Minimum Aisle Width Path Planning for Automated Guided Vehicles (AGVs)

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Abstract

Automated guided vehicle systems (AGVSs) are often used to transfer materials from one location to another in industrial environments. Due to the high cost of floor space and ever-present safety concerns, facilities that use automated guided vehicles (AGVs) must be particularly concerned with the path planning of these vehicles. A path that minimizes the required aisle widths for maneuvers can lower costs by improved floor space utilization and increased safety.

This paper introduces research that investigates solution procedures and algorithms for determining minimum aisle width paths for nonholonomic mobile robots. The goal will be to determine the path that results in a global minimum aisle width. The research will provide additional insight to understanding the interactions between vehicle kinematics, vehicle shape, environment characteristics, and the path. The research will enable new and old plant layouts as well as vehicle designs to be evaluated and improved. Environments and vehicles typically found in industrial settings will be studied. This research can also contribute to areas outside industrial environments such as roadway design and general mobile robots.

Introduction

Most industrial facilities with AGVs have wide aisles in which the vehicles traverse. The wide aisles contribute to personnel safety and reduce product and equipment damage by maintaining a large safety distance from people and other equipment. On the other hand, the wide aisles take up costly floor space that could better be utilized for additional equipment or storage. A path that minimizes the required aisle widths for a given vehicle can be determined that would reduce the width of the aisles while maintaining safety. The minimized aisle widths may correspond to actual physical aisle widths or may only be the aisle widths required by the vehicle (vehicle aisle widths) to make a turn that are contained within physical aisle widths as shown in Figure 1.

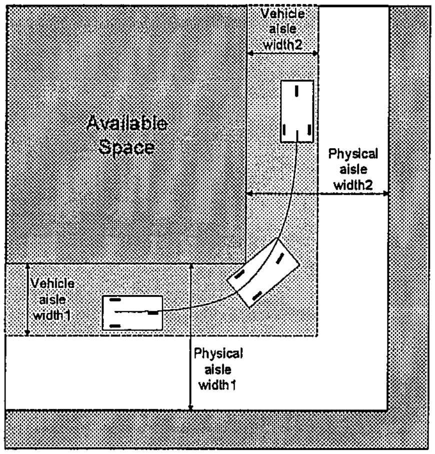

Figure 1 - Physical aisle widths versus vehicle aisle widths

Adding to the complexity of path planning, most moving vehicles, including AGVs, have nonholonomic constraints, which affect how the vehicle can move. Nonholonomic velocity constraints reduce the degrees of freedom locally but maintain all degrees of freedom globally. For example, when parallel parking a car, the driver cannot move beside the parking space and move the car sideways into the space. The driver must instead maneuver the car into the space due to the nonholonomic constraint. Locally, the driver has two degrees of freedom available, steering angle and linear velocity. Globally in a clear area, the driver can achieve any position and orientation, which results in three degrees of freedom. Under no-slip conditions, the constraint requires the center of the rear axle to move in the direction of the rear wheels, which reduces the local degrees of freedom. Since path planning for vehicles with nonholonomic constraints is difficult, the constraints are often ignored and paths that cannot be followed by the vehicle result. Typically, real-time control routines attempt to correct for the inaccurate paths. Feasible paths that nonholonomic vehicles can follow are more desirable and are developed in this research.

Typically in industry, path planning for AGVs is a trial and error process assisted by past experience. While some AGV suppliers have software to assist in path planning, this software typically requires the user to input a path and then only checks whether the vehicle path is collision free. If the vehicle path is not collision-free, the user must adjust the path and try again in an iterative process. User experience obviously reduces the number of iterations but the process can still take a considerable amount of time.

The research presented here takes a direct approach to solving the problem of AGV path planning while considering floor space utilization and safety and develops solution procedures and algorithms to determine paths that result in minimum aisle widths for particular vehicles in industrial facilities. Knowledge of the minimum aisle widths required by a vehicle is also useful for designing and improving facility layouts as well as defining and examining different vehicle specifications. Minimum aisle width paths can also be applied to roadway design, computer assisted parking and steering of automobiles and trucks, general mobile robot motion planning, and path planning for other nonholonomic devices such as missiles, boats, and submarines.

Related Research

While much work has been done in the general field of mobile robots, this section will only review research related to the spatial aspects of mobile robot path planning.

The "piano movers" problem is the classical problem of moving a body from one point to another point while avoiding obstacles and walls. The obstacles and walls are geometrical constraints. The body is free to translate and rotate characteristic of holonomic motion. This problem is often the basis for mobile robot path planning work. Schwartz and Sharir present a detailed general analysis of a two-dimensional "piano movers" problem with polygonal bodies and obstacles [6]. A connectivity graph is constructed and then searched for a path that connects the start point and end point. Many of the issues developed are from computational geometry on which O'Rourke's textbook provides a good background [5]. The "piano movers" problem is more specifically applied to mobile robotics in Latombe's book [4]. This classic problem forms the basis for the research presented here.

The discussion of the "space" a vehicle requires when following a path is sparse in the literature. There are brief mentions of spatial considerations in roadway and tractor-trailer design literature [see references in 1]. Typically, analysis is limited to circular arc paths and the works suggest an additional safety zone to provide for actual paths, which typically are not circular arc paths [1, p. 39]. Often, the work is primarily interested in the "offtracking" of the rear wheels of a bus or trailer when following a prescribed path, usually a circular arc path [2][3][1].

The papers by Wilfong, by Tournassoud and Jehl, and by Alexander and Maddocks provide the most depth into work related to this research [8][7][1]. In the paper by Wilfong, the entire region swept out by a rectangular shaped vehicle during a circular arc turn is investigated [8]. The swept out region is taken into account when determining the range of collision free radii for each turn. A global path is determined by selecting a radius from the range in each turn.

Tournassoud and Jehl determine a local "canonical contact trajectory" for a vehicle turning a corner. If the canonical trajectory is not free of collisions then there is no reversal free trajectory [7]. The vehicle is rectangular shaped with one kinematic constraint. The canonical trajectory is used to determine whether the vehicle can turn a particular corner without reversals. The global path is determined by minimizing a cost function using the path length and the number of reversals.

Alexander and Maddocks provide a general analysis for vehicles that move on rolling wheels [1]. They investigate general conditions for rolling and apply these conditions to motion of vehicles with one fixed axle. The optimal trajectory is defined to be one in which the vehicle "steers around the corner as close as possible to the outer boundary" [1, p. 48]. The optimal trajectory for a vehicle with zero width is developed and said to extend to vehicles with nonzero width turning around a corner with an interior angle greater than or equal to $\pi/2$ and less than π. It is noted in the paper that for vehicles of zero width in aisles of equal width, "the narrowest lanes can be traversed with the [fixed] axle at mid-length" [1, p. 50]. No formulas are developed and there is no mention of the "narrowest lanes" for vehicles other than ladder-like, single fixed axle vehicles. Clearly, this work is not complete but provides a good basis for this research.

Problem Statement

A vehicle path will be generated that minimizes a cost function of selected aisle widths. The vehicle must move collision free in a two-dimensional workspace. The vehicle shape and environment obstacles are modeled with simple polygons. The vehicle motion is subject to one or more nonholonomic constraints. An aisle is defined by two parallel edges of different polygons and an aisle width is defined as the perpendicular distance between the two edges of an aisle. The path is defined as a set of two-dimensional vector functions that are functions of time. In order for the path of a vehicle with nonholonomic constraints to be feasible, the path must be at least tangent continuous (no reversals) and piecewise twice differentiable [4, p. 438].

Solution Method

This research will develop solution procedures and algorithms that produce paths that minimize a cost function of aisle widths for nonholonomic vehicles in industrial environments. The solution procedures and algorithms will be based on the well researched "piano movers" problem and research mentioned in the section Related Research.

There are three types of solution paths of interest, holonomic motion, straight line and circular arc (SLCA), and general unrestricted, feasible (GUF) paths. The path that minimizes the aisle widths for a vehicle moving holonomically, as in the "piano movers" problem, will determine a lower bound solution for the general minimum aisle width problem. The "piano movers" problem has been well researched and therefore will not be presented here, although the research has not typically been concerned with determining the minimum aisle widths. The minimum aisle width solution to paths comprised of straight line and circular arc segments is also of interest for those AGVs where it is not feasible or practical to implement paths of complex curves, as may be found with AGVs that follow physical guidepaths. SLCA paths will not be presented here for brevity. The paths of primary interest are general, unrestricted, feasible paths that are useful for the more advanced self-guided vehicles (SGVs) that can follow complex software-generated paths. GUF paths provide the best solution for a given vehicle and environment and will always result in an equal or smaller aisle width cost function than the SLCA paths. In addition, GUF paths, unlike SLCA paths, will be "smooth" or continuous in curvature.

The solution procedures and algorithms to any of the minimum aisle width path problems depends on the interactions of four factors: vehicle kinematics, vehicle shape, environment characteristics, and the path. The balance between these four factors is determined by performance measures. The factor interaction diagram is shown in Figure 2. By taking into account these four aspects with a performance measure to minimize the aisle widths, a vehicle path is developed that improves floor space utilization and increases safety. These gains cannot be achieved by investigating each aspect individually.

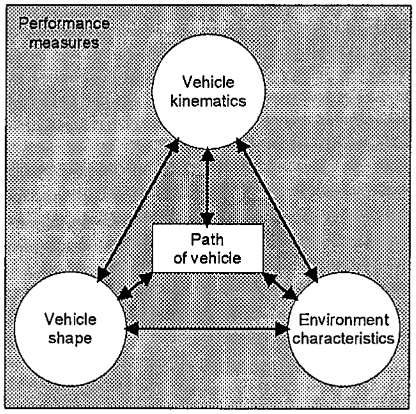

Figure 2 - Factor interaction diagram

The vehicle kinematics depend on the type of vehicle. The type of vehicles investigated for this research will have one nonholonomic equality constraint resulting from a single fixed axle. Single fixed axle vehicles include tricycle-like, automobile-like, and differential drive (dual independent drive wheels) vehicles.

In the general problem formulation, the vehicle's configuration is described by a point $r = \{x, y\}$, an orientation, θ, and combined in a configuration vector, $q = \{x, y, \theta\}$. The orientation of the vehicle is determined from the path and the vehicle kinematics. The vehicle's start location is at some point along a line, L_{start}, with a given orientation, θ_{start}. The vehicle traverses a path, *P*, and ends at some point along a second line, L_{end}, with a given orientation, θ_{end}. The vehicle shape is described by a simple polygon and the space occupied by the vehicle in a given configuration is *V(q)*.

The vehicle moves in an environment of simple polygons, E_i, described by the configuration vectors, $e_i = \{x, y, \theta\}$. The environment is structured such that the aisles have parallel sides. One of the aisle sides must be fixed while the other aisle side may be moved, subject to the constraint that the aisle sides remain parallel. The aisle widths, w_i, are measured perpendicular to the aisle sides. The general problem formulation is shown in Figure 3.

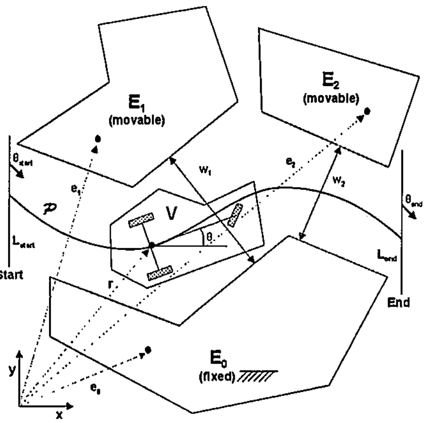

Figure 3 - General problem formulation of vehicle and environment

The two-dimensional vector path, *P*, is followed by some reference point on the vehicle. The path will be defined by steering control laws that are functions of the vehicle and environment parameters as well as time. An admissible path is one that adheres to the nonholonomic constraints of the vehicle and one in which the vehicle polygon is free of collisions with the environment polygons. Points in contact on the boundaries of the vehicle and environment are not considered collisions provided neither point is in the interior of the other object. Using set notation, where $E = \{E_1, E_2, \ldots\}$, the collision free requirement says the intersection of all polygons is the null set over all time, t,

$$V(q(t)) \cap E = \varnothing, \text{ all } t. \tag{1}$$

The general path that minimizes the aisle width cost function over all admissible paths is the solution to the problem. The cost function will be a scalar weighted sum of the aisle widths,

$$J = \sum_i a_i w_i, \tag{2}$$

where a_i is a scalar aisle width weighting factor. The cost function is minimized over the set of all admissible paths, P, that are collision free as described by,

$$S = \min_{\{P:\ V(q) \cap E = \varnothing\}} \left\{ \sum_i a_i w_i \right\}, \tag{3}$$

where S is the scalar value of the cost function at the solution. The solution path, P^*, is the path that minimizes Equation (3).

Minimizing the solution path equation over the infinite set of collision free paths is difficult. To reduce the complexity, two hypotheses are proposed.

The first hypothesis proposes that there is a critical portion of the path that defines the aisle widths. For example in the "piano movers" problem for a rectangular shaped vehicle moving in an L-shaped aisle, there is only one critical vehicle configuration that determines the aisle widths. In general, since the vehicles have nonholonomic constraints, the aisle widths will be determined from more than one vehicle configuration. The critical portion of the path is the part of the path between the two extreme critical configurations. Sections of the path not considered to be the critical path are called noncritical paths as shown in Figure 4. The noncritical paths must be such that the vehicle stays within the defined aisles and the path is feasible where each noncritical path meets with one of the endpoints of the critical path as defined in the problem statement. The noncritical paths may not be unique. It is further hypothesized that there is only one critical path that determines the minimum aisle widths. Since the critical path determines the aisle widths and those are the variables to be minimized, only the critical part of the path needs to be determined, which greatly reduces the computations required to determine the minimum aisle width path.

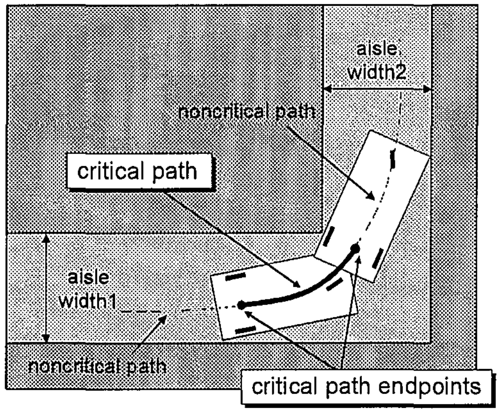

Figure 4 - Critical and noncritical portions of the path

The second hypothesis follows from the "piano movers" problem and the works of Alexander and Maddocks [1]. The minimum aisle width path for holonomic motion is one in which the moving object slides along the boundaries of the environment objects. Alexander and Maddocks steer optimally by steering as "close as possible to the outer boundary" [1, p. 48]. It is therefore hypothesized that the general minimum aisle width path for nonholonomic vehicles is one in which the vehicle maintains contact with the environment at one or more points or is subject to limitations imposed by the nonholonomic constraints.

To show how the critical portion of the minimum aisle width path can be determined, an example is developed for a rectangular shaped tricycle-like vehicle (one nonholonomic constraint) turning around a perpendicular, L-shaped corner. For brevity in this example, the vehicle configuration must be such that the instantaneous center of rotation (ICR) will be outside the vehicle body during the critical portion of the path. Otherwise, a more complex analysis is required.

By assuming a vehicle configuration such that at least one point of the vehicle must be in contact with an environment edge at all times, steering control laws can be developed from kinematics or geometry that cause either the rear outer corner point or the front outer corner point of the vehicle to follow the outer aisle walls. The steering control laws for rear and front wall following are given respectively by,

$$\phi_{rear} = \operatorname{atan}\left(\frac{L}{\frac{-W}{2} + R \cdot \cot(\theta)}\right) \text{ and} \tag{4}$$

$$\phi_{front} = \operatorname{atan}\left(\frac{L}{\frac{-W}{2} + (L+F)\cdot\tan(\theta)}\right), \qquad (5)$$

where ϕ_{rear} and ϕ_{front} are rear and front wall following steering angles,
L is the wheelbase of the vehicle,
W is the width of the vehicle,
R is the rear overhang behind the rear axle,
F is the front overhang past the front wheel, and
θ is the orientation of the vehicle.

By equating the two control law equations, a unique vehicle configuration can be determined. This unique vehicle configuration is where two distinct points on the vehicle contact the two environment edges and is the configuration where the vehicle would switch from rear wall following to front wall following, although no rear wall following is actually required in this case. The vehicle switching orientation is given by,

$$\theta_{switch} = \operatorname{atan}\left(\sqrt{\frac{R}{L+F}}\right). \qquad (6)$$

Physically, in order for the vehicle to move without collision from this switching point, the instantaneous motion of the contact points must move parallel to the environment edge. This motion requires the vehicle to instantaneously rotate about a point defined by the intersection of two lines normal to the obstacle edge through the contact point. The instantaneous center of rotation of the vehicle must be along a line extending from the fixed axle of the vehicle at all times. By continuity, there is a unique vehicle configuration where all three lines intersect [1].

The unique switching angle determines a critical vehicle configuration. This critical configuration defines the relationship of the vehicle configuration to the outer walls (see Figure 5). This is a unique configuration where the vehicle is in contact with both walls. There is no other configuration that enables the vehicle move from contacting both walls without colliding into the walls or violating the nonholonomic constraint. From the switching point in either direction, the control laws cause the vehicle to asymptotically approach the wall with a single contact point.

A second critical configuration can be found that defines a vehicle configuration relative to the inside corner. In general, this configuration will be different from the first critical configuration and will be on a part of the path where there is a single contact point. If the instantaneous center of rotation is restricted to be outside the vehicle's shape along the axle line, the vehicle orientation that defines the inner corner can be easily determined. By restricting the instantaneous center of rotation and for motion following a wall parallel to the y-axis, all points along the inside edge

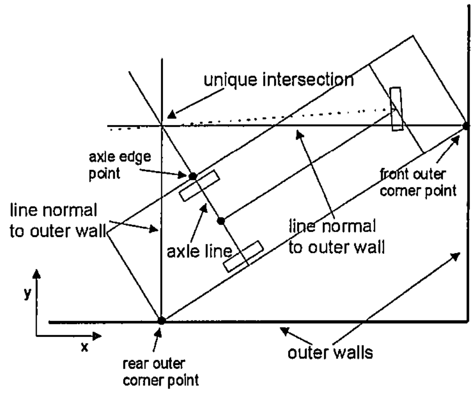

Figure 5 - Determining the vehicle configuration at one of the critical endpoints

of the vehicle in front of the rear axle will trace out curves that are monotonically increasing in y since the vehicle path is monotonically increasing in y. The points along the inside edge of the vehicle behind the axle will rotate away from the inner corner and therefore will not be a concern. It can be shown that the curve the axle edge point produces will be the inner most curve and therefore the inner corner will be located at some point along this curve in order for the vehicle to clear the corner. A cost function, J, of equally weighted aisle widths is given by,

$$J = (x_w - x_c) + (y_c - y_w), \tag{7}$$

where x_c, y_c is the variable location of the inner wall corner and
x_w, y_w is the fixed location of the outer wall corner.

The minimum aisle width inner corner location is determined by differentiating the cost function with respect to a path variable, t, and setting equal to zero. For the case of two equally weighted aisle widths, an expression for the tangent to the curve and the chain rule are used and result in the expression,

$$\tan(\alpha) = 1. \tag{8}$$

Solving for the tangent angle, α, the expression shows extremum points occurring at $\pi/4$ and $3\pi/4$. The path can be shown to be everywhere concave up with a derivative of the tangent always positive. By being concave up and taking the positive solution, $\pi/4$, the minimum solution is determined. The angle of the tangent of the axle edge path is the same angle as the vehicle orientation. Therefore, the minimum inner corner location is determined when the vehicle is at an orientation of $\pi/4$. This is the second critical configuration since the vehicle is asymptotically approaching the wall past this point and is clear of the inner corner. Since all walls have been defined relative to vehicle configurations, no more critical configurations need to be found and these two critical configurations determine the critical path's endpoints. The portion of the path between the two critical endpoints is the unique critical path. For nonequally weighted aisle widths, the angle will be different and can be easily determined following the same procedure.

Due to the nonholonomic constraint, the precise location of the inner corner must be determined by integrating along the path from the first critical endpoint with the steering angle input determined from the front wall following steering control law. Currently, the integration is performed numerically. Closed form solutions are being investigated. Knowledge of the critical endpoints is of great benefit since the region of the path that must be integrated to determine the minimum aisle width solution is reduced. Solving more general problems, such as with the ICR restriction removed and with different environments, requires a more rigorous development but follows from the basic outline presented here.

Conclusions

This research provides a contribution for mobile robot path planning in industrial environments where the spatial aspects are of concern. Solution procedures and algorithms are developed for determining a path that minimizes aisle widths. The research provides a better understanding of how the vehicle kinematics, vehicle shape, and environment characteristics interact and affect path planning for nonholonomic vehicles. The understanding of the interactions between the vehicle kinematics, vehicle shape, environment characteristics, and vehicle path indicate how to improve any one factor given the others. For example, if the aisle width, path, and kinematics are fixed, it can be determined how to modify the vehicle shape for the most advantage. This understanding gives more flexibility and even greater usefulness to this research in real applications.

This research can also contribute to areas outside industrial environments involving vehicles. Some of these areas include roadway design, driver assisted and automated automobiles and trucks, and general mobile robot path planning in confined areas. This research can also be useful for path planning of other devices that have nonholonomic constraints, such as missiles, boats, and submarines.

Future work

There is additional ongoing work not shown in this paper that include closed form solutions and proofs that the minimum aisle width solutions are global minimums. Closed form solutions will further provide additional ease of use by not requiring numerical methods to be used. Direct relationships and understanding between variables will be more evident with closed form solutions. While the solution procedures developed here are expected to be global minimums and not just local minimums, a formal proof has not been completely developed. A global minimum solution provides a clear benchmark and a "best" solution to an area in need of analytical methods for path planning real world industrial vehicles. Both of these ongoing developments will provide a practical and useful contribution to industrial vehicle path planning.

Future work will also include other typical industrial environments such as T- and X-intersection along with track siding or spur aisles. Also, other vehicles, such as vehicles with trailers, may be investigated.

Other additional work in the future will be tying together multiple turns in succession and the distance required to straighten the vehicle out after a turn. The analysis presented here assumes the turns are far enough apart to "set up" a minimum aisle width turn. While useful in many situations, the turns are not always separated far enough apart to set up the next turn. There are also times when a workstation is close to a turn and the vehicle must straighten up in order to dock with a workstation. The distance from the corner to the point where the vehicle straightens up is desired to be as small as possible in many cases. The research presented here can indirectly provide a path that causes the vehicle to rapidly straightening up by weighting the entry or second aisle width much higher than the exit or first aisle width.

Acknowledgments

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