W} * F^2 \qquad (8)$$

These expressions can be inserted into equations 5 and 6 for numerical evaluation.

The normal, or Gaussian, distribution can be similarly implemented. Define an intermediate variable, Z:

$$Z = W/\overline{W} \qquad (9)$$

and the distribution becomes:

$$f(W) = f(Z) = \frac{1}{F \cdot \sqrt{2\pi}} \cdot \exp[-(Z-1)^2/2F^2] \qquad (10)$$

which is readily inserted into formulas 3 and 4 for numerical evaluation. Negative basis weights have to be excluded. Hence, application of formula 10 has to be truncated at zero and the resulting value normalized by the expression:

$$\int_{0}^{\infty} f(Z) \cdot dZ$$

when used. At high formation intensities, above 30 to 40 percent depending on the desired accuracy, the mean and the type values differ significantly, so it is necessary to compute:

$$E(Z) = \int_{0}^{\infty} f(Z) \cdot Z \cdot dZ \qquad (11)$$

$$D^2(Z) = \int_{0}^{\infty} f(Z) \cdot Z^2 \cdot dZ \qquad (12)$$

and the resulting value of formation. If the resulting value is not close enough to the desired value of formation, a new starting value of F has to be selected and the process repeated. This is a fairly time consuming calculation.

Implementation of the log-normal distribution is straightforward:

$$f(W) = \frac{1}{W \cdot \sigma(W)\sqrt{2\pi}} \cdot \exp\left[\frac{-[\ln(W/\overline{W})]^2}{2\sigma^2(W)}\right] \qquad (13)$$

$$\sigma(W) = \sqrt{\ln(1 + F)} \qquad (14)$$

Insertion into formulas 3 and 4 for numerical integration requires no special procedure.

RESULTS

The results are presented in a series of diagrams. The first two are intended for direct comparison to Jordan's data (8) and to demonstrate the importance of the choice of amplitude distribution function. Both diagrams deal with papers having a reflectivity of 65% and a nominal printing opacity of 85% and 95%, i.e., values characteristic of some newsprint grades.

Figure 1 shows the opacity loss, compared to an ideally uniform sheet, as a function of the formation intensity (identical to Jordan's formation index but expressed as percent). It is clear that at "low" formation intensities, i.e., up to about 30%, the effects on opacity are quite small and the influence of the choice of amplitude distribution function is even smaller. At "high" formation intensities, however, both effects can be large. If the basis weight has a log-normal distribution, the influence on opacity is considerably less than if the distribution is of a Gaussian or Poisson type.

Figure 2 illustrates the effects of formation on the light scattering coefficient. The effects are larger than in the previous diagram but the trends are very similar. The effects start to become quite substantial at formation intensities around 30 percent. Again, the effects of formation are less if the basis weight has a log-normal distribution than if the distribution is Gaussian or of the Poisson type. These are, in fact, general trends observed in all cases.

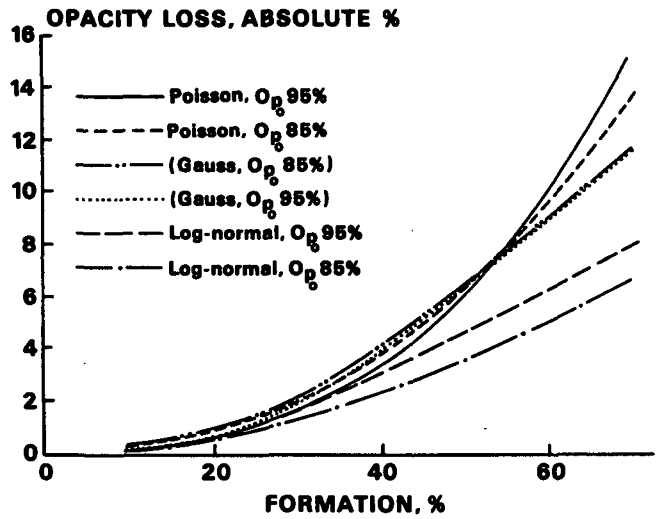

Figure 1. Opacity loss, absolute percent, as a function of formation intensity, percent. Results are shown for sheets having opacities of 85% and 95% when F = 0. Poisson, Gaussian, and log-normal distributions of basis weight were used.

It is noted that the effects of formation on the light scattering coefficient are stronger for the more opaque sheet. This was also noted by Jordan (8) and is a natural consequence of the nonlinear dependence of light transmission on opacity or basis weight.

It can also be observed in Figure 2 that the curves representing the Gaussian distribution start to level out at high formation values. That is a direct consequence of the necessity for truncation and normalization described above.

A more general plot, or "map" of the effects is shown in Figure 3. It is a plot of transmittance against reflectance for homogeneous sheets and for sheets with various levels of formation intensity. The thin, nearly vertical lines each represent a certain value of reflectivity, R_∞, for a homogeneous sheet. The points located on four of these lines (R_∞ = 15, 40, 65, and 90%) are the starting points for each of four series of calculated results. They represent an ideally uniform sheet, i.e., F = 0.

All the other points have been calculated as described above using formulas 5 to 8, i.e., using the Poisson distribution. The points represent sheets with various levels of formation, from 10 to 60 percent in 10 percent increments. It can be seen that the major effect of increasing the formation intensity is a significant increase of transmittance. At R_∞ = 15%, for instance, the transmittance increases from 22 to 34 percent, approximately, when the formation is increased from zero to 60 percent. There is a much smaller (relative) effect on the reflectance. The net effect, however, is to cause the points at higher formation to "fall off the line", i.e., not to be in perfect agreement with the Kubelka-Munk equations. Expressed differently: an increase of formation intensity, as well as a decrease of basis weight, cause the transmittance to increase and the reflectance to decrease. At any given level of transmittance increase, however, the effect of formation on reflectance is higher than the effect of a basis weight change.

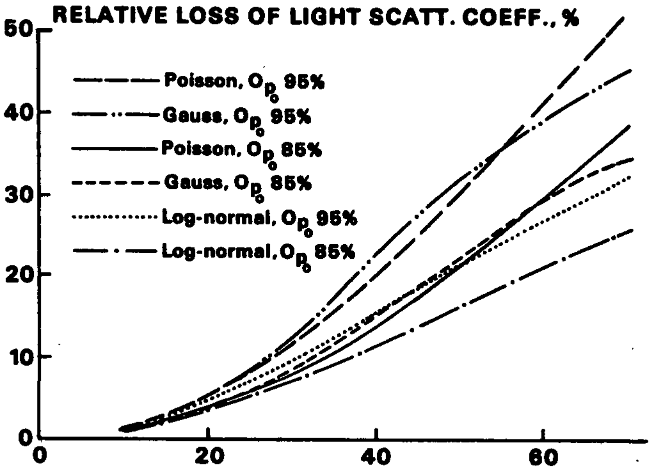

Figure 2. Relative loss of light scattering coefficient as a function of formation intensity. Results are shown for sheets having opacities of 85% and 95% when F = 0. Poisson, Gaussian, and log-normal distributions of basis weight were used.

Corresponding plots made for Gaussian and log-normal distributions of basis weight are very similar to those shown in Figure 3. The direction of change at all reflectivity levels is identical to those shown in Figure 3. The effects of formation are slightly smaller when using the Gaussian distribution. They are considerably smaller when using the log-normal distribution; at 60% formation intensity the effects do not quite reach the levels indicated for the Poisson distribution at 50% intensity.

One way to summarize the results is to define a "Reflectivity Difference", ΔR_∞, as the difference between the actual, measured reflectivity and that calculated from the values of R_0 and T for sheets with nonideal formation. In Figure 3, this corresponds to the deviation of the points representing sheets with formation larger than zero from the R_∞-line they approach as formation approaches zero. As can be seen in Figure 3, the reflectivity difference is largest in the midrange of reflectivity. Selecting R_∞ = 65%, it is found that, approximately:

$$\Delta R_\infty = 0.000316 * F^{2.25}$$

for a Poisson distribution, and

$$\Delta R_\infty = 0.00114 * F^{1.8}$$

for a log-normal distribution (both ΔR_∞ and F are expressed as absolute percent).

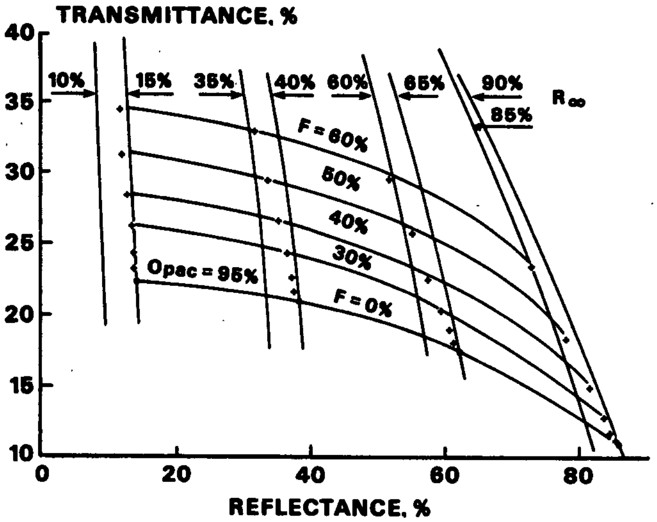

Figure 3. Transmittance as a function of reflectance for sheets of various reflectivities. Each thin, more or less vertical curve represents a constant value of reflectivity, R_∞. The bottom, nearly horizontal curve represents homogeneous sheets with 95% opacity. For four values of reflectivity (15, 40, 65, and 90 percent), transmittance and reflectance values have been calculated and plotted for sheets having formation intensities of 10 to 60 percent in increments of 10 percent. The nearly horizontal curves connect points with equal formation intensity, from zero at the bottom to 60% at the top. A Poisson distribution of basis weight was assumed.

These expressions are quite accurate, having a standard error of estimate of about 5 percent of the value of ΔR_∞ as calculated over the entire range from zero to over 70% formation intensity. As is readily evaluated from these expressions, the reflectivity difference effects are rather small. Using a log-normal distribution for example, the ΔR_∞ is less than two absolute percentage points at 60% formation intensity. The reflectivity difference effect is most pronounced in the range of opacities of major interest to newsprint, magazine, and many other printing papers. It falls off rather steeply at very high and low levels of opacity. Using a Poisson distribution for instance, virtually identical results are obtained at 85% and 95% opacity. For log-normal distribution of basis weight the reflectivity difference effect is approximately 40 percent higher at 85% opacity than it is at 95 percent opacity but then decreases rather sharply at lower opacity levels.

Since R_∞ = 65% and opacity levels between 85% and 95% seem to represent an area of maximum sensitivity, and it certainly is one of practical interest to newsprint manufacturers, a final example of results is presented in Figure 4 focusing on that area. Variables and parameters in Figure 4 are the same as in Figure 3, only scaling factors and detail selections differ.

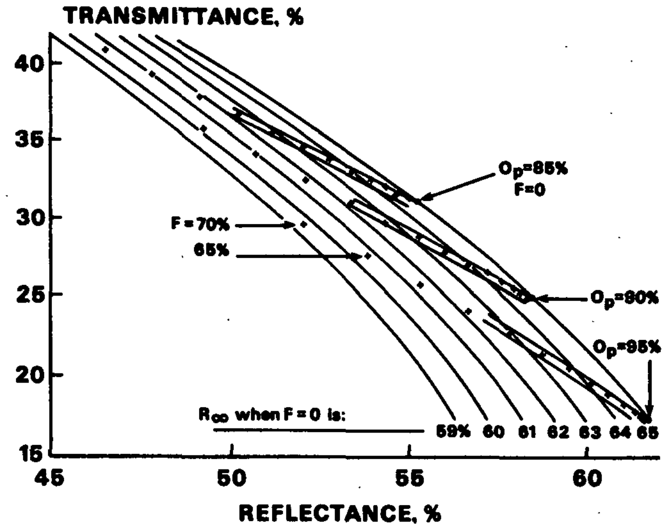

Figure 4. Transmittance as a function of reflectance. Each thin curve represents a constant value of reflectivity, R_∞. For three values of homogeneous sheet opacity (85, 90, and 95%), transmittance and reflectance values have been calculated and plotted as points for sheets having formation intensities of 10 to 70 percent in increments of 5 percent. A Poisson distribution of basis weight was assumed. The double-drawn curves represent data calculated over the same range but using log-normal distributions of basis weight.

Some significant uncertainties and potentially large practical effects are demonstrated in Table 1, which shows six combinations of values of light scattering and absorption coefficient, and formation which all would produce the same macroscopic optical properties in a sheet of newsprint.

The table demonstrates that if the basis weight has a Poisson distribution, it would not be possible to distinguish by reflectometric measurements between sheets constituted of a uniformly distributed, slightly light scattering mass, case 1, or of a very nonuniformly distributed and very strongly light scattering and absorbing material, case 6. If such a range of materials exists, they would probably differ widely in texture and end-use properties. Cases 1, 5, and 6 seem to be only hypothetical possibilities.

Case No.	1	2	3	4	5	6
Formation, %	0	42	51	57.7	58.7	58.72
Scattering coeff.	55.1	75	100	200	1000	10000
Absorption coeff.	5.2	7.065	9.43	18.85	94.25	942.5
Transmittance, %	18.2	15.2	12.5	8.0	5.5	5.5

Table 1. Hypothetical examples of possible structures underlying readily measurable properties of standard newsprint. The top three lines show six combinations of values which all would give a 48.8 g/m^2 sheet R_o = 61.4%, R_∞ = 65%, and opacity = 94.5% if sheets were characterized by reflectometric measurements. A Poisson distribution of basis weight was assumed. The bottom line shows values of transmittance calculated for the same samples.

The table also demonstrates a consequence of the fact that in the limit of strongly (infinitely) light scattering materials the transmittance equals one minus the opacity. In the case of a Poisson distribution of basis weight, eq. 7, the transmittance then approaches:

$$T_{n=0} = \exp(-1/F^2) \qquad (15)$$

and therefore:

$$F = (-\ln(1 - \text{opacity}))^{-0.5} \qquad (16)$$

Examples using the log-normal distribution are less extreme.

DISCUSSION

Formation is very important to the printability and appearance properties of paper. It has also been shown to exert considerable influence on some mechanical properties. This study was undertaken in support of answers to two questions

1. Does formation exert a significant influence on the average optical properties of paper as defined by the Kubelka-Munk equations?

2. Do the Kubelka-Munk equations produce internally consistent results even for highly nonuniform sheet materials?

It is clear from all the data that high levels of formation intensity can exert strong detrimental effects on opacity and light scattering power. It is equally clear that the levels of formation intensity reported in the literature, 10 to 20 percent, are not high enough to exert any substantial influence on the average optical properties. These measurements, however, are limited to nonuniformities larger than about 1/10th of a millimeter, whereas the influence of nonuniformities on optical properties ought to be influential down to sizes the order of the wavelength of light. If that were the case, would the Kubelka-Munk equations still yield internally consistent results?

As a background, the reader is reminded that homogeneous, nonlayered sheets can be completely characterized in the Kubelka-Munk sense and at any particular wavelength by any three independent optical parameters, e.g., reflectance, transmittance, and reflectivity. That fact is utilized in Figure 3, which is a plot of transmittance against reflectance. Each thin, more or less vertical curve in Figure 3 represents a constant value of reflectivity, R_∞. All possible combinations of reflectance and transmittance for a homogeneous sheet fall along one such curve. The basis weight and/or light scattering and absorption coefficients may vary along the curve in any number of combinations.

Suppose now that for a sample of paper, measurements are made of transmittance, reflectance, and reflectivity and that within the experimental uncertainty the results do not deviate from one of these curves. It would then be concluded that the measurements, as well as the Kubelka-Munk theory are good. Conversely, all would be questioned if the data deviate from the line.

In Figure 3, all data points shown in the diagram have been calculated on the assumption that the Kubelka-Munk theory is valid. Added in is only the factor of formation - and formation up to levels far beyond those reported in the literature. Yet, the points do not deviate very much from the curves. A few percentage points of transmission at high reflectivity levels or a fraction of one percentage point of reflectivity at low reflectivity levels is the net effect of a formation intensity of 60% percent.

As has been illustrated in another context (12), it is not yet a trivial task to make accurate transmittance measurements that adhere strictly to the assumptions made in the Kubelka-Munk theory. Even with the sophisticated instruments now available for reflectance measurements, a deviation of a fraction of one percentage point of reflectivity measured on a dark sheet may readily be lost in the general variability of the material and the procedures. Hence, chances are very high that deviations from the Kubelka-Munk theory for homogeneous sheets caused even by very high formation intensities would go unnoticed or be ascribed to experimental uncertainty.

Conversely, and most interesting, if the Kubelka-Munk equations are valid at a point in a large sheet, they should also be very nearly correct for sheets having a very high formation intensity. Attempts to test the equations should detect deviations when varying the basis weight, for example, because formation naturally worsens with decreasing basis weight. Still, the results should be very nearly internally consistent - as has been the general experience in such endeavors.

Considering the results just obtained by calculation, it is not inconceivable that some papers may have "true" formation intensities, i.e., including very small wavelengths, up in the range of 50 to 70 percent. This is consistent with the fact that refining and even very mild semichemical pulping treatments bring drastic reductions in the light scattering coefficient without necessarily modifying fiber dimensions or sheet structure. If that is true, it also appears obvious that any major effects on optical properties have to occur or be applied at the microscopic level. The macroscopic structure of the sheet can be geared toward optimization of printing, appearance, and other properties.

It should be possible to engineer such optimization by inducing changes at the micro and the macro levels rather independently. Colloidal phenomena in the suspension and physical changes to the fiber wall are major factors governing the microscopic structure. Fiber flocculation and small scale motions of fibers and fines induced by turbulence, shear, and consolidation forces are major factors governing the macroscopic visible structure.

One of the ideas causing this work to be done was that it might be possible to get a relatively simple measure of "true" formation. It would be done by measuring reflectance, reflectivity, and transmittance on a sample, noting the deviation, ΔR_∞, and converting it to the desired measure by means of formulas or diagrams such as those just described. The results of the calculations just presented indicate, however, that ΔR_∞ would be too small to make such a method feasible.

The hypothetical example shown in Table 1 demonstrates that virtually identical conventionally measured macroscopic optical properties may be obtained from a wide variety of structures at the microscopic level. Neither of the extreme cases, absolute homogeneity and almost infinite inhomogeneity, is likely to occur in practice but it is interesting that we really do not know where along this spectrum of possibilities paper products fit.

It is well known that the apparent light scattering coefficient of most products decreases with increasing wavelength of the light employed in the measurement. This is at least a qualitative indication that optical inhomogeneities of considerable magnitude extend down into the size range of the wavelength of light, but they need not necessarily be a direct reflection of corresponding inhomogeneities of mass distribution. Based on the type of reasoning demonstrated in this report it is not possible, a priori, to distinguish effects of mass distribution on optical properties from various types of optical inhomogeneity. It seems very likely, however, that both types of inhomogeneity occur in most paper products and that both types contribute to the overall properties. Hence, it may be difficult to distinguish even large effects of formation on optical properties from those induced by other factors.

No mention has been made in this report of the wavelength or frequency spectrum, i.e., the distribution of "floc sizes," because it is not necessary to consider anything but the amplitude distribution of basis weight when calculating average optical properties. In doing so, however, one is automatically integrating over all floc sizes, from the microscopic up to the size of the sample. It is quite probable that mass variations on a microscopic scale are very common. It is also quite probable that such geometrically small variations are small also with respect to the local variance of basis weight they represent. If such a coupling does in fact occur in paper sheets, then conditions are right for representing the distribution of local basis weight by a log-normal distribution. This was one of the reasons for including the log-normal distribution in this treatment.

It has been shown above that the log-normal distribution of basis weight yields results which differ significantly from those obtained with other distributions. One of the properties of the log-normal distribution is that the frequency of zero basis weight is zero, i.e., there are no pinholes. Hence, a papermaker might consider such a sheet to be "well closed." Another characteristic is that there exist areas of very high basis weight which might be perceived as occasional, heavy flocs. The Poisson distribution, on the other hand, seems to represent an open structure formed without interaction between particles or between particles and the forming or consolidation process (1).

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