DESCRIBING AIRSPACE COMPLEXITY: AIRSPACE RESPONSE TO DISTURBANCES

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DESCRIBING AIRSPACE COMPLEXITY: AIRSPACE RESPONSE TO DISTURBANCES

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To my family.

I love you.

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SUMMARY

The Air Traffic Management (ATM) system provides services for safe and efficient aircraft operations to transport people and goods. Much effort has been made to improve it to meet increased demand for air transportation. A fundamental aspect of ATM is monitoring and mitigating mismatches between air traffic demand and airspace capacity. Advanced Traffic Flow Management (TFM) concepts have been proposed to effectively manage traffic flows to balance air traffic demand with airspace capacity, and Dynamic Airspace Configuration (DAC) concepts have been examined to reconfigure the airspace structure to meet variations in air traffic demand.

In ongoing efforts to balance air traffic demand and airspace capacity, describing airspace complexity stands as a fundamental research problem. For example, traffic flow through multiple airspaces can be managed based on their complexity. Likewise, DAC requires evaluating different airspace configurations based on their complexity over a range of traffic situations. While much effort has been made to understand airspace complexity, most of the previous research has focused on airspace complexity measures referenced to controllers' workload.

Taking a more analytic approach, this thesis proposes that airspace complexity can be described in terms of how the airspace (together with the traffic inside it and the traffic control method) responds to disturbances. The response of the airspace to a disturbance is captured by the control activity required to accommodate that disturbance. Furthermore, since the response of the airspace depends on the disturbance, this thesis introduces a complexity map which shows how an airspace responds to a set of different disturbances. Among the many possible types of disturbances, this thesis considers an aircraft entering into the airspace, and the proposed method

of describing airspace complexity is illustrated with examples.

In addition, the proposed method is illustrated in relation to current and future traffic flow management concepts. A complexity map can help airspace managers to identify overly demanding traffic situations and determine effective restriction to traffic flows. Furthermore, using detailed information provided by the complexity map, airspace managers can assess how their decision to restrict traffic flows will affect the complexity of adjacent airspace. Moreover, a complexity map can provide detailed information about how environmental factors such as convective weather will affect airspace complexity and help airspace managers to identify the best way to use the available capacity of airspace.

The time evolution of a complexity map is investigated experimentally. The temporal evolution of a complexity map is of interest since applications need knowledge of airspace complexity over a period of time rather than at an instant in time.

It is also shown that the proposed method can be applied to airspace design problems. The induced complexity of a given air route structure is described by assessing its complexity over a wide range of traffic situations. Different air route structures are compared experimentally.

CHAPTER I

INTRODUCTION

1.1 Motivation & Background

The Air Traffic Management (ATM) system provides services for safe and efficient aircraft operations to transport people and goods.[62, 59] The airspace in the United States is divided into 22 Centers including Alaska and Hawaii, as shown in Fig. 1, and each of them has a facility called Air Route Traffic Control Center (ARTCC) to manage the traffic flows at the National Airspace System(NAS) level. Each center is subdivided into sectors, as shown in Fig. 2, and aircraft inside each sector are controlled by one or two sector controllers whose primary task is assuring proper separation between aircraft. In other words, the Traffic Management Coordinators (TMC) at ARTCC focus on orderly and efficient traffic flows through multiple airspaces, i.e., centers and sectors, to ensure efficient utilization of NAS, while sector controllers focus more on occasionally changing traffic situations inside sectors to avoid collisions between aircraft.

Increased demand on the air transportation system will degrade the quality of services in the near future. Therefore, much effort has been put into improving the current air traffic management system. For example, in December 2004, the Joint Planning and Development Office (JPDO) announced the plan for the Next Generation Air Transportation System (NextGen) [64] with the vision of tripling the capacity of the National Airspace System (NAS) by 2025.

In the current system, each sector has a capacity limit which is defined by the maximum number of aircraft that can be handled in the sector during a given time

Figure 1: Centers over the continental United States

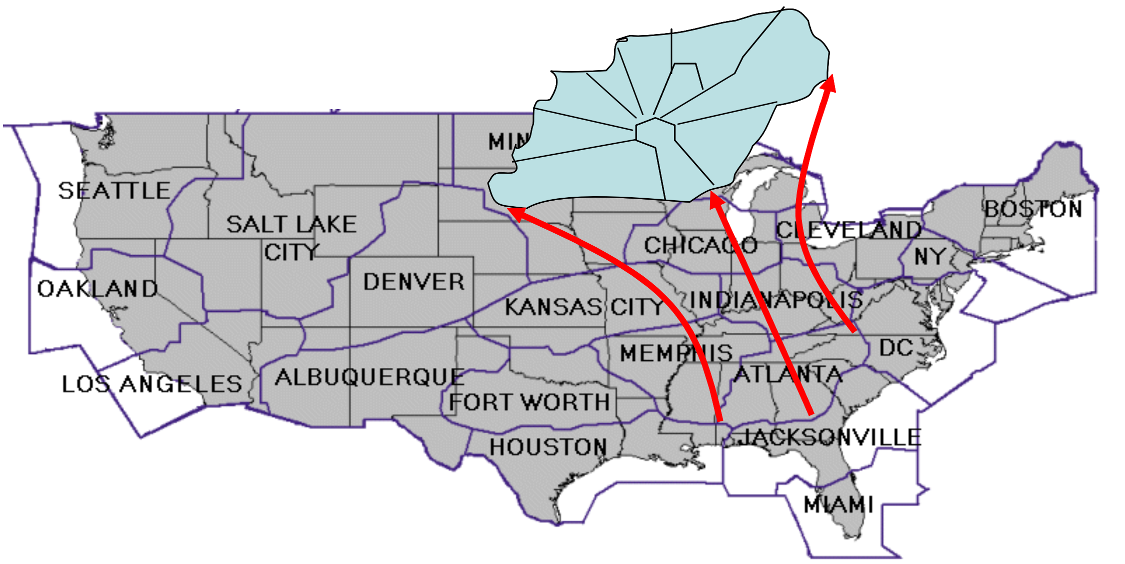


Figure 2: High level sectors in Atlanta Center

period.[30, 33] This maximum number of aircraft is called the Monitor Alert Parameter (MAP).[28] The MAP is primarily determined by the sector geometry such as the size of the sector, but it can also be affected by dynamic environmental factors such as convective weather. For example, a larger sector in general has a bigger MAP than a smaller sector, but its MAP can significantly drop in a bad weather condition. If the number of aircraft inside a sector is expected to be more than its MAP, air traffic managers can respond either by restricting incoming traffic flows or by increasing the capacity of the sector.

Traffic Flow Management (TFM) is the process that is used to balance air traffic demand with airspace capacity by regulating traffic flows. Much effort has been put into developing advanced traffic flow management techniques.[59, 63, 16, 47, 65, 14, 56, 57] Currently, Traffic Management Coordinators (TMCs) monitor congested traffic situations using flight data from the Enhanced Traffic Management System (ETMS) and restrict incoming traffic flows into the sector, if necessary. The restrictions on incoming traffic flows can be implemented either by changing the arrival times of incoming aircraft, which is called metering, or imposing the proper distances between aircraft, which is called spacing.[59] Furthermore, incoming traffic flows are sometimes rerouted to other sectors.

On the other hand, Dynamic Airspace Configuration (DAC) examines how airspace managers may reconfigure the airspace to meet variations in air traffic demand.[69, 28, 29, 40, 44, 64, 15] For example, airspace managers may increase the capacity of the sector by de-allocating Special Use Airspace (SUA) around it. In fact, large volumes of airspace are regularly restricted for different reasons, such as for the military or for space launch operations, and those restricted airspace decrease the capacity of sectors around them.[13]

However, airspace capacity, in fact, depends on the complexity of the traffic situation inside the airspace, and the current practice of counting aircraft in a sector inadequately captures the complexity of a given air traffic situation.[55, 7, 45, 49, 60, 67, 66] For example, it has been noticed that a sector sometimes accepts more aircraft than its actual capacity as currently defined by MAP; sometimes, the sector does not accept more aircraft, although the number of aircraft inside it does not reach its MAP.[10] As a consequence, several new concepts have been proposed to properly represent the complexity of the traffic situation inside the sector. Despite efforts devoted to airspace complexity, no research has yet moved substantially beyond the current practice of counting aircraft.[21, 41]

It should also be noted that a single scalar measure of airspace complexity might not be enough to describe multi-dimensional aspects of airspace complexity. To be useful, a method of describing airspace complexity should not only identify overly complex traffic situations but also suggest how airspace managers can resolve those overly demanding traffic situations. Therefore, a method of describing airspace complexity should provide more detailed information about the current airspace configuration.

1.2 Previous Research on Airspace Complexity

1.2.1 Dynamic Density

Most previous efforts to properly describing airspace complexity has attempted to find a measure of airspace complexity referenced to controller workload. This type of airspace complexity measure is often called Dynamic Density. This type of research has focused on identifying various factors affecting controllers' workload, i.e., the amount of mental or physical activities resulting from handling air traffic.[49, 18] Then, studies of dynamic density have built a measure of airspace complexity by combining those identified complexity factors. Numerous simulations have recorded physical activity measures of controller workload such as their self-reports of perceived workload, or the number of communications; since controller workload cannot be directly measured,[49, 18] corresponding effort has been put into identifying controller workload indicators.[38] The weighting of the traffic complexity factors was determined through either linear or nonlinear regressions relative to the workload measures. Fig.3 shows typical procedure of constructing a dynamic density measure.[62, 52, 39, 6, 45, 31]

For example, Laudeman *et al*[39] proposed the following measure of dynamic density.

$$\mathrm{DD} = \sum_{i=1}^{i=N} \mathrm{W}_i \mathrm{TC}_i + \mathrm{TD} \tag{1}$$

where DD represents dynamic density, N is the number of aircraft, TC_i represents the i^{th} traffic complexity factor, W_i represents the weighting for the i^{th} traffic complexity factor, and TD represents the traffic density. The set of traffic complexity factors includes:

- Heading Change (HC)-The number of aircraft that made a heading change greater than 15° during a sample interval.
- Speed Change (SC)-The number of aircraft with an airspeed change greater than 0.2 Mach during a sample interval.
- Minimum Distance 0-5 nautical miles (MD5)-The pairs of aircraft that had a Euclidean distance separation of 0-5 nautical miles.
- Minimum Distance 5-10 nautical miles (MD10)-The pairs of aircraft that had a Euclidean distance separation of 5-10 nautical miles.
- Conflict Predicted 0-25 nautical miles (CP25)-The number of aircraft predicted to be in conflict with another aircraft whose lateral distance separation at the end of each sample interval was 0-25 nautical miles.
- Conflict Predicted 25-40 nautical miles (CP40)-The number of aircraft predicted

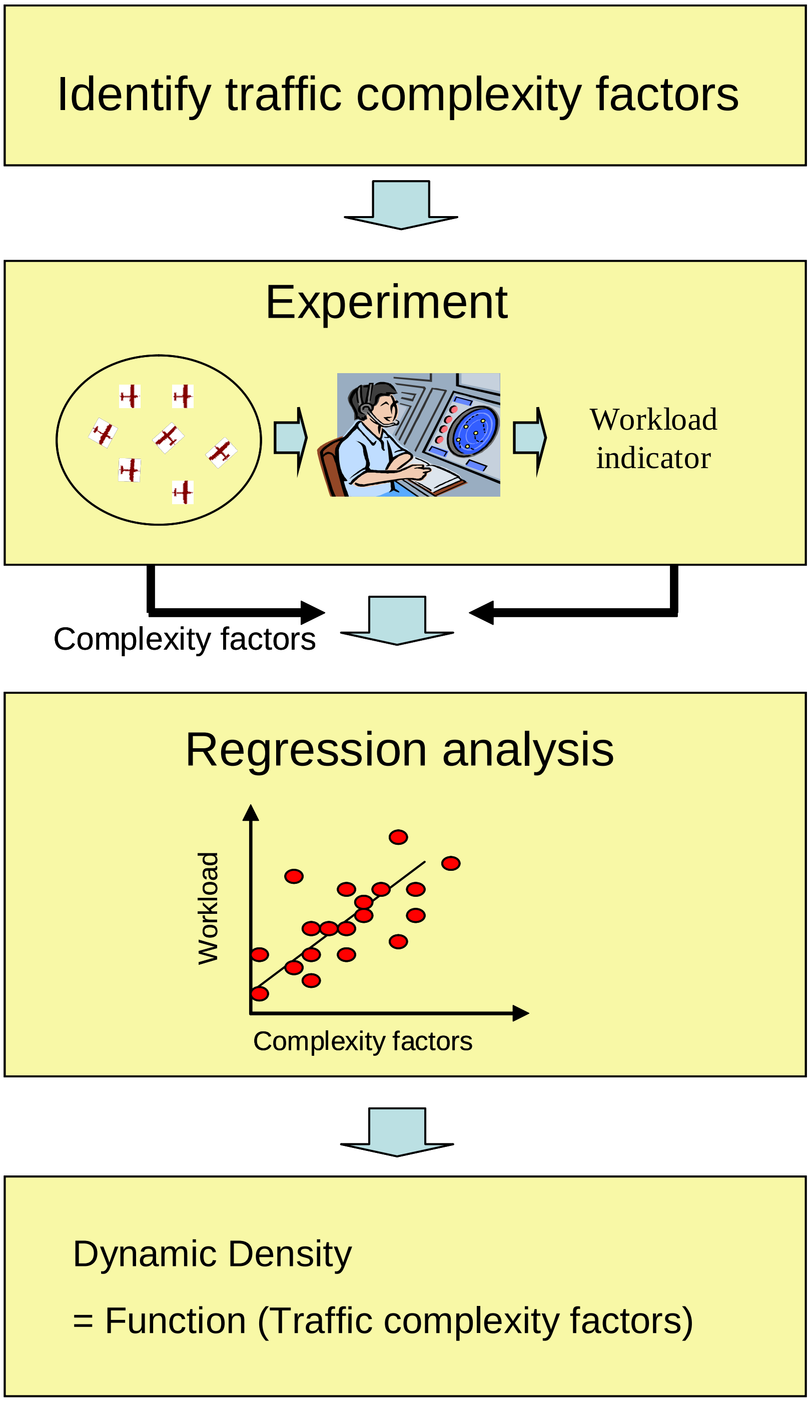


Figure 3: Previous efforts: Technical approach to compute Dynamic Density

to be in conflict with another aircraft whose lateral distance separation at the end of each sample interval was 25-40 nautical miles.

Then, Laudeman *et al*[39] performed numerous simulations and collected the number of communications between controllers and aircraft as an indication of controllers' workload. Linear regressions on these data determined the weightings for the traffic complexity factors to predict workload.

Despite many attempts to relate airspace complexity to measures referenced to controllers' cognitive workload,[62, 49, 32, 52, 39, 6, 45, 31] this type of measures can be idiosyncratic and situation specific. Although airspace complexity accounts for a large portion of controller's workload, controller's workload is also related to other mediating factors such as an individual's cognitive process or other factors such as traffic information displays.[19, 9, 68] Furthermore, as pointed out by previous research, air traffic controllers actively regulate their workloads by modifying their strategies.[61, 21, 41, 3, 22]

1.2.2 Analytical Approaches on Airspace Complexity

Other research has developed more analytic airspace complexity indicators. One project suggested a metric which essentially predicts the number of conflicts in the sector.[2] A similar metric represents the average number of crossing conflicts and overtake events.[58, 67] Other recent research has focused on geometric attributes of traffic. In one instance, metrics have been proposed to reflect the lack of structure of the traffic based on relative aircraft positions and velocities.[10] In another, the fractal dimensions of the traffic flows was measured,[50] while in still another, related aircraft are grouped into clusters.[8, 17]

Another approach mapped the airspace onto a simpler dynamical system to compute its Lyapunov exponents map.[10, 27] Other research analyzed the behavior of multiple interacting aircraft. For example, the performance and stability of intersecting aircraft flows[43, 42] and free flight traffic scenarios[5] under different traffic control methods were investigated. Likewise, the behavior of the airspace with multiple adaptive agents have been investigated.[12]

1.3 Thesis Objectives

This thesis focuses on providing an objective method to describe airspace complexity in a manner useful to applications of traffic flow management or dynamic airspace configuration. To this end, the following attributes were sought in the method:

- A method of describing airspace complexity should be able to identify complex traffic situations which initiate the traffic flow restriction or the airspace reconfiguration. Furthermore, it should help airspace managers to determine how to mitigate these overly demanding traffic situations.

- A method of describing airspace complexity should be able to incorporate environmental factors. For example, it should allow airspace managers to apprehend how convective weather affects airspace complexity.

- A method of describing airspace complexity should provide the capability of evaluating different airspace configurations.

- A method of describing airspace complexity should not provide assessments of complexity which evolve faster than any responses to them can be implemented.

- A scalar measure of airspace complexity is not always enough to fully describe airspace complexity. Therefore, this thesis focuses on multi-dimensional description of airspace complexity.

CHAPTER II

NEW NOTION OF AIRSPACE COMPLEXITY

This thesis will describe the complexity of a given sector for an instantaneous traffic situation. The airspace, together with the aircraft inside it, can be viewed as a closed loop controlled system in which many events, without intervention, can create safety violations between aircraft.[23] Air traffic controllers currently provide the response to such events; in the future, some of these events may be resolved automatically.

Definition 1: A **disturbance** is any event that may require intervention to maintain the safe, orderly flow of aircraft □

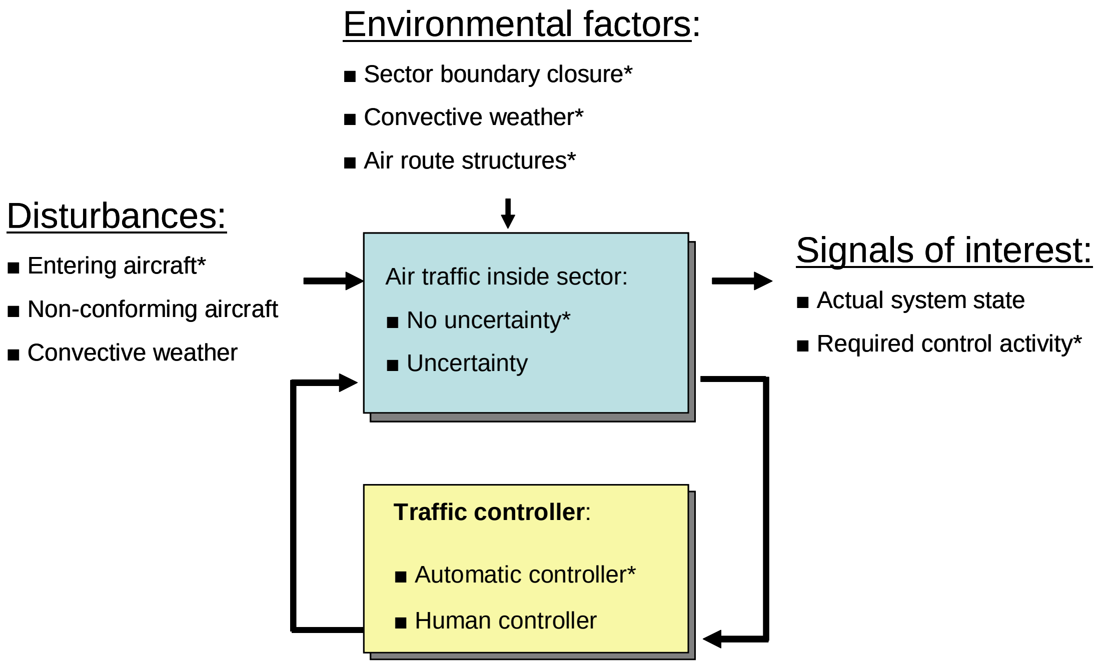


Figure 4: Air traffic control system as a closed loop controlled system (*: Specifically examined in this thesis)

Disturbances include new aircraft entering into the airspace, non-conforming aircraft inside the sector, and convective weather. It should be noted that convective weather can be considered as both a disturbance and an environmental factor. If we focus on the stochastic nature of convective weather and analyze how convective weather can disturb a current airspace, it should be considered as a disturbance. However, it should be considered as an environmental factor if we focus more on managing the traffic flows in the presence of known convective weather.

From this controlled system perspective, the complexity of a traffic situation can be described in terms of how the airspace, i.e. traffic configuration and control architecture, reacts to disturbances. Furthermore, the response of the airspace to the disturbance represents the difficulty of dealing with traffic in the face of such disturbances, and is captured by control activity defined in the following way:

Definition 2: **Control activity** is the degree of response required to resolve any conflict arising from a disturbance. □

Control activity depends on the control strategy used to manage the traffic. As shown in the previous studies[43, 42], a traffic situation can be extremely difficult to manage by some traffic control strategies, but it might not be difficult for others. Intrinsic airspace complexity might be described by computing the maximum lower bound of the control activity over all traffic control strategies available, but this thesis describes airspace complexity for each of a set of given traffic control strategies.

Different types of control activity can be defined for different analysis purposes. Although the arbitrariness of choosing control activity compromises the concreteness of the method, it provides broader insights into the air traffic control system. For example, although we do focus on a more analytical approach for airspace complexity, the results can be related to human controllers' workload by a proper choice of measure of control activity. Likewise, some traffic control algorithms can seek to predict how a human might control the traffic and their resulting control activity assessed.

Now the complexity of a given traffic situation can be described using the control activities corresponding to a given set of disturbances as follows:

Definition 3: A **complexity map** details the control activity as a function of the parameters describing the disturbances. □

The difficulty of dealing with a given traffic situation depends on the disturbance. In other words, a traffic situation can suffer significantly from some disturbances but might not be affected by others. A single scalar metric can not properly describe the traffic complexity, and the complexity map proposed in this thesis serves as a means for analyzing airspace's response to a set of disturbances.

A partial order between complexity maps can be defined:

Definition 4: One complexity map is less than the other if and only if control activity of one map is less than control activity of the other map over all disturbances. □

In addition to describing the airspace complexity of a given traffic scenario, it is very important to assess how airspace complexity is affected by environmental changes such as convective weather 'shutting down' a region within the airspace or a partial closure of a sector's boundary due to dynamic airspace management restrictions on traffic flow. Quantifying the degree of increased complexity caused by these environmental changes is necessary for traffic flow management and dynamic airspace configuration. For example, one of the main capabilities required for the dynamic airspace configuration is that airspace manager can dynamically allocate and de-allocate airspace for military and other special uses.[64, 13, 33] Therefore, airspace managers need to assess how their sector closing decisions will affect the complexity of other adjacent sectors. Likewise, it is important to assess how convective weather, the leading cause of delays in the NAS,[48, 33, 28] affects airspace complexity. These environmental factors also include static airspace configurations such as air route structures. We will see in a later chapter that the method presented in this thesis allows airspace managers to quickly apprehend in detail how airspace complexity is affected by those environmental changes.

CHAPTER III

AIRSPACE MODEL

In this chapter, we will explain the attributes of each element of Fig. 4 in more detail. We will explain how to model aircraft behavior within the sector, and the types of control architectures used to avoid conflicts between aircraft.

3.1 Aircraft & Sector Models

For simplicity, this thesis considers only the horizontal motion of aircraft. The following 2-D kinematical model is used for each aircraft inside the sector:

$$\dot{x}_i = V \cos\theta_i, \quad \dot{y}_i = V \sin\theta_i, \quad \text{where} \quad i = 1, 2, ..., N \tag{2}$$

Each pair of aircraft is not allowed to be closer than a given separation standard, d. In other words, a safety violation between aircraft i and j occurs if

$$\sqrt{[x_i(t) - x_j(t)]^2 + [y_i(t) - y_j(t)]^2} < d \tag{3}$$

Definition 5: Two aircraft are in **conflict** if propagation of their positions in time, without intervention, creates a safety violation between them at some point in time. □

Definition 6: A traffic situation is **conflict-free** if propagation of all aircraft positions in time, without intervention, do not create any safety violations at any time. □

In a conflict resolution problem involving multiple aircraft, resolving conflicts can create new conflicts which in turn may create additional conflicts during subsequent resolutions.[35] This fact leads us to use the following definition:

Definition 7: A **secondary conflict** is a conflict newly arising from resolving any conflict. □

We can consider each aircraft as a disc with radius $d/2$; safe separation is ensured where these discs do not intersect each other. In this thesis, all aircraft have the same constant velocity, and are only allowed to change their heading angle to avoid conflicts with others.

The sector boundary is approximated by a circle. The sector boundary should be explicitly defined since it defines when a disturbance, such as an additional aircraft, enters the system. The idea of using a pre-defined sector boundary is relevant to current operation of the NAS and has been used in previous research for mathematical analysis of air traffic control systems.[43, 42, 11]

It should be noted that the proposed method can be applied to describing local traffic complexity of groups of aircraft independent of geographic sector boundaries in the current National Airspace System(NAS). This type of complexity has been recently called Gaggle Density.[4] For example, in a free flight environment in which aircraft choose their own air routes, the air traffic management system may still need to prevent aircraft from entering any locally complex areas in which separation may be difficult to achieve without excessive control activity.[4]

The method proposed in this thesis does not preclude the use of more extensive models. For example we may use a three dimensional airspace model, and different speeds and altitude changes can be allowed for aircraft. Likewise, the method can accommodate different shapes for the sector boundary, as discussed later.

3.2 Control Architecture

The control architecture generates the system's reaction to a disturbance in the form of conflict avoidance maneuvers. There are several control architectures to choose from, and the description of airspace complexity depends on which control architecture is chosen.[41, 22, 43, 42] The goal is to build a tool for fast, computer-based complexity evaluations, which requires choice of a computer-based conflict resolution algorithm. These could consist of centralized or decentralized optimal conflict solvers or of rule-based solvers, and could resolve conflicts simultaneously or sequentially.[37] The choice of solver might itself be driven by the goal of the analysis or by the control architecture used in real operations. For illustrative purposes, this work uses the three different control architectures described in the following sub-sections to illustrate how this approach can incorporate different conflict solvers without difficulty.

3.2.1 Centralized Conflict Resolution Algorithm Using Mixed Integer Programming

The first control architecture is the centralized conflict solver proposed by Pallottino *et al* . [51] Most analyses in this thesis will be based on this conflict resolution algorithm. This traffic control architecture assumes that all conflicts are resolved simultaneously when a new aircraft enters the airspace. The available decision variable in this solver is aircraft heading only. In other words, to avoid conflicts, each aircraft is allowed to change its heading angle in one instantaneous heading change. This traffic control architecture does not distinguish between safety violations happening outside the sector from those happening inside the sector, although our method does not preclude this. As a brief summary, the problem can be formulated as a mixed integer linear programming problem and can be solved by fast optimization tools such as CPLEX.[26]

In Fig. 5, the circle around each aircraft represents its safety region. No safety region should intersect with another. For this, the direction of the relative velocity vector of aircraft 1 with respect to aircraft 2, i.e., $V_{1/2}$, should satisfy certain conditions. These conditions depend on the relative heading angles of each aircraft, i.e., $q_{1/2}$,$q_{2/1}$. For example, if $0 \leq q_{1/2}$, $0 \leq q_{2/1}$ and $q_{1/2} \leq q_{2/1}$, then the following

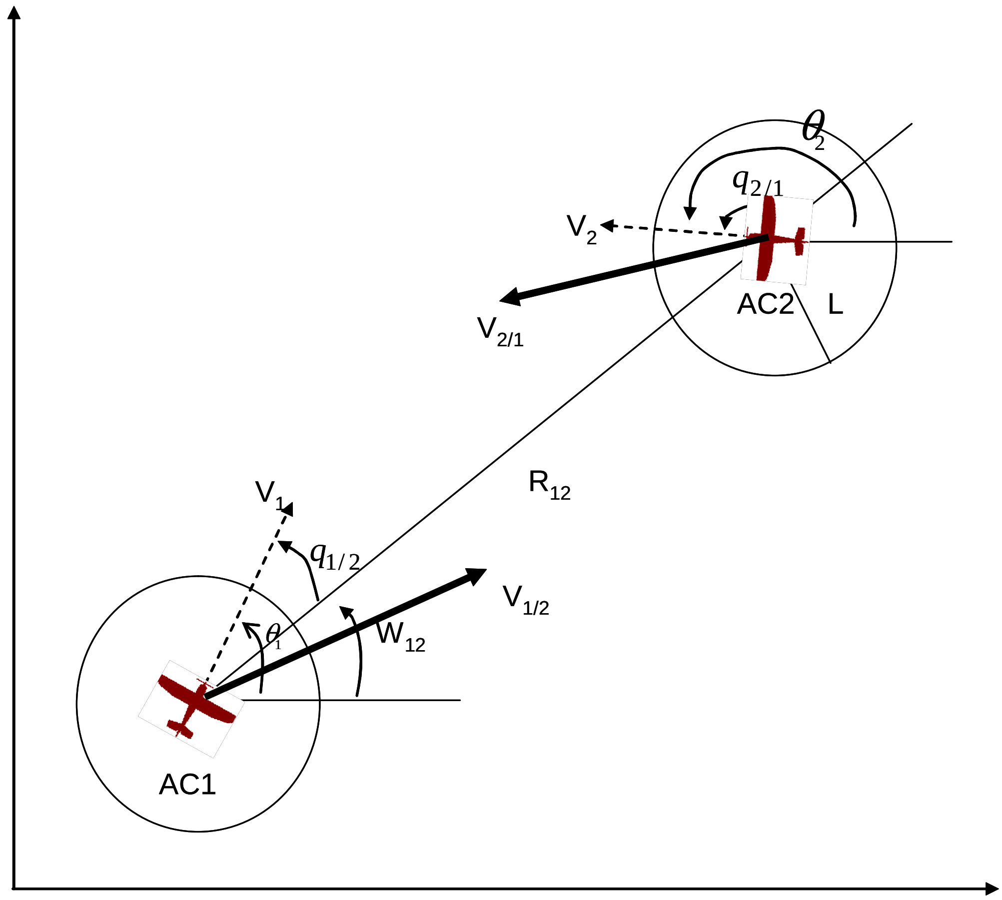


Figure 5: Conflict geometry between two aircraft

conditions should be satisfied:

$$\theta_s \leq \left| -\frac{\pi}{2} + \frac{1}{2}(q_{1/2} + q_{2/1}) \right|, \text{and} \tag{4}$$

$$\theta_s = \arcsin(\frac{2L}{R_{12}}). \tag{5}$$

Note that different configurations between aircraft require different conditions for the absence of conflicts. Likewise, trajectories are represented by absolute headings, θ_i, instead of $q_{i/j}$ in multiple aircraft scenarios because $q_{i/j}$ are relative quantities for each pair of aircraft. For example, aircraft 1 will have different q angles with respect to different aircraft. A relationship between θ_1 and $q_{1/2}$ is:

$$q_{1/2} = \theta_1 - W_{12} - 2\pi S_{12} C_1, \tag{6}$$

where $S_{12} = -\text{sgn}(W_{12})$ and C_1 are the binary variables, because non-conflict conditions are derived in the confined solution space, i.e., $-\pi \leq q_{1/2} \leq \pi$ and $-\pi \leq \theta_1 \leq \pi$. Similarly, we can derive the entire set of non-conflict constraints for all aircraft, as given in Appendix A.

If there are conflicts between aircraft, some of them should change their heading angles to satisfy non-conflict conditions. Among the many possible solutions, we can choose the one that minimizes the following objective function:

$$\text{Cost function} = \sum_{i=1}^{i=N} |\theta_{in} - \theta_i|, \tag{7}$$

where θ_{in} is the new heading angle for each aircraft. Other objective functions may be defined as appropriate.

3.2.2 Sequential Conflict Resolution Algorithm

Another control method that may stand closer to air traffic control by humans solves conflicts sequentially. The sequential conflict resolution algorithm is faster than the previous conflict resolution algorithm in scenarios involving many aircraft. In the previous section (Section 3.2.1), all conflicts are resolved simultaneously when a new aircraft enters into the airspace. However, in real operations, conflicts are resolved successively over a certain period of time. In other words, air traffic controllers command only one aircraft to change its heading at a time. Then air traffic controllers command another aircraft to change its heading after a certain time interval. Therefore, this thesis considers another conflict resolution algorithm which solves conflicts in the following way:

• Create a sequence of aircraft inside the sector, e.g., AC1-AC2-AC3-AC4-.... In this thesis, we assign a number to the entering aircraft first. Then we assign a number to each aircraft from the west-most aircraft to the east-most aircraft. If two aircraft are aligned vertically, we assign a lower number to the south-most aircraft. Other indexing rules are possible.

• AC1 maintains its trajectory while AC2 changes its trajectory in order to avoid a conflict with AC1.

• After a certain time interval, AC3 changes its trajectory in order to avoid conflicts with AC1 and AC2 while AC1 and AC2 maintain their trajectories.

• We proceed with each subsequent aircraft k and change its trajectory to avoid conflicts with the previous aircraft 1,2,...$k-1$. The previous aircraft maintain their trajectories as previously determined.

The available decision variable in this solver is only aircraft heading, and each heading change is instantaneous. This traffic control architecture does not distinguish between safety violations happening outside the sector from those happening inside the sector.

3.2.3 Conflict Resolution Algorithm with a Fixed Horizon

Another traffic control method perhaps closest to the current practice resolves conflicts with a fixed time horizon. In other words, at every time sequence, air traffic controllers resolve conflicts only if the conflict is expected to happen in a certain time interval. This algorithm periodically resolves conflicts between aircraft which are close each other within a given distance. At every time interval, e.g., every 10 seconds, conflicts between aircraft are resolved in the following way:

- Create a sequence of aircraft inside the sector, e.g., AC1-AC2-AC3-AC4-.... In this thesis, we assign a number to the entering aircraft first. Then we assign a number to each aircraft from the west-most aircraft to the east-most aircraft. If two aircraft are aligned vertically, we assign a lower number to the south-most aircraft. Other indexing rules are possible.

- AC1 maintains its trajectory while AC2 changes its trajectory in order to avoid a conflict with AC1 if the distance to AC1 is less than a given distance.

- AC3 changes its trajectory in order to avoid conflicts with AC1 and AC2 only if the distance to AC1 or AC2 is less than a given distance, while AC1 and AC2 maintain their trajectories.

- We proceed with each subsequent aircraft k and change its trajectory to avoid conflicts with the previous aircraft 1,2,...$k-1$ only if the distance to each of them is less than a given distance. The previous aircraft maintain their trajectories as previously determined.

3.3 Environmental Factors

We are often interested in analyzing how airspace complexity is affected by changes in environmental factors. In this thesis, we investigate how airspace complexity is affected by an adjacent sector closure by convective weather, and by air route structures. For the adjacent sector closure, the traffic inside the sector is not allowed to exit through the part of the sector boundary where the adjacent sector is closed for reasons such as congestion management or military operations. The case of adjacent sector closure is modeled as a partial closure of the sector's boundary. The case of convective weather is modeled as a circular 'shut down' region in the airspace, and the traffic inside the sector is not allowed to go through this part of airspace.

More static environmental factors such as a given air route structure are investigated in this thesis by assessing their complexity over a wide range of traffic situations.

CHAPTER IV

THE RESPONSE OF AIRSPACE TO ENTERING AIRCRAFT

This chapter investigates the response of the airspace to a specific type of disturbances, i.e., aircraft entering into the airspace. As mentioned before, the motivation of this study is intimately tied to traffic flow management, so a sector should be considered as a part of a large network of airspaces. Therefore the complexity of the airspace should be assessed in terms of how difficult to manage its traffic is in the face of an entering aircraft from an adjacent airspace.

First, I will explain how to model the disturbances, i.e., the entering aircraft, and the control activity from which we can describe the complexity of the traffic inside the sector. I will subsequently illustrate how to describe airspace complexity using a complexity map.

4.1 Details of the Disturbances

Among the many different types of disturbances, this thesis considers any aircraft entering the sector at any heading and location. In this thesis, an entering aircraft is at the sector boundary when it appears. Therefore, the complexity of an instantaneous traffic situation is described based on its response to additional aircraft entering into the airspace at that moment. The entering aircraft has the same speed as the other aircraft in the sector. A complexity map is computed by considering all possible entering aircraft spanning all entering points on the boundary and all headings. To define instances of entering aircraft, the following definitions are introduced and illustrated in Fig. 6.

Definition 8: The **entering aircraft position angle** is the entry location of an aircraft into the sector, defined in angular coordinates from North. □

Definition 9: The **entering aircraft bearing** is the relative track of an entering aircraft with respect to the radial line connecting the aircraft to the center of the sector. When the bearing is zero, the entering aircraft is moving straight toward the center of the sector. Fig. 6 illustrates a positive bearing. □

A complete set of disturbances encompasses all possible entering aircraft position angles and bearings. For a circular sector boundary, the entering aircraft position angle spans $0°$ to $360°$. We consider only entering aircraft which will be inside the sector for some period of time, i.e., with bearing angles between $-90°$ and $90°$.

As mentioned before, the method can be extended to using a different shape of sector boundary. For a non-circular sector boundary, we can define each instance of an entering aircraft in a different way. For example, for an arbitrary shape of a sector boundary, we can define the entry location of an aircraft by the parameter, 's', in Fig. 7 and entering aircraft bearing can be defined with respect to the true north or some reference point within the sector as shown in Fig. 7.

Finally, it should be noted that the method provided in the thesis can accommodate other types of disturbances such as non-conforming aircraft and convective weather. The flexibility provided by this method in analyzing different types of disturbances provides broad applicability.

4.2 Control Activity & Complexity Map

Control activity represents the "pain" experienced by airspace as it reacts to the disturbance. If no conflict arises from an entering aircraft, and no conflict was originally present among the aircraft inside the sector, the control activity due to this particular entering aircraft is zero. This thesis uses three different types of control activity. Most analyses in this thesis will compute the sum of the total heading changes over

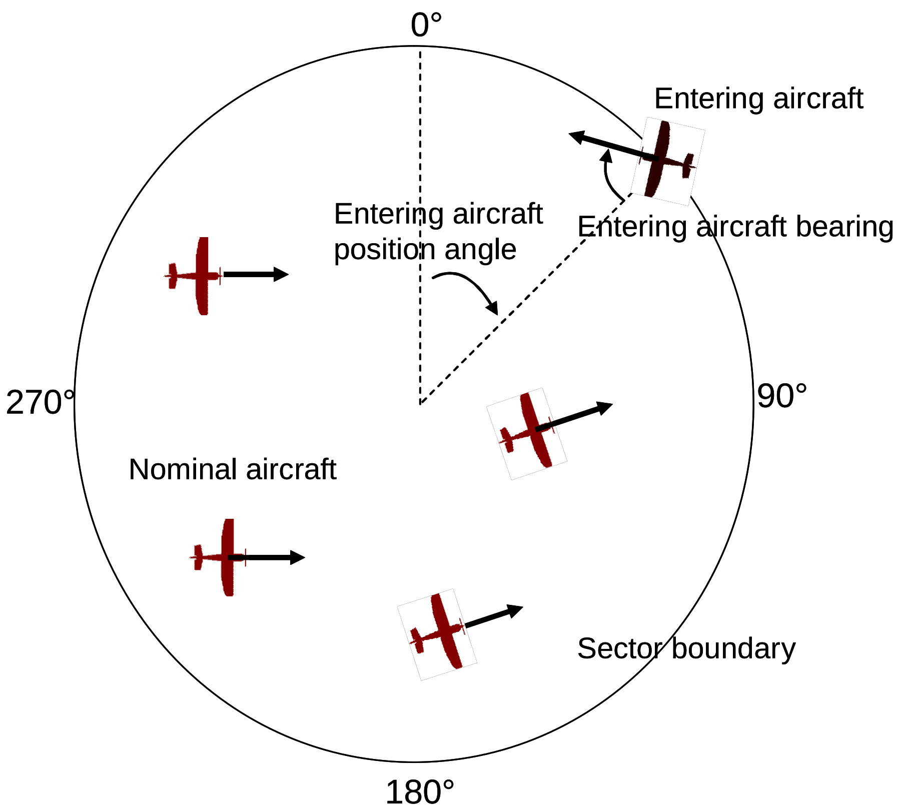


Figure 6: Parameters to define an entering aircraft

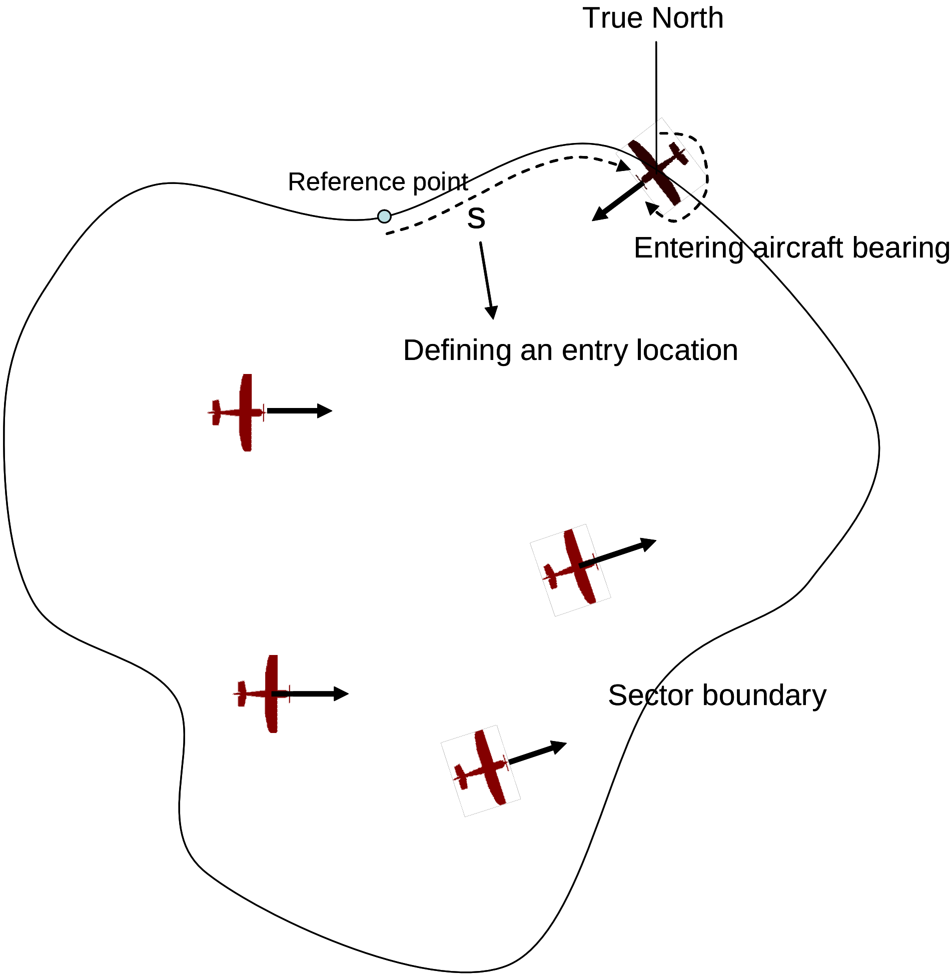


Figure 7: Parameters to define an entering aircraft: Arbitrary sector boundary shape

all aircraft inside the sector to maintain a conflict-free situation. With the traffic control architecture described in Section 3.2.3, which resolves conflicts over a fixed horizon, heading changes are aggregated from the time when a new aircraft enters into the airspace until it leaves.

Another type of control activity is the number of aircraft that underwent a heading change inside the sector. This measure might be a better indicator of the "pain" experienced by the traffic from the disturbances, because many small heading changes may, by some measures, require more control actions than a single large heading change.

The last type of control activity examined in this thesis is the sum of the heading changes due to secondary conflicts. In multiple aircraft scenarios, resolving conflicts can create new conflicts which, in turn, may create additional conflicts during subsequent resolutions. These newly arising conflicts resulting from resolving other conflicts are called secondary conflicts, and a measure of secondary conflicts can be a good indicator of how the aircraft trajectories are interrelated.

Alternate measures are possible, including non-linear weighting of heading changes (e.g., not including heading changes under a threshold considered to be negligible). For a more extensive airspace model, the control activity might be defined by combining different types of aircraft's maneuvers (e.g., the heading change, the speed change, and the altitude change) with proper weightings of each. This flexibility in using different types of control activity allows tailoring the analysis to specific purposes.

The loci of these values of control activity over all possible entering aircraft position angles and bearings are displayed as a complexity map of the immediate traffic situation. The horizontal axis of the complexity map represents entering aircraft position angle and the vertical axis of the complexity map represents entering aircraft bearing angle.

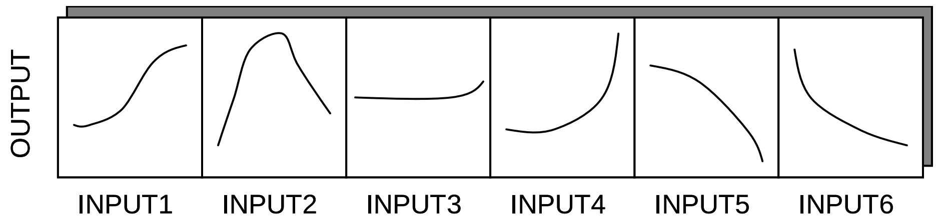


Figure 8: Prediction profile plot

More than two parameters might be needed to define each instance of entering aircraft. For example, we may consider entering aircraft with different entry locations, different heading angles and different speeds. In such a case, a single two dimensional complexity map can not fully describe complexity of a given airspace. Other types of graphical representations, e.g., a prediction profile plot (Fig. 8), might be used.[46] Two dimensional complexity maps can still be used by choosing two major parameters and, for each combination of major parameters, computing control activity over ranges of other parameters.

4.3 Numerical Examples

Two airspaces configurations in Fig. 9, called Sectors A and B respectively, will be used to demonstrate the method. The large circles represent sector boundaries and the small circles represent aircraft inside the sector. The velocity vectors of the aircraft are indicated by the line segments originating from the center of the small circles. There is no fixed air route structure. However, as shown later, this method can be applied to airspace with a fixed air route structure. The initial configurations of traffic in both sectors are conflict-free, which means that the undisturbed propagation of all aircraft in time will not create any safety violations between aircraft. However, this assumption can be relaxed. We will describe airspace complexities of both traffic situations for each of the three different control algorithms explained in the previous chapter.

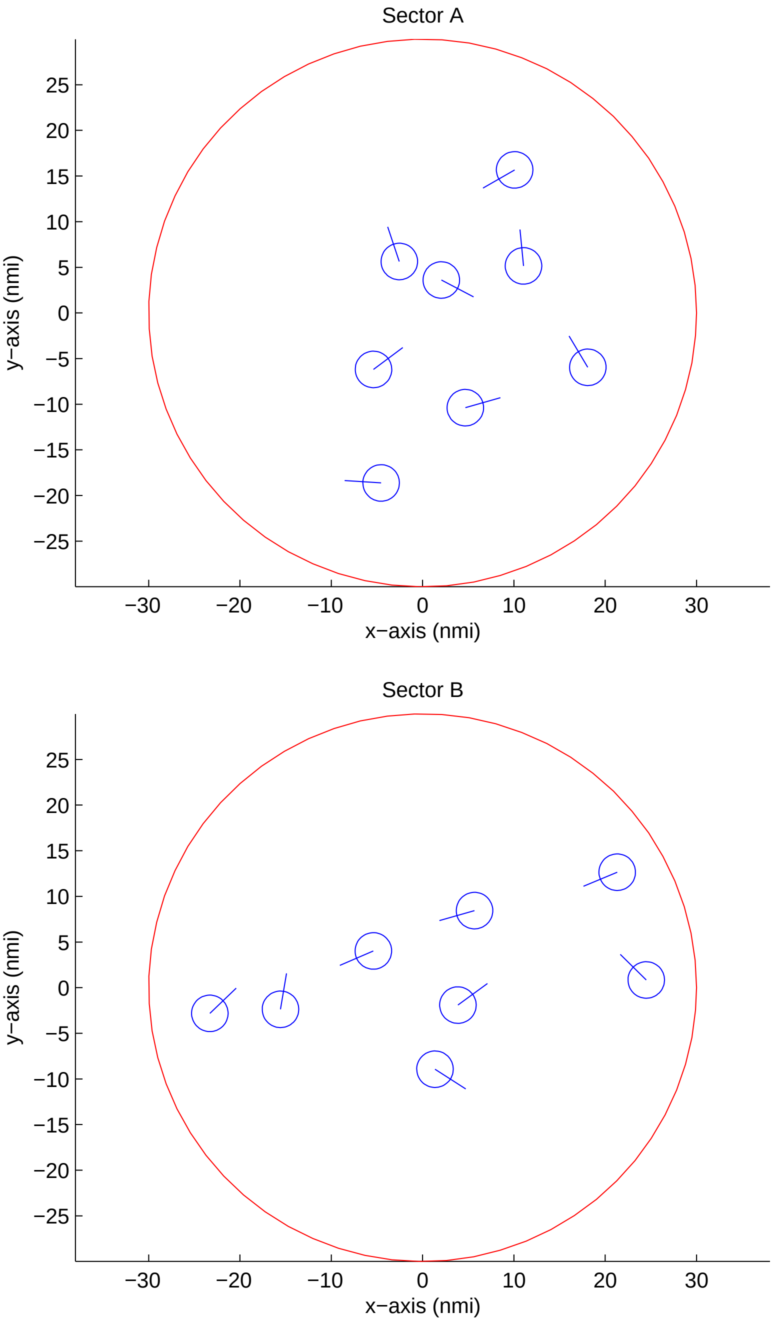


Figure 9: Traffic Situations in Sector A and Sector B

These traffic scenarios mimic the future free flight environment since they have no fixed air route structure. As shown in Fig. 10, the current NAS has fixed air route structures for aircraft above a certain altitude. Moreover, the traffic density used in this examples, i.e., 28 aircraft per 10000 Nm^2, is much higher than the usual traffic density in the current air traffic management system. In fact, the peak U.S. Center-wide traffic density ranges from 1 to 5 aircraft per 10000 Nm^2.[35]

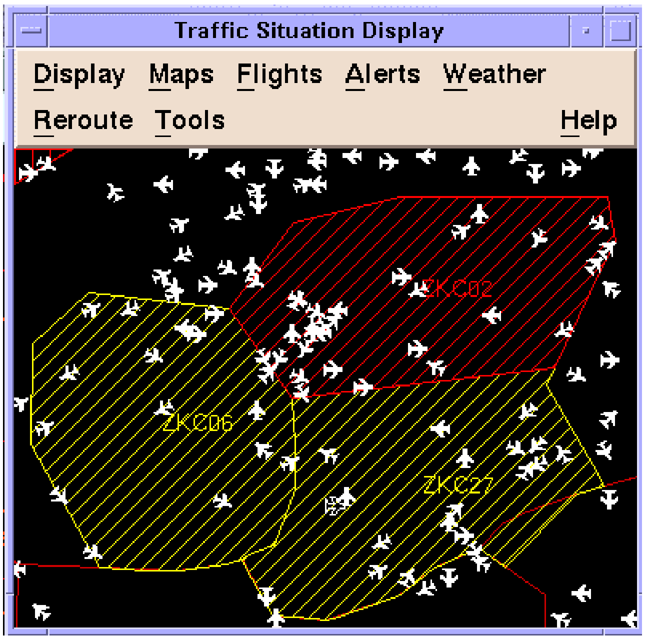


Figure 10: Example traffic situation in the Kansas City Center: Enhanced Traffic Management System(ETMS)

Two different traffic situations were investigated to illustrate how the proposed method can incorporate different traffic situations without difficulty. As will be shown later, the complexity map of one traffic situation has high control activity over a large range of entering aircraft, while the complexity map of the other does not. (It is not easy to see beforehand which traffic situation corresponds to the complexity map with high control activity.)

4.3.1 Complexity Map

Complexity maps for the traffic situations in Sector A and B were computed using the centralized, optimal conflict solver from Pallottino *et al* [51]. These are shown in Fig. 11. These complexity maps show how each traffic situation will suffer from a disturbance, i.e. an entering aircraft, in terms of control activity.

The complexity map for each sector shows the control activity, i.e. the total required heading changes by aircraft inside the sector, for all combinations of entering aircraft position angle and bearings. For example, consider a particular entering aircraft whose position angle is 120° and whose bearing is 10° for the traffic situation in Sector A. This entering aircraft is represented on the complexity map for the traffic in Sector A by the symbol '◯' in Fig. 11, and the required control activity resulting from this disturbance is 35°. However, an entering aircraft with a position angle of 150° and the same bearing, represented by the symbol '+' on the first complexity map in Fig. 11, requires zero control activity. Similarly, an entering aircraft with position angle of 120° and bearing of −20° requires zero control activity by the current traffic in the Sector A as represented by the symbol '□'.

Comparing two complexity maps in Fig. 11, we see that the traffic situation in Sector B requires a non-zero control activity over a noticeably larger range of entering aircraft position and bearing angles compared to the traffic situation in Sector A. Moreover, the largest control activity for the traffic situation in Sector A is 49° whereas this maximum value for Sector B is 79°.

More important information comes from the relative impact of the different entering aircraft on the airspace, as shown by the topology of the complexity map. For example, in the traffic situation in Sector B, entering aircraft with position angles between 30° and 80° and between 270° and 330° will require large control activity. Based on this information, the airspace manager can decide to restrict traffic coming through this part of the sector boundary, therefore aiding TFM based on traffic

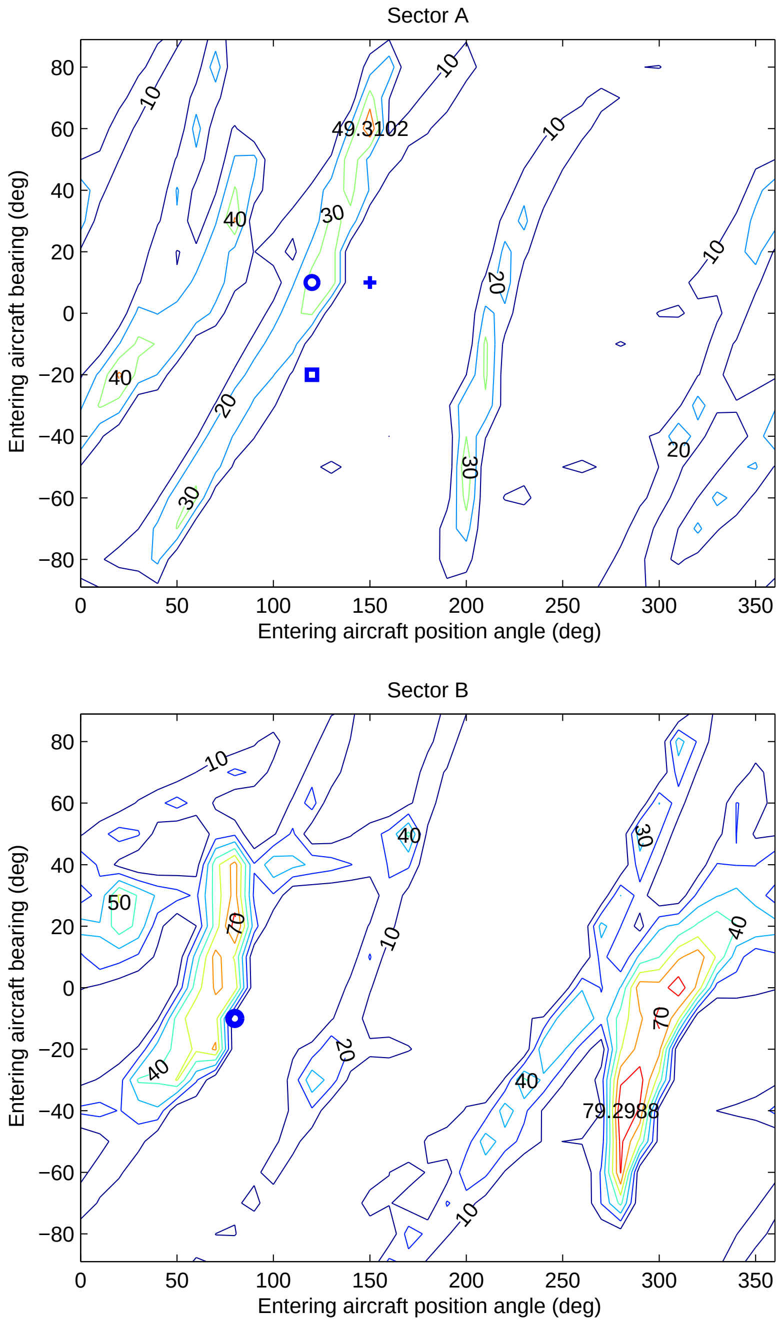


Figure 11: Complexity map for the traffic situations in Sectors A and B. The plot contours indicate the control activity for all combinations of entering aircraft bearing and position angles

complexity. As an extreme case, if an aircraft inside the sector is too close to the sector boundary to resolve a conflict with a certain range of entering aircraft, the corresponding part of the sector boundary must be closed. (This partial closure of a sector boundary will affect the complexity of the adjacent sector, as discussed in the next chapter.)

In a complexity map, a dense concentration of contour lines represents a rapid change of control activity with respect to the entry location and heading of the entering aircraft. For example, consider an entering aircraft whose position angle is 80° and whose bearing is $-10°$ in the traffic situation in Sector B. This entering aircraft is represented on the complexity map for Sector B by the symbol '◯'in Fig. 11, and the required control activity for this aircraft is zero. However, small increase in bearing or a small decrease in position angle can result in a high required control activity.

Now, assume that one of aircraft has been removed from the traffic situation in Sector B, as shown in Fig. 12. The corresponding complexity map is also shown in Fig. 12. Comparing this complexity map with the original complexity map for Sector B in Fig. 11, we can notice that control activity has decreased over all entering aircraft. The difference between those maps is given in Fig. 13. It should be noted that the complexity map with removed aircraft is not always less for every conflict resolution algorithm. This is true only when a given conflict resolution algorithm can find the minimum control activity to resolve all conflicts arising from an entering aircraft; the conflict resolution algorithm used in this section finds the minimum required heading changes to resolve all conflicts.

Finally, investigate how complexity is added up for Sector B by each of aircraft inside it. We constructed the traffic situation in Sector B by adding one aircraft by one aircraft from the west-most aircraft to the east-most aircraft and see how the complexity map changes. The evolution of the complexity map for Sector B are shown in Fig. 14 and Fig. 15.

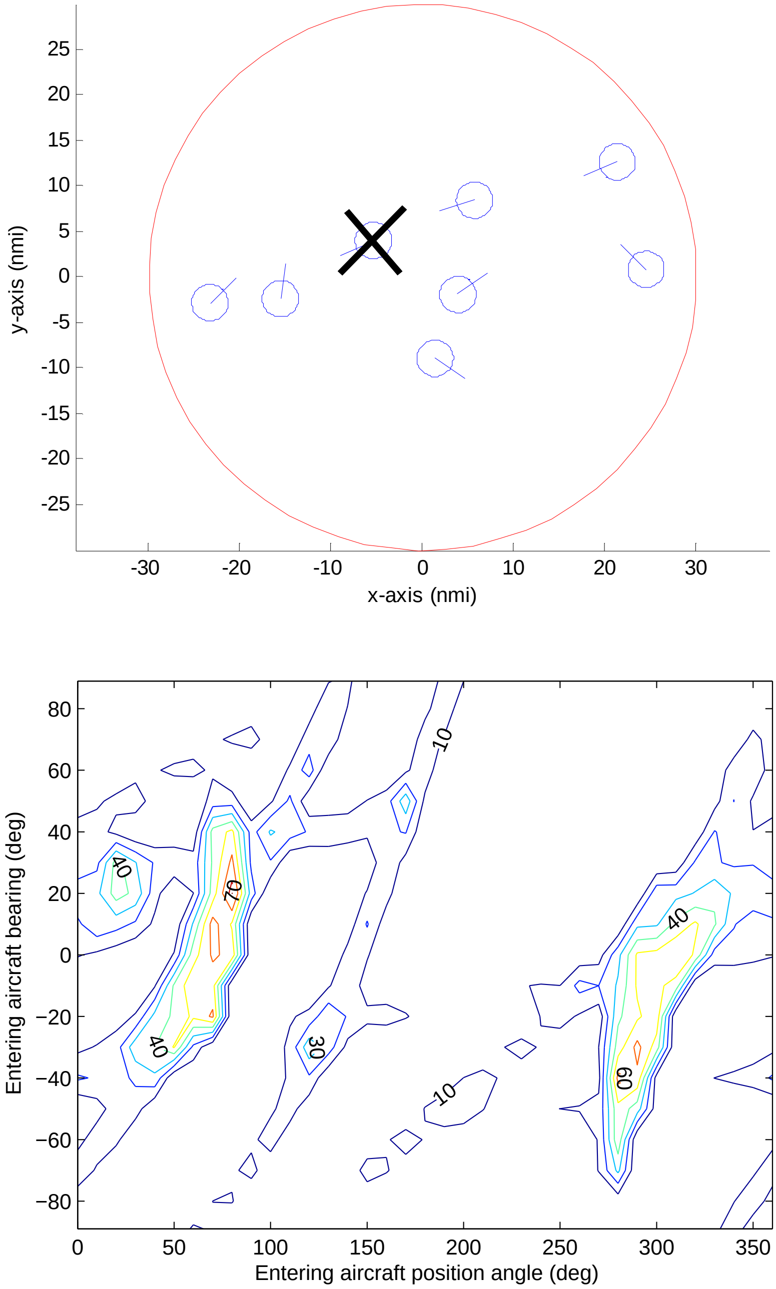


Figure 12: Removal of one aircraft from Sector B (Fig. 9) and corresponding complexity map

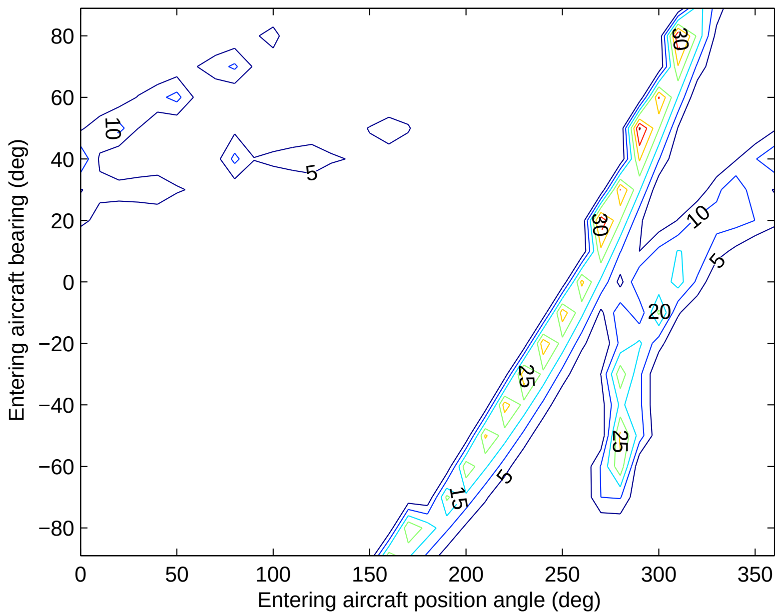


Figure 13: Difference in complexity for Sector B with one aircraft removed

As we can notice, high control activity for entering aircraft with position angles between 270° and 330° started popping up when the first west-most aircraft was added and stopped after the third west-most aircraft was added. Another area of high control activity (which corresponds to entering aircraft with position angles between 30° and 80°) did not appear until the east-most aircraft was added.

4.3.2 Complexity Map: Number of Heading Changes

Using the same control architecture, i.e., the optimal conflict solver from Pallottino *et al* [51], the complexity maps for the previous traffic situations shown in Fig. 9 were computed with a different measure of control activity, i.e. the number of aircraft that underwent a heading change. The complexity maps for traffic situations in Sector A and B are shown in Fig. 16.

Comparing two complexity maps in Fig. 16, we see that the traffic situation in Sector B requires more aircraft to change their heading angles over a noticeably

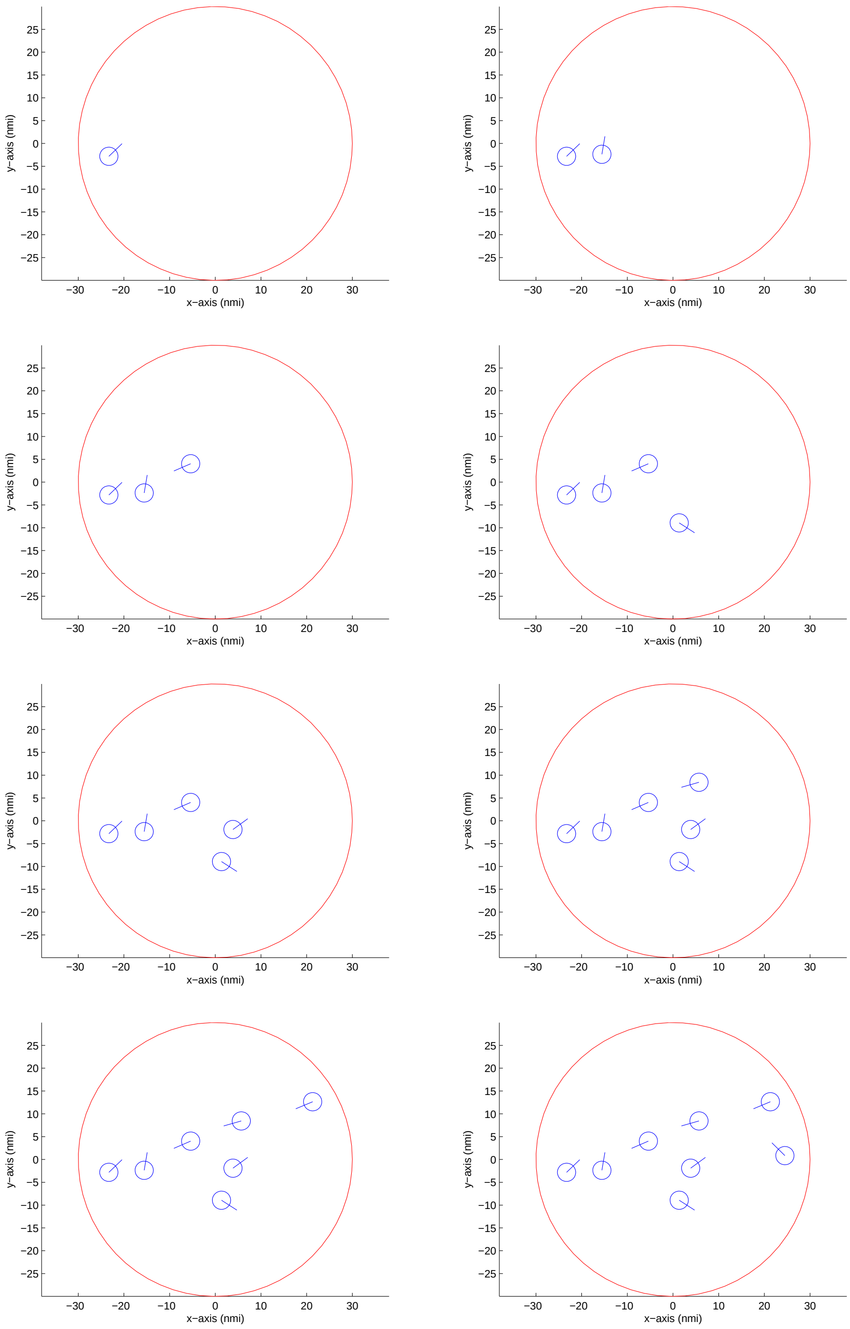


Figure 14: Construction of the traffic situation in Sector B by adding one aircraft by one aircraft

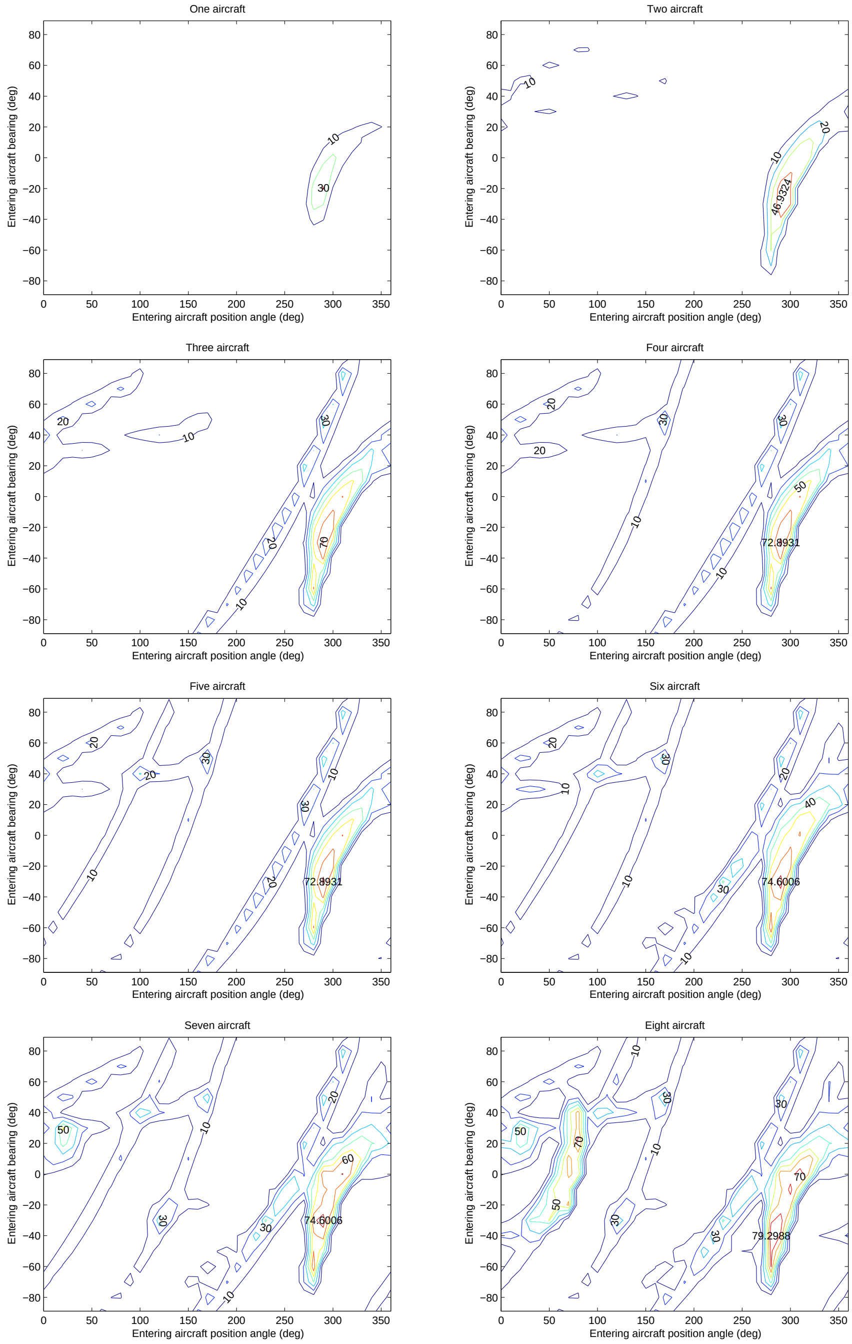


Figure 15: The evolution of the complexity map for Sector B by adding aircraft

large range of entering aircraft position and bearing angles compared to the traffic situation in Sector A. We can also notice that, for the traffic situation in Sector B, entering aircraft with position angles between 30° and 80° and between 270° and 330° (which corresponds to the problematic part of sector boundary identified in the previous section) still require a large control activity, i.e., a large number of aircraft are required to change their heading angles.

4.3.3 Complexity Map: Secondary Conflicts

Initial conflicts arise from entering aircraft when the initial configurations of traffic are conflict-free as in both Sectors A and B.) However, in multiple aircraft scenarios, resolving these initial conflicts can create new conflicts, and the sum of required heading changes for resolving these secondary conflicts can also serve as a measure of control activity.

Using this measure of control activity and the same control architecture, i.e., the optimal conflict solver from Pallottino *et al* [51], the complexity map for the previous traffic situations shown in Fig. 9 were computed as shown in Fig. 17.

Comparing the complexity map for Sector B in Fig. 17 with that in Fig. 11, a large portion of control activity in the complexity map in Fig. 11 was due to the secondary conflicts. In other words, only few conflicts arising from an entering aircraft create a large amount of control activity in Sector B. On the other hand, for the traffic situation in Sector A, entering aircraft did not create many secondary conflicts, as shown in Fig. 17.

4.4 Complexity Map Using Different Conflict Resolution Algorithms

This section investigates the complexity of the traffic situations in Sector A and Sector B when controlled by the different conflict resolution algorithm discussed in the previous chapter. This illustrates how the proposed approach can incorporate

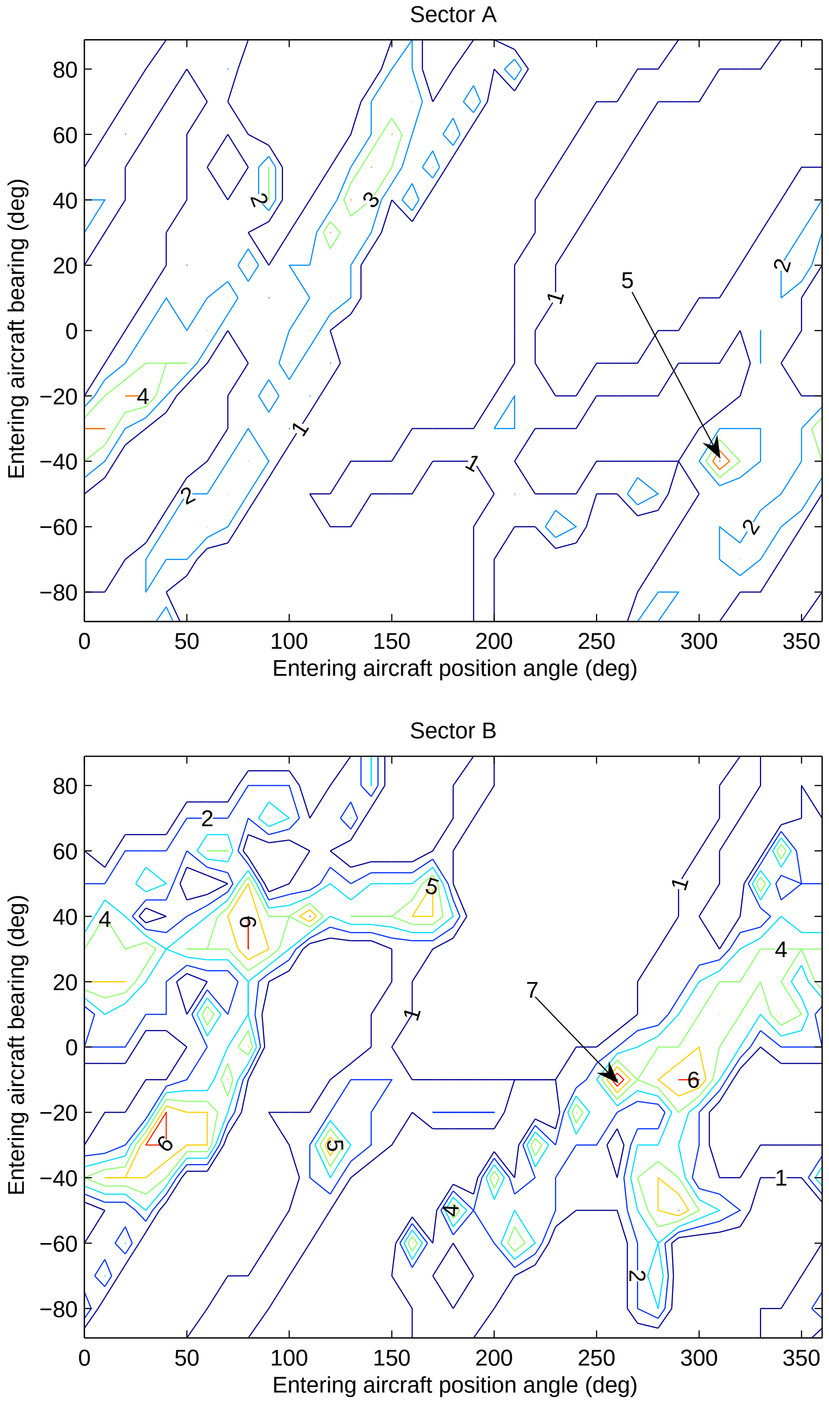


Figure 16: Complexity map for the traffic situation in Sectors A and B. The plot contours indicate the number of heading changes required by aircraft for all combinations of entering aircraft bearing and position angles

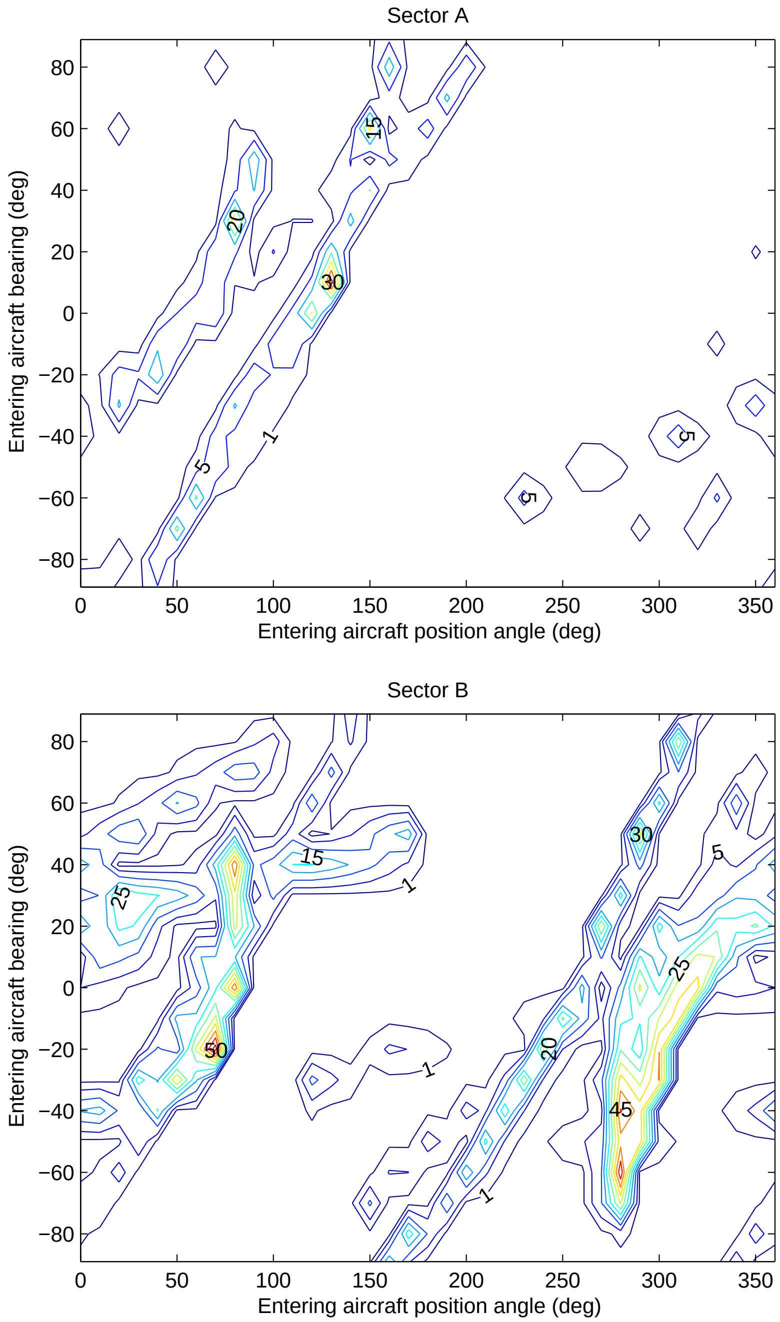


Figure 17: For the traffic situation in Sectors A and B, the plot contours indicate the control activity due to secondary conflicts for all combinations of entering aircraft bearings and position angles

different conflict solvers without difficulty. The measure of control activity used in this section was the total required heading changes by aircraft inside the sector.

4.4.1 Complexity Map: Sequential Conflict Resolution Algorithm

Complexity maps for traffic situations in Sectors A and B with the sequential conflict resolution algorithm described in Section 3.2.2 are shown in Fig. 18. Comparing these complexity maps with maps with the optimal conflict solver from Pallottino *et al* [51], the control activity increases for both traffic situations due to the sub-optimality of the sequential conflict resolution algorithm. While the affected areas for the traffic situation in Sector A are very limited, the amount of control activity required within those areas increases dramatically. Correspondingly, for the traffic situation in Sector B, the control activity increases dramatically for a wider range of entering aircraft position angles and bearings. The average control activity for traffic situation in Sector A is increased by 23% while the average control activity for traffic situation in Sector B is increased by 44% The difference between those maps for Sectors A and B are given in Fig. 19

4.4.2 Complexity Map: Sequential Conflict Resolution Algorithm with a Fixed Horizon

In this section, the sequential conflict resolution algorithm with a fixed horizon (Section 3.2.3) will be used to describe the complexity of the traffic situations. The corresponding complexity maps are shown for traffic situations in Sectors A and B in Fig. 20.

As entering aircraft bearing goes to $\pm 90°$, the required control activity decreases and eventually disappears since an entering aircraft with a bearing angle of greater (or less) than $\pm 90°$ does not travel through the airspace. For the traffic situation in both Sectors A and B, the control activity required for the entering aircraft with position angles between $0°$ and $180°$ increases dramatically. However, for the traffic

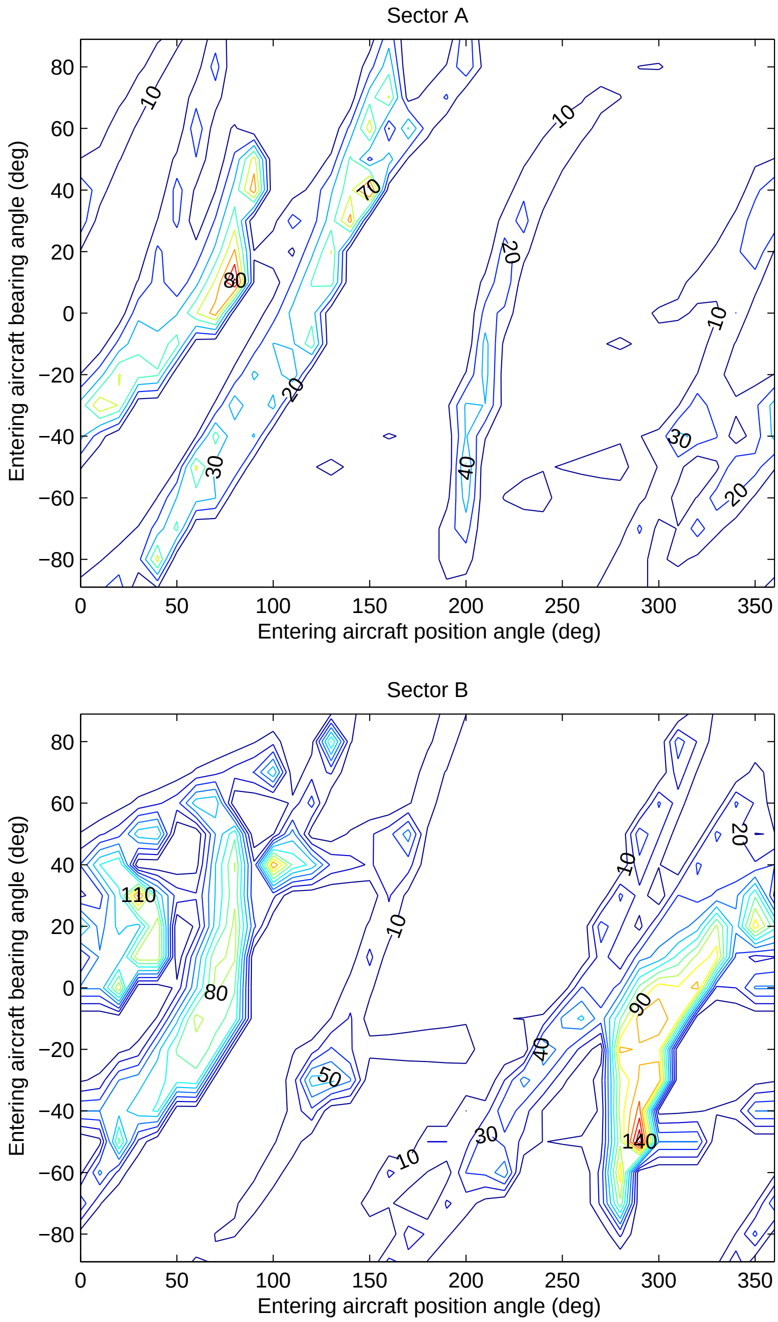


Figure 18: Complexity map for the traffic situation in Sectors A and B: Sequential conflict resolution algorithm

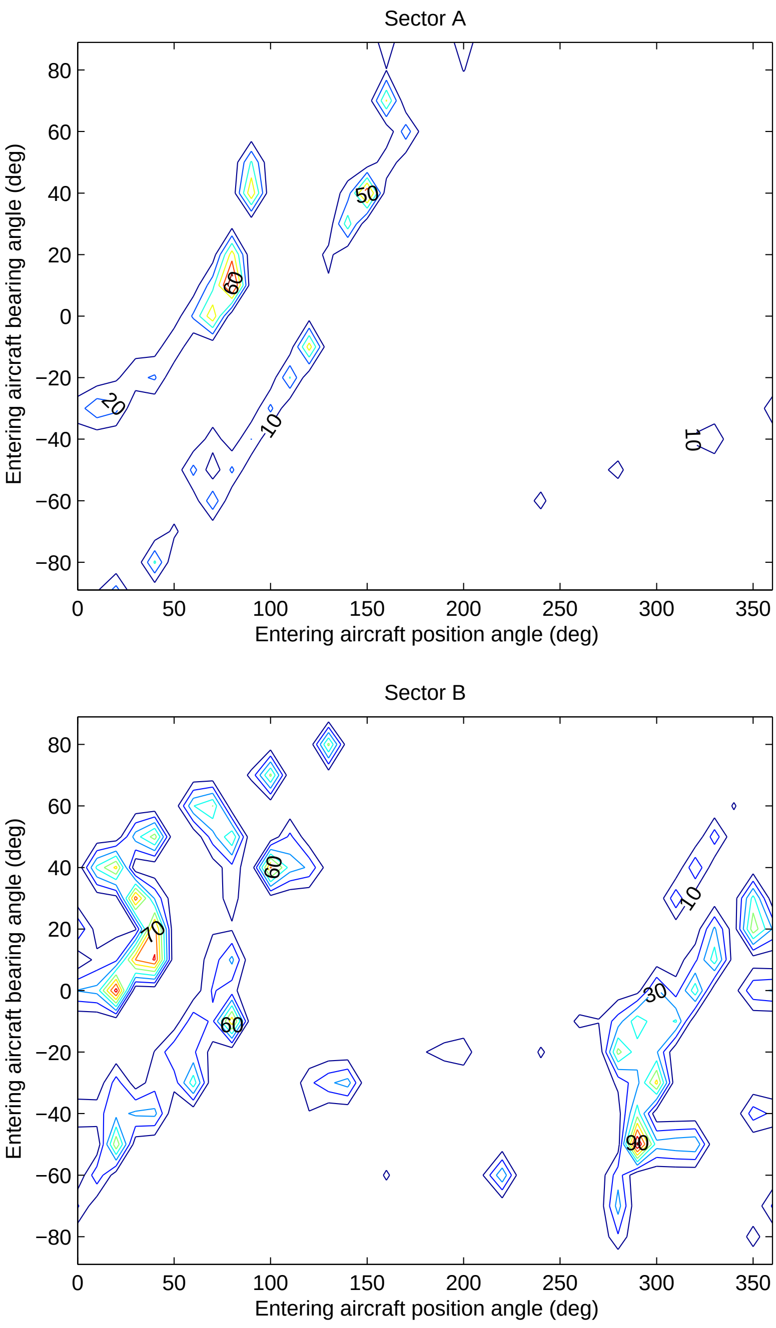


Figure 19: Difference between complexity maps with MILP solver and sequential solver for Sectors A and B

situation in Sector B, the control activity for the entering aircraft with position angles between 270° and 330° also increases dramatically. The difference between those maps for Sectors A and B are given in Fig. 21 It should be noted that sequential conflict resolution algorithm with a fixed horizon sometimes requires less control activity since heading changes are aggregated only from the time when a new aircraft enters into the airspace until it leaves.

4.5 Scalar Measures of Airspace Complexity

The complexity map provides detailed information about the control authority required within a sector in response to disturbances. However, this detailed analysis may be aggregated into one of a variety of scalar measures as appropriate to different applications. For example, the worst-case value for required control activity may be used as an indication of the sector's sensitivity to disturbances. Alternatively, the area enclosed on a complexity map representing conditions requiring a control activity exceeding some minimum threshold provides an estimate of the range of situations requiring a significant response. One may also use the average measure of control activities. Many other methods are, of course, possible for reducing the complexity maps to scalar values, each with its own operational relevance. Table 1 shows several scalar complexity measures corresponding to the previous complexity maps. Table 2 also shows how complexity of Sector B is added up by each of its aircraft.

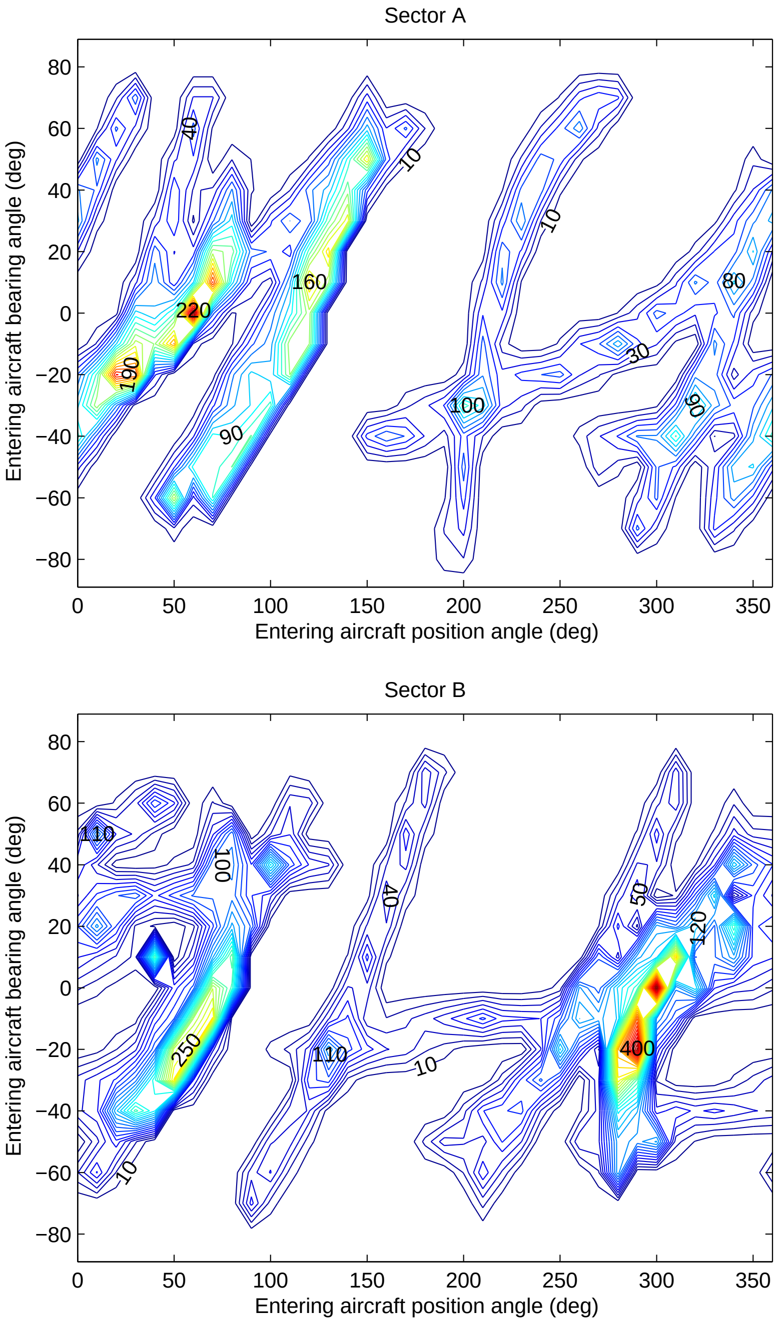


Figure 20: Complexity map for the traffic situation in Sectors A and B: Sequential conflict resolution algorithm with a fixed horizon

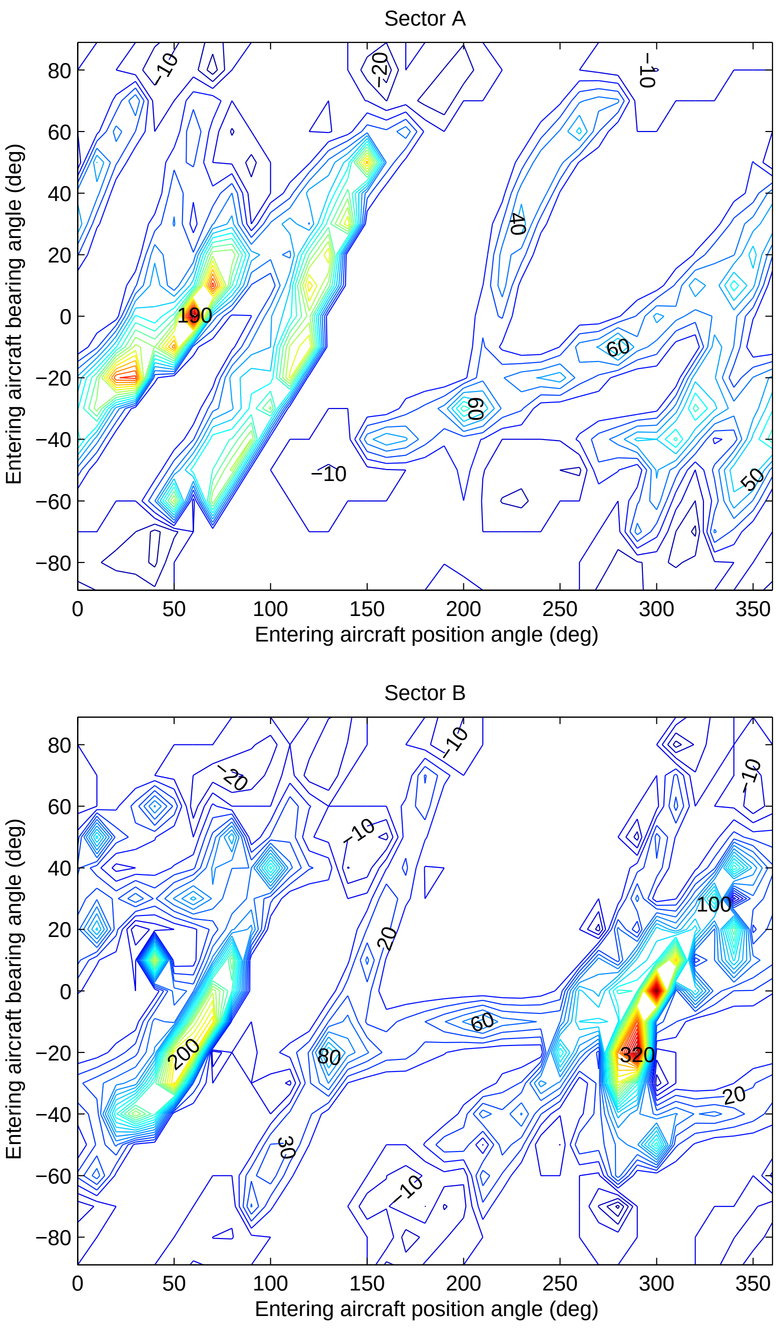


Figure 21: Difference between complexity maps with MILP solver and sequential solver with a fixed horizon for Sectors A and B

Table 1: Scalar measures of airspace complexity (C:Centralized conflict resolution algorithm using integer programming by Pallottino, S:Sequential conflict resolution algorithm, SF:Sequential conflict resolution algorithm with a fixed horizon)

	Average(deg)	Worst-case(deg)
Sector A(C) (The first map in Fig. 11)	6.6	49.3
Sector A(S) (The first map in Fig. 18)	8.2	73.3
Sector A(SF) (The first map in Fig. 20)	19.7	235.5
Sector B(C) (The second map in Fig. 11)	10.4	79.3
Sector B(S) (The second map in Fig. 18)	14.9	151.3
Sector B(SF) (The second map in Fig. 20)	25.7	413.7

Table 2: Complexity added by each aircraft in Sector B

	Average(deg)	Worst-case(deg)
Sector B (No aircraft removed)	10.4	79.3
Sector B (One aircraft removed)	8.1	74.6
Sector B (Two aircraft removed)	6.9	74.6
Sector B (Three aircraft removed)	5.8	72.9
Sector B (Four aircraft removed)	5.2	72.9
Sector B (Five aircraft removed)	4.5	72.9
Sector B (Six aircraft removed)	2.0	46.9
Sector B (Seven aircraft removed)	0.8	30.5

CHAPTER V

TIME EVOLUTION OF A COMPLEXITY MAP

This chapter investigates how a complexity map changes with respect to time. So far, we have described airspace complexity at an instant in time. In other words, the complexity of an instant traffic situation is described based on its response to aircraft entering into the airspace at that moment. However, in practical applications, the complexity of the traffic should be described over a certain period of time because of uncertainties in the system. For example, due to the limitations of current navigation technologies, it is difficult to present the each aircraft's position along its trajectory. For a similar reason, we should consider aircraft entering into the airspace over a certain time period rather than an instant in time. Further, the airspace requires some time to act upon airspace managers' decisions to regulate traffic flow. Therefore, airspace complexity should be described over a certain period of time, and a complexity map used by the airspace managers should not change significantly for that time interval.

In this chapter, a statistical approach was used to evaluate the time evolution of the complexity map. We randomly generated traffic situations, created complexity maps, and assessed how they change over time.

Throughout, it should be noted that a complexity map represents the control activity as a function of two parameters, i.e.,the entering aircraft position angle and the entering aircraft bearing angle. These parameters are called x and y respectively. Therefore, a complexity map at time t_i, which represents the i^{th}-time in the discrete time sequence, can be written as $f(x, y, t_i)$, where $x \in \{x_1 = 0°, ..., x_N = 360°\}$ and $y \in \{y_1 = -90°, ..., y_M = 90°\}$. Now, the difference between complexity maps at

t_i and t_{i+1}, which represents how the complexity map changes over $[t_i\ t_{i+1}]$, can be evaluated by the following metric:

$$\text{difference metric} = \frac{1}{NM} \sum_{y=y_1}^{y=y_M} \sum_{x=x_1}^{x=x_N} |f(x, y, t_{i+1}) - f(x, y, t_i)| \tag{8}$$

For a given traffic situation, the complexity map at t_{i+1} will be different with the complexity map at t_i due to aircraft movement over the time interval, and the metric defined in Eq. 8 represents the difference between those complexity maps at t_i and t_{i+1}. In this chapter, the confidence interval for the average value of the metric (Eq. 8) will be constructed.

5.1 Statistical Inferences: Confidence Interval

The Confidence intervals, common in statistical analysis, were used to describe how fast a complexity map changes over time in terms of the previously defined metric (Eq. 8). Since we can not test all possible traffic situations, we have to infer the the average value of the metric, μ, from a sample data set. This technique is often referred to as "confidence interval construction" since we construct an interval that contains a set of plausible values of the μ. A confidence interval is associated with the confidence level, α, selected as the required probability$(1-\alpha)$ that the confidence interval actually contains the μ. In other words, we can conclude that the average value of the metric(Eq. 8) over all possible traffic situations is in a certain interval with confidence $1 - \alpha$. This technique can be applied to the population which is not normally distributed if the sample size is big enough.[20]

In this thesis, the one-sided confidence interval was constructed:

$$\mu \in (-\infty, \overline{x} + \frac{t_{\alpha,n-1}s}{\sqrt{n}}) \tag{9}$$

where n represents the sample size, $\overline{x}$ represents the sample mean, and s represents the sample standard deviation, which can be computed as the square root of the variance:

$$s^2 = \frac{\sum_{i=1}^{i=n} (x_i - \overline{x})^2}{n-1} \tag{10}$$

where x_i represents the i^{th} experimental observation.

In the equation 9, $t_{\alpha,n-1}$ is the critical point of a t-distribution, which is defined in the following way:

$$P(X \geq t_{\alpha,n-1}) = \alpha \tag{11}$$

where X has a t-distribution with n-1 degrees of freedom.

5.2 Experimental Result

5.2.1 Time Evolution of a Complexity Map over 1 Second

This section investigates how fast a complexity map changes within 1 second, representative of a potential update rate of a map in operation. Therefore, how a complexity map changes over the time period of 1 second will be investigated. For each simulation, four aircraft were randomly (with a uniform distribution over the airspace) created within a 30 Nm radius circular airspace, which corresponds to approximately 14 aircraft per 10000 Nm^2. It should be noted that the metric defined in Eq. 8 may depend on the traffic density. However, the traffic density used in the experiments is much higher than the usual traffic density in the current air traffic management system. In fact, the peak U.S. Center-wide traffic density ranges from 1 to 5 aircraft per 10000 Nm^2.[35]

For each traffic situation, a complexity map was created. Then, the trajectories of aircraft were propagated in time for 1 second, and the complexity map was created again for the traffic after 1 second. Using these complexity maps, the metric defined in Eq. 8 was computed.

Sample data (from 1000 runs) of these computed metrics is described by the "box plot" shown in Fig. 22. The mean value is 0.28°, and 75% of the data has a value smaller than 0.25°.

For our analysis, we decided to set the confidence level, α, to be 0.01 and the following confidence interval was computed:

$$\mu \in (0, 0.30^\circ) \tag{12}$$

This means that the average value of the metric(Eq. 8) over all simulated traffic situations is less than 0.30° with the probability of 99%.

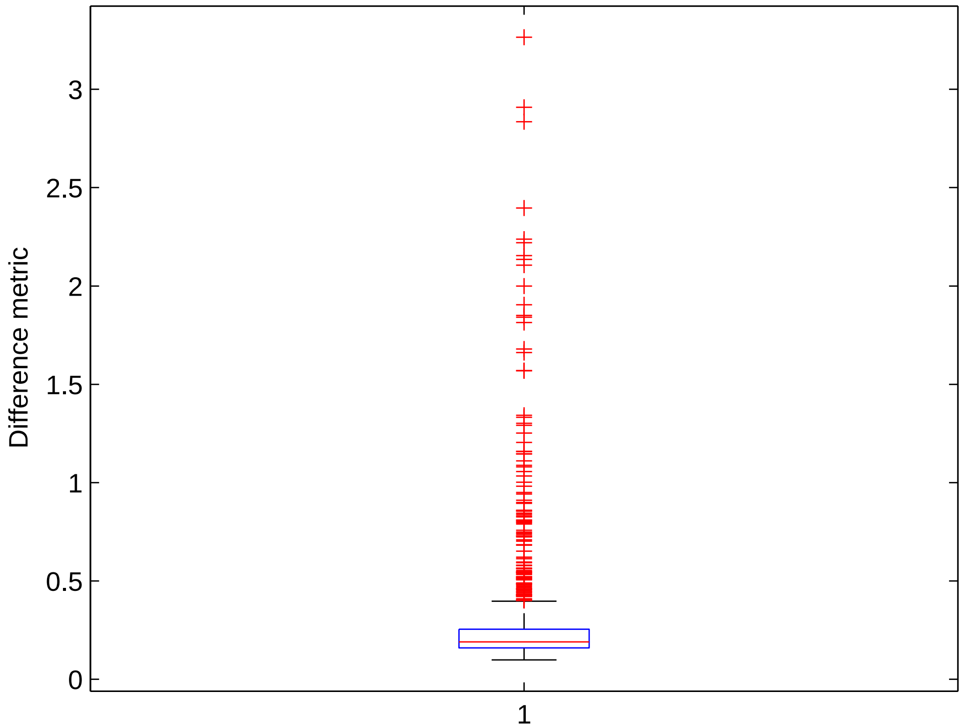


Figure 22: Box plot of the difference metric over 1 second in 1000 traffic situations

So far, we have shown that the average value of the metric(Eq. 8) over all possible traffic situations is most likely to be smaller than 0.30°. We now need to understand how this difference is distributed over a complexity map. As an example, consider the traffic situation at current time and its configuration after 1 second are shown in Fig. 23. The complexity maps for these traffic situations are shown in Fig. 24. Using these complexity maps, the metric(Eq. 8) was computed to be $0.31°$. The difference between those maps is shown in Fig. 25. As we can see, the difference between the two does not correspond to noticeably different implications for traffic flow management; the same attributes of entering aircraft generate the highest control activity.

The sample data sometimes has very high values for the difference metric. These data points, i.e., outliers represented by the symbol '+' in Fig. 22, correspond to cases where two conflicting aircraft are too close each other. For example, consider he traffic situation at current time and its configuration after 1 second as shown in Fig. 26. The complexity maps for these traffic situations are shown in Fig. 27, and have a difference metric(Eq. 8) of $2.14°$. This high value comes from the proximity of aircraft 1 and 2. The difference between those maps is shown in Fig. 28. Fortunately, this kind of traffic scenario must rarely happen in real situation. In addition, we can notice that even having $2.14°$ for the metric does not correspond to very rapid time evolution of the complexity map as shown in Fig. 27 and Fig. 28.

5.2.2 Time Evolution of a Complexity Map over 60 Seconds

The time evolution of a complexity map should fit within the time required to make decisions based upon it. The time interval for the airspace to respond to the airspace managers' decision is much longer than the time interval when a complexity map is updated (i.e., 1 second in the previous section). Therefore, this section investigates how a complexity map changes over a 60 second time period. Because of the small size of a sector, i.e., 30 Nm radius circular airspace, and the high traffic density, i.e., 14 aircraft per 10000 Nm^2, used in our simulations, 60 seconds can be considered long enough to see how much a complexity map changes over a time scale corresponding

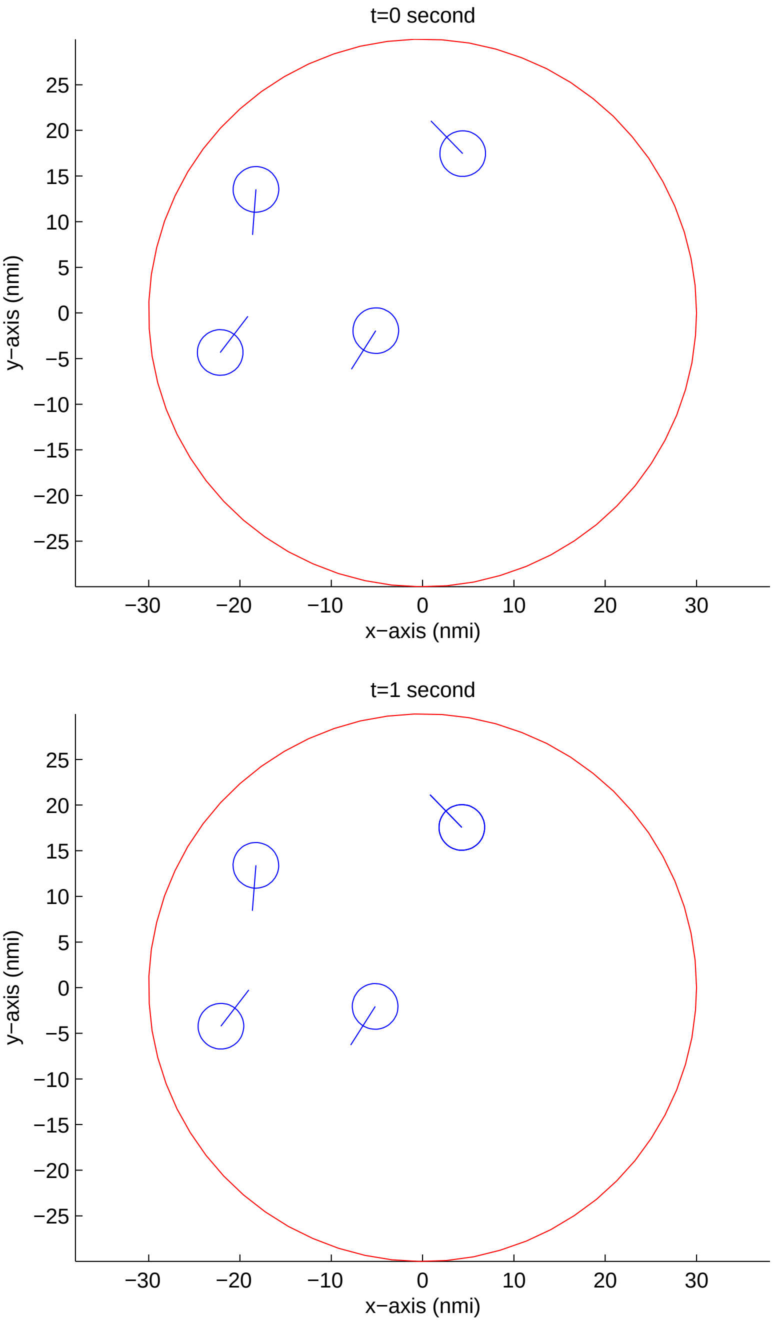


Figure 23: Example traffic situation

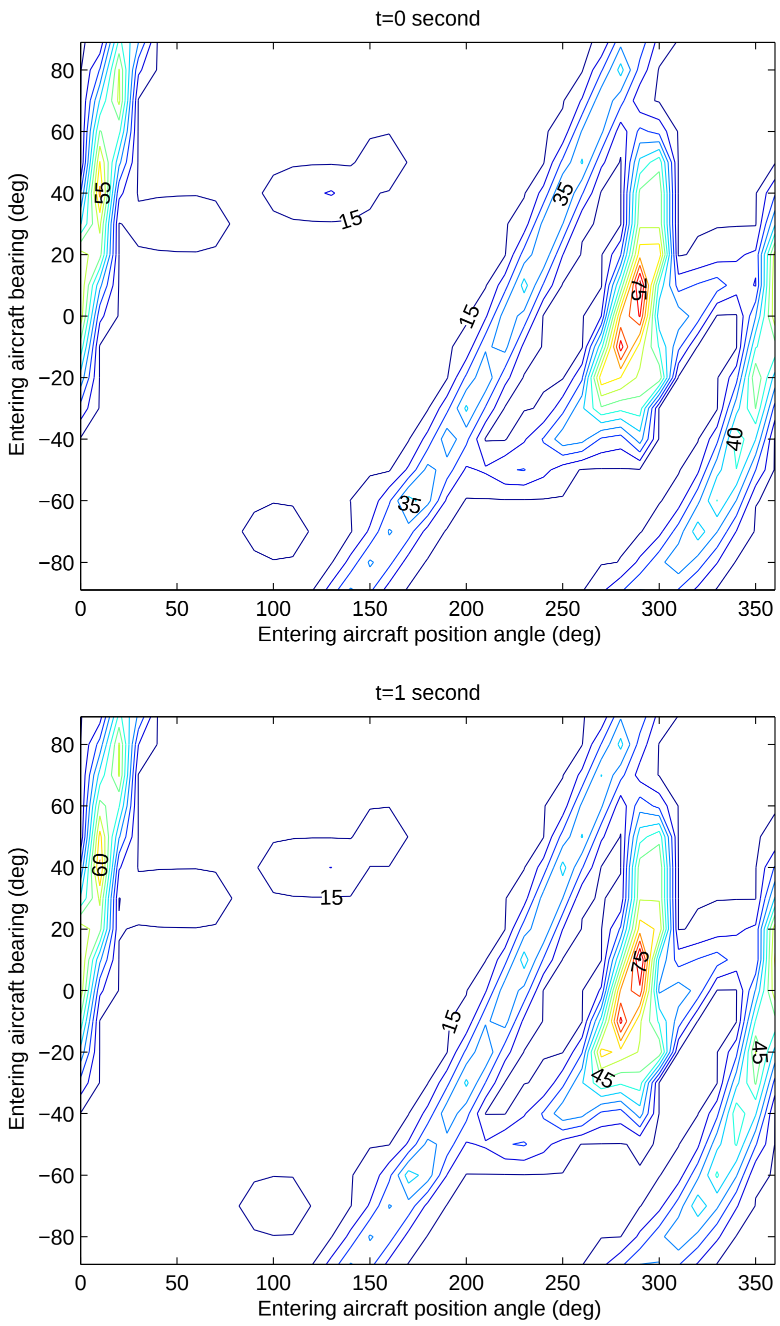


Figure 24: Complexity map for the traffic situation in Fig. 23 at current time and after 1 second

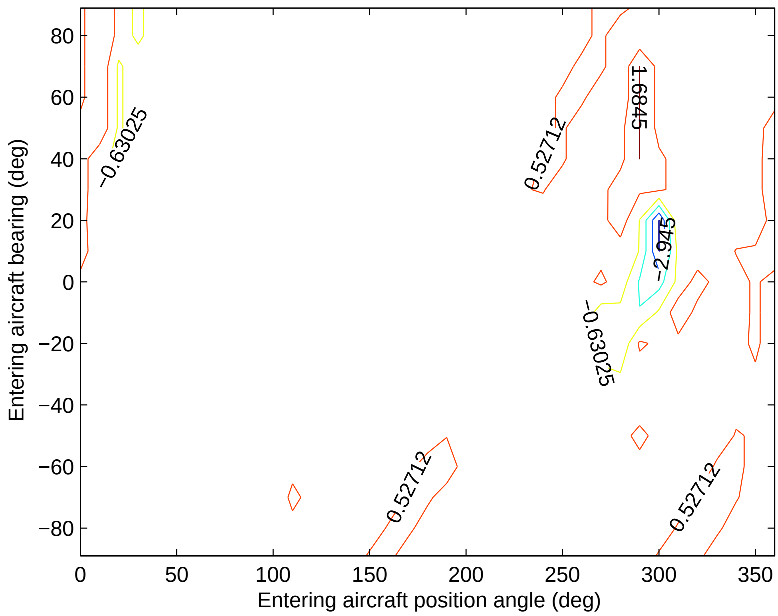


Figure 25: Difference between complexity maps taken 1 second apart

to traffic flow management applications.

For each simulation, four aircraft were randomly (with a uniform distribution over the airspace) created within a 30 Nm radius circular airspace, and the corresponding complexity map was created. Then, the trajectories of aircraft were propagated in time for 60 seconds, and the complexity map was created again for the traffic after 60 seconds. Using these complexity maps, the difference metric defined in Eq. 8 was computed.

After 1000 runs, the mean value of the sample data was found to be 11.65°. We decided to set the confidence level, α to be 0.01 and the following confidence interval was computed:

$$\mu \in (0, 12.52^\circ) \tag{13}$$

This means that the average value of the difference metric(Eq. 8 with the time interval of 60 seconds) over all possible traffic situations is less than 12.52° with the

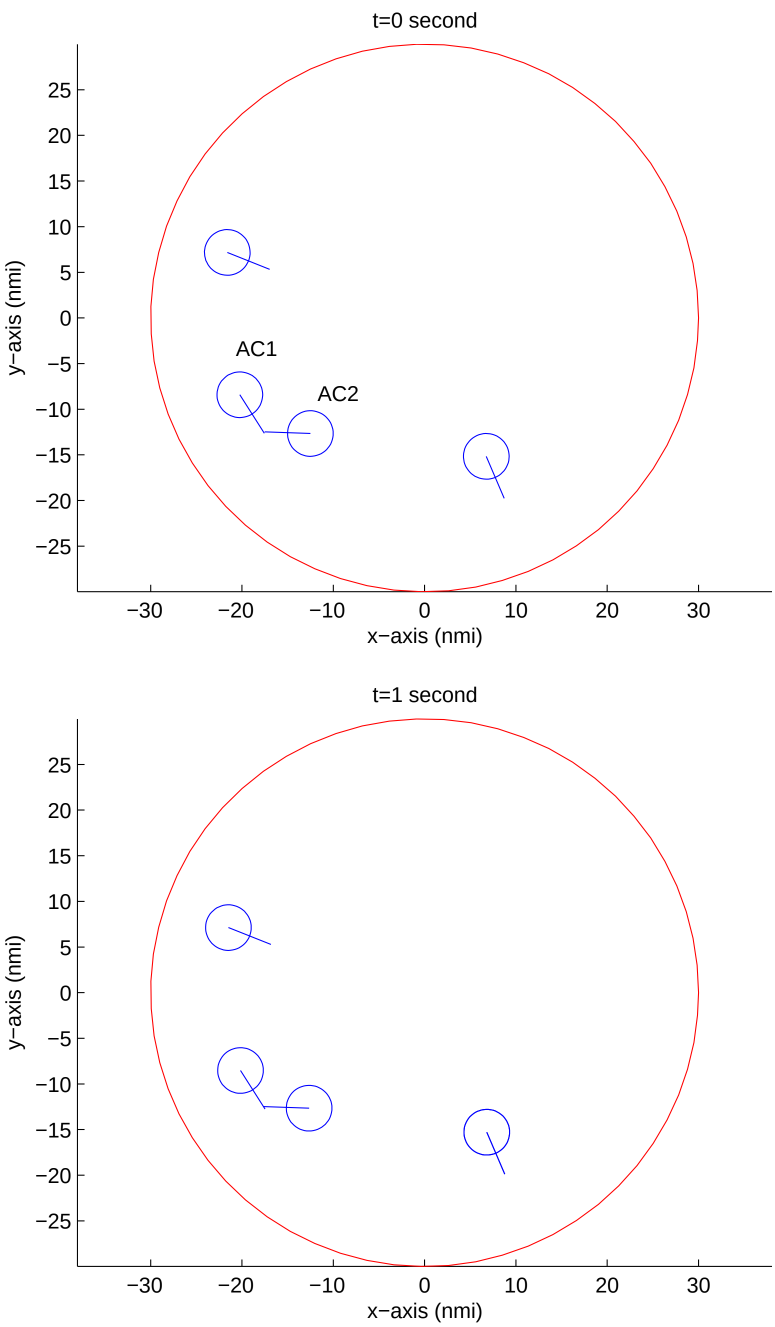


Figure 26: Traffic situation with a high difference metric

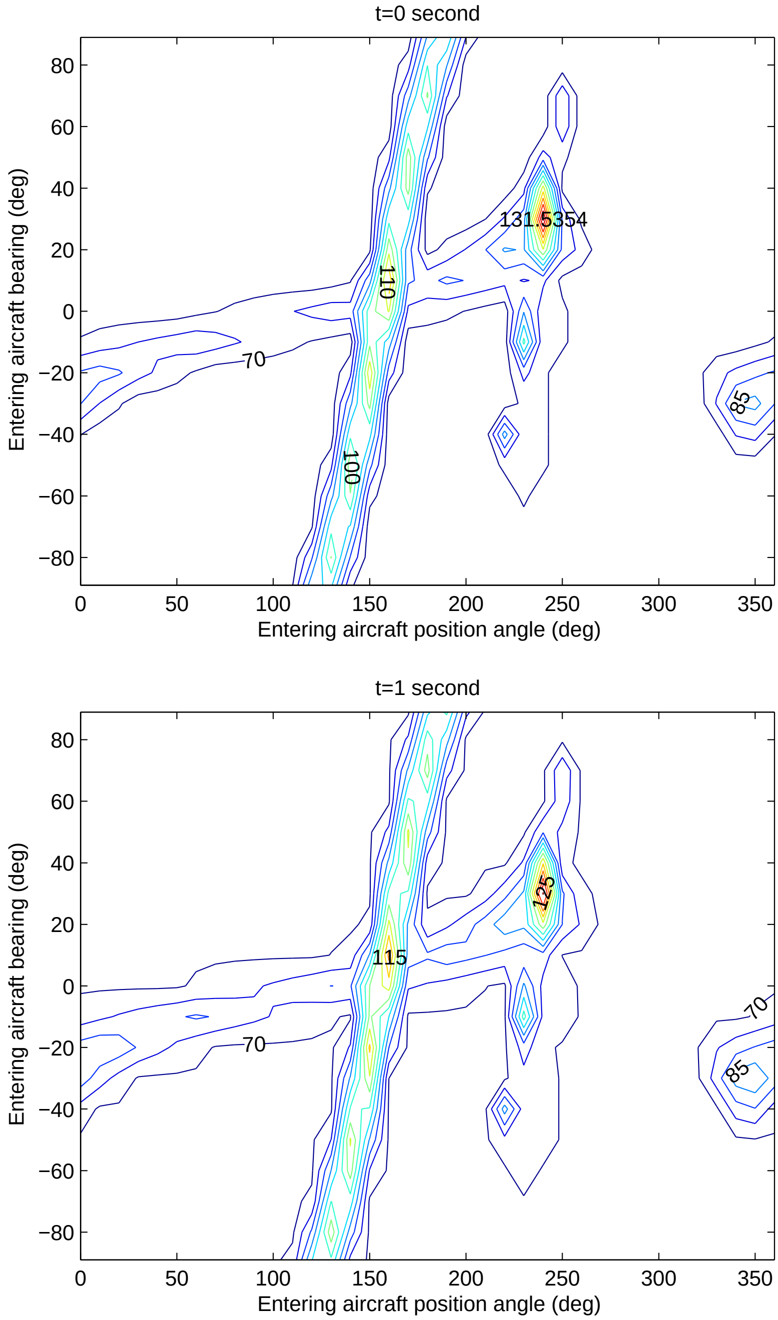


Figure 27: Complexity map for the traffic situation in Fig. 26 at current time and after 1 second

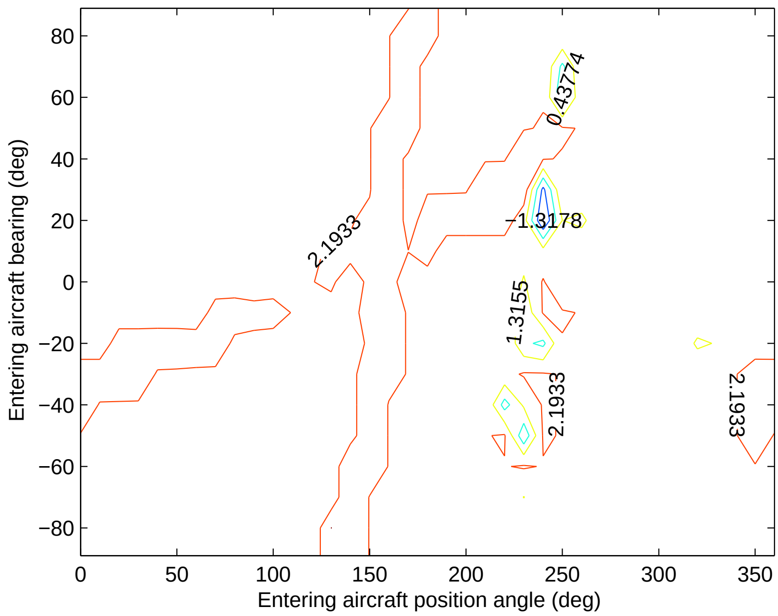


Figure 28: Difference between complexity maps for high difference probability of 99%.

We now need to understand how this number, i.e., 12.52°, is distributed over the complexity map. As an example, consider the traffic situation at current time and its configuration after 60 seconds shown in Fig. 29, and their complexity maps (Fig. 30). Their difference metric(Eq. 8) was computed to be 11.62°.

5.2.3 Time Evolution of a Complexity Map with a Higher Traffic Density

Although the traffic density used in the previous sections is much higher than the usual traffic density in the current air traffic management system, it is worth investigating the time evolution of a complexity map with a higher traffic density. In this section, the traffic situation in Sector B in Chapter 4 will be investigated.

The traffic situation in Sector B and its complexity map is given in Fig. 32. The time evolution of traffic situation in Sector B and the corresponding complexity maps over 30 seconds are shown in Fig. 33 and Fig. 34 at 5 second intervals.

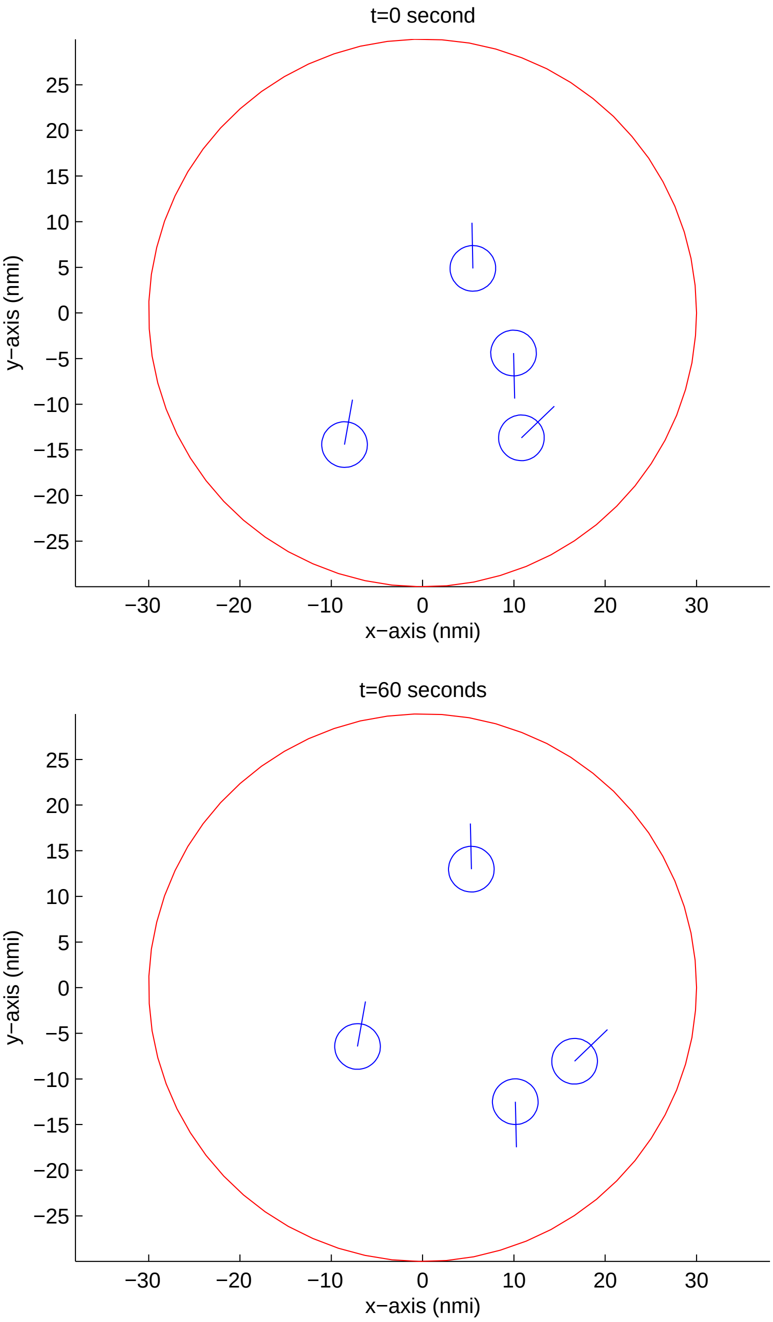


Figure 29: Traffic situation at current time and after 60 seconds

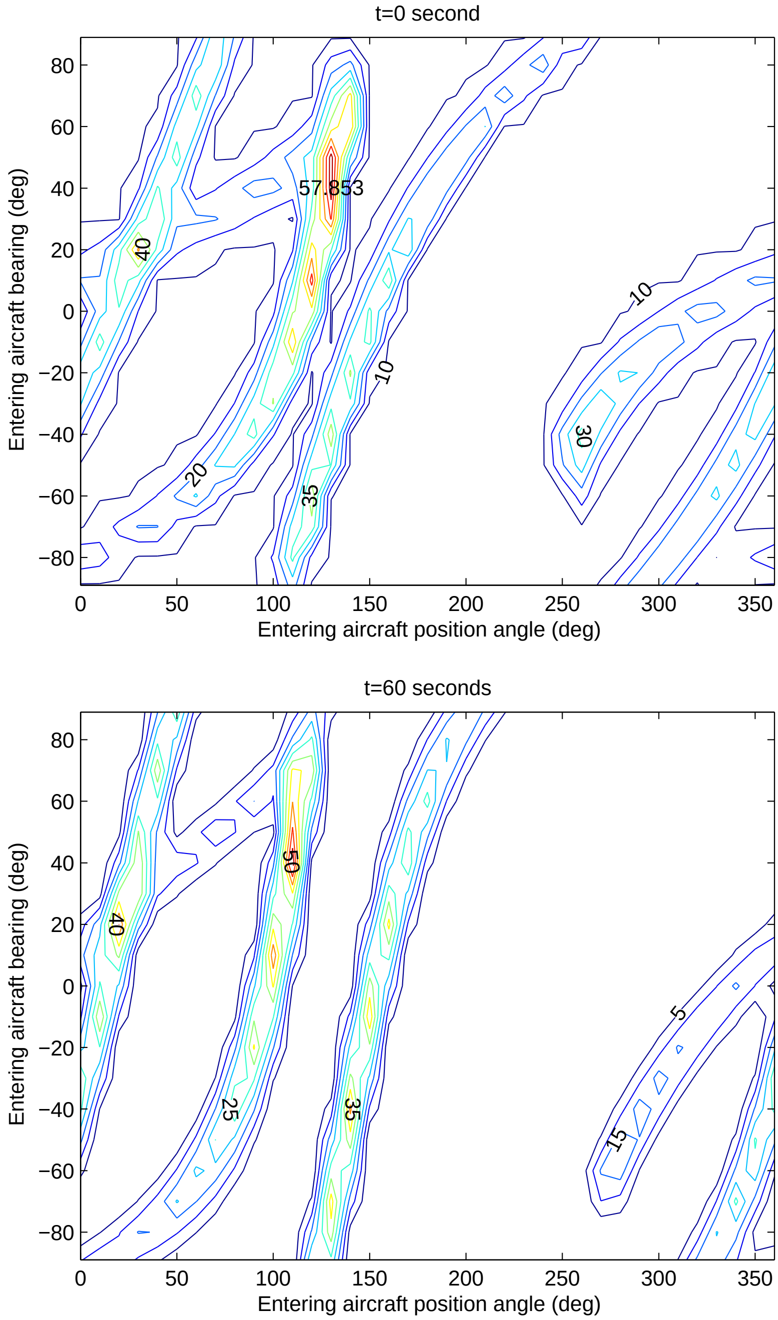


Figure 30: Complexity map for the traffic situation in Fig. 29 at current time and after 60 seconds

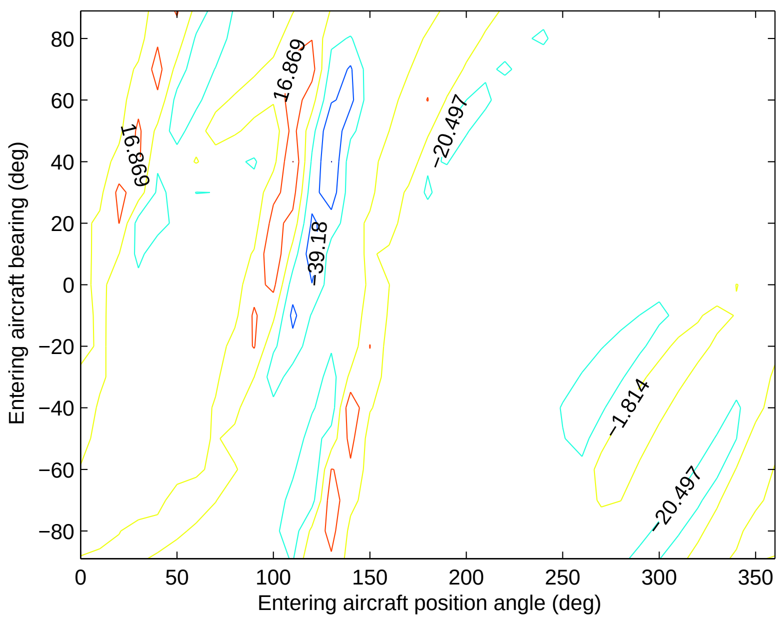


Figure 31: Difference in complexity between current time and 60 seconds later

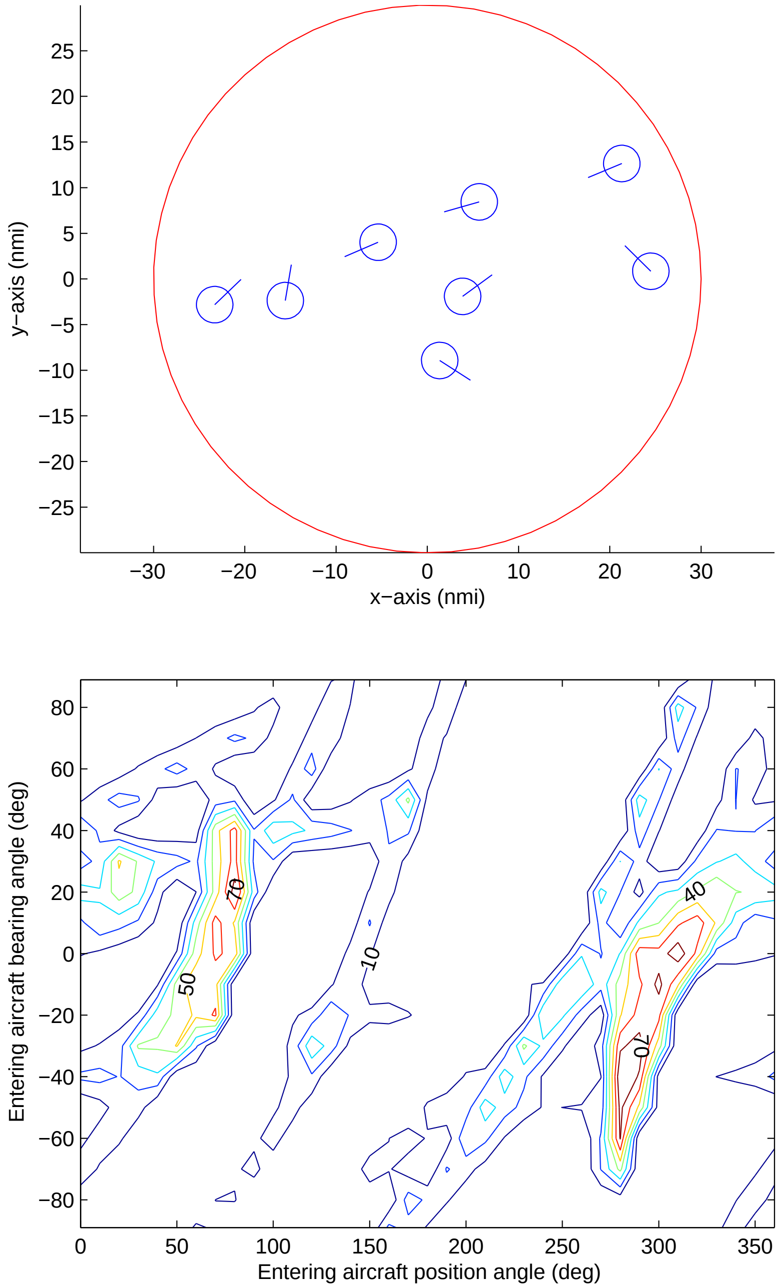


Figure 32: Traffic situation in Sector B at current time and its complexity map

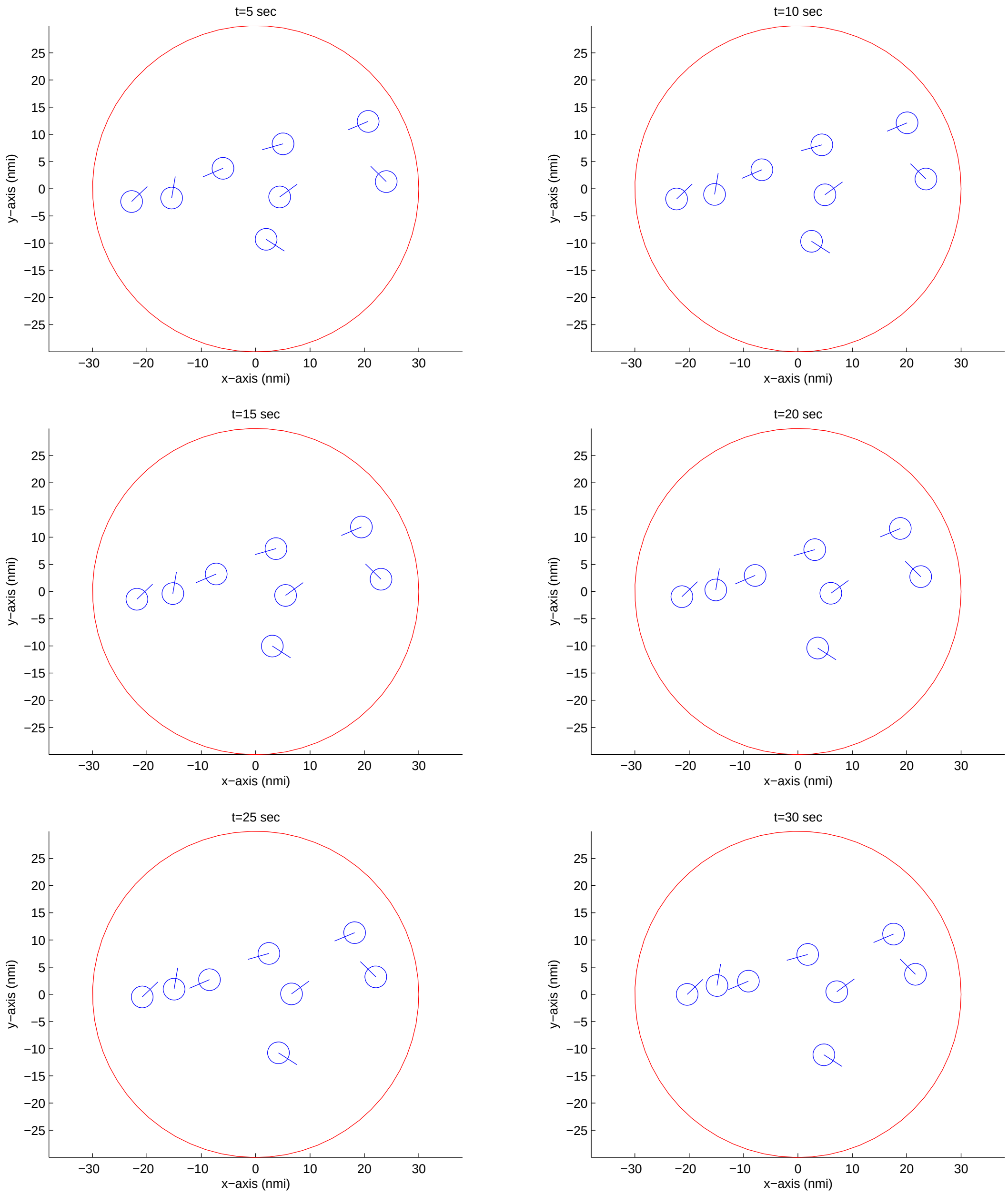


Figure 33: The time evolution of the traffic situation in Sector B over 30 seconds

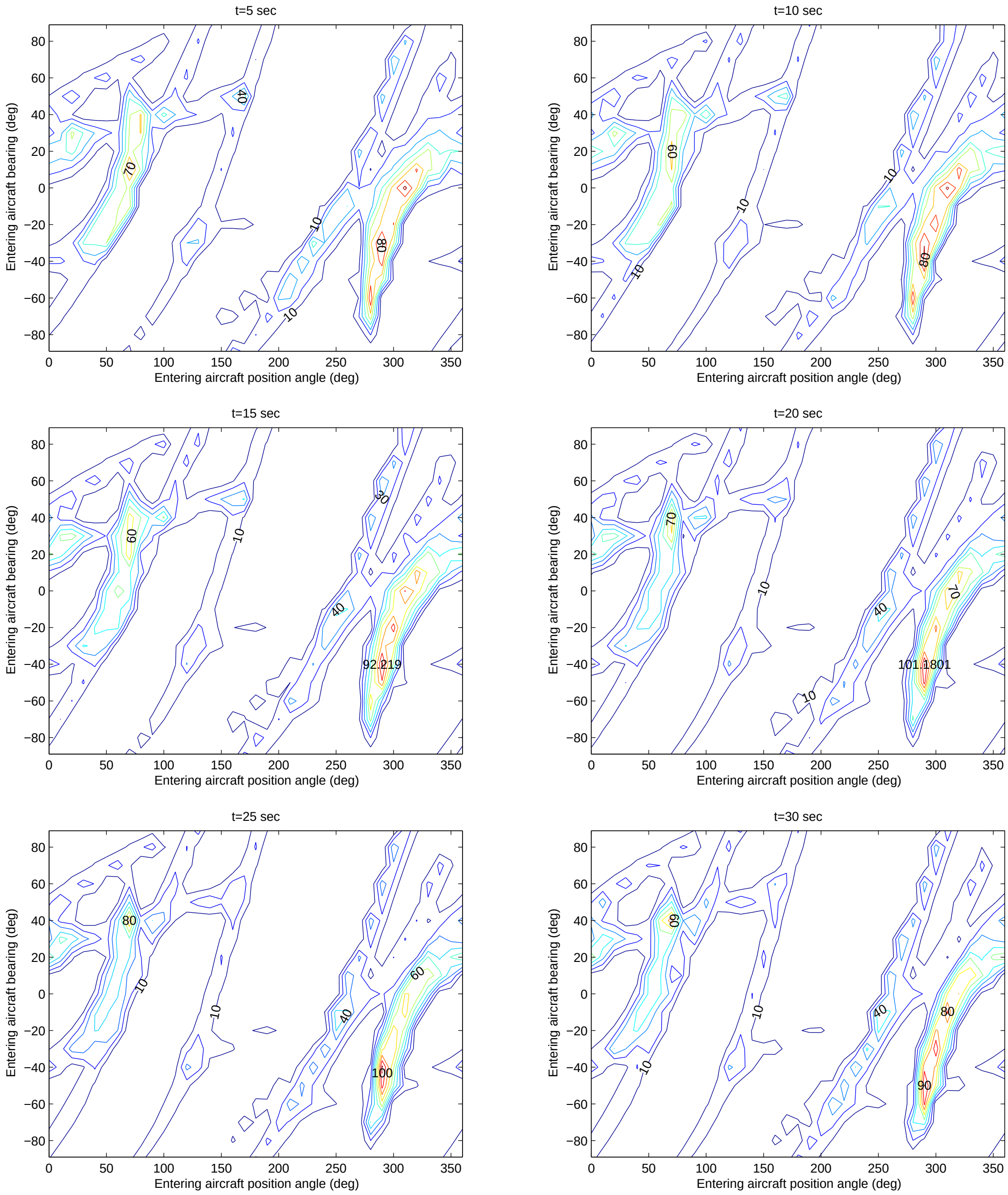


Figure 34: Complexity maps for Sector B over 30 seconds (Fig. 33)

CHAPTER VI

APPLICATIONS TO CURRENT & FUTURE TRAFFIC FLOW MANAGEMENT CONCEPTS

This chapter discusses applying our method for assessing airspace complexity to current and future traffic flow management concepts. Among advanced Traffic Management concepts that have been suggested, the multi-sector (or center) traffic flow management concepts have drawn attention, and their benefits have been previously identified. [57, 65, 14, 25] The multi-sector traffic management concept stresses cooperation between controllers in different sectors to best use airspace capacity. Therefore, controllers need to capture how incoming traffic from other airspace impacts their airspace, and the method proposed in this thesis can provide this capability, as discussed in the following sections

6.1 Airspace Restriction

Air traffic flow managers restrict incoming traffic flows into an airspace based on its current and future traffic configurations, i.e., airspace complexity. However, since each airspace receives traffic from many others, the most effective restriction is not easily identified. The current practice is based on intuition or historical data and often results in over-restriction or under-restriction.[30]

Because a complexity map provides detailed information about airspace response to any entering aircraft, air traffic flow managers can easily identify the problematic part of the sector boundary that should be closed. For example, consider Sectors A and B as shown in Fig. 37. Based on the predicted traffic situation in Sector B, the average complexity map for Sector B over the next 30 seconds is computed. The complexity map of Sector B with its instantaneous traffic situation and the average complexity map of Sector B over 30 seconds are presented in Fig. 35. The difference between these maps are also presented in Fig. 36.

From this complexity map, air traffic flow managers can easily identify a range of entering aircraft which require high control activity inside Sector B. Assume that, based on this average complexity map, air traffic managers decide to restrict the traffic flows into Sector B through the sector boundary corresponding to aircraft entering with position angles between 290° and 360° (as shown in Fig. 37), because an entering aircraft coming into Sector B through this part of the boundary will require high control activity.

Such restrictions on the incoming traffic flows impact the complexity of the upstream airspace. We assumed that Sector A is one of the adjacent sectors of Sector B, so that a part of the Sector A's boundary is closed as shown in Fig. 37. Therefore, aircraft inside the Sector A are not allowed to exit through that part of the sector boundary. The complexity of Sector A will be affected by this boundary constraint. To show how the complexity map for Sector A will be affected by this boundary constraint, let us consider the current traffic situation inside Sector A as shown in Fig. 38. Note that this traffic is not conflicting with this boundary closure constraint since their routes should be already adjusted to meet this constraint. However the complexity of this traffic will be affected since its response to a disturbance is limited by this boundary closure. The complexity map for Sector A with this partially closed boundary is shown in Fig. 39. Comparing this complexity map with the complexity map without partial sector boundary closure as shown in Fig. 38, the control activity increases but the affected areas are limited in range. Difference between complexity maps with and without this boundary closure is given in Fig. 40 This information helps the air traffic flow manager to identify how their decision will affect other airspaces and decide whether or not to partially close this segment of the boundary

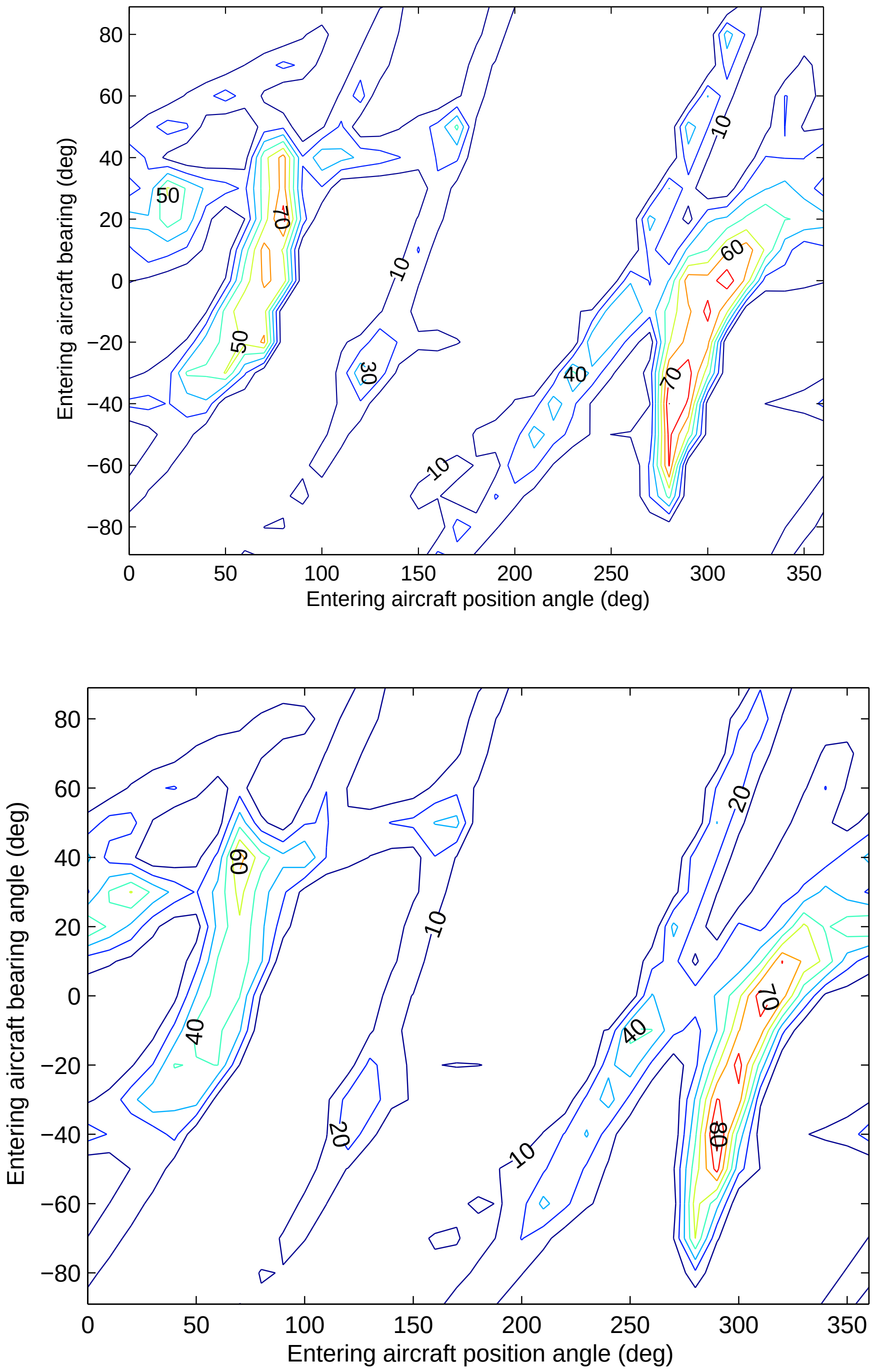


Figure 35: Complexity map for Sector B at instant in time and its average complexity map over the next 30 seconds

between Sector A and B. For example, the air traffic flow managers might be able to decide to close this part of the sector boundary since the boundary closure does not increase the complexity of the adjacent Sector A.

This example illustrates capabilities of the proposed method by which air traffic flow managers can identify overly demanding traffic situations and determine the effective flow restrictions. The proposed method of a complexity map also allows air traffic managers to assess how their sector closure decisions will affect the complexity of the adjacent airspace. Detailed information about how a particular change in one airspace affects the complexity of other airspace is necessary for the cooperation between multiple airspace, and the proposed method can provide this capability.

Although this section focused on the traffic flow management, quantifying the degree of increased complexity caused by partial closure of a sector's boundary is also necessary for decision makers in the dynamic airspace configuration. For example, in the dynamic airspace configuration, the large volumes of airspace are regularly restricted by airspace managers for other reasons such as military or space launch operations[13], and airspace managers need to assess how their sector closing decisions will affect the complexity of other sectors.

6.2 Traffic Flow Management in Convective Weather

As mentioned before, convective weather is the leading cause of delays in the National Airspace System,[48, 33, 28] and it is important to assess how convective weather affects airspace complexity. In this section, it will be shown that the proposed method can incorporate convective weather scenario and allow air traffic flow managers to identify the effective way to manage traffic flows over multiple sectors in the presence of convective weather.

For example, consider the airspace, called Sector C, in Fig. 41. Its complexity map is also shown in Fig. 41. Now assume that convective weather exists in Sector

C. Convective weather is modeled as a circular 'closed' region in the airspace as shown in Fig. 42, and the traffic inside the sector is not allowed to go through it. We assume that the current traffic in Sector C is not conflicting with this convective weather constraint, but the complexity of this traffic will be affected since its response to a disturbance is limited by this constraint. The complexity map for Sector C with this convective weather is also shown in Fig. 42. Comparing the complexity maps with and without this convective weather, the affected areas for the traffic situation in Sector C by this convective weather scenario are very limited. Difference between complexity maps with and without this convective weather is given in Fig. 43. However, the control activity for entering aircraft with position angles between 230° and 250° significantly increased over a wide range of entering aircraft bearing angles. Therefore, we assume that air traffic flow managers decided to reroute the traffic flow into the Sector C through this part of the sector boundary, as shown in Fig. 44. Based on the complexity maps of Sector C and its adjacent sectors (Sectors D and E), air traffic flow managers can identify the most efficient way to reroute traffic flows. If rerouting the traffic flow through Sector D requires high control activity in Sector D, air traffic flow managers might need to reroute the traffic flow through Sector E instead. Furthermore, if rerouting the traffic flow through Sector E also requires high control activity in Sector E, air traffic flow managers might even decide to keep the original route through Sector C.

6.3 Co-operative Air Traffic Management

Co-operative Air Traffic Management (CO-ATM) is a concept which is currently being investigated for a gradual transition from the current air traffic management system to the next generation air traffic management system.[53] In the CO-ATM environment, conventional aircraft are managed by sector controllers in the traditional manner, and aircraft equipped with advanced avionics, managed by area controllers, strategically determine their routes to fully utilize the available airspace capacity. Flight crews in the equipped aircraft request their preferred trajectories, and area controllers determine whether or not to approve their requests. Thus, area controllers need to identify how each sector (or any local area of airspace) can suffer from the entrance of those equipped aircraft. Further, area controllers can negotiate trajectories of the equipped aircraft by identifying the available capacity of other sectors. A complexity map can provide this information to area controllers by showing how each airspace responds to different entering of equipped aircraft. For example, consider a particular equipped aircraft who wants to enter Sector B after 120 seconds, as denoted by "Original plan" in Fig. 46. The traffic inside Sector B after 120 seconds is expected as shown in Fig. 46, and its complexity map is given in Fig. 47. From this complexity map, area controllers can identify that the required control activity resulting from the entrance of the equipped aircraft will be very high (i.e. Sector B will seriously suffer). However, area controllers also can notice that the traffic inside Sector A (one of adjacent sectors of Sector B) after 120 seconds is expected as shown in Fig. 46, and its complexity map (Fig. 48) shows that Sector A can accept new aircraft coming through a certain part of its boundary (as shown in the red line in Fig. 46) without much difficulty. Therefore, area controllers can negotiate with the equipped aircraft to modify its trajectory as shown in "Alternative plan" in Fig. 46. Further, area controllers can check how the complexity of Sector B changes over time and might ask the equipped aircraft to speed up or down to adjust its entering time into Sector B so that its entrance into Sector B would be less disruptive.

This example demonstrates capabilities of the proposed approach with which an airspace manager can identify the best way to use the available capacities of the sectors and to negotiate trajectories of the equipped aircraft. While the first application discussed in this chapter, i.e., airspace restriction, utilizes a complexity map to identify over-demand situation and effectively restrict traffic flows, CO-ATM focuses more on identifying how to strategically manage the trajectories of equipped aircraft to fully utilize airspace's capacity.

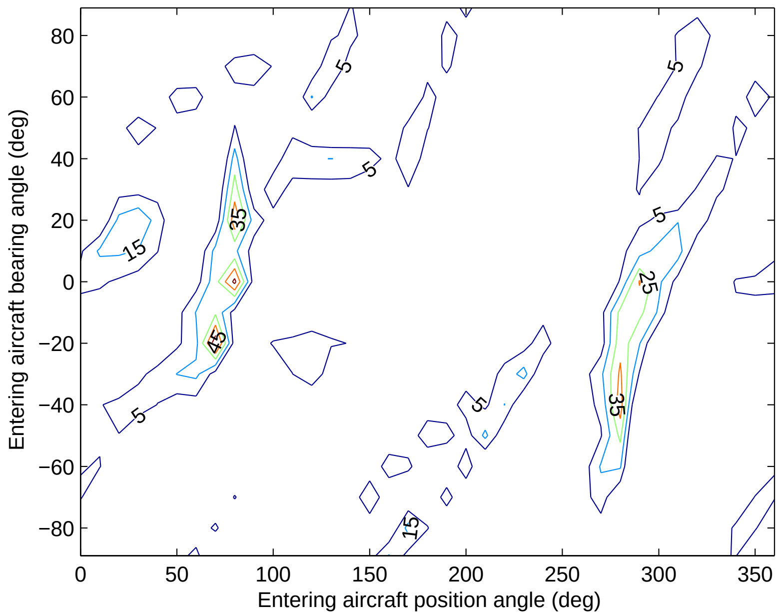


Figure 36: Difference between complexity maps in Fig. 35

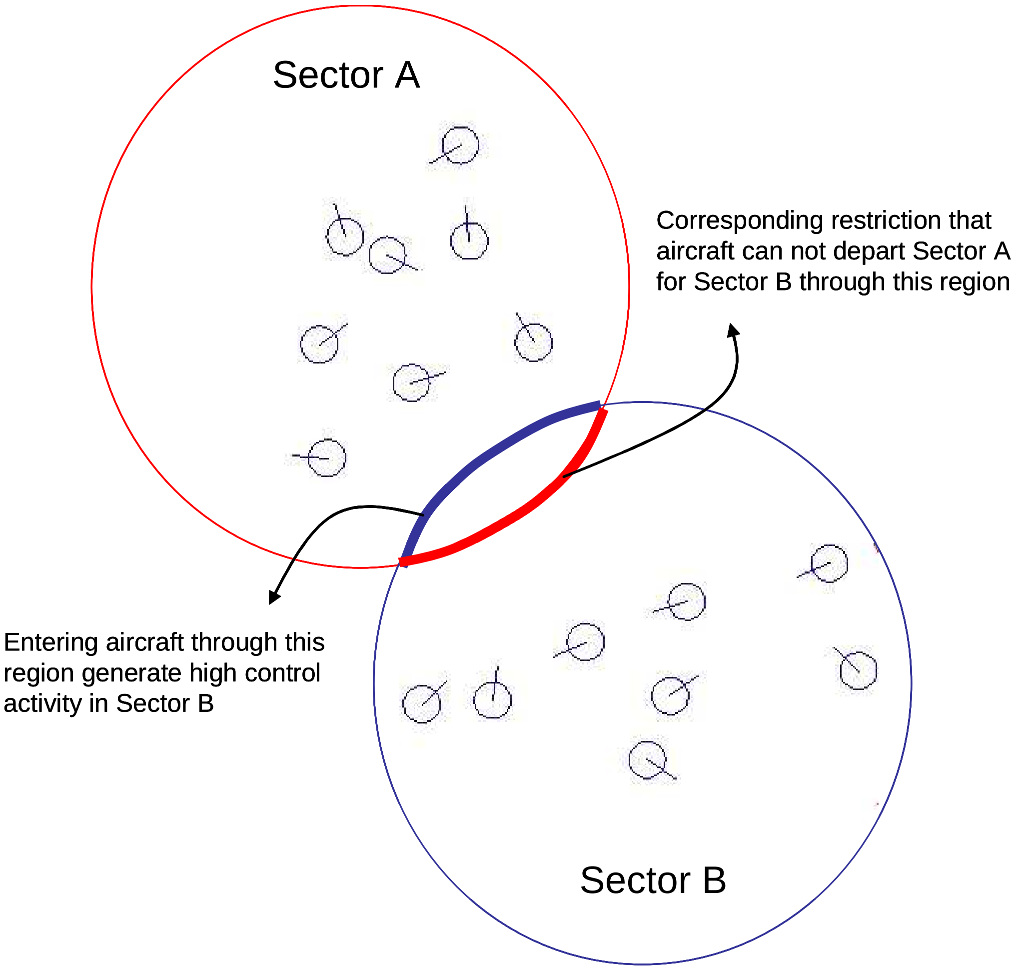


Figure 37: Partial closure of the sector boundary

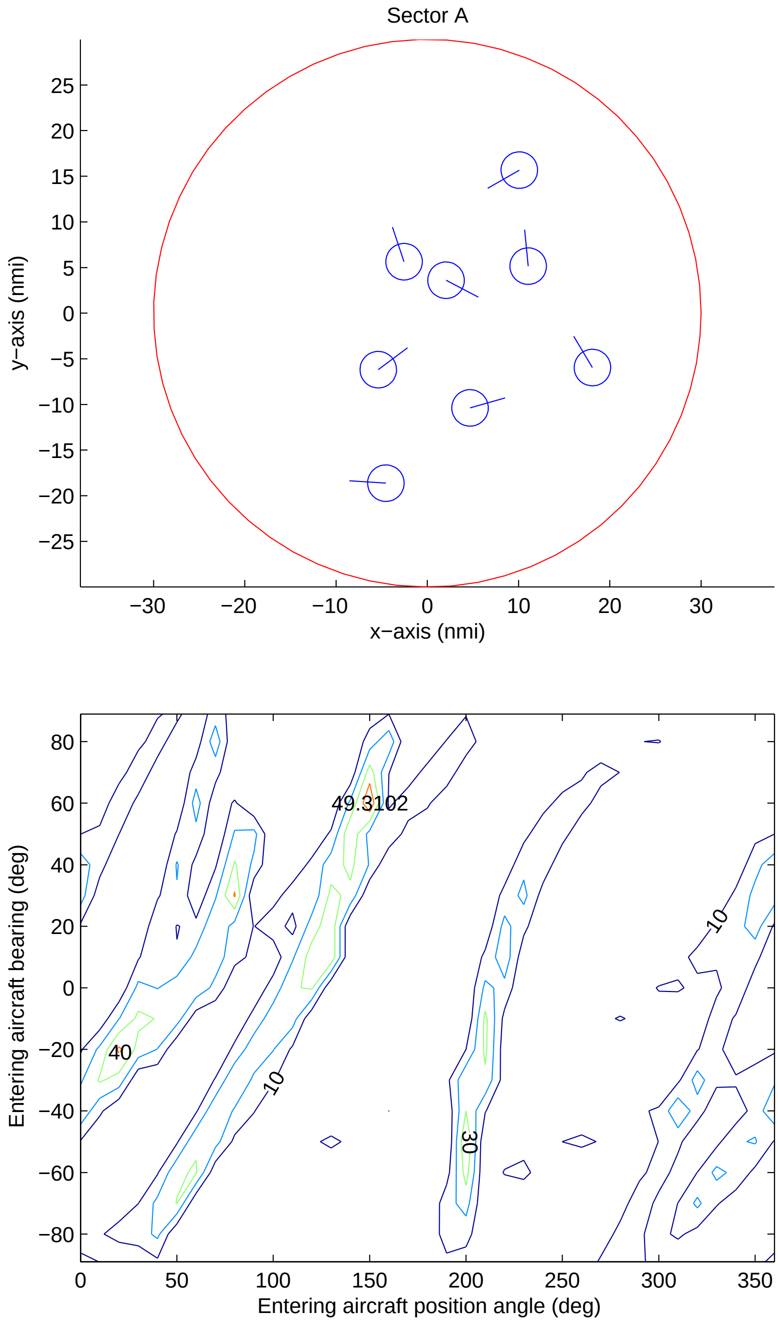


Figure 38: The current instantaneous traffic situation in Sector A and its complexity map

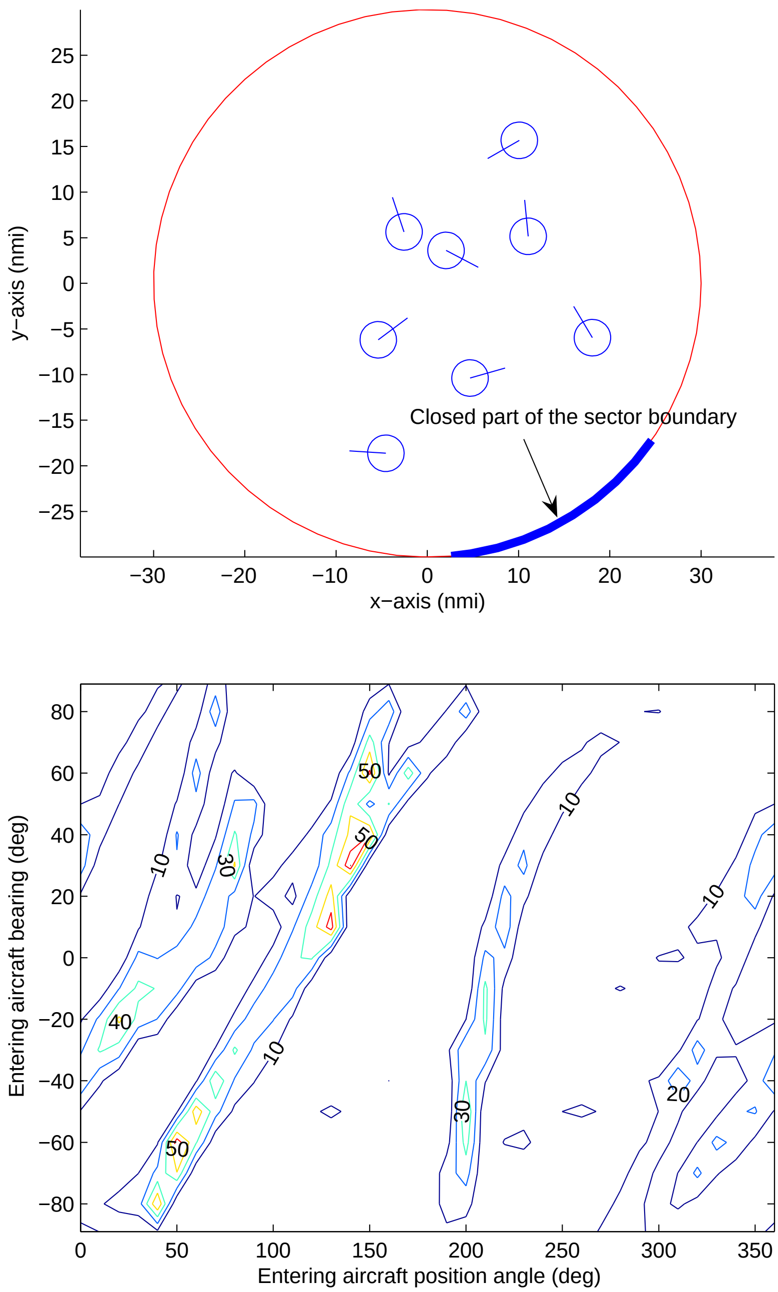


Figure 39: Complexity map for Sector A with partially closed boundary

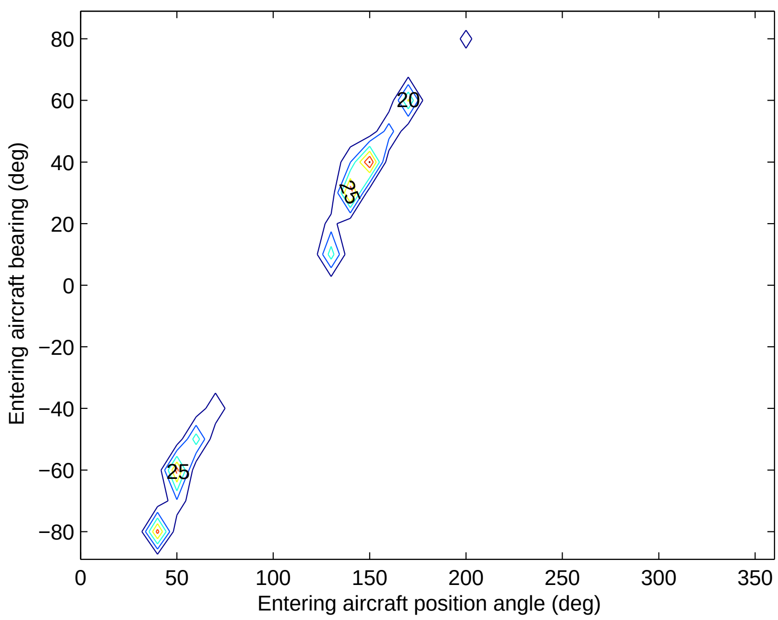


Figure 40: Difference in complexity maps for Sector A with and without partially closed boundary

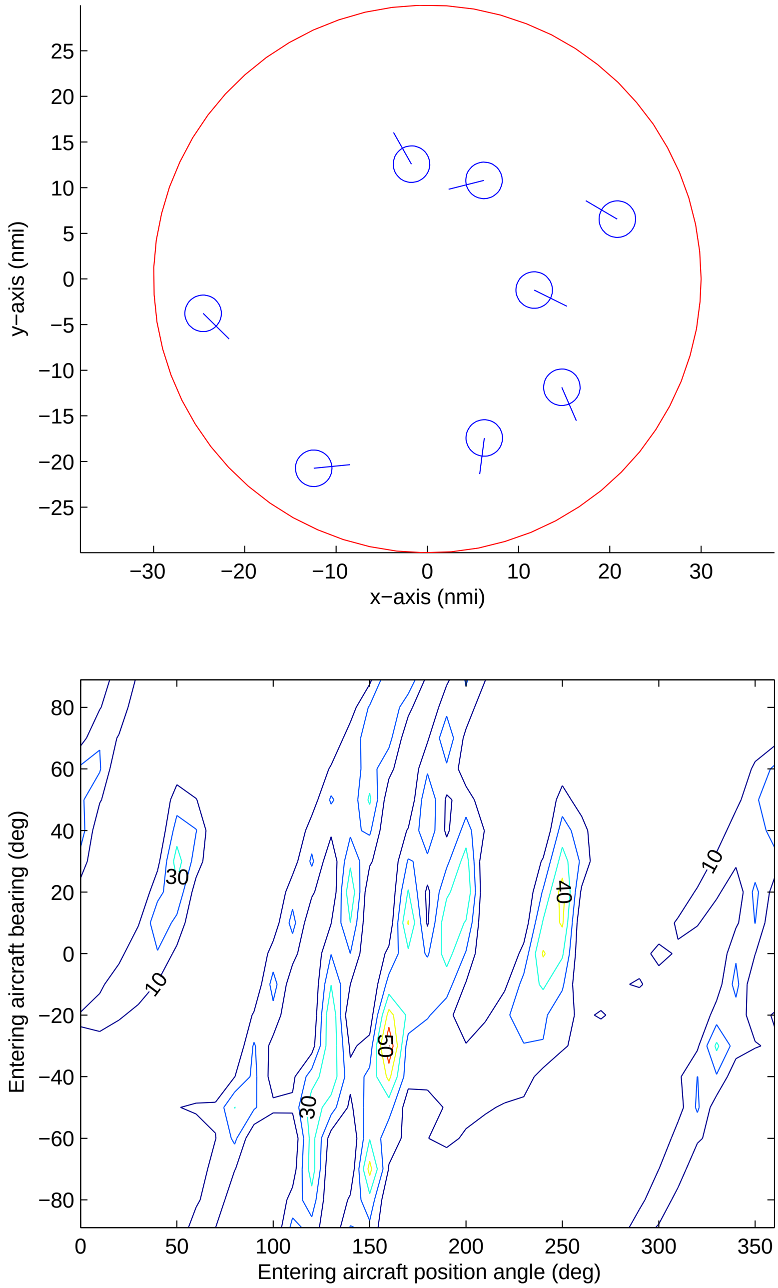


Figure 41: Traffic situation in Sector C and its complexity map

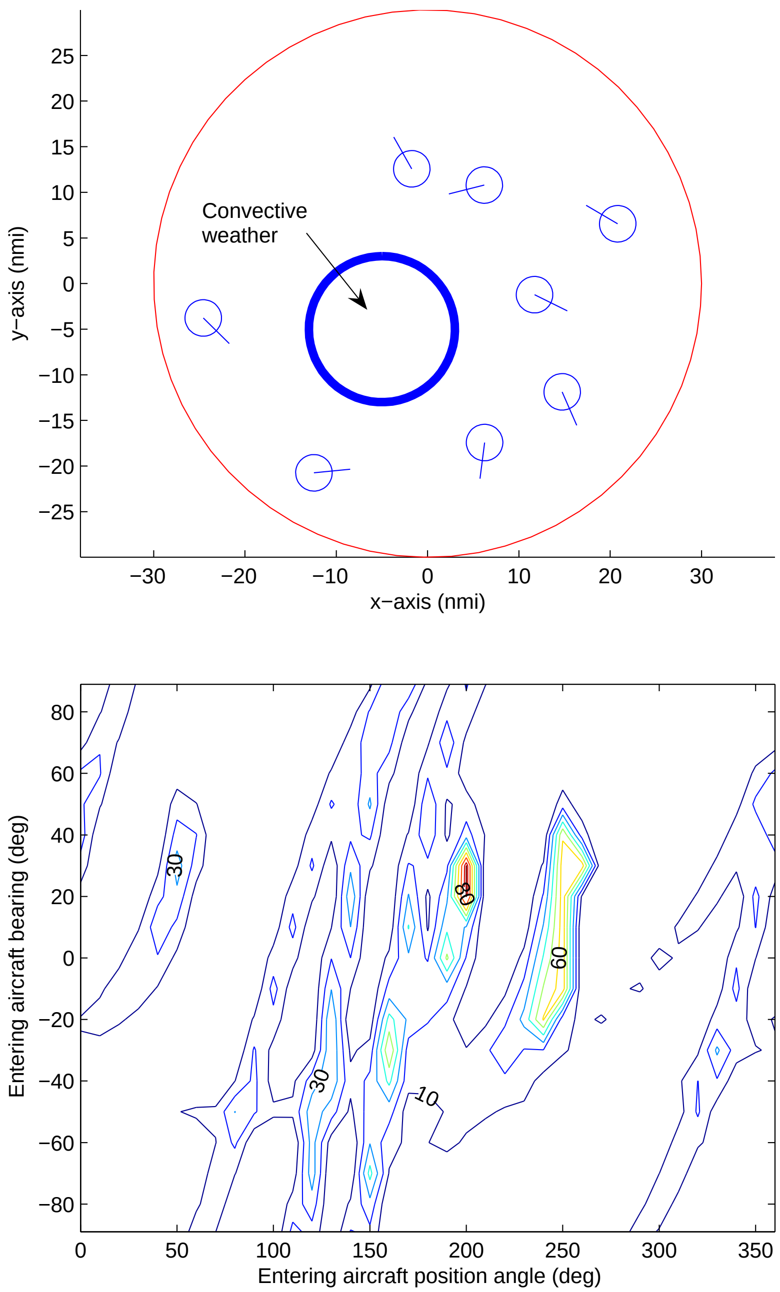


Figure 42: Traffic situation in Sector C with convective weather and its complexity map

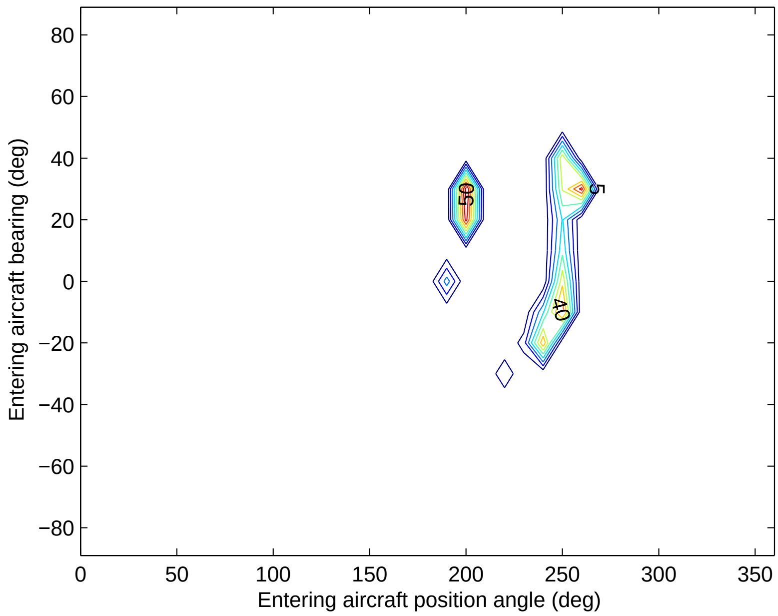


Figure 43: Difference in complexity maps for Sector C with and without convective weather

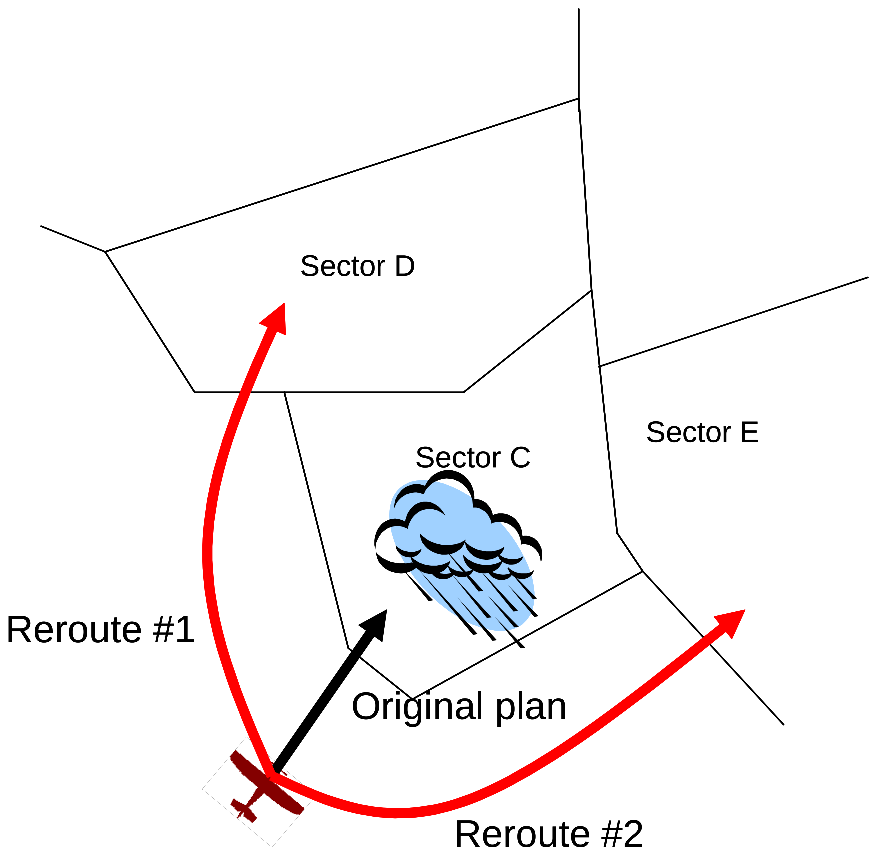


Figure 44: Application to rerouting traffic flows in convective weather

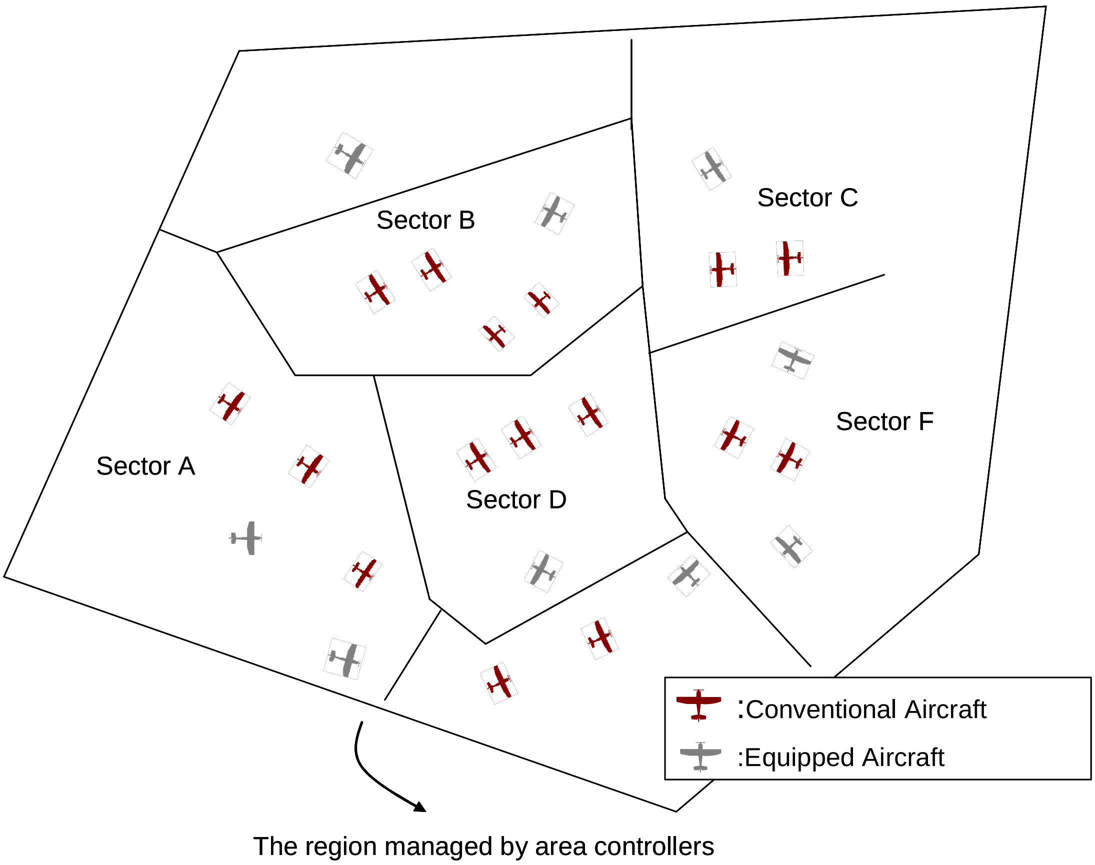


Figure 45: Co-Operative Air Traffic Management :Ref.[53]

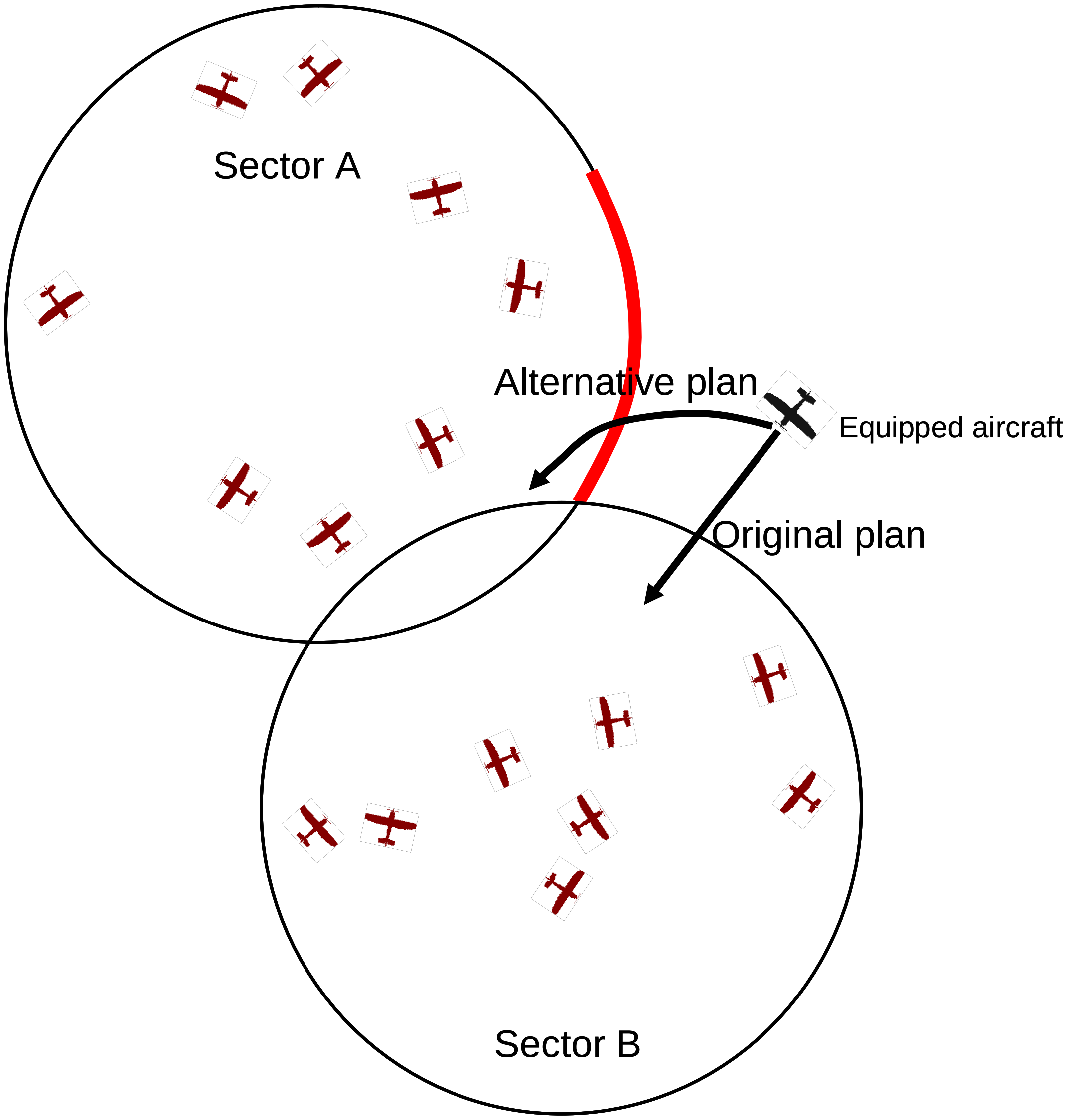


Figure 46: Application to CO-Operative Air Traffic Management

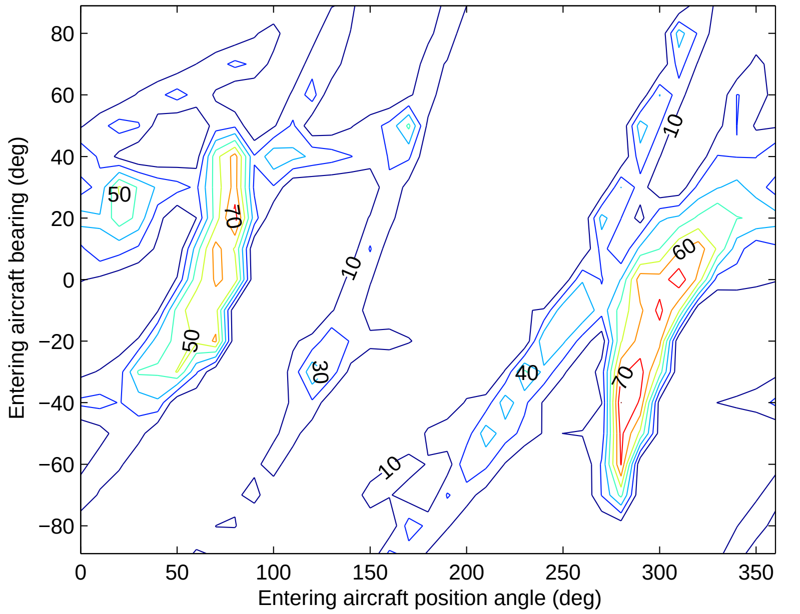


Figure 47: Complexity map for Sector B after 120 seconds

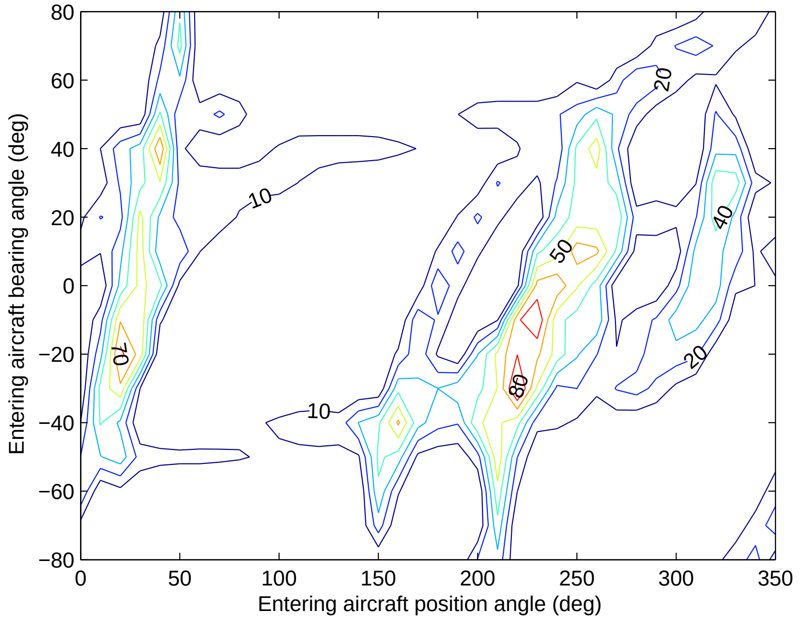


Figure 48: Complexity map for Sector A after 120 seconds

CHAPTER VII

THE INDUCED COMPLEXITY OF AN AIR ROUTE STRUCTURE

So far, we have described the complexity of a given traffic situation by computing the required control activity for a set of disturbances. However, the study of airspace complexity can also be applied to airspace design problems. For example, sector boundaries or air route structures can be designed to equally distribute complexity across multiple sectors.[69, 15, 44, 22] The complexity of a given airspace structure, e.g., sector boundaries or air route structures, can be defined by its complexity over a wide range of traffic situations.

In this chapter, the induced complexity of a given air route structure will be described by its responses to disturbances over a wide range of traffic situations within the route structure. The traffic density used in the experiments in this chapter is five aircraft in a 30 nautical miles radius circular airspace, which corresponds to approximately 18 aircraft per 10000 Nm^2. This traffic density is much higher than the usual traffic density in the current air traffic management system. Although this chapter is to show the applicability of the proposed method to airspace design problems, some results of analysis will be presented demonstrating the insights it provides.

7.1 No Air Route Structure

Before investigating the induced complexity of an air route structure, we investigated the case of no air route structure. For each simulation, a traffic situation with five aircraft were randomly placed with a uniform distribution in a 30 nautical miles radius circular airspace, and the complexity map for each traffic situation was computed. The separation standard between aircraft was 5 nautical miles, and 800 runs were performed.

The average complexity map over a wide range of traffic situations with no air route structure should be uniform for each entering aircraft position angle, i.e., average control activity over a wide range of traffic situations should be independent to entering aircraft position angle. Thus, for each entering aircraft bearing angle, the average control activity over entering aircraft position angles was computed, and the one dimensional induced complexity map was computed, as shown in Fig. 49. The map is symmetric along the entering aircraft bearing angle due to the circular sector boundary. As entering aircraft bearing goes to zero, i.e., entering aircraft going toward the center of the airspace, the required control activity increases. The average control activity is 21.03°. It should be noted that the control activity comes both from conflicts among existing aircraft and from conflicts arising from the entering aircraft. For each simulation, i.e., each traffic situation, the control activity due to conflicts among existing aircraft can be found by the minimum control activity over all entering aircraft. Then the average (over all traffic situations) control activity due to conflicts among existing aircraft was found to be 14.87°.

7.2 The Induced Complexity of an Air Route Structure

7.2.1 One Air Route

A wide range of traffic situations (with five aircraft) in the air route structure, as shown in Fig. 50, was randomly (with a uniform distribution) created, and the corresponding complexity maps for those traffic situations were computed. The separation standard between aircraft was 5 nautical miles, and 100 runs were performed. Then, the complexity map was found as shown in Fig. 50. In fact, this complexity map shows a skew symmetric pattern, as may be expected; note that we can expect a

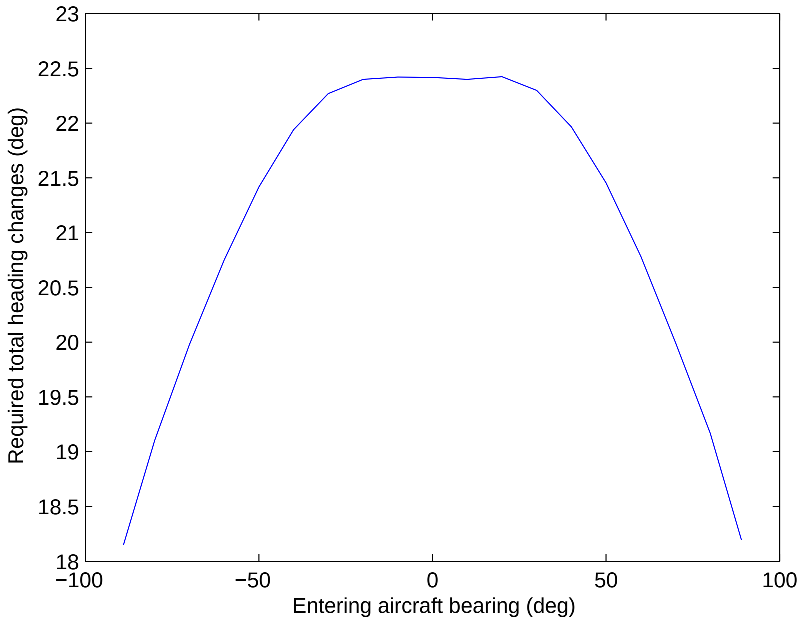


Figure 49: Complexity as a function of entering aircraft bearing

skew symmetric pattern in the induced complexity map since we assume a circular sector boundary. Compared to the case with no air route structure (Fig. 49), the overall required control activity decreased since conflicts among existing aircraft are removed by putting them on the same air route. In fact, the average control activity over all entering aircraft for the induced complexity map shown in Fig. 50 is 6.09°, as opposed to 21.03° for the previous case with no air route structure. However, an entering aircraft with position angles of 90° and bearing of 0° (which corresponds to aircraft entering at the exit location of the air route) will require very significant control activity. The maximum value of the control activity is 149.44° with an air route, as opposed to 23.48° for the no air route structure case shown in Fig. 49.

7.2.2 Two Air Routes

An air route structure with two air routes (Fig. 51) was investigated over a wide range of traffic situations. In each simulation, three aircraft on the route #1 and two aircraft

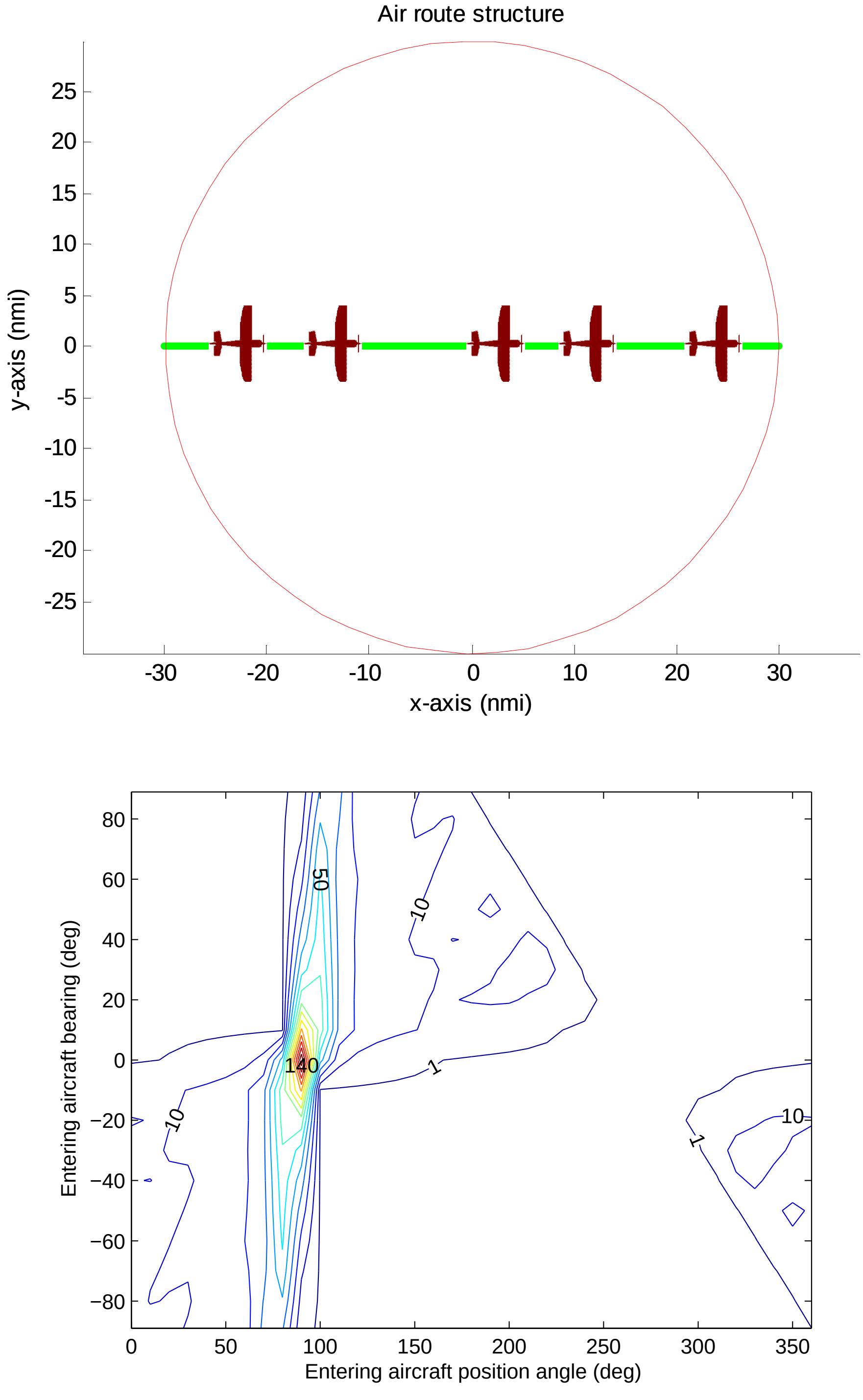


Figure 50: Air route structure with one air route and its induced complexity map

on the route #2 were randomly placed (with a uniform distribution). The separation standard between aircraft was 5 nautical miles, and 100 runs were performed. The corresponding complexity maps for each traffic situation were computed, and the average complexity map was found as shown in Fig. 51. For better visual clarity, a three dimensional map is provided in Fig. 52.

Comparing the complexity map in Fig. 51 (which corresponds to two air routes) with the map in Fig. 50 (which corresponds to one air route), the overall required control activity increased since conflicts among existing aircraft can not be totally removed. In fact, the control activity due to conflicts among existing aircraft in the case of two air routes was found to be 14.74°, which is almost same as the one with no air route structure, i.e., 14.87°. The average control activity over all entering aircraft for two air routes (Fig. 51) is 20.16°, which is almost same as the one for the previous case with no air route structure, i.e., 21.03°. Also the induced complexity map in Fig. 51 still has areas with high control activity around the exit locations of both air routes (which correspond to entering aircraft with position angles between 15° and 45° and between 135° and 165°). Moreover, we can identify other areas with high control activity. For example, entering aircraft with position angles between 250° and 290° will also require a high control activity. Although airspace with the air route structure shown in Fig. 51 can not decrease the complexity (in terms of the average control activity over all entering aircraft) compared to the case of no route structure, it concentrates control activity on a specific range of position angles for entering aircraft; air traffic controllers can focus on those parts of the sector boundary instead of equally distributing their attention around the boundary.

7.2.3 Three Air Routes

An air route structure with three air routes (Fig. 53) was investigated over a wide range of traffic situations. For each simulation, two aircraft on route #1, two aircraft

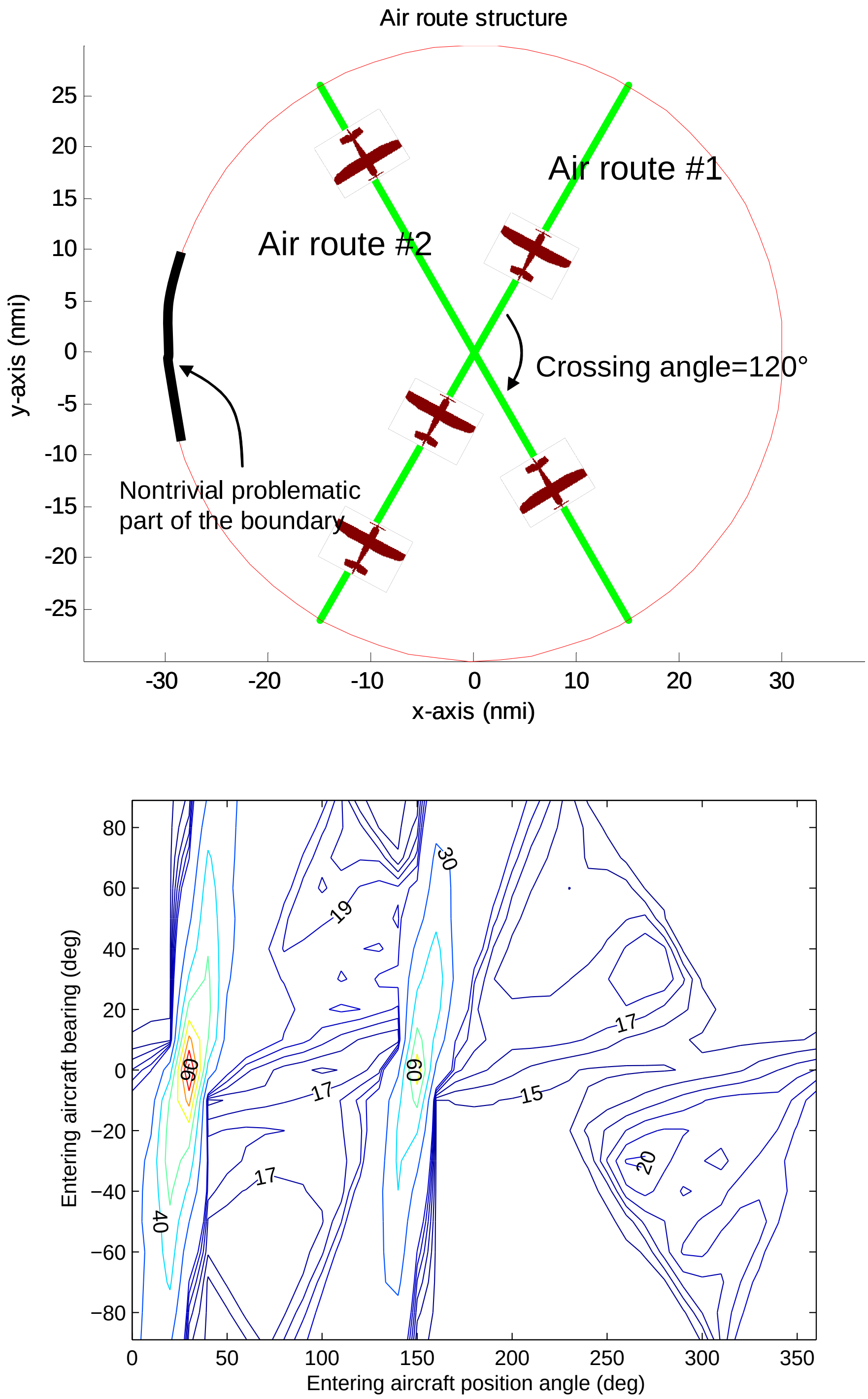


Figure 51: Air route structure (two air routes with crossing angles of 120°) and the averaged over 100 traffic situations

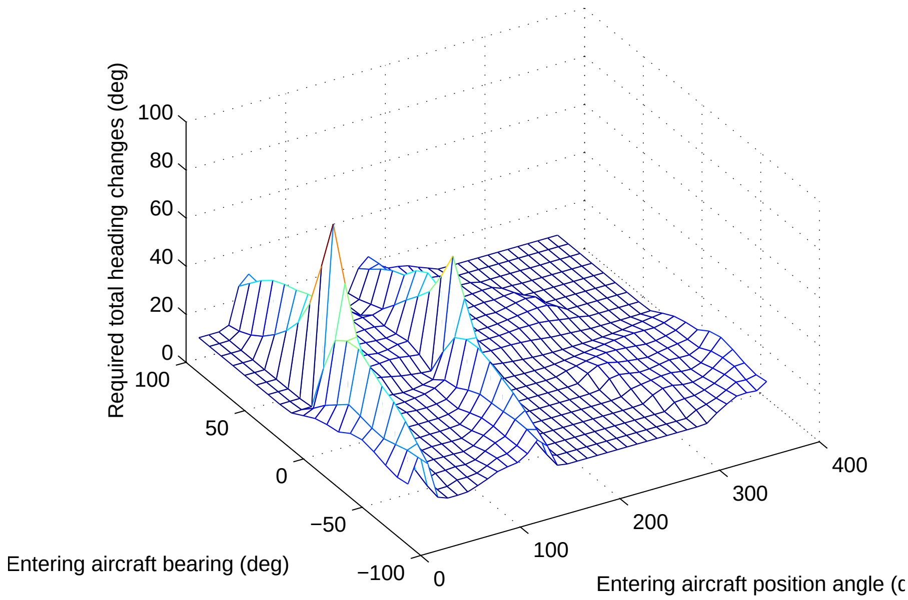


Figure 52: Induced complexity map (3-dimensional) for the case of two air routes with crossing angles of 120°

on route #2 and one aircraft on route #3 were randomly placed (with a uniform distribution). The corresponding complexity map for each traffic situation was computed, and the complexity map was found as shown in Fig. 53. The separation standard between aircraft was 5 nautical miles, and 100 runs were performed.

The average control activity over all entering aircraft for the induced complexity map shown in Fig. 53 was found to be 20.83°, and the control activity due to conflicts among existing aircraft was found to be 15.33°. The induced complexity map in Fig. 53 still has areas with high control activity around the exit locations of air routes. Moreover, we can identify some other areas with high control activity. For example, the induced complexity map shows that entering aircraft with position angles around 270° and bearing angle around ±20° will also require a high control activity. This example shows that the same technique can be applied to a more complex air route structure.

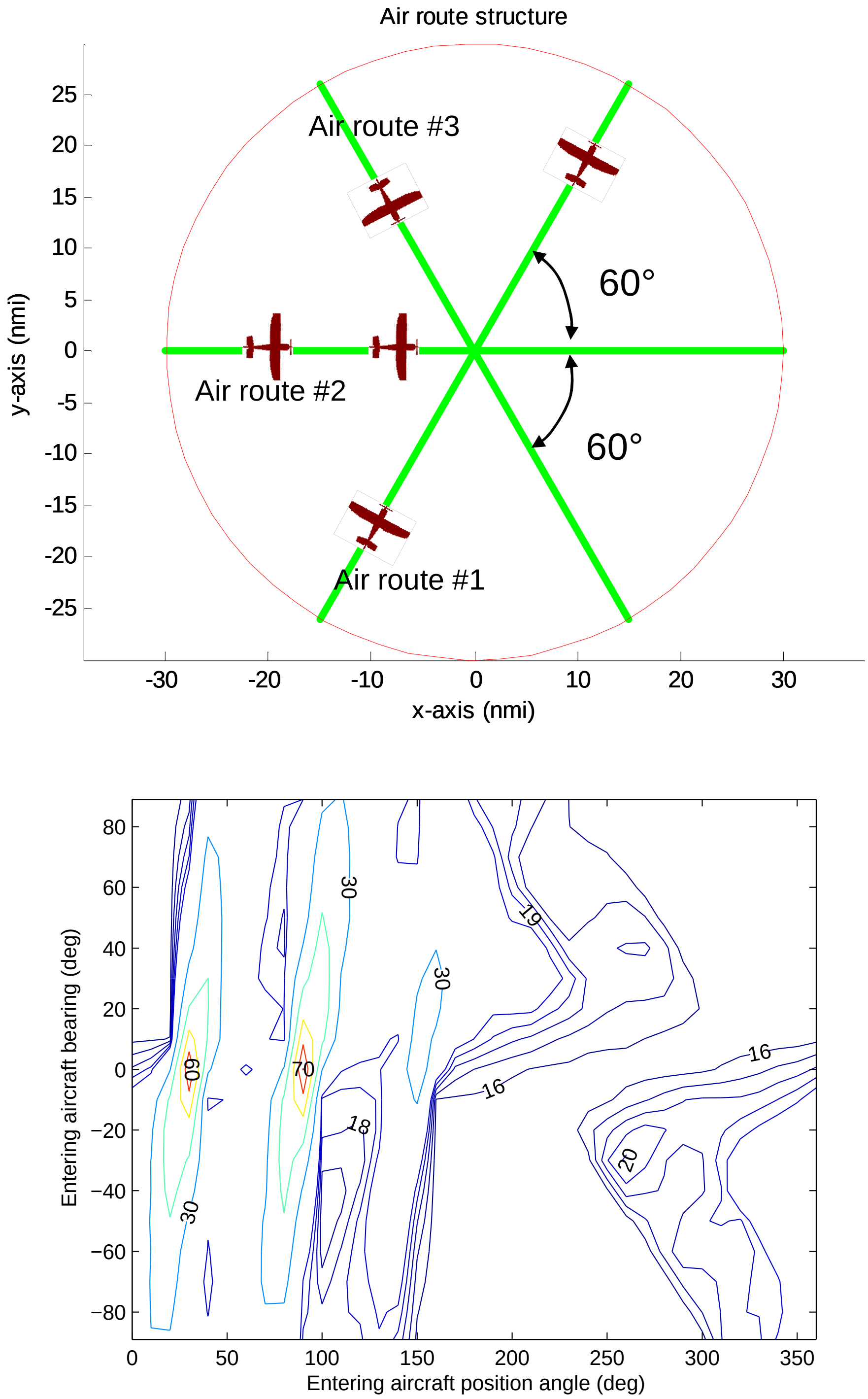


Figure 53: Air route structure with three air routes and its complexity map averaged over 100 traffic situations

7.3 Comparing Air Route Structures

7.3.1 Examples

In this section, the complexities of air route structures were compared. Each air route structure has two air routes with a different crossing angle as shown in Fig. 54. The complexity maps for these air route structures are shown in Fig. 55

Each complexity map (in Fig. 55) has areas with high control activity around exit locations of its air routes. Table 3 shows several important scalar measures of their complexity. The upper part of the table shows the average and maximum control activity over all entering aircraft for each map. As mentioned before, the control activity comes both from conflicts among existing aircraft and from conflicts arising from entering aircraft. The lower part of the table shows the division between these two types of control activity.

These results are charted in Fig. 56 to Fig. 58, which lead us to several observations. First, the average control activity (over all entering aircraft) increases as crossing angle increases, as shown in Fig. 56. Control activity comes both from conflicts among existing aircraft and from conflicts arising from entering aircraft, and these two types of control activity are shown in Fig. 57 and Fig. 58. From these graphs, a larger average control activity for a larger crossing angle is mainly due to conflicts among existing aircraft. In fact, the average number of conflicts among existing aircraft increases as crossing angle increases, as shown in Fig. 59. In a previous study[59], in which an analytic encounter model was used to estimate the number of conflicts given the density of traffic flow rate on each air route, the conflict rate between aircraft increases as a crossing angle between two air routes increases. Fig. 60 shows the conflict rate for different crossing angles using this encounter model for speeds of 560 miles/hour. To compare Fig. 59 with Fig. 60, the Chi-square test was performed. The p-value was computed to be 0.9936, which is high enough to conclude those two distributions are similar.

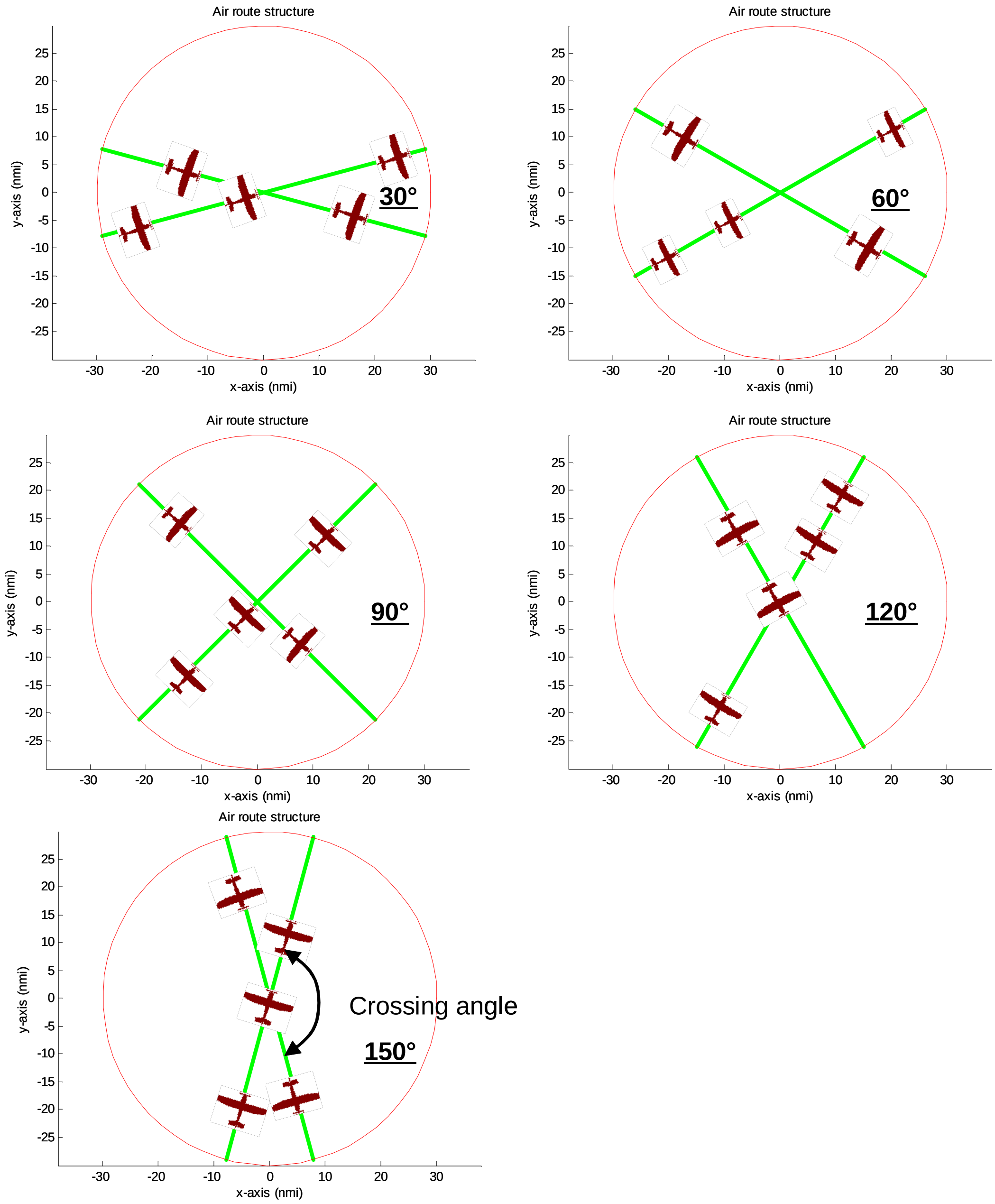


Figure 54: Two air routes with different crossing angles from 30° to 150°

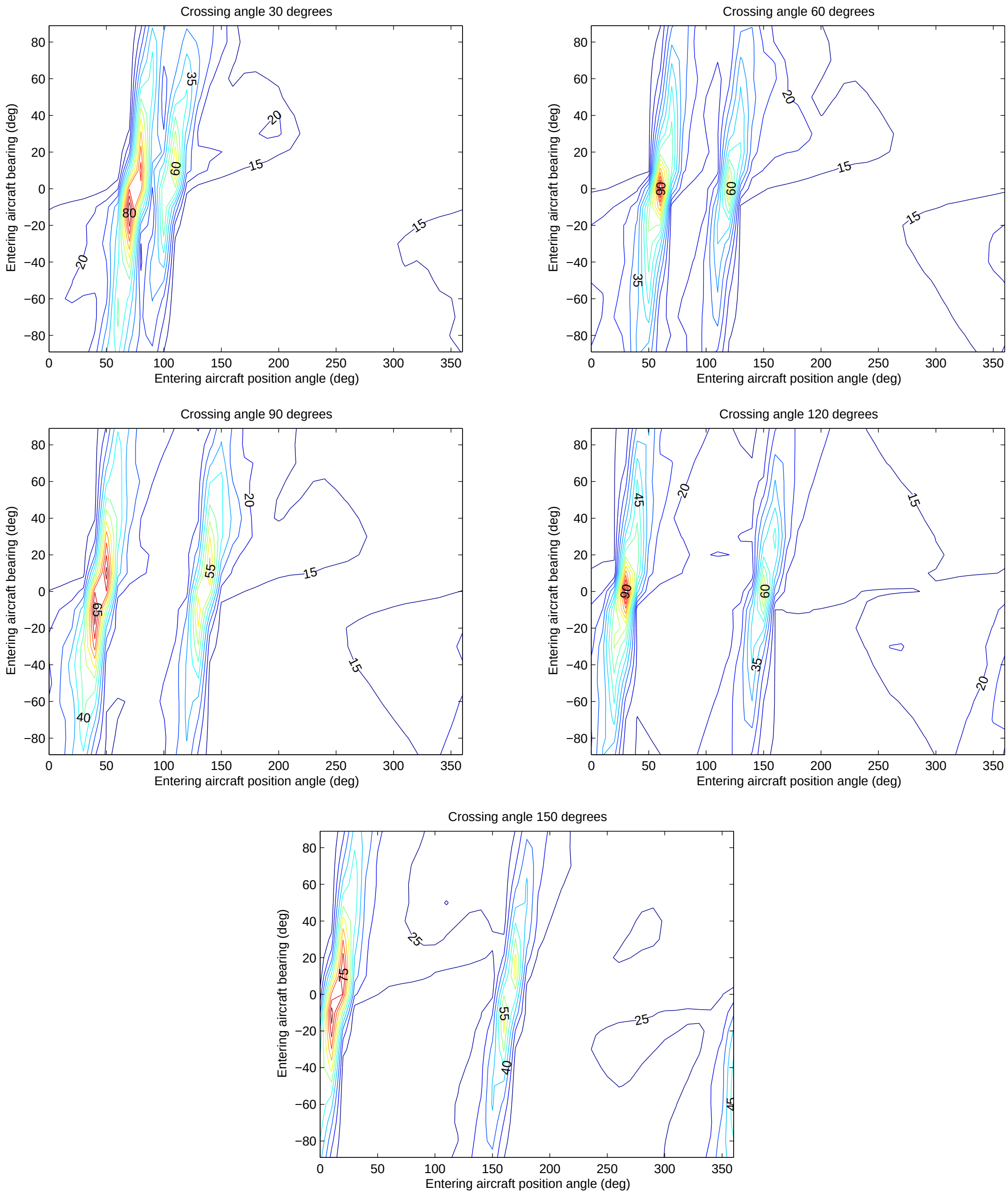


Figure 55: Induced complexity map for the air route structures in Fig. 54

Table 3: Scalar measures of air route complexity (Control activity (E): Control activity due to conflicts among existing aircraft, Control activity (I): Control activity induced by entering aircraft)

	Average control activity(deg)	Maximum control activity(deg)
Crossing angle:30°	15.79	83.62
Crossing angle:60°	18.80	96.10
Crossing angle:90°	18.59	73.96
Crossing angle:120°	20.16	94.17
Crossing angle:150°	26.32	78.45
	Control activity(E)(deg)	**Control activity(I)(deg)**
Crossing angle:30°	10.03	5.76
Crossing angle:60°	13.27	5.53
Crossing angle:90°	13.00	5.59
Crossing angle:120°	14.74	5.42
Crossing angle:150°	21.03	5.29

Another interesting thing is that control activity due to conflicts arising from entering aircraft decreases as a crossing angle increases, as shown in Fig. 58. This result might imply that traffic flows with a shallower crossing angle are difficult to control in the face of disturbances, i.e., entering aircraft, due to geometrical interrelation between aircraft. This agrees with another previous study, in which the stability of traffic flows (i.e., the maximum deviation of aircraft from its nominal air route) decreases as a crossing angle between two air routes decreases.[43] Other previous research[10] also pointed out that the Kolmogorov entropy of traffic flows with a shallower crossing angle is larger than one with a wide crossing angle.

Finally, each data set of different air route structures has significant variance, as shown from Fig. 56 to Fig. 58. Therefore, we need a more rigorous statistical analysis to compare those air route structures, which will be presented in the following section.

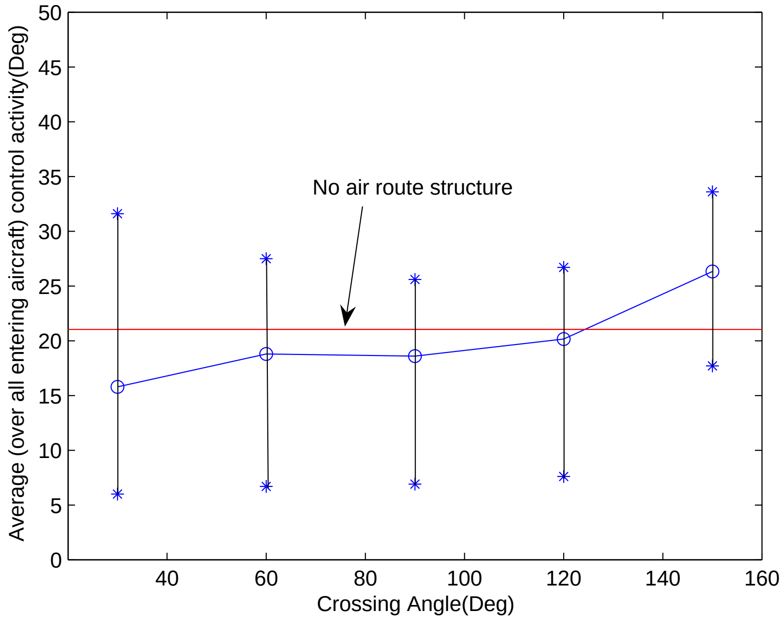


Figure 56: Average control activity (over all entering aircraft) for two air routes by crossing angle (Fig. 55): The upper quartile and lower quartile of each data are denoted by '*'

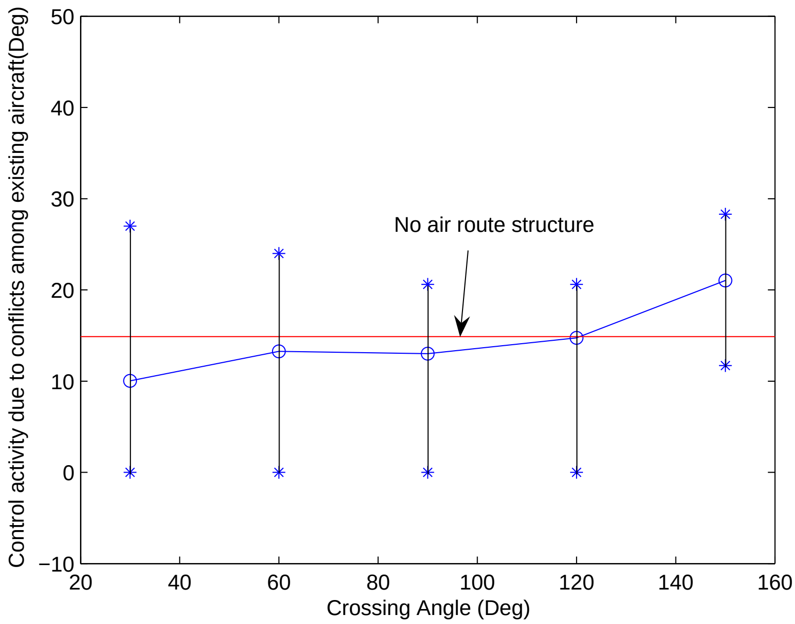


Figure 57: Control activity due to conflicts among existing aircraft for two air routes by crossing angle (Fig. 55): The upper quartile and lower quartile of each data are denoted by '*'

7.3.2 Hypothesis Test

In this section, more rigorous statistical analysis was performed to compare route structures. Two extreme cases, i.e., with the crossing angle of 30° and 150°, were compared. Comparing the complexity map in Fig. 61 with the complexity map in Fig. 62, the average control activity over all entering aircraft with a crossing angle of 30° is 15.79°, while with a crossing angle of 150° it is 26.32°.

A 't-test' was used to compare the induced complexities of both air route structures.

Sample data for each air route structure consisted of the average control activities (over all entering aircraft) for the complexity maps produced from 100 traffic situations. The following hypothesis was made:

$$\text{Hypothesis: } \mu_A = \mu_B \tag{14}$$

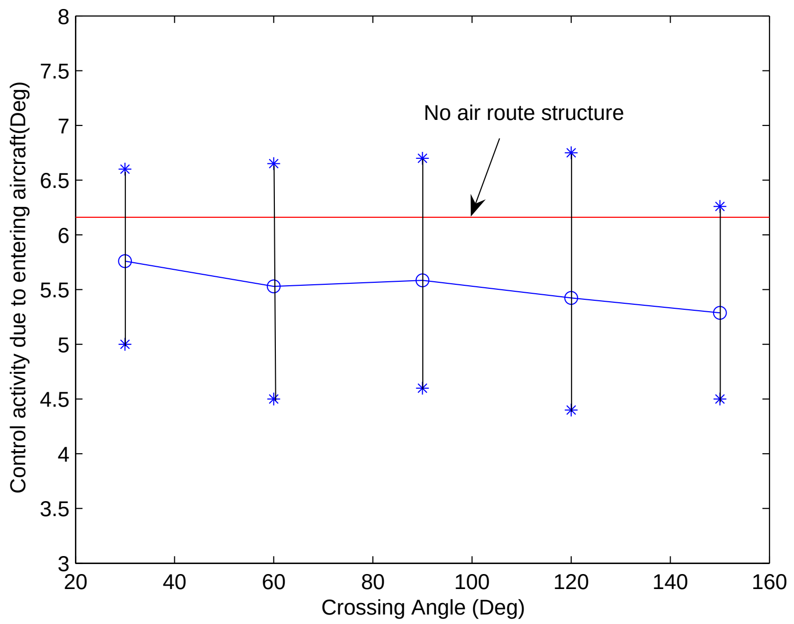


Figure 58: Control activity due to conflicts arising from entering aircraft for two air routes by crossing angle (Fig. 55): The upper quartile and lower quartile of each data are denoted by '*'

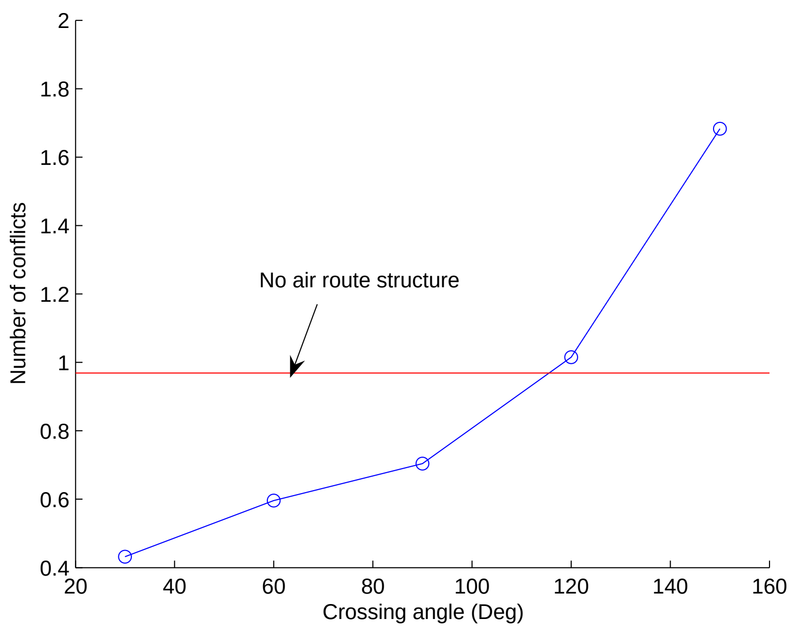


Figure 59: The number of conflicts found among existing aircraft by crossing angles

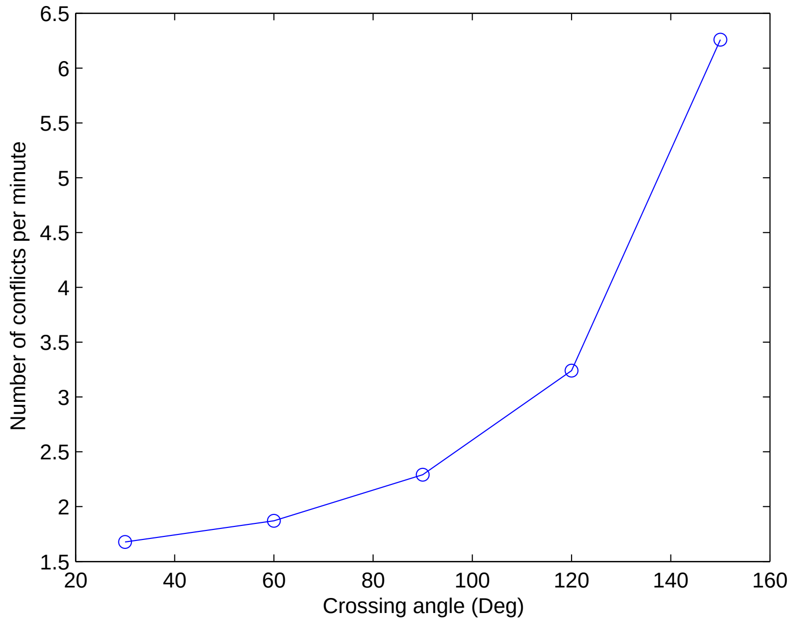


Figure 60: The number of conflicts per minute provided by an analytic encounter model

where μ_A represents the average control activity (over a range of different traffic situation and over all entering aircraft) for the air route structure in Fig. 61, and μ_B is for the air route structure in Fig. 62. In order to determine whether or not we reject the hypothesis, the t-statistic can be computed:

$$\text{t-statistic} = \frac{(\overline{x} - \overline{y})}{\sqrt{\frac{{s_A}^2}{n} + \frac{{s_B}^2}{m}}} \tag{15}$$

where n represents the sample size for the air route structure in Fig. 61, and m is for the air route structure in Fig. 62. $\overline{x}$ represents the sample mean for the air route structure in Fig. 61, and $\overline{y}$ is one for the air route structure in Fig. 62. s_A represents the sample standard deviation for the air route structure in Fig. 61, and s_B is one for the air route structure in Fig. 62.

We can use the t-statistic to compute the "p-value" in the following way:

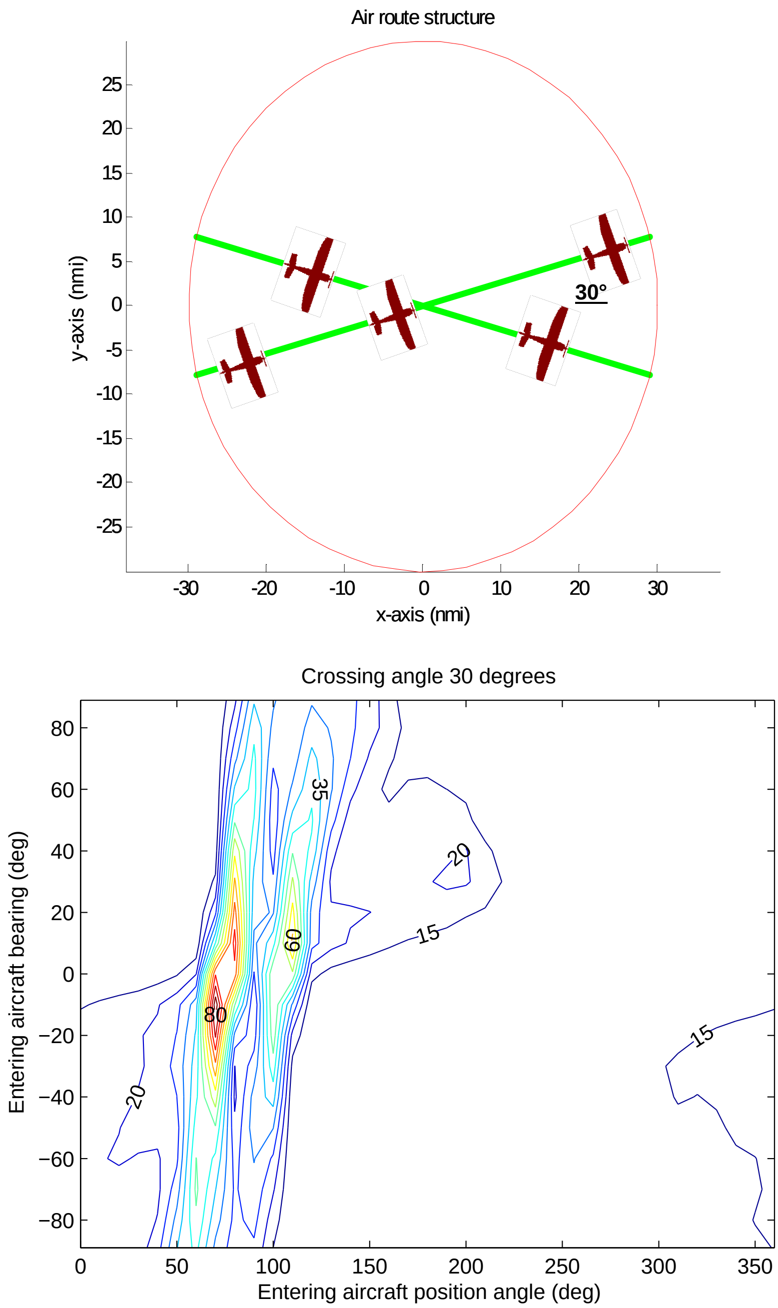


Figure 61: Air route structure (two air routes with crossing angles of 30°) and its complexity map averaged over 100 traffic situation

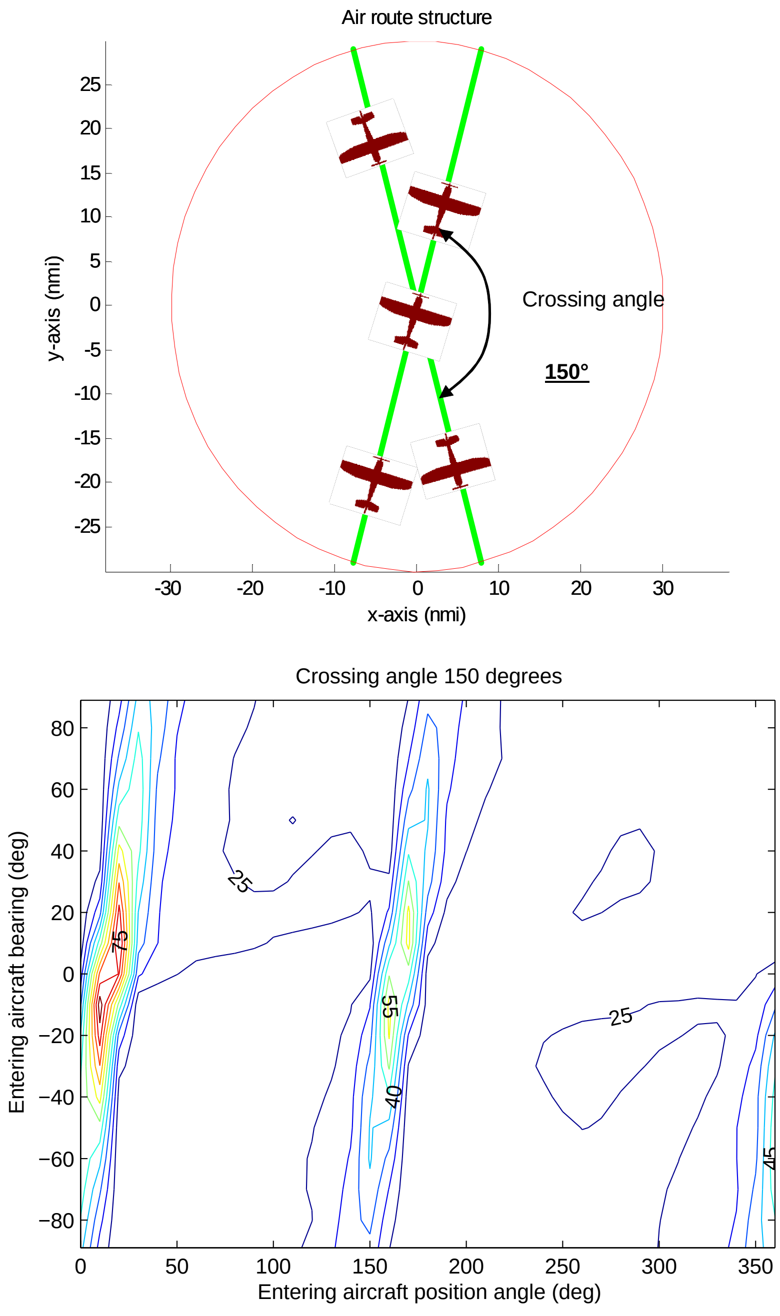


Figure 62: Air route structure (two air routes with crossing angles of 150°) and its complexity map over 100 traffic situations

$$\text{p-value } = P(X \leq \text{t-statistic}) \tag{16}$$

where X has a t-distribution with ν degrees of freedom. The ν was computed in the following way:

$$\nu = \frac{\frac{{s_A}^2}{n} + \frac{{s_B}^2}{m}^2}{\frac{{s_A}^4}{n^2(n-1)} + \frac{{s_B}^4}{m^2(m-1)}} \tag{17}$$

The p-value represents the probability of the hypothesis. The smaller the p-value, the less plausible the hypothesis. Therefore, if a p-value is too small, we reject the hypothesis. Determining when a p-value is "small" depends on the confidence level, α, selected as the required probability of the hypothesis actually being true.

The p-value was found to be 3.49×10^{-7}, which we considered to be small enough for us to reject the hypothesis. Thus, we concluded that the induced complexity (in terms of the average value over all entering aircraft) of the air route structure with the crossing angle of 150° is bigger than one for the air route structure with the crossing angle of 30°.

Now compare those air route structures in terms of control activity due to conflicts arising from entering aircraft. Another statistical analysis was performed, and the p-value was found to be 0.01, which was also considered to be small enough for us to reject the hypothesis. Thus, we might conclude that a traffic in the air route structure with the crossing angle of 30° is more difficult to control than a traffic in the air route structure with the crossing angle of 150° in the face of entering aircraft. Finally, it should be noted that the results would be different with another traffic density.

CHAPTER VIII

CONCLUSIONS & FUTURE WORK

8.1 Summary

8.1.1 New Notion of Airspace Complexity

With increased demand on the air transportation system, much effort has been made to improve the current Air Traffic Management(ATM) system. The fundamental function of ATM is monitoring and mitigating mismatches between air traffic demand and airspace capacity. Many advanced Traffic Flow Management(TFM) concepts have been proposed to effectively manage traffic flows to balance air traffic demand with airspace capacity, and Dynamic Airspace Configuration(DAC) concepts have examined how to reconfigure the airspace to meet variations in air traffic demand. In ongoing efforts to balance air traffic demand and airspace capacity, describing airspace complexity stands as a fundamental research problem because traffic flow management over multiple airspaces can be done based on the complexity of those airspaces. Likewise, dynamic airspace configuration can be performed by evaluating different airspace configurations based on their complexity. Much effort has been made to understand airspace complexity, typically focusing on airspace complexity measures referenced to controllers' workload. However, no research has yet moved substantially beyond the current practice of counting aircraft, largely due to the inherent difficulty in understanding human cognition. Furthermore, approaches that heavily rely on human controllers' subjective responses have not provided consistent results.

Taking a more analytical approach, this thesis suggested that airspace complexity can be described in terms of how the airspace (together with the traffic inside it and the traffic control algorithm) responds to disturbances. This new notion of traffic complexity takes the viewpoint that an airspace is a closed loop controlled system responding to various disturbances. The difficulty experienced by airspace in the face of such disturbance is captured by required control activity to resolve conflicts arising from the disturbance. Since the response of the airspace depends on the disturbance, this thesis introduced complexity maps, each showing how an airspace responds to a set of different disturbances.

Among the many possible disturbances, this thesis considers an aircraft entering into the airspace. A two dimensional airspace model (representing a single flight level) and three different traffic control methods (i.e., the centralized conflict resolution algorithm using mixed integer programming, the sequential conflict resolution algorithm and the sequential conflict resolution algorithm with a fixed horizon) were applied. Using different types of control activity measures (i.e., the sum of the total heading changes over all aircraft inside the sector, the number of heading changes done by aircraft and the sum of the heading changes due to secondary conflicts), numerical simulations were performed to illustrate this method of describing airspace complexity. Examples of complexity maps for different airspaces were presented, and extracting information relevant to traffic flow management was discussed.

The time evolution of a complexity map was investigated. The time evolution of a complexity map is of interest since, in a real application, airspace complexity should be described over a certain period of time rather than at an instant in time. Describing airspace complexity over a certain time period is more appropriate because of various uncertainties in the system and the time required for airspace to act upon the airspace managers' decision. The Monte Carlo approach and the statistical analysis (i.e, confidence interval) were performed to examine their time evolution experimentally.

8.1.2 Applications

Applications of the proposed method to traffic flow management were suggested. Air traffic flow managers sometimes restrict incoming traffic flows into an airspace to prevent overly demanding traffic situations. However, since each airspace receives traffic from many others, the most effective restriction is not easily identified. Because a complexity map provides detailed information about an airspace's response to a range of entering aircraft, air traffic flow managers can easily identify the problematic part of the sector boundary that should be closed. Furthermore, the proposed method also allows air traffic managers to assess how their sector closure decisions will affect the complexity of the adjacent airspace. Detailed information about how a particular change in one airspace affects complexity of others is necessary for cooperation between multiple airspaces.

Similarly, the proposed method can be applied to assess how airspace complexity is affected by convective weather. Detailed information on how airspace complexity will be affected by convective weather allows more efficient management of traffic flows during convective weather events.

The proposed complexity map also can be used for the co-operative air traffic management concept, which is currently being investigated for a gradual transition to the next generation air traffic management system. Using complexity maps, area controllers can determine efficient trajectories of aircraft (equipped with advanced avionics) to fully utilize the available airspace capacity, since a complexity map allows area controllers to identify how each sector will suffer from its entrance.

The proposed method can also be used to evaluate different airspace configurations. For example, this thesis investigated the complexity of an air route structure, over a wide range of corresponding traffic situations. To show the applicability of the proposed method to airspace design problems, some analysis was presented. The complexity of a set of air route structures were compared. Each had two air routes with different crossing angles. Statistical analysis showed that the induced complexity (in terms of the average control activity over all entering aircraft) of the air route structure increases as the crossing angle increases. The higher complexity for a larger crossing angle mainly comes from conflicts among existing aircraft. On the other hand, the control activity for conflicts arising from entering aircraft decreases as the crossing angle increases. In other words, traffic in an air route structure with a shallower crossing angle might be more difficult to control in the face of entering aircraft.

Finally, quantifying the degree of increased complexity caused by partial closure of a sector boundary can be applied to dynamically allocating or de-allocating special use airspace to increase the capacity of its adjacent sectors. Large volumes of airspace are regularly restricted by airspace managers for other reasons such as military and space launch operations, and airspace managers need to assess how these restricted airspace will affect complexity of other airspaces.

8.2 Contributions

• This thesis suggests that airspace complexity can be described in terms of the response of the airspace to disturbances. This innovative notion of airspace complexity takes the viewpoint that an airspace is a closed loop controlled system responding to various disturbances.

• This thesis also suggests that a single scalar metric can not properly describe the complexity of a given traffic. Therefore, the thesis proposes a complexity map to describe an airspace's response to a set of disturbances. The detailed information provided by the complexity map is relevant to the traffic flow management and dynamic airspace configuration.

• The proposed complexity map provides capabilities for airspace managers to; identify overly complex traffic situations and determine the most effective way to restrict traffic flows; assess how airspace complexity will be affected by environmental factors such as convective weather; identify the best way to use the available capacity of airspace, which helps airspace managers to effectively reroute traffic flows to fully utilize the available airspace capacity in case of convective weather; and assess how a particular change in one airspace (i.e., a partial sector boundary closure) affects the complexity of another airspace. These capabilities are necessary for cooperation between airspace managers.

- Unlike previous studies on airspace complexity, this thesis investigates how airspace complexity changes over time by using the Monte Carlo approach and the statistical analysis. This investigation is important since, in a real application, airspace complexity should be described over a certain time interval.

- The proposed method can be applied to airspace design problems. The thesis shows that the complexity of a given air route structure can be assessed its complexity over a wide range of traffic situations for comparison with other structures.

8.3 Future Work

Future work should incorporate more extensive models for each component in the air traffic control system. The simple airspace model used in this thesis might not fully capture all relevant behaviors. The two dimensional airspace model (representing a single flight level) can be extended to a three dimensional model, and different speeds should be allowed for each aircraft. Furthermore, the traffic control methods used in current or future operations should be identified and their computational models need to be developed. The details of the model should fit the needs of the application of the method.

Working with the operational community, we need to identify proper measures of control activity that represent the "pain" experienced by the system from disturbances. Those measures of control activity need not necessarily represent human controllers' workload. Different measures of control activity can be used for different analysis. For example, a complexity map of time-to-conflict can be computed. Fig. 63 shows the complexity map of time-to-conflict for the traffic situation in Sector A examined throughout this thesis.

We also need to identify and characterize other types of disturbances, and the choice of disturbance should be closely tied with its practical application. For example, non-conforming aircraft can be considered as a disturbance in the airspace. Conformance monitoring is critical since any excessive deviation of aircraft from their planned trajectories might seriously degrade system performance.[54] By evaluating an airspace's response to different non-conformance events, traffic controllers can focus on aircraft whose non-conformance creates the most serious problem in the system. As another example, convective weather can also be considered as a disturbance by focusing on its stochastic nature and analyzing how convective weather can disturb airspace. Then, the proposed method can be applied to evaluate how convective weather can disturb a given traffic situation. Monte Carlo methods can be used to evaluate a given traffic situation over a wide range of convective weather (in terms of its size and location). If convective weather is expected, airspace managers might implement reroutes so that the traffic would be less disturbed by likely convective weather. This detailed information allows for more accurate strategies for airspace managers to prepare for convective weather events.

Various applications of the proposed method should also be further investigated. For example, the method proposed in this thesis can also provide a unified approach to evaluate any type of control method in terms of how it can handle different types of disturbances. As shown in this thesis, the proposed method can incorporate different traffic control methods without difficulty.

The stochastic natures of the system must be considered. First we need to identify parts of the system that cannot be exactly known. For example, each aircraft's

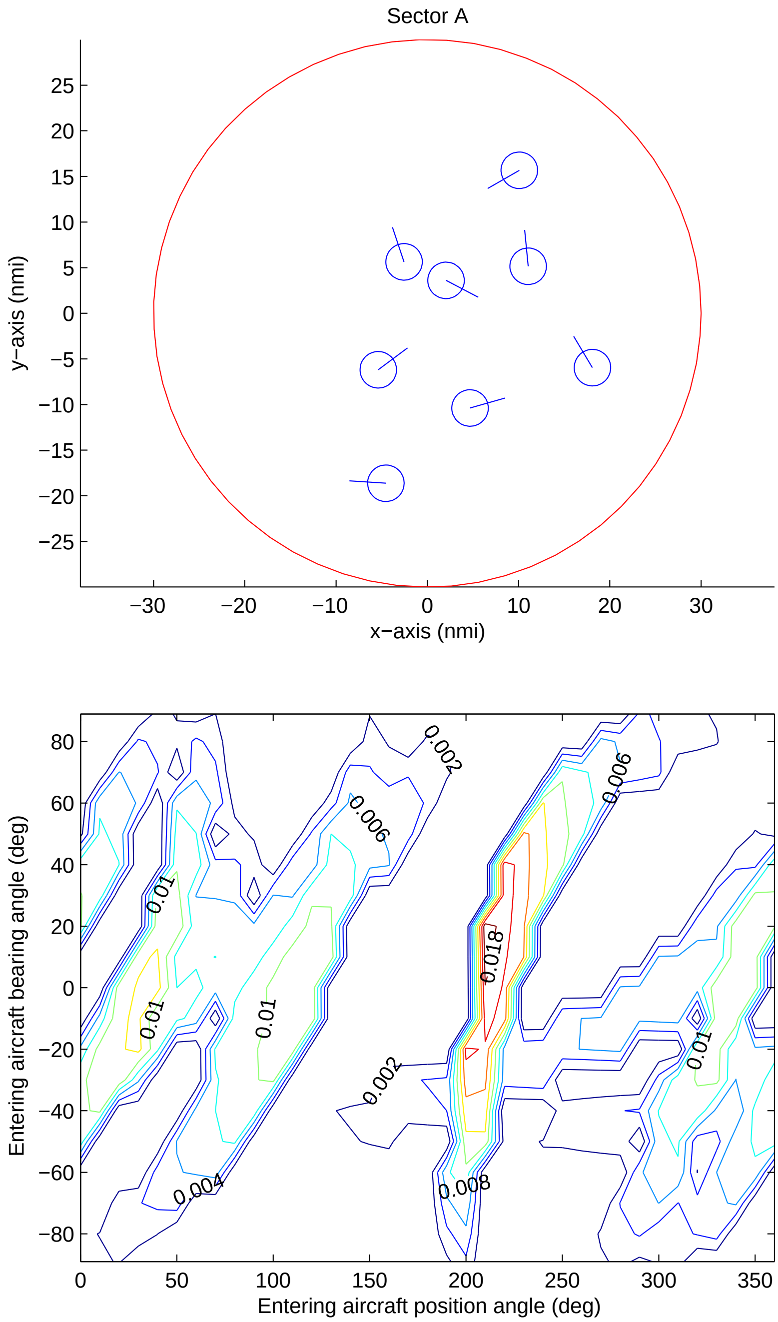


Figure 63: Complexity map for the traffic situation in Sector A. The plot contours indicate the inverse of time-to-conflict for all combinations of entering aircraft bearing and position angles

position and heading can not be exactly known due to limits on current navigation systems. A sensitivity analysis for these unknown parts of the system should be performed. Approaches similar to those used for investigating the time evolution of a complexity map can be used. Then, we need to investigate how to incorporate these stochastic parts of the system into the complexity analysis.

This thesis describes the complexity of airspace in terms of how its traffic situation will suffer from disturbances, i.e., entering aircraft. Future investigations should examine how airspace complexity changes after a new aircraft enters into the sector. For example, consider the traffic situation in Sector F, as shown in Fig. 64. Based on its complexity map, traffic flow managers may decide to accept an entering aircraft with position angle of 300° and bearing of 0°, since this entering aircraft will not require any control activity by aircraft inside Sector F. At that point, this entering aircraft is also considered part of Sector F, and the complexity of Sector F should be described with this aircraft. The new complexity map of Sector F with this new aircraft is shown in Fig. 65. For this new complexity map of Sector F, entering aircraft with position angles between 290° and 310° can not be accepted since Sector F just accepted a new aircraft with position angle of 300°.

Comparing complexity maps in Fig. 64 and Fig. 65, control activity has been increased by accepting the new aircraft. However, as shown in Fig. 66, the affected area are limited to the part of sector boundary from which Sector F just accepted the new aircraft. This investigation will lead us to the concept of closed loop traffic flow management. For example, when traffic flow mangers decide whether or not to accept an entering aircraft, they need to consider not only how an airspace will suffer from the entrance of an aircraft but also how airspace complexity will change after accepting an entering aircraft.

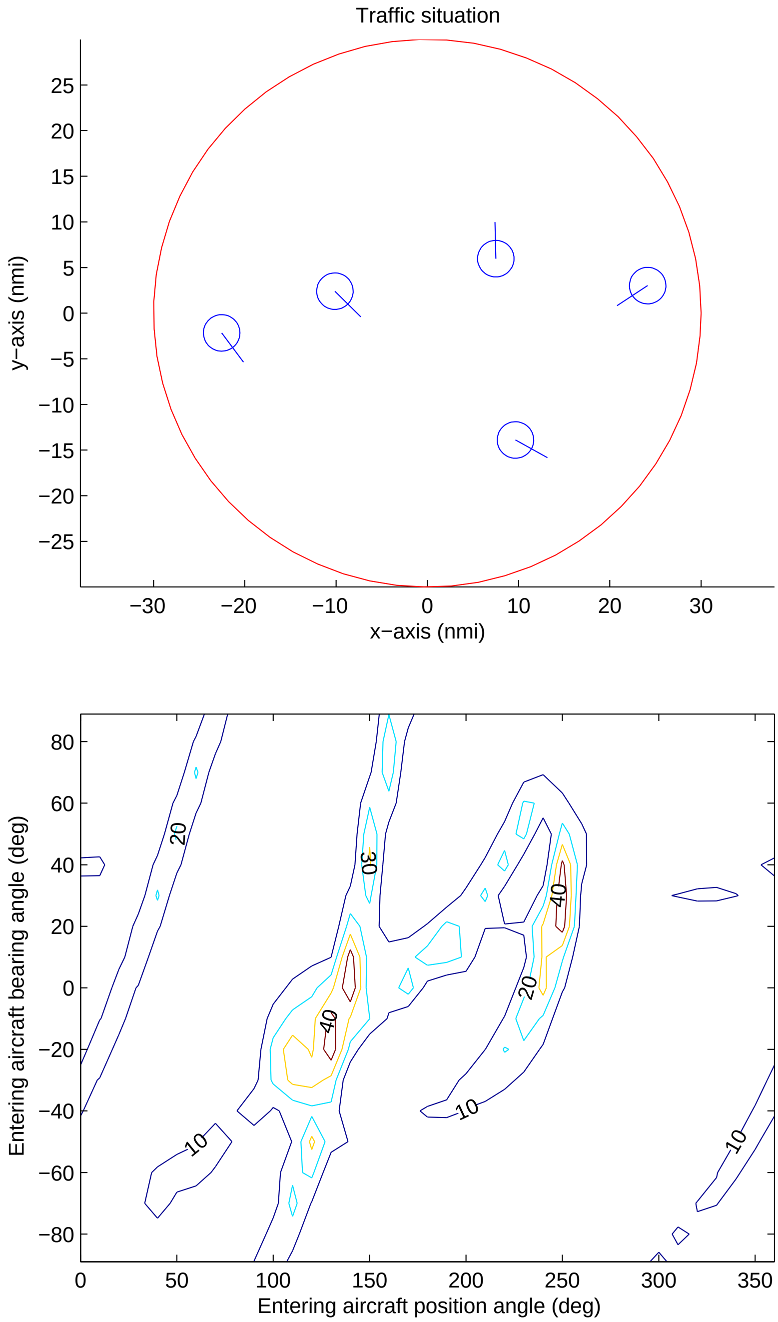


Figure 64: Complexity map for the traffic situation in Sector F and its complexity map

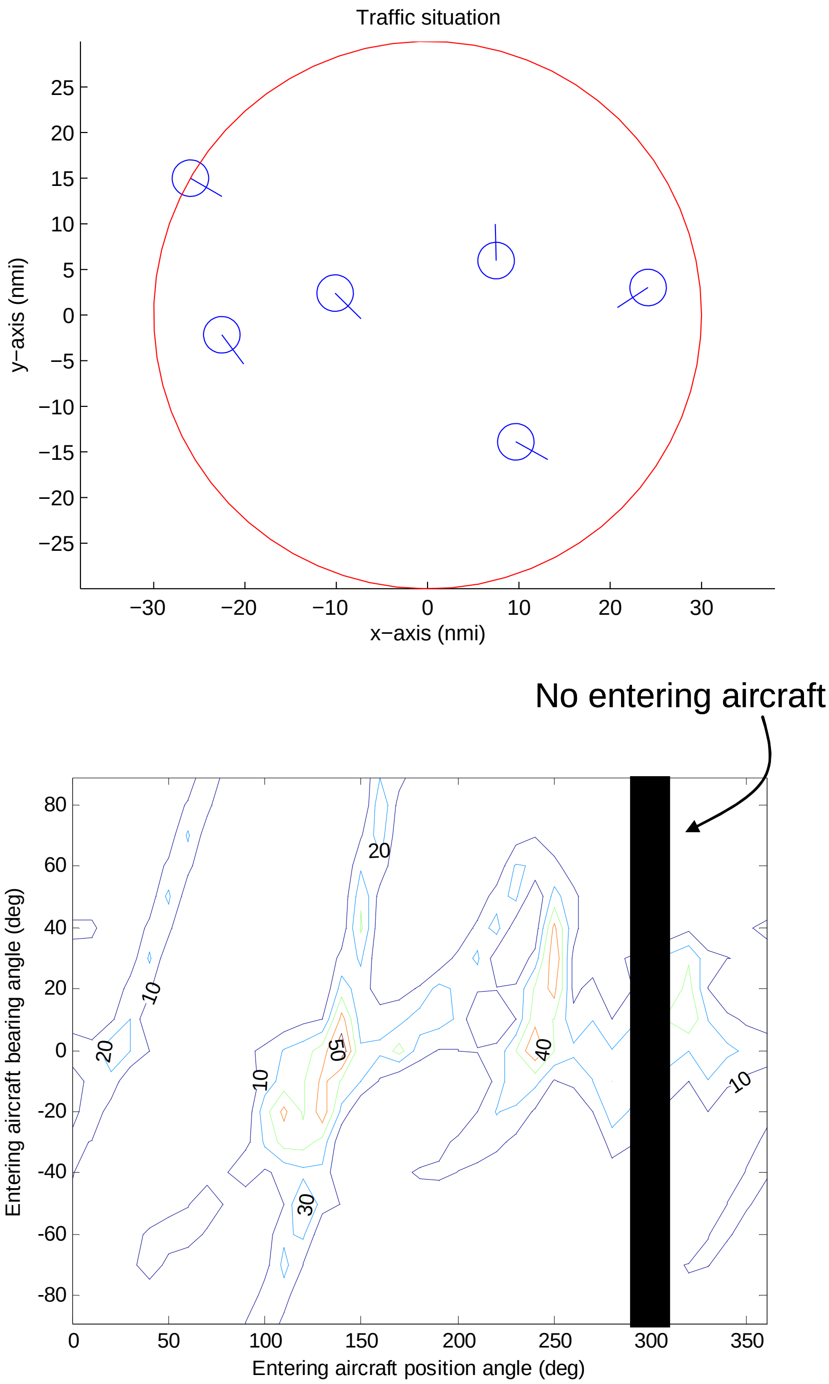


Figure 65: Complexity map for the traffic situation in Sector F with the new aircraft. Entering aircraft with position angles between 290° and 310° can not be accepted

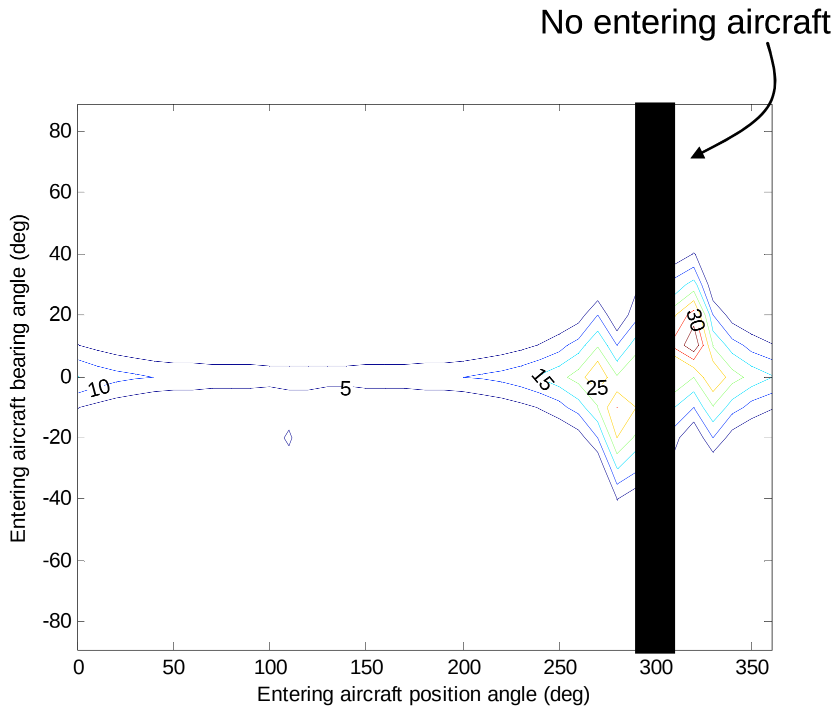


Figure 66: Difference between two complexity maps in Fig. 64 and Fig. 65, i.e., before an aircraft enters at a position angle of 300°

Finally, with appropriate computational airspace models, traffic control methods, disturbances and operational concepts of applications, we can develop computational tools for different applications. Various operational concepts using the proposed method have already been proposed in this thesis. Usefulness of these tools should be validated with the operational community. These tools can be used not only for real operations, but also for studying fundamental issues in air traffic systems. For example, this method enables the examination of trade-offs between traffic volumes and system performance, which gives us further insight about the nature of the air traffic control system.

APPENDIX A

AIR TRAFFIC CONFLICT RESOLUTION TOOL

This conflict resolution algorithm has been fully described previously.[51] Let's consider the conflict geometry for two aircraft. The direction of $V_{2/1}$ (or $V_{1/2}$) should satisfy certain conditions for the absence of conflicts. Exact mathematical conditions for this statement depend on the positions and velocities of the two aircraft. First, assign a number to each aircraft from the left most aircraft to the right most aircraft. If two aircraft are aligned vertically, assign the number to the aircraft below first. From this indexing rule, we can get the mathematical condition that the w_{12} angle shown in Fig. 67 must satisfy $|w_{12}| \leq \frac{\pi}{2}$.

Let's derive non-conflict conditions based on the relative heading angles, i.e. $q_{1/2}$ and $q_{2/1}$ in Fig. 67. In order to avoid a conflict, $q_{1/2}$ and $q_{2/1}$ should satisfy one of the following conditions:

1. Case1: $q_{1/2} \geq 0$, $q_{2/1} \geq 0$, $q_{1/2} \geq q_{2/1}$
2. Case2: $q_{1/2} \geq 0$, $q_{2/1} \geq 0$, $q_{1/2} \leq q_{2/1}$, $\left| -\frac{\pi}{2} + \frac{q_{1/2}+q_{2/1}}{2} \right| \geq \theta_s$
3. Case3: $q_{1/2} \geq 0$, $q_{2/1} \leq 0$, $q_{1/2} \geq -q_{2/1}$
4. Case4: $q_{1/2} \geq 0$, $q_{2/1} \leq 0$, $q_{1/2} \leq -q_{2/1}$, $\left| \frac{\pi}{2} + \frac{q_{1/2}+q_{2/1}}{2} \right| \geq \theta_s$
5. Case5: $q_{1/2} \leq 0$, $q_{2/1} \geq 0$, $-q_{1/2} \geq q_{2/1}$
6. Case6: $q_{1/2} \leq 0$, $q_{2/1} \geq 0$, $-q_{1/2} \leq q_{2/1}$, $\left| -\frac{pi}{2} + \frac{q_{1/2}+q_{2/1}}{2} \right| \geq \theta_s$
7. Case7: $q_{1/2} \leq 0$, $q_{2/1} \leq 0$, $-q_{1/2} \geq -q_{2/1}$
8. Case8: $q_{1/2} \leq 0$, $q_{2/1} \leq 0$, $-q_{1/2} \leq -q_{2/1}$, $\left| -\frac{pi}{2} + \frac{q_{1/2}+q_{2/1}}{2} \right| \geq \theta_s$

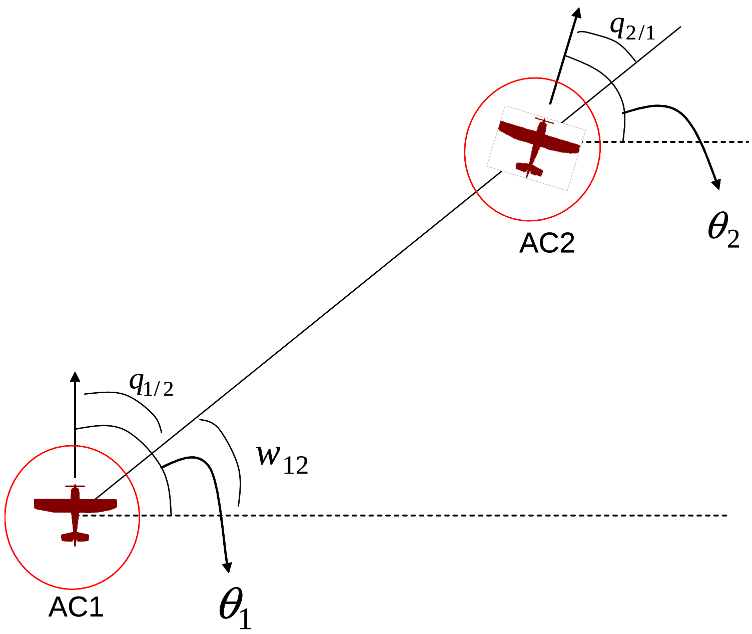


Figure 67: Geometry of conflict resolution(1)

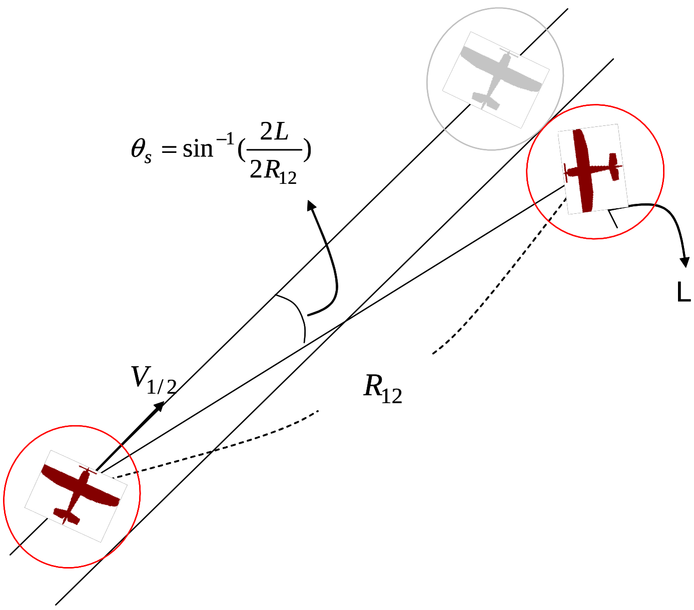


Figure 68: Geometry of conflict resolution(2)

However, we have to deal with these conditions using the absolute heading angles, θ_i instead of the $q_{i/j}$ angles since the $q_{i/j}$ angles are relative quantities based on the aircraft being compared. In other words, each aircraft can have different $q_{i/j}$ angles. The relationship between these angles for aircraft 1 and 2 in Fig. 67 is:

$$q_{1/2} = \theta_1 - W_{12} - 2\pi S_{12} C_{left,12} \tag{18}$$

where $S_{12} = -sgn(W_{12})$ and $C_{left,12}$ can have 0 or 1. These integer variables, S_{12} and $C_{left,12}$, are introduced in order to restrict the solution to the domain $-\pi \leq q_{1/2} \leq \pi$ and $-\pi \leq \theta_1 \leq \pi$.

Among the many possible sets of heading changes, we choose the one which minimizes the following objective function:

$$Cost\ function = \sum_{i=1}^{i=N} |\theta_{in} - \theta_i| \tag{19}$$

where θ_{in} is the new heading angle for each aircraft.

Here is pseudo-code for the CPLEX (Mixed integer linear programming package) implementation of the described optimization problem.

1. **Calculate parameters**

   (a) W_{ij}, S_{ij} , i=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

   :Parameters used for converting θ to q angles.

2. **Define variables**

   (a) θ_i, i=1,2,...,N

   (b) $C_{left,ij}$, $C_{right,ij}$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

   :Variables for converting θ to q angles. These are binary variables.

(c) $Case1_{ij}$, $Case2_{ij}$,..., $Case8_{ij}$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

:Variables for choosing one of non-conflict conditions. Binary variables indicating satisfaction of the non-conflict conditions.

(d) $Case2-1_{ij}$, $Case2-2_{ij}$,..., $Case8-1_{ij}$, $Case8-2_{ij}$,

ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

:These are binary Variables for implementing absolute value in non-conflict conditions in CPLEX.

(e) aux_i, $pCase1_i$, $pCase2_i$, $pCase3_i$, i=1,2,...,N

:These are variables for absolute value implementation in performance index in CPLEX.

3. **Write non-conflict conditions**

(a) Case1:

i. $-M(1-Case1_{ij}) \leq \theta_i - W_{ij} - 2\pi S_{ij}C1_{ij}$,

ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

ii. $-M(1-Case1_{ij}) \leq \theta_j - W_{ij} - 2\pi S_{ij}C2_{ij}$,

ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

iii. $-M(1-Case1_{ij}) + \theta_j - 2\pi S_{ij}(C2_{ij} - C1_{ij}) \leq \theta_i$,

ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

In the above equations, M is an arbitrarily large number. Note that if $Case1_{ij} = 0$, the constraint equations for Case 1 are trivially satisfied and therefore meaningless (relaxed), while if $Case1_{ij} = 1$ then the constraint equations in case1 are valid and must be satisfied.

We can make constraint equations for other cases in a similar manner.

4. **Exclusive condition**

(a) $Case1_{ij} + Case2_{ij} + Case3_{ij} + Case4_{ij} + Case5_{ij} + Case6_{ij} + Case7_{ij} + Case8_{ij} = 1$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

From this constraint equation, only one of non-conflict conditions can be valid.

5. **Conditions for confined solution space**

   (a) $\theta_i - W_{ij} - 2\pi S_{ij} C1_{ij} \leq \pi$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

   (b) $-\pi \leq \theta_i - W_{ij} - 2\pi S_{ij} C1_{ij}$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

   (c) $\theta_j - W_{ij} - 2\pi S_{ij} C2_{ij} \leq \pi$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

   (d) $-\pi \leq \theta_j - W_{ij} - 2\pi S_{ij} C2_{ij}$, ij=(1,2),(1,3),...,(1,N),(2,3),(2,4),...,(N-1,N)

6. **Conditions for performance index**

   (a) $\theta_i - \theta_{0i} \leq aux_i + M(1 - pCase1_i)$, i=1,2,...,N

   (b) $-aux_i - M(1 - pCase1_i) \leq \theta_i - \theta_{0i}$, i=1,2,...,N

   (c) $2\pi - (\theta_i - \theta_{0i}) \leq aux_i + M(1 - pCase2_i)$, i=1,2,...,N

   (d) $2\pi + (\theta_i - \theta_{0i}) \leq aux_i + M(1 - pCase3_i)$, i=1,2,...,N

   (e) $pCase1_i + pCase2_i + pCase3_i = 1$, i=1,2,...,N

   Performance index is 'Minimize $\sum_{i=1}^{i=N} aux_i$'

APPENDIX B

THE JUMP DISCONTINUITY IN MULTIPLE AIRCRAFT CONFLICT RESOLUTIONS

B.1 Conflict Geometry

Research has examined the relative motion of a pair of aircraft.[1, 36, 34, 59] However these results can not easily be extended to multiple aircraft scenarios. For example, conflict resolution problems involving many aircraft must address a critical issue known as the domino effect: Resolving conflicts can create new conflicts which in turn may create additional conflicts during subsequent resolutions [35]. Therefore, pairwise conflict resolution algorithms can not guarantee solutions to multiple aircraft conflicts [24].

We address another property of a conflict resolution problem with multiple aircraft. First of all, we need to define our notation. For a pair of aircraft, there are two way to resolve a conflict. One is a clockwise pattern and the other is a counter-clockwise pattern. These patterns are described in Fig. 69. $V_{2/1}$ represents relative velocity of aircraft 2 with respect to aircraft 1. In order to avoid a conflict, the direction of $V_{2/1}$ should be placed outside of the forbidden area.

Definition 10: Two aircraft are **adjacent** if the relative velocity of one with respect to the other, e.g. $V_{2/1}$ in Fig. 69, is aligned with either the clockwise crossing line or the counter-clockwise crossing line as shown in Fig. 69. If it is parallel with the clockwise crossing line, the two aircraft are called **clockwise adjacent** If it is parallel with the counter-clockwise crossing line, the two aircraft are called **counter-clockwise adjacent** . □

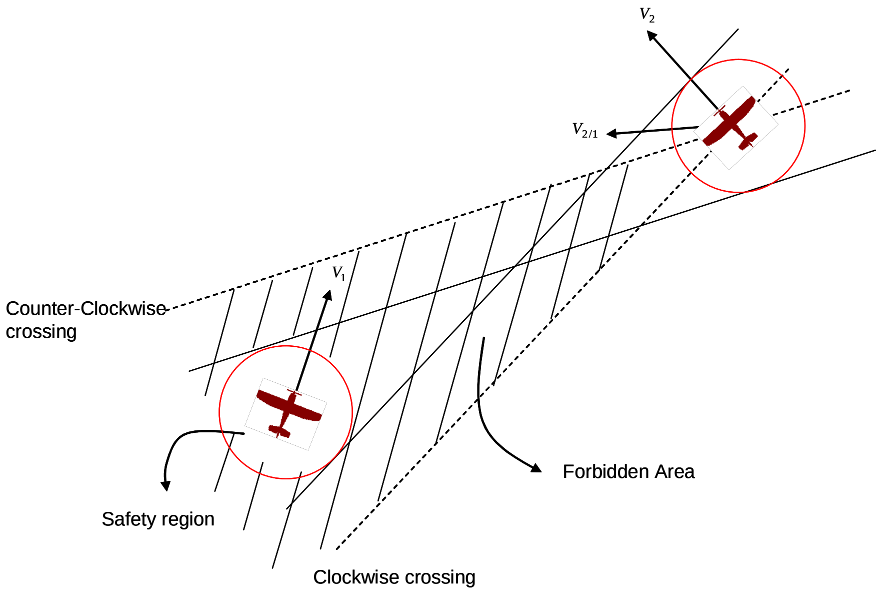


Figure 69: Crossing patterns

Observation 1: Consider two conflict-free aircraft. If two aircraft are not adjacent, sufficiently small heading changes on one aircraft will not introduce any conflict. If two aircraft are clockwise adjacent, any small clockwise heading changes by one aircraft introduces a conflict. In this case, the other aircraft could turn in a counter-clockwise direction in order to resolve a conflict with minimum heading deviation.

If two aircraft are counter-clockwise adjacent, any small counter-clockwise heading changes by one aircraft introduces a conflict. In this case, the other aircraft could turn in a clockwise direction in order to resolve a conflict with minimum heading deviation. □

B.2 The jump discontinuity by heading perturbations

In this section, we investigate the jump discontinuity behavior by heading perturbations on one aircraft.

Definition 11: The **jump discontinuity** is a visible and sudden jump from one conflict free configuration to another conflict free configuration by small perturbations. Perturbations can be applied to positions of aircraft or heading angles of

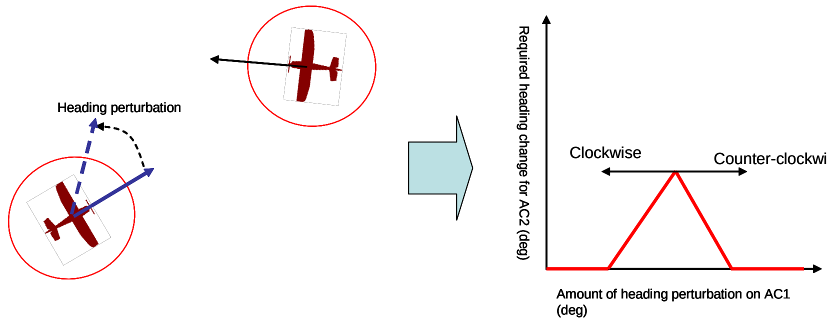


Figure 70: Heading perturbation: Two aircraft

aircraft. □

Fact 1: Consider two aircraft without any conflict. Assume that there is a heading perturbation by AC1. Then the other aircraft, i.e. AC2, may need to change its heading angle to avoid a conflict. The required heading changes for AC2 is continuous with respect to the perturbation.

Proof: Until the magnitude of the perturbation reaches a certain point such that the two aircraft are adjacent, the required heading changes for AC2 is zero. After this point, the required heading changes for AC2 increases continuously. At some point, AC2 may turn in either way, i.e., in either a clockwise or a counter-clockwise, with same amount of angle. After this point, AC2 should turn the other way, and the required heading changes decreases continuously. Fig. 70 shows the general shape of required heading changes of AC2 against the perturbation on AC1. □

Now consider multiple aircraft scenarios.

Property 1: If the heading of one of the aircraft is perturbed, the total required heading change for the other aircraft may be not continuous with respect to the perturbation. Required control activities suddenly jump at the critical point.

Proof : We can prove this property by the counter example in Fig. 71, where the total required heading changes for AC2 and AC3 suddenly jump at the critical amount of perturbation. Basically, this kind of jump discontinuity phenomenon happens because multiple aircraft, i.e. AC2 and AC3 in this example, compete against each other to get their own optimal solutions, which may conflict. In this example, after some point, both AC2 and AC3 conflict by avoiding conflict to AC1's perturbation. Both AC2 and AC3 want to change their heading angle in a counter-clockwise direction in order to avoid conflicts with AC1 with minimum heading deviations. However, they both can not take a counter-clockwise turns without creating a conflict between themselves. One of them must turn in a clockwise direction with finite amount of angle. The required heading deviations for AC2 and AC3 jump at this point. □

B.3 The jump discontinuity by position perturbations

In this section, we investigate the jump discontinuity behavior by position perturbations on one aircraft.

Observation 2: If the position of AC2 is perturbed into the 1st quadrant in Fig. 72, the clockwise crossing line between AC1 and AC2 will rotate in a counter-clockwise direction and the counter-clockwise crossing line will also rotate in a counter-clockwise direction.

If the position of AC2 is perturbed into the 2nd quadrant, the clockwise crossing line will rotate in a counter-clockwise direction. However, in this case, the counter-clockwise crossing line will rotate in a clockwise direction.

If the position of AC2 is perturbed into the 3rd quadrant, the clockwise crossing line will rotate in a clockwise direction and the counter-clockwise crossing line will also rotate in a clockwise direction.

If the position of AC2 is perturbed into the 4th quadrant, the clockwise crossing line will rotate in a clockwise direction. However, in this case, the counter-clockwise crossing line will rotate in a counter-clockwise direction. □

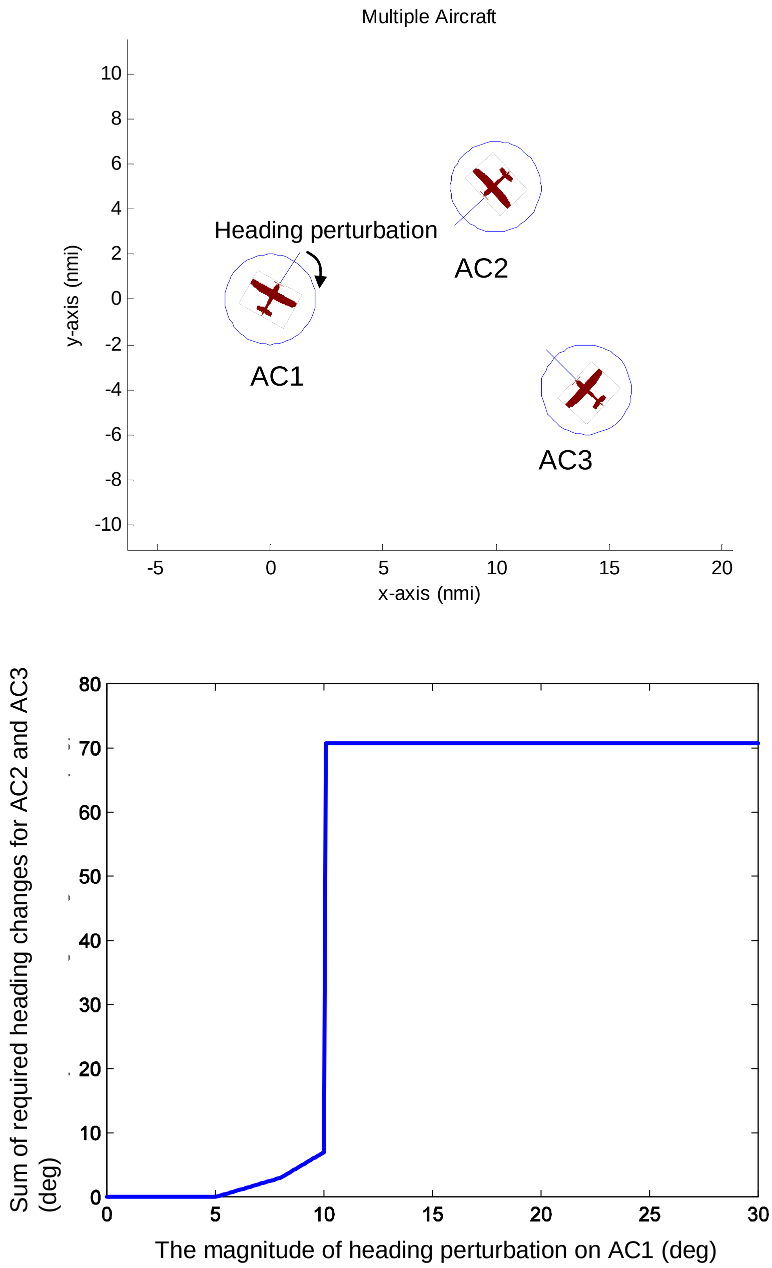


Figure 71: Heading perturbation:10 deg is the point after which AC2 and AC3 conflict in their individually optimal solution

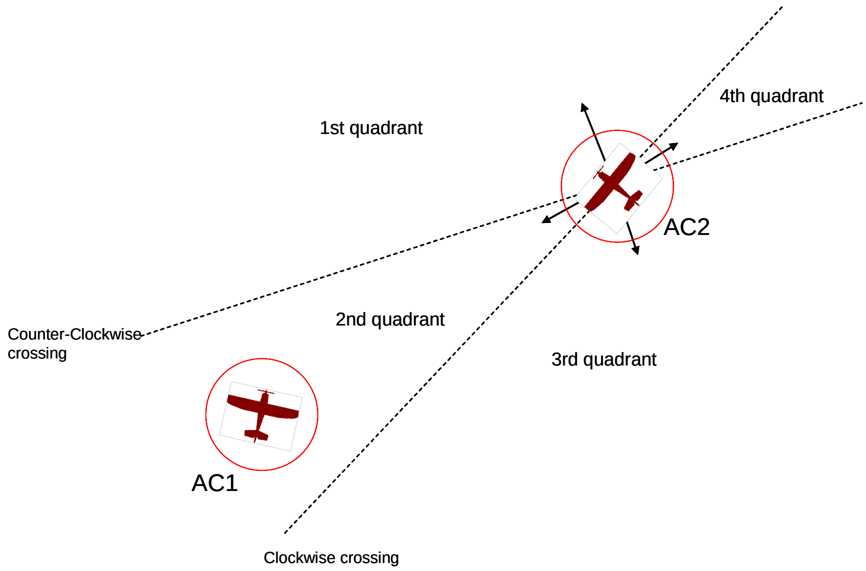


Figure 72: Changes of the orientations of crossing lines due to the position perturbation

From this observation, we can expect that the position perturbations of one aircraft may introduce a conflict with another aircraft.

Fact 2 : The magnitude of the angle perturbation on a clockwise crossing line or a counter-clockwise crossing line is linear with respect to the position perturbations on the aircraft.

Proof :Consider two aircraft in Fig. 73. Each circle represents safety regions for each aircraft. Note that line L is parallel to the clockwise crossing line between AC1 and AC2. Therefore θ can define the orientation angle of the clockwise crossing line. We can calculate θ in the following way:

$$\theta = \alpha + \beta \tag{20}$$

$$\alpha = \frac{2d_s}{R} = \frac{2d_s}{\sqrt{x^2+y^2}}, \quad \beta = \frac{y}{R} = \frac{y}{\sqrt{x^2+y^2}} \tag{21}$$

In equations 20 and 21, x and y represent the position of AC2 relative to AC1.

Now consider the position perturbation of AC2. Without loss of generality, we can

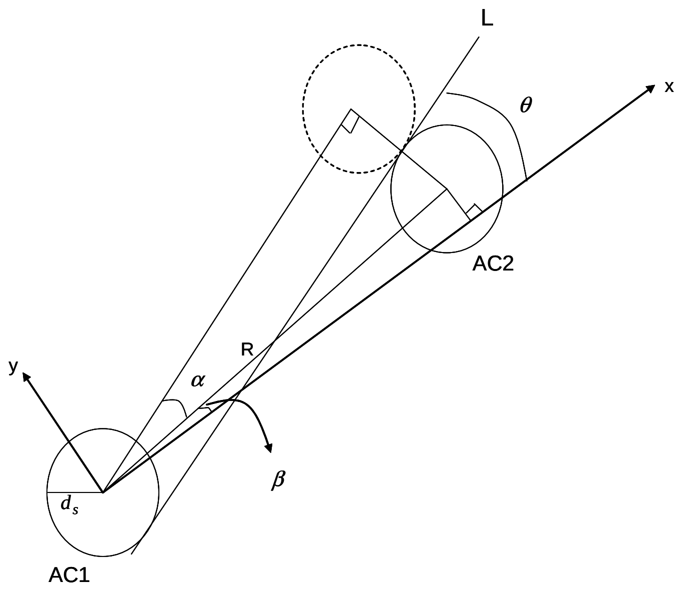


Figure 73: The magnitude of orientation changes on crossing lines due to the position perturbation

say the position is perturbed in the y-direction by choice of reference frame. From equation 21, we can get following equations:

$$\delta\alpha = \frac{-2d_s y_0}{cos(\alpha_0)({x_0}^2 + {y_0}^2)\sqrt{{x_0}^2 + {y_0}^2}}\delta y, \quad \delta\beta = \frac{{x_0}^2}{cos(\beta_0)({x_0}^2 + {y_0}^2)\sqrt{{x_0}^2 + {y_0}^2}}\delta y \quad (22)$$

In equation 22, x_0, y_0, α_0 and β_0 represent current values of positions and angles. From equation 22, perturbations on α and β are linear to the perturbation on the position, i.e., δy.

Therefore, the angle perturbations on the clockwise crossing line, i.e., $\delta\theta$, is linear to the position perturbation on AC2.

We can use the same argument for the counter-clockwise crossing line. □

From Fact 2, we can easily find the following fact.

Fact 3: Consider two aircraft without any conflicts. Assume there is a perturbation in the position of AC1. Then the other aircraft, i.e. AC2, may need to turn its heading to avoid a conflict. The required heading change for AC2 is continuous with respect to the perturbation.

Proof: From Fact 2, the amount of angle perturbations on the clockwise crossing line or the counter-clockwise crossing line is linear with respect to the position perturbation on AC1. Using similar arguments from the proof of Fact 1, we can see the required heading change for AC2 is continuous with respect to the position perturbation by AC1. □

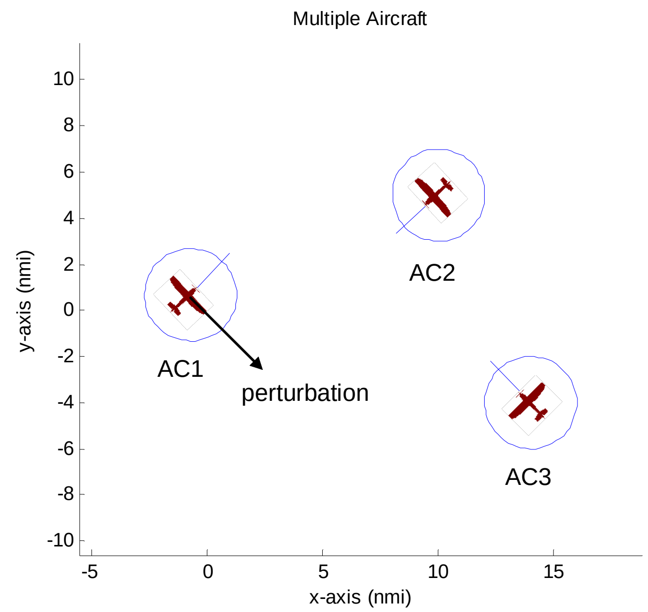
Now consider a scenario with multiple aircraft. We can prove the following property.

Property 2: If one of aircraft's position is perturbed, the total required heading changes for the other aircraft may be not continuous with respect to the perturbation. Required control activities suddenly jump at the critical point.

Proof : We can prove this property by the counter example in Fig. 74, where the total required heading changes for AC2 and AC3 suddenly jump at the critical amount of perturbation. The mechanism of this jump discontinuity behavior is same as property 1. □

B.4 Implication

Properties 1 and 2 represent another property of multiple aircraft conflict resolution problems. The result of a conflict resolution with multiple aircraft is possibly very sensitive to small changes on each aircraft's position or heading. This property results from the fact that one aircraft compete against another for their optimal way to avoid conflicts. This property intimately ties with the robustness of any conflict resolution algorithm with multiple aircraft. Compared to two-aircraft scenarios, relatively small uncertainties might erode the safety in multiple aircraft scenarios. This property needs to be investigated further.



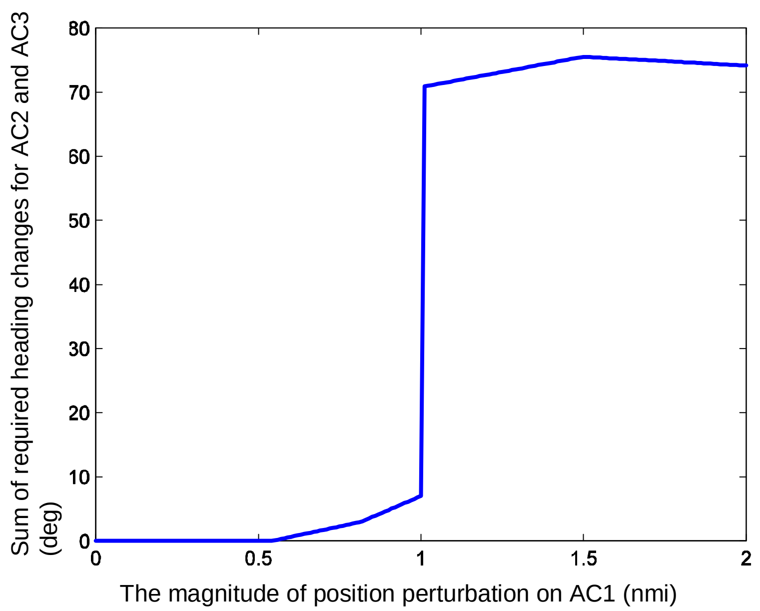


Figure 74: Position perturbation:1 Nmi is the point after which AC2 and AC3 conflict in their individually optimal solution

REFERENCES

[1] ANDREWS, J. W., "A relative motion analysis of horizontal collision avoidance," tech. rep., MIT Lincoln Laboratory, 1973.

[2] ARAD, B., "The control load and sector design," *Journal of Air Traffic Control*, 1964.

[3] ATHENES, S., AVERTY, P., PUECHMOREL, S., DELAHAYE, D., and COLLET, C., "ATC complexity and controller workload: Trying to bridge the gap," *International Conference on Human-Computer Interaction in Aeronautics*, 2002.

[4] BILIMORIA, K. D. and LEE, H. Q., "Analysis of aircraft clusters to measure sector-independent airspace congestion," *5th Aviation, Technology, Integration, and Operations Conference*, 2005.

[5] BILIMORIA, K. D., LEE, H. Q., MAO, Z., and FERON, E., "Comparison of centralized and decentralized conflict resolution strategies for multiple-aircraft problems," *AIAA Guidance Navigation and Control Conference*, 2000.

[6] CHATTERJI, G. B. and SRIDHAR, B., "Neural network based air traffic controller workload prediction," *American Control Conference*, 1999.

[7] CHATTERJI, G. B. and SRIDHAR, B., "Measures for air traffic controller workload prediction," *1th AIAA Aircraft Technology, Integration and Operations Forum*, 2001.

[8] CLOEREC, A., ZEGHAL, K., and HOFFMAN, E., "Traffic complexity analysis to evaluate the potential for limited delegation of separation assurance to the cockpit," *IEEE Digital Avionics System Conference*, 1999.

[9] CUMMINGS, M. and TSONIS, C., "Deconstructing complexity in air traffic control," *HFES Annual Conference*, 2005.

[10] DELAHAYE, D. and PUECHMOREL, S., "Air traffic complexity:towards intrinsic metrics," *3rd USA/Europe Air Traffic Management R/D Seminar*, 2000.

[11] DEVASIA, S. and MEYER, G., "Automated conflict resolution procedures for air traffic management," *IEEE Conference on Decision and Control*, 1999.

[12] EREV, I., BARRON, G., and REMINGTON, R., "Right of way in the sky: Two problems in aircraft self-separation and the auction-based solution," *Human Factors*, vol. 46, no. 2, 2004.

[13] Falker, J. M. and Kuchar, J. K., "Analytical and empirical analysis of the impacts of restricting airspace," *4th USA/Europe Air Traffic Management R/D Seminar*, 2001.

[14] Farley, T. C., Landry, S. J., Hoang, T., Nickelson, M., Levin, K. M., Rowe, D., and Welch, J. D., "Multi-center traffic management advisor: Operational test results," *AIAA Aviation Technology,Integration and Operations Conference*, 2005.

[15] Gianazza, D., "Airspace configuration using air traffic complexity metrics," *7th USA/Europe Air Traffic Management R/D Seminar*, 2007.

[16] Grabbe, S. and Sridhar, B., "Congestion management with an aggregate flow model," *AIAA Guidance, Navigation, and Control Conference*, 2005.

[17] Granger, G. and Durand, N. K., "A traffic complexity approach through cluster analysis," *5th USA/Europe Air Traffic Management R/D Seminar*, 2003.

[18] Grossberg, M., "Relation of sector complexity to operational errors," tech. rep., FAA, 1989.

[19] Hart, S., "Effect of VFR aircraft on approach traffic with and without cockpit displays of traffic information," *18th Annual Conference on Manual Control*, 1982.

[20] Hayter, A., *Probability and Statistics for Engineers and Scientists.* THOMSON, 2007.

[21] Hilburn, B., "Cognitive complexity in air traffic control: A literature review," Tech. Rep. 04/04, EUROCONTROL Experimental Centre Note, 2004.

[22] Histon, J., Aigoin, G., Delahaye, D., Hansman, J., and Puechmorel, S., "Introducing structural considerations into complexity metrics," *Air Traffic Control Quarterly*, vol. 10, no. 2, 2002.

[23] Hunt, V. and Zellweger, A., "Strategies for future air traffic control," *IEEE Computer*, 1987.

[24] Hwang, I. and Tomlin, C. J., "Protocol based conflict resolution for air traffic control," tech. rep., Stanford University, 2002.

[25] Idris, H. and Evans, A., "Benefits assessment of multi-center traffic management advisor for Philadelphia and New York," *AIAA Guidance, Navigation, and Control Conference*, 2003.

[26] ILOG, *CPLEX User's Guide.* ILOG, 1999.

[27] Ishutkina, M., Feron, E., and Bilimoria, K., "Describing air traffic complexity using mathematical programming," *AIAA 5th Aviation, Technology, Integration, and Operations Conference*, 2005.

[28] Klein, A., Kopardekar, P., Rodgers, M. D., and Kaing, H., ““Airspace Playbook”: Dynamic airspace reallocation coordinated with the national severe weather playbook,” *AIAA Aviation Technology, Integration and Operations Conference*, 2007.

[29] Kopardekar, P., Bilimoria, K., and Sridhar, B., “Initial concepts for dynamic airspace configuration,” *AIAA Aviation Technology,Integration and Operations Conference*, 2007.

[30] Kopardekar, P. and Green, S., “Airspace restriction planner for sector congestion management,” *AIAA Aviation Technology,Integration and Operations Conference*, 2005.

[31] Kopardekar, P. and Magyarits, S., “Measurement and prediction of dynamic density,” *5th USA/Europe Air Traffic Management R/D Seminar*, 2003.

[32] Kopardekar, P., Schwartz, A., Magyarits, S., and Rhodes, J., “Airspace complexity measurement: An air traffic control simulation analysis,” *7th USA/Europe Air Traffic Management R/D Seminar*, 2007.

[33] Krozel, J. and Mitchell, J. S. B., “Airspace capacity estimation with convective weather constraints,” *AIAA Guidance, Navigation, and Control Conference*, 2007.

[34] Krozel, J. and Peters, M., “Strategic conflict detection and resolution for free flight,” *IEEE Conference on Decision and Control*, 1997.

[35] Krozel, J., Peters, M., Bilimoria, K. D., Lee, C., and Mitchell, J. S., “System performance characteristics of centralized and decentralized air traffic separation strategies,” *4th USA/Europe Air Traffic Management R/D Seminar*, 2001.

[36] Kuchar, J. and Yang, L., “Incorporation of uncertain intent information in conflict detection and resolution,” *IEEE Conference on Decision and Control*, 1997.

[37] Kuchar, J. and Yang, L., “A review of conflict detection and resolution modeling methods,” *IEEE Trans. Intelligent Transportation System*, vol. 1, no. 4, 2000.

[38] Kuhar, W., Gavel, P., and Moreland, J., “Impact of automation upon air traffic control system productivity/capacity(ARTS-III),” Tech. Rep. FAA-RD-77-39, FAA, 1976.

[39] Laudeman, I., Shelden, S., Branstrom, R., and Brasil, C., “Dynamic density: An air traffic management metric,” Tech. Rep. NASA-TM-1998-112226, NASA, 1998.

[40] Leiden, K., Kamienski, J., and Kopardekar, P., "Initial implications of automation on dynamic airspace configuration," *AIAA Aviation Technology,Integration and Operations Conference*, 2007.

[41] Loft, S., Sanderson, P., Neal, A., and Mooij, M., "Modeling and predicting mental workload in en route air traffic control: Critical review and broader implications," *Human Factors*, vol. 49, no. 3, 2007.

[42] Mao, Z., Dugail, D., Feron, E., and Bilimoria, K. D., "Stability of intersecting aircraft flows using heading change maneuvers for conflict avoidance," *IEEE Trans. Intelligent Transportation Systems*, vol. 6, no. 4, 2005.

[43] Mao, Z., Feron, E., and Bilimoria, K. D., "Stability and performance of intersecting aircraft flows under decentralized conflict avoidance rules," *IEEE Trans. Intelligent Transportation Systems*, vol. 2, no. 2, 2001.

[44] Martinez, S. A., Chatterji, G. B., Sun, D., and Bayen, A. M., "A weighted-graph approach for dynamic airspace configuration," *AIAA Aviation Technology,Integration and Operations Conference*, 2007.

[45] Masalonis, A. J., Callaham, M. B., and Wanke, C. R., "Dynamic density and complexity metrics for realtime traffic flow management," tech. rep., CAASD, MITRE Corporation, 2003.

[46] Mavris, D. N. and DeLaurentis, D., "Methodology for examining the simultaneous impact of requirements, vehicle characteristics, and technologies on military aircraft design," *22th Congress of the International Council on the Aeronautical Sciences*, 2000.

[47] Menon, P. K., Sweriduk, G. D., and Bilimoria, K. D., "A new approach for modeling, analysis and control of air traffic flow," *AIAA Guidance, Navigation, and Control Conference*, 2002.

[48] Mitchell, J. S. B., Polishchuk, V., and Krozel, J., "Airspace throughput analysis considering stochastic weather," *AIAA Guidance, Navigation, and Control Conference*, 2006.

[49] Mogford, R., Guttman, J., Morrow, S., and Kopardekar, P., "The complexity construct in air traffic control: A review and synthesis of the literature," Tech. Rep. DOT/FAA/CT-TN95/22, FAA, 1995.

[50] Mondoloni, S. and Liang, D., "Airspace fractal dimension and application," *4th USA/Europe Air Traffic Management R/D Seminar*, 2001.

[51] Pallottino, L., Feron, E., and Bicchi, A., "Conflict resolution problems for air traffic management systems solved with mixed integer programming," *IEEE Trans. Intelligent Transportation System*, vol. 3, no. 1, 2002.

[52] Pawlak, W. S., Brinton, C. R., Crouch, K., and Lancaster, K. M., "A framework for the evaluation of air traffic control complexity," *AIAA Guidance Navigation and Control Conference*, 1996.

[53] Prevot, T., Callantine, T., Lee, P., Mercer, J., Battiste, V., Palmer, E., and Smith, N., "Co-operative air traffic management: Concept and transition," *AIAA Guidance, Navigation, and Control Conference*, 2005.

[54] Reynolds, T. G. and Hansman, R. J., "Investigating conformance monitoring issues in air traffic control using fault detection techniques," *Journal of Aircraft*, vol. 42, 2005.

[55] Rodgers, M. D., Mogford, R., and Mogford, L. S., "The relationship of sector characteristics to operational errors," Tech. Rep. DOT/FAA/AM-98/14, Office of Aviation Medicine, 1995.

[56] Roy, K. and Tomlin, C. J., "Solving the aircraft routing problem using network flow algorithms," *Proceedings of the 2007 American Control Conference*, 2007.

[57] Schaefer, D., Meckiff, C., Magill, A., Pirard, B., and Aligne, F., "Air traffic complexity as a key concept for multi-sector planning," *IEEE 20th Digital Avionics System Conference*, 2001.

[58] Schmidt, D. K., "On modeling atc work load and sector capacity," *Journal of Aircraft*, no. 7, 1976.

[59] Simpson, R. W., "Engineering of air traffic control systems," tech. rep., Flight Transportation Laboratory, MIT, 1993.

[60] Song, L., Wanke, C., and Greenbaum, D. P., "Predicting sector capacity for tfm decision support," *6th AIAA Aviation Technology, Integration and Operations Conference*, 2006.

[61] Sperandio, J., "Variation of operator's strategies and regulating effects on workload," *Ergonomics*, 1971.

[62] Sridhar, B., Sheth, K. S., and Grabbe, S., "Airspace complexity and its application in air traffic management," *2nd USA/Europe Air Traffic Management R/D Seminar*, 1998.

[63] Sridhar, B., Soni, T., Sheth, K., and Chatterji, G., "An aggregate flow model for air traffic management," *AIAA Guidance, Navigation, and Control Conference*, 2004.

[64] Swenson, H., Barhydt, R., and Landis, M., "Next generation air transportation system(NGATS) air traffic management(ATM)-airspace project," tech. rep., NASA, 2006.

[65] Thipphavong, J. and Landry, S. J., "The effects of the uncertainty of departures on multi-center traffic management advisor scheduling," *AIAA Aviation Technology,Integration and Operations Conference*, 2005.

[66] Wanke, C., Song, L., Zobell, S., Greenbaum, D., and Mulgund, S., "Probabilistic congestion management," *6th USA/Europe Air Traffic Management R/D Seminar*, 2005.

[67] Welch, J. D., Andrews, J. W., Martin, B. D., and Sridhar, B., "Macroscopic workload model for estimating en route sector capacity," *7th USA/Europe Air Traffic Management R/D Seminar*, 2007.

[68] Xing, J. and Manning, C. A., "Complexity and automation displays of air traffic control: Literature review and analysis," Tech. Rep. DOT/FAA/AM-05/4, FAA, 2005.

[69] Yousefi, A. and Donohue, G. L., "Temporal and spatial distribution of airspace complexity for air traffic controller workload-based sectorization," *AIAA Aviation Technology,Integration and Operations Conference*, 2004.

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